Solución: Las coordenadas de un punto P colineal con P_1 y P_2 son $\overrightarrow{P_1P} = \lambda\overrightarrow{P_1P_2} \implies \overrightarrow{P} = \overrightarrow{P_1} + \lambda\overrightarrow{P_1P_2} = (1,-2,0) + \lambda\left[(3,1,-1) - (1,-2,0)\right] = (1,-2,0) + \lambda\,(2,3,-1) = (2\lambda+1, 3\lambda-2, -\lambda)$

Los tres vectores involucrados en el tetraedro son $\overrightarrow{P_3P_4}$, $\overrightarrow{P_3P_5}$, $\overrightarrow{P_3P}$ este último vector es variable en lambda lo que sugiere que puede haber más de una solución. Ver Figura 4

$$\begin{cases} \overrightarrow{P_3P_4} = (3,0,1) - (2,1,-1) = (1,-1,2) \\ \overrightarrow{P_3P_5} = (2,-1,3) - (2,1,-1) = (0,-2,4) \\ \overrightarrow{P_3P} = (2\lambda+1, 3\lambda-2, -\lambda) - (2,1,-1) = (2\lambda-1, 3\lambda-3, 1-\lambda) \end{cases}$$

$$Volum\ tetraedro = \frac{\left|\left(\overrightarrow{A}\times\overrightarrow{B}\right)\cdot\overrightarrow{C}\right|}{6} = \frac{\left|\det\begin{bmatrix} 2\lambda-1 & 3\lambda-3 & 1-\lambda \\ 1 & -1 & 2 \\ 0 & -2 & 4 \end{bmatrix}\right|}{6}$$

$$Volum\ tetraedro = \frac{1}{6}\left|10-10\lambda\right| = 6 \implies \left|10-10\lambda\right| = 36$$

La cual efectivamente tiene dos soluciones

$$\begin{cases} 10-10\lambda = 36 \\ 10-10\lambda = -36 \end{cases} \quad \begin{cases} \lambda_1 = -\dfrac{13}{5} \\ \lambda_2 = \dfrac{23}{5} \end{cases}$$

Resultado:

$$P_{S_1}\,(2\lambda+1, 3\lambda-2, -\lambda)\Big|_{\lambda=\frac{-13}{5}} = \left(2\left(\frac{-13}{5}\right)+1, 3\left(\frac{-13}{5}\right)-2, -\left(\frac{-13}{5}\right)\right) =$$

$$P_{S_1}\left(-\frac{21}{5}, -\frac{49}{5}, \frac{13}{5}\right)$$

$$P_{S_2}\,(2\lambda+1, 3\lambda-2, -\lambda)\Big|_{\lambda=\frac{23}{5}} = \left(2\left(\frac{23}{5}\right)+1, 3\left(\frac{23}{5}\right)-2, -\left(\frac{23}{5}\right)\right) =$$

$$P_{S_2}\left(\frac{51}{5}, \frac{59}{5}, -\frac{23}{5}\right)$$