Monotonic Target Assignment For Robotic Networks

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The Target Assignment Problem

- n mobile robots, m target locations, in $\mathbb{R}^2$
- Each robot with unique identifier
  - moves $\dot{p}^{[i]} = u^{[i]}$, $|u^{[i]}| \leq v_{\max}$
  - knows, or can sense, the target locations
- Discrete-time communication model
  - communication range r_{comm}
  - max message length $O(\log n)$

Problem: distributed algorithm to

- allow group of agents to divide m targets among themselves;
- lead each agent to its unique target in minimum time.

Related Combinatorics and Robotics Literature

Centralized assignment problems:

- Max. matching in bipartite graphs (Hopcroft and Karp, '73)
- Sum assignment problem (Kuhn, '55)
- Bottleneck assignment problem (Derigs and Zimmermann '78)

Parallel/Decentralized assignment problems

- The auction algorithm (Bertsekas, '88)
- Others include Zavlanos and Pappas, Castañón and Wu, Moore and Passino, Arslan, Marden and Shamma.

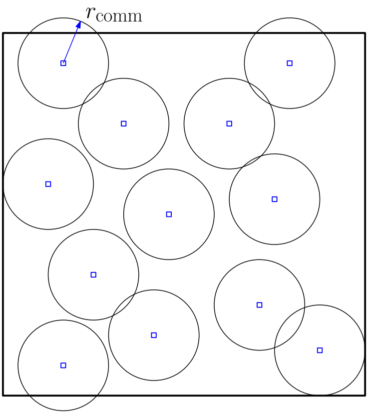
Our Goals:

- Develop efficient algorithms for target assignment problem.
- Evaluate scalability/asymptotic performance.

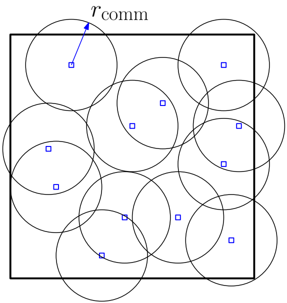
Key Challenge: Optimize completion time while satisfying

1. range constraint: compute distributed assignment, possibly without connectivity.
2. bandwidth constraint: share assignment data sparingly.

Size of Environment $\mathcal{E}(n)$ as $n \to +\infty$



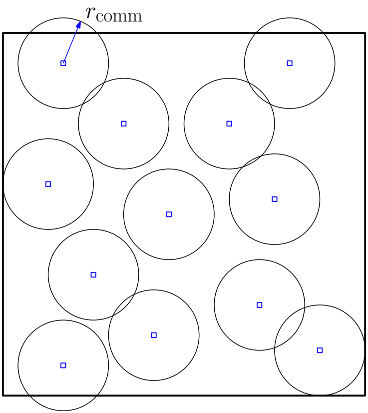
Sparse: $|\mathcal{E}(n)|/n \to +\infty$



Dense: $|\mathcal{E}(n)|/n \to 0$

Critical: $|\mathcal{E}(n)|/n \to C \in \mathbb{R}_{>0}$

Size of Environment $\mathcal{E}(n)$ as $n \to +\infty$



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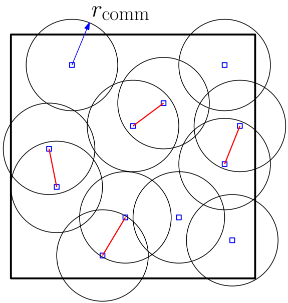


Dense: $|\mathcal{E}(n)|/n \to 0$

Critical: $|\mathcal{E}(n)|/n \to C \in \mathbb{R}_{>0}$

Monotonic Algorithms for Target Assignment

Definition (Monotonic algorithms)

- deterministic algorithm
- target j occupied at time $t_1 \Rightarrow$ target j occupied for all $t > t_1$.

Theorem (Worst-case lower bound on Monotonic Algs)

n agents and n targets in square $\mathcal{E}(n)$:

$\mathcal{E}(n)$ size	*Worst-case completion time*
Sparse $\left(\frac{\lvert\mathcal{E}(n)\rvert}{n} \to +\infty\right)$	$\Omega(\sqrt{n\lvert\mathcal{E}(n)\rvert})$
Critical $\left(\frac{\lvert\mathcal{E}(n)\rvert}{n} \to C\right)$	$\Omega(n)$
Dense $\left(\frac{\lvert\mathcal{E}(n)\rvert}{n} \to 0\right)$	$\Omega(\lvert\mathcal{E}(n)\rvert)$

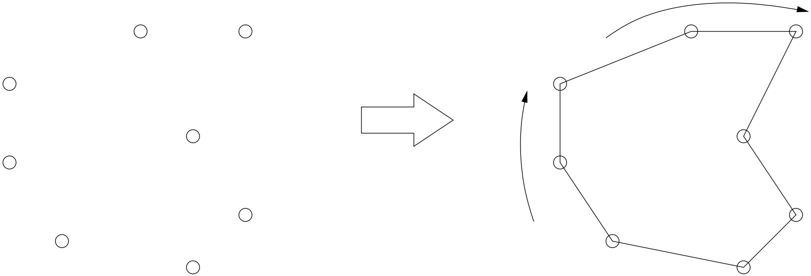
ETSP ASSGMT for Sparse Environments

- Target locations known a priori
- Maintain "available/taken" bit for each target.

The Basic Ideas:

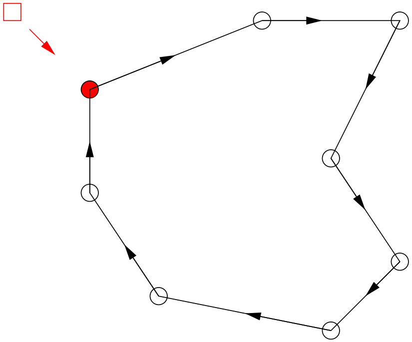
1. all agents turn the cloud of targets into ordered ring
2. move toward the closest target on the ring
3. if agent loses conflict, move to next available target on ring
4. agents exchange segments of tour that are "taken."

Idea 1: create ordered ring



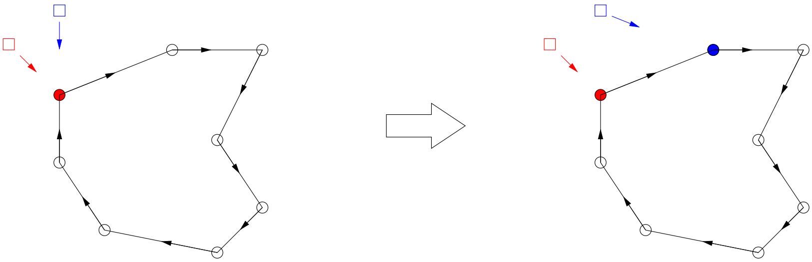
- constant factor ETSP
- same tour for all agents, same order

Idea 2: move toward the closest target on ring



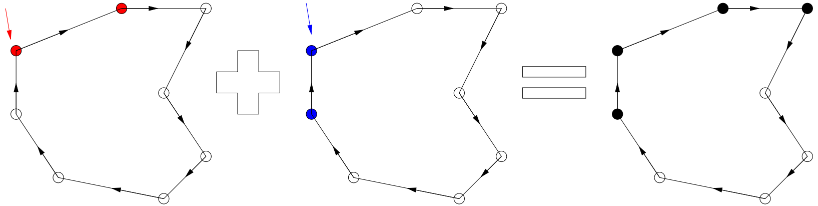
Agent keeps “current” pointer and moves accordingly

Idea 3: lose conflict, move to next available target on ring



- closest agent wins conflict,
- loser selects next target on ring which may be available.

Idea 4: transmit a segment of the tour



- message transmission $O(\log n)$ bits.
- merge "taken" segments.

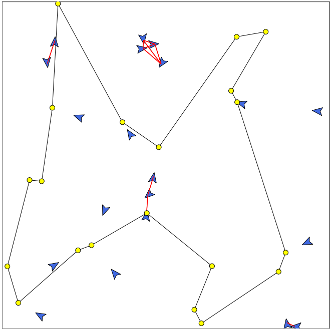
Time Complexity for ETSP Assgmt

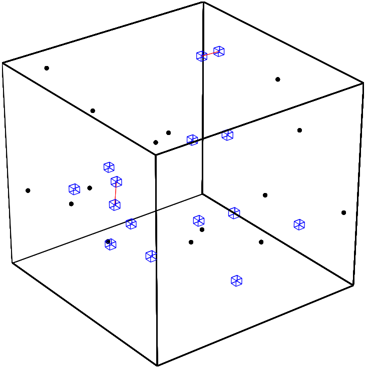
Theorem (Worst-case upper bound)

- *Assume n agents, n targets in $\mathcal{E}(n)$,*
- *then worst-case completion time in $O(\sqrt{|\mathcal{E}(n)|n})$.*

Sparse/critical $\mathcal{E}(n) \Rightarrow$ ETSP Assgmt is an asymptotically optimal monotonic algorithm

Simulations for ETSP Assgmt





GRID ASSGMT Algorithm for Dense Environments

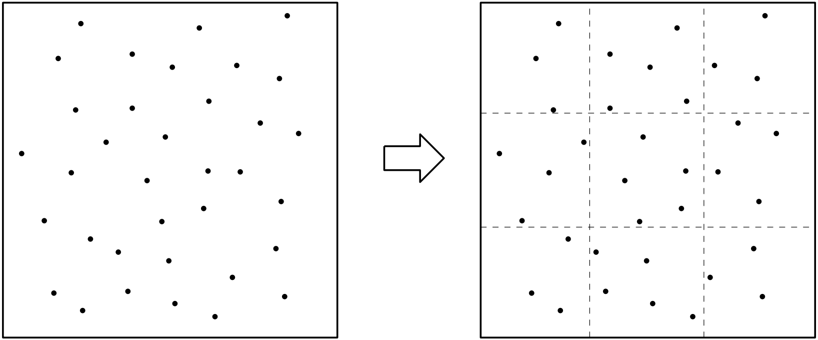
The Basic Ideas:

1. All agents partition environment into small cells.
2. In each cell, agents find maximum matching and elect leader.
3. Leaders communicate to determine location of free targets.
4. Unassigned agents are directed to free targets by leaders.

Assumes either

- Each agent knows target locations *a priori*, or
- no *a priori* knowledge but $r_{\text{sense}} \geq \sqrt{2/5}\, r_{\text{comm}}$ to sense.

Idea 1: partition the environment

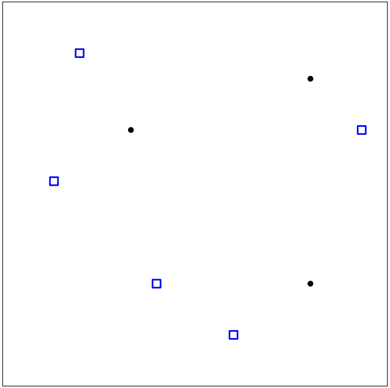


Choose grid size, based on $\mathcal{E}(n)$ and r_{comm} so that:

- Communication graph in a cell is complete.
- Communication between adjacent cells is possible.

Idea 2: leader election and maximum matching in each cell

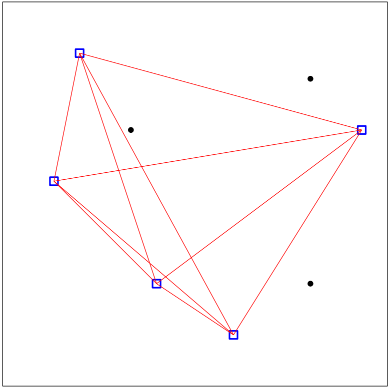
- Match agents to targets
- Elect leader



Example cell

Idea 2: leader election and maximum matching in each cell

- Match agents to targets
- Elect leader



Example cell

Idea 2: leader election and maximum matching in each cell

- Match agents to targets
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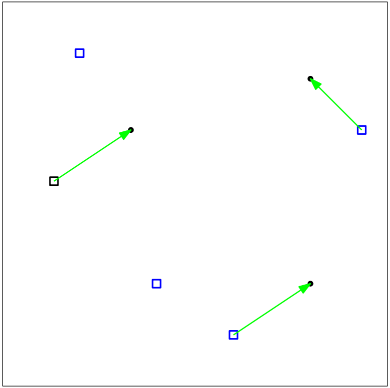
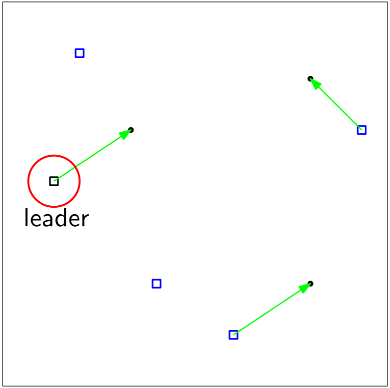


Example cell

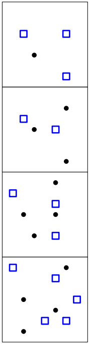
Idea 2: leader election and maximum matching in each cell

- Match agents to targets
- Elect leader



Example cell

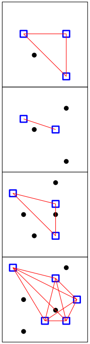
Idea 3: leaders estimate free target locations



Example column

Leaders estimate number of available targets.

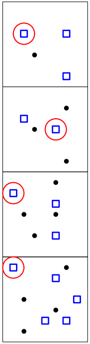
Idea 3: leaders estimate free target locations



Example column

Electing leader in each cell

Idea 3: leaders estimate free target locations



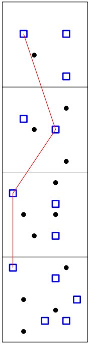
Example column

$\Delta^{[1]}$	-2
$\Delta^{[1]}_{\text{blw}}$	$+11$
$\Delta^{[2]}$	$+1$
$\Delta^{[2]}_{\text{blw}}$	$+8$
$\Delta^{[3]}$	$+1$
$\Delta^{[3]}_{\text{blw}}$	$+4$
$\Delta^{[4]}$	-1
$\Delta^{[4]}_{\text{blw}}$	0

Initialization

$\Delta^{[i]} = (\bullet - \square)$ in cell i $\qquad$ ${\Delta_{\text{blw}}}^{[i]} =$ est. of $(\bullet - \square)$ in cells below i

Idea 3: leaders estimate free target locations



Example column

$\Delta^{[1]}$	-2
$\Delta^{[1]}_{\text{blw}}$	$+11$
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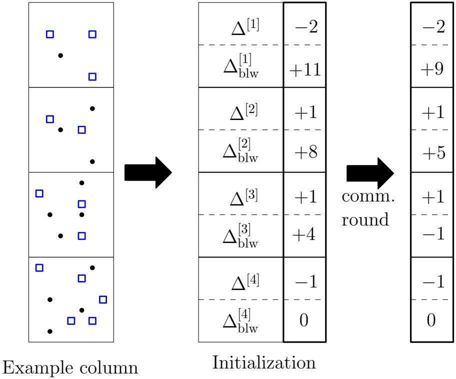
$\Delta^{[3]}$	$+1$
$\Delta^{[3]}_{\text{blw}}$	$+4$
$\Delta^{[4]}$	-1
$\Delta^{[4]}_{\text{blw}}$	0

Initialization

comm.
round

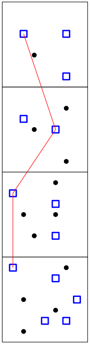
Update estimates through communication

Idea 3: leaders estimate free target locations



Update estimates through communication

Idea 3: leaders estimate free target locations



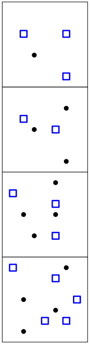
Example column

$\Delta^{[1]}$	-2	-2
$\Delta^{[1]}_{\mathrm{blw}}$	$+11$	$+9$
$\Delta^{[2]}$	$+1$	$+1$
$\Delta^{[2]}_{\mathrm{blw}}$	$+8$	$+5$
$\Delta^{[3]}$	$+1$	$+1$
$\Delta^{[3]}_{\mathrm{blw}}$	$+4$	-1
$\Delta^{[4]}$	-1	-1
$\Delta^{[4]}_{\mathrm{blw}}$	0	0

Initialization $\rightarrow$ comm. round $\rightarrow$ comm. round

Update estimates through communication

Idea 3: leaders estimate free target locations



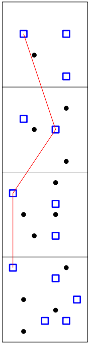
Example column

$\Delta^{[1]}$	-2	-2	-2
$\Delta^{[1]}_{\text{blw}}$	$+11$	$+9$	$+6$
$\Delta^{[2]}$	$+1$	$+1$	$+1$
$\Delta^{[2]}_{\text{blw}}$	$+8$	$+5$	0
$\Delta^{[3]}$	$+1$	$+1$	$+1$
$\Delta^{[3]}_{\text{blw}}$	$+4$	-1	-1
$\Delta^{[4]}$	-1	-1	-1
$\Delta^{[4]}_{\text{blw}}$	0	0	0

Initialization → comm. round → comm. round

Update estimates through communication

Idea 3: leaders estimate free target locations



Example column

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$\Delta^{[2]}_{\text{blw}}$	$+8$	$+5$	0
$\Delta^{[3]}$	$+1$	$+1$	$+1$
$\Delta^{[3]}_{\text{blw}}$	$+4$	-1	-1
$\Delta^{[4]}$	-1	-1	-1
$\Delta^{[4]}_{\text{blw}}$	0	0	0

Initialization

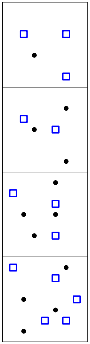
comm. round

comm. round

comm. round

Update estimates through communication

Idea 3: leaders estimate free target locations

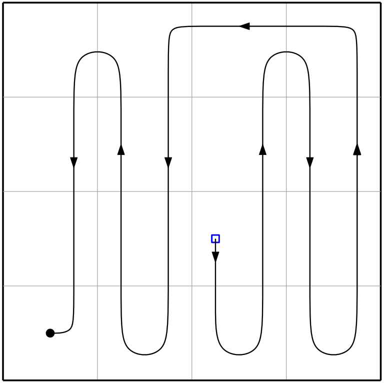


Example column

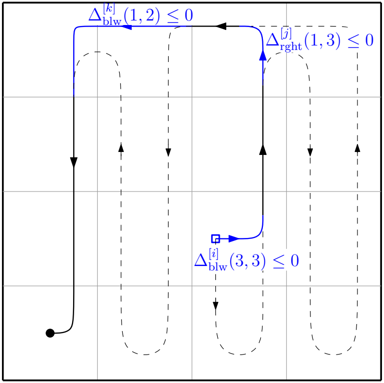
	Initialization	comm. round	comm. round	comm. round / Fixed point
$\Delta^{[1]}$	-2	-2	-2	-2
$\Delta^{[1]}_{\text{blw}}$	$+11$	$+9$	$+6$	$+1$
$\Delta^{[2]}$	$+1$	$+1$	$+1$	$+1$
$\Delta^{[2]}_{\text{blw}}$	$+8$	$+5$	0	0
$\Delta^{[3]}$	$+1$	$+1$	$+1$	$+1$
$\Delta^{[3]}_{\text{blw}}$	$+4$	-1	-1	-1
$\Delta^{[4]}$	-1	-1	-1	-1
$\Delta^{[4]}_{\text{blw}}$	0	0	0	0

Only let unassigned agents "down" if estimates are positive

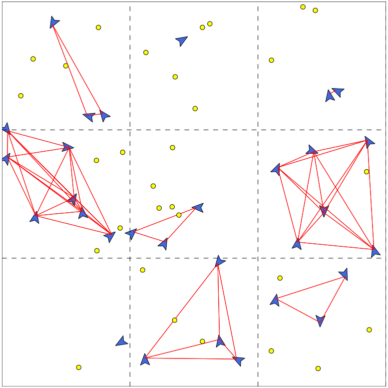
Idea 4: unassigned agent motion

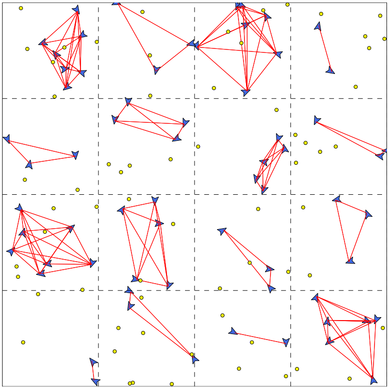


Nominal search order



Search order with leader comm.





Worst-case upper bound for Grid Assgmt

Theorem (Worst-case upper bound)

- *Assume n agents, n targets in $\mathcal{E}(n)$.*
- *then worst-case completion time in $O(|\mathcal{E}(n)|)$.*

Worst-case performance comparison:

	Sparse	**Critical**	**Dense**
Monotonic	$\Omega(\sqrt{\lvert\mathcal{E}(n)\rvert n})$	$\Omega(n)$	$\Omega(\lvert\mathcal{E}(n)\rvert)$
ETSP Assgmt	$O(\sqrt{\lvert\mathcal{E}(n)\rvert n})$	$O(n)$	$O(\sqrt{\lvert\mathcal{E}(n)\rvert n})$
Grid Assgmt	$O(\lvert\mathcal{E}(n)\rvert)$	$O(n)$	$O(\lvert\mathcal{E}(n)\rvert)$

Stochastic Bounds on Grid Assgmt

- Recall, dense $\mathcal{E}(n) \Rightarrow \frac{|\mathcal{E}(n)|}{n} \longrightarrow 0$ as $n \longrightarrow +\infty$.
- Connectivity regime: $\frac{|\mathcal{E}(n)|}{n} \in O\left(\frac{1}{\log n}\right)$.

Theorem (Stochastic performance)

- *n agents, m targets uniformly randomly distributed in $\mathcal{E}(n)$.*
- *Assume $\mathcal{E}(n)$ is in connectivity regime.*
- *If $m = n$, then w.h.p. completion time in $O(\sqrt{|\mathcal{E}(n)|})$.*
- *If $m = n/\log n$ then w.h.p., completion time in $O(1)$.*

Conjectured properties

Stochastic properties of ETSP ASSGMT

- In sparse $\mathcal{E}(n)$:

  If $m = n$, then stochastic performance is same as worst case

- In critical or sparse $\mathcal{E}(n)$:

  If $m = n/\log n$, then completion time is $O(\log n)$.

Stochastic properties of GRID ASSGMT in connectivity regime

- If $m = cn$ for some $c \in (0, c_{\mathrm{crit}})$, then compltn time is $O(1)$.

  i.e., constant factor additional agents $\implies$ for $O(1)$

Conclusions and Related Problems

In this talk, introduced:

- a broad class of algorithms for static target assignment;
- asymp. opt. algorithms for dense and sparse environments;
- a sensor based target assignment problem.

Variations and other problems:

- Nonholonomic vehicles (w/ EF and KS)
- Consistent knowledge assumption
- Related problems
  1. Targets arriving sequentially/dynamically over time (w/ EF)
  2. Search and assignment problems
  3. Moving targets

References

1. S. L. Smith and F. Bullo. Target assignment for robotic networks: Asymptotic performance under limited communication. In *American Control Conference*, pages 1155–1160, New York, July 2007
2. S. L. Smith and F. Bullo. Target assignment for robotic networks: Worst-case and stochastic performance in dense environments. In *IEEE Conf. on Decision and Control*, pages 3585–3590, New Orleans, LA, December 2007
3. S. L. Smith and F. Bullo. Monotonic target assignment for robotic networks. *IEEE Transactions on Automatic Control*, June 2007. To appear