

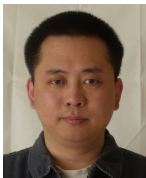
Opinion Dynamics and the Evolution of Influence Networks



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2014 SoCal Symp on Network Economics and Game Theory



Peng Jia

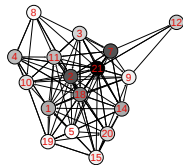


Ana MirTabatabaei



Noah Friedkin

- drivers
 - “big data” increasingly available
 - quantitative methods in social sciences
 - applications in marketing and (in)-security
- dynamical processes over social networks
 - opinion dynamics, info propagation
 - network formation and evolution
 - co-evolutionary processes
- key novelty: sequence of issues



Krackhardt's advice network

Small deliberative groups

- small deliberative groups are assembled in most social organization to deal with sequences of issues in particular domains:
 - judicial, legislative and executive branches: grand juries, federal panels of judges, Supreme Court – standing policy bodies, congressional committees – advisory boards
 - corporations: board of directors/trustees
 - universities: faculty meetings
- group properties may evolve over its issue sequence according to natural social processes that modify its internal social structure
- possible systematic changes:
 - ① a stabilization of individuals' levels of openness and closure to interpersonal influences on their initial preferences,
 - ② a stabilization of individuals' ranking of, and influence accorded to, other members'

Opinions, influence networks and centrality

Dynamics and Formation of Opinions

- convex combinations of opinions
- model by French ('56), Harary ('65), and DeGroot ('74)

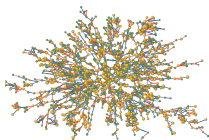


Opinion formation

Dynamics of Influence Networks and Social Power

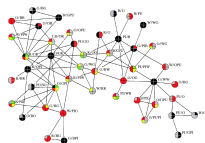
- reflected appraisal hypothesis by Cooley, 1902

individual' self-appraisal (e.g., self-confidence, self-esteem, self-worth) is influenced by the appraisal of other individuals of her



Social network for obesity study (Christakis and Fowler, 2007)

- mathematization by Friedkin, 2012:
 - varying social power and self-confidence
 - constant relative interpersonal relations



Social network for male wire-tailed manakins (Ryder et al. 2008)

Network centrality

- centrality measure of network nodes, e.g., eigenvector centrality by Bonacich, 1972

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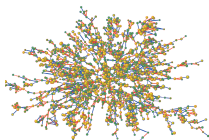


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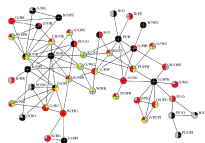
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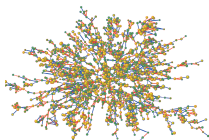


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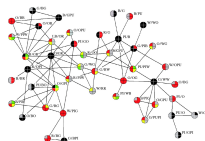
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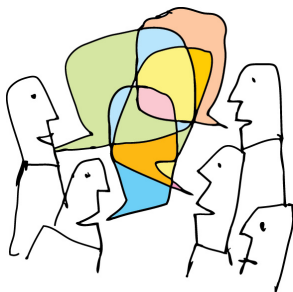
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The dynamics of opinions

DeGroot opinion dynamics model

$$y(t+1) = W y(t)$$

- Opinions $y \in \mathbb{R}^n$
- Influence network = row-stochastic W
- by P-F: $\lim_{t \rightarrow \infty} y(t) = (w^T y(0)) \mathbf{1}_n$
where w is dominant left eigenvector of W
- Self-weights $W_{ii} =: x_i$
- Interpersonal accorded weights W_{ij}
- Relative interpersonal accorded weights C_{ij} ,
where $W_{ij} = (1 - x_i) C_{ij}$



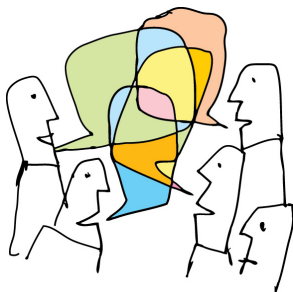
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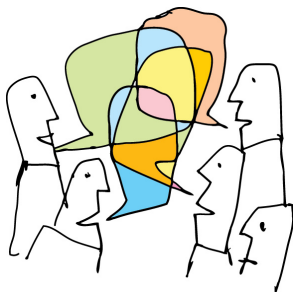
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The dynamics of social power and self-confidence

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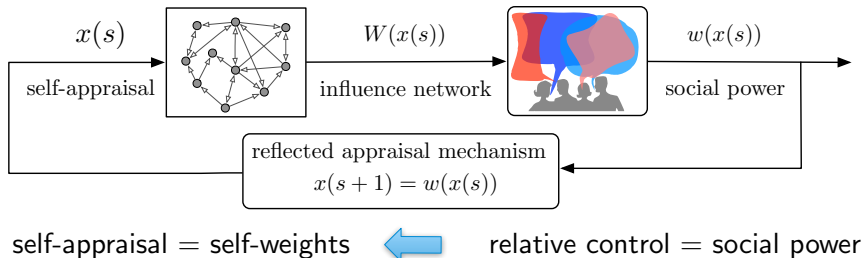
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The dynamical system

- DeGroot dynamics about an issue: $y(t+1) = W(x)y(t)$
- Influence network $W(x) = \text{diag}(x)I_n + \text{diag}(\mathbf{1}_n - x)C$
- Reflected appraisal across issues:

$$x(k+1) = w(x(k)) = F(x(k))$$

DeGroot-Friedkin dynamics

$$F(x) = \begin{cases} e_i, & \text{if } x = e_i \text{ for all } i \\ \left(\frac{c_1}{1-x_1}, \dots, \frac{c_n}{1-x_n} \right) / \sum_{i=1}^n \frac{c_i}{1-x_i}, & \text{otherwise} \end{cases}$$

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The map and the eigenvector centrality parameter

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- $F : \Delta_n \rightarrow \Delta_n$ locally Lipschitz
- The vertices $\{e_i\}$ are fixed points under F
- relative interpersonal weights C play role only through c
- c = appropriate eigenvector centrality (dominant left eigenvector)

Lemma (Eigenvector centrality)

For any C row-stochastic, irreducible with zero diagonal and $c \in \Delta_n$,

- $\max\{c_i\} \leq 0.5$
- $c_i = 0.5 \iff G(C)$ is with star topology and i is the center

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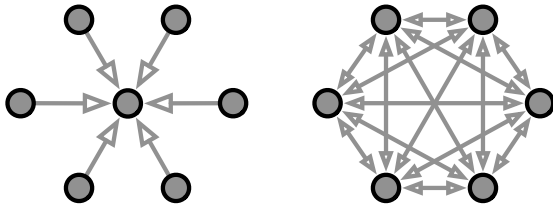
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Problem: dynamical system analysis and sociological interpretation



- Existence and stability of equilibria for the D-F model?
- Role of network structure and parameters?
- Conditions of emergence of *autocracy* and *democracy*?
- Insight into “iron law of oligarchy” by Michels 1915?

Main results

for generic “relative interpersonal accorded weights”

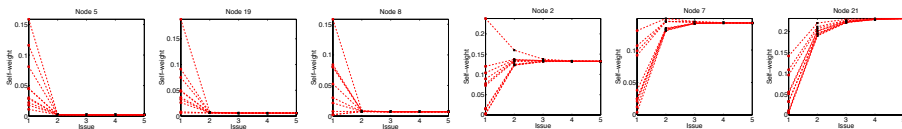
① **unique non-trivial fixed point:** $x^* = x^*(c)$ in interior of Δ_n

② **convergence = forgetting initial conditions**
for all non-trivial initial conditions,

$$\lim_{k \rightarrow \infty} x(k) = \lim_{k \rightarrow \infty} w(x(k)) = x^*$$

③ **accumulation of social power and self-appraisal**

- fixed point $x^* > 0$ has same ordering of c
- social power threshold T such that: $x_i^* \geq c_i \geq T$ or $x_i^* \leq c_i \leq T$



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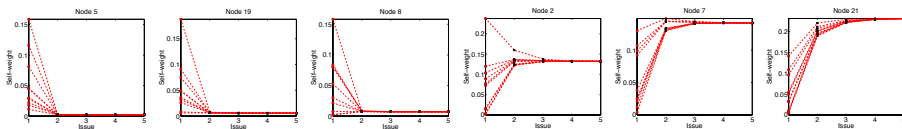
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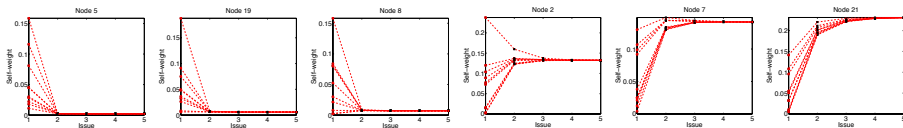
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Doubly-stochastic C : emergency of democracy

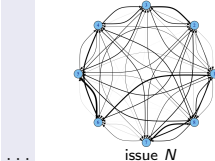
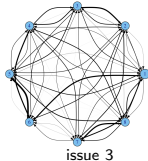
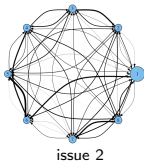
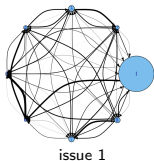
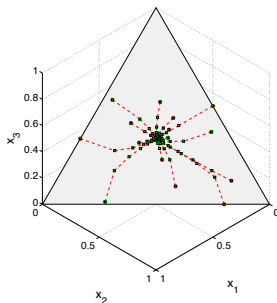
Lemma (Convergence to democracy)

Iff C is doubly-stochastic:

- 1 the non-trivial fixed point of F is $\frac{\mathbb{1}_n}{n}$,
- 2 for all non-trivial initial conditions,
 $\lim_{k \rightarrow \infty} x(k) = \lim_{k \rightarrow \infty} w(x(k)) = \frac{\mathbb{1}_n}{n}$.

- Uniform social power
- No power accumulation

$$c = [1/3, 1/3, 1/3]$$



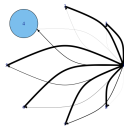
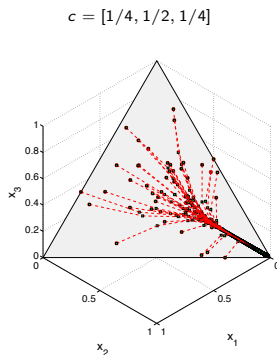
Star topology: emergency of autocracy

Lemma (Convergence to autocracy)

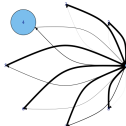
Iff graph has star topology with center j :

- ❶ *there are no non-trivial fixed points of F*
- ❷ *for all initial non-trivial conditions,*
 $\lim_{k \rightarrow \infty} x(k) = \lim_{s \rightarrow \infty} w(x(k)) = e_j.$

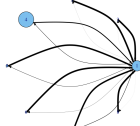
- Autocrat appears in star center
- Extreme power accumulation



issue 1

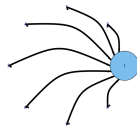


issue 2



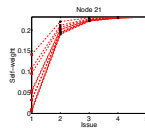
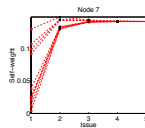
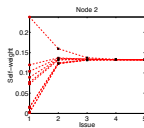
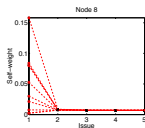
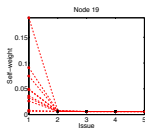
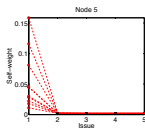
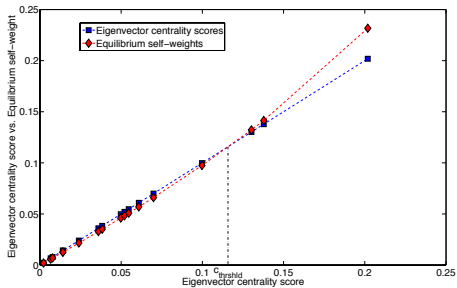
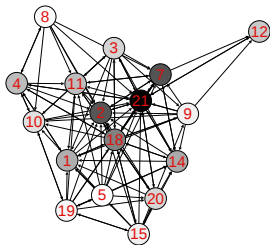
issue 3

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issue N

D-F on Krackhardt's advice network



- ① existence via Brouwer fixed point theorem (F continuous on compact)
- ② ranking and uniqueness: elementary steps and contradictions
- ③ monotonicity: i_{\max} and i_{\min} are invariant

$$i_{\max} = \operatorname{argmax}_j \frac{x_j(0)}{x_j^*} \implies i_{\max} = \operatorname{argmax}_j \frac{x_j(s)}{x_j^*}, \quad \forall s$$

- ④ convergence: Lyapunov function decreasing everywhere $x \neq x^*$

$$V(x) = \max_j \left(\ln \frac{x_j}{x_j^*} \right) - \min_j \left(\ln \frac{x_j}{x_j^*} \right)$$

Ongoing experiment

- 30 groups of 4 subjects in a face-to-face discussion
- opinion formation on a sequence of 15 issues
- issues in the domain of choice dilemmas:
what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff
- 15 groups under pressure to reach consensus, other 15 no
- On each issue, each subject privately recorded (in following temporal order):
 - 1 an initial opinion on the issue prior to the group-discussion,
 - 2 a final opinion on the issue upon completion of the group-discussion (which ranged from 3-27 minutes), and
 - 3 an allocation of 100 influence units (under the instruction that these allocations should represent their appraisals of the relative influence of each group member's opinion on their own opinion).

Contributions

- a new perspective and a novel dynamical model
for *social power, self-appraisal, influence networks*
- dynamics and feedback in sociology
- a new potential explanation for the emergence of autocracy
see “iron law of oligarchy” by Michels 1911

Future work

- Robustness of results for distinct models of opinion dynamics
- Robustness of results for higher-order models of reflected appraisal

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Funding: Institute for Collaborative Biotechnology through grant W911NF-09-D-0001 from the U.S. Army Research Office

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