

Current Epidemiological Models: Scientific Basis and Evaluation



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Issues, Approaches, and Consequences of the COVID-19 Crisis

<https://www.cits.ucsb.edu/spring2020>



With COVID-19,
modeling takes on life and death importance

"But on March 16th, the Imperial College group published a dramatically revised model that concluded [...] that even a reduced peak would fill twice as many intensive care beds as estimated previously." Science, March 27th

The screenshot shows a news article from Science magazine. The headline is "With COVID-19, modeling takes on life and death importance". The article discusses how a revised model by the Imperial College group predicted a much higher peak in intensive care admissions than previously estimated. The article includes quotes from experts like Anthony Fauci and Jon Cohen, and features a photograph of medical staff in protective gear moving a patient on a stretcher.

Acknowledgments

Terrible impact of this pandemic

The complex, dangerous, critical work by healthcare professionals all over the world on the front line of this battle

All essential frontline workers, including first responders, grocery-store workers, and transit workers

We owe them all a great deal of gratitude

Relevant links:

- "Life and Death in the 'Hot Zone'" (article and video) by Nicholas Kristof, New York Times, 4/11/2020
<https://nyti.ms/3a1GATB>
- Webinar by Dr Carolina Arias Gonzales (UCSB MCDB) and Dr Lynn N. Fitzgibbons (Cottage Health), 4/14/2020
<https://www.cits.ucsb.edu/spring2020>

Outline

- ➊ historical notes
- ➋ introduction to mathematical epidemiology
 - ➌ the simplest SIR model
 - ➍ stochastic SIR models
 - ➎ direct statistical estimation
- ➏ summary evaluation
- ➐ conclusion on non-pharmaceutical interventions (NPIs)

Warnings: elementary intro, no new model

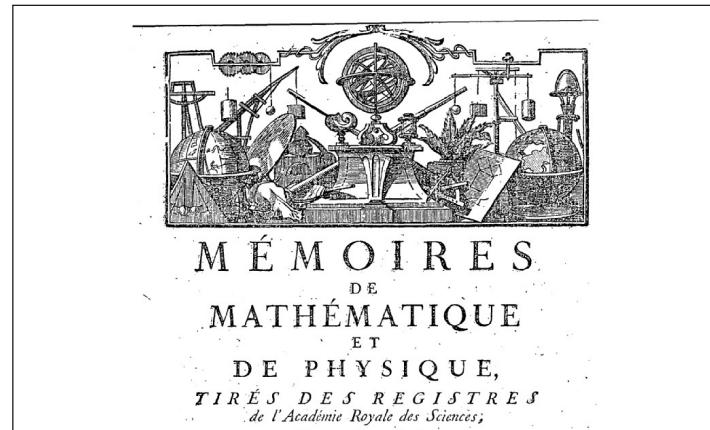
My qualifications:

- F. Bullo. *Lectures on Network Systems*. Kindle Direct Publishing, 1.3 edition, July 2019.
URL: <http://motion.me.ucsb.edu/book-lns>
- W. Mei, S. Mohagheghi, S. Zampieri, and F. Bullo. On the dynamics of deterministic epidemic propagation over networks. *Annual Reviews in Control*, 44:116–128, 2017.
doi:10.1016/j.arcontrol.2017.09.002

Daniel Bernoulli 1760: controversial smallpox variolation

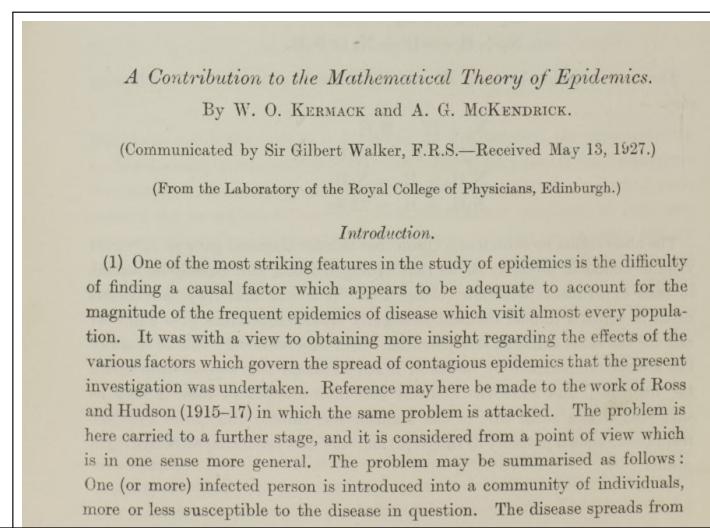
- "the greatest killer in history"
- variolation, i.e., inoculation with a mild strain
- controversy: long-term benefit vs risk of immediate death

using empirical data, mathematical proof that inoculation could increase life expectancy at birth up to three years



Kermack and McKendrick 1927: epidemic thresholds and outbreaks

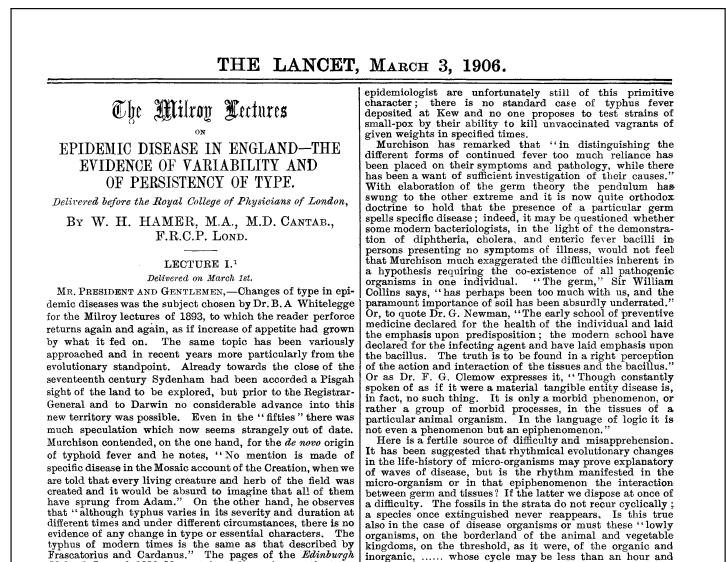
- **epidemic threshold:** the density of susceptibles must exceed a critical value in order for an **epidemic outbreak** to occur
- differential equations, calculus



(1) One of the most striking features in the study of epidemics is the difficulty of finding a causal factor which appears to be adequate to account for the magnitude of the frequent epidemics of disease which visit almost every population. It was with a view to obtaining more insight regarding the effects of the various factors which govern the spread of contagious epidemics that the present investigation was undertaken. Reference may here be made to the work of Ross and Hudson (1915-17) in which the same problem is attacked. The problem is here carried to a further stage, and it is considered from a point of view which is in one sense more general. The problem may be summarised as follows: One (or more) infected person is introduced into a community of individuals, more or less susceptible to the disease in question. The disease spreads from

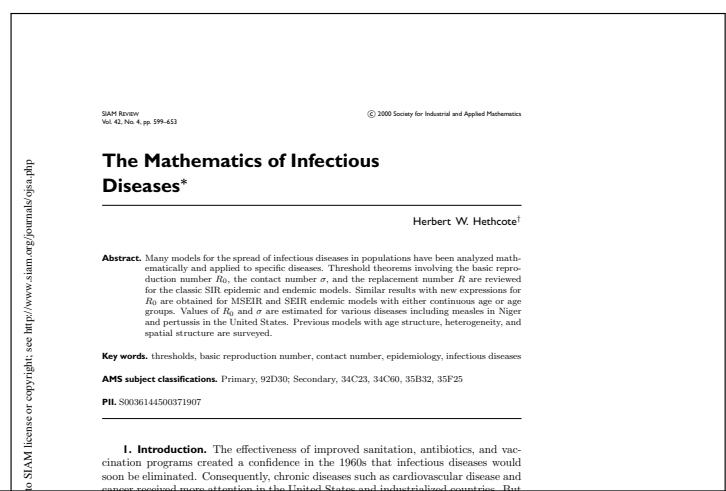
W. Hamer 1906: nonlinear incidence

- **compartments:** S, I and R
- incidence = number of new cases per unit time
depends on the **product of the densities of S and I**



Hethcote's leading survey in 2000

motivated by a range of infectious diseases and outbreaks,
one thousand and one models have been analyzed mathematically
e.g., models with age structure, heterogeneity, and spatial structure
threshold theorems for **epidemic outbreaks**



- Daniel Bernoulli. Essai d'une nouvelle analyse de la mortalité causée par la petite vérole, et des avantages de l'inoculation pour la prévenir. *Mémoires de Mathématiques et de Physique, Académie Royale des Sciences*, pages 1–45, 1760
- W. H. Hamer. On epidemic disease in England. *The Lancet*, 167(4305):569–574, 1906. doi:10.1016/S0140-6736(01)80187-2
- W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society A*, 115:700–721, 1927. doi:10.1098/rspa.1927.0118
- N. T. J. Bailey. *The Mathematical Theory of Infectious Diseases*. Griffin, 1957
- H. W. Hethcote. The mathematics of infectious diseases. *SIAM Review*, 42(4):599–653, 2000. doi:10.1137/S0036144500371907

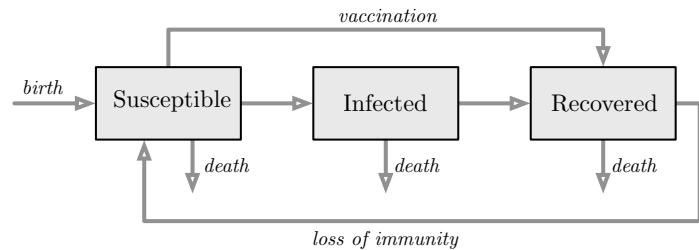
Concept #1: Compartmental Models

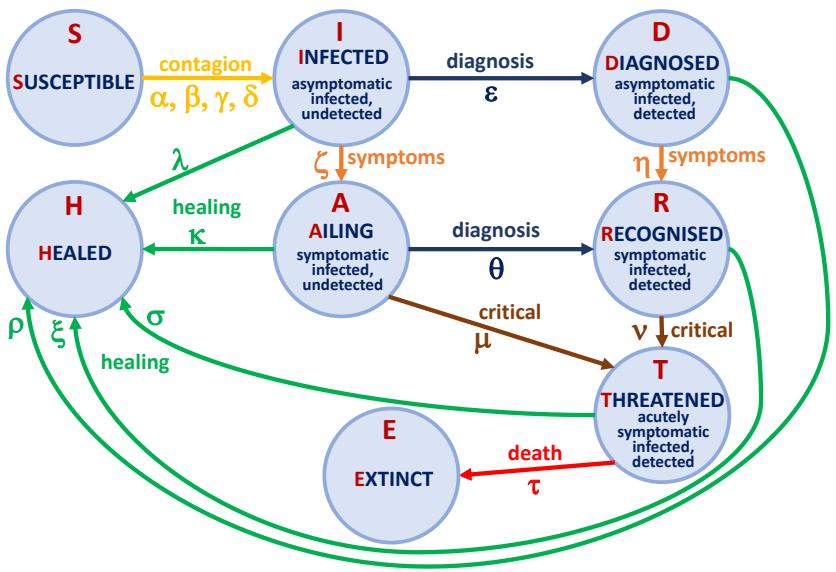
each individual is in one of multiple possible states:



Two types of transitions:

- ① $S \rightarrow I$: interaction between a susceptible and an infected
- ② $I \rightarrow R$: spontaneous, independent of interactions





G. Giordano, F. Blanchini, R. Bruno, P. Colaneri, A. Di Filippo, A. Di Matteo, and M. Colaneri. A SIDARTHE model of COVID-19 epidemic in Italy, 2020. Arxiv preprint. URL: <https://arxiv.org/pdf/2003.09861>

Scope of simplest SIR model

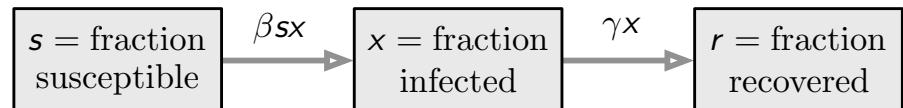
In a population of n individuals, on average:

- ➊ contacts between uniformly randomly selected individuals
 - **contact rate** $\beta_c > 0$ so that
during $(t, t + \Delta t)$, $n\beta_c\Delta t$ individuals meet other $n\beta_c\Delta t$
i.e., each individual meets $\beta_c\Delta t$
 - **transmission fraction** $0 < \beta_t < 1$ resulting in infection
- ➋ **recovery rate** $\gamma > 0$ so that
during $(t, t + \Delta t)$, $n\gamma\Delta t$ individuals recover
i.e., **infective period** $= 1/\gamma$

Therefore, on average

$$\frac{x(t + \Delta t) - x(t)}{\Delta t} = + \underbrace{2\beta_t\beta_c}_{\text{rate } \beta} \underbrace{x(t)s(t)}_{\text{Hamer's product}} - \gamma x(t)$$

Concept # 2: Simplest SIR model



differential equation = fundamental mechanism to compute an evolution

given infection rate β and recovery rate γ ,
given initial values $s(0)$, $x(0)$, $r(0)$:

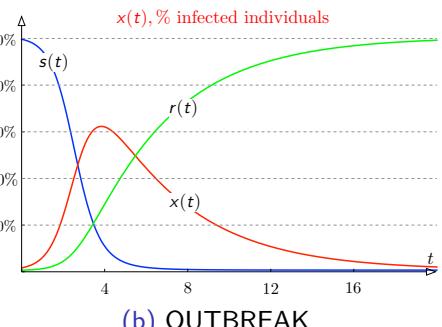
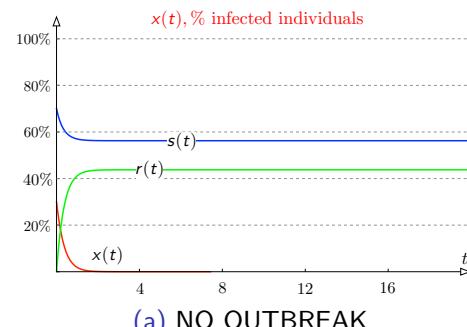
$$\dot{s} = -\beta sx$$

$$\dot{x} = \beta sx - \gamma x$$

$$\dot{r} = \gamma x$$

Predictions of simplest SIR model

$$\begin{aligned} \dot{s} &= -\beta sx \\ \dot{x} &= \beta sx - \gamma x = \gamma \left(\frac{\beta}{\gamma} s - 1 \right) x \\ \dot{r} &= \gamma x \end{aligned}$$



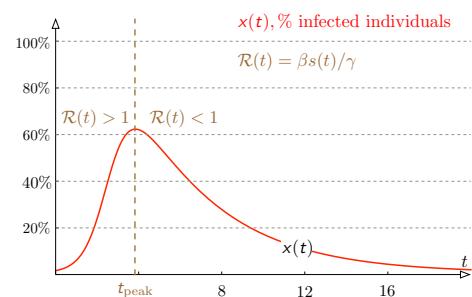
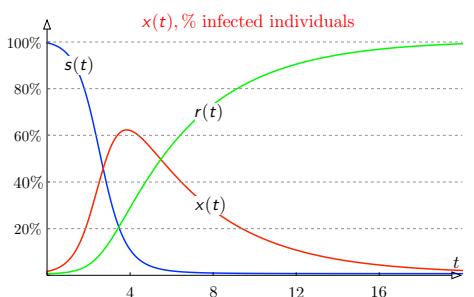
Concept #3: Reproduction number and epidemic threshold

Basic reproduction number \mathcal{R}_0 = expected number of secondary cases produced by a typical infective individual, at start of epidemic

$$\mathcal{R}_0 = \beta \times 1/\gamma \times s(0)$$

$$\approx ((\text{contacts/day}) \times (\text{transmission})) \times (\text{infective days}) \times s(0)$$

$\mathcal{R}_0 > 1 \implies \text{exponential growth}$



Question 1: what are individual factors in \mathcal{R}_0 ? For thought experiments – without further evidence – imagine

$$\underbrace{\mathcal{R}_0}_{2.5 \text{ persons}} \approx \underbrace{((\text{contacts/day}) \times (\text{transmission}))}_{2 \text{ persons/day}} \times \underbrace{(\text{infective days})}_{25\%} \times \underbrace{s(0)}_{5 \text{ days}} = 100\%$$

Question 2: how to compute the doubling time? While $s \approx 1$,

$$t_{\text{doubling}} \approx \frac{\ln(2)}{(\beta - \gamma)} = \frac{\ln(2)}{1/2 - 1/5} \approx 2.3 \text{ days}$$

Question 3 (Herd Immunity): what percentage of the population x^* needs to have immunity in order for $\mathcal{R}(t) = 1$? Assume all population is susceptible $s(0) = 100\%$, then

$$1 = \mathcal{R}(t) = \mathcal{R}_0 s(t^*) \implies x^* = 1 - s(t^*) = 1 - \frac{1}{\mathcal{R}_0} = 60\%$$

Speculations about uncertain COVID-19 parameters

Values before: social distancing, other NPI measures, and fear

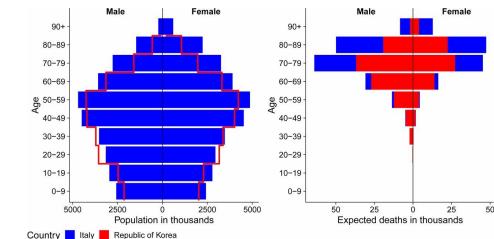
Quantity	Value	Explanation
\mathcal{R}_0	2.2-2.7 persons	Highly dependent upon region, age group, etc. Some estimates are much higher. (source: wikipedia)
incubation period	5 days	(median), between exposure and first symptoms, 97.5% before 12 days. (source: wikipedia)
infective period $1/\gamma$	5 days	"people can test positive for COVID-19 from 1-3 days before they develop symptoms" (source: Report WHO China Joint Mission). includes asymptomatic infective people.
doubling time	2-7 days	(source: Imperial College report and "Epidemic doubling time of the COVID-19 epidemic by Chinese province")
asymptom cases	5% - 80%	

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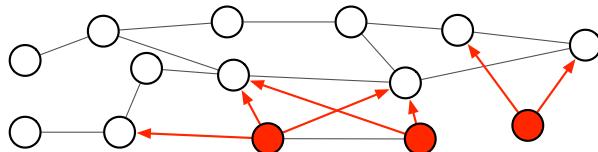
(c) Heterogeneity by spatial position



(d) Heterogeneity by age structure

More accurate models: Structured Multi-group SIR

n = number of homogeneous groups in heterogeneous population
based on spatial position, age, social behavior



- ① for each group, s_i susceptible, x_i infected, or r_i recovered
- ② heterogeneous recovery rate γ_i
- ③ heterogeneous meeting/contact rate $(\beta_c)_{ij}$ between i and j

$$\dot{x} = \beta s x - \gamma x \implies \dot{x}_i = \sum_{j=1}^n \beta_t (\beta_c)_{ij} s_i x_j - \gamma_i x_i$$

Parameters: infection matrix $\beta_t \beta_c$, recovery rates γ_i

N. M. Ferguson et al. Impact of non-pharmaceutical interventions (NPIs) to reduce COVID19 mortality and healthcare demand. Technical report, Imperial College, March 2020. [doi:10.25561/77482](https://doi.org/10.25561/77482)

16 March 2020

Imperial College COVID-19 Response Team

Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand

Neil M Ferguson, Daniel Laydon, Gemma Nedjati-Gilani, Natsuko Imai, Kylie Ainslie, Marc Baguelin, Sangeeta Bhatia, Adhirath Boonyasiri, Zulma Cucunubá, Gina Cuomo-Dannenburg, Amy Dighe, Ilaria Dorigatti, Han Fu, Katy Gaythorpe, Will Green, Arran Hamlet, Wes Hinsley, Lucy C Okell, Sabine van Elsland, Hayley Thompson, Robert Verity, Erik Volz, Haowei Wang, Yuanrong Wang, Patrick GT Walker, Caroline Walters, Peter Winskill, Charles Whittaker, Christl A Donnelly, Steven Riley, Azra C Ghani.

On behalf of the Imperial College COVID-19 Response Team

WHO Collaborating Centre for Infectious Disease Modelling
MRC Centre for Global Infectious Disease Analysis
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Summary

The global impact of COVID-19 has been profound, and the public health threat it represents is the most serious seen in a respiratory virus since the 1918 H1N1 influenza pandemic. Here we present the results of epidemiological modelling which has informed policymaking in the UK and other countries in recent weeks. In the absence of a COVID-19 vaccine, we assess the potential role of a number of public health measures – so-called non-pharmaceutical interventions (NPIs) – aimed at reducing

Stochastic SIR models

- In the spirit of “simplest SIR” = compartments with transitions
- No explicit estimation/computation of contact rates
- From differential equations to **stochastic virtual worlds**

Imperial College model, Report March 16th, 2020

- ① synthetic individuals – by spatial position, age, social behavior
- ② synthetic contacts at: (1) home/residence, (2) central hubs (work, schools, markets, churches), (3) local neighborhoods
- ③ parameters of person-to-person contact based on large tuning data
- ④ **stochastic individual-based simulation**

large-scale Monte-Carlo simulations on HPC clusters

Stochastic simulation + visualization: <https://youtu.be/gxAa02rsdIs>

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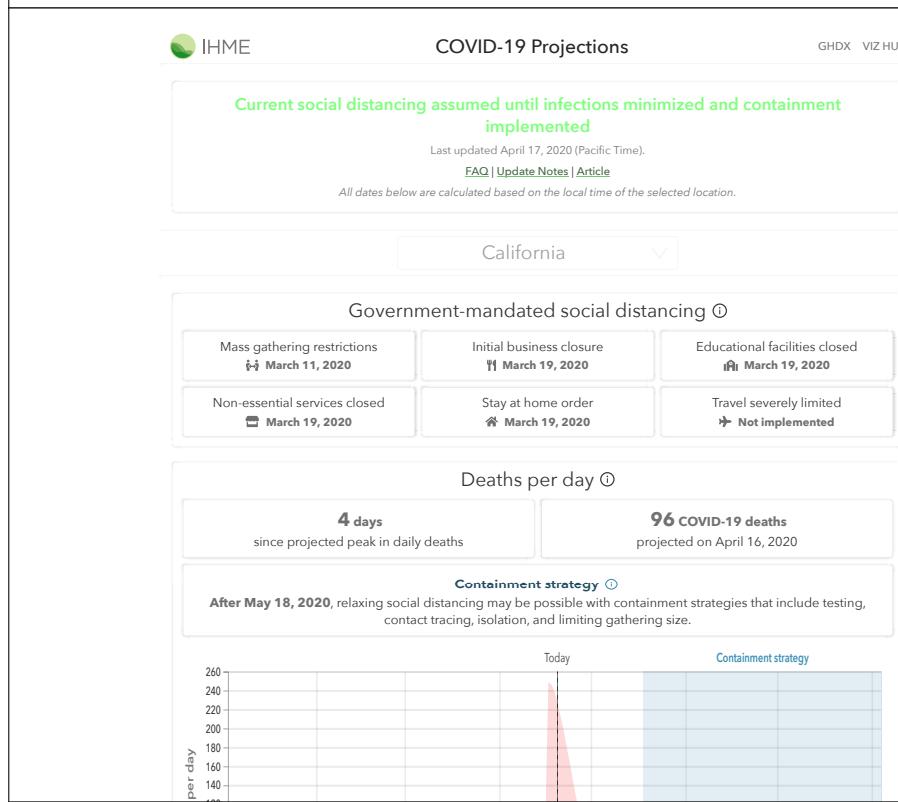
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Statistical model estimation

- do not impose a mechanism for transmission, do not model micro-interactions and micro-transitions between compartments
- direct interpolation of signals
- model empirically-observed death rate curves

Institute for Health Metrics and Evaluation (IHME), lead by Dr Murray.
MedRxiv paper March 30th, 2020.

- ① collected: age-specific deaths by day, start date for NPIs, hospital beds and ICU capacity, & (starting April 17) mobile phone data
- ② indirect standardization of age structure
- ③ only “admin 1 locations” with .31 death/million and time-referenced
- ④ curve-fitting: cumulative death rate as Gaussian error function
- ⑤ statistical covariate: # days from .31 threshold to NPI day estimated from Wuhan data (before and after NPI impositions)



IHME COVID-19 health service utilization forecasting team, C. Murray. Forecasting COVID-19 impact on hospital bed-days, ICU-days, ventilator-days and deaths by US state in the next 4 months. *medRxiv*, 2020. URL: <https://covid19.healthdata.org/>, doi:10.1101/2020.03.27.20043752

Forecasting COVID-19 impact on hospital bed-days, ICU-days, ventilator-days and deaths by US state in the next 4 months

IHME COVID-19 health service utilization forecasting team

Key Points

Question: Assuming social distancing measures are maintained, what are the forecasted gaps in available health service resources and number of deaths from the COVID-19 pandemic for each state in the United States?

Findings: Using a statistical model, we predict excess demand will be 64,175 (95% UI 7,977 to 251,059) total beds and 17,380 (95% UI 2,432 to 57,955) ICU beds at the peak of COVID-19. Peak ventilator use is predicted to be 19,481 (95% UI 9,767 to 39,674) ventilators. Peak demand will be in the second week of April. We estimate 81,114 (95% UI 38,242 to 162,106) deaths in the United States from COVID-19 over the next 4 months.

Meaning: Even with social distancing measures enacted and sustained, the peak demand for hospital services due to the COVID-19 pandemic is likely going to exceed capacity substantially. Alongside the implementation and enforcement of social distancing measures, there is an urgent need to develop and implement plans to reduce non-COVID-19 demand for and temporarily increase capacity of health facilities.

References

Sample references about SIR models

- ① N. M. Ferguson et al. Impact of non-pharmaceutical interventions (NPIs) to reduce COVID19 mortality and healthcare demand. Technical report, Imperial College, March 2020. doi:10.25561/77482
- ② Jose Lourenco, Robert Paton, Mahan Ghafari, Moritz Kraemer, Craig Thompson, Peter Simmonds, Paul Klenerman, and Sunetra Gupta. Fundamental principles of epidemic spread highlight the immediate need for large-scale serological surveys to assess the stage of the SARS-CoV-2 epidemic. *medRxiv*, 2020. doi:10.1101/2020.03.24.20042291
- ③ A. J. Kucharski et al. Early dynamics of transmission and control of COVID-19: a mathematical modelling study. *The Lancet Infectious Diseases*, 2020. doi:10.1016/S1473-3099(20)30144-4

Sample references about statistical models

- ① IHME COVID-19 health service utilization forecasting team, C. Murray. Forecasting COVID-19 impact on hospital bed-days, ICU-days, ventilator-days and deaths by US state in the next 4 months. *medRxiv*, 2020. URL: <https://covid19.healthdata.org/>, doi:10.1101/2020.03.27.20043752
- ② G. Sotgiu, G. A. Gerli, S. Centanni, M. Miozzo, G. W. Canonica, J. B. Soriano, and C. Virchow. Advanced forecasting of SARS-CoV-2 related deaths in Italy, Germany, Spain, and New York State. *Allergy*. doi:10.1111/all.14327

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From IHME MedRxiv on March 30th, criticism of SIR model:

- ① “random mixing between all individuals in a given population”
- ② given current estimates of \mathcal{R}_0 , SIR models “generally” over-predict
- ③ “results of these models are sensitive to starting assumptions”
- ④ “SIR models with assumptions of random mixing can overestimate [...] by not taking into account behavioral change and government-mandated action”

Evaluation of Statistical models by UK scientists

From CNN article on April 9th:

- From IHME website as of April 9th, prediction of 66K deaths in the UK by early August. (As of April 20th, IHME predicts 37.5K deaths)
- Imperial College model predicts 20K-30K, if NPIs are imposed

Professor Sylvia Richardson, Cambridge University and co-chair of the Royal Statistical Society Task Force on Covid-19, says

- ① IHME's projections are based on "very strong assumptions about the way the epidemic will progress."
- ② "based mostly on using the experience in other countries to fit a smooth curve to the counts of deaths reported so far in the UK, rather than any modeling of the epidemic itself."
- ③ "Methods like this are well known for being extremely sensitive, and are likely to change dramatically as new information comes in"

Summary

- 260 years old mathematical journey. Results have been stellar.
- simplest SIR model explains emerging phenomena
salient features: $\mathcal{R}(t)$, growth/decay, and explains NPIs
- more realistic, but still extremely data-dependent, models:
 - ① stochastic structured/multi-group SIR models
 - ② statistical models based on data fitting

name	description	scope
simplest SIR	low complexity explanation	crucial basic understanding
Stochastic SIR	mechanistic explanation	assessment of existing and novel NPIs
Statistical models	direct data fitting	prediction

- **from data to parameters and state** – next webinars in series

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Concluding Question: how can we safely reopen UCSB?

- What if we were to perform extensive testing, contact tracing and other measures — for those students willing to consent?
- What models and what data would we need?
- What would a comprehensive approach entail?
 - campus infrastructure = health center, classroom, dining
 - digital infrastructure = mobile app, backend ...



Recall

$$\begin{aligned}\mathcal{R}(t) &= (\beta_m \times \beta_t) \times 1/\gamma \times s(t) \\ &\approx ((\text{contacts/day}) \times (\text{transmission})) \times (\text{infective days}) \times s(t)\end{aligned}$$

Non-pharmaceutical interventions aimed at decreasing $\mathcal{R}(t)$:
NPI effect

washing hands and wearing masks	decrease infection transmission β_t
social distancing and travel restrictions	decrease contact rates β_m
testing leading to quarantine	decreases infective duration $1/\gamma$
contact tracing leading to quarantine	decreases infective duration $1/\gamma$