Opinion Dynamics

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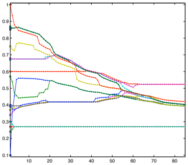

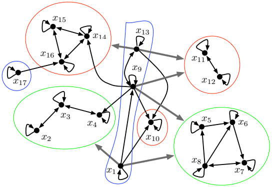

Motivations

- Economics (Sznajd-Weron, 2002)
- Politics (Ben-Naim, 2005)
- Physics (Ben-Naim et al., 2003)
- Sociology (Tavares, 2007)

Can we convince others to follow us?

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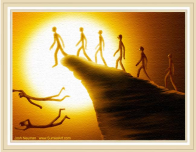
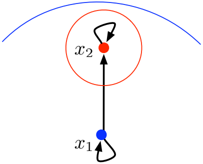


Can we convince others to follow us?

Synchronized Bounded Confidence (SBC) Model

- n agents, with n bounds of confidence $\{r_1, \ldots, r_n\}$
- agent i has opinion $x_i(t) \in \mathbb{R}$
- j is i's out-neighbor if $|x_i(t) - x_j(t)| \leq r_i$

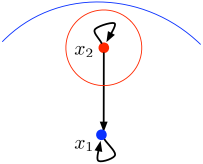


$$x(t+1) = A(x(t))x(t)$$

$$a_{ij}(x(t)) = \begin{cases} \dfrac{1}{\#\text{ of } i\text{'s neighbors}} & \text{if } j \text{ is } i\text{'s out-neighbor} \\ 0 & \text{otherwise} \end{cases}$$

Synchronized Bounded Influence (SBI) Model

- n agents, with n bounds of confidence $\{r_1, \ldots, r_n\}$
- agent i has opinion $x_i(t) \in \mathbb{R}$
- j is i's out-neighbor if $|x_i(t) - x_j(t)| \leq r_j$



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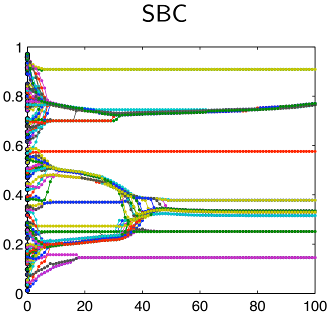
For equal initial opinion vector and bounds vector, an SBC system exhibits more complex behavior than an SBI system

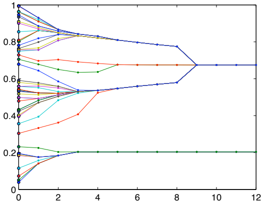


- System with homogeneous bounds

  converges in finite time

  the order of opinions is preserved

  in steady state agents are in agreement or disconnected

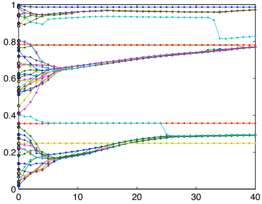


- System with heterogeneous bounds

  may converge in infinite time

  pseudo-stable configurations

  disconnected clusters may reconnect

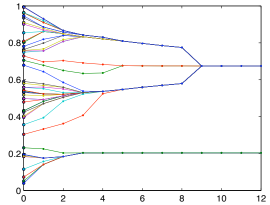


- Homogeneous SBC (Hegselman-Krause) Model

  time complexity of convergence (Bullo et al. 2007)

  consensus (Olshevsky et al. 2009)

  final opinion vector (Blondel et al. 2009)

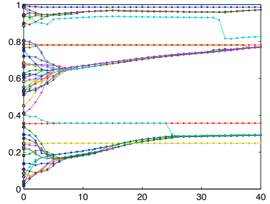


- Heterogeneous SBC (Hegselman-Krause) Model

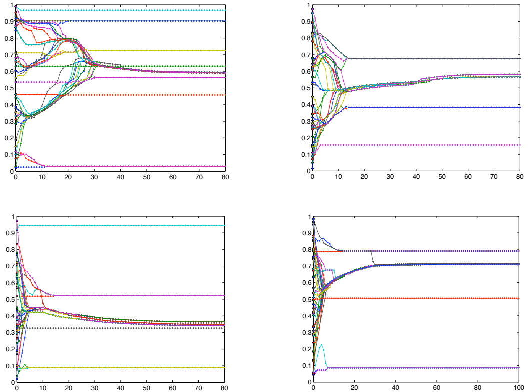
  enhances the chances for consensus (Lorenz 2008)

  classification of agents (Lorenz 2006)

  product of infinite adjacency matrices (Lorenz 2006)



Conjectures



when does convergence start?

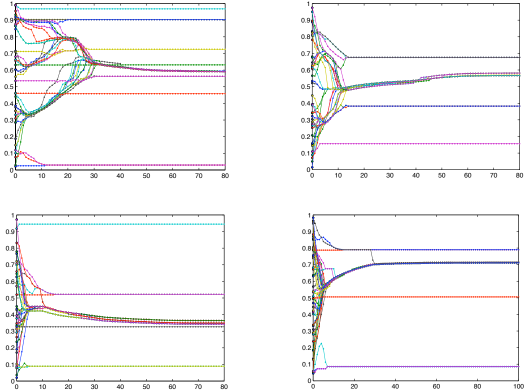
what is the final value?

who moves and who stops?

rate and direction of moving ones?

do all SBC and SBI systems converge?

Conjectures



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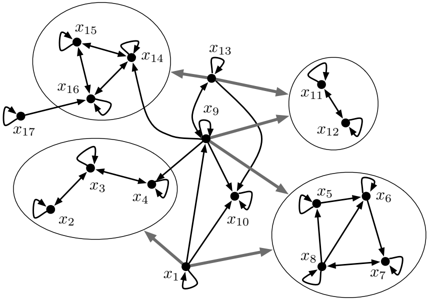
Classification of Agents



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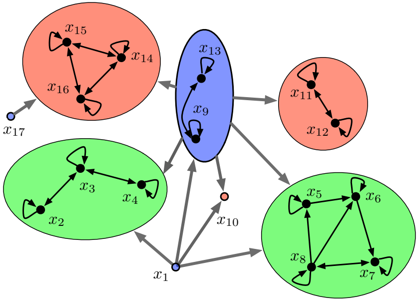

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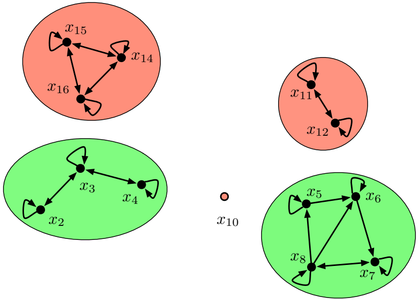
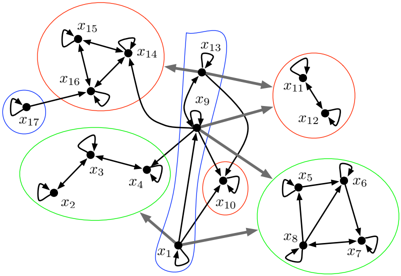


Classification of Agents

Open-minded

Moderate-minded

Closed-minded



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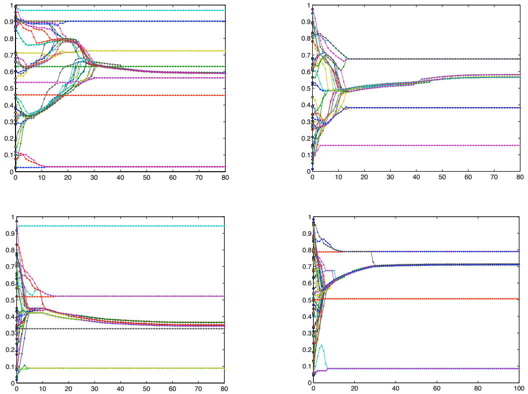

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$$A(x) = \begin{bmatrix} C & 0 & 0 \\ 0 & M & 0 \\ \Theta_C & \Theta_M & \Theta \end{bmatrix}$$

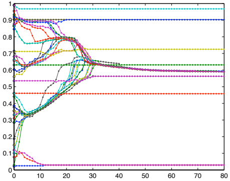
Final value at constant topology

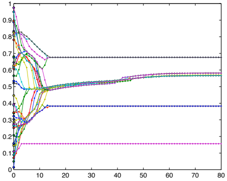
$$\begin{aligned} x^*(x) &= \lim_{t \to \infty} A(x)^t x \\ &= \begin{bmatrix} C & 0 & 0 \\ 0 & M^* & 0 \\ (I - \Theta)^{-1}\Theta_C C & (I - \Theta)^{-1}\Theta_M M^* & 0 \end{bmatrix} x \end{aligned}$$

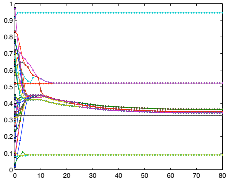
If $A(x^*(x)) = A(x)$, then

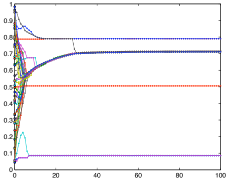
- $x^*(x)$ is an equilibrium vector
- their proximity graph contains no moderate-minded

Conjectures









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Equi-topology neighborhood of $z \in \mathbb{R}^n$

The set of vectors $y \in \mathbb{R}^n$ such that for all $i \in \{1, \ldots, n\}$

$$\begin{aligned} |y_i - z_i| &< \epsilon_i(z), \qquad \text{if } \epsilon_i(z) > 0, \text{and} \\ |y_i - z_i| &= \epsilon_i(z), \qquad \text{if } \epsilon_i(z) = 0, \end{aligned}$$

where

$$\epsilon_i(z) = 0.5 \min\{||z_i - z_j| - R| \mid j \in \{1, \ldots, n\} \setminus \{i\}, R \in \{r_i, r_j\}\}.$$

Invariant equi-topology neighborhood of $z \in \mathbb{R}^n$

The set of vectors $y \in \mathbb{R}^n$ such that for all $i \in \{1, \ldots, n\}$

$$\begin{aligned} |y_i - z_i| &< \delta_i(z), \qquad \text{if } \delta_i(z) > 0, \text{and} \\ |y_i - z_i| &= \delta_i(z), \qquad \text{if } \delta_i(z) = 0. \end{aligned}$$

where

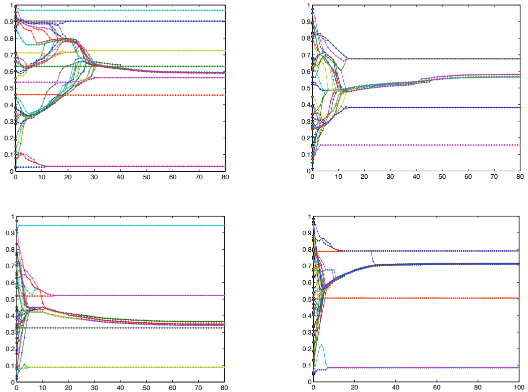
$$\delta_i(z) = \min\{\epsilon_j(z) \mid j \text{ is a predecessor of } i\}.$$

Theorem (Sufficient condition for constant topology and convergence)

Consider a trajectory $x(t)$ of an SBC or SBI system. Assume that there exists an equilibrium opinion vector $z \in \mathbb{R}^n$ for the system such that $x(0) \in \mathbb{R}^n$ belongs to the invariant equi-topology neighborhood of z. Then, for all $t \geq 0$:

1. *$x(t)$ takes value in the equi-topology neighborhood of z;*
2. *the proximity digraphs $G_r(x(t))$ and $G_r(z)$ are equal;*
3. *$G_r(x(t))$ contains no moderate-minded component; and*
4. *$x(t)$ converges to the final value at constant topology of $x(0)$ as $t \to \infty$.*

Conjectures



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Rate of Convergence

Agent's per-step convergence factor

$$k_i(x(t)) = \frac{x_i(t+1) - x_i^*(x(t))}{x_i(t) - x_i^*(x(t))}$$

$$k_i = 1 - \text{ rate of convergence of agent } i$$

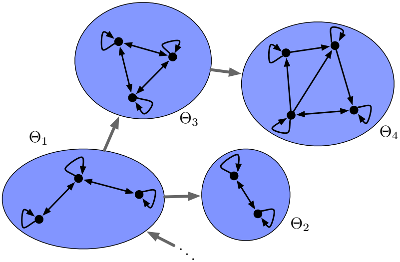
$$\text{monotonic convergence toward final value} \quad \equiv \quad 0 \leq k_i \leq 1$$

Theorem (Evolution under Constant Topology after τ)

- *$x(t)$ converges to $x^*(x(\tau))$*
- *no moderate minded at τ*
- *if $\rho(\Theta_4) \geq \rho(\Theta_{1,2,3})$, then for all $i \in G_{\Theta_1}$ and $j \in G_{\Theta_4}$*

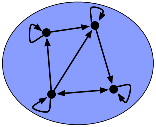
  a) $\lim_{t\to\infty} k_i(x(t)) = \rho(\Theta_4)$

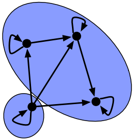
  b) there exists $T \geq \tau$, after which $(x_i(t) - x_i^*)(x_j(t) - x_j^*) \geq 0$



In a Real Society

- the initial opinion of an open-minded has no effect, $x^* = \begin{bmatrix} C & 0 \\ \Theta_C^* & 0 \end{bmatrix} x.$
- an individual converges to his final decision as slow as the slowest group.
- the leader govern followers direction and rate.
- one can become a leader by joining a large strongly connected group:





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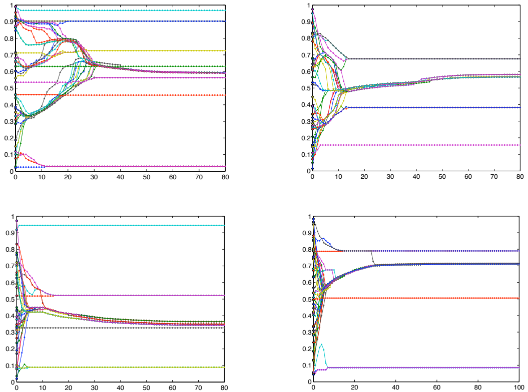

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Future Work

- Convergence of SBC and SBI systems
  1. what is the basin of attraction of x^*?
  2. what are possible x^*'s for a system?
  3. does the system fall into one of those basins of attraction?

Theorem (Convergence of products of stochastic matrices, Lorenz '06)

$$\lim_{t \to \infty} A(t,0) = \begin{bmatrix} C_1 & & & \\ & \ddots & & \\ & & C_m & \\ & ? & & 0 \end{bmatrix} A(t_0, 0)$$

- How can one become a leader by changing his r?