Synchronization in Power Networks

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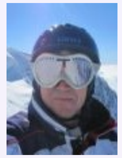
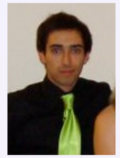


Misha Chertkov



John Simpson-Porco

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2. F. Dörfler and F. Bullo. On the critical coupling for Kuramoto oscillators. *SIAM J Appl Dyn Sys*, 10(3):1070–1099, 2011
3. F. Dörfler and F. Bullo. Synchronization and transient stability in power networks and non-uniform Kuramoto oscillators. *SIAM JCO*, 2012. To appear
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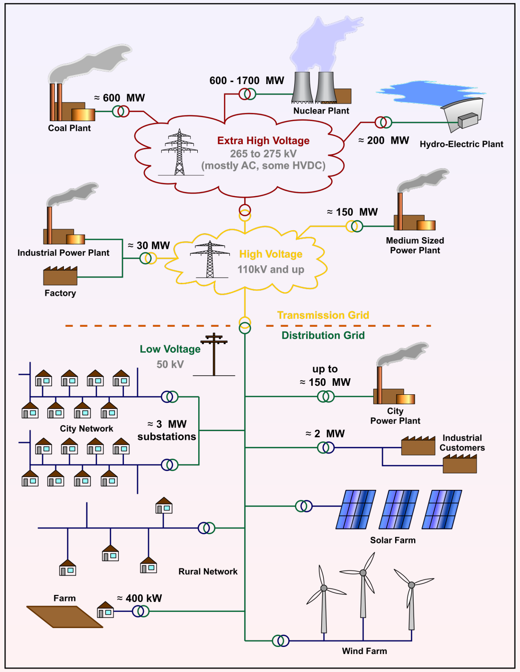
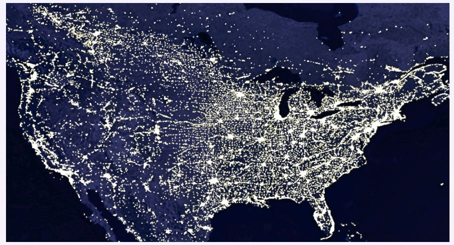
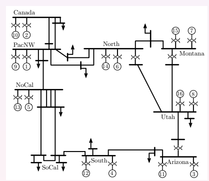
Outline

1. Models
2. Problem statement
3. Synchronization Tests
4. Case Study

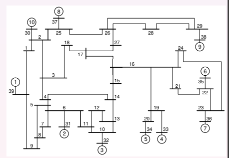
Power Generation and Transmission Network





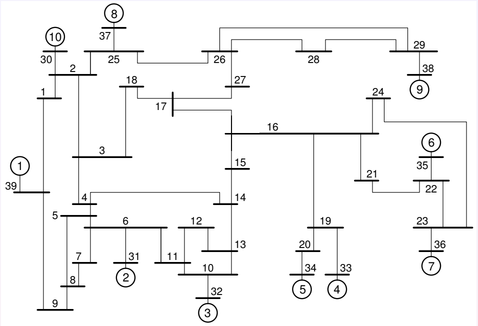


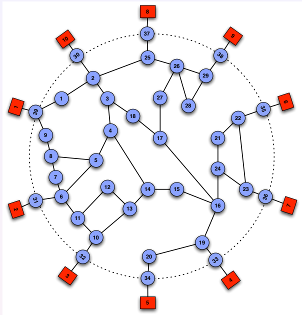
Western US
(WECC 16-m, 25-b)



New England
(10-m, 13-b)

Mathematical Model of a Power Transmission Network





1. n **generators** ■ and m **load buses** ●
2. **admittance matrix** $Y \in \mathbb{C}^{(n+m)\times(n+m)}$, symmetric, sparse, lossless

Central task: generators provide power for loads

Problems: stability in face of disturbances, security from cyber attacks

Mathematical Model of a Power Transmission Network

1. power transfer on line $i \rightsquigarrow j$: $\underbrace{|V_i||V_j||Y_{ij}|}_{a_{ij}=\text{max power transfer}} \cdot \sin(\theta_i - \theta_j)$
2. power balance at node i: $\underbrace{P_i}_{\text{power injection}} = \sum_j a_{ij} \sin(\theta_i - \theta_j)$

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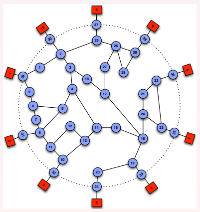
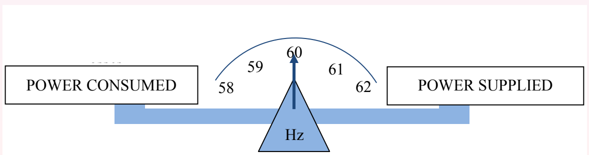
Structure-Preserving Model [Bergen & Hill '81]

for ■, swing eq with $P_i > 0$ $\quad M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j)$

for ●, const $P_i < 0$ and $D_i \geq 0$ $\quad D_i\dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j)$

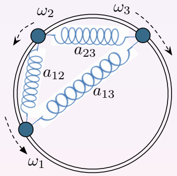




Outline

Synchronization in Power Networks

1. power networks are coupled oscillators
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum\nolimits_j a_{ij}\sin(\theta_i - \theta_j)$$
$$D_i\dot{\theta}_i = P_i - \sum\nolimits_j a_{ij}\sin(\theta_i - \theta_j)$$
2. synchronization: coupling strength vs. frequency non-uniformity



3. graph theory provides notions of
"coupling/connectivity" and "non-uniformity"

> power networks **should** synchronize
> for large "coupling/connectivity" and small "non-uniformity"

The Synchronization Problem

Determine conditions on the power injections $(P_1, \dots, P_{n+m})$, network admittance Y, and node parameters (M_i, D_i), such that:

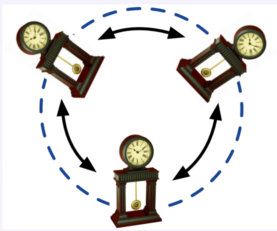
$$|\theta_i - \theta_j| \text{ bounded} \quad \text{and} \quad \dot{\theta}_i = \dot{\theta}_j$$

Literature

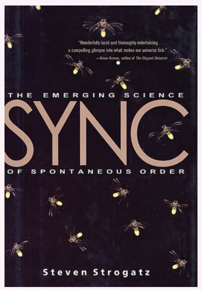
1. **Classic security analysis:** load flow Jacobian & network theory
[S. Sastry et al. '80, A. Araposthatis et al. '81, F. Wu et al '82, M. Ilić '92, ...]
2. **Broad interest for Complex Networks, Network Science** [Ilić '92, Hill & Chen '06] stability, performance, and robustness of power network $\overset{?}{\leftrightsquigarrow}$ underlying graph properties (topological, algebraic, spectral, etc.)

Coupled Oscillators in Technology and Science

Kuramoto model of coupled oscillators:
$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij}\sin(\theta_i - \theta_j)$$



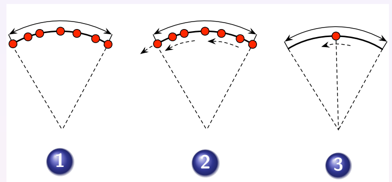
- Sync in Josephson junctions [S. Watanabe et. al '97, K. Wiesenfeld et al. '98]
- Sync in a population of fireflies [G.B. Ermentrout '90, Y. Zhou et al. '06]
- Coordination of particle models
[R. Sepulchre et al. '07, D. Klein et al. '09]
- Deep-brain stimulation and neuroscience
[P.A. Tass '03, E. Brown et al. '04]
- Countless other sync phenomena
[A. Winfree '67, S.H. Strogatz '00, J. Acebrón '01]



Synchronization Notions

$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij}\sin(\theta_i - \theta_j)$$

1. phase cohesive: $|\theta_i(t) - \theta_j(t)| < \gamma$
for small $\gamma < \pi/2$ ... arc invariance
2. frequency synchrony: $\dot{\theta}_i(t) = \dot{\theta}_j(t)$
3. phase synchrony: $\theta_i(t) = \theta_j(t)$



- $\{a_{ij}\}_{\{i,j\}\in\mathcal{E}}$ small & $|\omega_i - \omega_j|$ large $\Longrightarrow$ no synchronization
- $\{a_{ij}\}_{\{i,j\}\in\mathcal{E}}$ large & $|\omega_i - \omega_j|$ small $\Longrightarrow$ cohesive + freq sync

Challenges: proper notions of sync, coupling & phase transition
[A. Jadbabaie et al. '04, P. Monzon et al. '06, Sepulchre et al. '07, S.J. Chung et al. '10, J.L. van Hemmen et al. '93, F. de Smet et al. '07, N. Chopra et al. '09, G. Schmidt et al. '09, F. Dörfler et al. '09 & '11, S.J. Chung et al. '10, A. Franci et al. '10, S.Y. Ha et al. '10, D. Aeyels et al. '04, R.E. Mirollo et al. '05, M. Verwoerd et al. '08, L. DeVille '11, ...]

Outline

Primer on Algebraic Graph Theory

Graph: weights $a_{ij} > 0$ on edges $\{i, j\}$, values x_i at nodes i

- adjacency matrix $A = (a_{ij})$
- degree matrix D is diagonal with $d_{ii} = \sum_{j=1}^{n} a_{ij}$
- Laplacian matrix $L = L^T = D - A \geq 0$

Notions of Connectivity

topological: connectivity, average and worst-case path lengths
spectral: second smallest eigenvalue λ_2 of L is "algebraic connectivity"

Notions of Dissimilarity

$\|x\|_{\infty,\text{edges}} = \max_{\{i,j\}} |x_i - x_j|$, $\qquad \|x\|_{2,\text{edges}} = \left(\sum_{\{i,j\}} |x_i - x_j|^2\right)^{1/2}$

(graph edges $\{i, j\} \in \mathcal{E}$) or (all edges $\{i, j\}$ satisfy $i < j$)

Sync Tests: Coupling vs. Power Imbalance

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum\nolimits_j a_{ij} \sin(\theta_i - \theta_j)$$

$$D_i \dot{\theta}_i = P_i - \sum\nolimits_j a_{ij} \sin(\theta_i - \theta_j)$$

Tight condition for homogeneous graphs ($a_{ij} = K > 0$), trees, graphs with disjoint 3- and 4-cycles, graphs with $L^\dagger P$ bipolar or symmetric

$$\left\| L^\dagger P \right\|_{\infty,\text{graph edges}} < 1 \quad \Longleftrightarrow \quad \text{sync}$$

Necessary and sufficient conditions for connected weighted graphs

$\sum_j a_{ij} \leq |P_i| \ \Rightarrow$ no sync $\qquad \lambda_2 > \|P\|_{2,\text{all edges}} \ \Rightarrow$ sync

sharpest general conditions known to date

A Nearly Exact Synchronization Condition – Accuracy

Randomized power network test cases

with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Correctness of condition: $\|L^\dagger \omega\|_{\infty,\text{g. edges}} \leq \sin(\gamma)$ $\Rightarrow \max_{\{i,j\}\in\mathcal{E}} |\theta_i^* - \theta_j^*| \leq \gamma$	Accuracy of condition: $\max_{\{i,j\}} |\theta_i^* - \theta_j^*| - \arcsin(\|B^T L^\dagger \omega\|_\infty)$	Phase cohesiveness: $\max_{\{i,j\}\in\mathcal{E}} |\theta_i^* - \theta_j^*|$
9 bus system	always true	$4.1218 \cdot 10^{-5}$ rad	0.12889 rad
IEEE 14 bus system	always true	$2.7995 \cdot 10^{-4}$ rad	0.16622 rad
IEEE RTS 24	always true	$1.7089 \cdot 10^{-3}$ rad	0.22309 rad
IEEE 30 bus system	always true	$2.6140 \cdot 10^{-4}$ rad	0.1643 rad
New England 39	always true	$6.6355 \cdot 10^{-5}$ rad	0.16821 rad
IEEE 57 bus system	always true	$2.0630 \cdot 10^{-2}$ rad	0.20295 rad
IEEE RTS 96	always true	$2.6076 \cdot 10^{-3}$ rad	0.24593 rad
IEEE 118 bus system	always true	$5.9959 \cdot 10^{-4}$ rad	0.23524 rad
IEEE 300 bus system	always true	$5.2618 \cdot 10^{-4}$ rad	0.43204 rad
Polish 2383 bus system (winter peak 1999/2000)	always true	$4.2183 \cdot 10^{-3}$ rad	0.25144 rad

condition $\left\| L^\dagger \omega \right\|_{\infty,\text{graph edges}} \leq \sin(\gamma)$ is extremely accurate for $\gamma \leq 25°$

AC power flow, DC power flow and our new condition

AC power flow

$$P_i = \sum_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j), \qquad |\theta_i - \theta_j| < \gamma_{ij}$$

DC power flow approximation

$$P_i = \sum_{j=1}^{n} a_{ij} (\delta_i - \delta_j), \qquad |\delta_i - \delta_j| < \gamma_{ij}$$

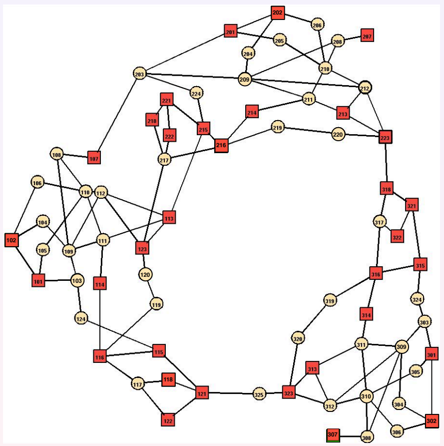
Novel test

$$P_i = \sum_{j=1}^{n} a_{ij} (\delta_i - \delta_j), \qquad |\delta_i - \delta_j| < \sin(\gamma_{ij})$$

Outline

Case Study: Predicting Transition to Instability

IEEE Reliability Test System '96 (33-m 44-b)



Optimal power dispatch

$$\text{minimize} \quad \sum (\text{cost})_{i,\text{gen}} P_{i,\text{gen}}$$

$$P_i = \sum\nolimits_j a_{ij} \sin(\theta_i - \theta_j)$$

$$|\theta_i - \theta_j| \leq (\text{thermal limit})_{ij}$$

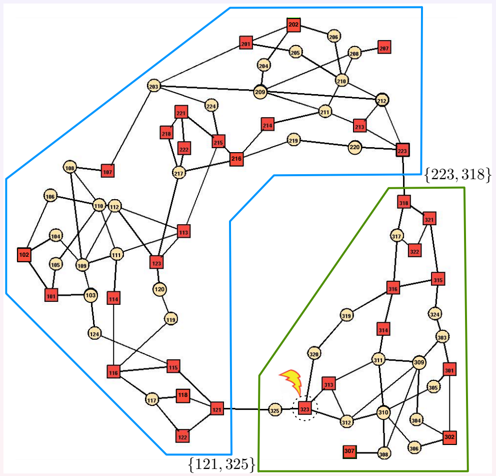
$$P_{\text{gen},i} \in (\text{feasible range})_{i,\text{gen}}$$

Power flow: periodically, solve optimal power dispatch problem, & real-time perturbations handled via generation adjustments

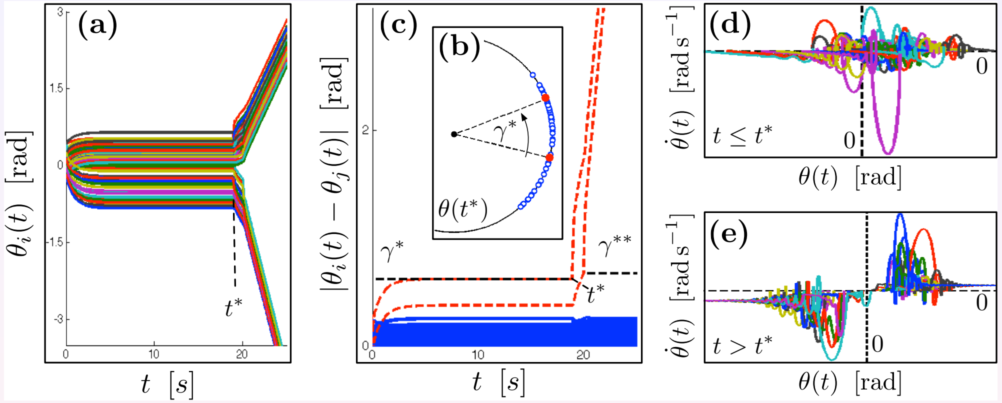
Case Study: Predicting Transition to Instability

IEEE Reliability Test System '96 (33-m 44-b)

Two contingencies:



1) generator 323 is tripped
2) increase loads & generation



Continuously increase loads:

$\Rightarrow$ condition $\left\|B^T L^\dagger \omega\right\|_\infty \leq \sin(\gamma)$ predicts that thermal limit γ^* of line $\{121, 325\}$ is violated at 22.23 % of additional loading

$\Rightarrow$ line $\{121, 325\}$ is tripped at 22.24% of additional loading

Conclusions

Summary:

1. connection between power networks and coupled Kuramoto oscillators
2. necessary and sufficient sync conditions
3. sharp sync conditions

Ongoing and future work:

1. general proof for sharp conditions
2. more detailed & stochastic models
3. region of attraction

2012 IFAC NecSys Workshop, Santa Barbara
CDC 2012, Maui Hawaii: Tutorial Session on Coupled Oscillators