Identifying Cyber Attacks via Local Model Information

Fabio Pasqualetti
Ruggero Carli Antonio Bicchi Francesco Bullo

Center for Control, Dynamical systems and Computation
University of California, Santa Barbara

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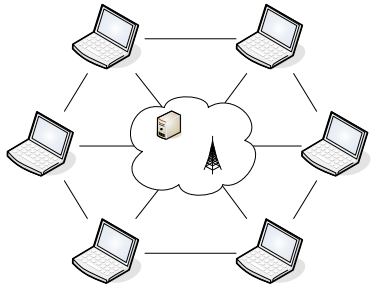
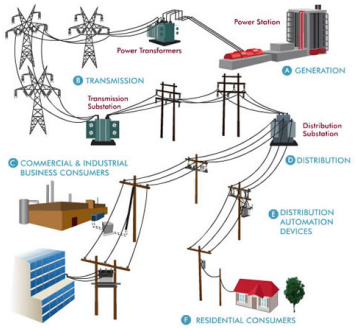
Network systems and cyber-physical attacks





- *Physical attack:* compromise networks dynamics
- *Cyber attack:* compromise integrity of data

How do we enforce security and reliability?

Linear networks with misbehaving components

Network dynamics

$$x(t+1) = Ax(t) + Bu(t)$$

- if A is row stochastic and primitive, then *consensus* algorithm
- physical and cyber attacks are modeled by an exogenous input
- the nonzero entries of B determine the *misbehaving* components

Monitor measurements model

$$y_j(t) = C_j x(t)$$

- the output matrix is determined by the interaction topology

Detection and identification: known results

By knowing the system matrix A, observer j

- detects the attack B if and only if (A, B, C_j) has no zero dynamics
- identifies the attack B if and only if $(A, [B\ \bar{B}], C_j)$ has no zero dynamics, where $\bar{B}$ is any possible attack matrix
- implements effective combinatorial security controls

For a network A with connectivity k, there exists

- an undetectable attack if $\text{rank}(B) \geq k$
- an unidentifiable attack if $\text{rank}(B) \geq \lceil k/2 \rceil$

S. Sundaram and C. Hadjicostis, "Distributed function calculation via linear iterative strategies in the presence of malicious agents," *IEEE Transactions on Automatic Control*

F. Pasqualetti, A. Bicchi, and F. Bullo, "Consensus computation in unreliable networks: A system theoretic approach," *IEEE Transactions on Automatic Control* (submitted)

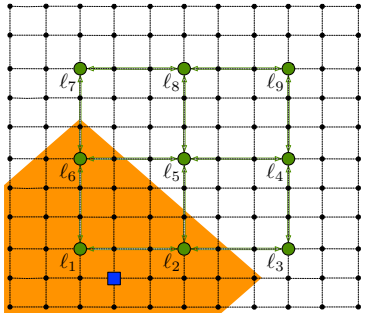
Detection and identification via local model information

Limitation of existing procedures

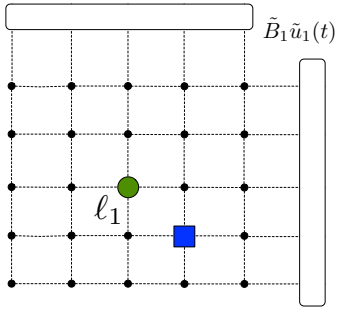
- the whole network topology has to be known by each observer
- high computational complexity
- numerically unreliable

Our approach

- geographically deploy leaders
- cooperation through a leader graph
- implement local security controls



Difficulty of a local approach

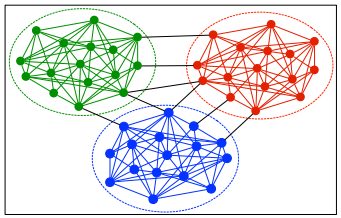


- the unmodeled dynamics act as an unknown and unmeasurable input

$$x_1(t+1) = A_1 x_1(t) + B_1 u_1(t) + \tilde{B}_1 \tilde{u}_1(t)$$

- Detection via network decomposition: $\tilde{B}_1\tilde{u}_1(t)$ is "small"
- Detection via leaders cooperation: $\tilde{B}_1\tilde{u}_1(t)$ is "large"

Weakly interconnected subnetworks



- Identify groups of strongly interacting components
- assign one leader to each group

$$A = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_m \end{bmatrix} + \varepsilon \begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1m} \\ \Delta_{21} & \ddots & \ddots & \Delta_{2m} \\ \vdots & \ddots & \ddots & \vdots \\ \Delta_{m1} & \Delta_{m2} & \cdots & \Delta_{mm} \end{bmatrix}$$

- if ε is "small", then different groups are weakly coupled
- determine the network partitioning that minimizes ε
- if A is a consensus matrix, then $\|\Delta\|_\infty = 2$

Local detection and identification

Each leader performs detection of misbehaving parts inside block

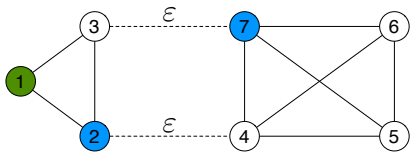
- neglect interaction with other blocks

Theorem (Local detectability)

Given $u_{\max}$, there exists α such that, if each input signal takes value in $\{u : \varepsilon\alpha u_{\max} \leq \|u\|_\infty \leq u_{\max}\}$, then local detection is successful

- there exists a critical value ε^* below which the local effect of misbehaving parts is larger than the effect of unmodeled dynamics

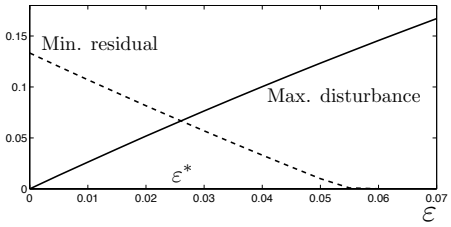
An example of detection via network decomposition



Residual generator

$$w(t+1) = Fw(t) + Ey_{\ell_1}(t)$$
$$r(t) = Mw(t) + Hy_{\ell_1}(t)$$

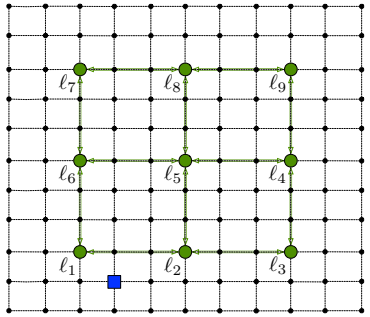
$$A = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3}-\varepsilon & \frac{1}{3} & \varepsilon & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}-\varepsilon & 0 & 0 & 0 & \varepsilon \\ 0 & 0 & \varepsilon & \frac{1}{4} & \frac{1}{4}-\varepsilon & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \varepsilon & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}-\varepsilon \end{bmatrix}$$



- if $\varepsilon < \varepsilon^*$, then observer 1 detects and identifies the misbehaving component 2 by means of a local residual generator

Hierarchical framework

For networks without clusters ($\varepsilon > \varepsilon^*$), leaders cooperation is needed



$$C = \begin{bmatrix} C_{\ell_1} \\ C_{\ell_2} \\ \vdots \\ C_{\ell_m} \end{bmatrix} \quad O^s = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix}$$

$$O_i^s = \begin{bmatrix} C_{\ell_i} \\ C_{\ell_i}A \\ \vdots \\ C_{\ell_i}A^{s-1} \end{bmatrix} \quad O^s = \begin{bmatrix} O_1^s \\ O_2^s \\ \vdots \\ O_m^s \end{bmatrix}$$

- extension to process and measurement noise

"Distributed estimation and detection under local information," in *IFAC Workshop on Distributed Estimation and Control in Networked Systems*,

State estimation

Measurements

$$Y_i^s = \begin{bmatrix} y_{\ell_i}(0) \\ y_{\ell_i}(1) \\ \vdots \\ y_{\ell_i}(s-1) \end{bmatrix} \quad Y^s = \begin{bmatrix} Y_1^s \\ Y_2^s \\ \vdots \\ Y_m^s \end{bmatrix}$$

In the absence of misbehaving components

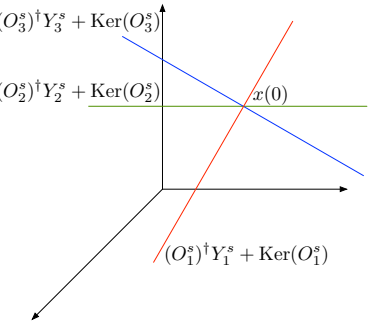
$$Y^s = O^s x(0)$$

- if $\mathrm{Ker}(O^s) = \{0\}$, then the state $x(0)$ can be recovered from the measurements Y^s

A geometric interpretation

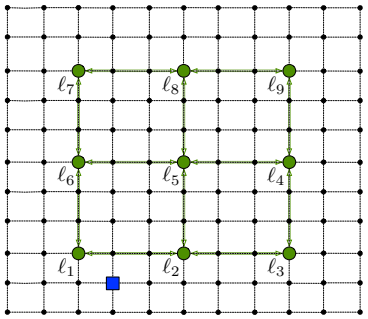
If $\mathrm{Ker}(O^s) = \{0\}$, the unique solution to $Y^s = O^s x(0)$ is the intersection of the affine subspaces defined by the block-rows of $Y^s = O^s x(0)$

$$\begin{bmatrix} Y_1^s \\ Y_2^s \\ \vdots \\ Y_m^s \end{bmatrix} = \begin{bmatrix} O_1^s \\ O_2^s \\ \vdots \\ O_m^s \end{bmatrix} x(0)$$



- similarities with iterative Kaczmarz method

An algorithmic solution



- the leader graph is connected

Leaders iterate the following operations

- transmit local estimate and uncertainty subspace
- receive estimates from neighbors
- update local estimate and uncertainty subspace

(Diffusive) Estimation algorithm

Input : Local observability matrix O_i^s, Local measurements Y_i^s;
Require : Global observability, i.e., $\mathrm{Ker}(O^s) = 0$;

transmit $\hat{x}_i = (O_i^s)^\dagger Y_i^s$, $\quad \mathcal{V}_i = \mathrm{Ker}(O_i^s)$;

while $\mathcal{V}_i \neq \{0\}$ **do**
  for $j \in N_i$ **do**
    receive $\hat{x}_j$ and $\mathcal{V}_j$;
    set $\hat{x}_i = \hat{x}_i \perp \left((\hat{x}_i + \mathcal{V}_i) \cap (\hat{x}_j + \mathcal{V}_j)\right)$;
    set $\mathcal{V}_i = \mathcal{V}_i \cap \mathcal{V}_j$;
  transmit $\hat{x}_i$ and $\mathcal{V}_i$;
return $\hat{x}_i$;

Detecting misbehaving components

If $Bu(t) \neq 0$, the measurements equation becomes

$$Y^s = O^s x(0) + F^s U^s$$

$$F_i^s = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_{\ell_i} B & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ C_{\ell_i} A^{s-2} B & \cdots & C_{\ell_i} B & 0 \end{bmatrix} \quad F^s = \begin{bmatrix} F_1^s \\ F_2^s \\ \vdots \\ F_m^s \end{bmatrix} \quad U^s = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(s-1) \end{bmatrix}$$

Assume that the attack is detectable from the measurements vector

- $\mathrm{Im}(O^s) \cap \mathrm{Im}(F^s) = \emptyset$

(Diffusive) Detection algorithm

If $F^s U^s \neq 0$, the system $Y^s = O^s x$ has no solution

- the affine subspaces have empty intersection

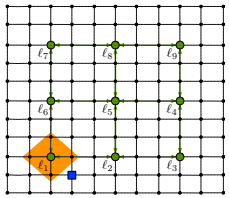
Input : O_i^s, Y_i^s;
Require : Global observability, i.e., $\mathrm{Ker}(O^s) = 0$,
Attack detectability, i.e., $\mathrm{Im}(O^s) \cap \mathrm{Im}(F^s) = \{0\}$;

transmit $\mathcal{S}_i = (O_i^s)^\dagger Y_i^s + \mathrm{Ker}(O_i^s)$;

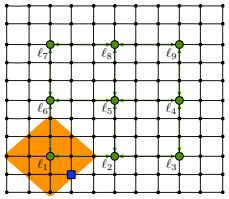
for *diam*$(G^{(\ell)})$ *iterations* **do**
  for $j \in N_i$ **do**
    set $\mathcal{S}_i = \mathcal{S}_i \cap \mathcal{S}_j$;
  transmit $\mathcal{S}_i$;
if $\mathcal{S}_i = \emptyset$ **then return** *Alarm*

Why local model information

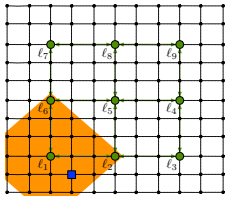
The computation of the matrix O_i^s only requires local model information



C_{ℓ_1}



$C_{\ell_1} A$



$C_{\ell_1} A^2$

- for the matrix $C_{\ell_1} A^s$, only the colored regions are needed
- place leaders such that $\mathrm{Ker}(O^s) = \{0\}$ with $s < \mathrm{diam}(G)$
- scalability properties against network dimension

Summary and extensions

Attack detection via local model information

- if the network is sufficiently weakly interconnected, then each leader performs detection of a certain class of attacks
  - local model information
  - finite time detection of a class of attacks
  - no communication overhead
- if a hierarchical structure of leaders is present, then state estimation and attack detection are possible
  - local model information
  - finite time estimation and detection
  - communication overhead

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