John, Richard and the Evolution of Influence Networks

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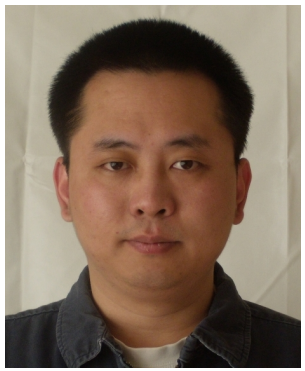

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Ana MirTabatabaei

Noah Friedkin

A sociological investigation of CDS at 20

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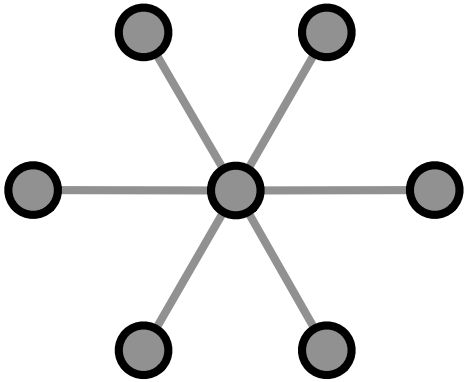

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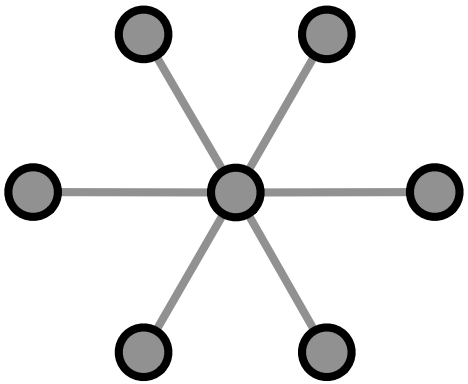
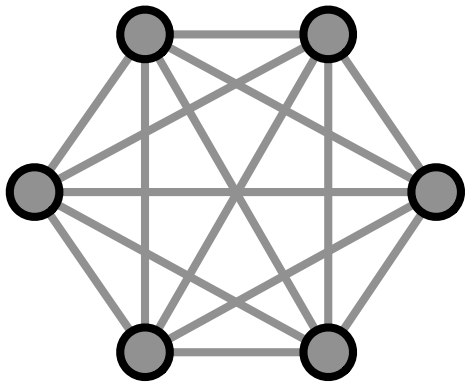


Richard implements perfect plans:



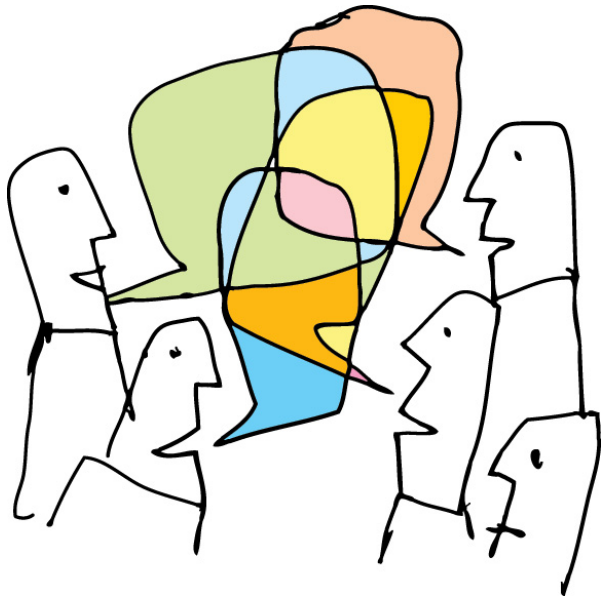
What are the consequences of these social structures?

The dynamics of opinions

DeGroot opinion dynamics model

$$y(t+1) = W\,y(t)$$

- Opinions $y \in \mathbb{R}^n$
- Influence network = row-stochastic W
- by P-F: $\lim_{t\to\infty} y(t) = (w^T y(0))\mathbb{1}_n$
  where w is dominant left eigenvector of W



- Self-weights $W_{ii} =: x_i$
- Interpersonal accorded weights W_{ij}
- Relative interpersonal accorded weights C_{ij},
  where $W_{ij} = (1 - x_i)C_{ij}$

$$W(x) = \mathrm{diag}(x)I_n + \mathrm{diag}(\mathbb{1}_n - x)C$$

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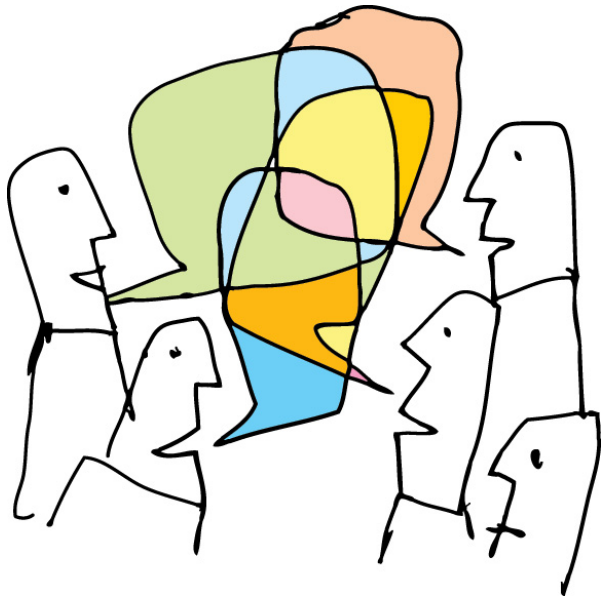

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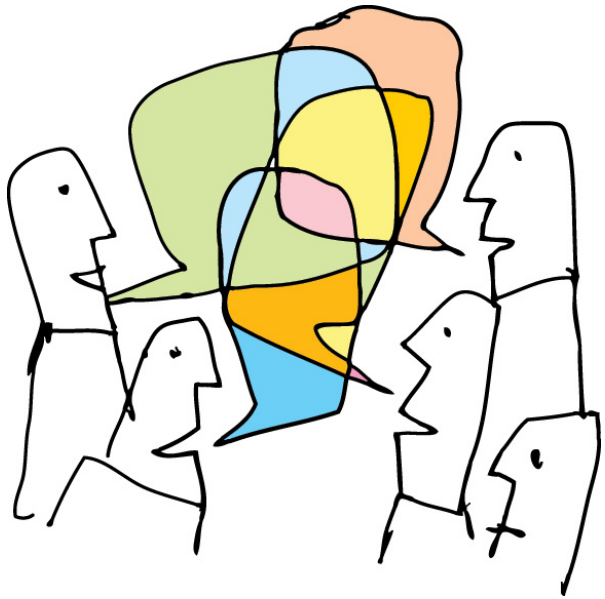

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The dynamics of social power and self-confidence

Reflected appraisal hypothesis by Cooley, 1902:

> *individual' self-appraisal (e.g., self-confidence, self-esteem, self-worth) is influenced by the appraisal held by others of her*

Mathematization: along a sequence of issues, individual dampens/elevates self-weight x_i according to her relative prior control

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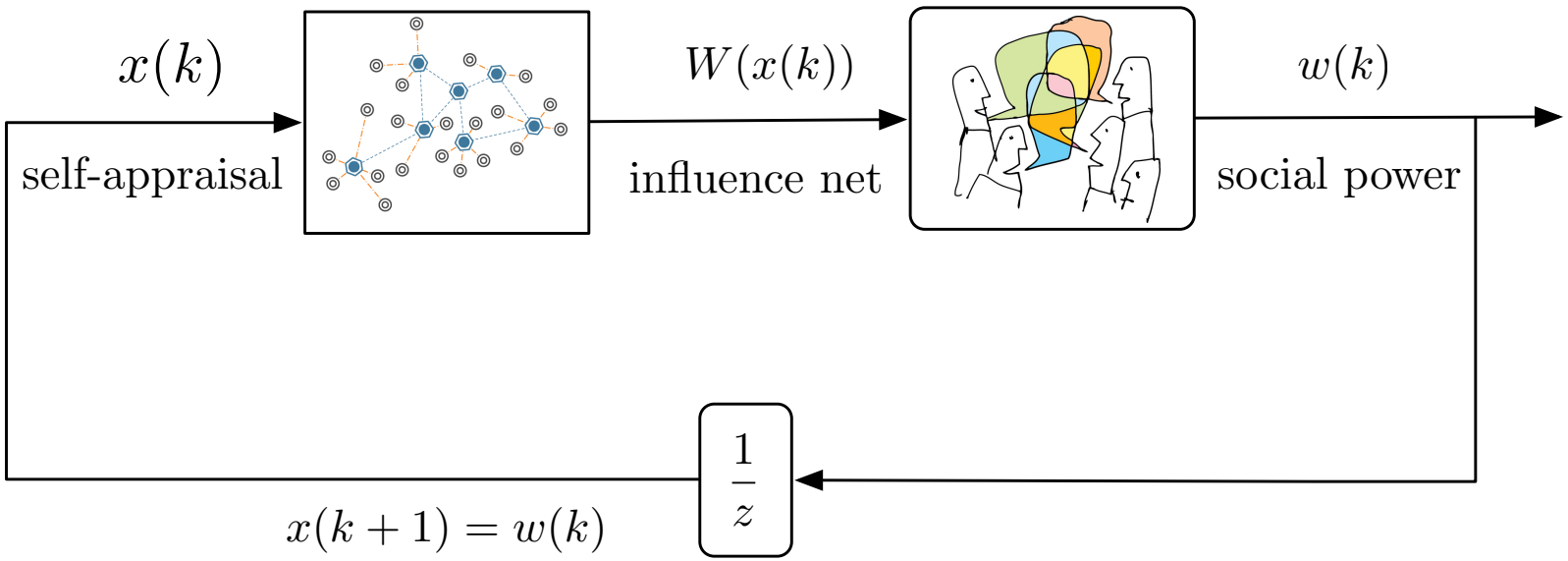


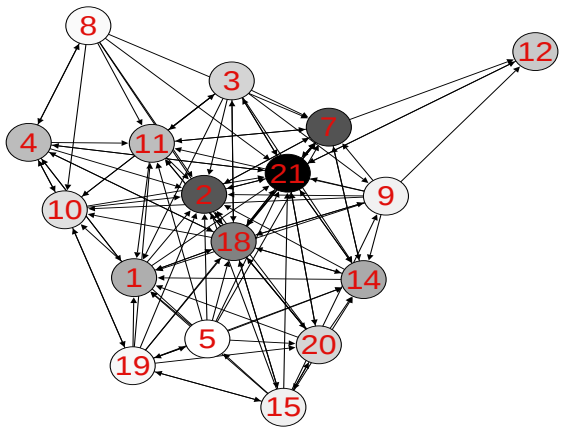
self-appraisal = self-weights $\leftarrow$ relative control = social power

The closed-loop system

- DeGroot dynamics about an issue: $y(t+1) = W(x))y(t)$
- Influence network $W(x) = \mathrm{diag}(x)I_n + \mathrm{diag}(\mathbb{1}_n - x)C$
- Reflected appraisal across issues: $x(k+1) = w(x(k))$ $=: F(x(k))$

$x(k+1) = F(x(k))$

$$F(x) = \begin{cases} \mathbb{e}_i, & \text{if } x = \mathbb{e}_i \text{ for all } i \in \{1, \dots, n\} \\ \left(\frac{c_1}{1-x_1}, \dots, \frac{c_n}{1-x_n} \right) / \sum_{i=1}^{n} \frac{c_i}{1-x_i}, & \text{otherwise} \end{cases}$$



c is the dominant left eigenvector of C

In Theory ...

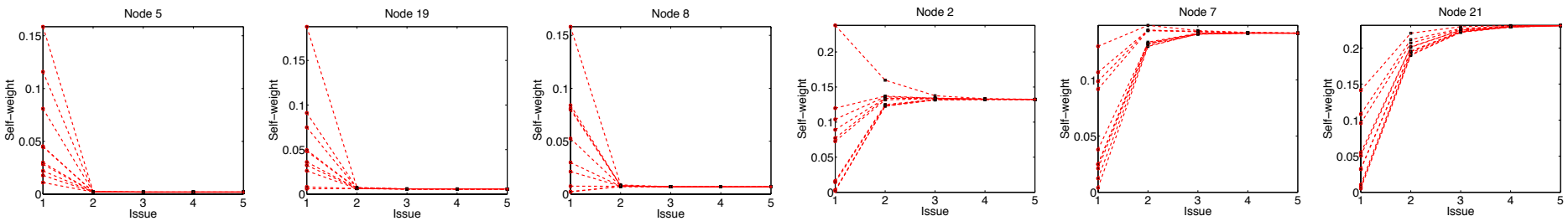
Theorem (General "relative interpersonal accorded weights" C)

1. **convergence = forgetting initial conditions**

   for all non-trivial initial conditions,

$$\lim_{k\to\infty} x(k) = \lim_{k\to\infty} w(x(k)) = x^*$$

2. **accumulation of social power and self-appraisal**
   - *fixed point* $x^* > 0$ *has same ordering of* c
   - *social power threshold* T *such that:* $x_i^* \geq c_i \geq T$ *or* $x_i^* \leq c_i \leq T$



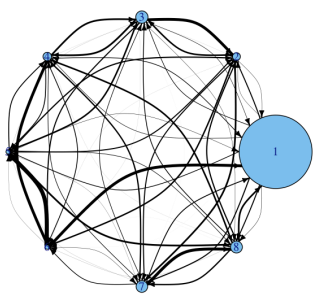
Doubly-stochastic C: emergency of democracy

Lemma (Convergence to democracy)

Iff C is doubly-stochastic:

1. *the non-trivial fixed point of F is $\frac{\mathbb{1}_n}{n}$,*
2. *for all non-trivial initial conditions, $\lim_{k\to\infty} x(k) = \lim_{k\to\infty} w(x(k)) = \frac{\mathbb{1}_n}{n}$.*

- Uniform social power
- No power accumulation



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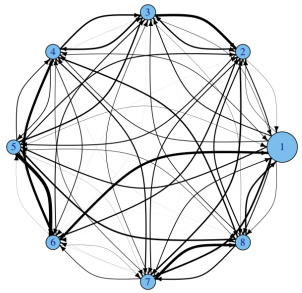

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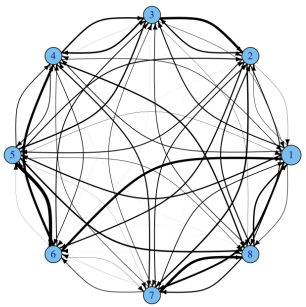

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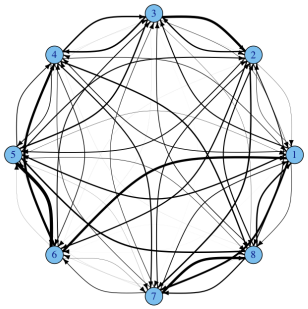


issue N

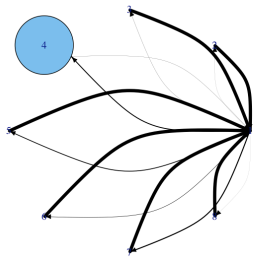
Star topology: emergency of autocracy

Lemma (Convergence to autocracy)

Iff graph has star topology with center j:

1. *there are no non-trivial fixed points of F*
2. *for all initial non-trivial conditions,* $\lim_{k\to\infty} x(k) = \lim_{s\to\infty} w(x(k)) = \mathbb{e}_j$.

- Autocrat appears in center of star topology
- Extreme power accumulation!



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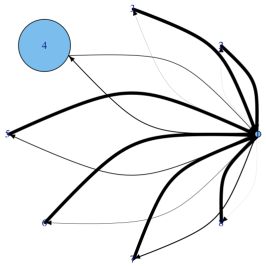

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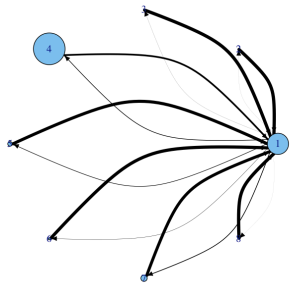

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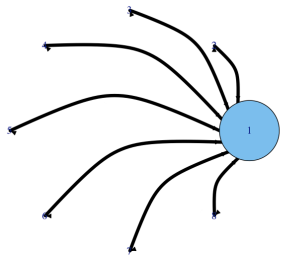

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Conclusions

Sociological investigation of CDS at 20

- John: self-confident and influential
- Richard: great manager and collaborator
- ... inexorable consequences of their surrounding interpersonal nets!

Coworkers: Peng Jia (Mech Eng, UCSB), Ana MirTabatabaei (Bosch), Noah Friedkin (Sociology, UCSB)

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