A005448 Centered triangular numbers: a(n) = 3n(n-1)/2 + 1.

(Formerly M3378)

1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199, 235, 274, 316, 361, 409, 460, 514, 571, 63
694, 760, 829, 901, 976, 1054, 1135, 1219, 1306, 1396, 1489, 1585, 1684, 1786, 1891, 1999, 2
2224, 2341, 2461, 2584, 2710, 2839, 2971, 3106, 3244, 3385, 3529 (list; graph; refs; listen; histor
text; internal format)

OFFSET 1,2

COMMENTS These are Hogben's central polygonal numbers

2

.P

3 n

Also the sum of three consecutive triangular numbers (A000217); i.e., a(4) = 1
T4 + T3 + T2 = 10 + 6 + 3. - Robert G. Wilson v, Apr 27 2001

For k>2, Sum_{n=1..k} a(n) gives the sum pertaining to the magic square of ord
E.g., Sum_{n=1..5} a(n) = 1 + 4 + 10 + 19 + 31 = 65. In general, Sum_{n=1..k
a(n) = k*(k^2 + 1)/2. - Amarnath Murthy, Dec 22 2001

Binomial transform of (1,3,3,0,0,0,...). - Paul Barry, Jul 01 2003

a(n) is the difference of two tetrahedral (or pyramidal) numbers: C(n+3,3) =
(n+1)(n+2)(n+3)/6. a(n) = A000292(n) - A000292(n-3) = (n+1)(n+2)(n+3)/6 -
(n-2)(n-1)(n)/6. - Alexander Adamchuk, May 20 2006

Partial sums are A006003(n) = n(n^2+1)/2. Finite differences are a(n+1) - a(n)
A008585(n) = 3n. - Alexander Adamchuk, Jun 03 2006

If X is an n-set and Y a fixed 3-subset of X then a(n-2) is equal to the numbe
3-subsets of X intersecting Y. - Milan Janjic, Jul 30 2007

Equals (1, 2, 3, ...) convolved with (1, 2, 3, 3, 3, ...). a(4) = 19 = (1, 2,
dot (3, 3, 2, 1) = (3 + 6 + 6 + 4). - Gary W. Adamson, May 01 2009

Equals the triangular numbers convolved with [1, 1, 1, 0, 0, 0, ...]. - Gary W
Adamson and Alexander R. Povolotsky, May 29 2009

a(n) is the number of triples (w,x,y) having all terms in {0,...,n} and
min(w+x,x+y,y+w) = max(w,x,y). - Clark Kimberling, Jun 14 2012

a(n) = number of atoms at graph distance <= n from an atom in the graphite or
graphene network (cf. A008486). - N. J. A. Sloane, Jan 06 2013

In 1826, Shiraishi gave a solution to the Diophantine equation a^3 + b^3 + c^3
d^3 with b = a(n) for n > 1; see A226903. - Jonathan Sondow, Jun 22 2013

For n > 1, a(n) is the remainder of n^2 * (n-1)^2 mod (n^2 + (n-1)^2). - J. M.
Bergot, Jun 27 2013

The equation A000578(x) - A000578(x-1) = A000217(y) - A000217(y-2) is satisfie
y=a(x). - Bruno Berselli, Feb 19 2014

A242357(a(n)) = n. - Reinhard Zumkeller, May 11 2014

A255437(a(n)) = 1. - Reinhard Zumkeller, Mar 23 2015

The first differences give A008486. a(n) seems to give the total number of
triangles in the n-th generation of the six patterns of triangle expansion s
in the link. - Kival Ngaokrajang, Sep 12 2015

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B. K. Teo and N. J. A. Sloane, Magic numbers in polygonal and polyhedral clust Inorgan. Chem. 24 (1985), 4545-4558.
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Index entries for sequences related to centered polygonal numbers
Index entries for linear recurrences with constant coefficients, signature (3,-3,1).

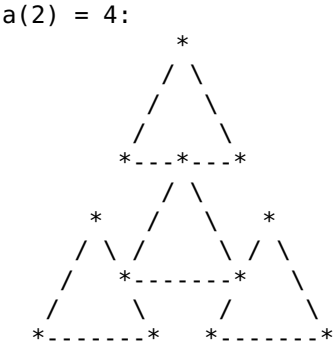
FORMULA

Expansion of x*(1-x^3)/(1-x)^4.
a(n) = C(n+3, 3)-C(n, 3) = C(n, 0)+3*C(n, 1)+3*C(n, 2). - Paul Barry, Jul 01 2
a(n) = 1 + Sum_{j=0..n-1} (3*j). - Xavier Acloque, Oct 25 2003
a(n) = A000217(n) + A000290(n-1) = (3*A016754(n) + 5)/8. - Lekraj Beedassy, Nc 2005
Euler transform of length 3 sequence [4, 0, -1]. - Michael Somos, Sep 23 2006
a(1-n) = a(n). - Michael Somos, Sep 23 2006
a(n) = binomial(n+1,n-1) + binomial(n,n-2) + binomial(n-1,n-3). - Zerinvary La Sep 03 2006
Row sums of triangle A134482. - Gary W. Adamson, Oct 27 2007
Narayana transform (A001263) * [1, 3, 0, 0, 0, ...]. - Gary W. Adamson, Dec 29
a(n) = 3*a(n-1) - 3*a(n-2) + a(n-3), a(1)=1, a(2)=4, a(3)=10. - Jaume Oliver Lafont, Dec 02 2008
a(n) = A000217(n-1)*3 + 1 = A045943(n-1) + 1. - Omar E. Pol, Dec 27 2008
a(n) = a(n-1) + 3*n-3. - Vincenzo Librandi, Nov 18 2010
Sum_{n>=1} 1/a(n) = A306324. - Ant King, Jun 12 2012
a(n) = 2*a(n-1) - a(n-2) + 3. - Ant King, Jun 12 2012
a(n) = A101321(3,n-1). - R. J. Mathar, Jul 28 2016
E.g.f.: -1 + (2 + 3*x^2)*exp(x)/2. - Ilya Gutkovskiy, Jul 28 2016
a(n) = A002061(n) + A000217(n-1). - Bruce J. Nicholson, Apr 20 2017
From Amiram Eldar, Jun 20 2020: (Start)
Sum_{n>=1} a(n)/n! = 5*e/2 - 1.
Sum_{n>=1} (-1)^n * a(n)/n! = 5/(2*e) - 1. (End)
a(n) = A000326(n) - n + 1. - Charlie Marion, Nov 21 2020

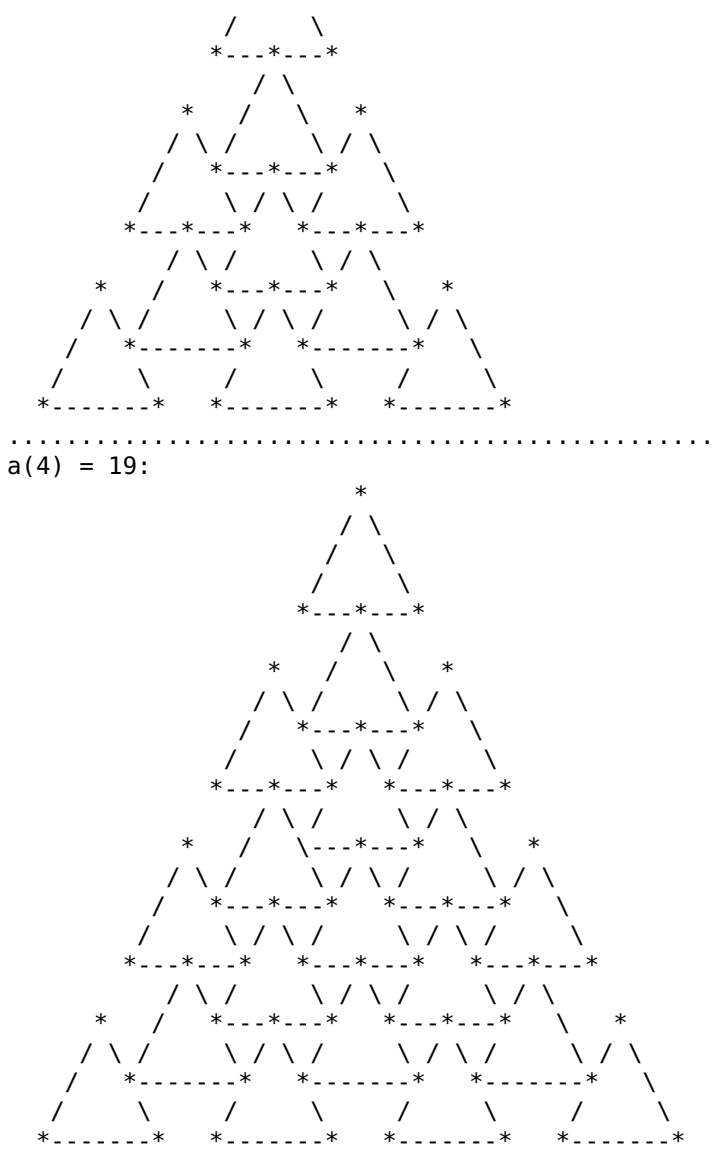
EXAMPLE

From Seiichi Manyama, Aug 12 2017: (Start)
a(1) = 1:

.....
a(2) = 4:



.....
a(3) = 10:



a(4) = 19:

(End)

MAPLE

```
A005448 := n->(3*(n-1)^2+3*(n-1)+2)/2: seq(A005448(n), n=1..100);
A005448 := -(1+z+z**2)/(z-1)^3; # Simon Plouffe in his 1992 dissertation for c
  0
```

MATHEMATICA

```
FoldList[#1 + #2 &, 1, 3 Range@ 50] (* Robert G. Wilson v, Feb 02 2011 *)
Join[{1, 4}, Total/@Partition[Accumulate[Range[50]], 3, 1]] (* Harvey P. Dale,
  17 2012 *)
LinearRecurrence[{3, -3, 1}, {1, 4, 10}, 50] (* Vincenzo Librandi, Sep 13 2015
```

PROG

```
(PARI) {a(n)=3*(n^2-n)/2+1} /* Michael Somos, Sep 23 2006 */
(PARI) isok(n) = my(k=(2*n-2)/3, m); (n==1) || ((denominator(k)==1) &&
  (m=sqrtint(k)) && (m*(m+1)==k)); \\ Michel Marcus, May 20 2020
(Haskell)
a005448 n = 3 * n * (n - 1) `div` 2 + 1
a005448_list = 1 : zipWith (+) a005448_list [3, 6 ..]
-- Reinhard Zumkeller, Jun 20 2013
(MAGMA) I:=[1, 4, 10]; [n le 3 select I[n] else 3*Self(n-1)-3*Self(n-2)+Self(n
  n in [1..60]]; // Vincenzo Librandi, Sep 13 2015
```

CROSSREFS

```
Cf. A000217, A000292, A001263, A001844, A002061, A006003 = partial sums, A0084
  A008585 = first differences, A045943, A134482, A226903, A242357, A255437.
Sequence in context: A162505 A025720 A022793 * A301247 A037040 A007077
Adjacent sequences:  A005445 A005446 A005447 * A005449 A005450 A005451
```

KEYWORD

```
nonn,easy,nice
```

AUTHOR

```
N. J. A. Sloane, R. K. Guy, Dec 12 1974
```

STATUS

```
approved
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