Nonholonomic Vehicle Routing and the Dubins TSP

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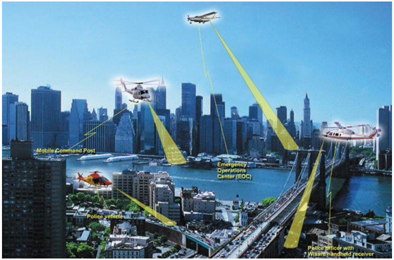
Emergent Unmanned Aerial Vehicle (UAV) technology



Advantages

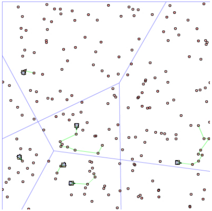
- surveillance
- data acquisition
- communication relays
- disaster and emergency management

Key scientific challenges

- scalability in performance and robustness
- sensor models and dynamics
- how to integrate control, sensing, communication

Vehicle Routing

Service dynamically arriving targets
via target assignment + path planning



vehicle routing by Frazzoli and Bullo, 2004

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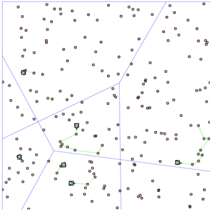

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Problem setup: Dynamic Traveling Repairperson Problem (DTRP)

- m vehicles with unit speed

  single integrator or Dubins nonholonomic

- random targets with time intensity: $\lambda > 0$ — spatial density: uniform

Objective: a *stabilizing* policy with *minimum system time*

Key requirement for stability

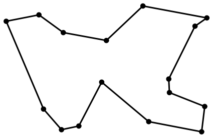
Suppse $n = \#$ outstanding targets:

$$\underbrace{\lambda}_{\text{target generation rate}} \quad - \quad \underbrace{\frac{n}{\text{TSPlength(n)}}}_{\text{target service rate}} \quad = \quad \text{target growth rate}$$

If TSPlength(n) depends on n strictly sub-linearly, then growth rate becomes negative

Euclidean TSP and Dubins TSP

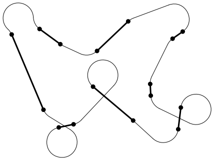
Euclidean TSP (ETSP)



- NP-hard
- effective heuristics available
- length$(\mathrm{ETSP}) \in O(\sqrt{n})$ (Supowit et. al. '83)

Dubins TSP (DTSP)

Given a set of points find the shortest tour with bounded curvature



- not a finite dimensional problem
- no prior algorithms or results
- length(DTSP) sub-linear in n ?

Stochastic DTSP

Problem Statement

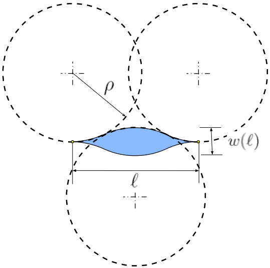
Given a set of n *independently and uniformly distributed* points, design algorithms with smallest *expected* DTSP tour length

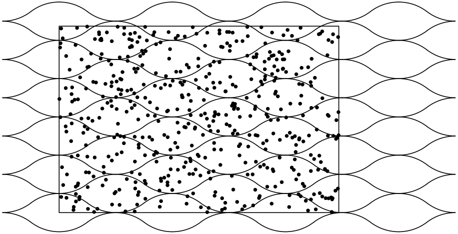
Lower bound

For n iid uniformly distributed points:

$$\mathrm{E}[\mathrm{DTSP}] \in \Omega(n^{2/3})$$

Bead Tiling of the plane





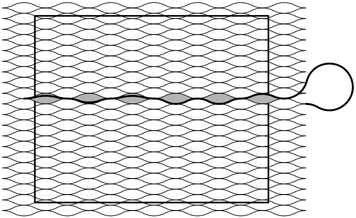
ρ: minimum turning radius, ℓ: length

Key properties of the bead

1. Beads tile the plane
2. Approaching and leaving a bead horizontally, Dubins can service a target anywhere in the bead (while remaining inside it)

Recursive Bead Tiling Algorithm (RecBTA)

Pick ℓ so that #beads $= n$



Recursive Bead Tiling Algorithm (RecBTA)

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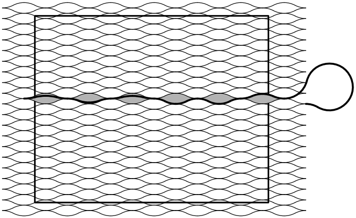
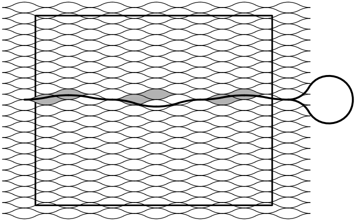
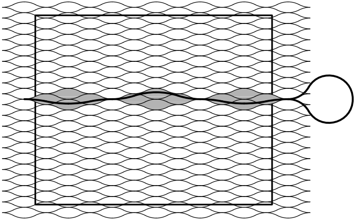


Phase 1



Phase 2



Phase 3

and so on ...

Analysis of RecBTA

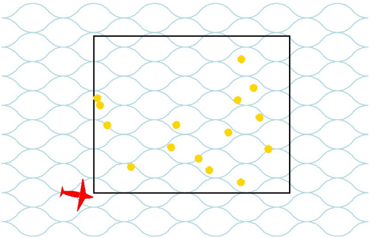
1. path length to execute all phases of RecBTA $\in O(n^{2/3})$
2. $\#$ targets remaining after all phases $\in O(\log n)$ with high probability
   (occupancy problem, stochastic analysis)
3. Hence, RecBTA is an asymptotic constant factor approximation whp

DTRP algorithms

Single vehicle case

Bead Tiling Algorithm (BTA)

1: Tile with *appropriate* resolution
2: Traverse all non-empty beads once, visiting one target per bead
3: Repeat step 2



Multiple vehicle case

Strip Tiling Algorithm (STA)

1: Divide the plane into m equal strips along the height
2: Each vehicle executes Bead Tiling Algorithm in its strip

Summary of prior and novel results

	Simple vehicle	Double integrator	Dubins vehicle
Length of TSP tour (worst-case)	$\Theta(n^{\frac{1}{2}})$	$\Omega(n^{\frac{1}{2}})$ $O(n^{\frac{3}{4}})$	$\Theta(n)$
Exp. Length of TSP tour (stochastic)	$\Theta(n^{\frac{1}{2}})$	$\Theta(n^{\frac{2}{3}})$ w.h.p.	$\Theta(n^{\frac{2}{3}})$ w.h.p.
System time for DTRP	$\Theta(\frac{\lambda}{m^2})$	$\Theta(\frac{\lambda^2}{m^3})$	$\Theta(\frac{\lambda^2}{m^3})$

The upper bounds are constructive

References

1. K. Savla, E. Frazzoli, and F. Bullo. On the point-to-point and traveling salesperson problems for Dubins' vehicle. In *American Control Conference*, pages 786–791, Portland, OR, June 2005
2. K. Savla, E. Frazzoli, and F. Bullo. Asymptotic constant-factor approximation algorithms for the traveling salesperson problem for Dubins' vehicle, March 2006. Available electronically at `http://arxiv.org/abs/cs/0603010`
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4. K. Savla. *Multi UAV Systems with Motion and Communication Constraints*. PhD thesis, Electrical and Computer Engineering Department, University of California at Santa Barbara, Santa Barbara, August 2007. Available electronically at `http://ccdc.mee.ucsb.edu`

Emerging discipline: motion-enabled networks

- **network modeling**

  network, ctrl+comm algorithm, task, complexity
- **coordination algorithm**

  deployment, task allocation, boundary estimation

Papers available at `http://motion.mee.ucsb.edu`