

Geometry, Optimization and Control in Robot Coordination

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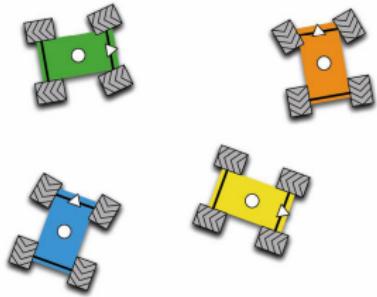
Acknowledgements

- **Jorge Cortés** (UCSD): robotic networks, multi-center optimization, nonconvex deployment and rendezvous
 - **Sonia Martínez** (UCSD): robotic networks, multi-center optimization, boundary estimation
 - **Emilio Frazzoli** (MIT): robotic networks, dynamic vehicle routing
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- **Marco Pavone** (MIT) and **Stephen Smith** (UCSB): dynamic vehicle routing
 - **Ruggero Carli** (UCSB), **Joey W. Durham** (UCSB), and **Paolo Frasca** (Universitá di Roma): peer-to-peer coordination
 - **Karl J. Obermeyer** (UCSB): nonconvex deployment



Distributed Control of Robotic Networks

A Mathematical Approach
to Motion Coordination Algorithms



Francesco Bullo
Jorge Cortés
Sonia Martínez

- ① intro to distributed algorithms (graph theory, synchronous networks, and averaging algos)
- ② geometric models and geometric optimization problems
- ③ model for robotic, relative sensing networks, and complexity
- ④ algorithms for rendezvous, deployment, boundary estimation

Manuscript by F. Bullo, J. Cortés, and S. Martínez. Princeton Univ Press, 2009, ISBN 978-0-691-14195-4. Freely downloadable at <http://coordinationbook.info> with tutorial slides and (ongoing) software libraries.

What kind of systems?

Groups of agents with control, sensing, communication and computing

What kind of abilities?

- each agent **senses** its immediate environment,
- **communicates** with others,
- **processes** information gathered, and
- **takes local action** in response



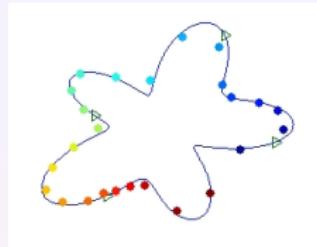
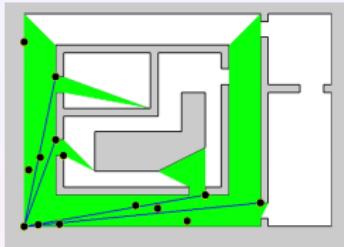
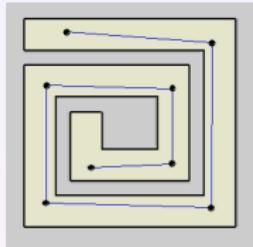
AeroVironment Inc., "Raven"
unmanned aerial vehicle



iRobot Inc., "PackBot"
unmanned ground vehicle

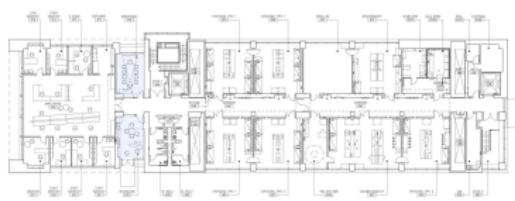
Cooperative systems: technologies and applications

What kind of tasks?



What scenarios?

Security systems, disaster recovery, environmental monitoring, study of natural phenomena and biological species, science imaging

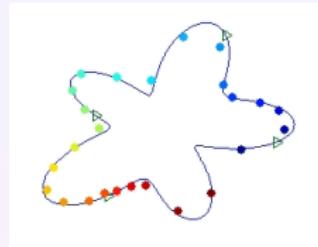
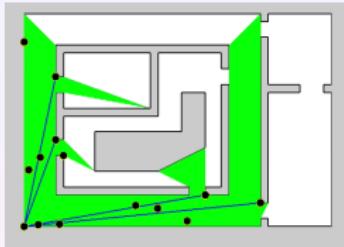
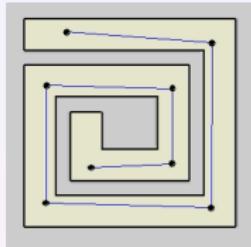


Security systems

Building monitoring and evac

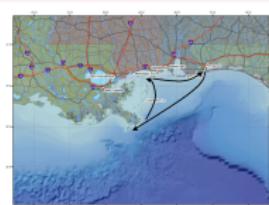
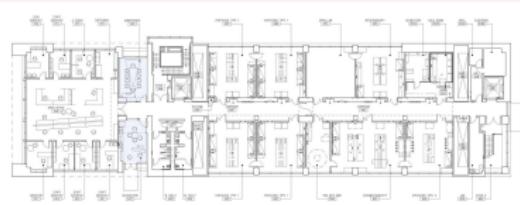
Environmental monitoring

What kind of tasks?



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Security systems, disaster recovery, environmental monitoring, study of natural phenomena and biological species, science imaging



Security systems

Building monitoring and evac

Environmental monitoring

① vehicle routing problems

via queueing theory and combinatorics

② territory partitioning

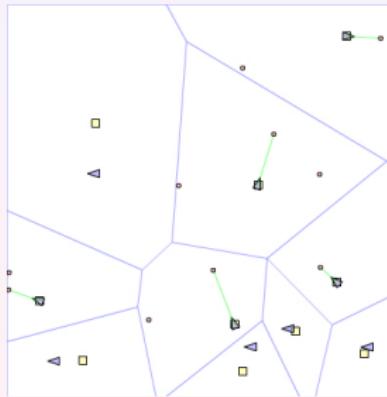
via emerging behaviors and geometric optimization

③ peer-to-peer coordination

via invariance principle on metric space

Dynamic Vehicle Routing

- customers appear randomly space/time
- robotic network knows locations and provides service
- Goal: distributed adaptive algos, delay vs throughput



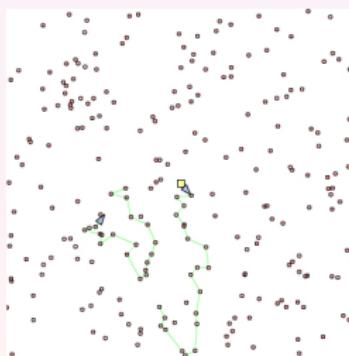
M. Pavone, E. Frazzoli, and F. Bullo. Decentralized algorithms for stochastic and dynamic vehicle routing with general target distribution. In *IEEE Conf. on Decision and Control*, pages 4869–4874, New Orleans, LA, December 2007

Receding-horizon Shortest-Path (RH-SP)

For $\eta \in (0, 1]$, single agent performs:

- 1: while no customers, move to center
- 2: while customers waiting

- ① compute shortest path through current customers
- ② service η -fraction of path



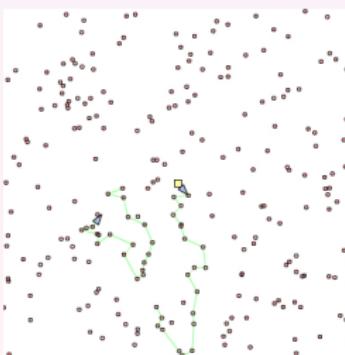
- shortest path is NP-hard, but effective heuristics available
- delay is optimal in light traffic
- delay is constant-factor optimal in high traffic

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Algo #1: Sketch of RH-SP analysis

via combinatorics in Euclidean space

- ① queue is stable if $\text{service time} < \text{interarrival time}$
- ② service time =
$$\frac{\text{length shortest path}(n)}{n} \quad (n = \# \text{ customers})$$
- ③ queue is stable if $(\text{length of shortest path}) = \text{sublinear } f(n)$



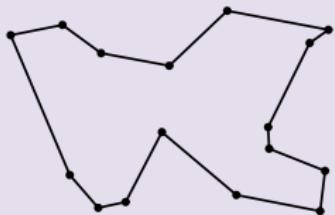
length shortest path(n) $\sim \sqrt{n}$

J. Beardwood, J. Halton, and J. Hammersley. The shortest path through many points. In *Proceedings of the Cambridge Philosophy Society*, volume 55, pages 299–327, 1959

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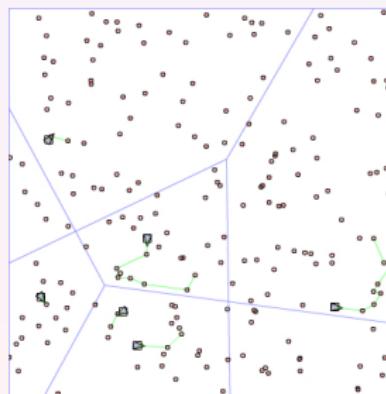
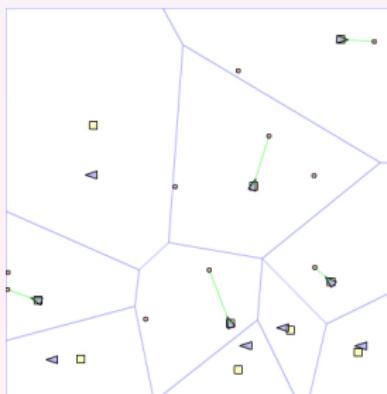
Algo #2: Load balancing via territory partitioning

RH-SP + Partitioning

For $\eta \in (0, 1]$, agent i performs:

- 1: compute own cell v_i in optimal partition
- 2: apply RH-SP policy on v_i

Asymptotically constant-factor optimal in light and high traffic



① vehicle routing problems

via queueing theory and combinatorics

② territory partitioning

via emerging behaviors and geometric optimization

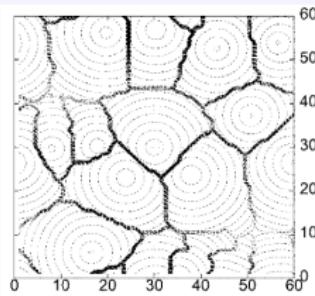
③ peer-to-peer coordination

via invariance principle on metric space

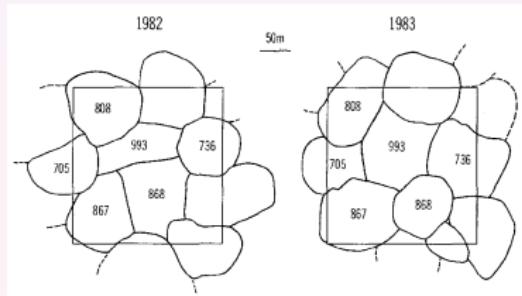
Territory partitioning akin to *animal territory dynamics*



Tilapia mossambica, "Hexagonal Territories," Barlow et al, '74



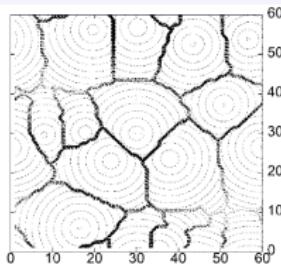
Red harvester ants, "Optimization, Conflict, and Nonoverlapping Foraging Ranges," Adler et al, '03



Sage sparrows, "Territory dynamics in a sage sparrows population," Petersen et al '87

ANALYSIS of cooperative distributed behaviors

- ① how do animals share territory?
 - how do they decide foraging ranges?
 - how do they decide nest locations?
- ② what if each robot goes to “center” of own dominance region?
- ③ what if each robot moves away from closest vehicle?



DESIGN of performance metrics

- ④ how to cover a region with n minimum-radius overlapping disks?
- ⑤ how to design a minimum-distortion (fixed-rate) vector quantizer?
- ⑥ where to place mailboxes in a city / cache servers on the internet?

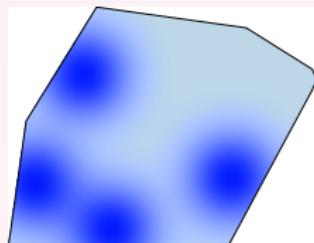
Expected wait time

$$H(p, v) = \int_{V_1} \|q - p_1\| dq + \cdots + \int_{V_n} \|q - p_n\| dq$$

- n robots at $p = \{p_1, \dots, p_n\}$
- environment is partitioned into $v = \{v_1, \dots, v_n\}$

$$H(p, v) = \sum_{i=1}^n \int_{V_i} f(\|q - p_i\|) \phi(q) dq$$

- $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}_{\geq 0}$ density
- $f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ penalty function

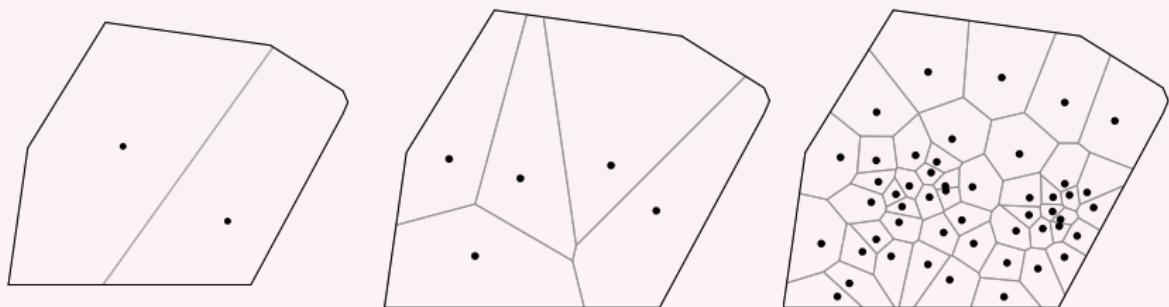


Optimal partitioning by Georgy Fedoseevich Voronoy (PhD from Saint Petersburg State University in 1896)

The **Voronoi partition** $\{V_1, \dots, V_n\}$ generated by points (p_1, \dots, p_n)

$$V_i = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \forall j \neq i\}$$

$$= Q \bigcap_j (\text{half plane between } i \text{ and } j, \text{ containing } i)$$



Optimal centering (for region v with density ϕ)

function of p

$$p \mapsto \int_v \|q - p\|^2 \phi(q) dq$$

$$p \mapsto \int_v \|q - p\| \phi(q) dq$$

$$p \mapsto \text{area}(v \cap \text{disk}(p, r))$$

$p \mapsto$ radius of largest disk centered
at p enclosed inside v

$p \mapsto$ radius of smallest disk cen-
tered at p enclosing v

minimizer = center

centroid (or **center of mass**)

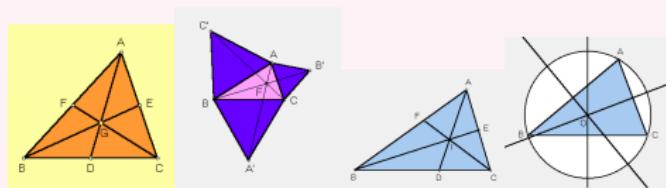
Fermat–Weber point (or **median**)

r -area center

incenter

circumcenter

From online
Encyclopedia of
Triangle Centers



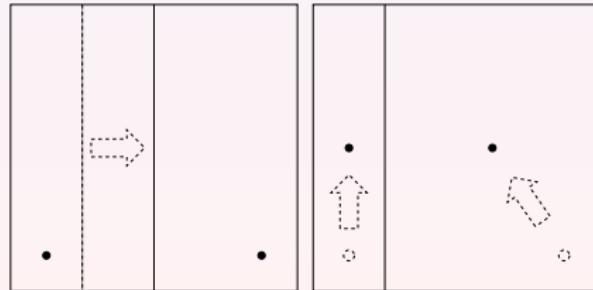
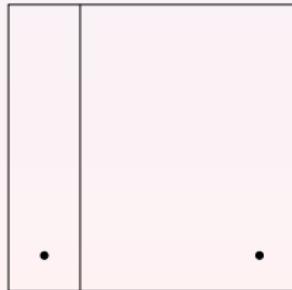
From optimality conditions to algorithms

$$H(p, v) = \sum_{i=1}^n \int_{v_i} f(\|q - p_i\|) \phi(q) dq$$

Theorem (Alternating Algorithm, Lloyd '57)

- ① at fixed positions, optimal partition is Voronoi
- ② at fixed partition, optimal positions are “generalized centers”
- ③ alternate v - p optimization

⇒ local optimum = center Voronoi partition

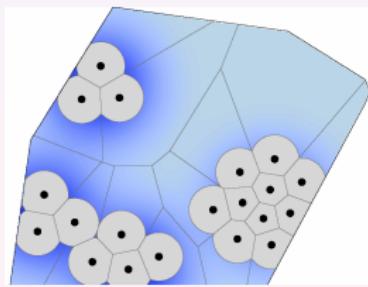
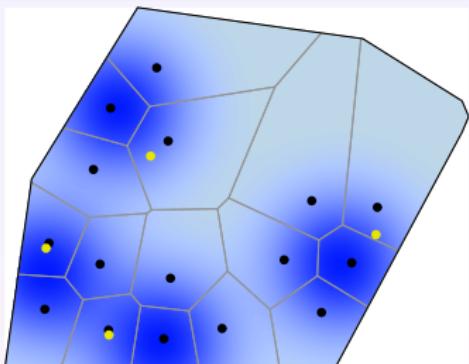


Voronoi+centering algorithm

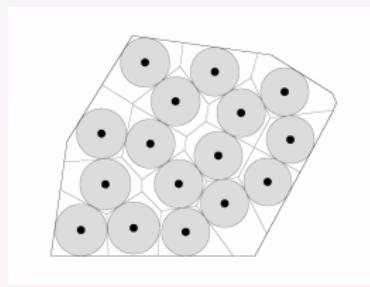
Voronoi+centering law

At each comm round:

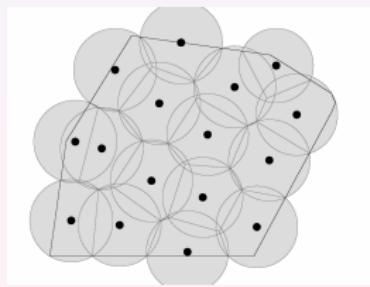
- 1: acquire neighbors' positions
- 2: compute own dominance region
- 3: move towards center of own dominance region



Area-center



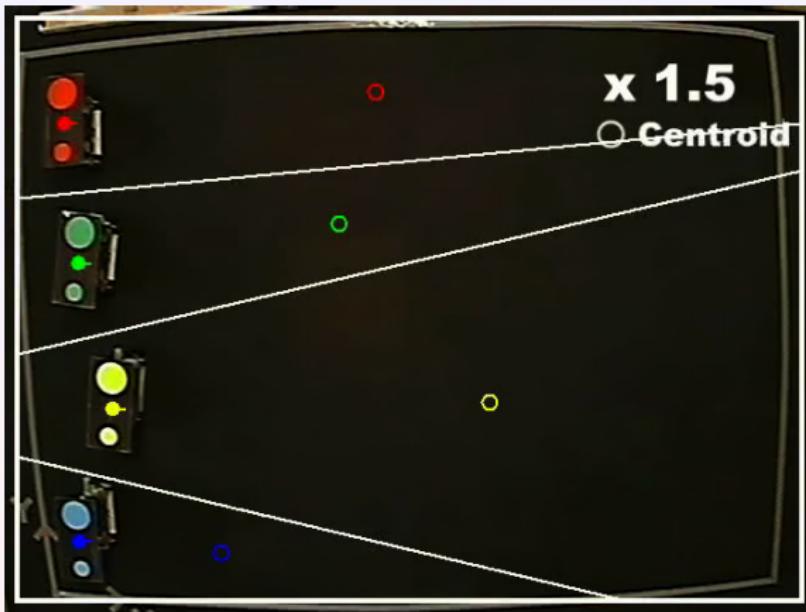
Incenter



Circumcenter

J. Cortés, S. Martínez, and F. Bullo. Spatially-distributed coverage optimization and control with limited-range interactions. *ESAIM: Control, Optimisation & Calculus of Variations*, 11:691–719, 2005

Experimental Territory Partitioning



Takahide Goto, Takeshi Hatanaka, Masayuki Fujita
Tokyo Institute of Technology

Optimal Distributed Coverage Control for Multiple Hovering Robots with Downward Facing Cameras

Mac Schwager

Brian Julian

Daniela Rus

Distributed Robots Laboratory, CSAIL

Mac Schwager, Brian Julian, Daniela Rus

Distributed Robots Laboratory, MIT

① vehicle routing problems

via queueing theory and combinatorics

② territory partitioning

via emerging behaviors and geometric optimization

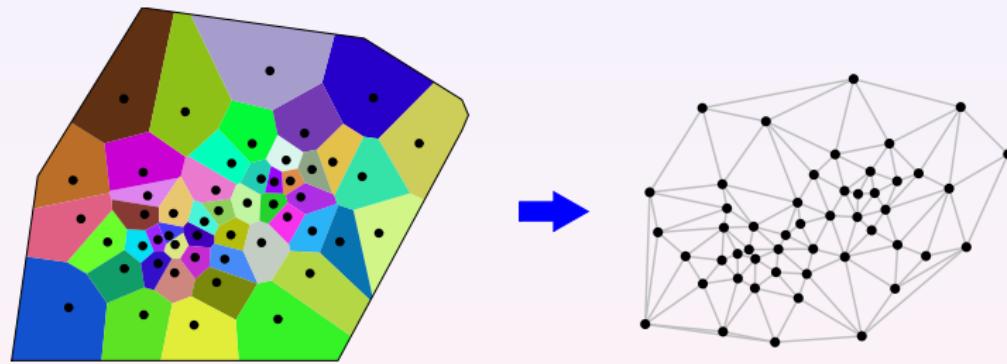
③ peer-to-peer coordination

via invariance principle on metric space

Partitioning with minimal communication requirements

Voronoi+centering law requires:

- ① synchronous communication
- ② communication along edges of dual graph

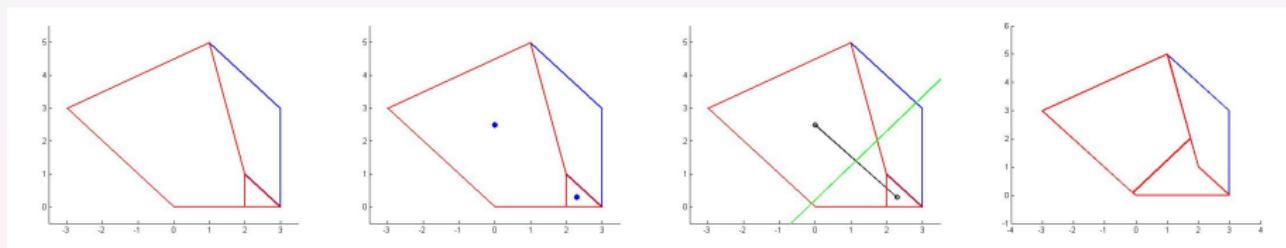


Minimalist coordination

- is synchrony necessary?
- is it sufficient to communicate peer-to-peer (gossip)?
- what are minimal requirements?

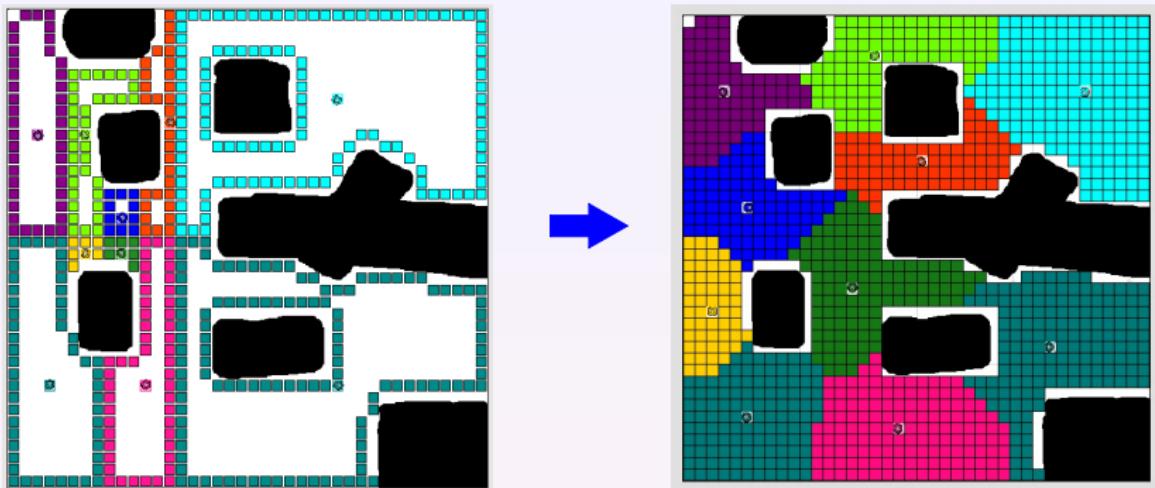
Peer-to-peer partitioning policy

- ① Random communication between two regions
- ② Compute two centers
- ③ Compute bisector of centers
- ④ Partition two regions by bisector



P. Frasca, R. Carli, and F. Bullo. Multiagent coverage algorithms with gossip communication: control systems on the space of partitions. In *American Control Conference*, pages 2228–2235, St. Louis, MO, June 2009

Indoor example implementation



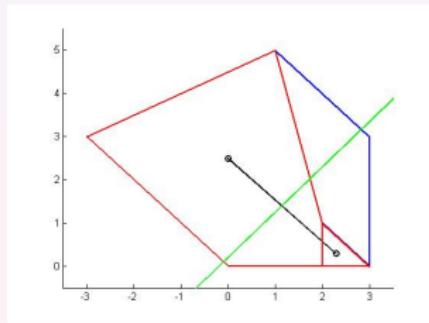
- Player/Stage platform
- realistic robot models in discretized environments
- integrated wireless network model & obstacle-avoidance planner

J. W. Durham, R. Carli, P. Frasca, and F. Bullo. Discrete partitioning and coverage control with gossip communication. In *ASME Dynamic Systems and Control Conference*, Hollywood, CA, October 2009. to appear

Lyapunov function for peer-to-peer territory partitioning

$$H(v) = \sum_{i=1}^n \int_{v_i} f(\|\text{center}(v_i) - q\|) \phi(q) dq$$

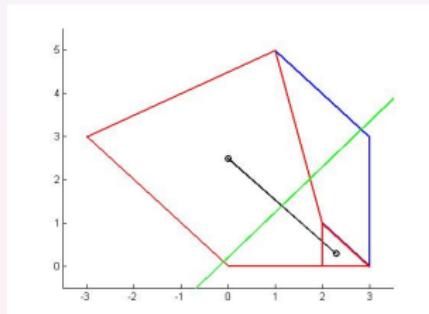
- ① state space is not finite-dimensional
non-convex disconnected polygons
arbitrary number of vertices
- ② peer-to-peer map is not deterministic, ill-defined and discontinuous
two regions could have same centers



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 - non-convex disconnected polygons
 - arbitrary number of vertices
- ② **peer-to-peer map** is not deterministic, ill-defined and discontinuous
 - two regions could have same centers



The space of partitions (proof sketch 2/3)

Definition (space of n -partitions)

v is collections of n subsets of Q , $\{v_1, \dots, v_n\}$, such that

- ① $v_1 \cup \dots \cup v_n = Q$,
- ② $\text{interior}(v_i) \cap \text{interior}(v_j) = \emptyset$ if $i \neq j$, and
- ③ each v_i is closed, has non-empty interior and zero-measure boundary

Given sets A, B , symmetric difference and distance are:

$$d_\Delta(A, B) = \text{area} \left((\text{points in } A \text{ that are not in } B) \cup (\text{vice versa}) \right)$$

Theorem (topological properties of the space of partitions)

Partition space with $(u, v) \mapsto \sum_{i=1}^n d_\Delta(u_i, v_i)$ is metric and precompact

- X is metric space
- finite collection of maps $T_i : X \rightarrow X$ for $i \in I$
- consider sequences $\{x_\ell\}_{\ell \geq 0} \subset X$ with

$$x_{\ell+1} = T_{i(\ell)}(x_\ell)$$

Assume:

- ① $W \subset X$ compact and positively invariant for each T_i
- ② $U : W \rightarrow \mathbb{R}$ decreasing along each T_i
- ③ U and T_i are continuous on W
- ④ there exists probability $p \in]0, 1[$ such that, for all indices $i \in I$ and times ℓ , we have $\text{Prob}[x_{\ell+1} = T_i(x_\ell) | \text{past}] \geq p$

If $x_0 \in W$, then almost surely

$$x_\ell \rightarrow (\text{intersection of sets of fixed points of all } T_i) \cap U^{-1}(c)$$

Robotic Network Theory

① network modeling

network, ctrl+comm algorithm, task, complexity

② coordination algorithm

partitioning, vehicle routing, task allocation

Open problems

① algorithmic design for minimalist robotic networks

scalable, adaptive, asynchronous, agent arrival/departure

rich task set, e.g., cooperative estimation

② mixed robotic-human networks

③ high-fidelity sensing/actuation scenarios