Visibility-based multiagent deployment in orthogonal environments

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Outline

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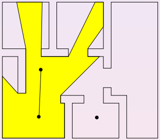
1. Robotic agents with visibility sensors
2. Deployment of multiple agents in orthogonal environments
3. Conclusions

Robotic agents with visibility sensors

- **Orthogonal polygon**
  Q: adjacent edges perpendicular to each other
- **Visibility**

  Visibility polygon
  $\mathcal{V}(p, Q) = \{q \in Q \mid q \text{ is visible from } p\}$



- **Robotic agent**
  First order dynamics: $p(t+1) = p(t) + u$
  Point robot with omnidirectional visibility sensing
  Line of sight communication: visibility graph

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Art Gallery Problem (Klee '73):

Imagine placing guards inside a nonconvex polygon with n vertices: how many guards are required and where should they be placed in order for each point in the polygon to be visible by at least one guard?

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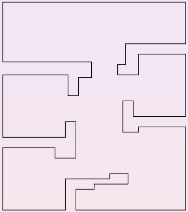
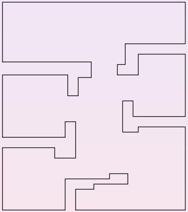


- Kahn et al '93
- $\lfloor \frac{n}{4} \rfloor$ sufficient and occasionally necessary



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- $\frac{n}{2} - 2$ sufficient and occasionally necessary

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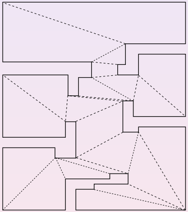

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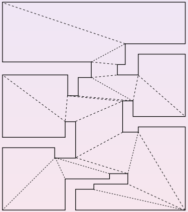

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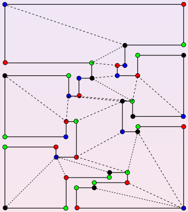

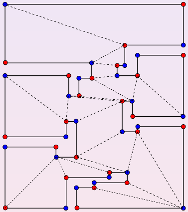

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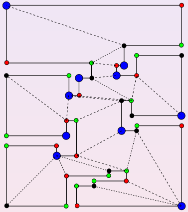

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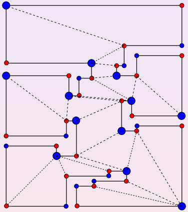

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Robotic network model

- Communicate within line-of-sight and within bounded distance
- Each agent has a unique identifier i
- p_i denotes position; $p_i(t + \Delta t) = p_i(t) + u_i$, $\|u_i\| \leq 1$
- $\mathcal{M}_i$ denotes memory ("limited") contents

Deployment problems

Nonconvex deployment problem

Design a provably correct distributed algorithm:

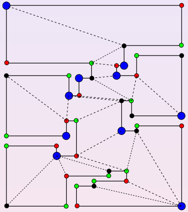
1. achieve complete visibility;
2. minimize the number of agents used

Nonconvex deployment problem with connectivity

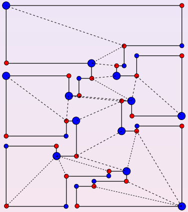
Design a provably correct distributed algorithm:

1. achieve complete visibility;
2. ensure that the visibility graph of final configuration is connected; and
3. minimize the number of agents used

Statement of results

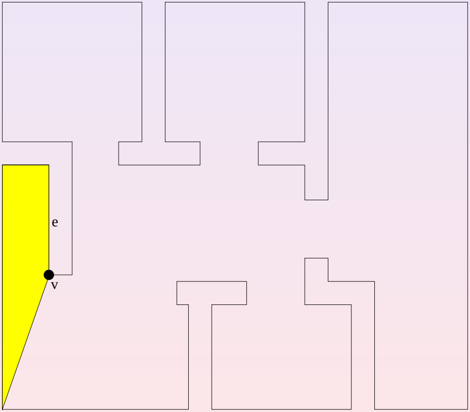


Starting from a single location, $\lfloor \frac{n}{4} \rfloor$ agents are always sufficient and occasionally necessary



Starting from a single location, $\frac{n}{2} - 2$ are always sufficient and occasionally necessary

Incremental Partition Algorithm



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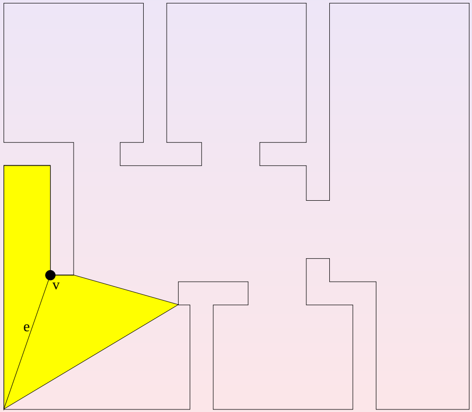

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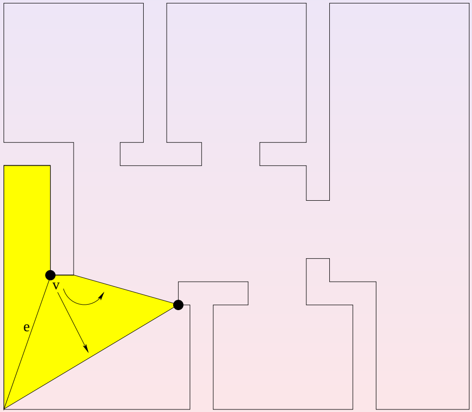

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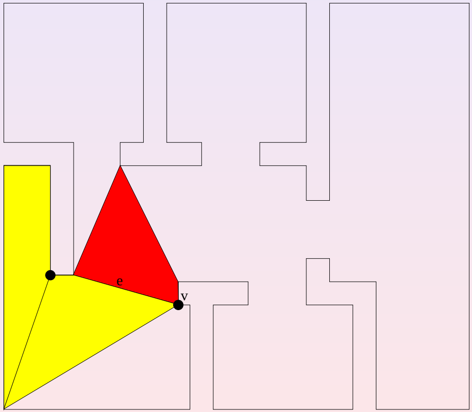

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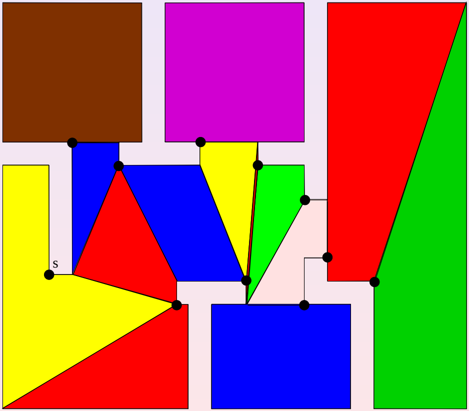
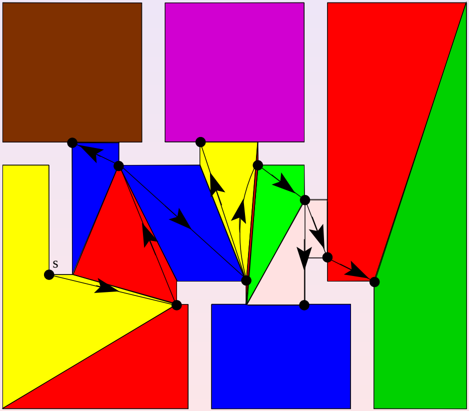
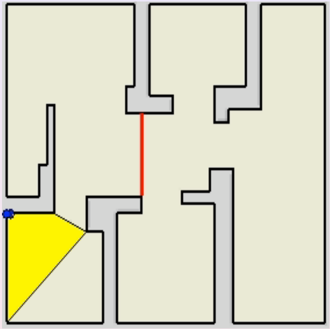


Vertex-induced tree



Incremental algorithm for connected deployment

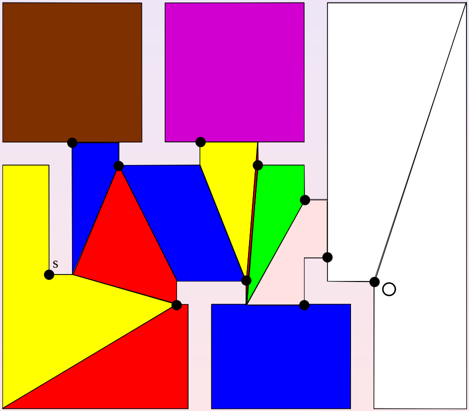


Robustness properties

Robust to agent failures

Changing environments

Sparse point set for deployment without connectivity



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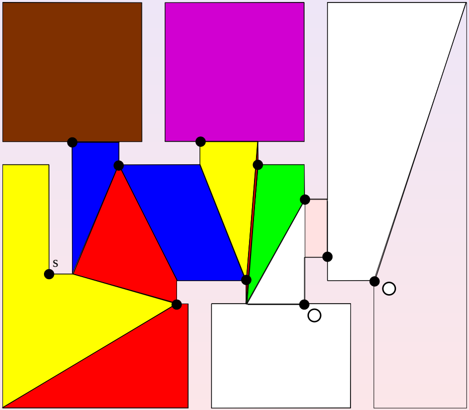

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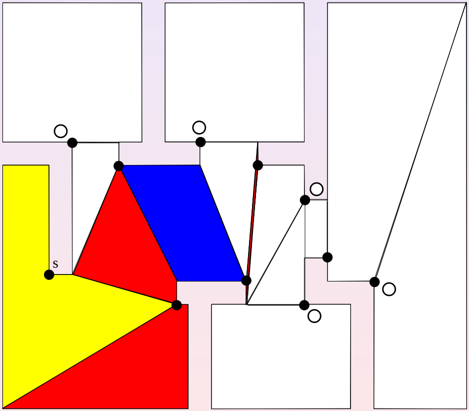

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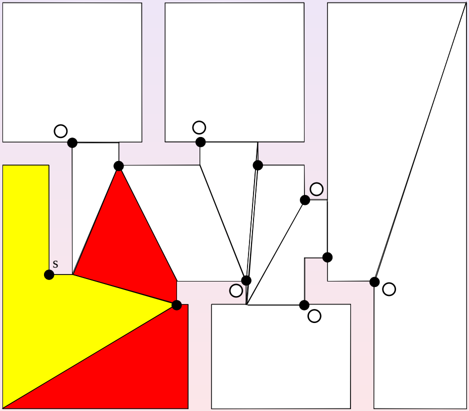

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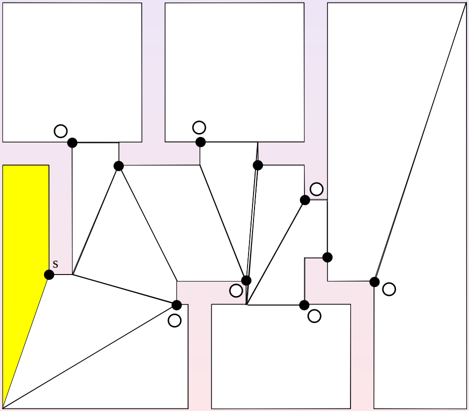

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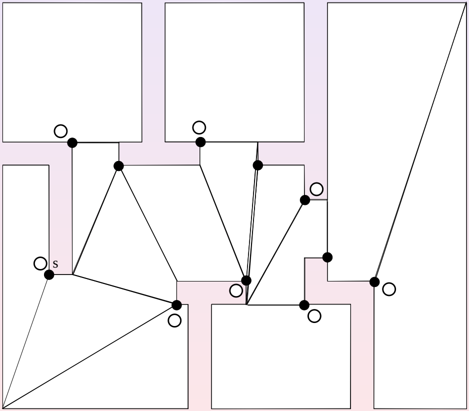

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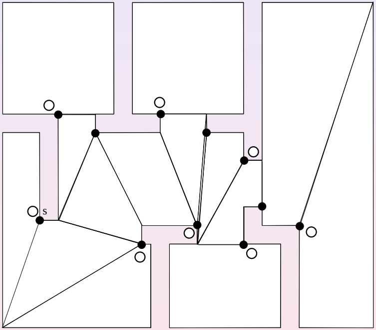
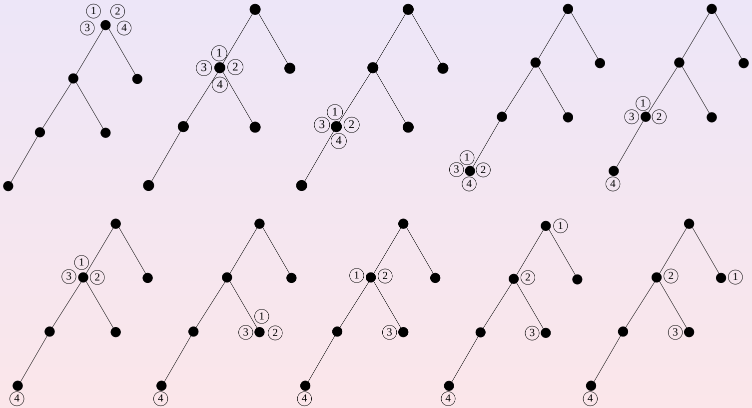


Every point in the kernel "owns" at least two quadrilaterals or four triangles
Total number of triangles is $n-2$

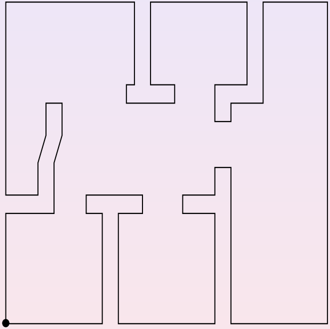
Therefore, number of points in the kernel is $n/4$.

Depth-first deployment



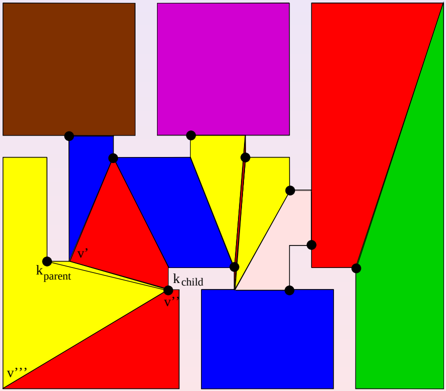
Assume: (i) Each node is a star-shaped set; (ii) Sets corresponding to non-leaf nodes are composed of a union of quadrilaterals equal in number to the number of children

Depth-first deployment



Depth-first deployment in general simply connected environments

Local navigation and distributed information processing



- Straight line paths between adjacent nodes
- Required memory: $\mathcal{M}_i : \{p_{\text{parent}}, p_{\text{last}}, g_1, g_2\}$
- After moving from k_{parent} to k_{child}, k_{parent} is added to the beginning of list p_{parent}, (v', v'') is added to list g_1, (v'', v'') is added to list g_2 and $p_{\text{last}} := k_{\text{parent}}$
- After moving from k_{child} to k_{parent}, the first elements of p_{parent}, g_1 and g_2 are deleted and $p_{\text{last}} := k_{\text{child}}$

Main results

Connected deployment

1. If # agents < cardinality of the sparse kernel point set,
   then in finite time each agent comes to rest at a unique kernel point
   else in finite time every kernel point contains an agent at rest
2. $\lfloor \frac{n}{4} \rfloor$ agents are always sufficient and occasionally necessary for the task

Deployment without connectivity

1. If # agents < cardinality vertex-induced tree,
   then in finite time each agent comes to rest at a unique node
   else in finite time every node contains an agent at rest
2. $\frac{n-2}{2}$ agents are always sufficient and occasionally necessary for the task

Outline

Conclusions

Summary

- distributed algorithms to achieve coverage in nonconvex orthogonal environments
- number of agents required is optimal in the worst case
- robustness to agent failures and changing environments

Future directions

- environments with holes
- 3D scenarios
- other notions of optimality: time taken, other complexity measures other than the number of vertices