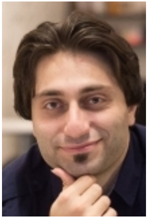
Wisdom of Crowds and Social Influence

Francesco Bullo

Center for Control,
Dynamical Systems & Computation
University of California at Santa Barbara
http://motion.me.ucsb.edu

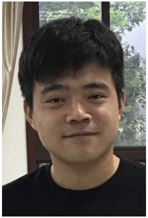
Workshop on Dynamics in Social and Economic Networks – URL
IEEE Conf Decision & Control, Jeju Island, Korea, Dec 13, 2020



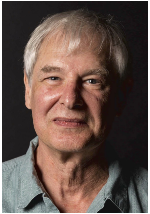
Omid Askarisichani
UC Santa Barbara

Barbara Franci
Technische Universiteit Delft

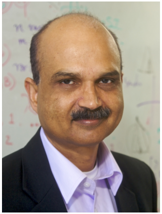
Elizabeth Huang
UC Santa Barbara



Ye Tian
Xidian University

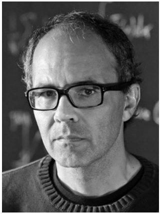


Noah Friedkin
UC Santa Barbara



Ambuj Singh
UC Santa Barbara

Giacomo Como
Politecnico di Torino



Fabio Fagnani
Politecnico di Torino

References

1. N. E. Friedkin and F. Bullo. How truth wins in opinion dynamics along issue sequences. *Proceedings of the National Academy of Sciences*, 114(43):11380–11385, 2017

2. F. Bullo, F. Fagnani, and B. Franci. Finite-time influence systems and the wisdom of crowd effect. *SIAM Journal on Control and Optimization*, 58(2):636–659, 2020

3. O. Askarisichani, E. Y. Huang, K. S. Sato, N. E. Friedkin, F. Bullo, and A. K. Singh. Expertise and confidence explain how social influence evolves along intellective tasks. *Submitted*, 2020

4. F. Bullo, G. Como, F. Fagnani, and N. E. Friedkin. Expertise, appraisals, influence systems, and the wisdom of crowds, 2020. Working paper

5. Y. Tian, G. Peluffo, L. Wang., F. Fagnani, and F. Bullo. On the wisdom of crowds problem, 2020. Working paper

Wisdom of crowd

1. B. Golub and M. O. Jackson. Naïve learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics*, 2(1):112–149, 2010

2. B. Bahrami, K. Olsen, P. E. Latham, A. Roepstorff, G. Rees, and C. D. Frith. Optimally interacting minds. *Science*, 329(5995):1081–1085, 2010

3. J. Lorenz, H. Rauhut, F. Schweitzer, and D. Helbing. How social influence can undermine the wisdom of crowd effect. *Proceedings of the National Academy of Sciences*, 108(22):9020–9025, 2011

4. J. Becker, D. Brackbill, and D. Centola. Network dynamics of social influence in the wisdom of crowds. *Proceedings of the National Academy of Sciences*, 114(26):E5070–E5076, 2017

A longstanding problem in the social, biological, and computational sciences is to determine how groups of distributed individuals can form intelligent collective judgments.

psychologist Ralph Hertwig, Science 2012:

[...] the group (also known as jury, team, crowd, and swarm) has been

deplored as a source of intellectual inferiority and disastrous policy decisions

hailed [...] magical creativity, unparalleled wisdom and forecast accuracy.

PSYCHOLOGY

Tapping into the Wisdom of the Crowd—with Confidence

The subjective confidence of individuals in groups can be a valid predictor of accuracy in decision-making tasks.

Ralph Hertwig

If research in psychology had a Dr. Jekyll and Mr. Hyde Award, it would go to—drum roll, please—the group as a decision-making instrument. Since the late 19th century, the group (also known as jury, team, crowd, and swarm) has been deplored as a source of intellectual inferiority (*1*) and disastrous policy decisions (*2*) and hailed as a source of near-magical creativity (*3*) and unparalleled wisdom and forecast accuracy (*4*, *5*). Some of these attributions have proved to be unfounded. For instance, with respect to creative potential, groups that engage in brainstorming lag hopelessly behind the same number of individuals working alone (*6*). The key to benefiting from other minds is to know when to rely on the group and when to walk which of two countries has a larger area), he shows that members of dyads—and, by extension, larger groups—can tap into the wisdom of two heads even in the absence of social interaction by using a simple heuristic: Select the response expressed with the higher—or in the case of more than two heads, highest—degree of confidence.

This maximum-confidence slating (MCS) heuristic enables humans to benefit from the presence of two or more opinions in choice tasks. Another simple and highly adaptive combination tool in choice tasks is the majority rule, but it requires at least three opinions (*8*). In estimation tasks, no combination strategy rivals the intelligent simplicity of averaging, which exploits the benefit

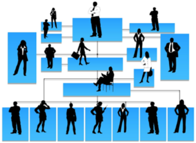
Why and when does the MCS heuristic work? By using the subjective confidence of each judge in the accuracy of their response, the heuristic flexibly adopts the opinion of one or the other judge. It does not bet that the same person will always be the best judge (while not precluding this possibility), but rather adaptively aligns itself with the judge who produces the most confident response in a given trial. In his first two experiments, Koriat shows that using this heuristic enables a level of inferential accuracy that is substantially higher than that achieved by the dyad's higher-performing member. Furthermore, a person who responds to the same task twice, separated by an interval and thus enabling variability (for example

biologist Charles Darwin (1809-1882):
Ignorance more frequently begets confidence than does knowledge.

philosopher Bertrand Russell (1872-1970):
The whole problem with the world is that
fools and fanatics are always so certain of themselves,
and wiser people so full of doubts.

"fools and fanatics"	vs.	"wiser"	:	expertise
"full of doubts"	vs.	"certain"	:	confidence
"whole problem"			:	loss of accuracy in intellective tasks and group decision making
"world"			:	**influence system**

deliberative groups



organizations

online networks

Outline

Towards a theory of wise influence systems

1. F. Bullo, G. Como, F. Fagnani, and N. E. Friedkin. Expertise, appraisals, influence systems, and the wisdom of crowds, 2020. Working paper

   Y. Tian, G. Peluffo, L. Wang., F. Fagnani, and F. Bullo. On the wisdom of crowds problem, 2020. Working paper

Influence system dynamics along memory tasks

2. O. Askarisichani, E. Y. Huang, K. S. Sato, N. E. Friedkin, F. Bullo, and A. K. Singh.

   Expertise and confidence explain how social influence evolves along intellective tasks. *Submitted*, 2020

Towards a theory of wise influence systems

- simple mechanisms suffice to explain macroscale longitudinal behavior
- from expertise, cognitive biases, and social influence
  to rationality, accuracy and fragility (forthcoming)

- datasets from our controlled experiments:
  1. **risky decision making** @ NEF-FB, Sociological Science 2016
  2. **problem solving** @ NEF-FB, PNAS 2019
  3. **memory questions** @ OA-AKS-etal, 2020

Accuracy in estimation tasks

n individuals with initial estimates:

$$y_i(0) = \text{truth} + \text{noise}$$

noise has variance σ_i^2 call σ_i^{-2} the **expertise of individual i**

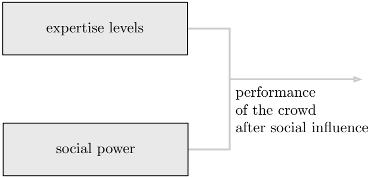
1. exact average = **democracy**:

$$\text{group decision} = \sum_{i=1}^{n} \frac{1}{n} y_i(0)$$

2. weighted average = influence system with **social power π_i**:

$$\text{group decision} = \sum_{i=1}^{n} \pi_i \, y_i(0)$$

Optimal social power is proportional to expertise



Theorem #1: optimal social power is proportional to expertise

$$\pi_i \propto \sigma_i^{-2}$$

(1) **technocracy** better than **democracy**

(2) a **rational group** would:

1. learn expertise levels $\sigma_1^{-2}, \ldots, \sigma_n^{-2}$
2. assign social power based only on expertise: $\pi_i \propto \sigma_i^{-2}$

Detour 1/3: Bases of social power

From social psychology, power is accorded based on:

1. several dimensions: coercive, reward, legitimate (position of authority), referent, persuasive, ...
2. **expert dimension**: power accorded because of **perceived expertise** by way of past actions, reputation, or credentials
3. all other dimensions are **sources of irrational behavior**

J. R. P. French Jr. and B. Raven. The bases of social power. In D. Cartwright, editor, *Studies in Social Power*, pages 150–167. Institute for Social Research, University of Michigan, 1959

Detour 2/3: Empirical lessons from our experiments

@ NEF-FB, PNAS 2019: **explainable problem solving**
@ OA-AKS-etal, 2020: **memory questions with immediate feedback**

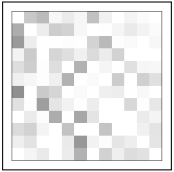
Most often, a relatively accurate **memory/appraisal system** develops:

1. *individuals with high expertise display high self-weight and confidence*
2. *group correctly learns the expertise of individuals and accords interpersonal influence accordingly*

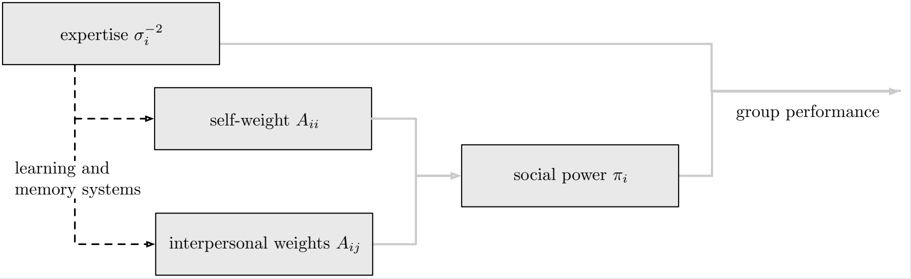
- overconfidence effect (Oskamp 1965):
  - overestimation of one's performance relative to others
- Dunning–Kruger effect (Kruger, Dunning 1999):
  - people with low skill at a task overestimate their skill

Do individual biases get amplified or attenuated in the group discussion?

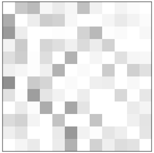
French-DeGroot model of social influence:

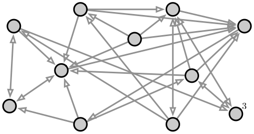


$$y(t+1) = \underbrace{\quad\quad}_{A} y(t), \qquad \implies \qquad \pi = \text{centrality vector of } A$$



- **expertise** σ_i^{-2}





- **influence topology**

- **self-appraisal bias**

$\beta_i > 1$: stubbornness, "certain of themselves"
$\beta_i = 1$: unbiased
$\beta_i < 1$: low self-confidence, "full of doubts"

Social influence model from expertise+bias+topology

self weight: $A_{ii} = \dfrac{\beta_i \sigma_i^{-2}}{\beta_i \sigma_i^{-2} + \sum_j \sigma_j^{-2}} \approx \dfrac{\text{own}}{\text{own} + \text{neighbors}}$

for neighbor j: $A_{ij} \propto \sigma_j^{-2} \approx \text{neighbor}$

Theorem #2: Social power for expertise+bias+topology influence:

$$\pi_i \;\propto\; \sigma_i^{-2}\Big(\beta_i\sigma_i^{-2} + \sum\nolimits_j \sigma_j^{-2}\Big)$$

- small unbiased group: $\beta_i = 1$ and all-to-all topology: $\pi_i \;\propto\; \sigma_i^{-2}$
- groups with homogeneous expertise: $\pi_i \;\propto\; \beta_i + d_i$
- **bias and topology lead to irrationality, in general**

Applications of Theorem #2:

$$\pi_i \;\propto\; \sigma_i^{-2}\left(\beta_i\sigma_i^{-2} + \sum_j \sigma_j^{-2}\right)$$

optimal $\iff$ **each ego-network has homogeneous expertise**

$$\beta_i\sigma_i^{-2} + \sum_j \sigma_j^{-2} = \text{constant for all } i$$

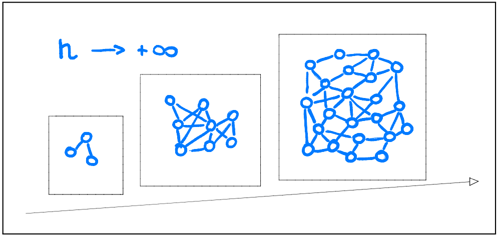
strategies for small groups:

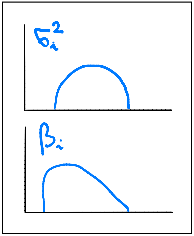
1. facilitate expertise recognition
2. adopt debiasing techniques

strategies for large groups:

1. adopt calibrated skepticism ... $\beta_i = \sum_{j \notin N_i} \sigma_j^{-2} / \sigma_i^{-2}$
2. rewire network to seek expertise

Large population limit





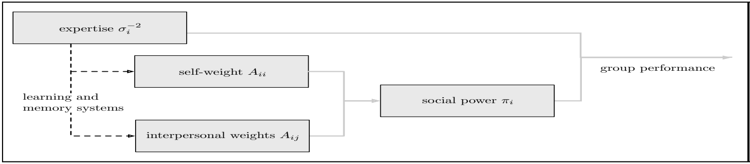
Theorem #3:

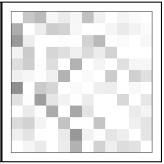
limited inaccuracy and bias: β_i and σ_i^2 have compact support

no dominant individual: $$\lim_{n\to\infty} \frac{\text{max degree}}{\text{sum of degrees}} = 0$$

⇒ mean square estimation error $\to 0$ as $n \to \infty$

B. Golub and M. O. Jackson. Naïve learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics*, 2(1):112–149, 2010





Towards a theory of wise influence systems

1. cognitive foundations of self and interpersonal appraisals
2. influence system based on expertise+bias+topology
3. predictions about
    1. team learning and performance
    2. rational behaviors and interventions
    3. wisdom and fragility in large population

Outline

Towards a theory of wise influence systems

1. F. Bullo, G. Como, F. Fagnani, and N. E. Friedkin. Expertise, appraisals, influence systems, and the wisdom of crowds, 2020. Working paper

   Y. Tian, G. Peluffo, L. Wang., F. Fagnani, and F. Bullo. On the wisdom of crowds problem, 2020. Working paper

Influence system dynamics along memory tasks

2. O. Askarisichani, E. Y. Huang, K. S. Sato, N. E. Friedkin, F. Bullo, and A. K. Singh.

   Expertise and confidence explain how social influence evolves along intellective tasks. *Submitted*, 2020

Experiment Design

- 31 teams of 4 individuals
- 45 memory questions per team (jeopardy)

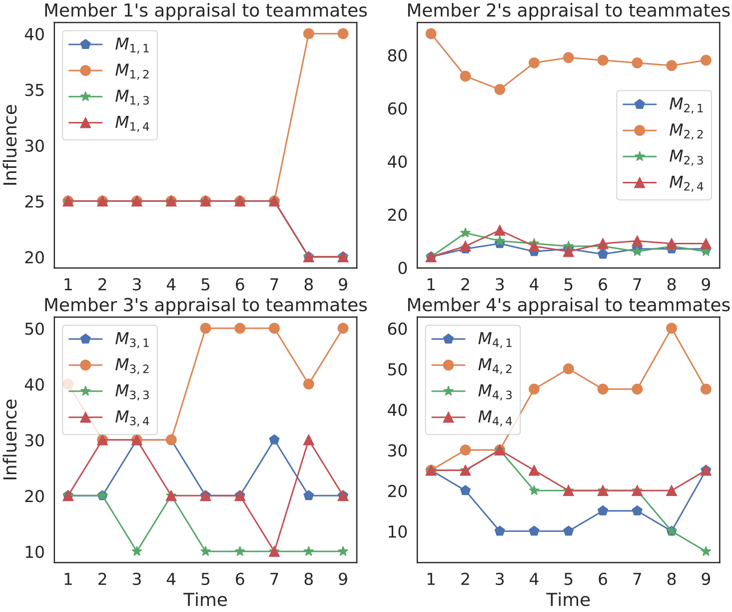
- Experiment design for each question:

each team member answers individually	$\Rightarrow$	individual answers revealed to team	$\Rightarrow$	team communicates over chat to choose 1 answer	$\Rightarrow$	correct answer revealed	$\Rightarrow$	questionnaire after every 5 questions

- Questionnaire after every 5 questions
  - 5 questions = 1 round
  - $t \in \{1, \ldots, 8\}$ denotes round
  - team reports $\hat{A}(t)$
  - performance as cumulative correctness rate

$$p_i(t) = \frac{\#\text{ of correct answers up to round } t}{\#\text{ of questions up to round } t}$$

A selected run



Bases of social power

Hypothesis 1: differentiation according to performance

influence is accorded based on differentiation of performance

if p_j is large, then $a_{ij} \nearrow$ for all i

Hypothesis 2: perceived performance skewed by confidence

individuals with high confidence have a higher perceived performance

if a_{jj} is large, then $a_{ij} \nearrow$ for all j

Hypothesis 3: reversion to the mean for low performers

low performing individuals have diminished ability to recognize experts and show central tendency

if p_i is small, then $a_{ij} \to \frac{1}{n}$ for all j

if p_i is large, then $a_{ij} \to p_j$ for all j

Regression Study Supporting Hypotheses

Hypothesis 1: differentiation according to performance

influence is accorded based on differentiation of performance

if p_j is large, then $a_{ij} \nearrow$ for all i

Hypothesis 2: perceived performance skewed by confidence

individuals with high confidence have a higher perceived performance

if a_{jj} is large, then $a_{ij} \nearrow$ for all j

Regression result on specified variables vs. total influence $\sum_{j=1}^{n} \hat{a}_{ij}$

	Feature-set 1	Feature-set 2	Feature-set 3
Intercept	0.13 ***	0.19 ***	0.12 ***
Performance	**0.20** ***		**0.17** ***
Confidence		**0.14** ***	**0.11** ***

statistical significance is portrayed with *** for $p < 0.01$ and ** for $p < 0.05$

Regression study supporting Hypotheses

Hypothesis 3: reversion to the mean for low performers

low performing individuals have diminished ability to recognize experts in the group with influence values reverting to the mean

if p_i is small, then $a_{ij} \to \frac{1}{n}$ for all j

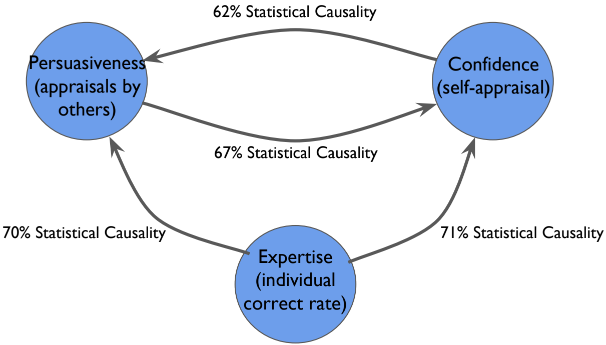
if p_i is large, then $a_{ij} \to p_j$ for all j

Regression result on performance vs. reversion to mean

reversion to mean for $i = \sum_{j=1}^{n} |\hat{a}_{ij} - \frac{1}{n}|^2$

	Regression Coefficients
intercept	0.10 ***
performance	**0.07** **

statistical significance is portrayed with *** for $p < 0.01$ and ** for $p < 0.05$



O. Askarisichani, E. Y. Huang, K. S. Sato, N. E. Friedkin, F. Bullo, and A. K. Singh.

Expertise and confidence explain how social influence evolves along intellective tasks. *Submitted*, 2020

Differentiation Model (D based on H1)

$$a_{ij}(t+1) = (1-\tau)a_{ij}(t) + \tau p_j(t)$$

Differentiation-Reversion Model (DR based on H1+H3)

$$a_{ij}(t+1) = (1-\tau)a_{ij}(t) + \tau\left(p_i(t)p_j(t) + \big(1-p_i(t)\big)\frac{1}{n}\right)$$

Differentiation-Reversion-Perceived Performance (DRP based on H1+H2+H3)

$$\text{perceived performance} \quad \hat{p}_i\big(p(t), A(t)\big) \propto a_{ii}(t)p_i(t)$$

Dynamical Behavior of DRP Model

Differentiation-reversion-perceived performance model:

$$a_{ij}(t+1) = (1-\tau)a_{ij}(t) + \tau\Big(\hat{p}_i(t)\hat{p}_j(t) + \big(1-\hat{p}_i(t)\big)\frac{1}{n}\Big)$$

$$\hat{p}_i(t) = \frac{a_{ii}(t)p_i}{\sum_{k=1}^{n} a_{kk}(t)p_k}$$

with only one parameter: τ

Dynamical Behavior of DRP Model

Reduced order dynamics

$$a_{ii}(t+1) = (1-\tau)a_{ii}(t) + \tau\Big(\hat{p}_i(t)^2 + \big(1-\hat{p}_i(t)\big)\frac{1}{n}\Big)$$

$$a_{ij}(t+1) = T_{ij}\big(A(0), \hat{p}_i(t), \hat{p}_j(t)\big) \qquad i \neq j$$

Existence, uniqueness and attractivity of equilibrium

Consider DRP model with constant accuracy rate $p \in \Delta_n$

- $\exists!$ an equilibrium $A^* \in (0,1)^{n\times n}$
- if additionally $p = \frac{1}{n}\mathbb{1}_n$, then $\lim_{t\to\infty} A(t) = \frac{1}{n}\mathbb{1}_n\mathbb{1}_n^\top$

Properties at equilibrium

$$p_i < p_j \quad \not\Longrightarrow \quad a^*_{ii} < a^*_{jj}$$

$$p_i < p_j \quad \Longrightarrow \quad \lim_{t\to\infty} \hat{p}_i(t) < \lim_{t\to\infty} \hat{p}_j(t) \quad \Longrightarrow \quad \sum_{k=1}^{n} a^*_{ki} < \sum_{k=1}^{n} a^*_{kj}$$

Error measure between reported and estimated matrix
Kullback-Leibler (KL) Divergence

$$\mathrm{KL}(\hat{A}, A) = \frac{1}{n} \sum_{i=1}^{n} \Big(\sum_{j=1}^{n} \hat{a}_{ij} \log \frac{\hat{a}_{ij}}{a_{ij}} \Big)$$

Model Validation

Multi-Round forecast

Input: $\hat{A}(0)$ and cumulative performance $p(t)$ from data

For $t \in \{0, \ldots, N-1\}$:

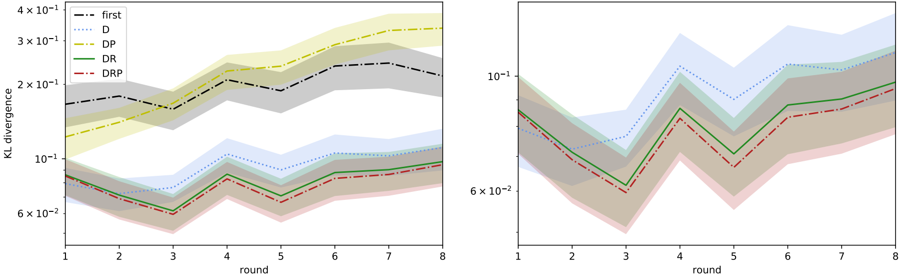
If t $= 0$: $\quad \hat{p}(1) = \hat{p}(0, p(0), \hat{A}(0))$

$$a_{ij}(1) = (1-\tau)\hat{a}_{ij}(0) + \tau\left(\hat{p}_i(0)\hat{p}_j(0) + (1-\hat{p}_i(0))\frac{1}{n}\right)$$

Else: $\quad \hat{p}_i(t) = \hat{p}_i(t, p(t), A(t))$

$$a_{ij}(t+1) = (1-\tau)a_{ij}(t) + \tau\left(\hat{p}_i(t)\hat{p}_j(t) + (1-\hat{p}_i(t))\frac{1}{n}\right)$$

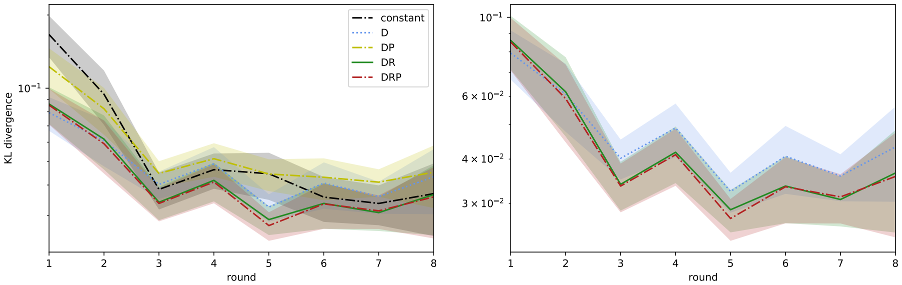
Output: $A(t+1) =$ estimate of $\hat{A}(t+1)$



Model Validation

Single-Round forecast

Input: $\hat{A}(t)$ and cumulative performance $p(t)$ from data
For $t \in \{0, \dots, N-1\}$:
$\hat{p}_i(t) = \hat{p}_i(t, p(t), \hat{A}(t))$
$a_{ij}(t+1) = (1-\tau)\hat{a}_{ij}(t) + \tau\left(\hat{p}_i(t)\hat{p}_j(t) + \left(1 - \hat{p}_i(t)\right)\frac{1}{n}\right)$
Output: $A(t+1) =$ estimate of $\hat{A}(t+1)$



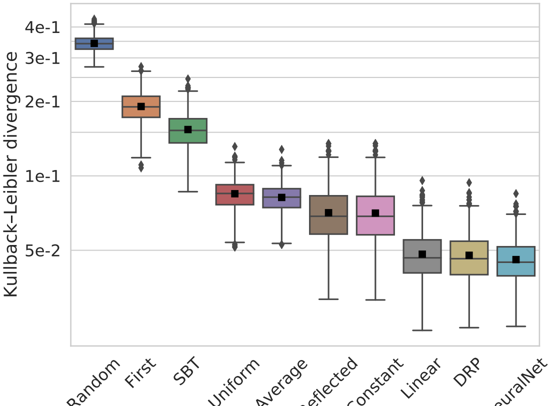
constant model $a_{ij}(t+1) = \hat{a}_{ij}(t)$

Model Validation

Single-Round forecast

Additional inputs for machine-learning models (Linear, NeuralNet)

- communication logs (time & content)
- trained on first 80



Random	$a_{ij}(t+1) = \text{rand}$
First	$a_{ij}(t+1) = \hat{a}_{ij}(0)$
SBT	$a_{ij}(t+1) = \sum_{k=1}^{n} \hat{a}_{ik}(t)\hat{a}_{kj}(t)$
Uniform	$a_{ij}(t+1) = \frac{1}{n}$
Average	$a_{ij}(t+1) = \text{ avg over data}$
Reflected	$\Delta p_i = p_i(t) - \sum_{k=1}^{n} \hat{a}_{ik}(t)p_k(t)$
	$a_{ii}(t+1) = \hat{a}_{ii}(t)(1 + (1 - \hat{a}_{ii}(t))\Delta p_i)$
	$a_{ij}(t+1) = \hat{a}_{ij}(t)(1 - \hat{a}_{ii}(t)\Delta p_i)$
Constant	$a_{ij}(t+1) = \hat{a}_{ij}(t)$

Outline

1.

Towards a theory of wise influence systems

F. Bullo, G. Como, F. Fagnani, and N. E. Friedkin. Expertise, appraisals, influence systems, and the wisdom of crowds, 2020. Working paper

Y. Tian, G. Peluffo, L. Wang., F. Fagnani, and F. Bullo. On the wisdom of crowds problem, 2020. Working paper

2.

Influence system dynamics along memory tasks

O. Askarisichani, E. Y. Huang, K. S. Sato, N. E. Friedkin, F. Bullo, and A. K. Singh.

Expertise and confidence explain how social influence evolves along intellective tasks. *Submitted*, 2020

Summary: From expertise and confidence to social influence

Towards a theory of wise influence systems	Influence system dynamics along memory tasks
static model	dynamic model
estimation task	memory task
describe rational influence	predict influence

Future work

1. empirically-validated mathematical models
2. nexus of expertise, influence systems, social power and performance
3. scope: intellective issues of various types
4. scope: small groups vs organizations vs online networks