27/08/22

ψ^{S_n}, ψ^{n}, ψ^{L_n}

$S_n \rightarrow$ ORDERED SYMBOLS

$L_n \rightarrow$ LABELS

$n \rightarrow$ INDEX

¿WHAT IS ~~THE~~ A NATURAL RELATION BETWEEN INDEXES, SYMBOLS AND LABELS?

LET'S THINK OF THE MAIN INDEX AS OUR IDEA OF TIME, OR IN A DSP CONTEXT AS A 'DISCRETE-TIME' INDEX. WE ARE NOT GOING TO WORRY ABOUT SAMPLING, NEITHER ABOUT DISCRETIZATION, BUT I'D LEAVE SOME LINKS IN THE DESCRIPTION TO MIT

n : $\{0, 1, 2, 3, \cdots, N-1\}$ $\quad t\ \{0, \Delta t, 2\Delta t, \cdots (N-1)\Delta t\}$

$\Delta t \rightarrow T_s \rightarrow$ SAMPLING RATE

$S_n\ \{S_{min}, S_{min}+\Delta_s, S_{min}+2\Delta_s, + \cdots, \underbrace{S_{min}+M_s\Delta_s}_{M_0}\}$

$0 < \Delta_s \leq 1$

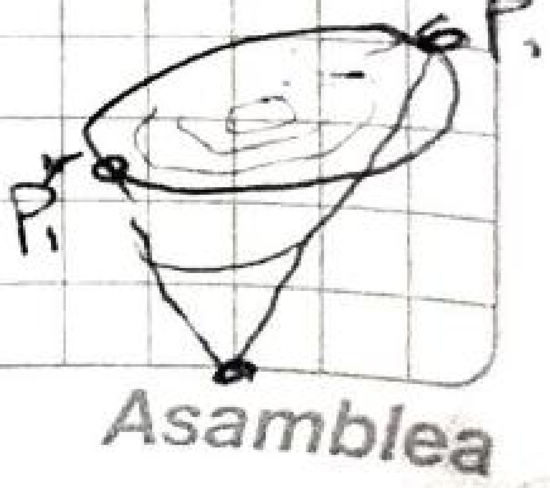
¿CAN YOU FIND CTB LIKE IMAGES IN ANY RANDOM MATRIX?

¿WITH ANY INITIAL CONDITIONS?

HOW TO TRACK P-POSITIONS EVOLUTION USING ONLY ONE PARAMETER IN

STEREO-RADIANS

(SOLID ANGLE)



SHOR BY DON WALLMAN

P Q
7177 x 3001

21538177
S

21538177 S

P Q

PROBLEM ¿COMPLEXITY?

FACTORIZE
S (S HAS
N DIGITS)

→ HOW MUCH
HARDER
CAN B
TO FACTORIZE
(S+T)

$\left(2^{\frac{N}{2}}\right)$

$\left(2^{\frac{N}{2}}\right)$ → SCALING
OF FACTORIZATI
↓
MEANS
HARD
WHEN
N IS IN
THE EXPONENT

CLASSICAL

QC

SHOR'S IS
POLYNOMIAL
log(N)

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SERGE LANG ALGEBRA

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SHANNON AND NYQUIST PARADOX (Roy Spielberg)

FACT: LET $s(t)$ BE ANY CONTINUOUS TIME SIGNAL, WE SAMPLE AND DISCRETIZE TO s_n, $s(nT_s)$

FOR EACH SAMPLE WE TAKE TO THE DISCRETE-TIME WORLD, WE LOOSE AN '∞' AMOUNT OF CONTINUOUS-TIME DATAPOINTS

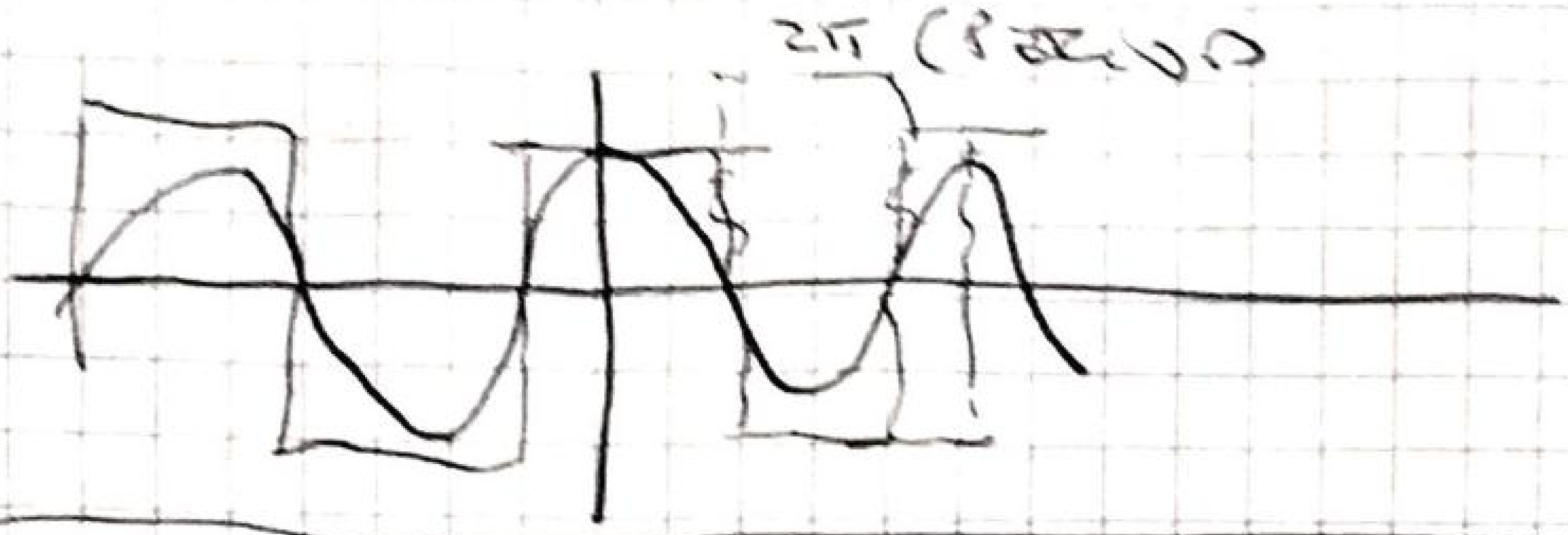
SH AND NY SAID, IF THE SAMPLING FREQ IS HIGHER THAN TWICE THE MAXIMUM FREQ OF THE SIGNAL, THE ORIGINAL SIGNAL IS COMPLETELY RECOVERABLE FROM ITS SAMPLES

$$\boxed{X(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \text{sinc}\left(\frac{\omega_s}{2}(t - nT)\right)}$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos nx + b_n \sin nx \right\}}$$ F. SERIES

$$\boxed{\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t}\, dt}$$ F. TRANSFORM

SQUARE WAVE FOURIER SERIES



$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin(nx) = 2\sin(x) - \sin(2x) + \frac{2}{3}\sin(3x)$$

AND A PERIODIC EXTENSION OF

$f(x) = x$

(SAW TOOTH)

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi(2n-1)} \sin\left((2n-1)x\right) = \frac{4}{\pi}\sin(x) + \frac{4}{3\pi}\sin(3x) + \cdots$$

IF THE FUNCTION IS NOT PERIODIC, YOU CAN

THINK OF IT HAVING A $\overset{\text{(PERIOD)}}{T} \to \infty$

$f \to 0$

THIS CREATES INTEGRALS FROM SUMS

AND FUNCTIONS FROM COEFFICIENTS

$$\sum \to \int$$

$$c_n \to \hat{f}(w)$$

F. TRANSFORM $\to$ PLOT THIS TO GET THE FEELING

$$g(x) = \frac{\sin(x)}{x}$$

$$f(x) = \int_{-e}^{e} g(t) \cos(xt)\, dt$$

$$f(x) = \cos(x)$$

$$h(w) = \int_{-e}^{e} f(x) \cos(wx)\, dx$$

Asamblea

DIRAC DELTA

$$\int_{-\infty}^{\infty} \delta(x-a) f(x)\, dx = F(a)$$

IMPULSE TRAIN

$$I_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$I_T(t) = 1 + 2 \sum_{n=-\infty}^{\infty} \cos\left(\frac{2\pi n}{T} t\right)$$

FOURIER SERIES FOR THE IMPULSE TRAIN

$$I_T(t) \cdot x(t) = x(t) + 2 \sum_{n=-\infty}^{\infty} \cos(n \omega_0 t) x(t)$$

$$Y(\omega) = \mathcal{F}\left[\frac{1}{T}\left(x(t) + 2 \sum_{n=-\infty}^{\infty} x(t) \cos(n \omega_0 t)\right)\right]$$

$$Y(\omega) = \mathcal{F}\left[x(t) \cdot I_T(t)\right]$$

Asamblea

$$Y(\omega) = \frac{1}{T}\left(X(\omega) + 2\sum_{n=1}^{\infty} \underline{X(\omega - n\omega_s)}\right.$$

$$Y(\omega) = \frac{1}{T}\left(X(\omega) + \cancel{2}\sum_{n=1}^{\infty} \frac{X(\omega - n\omega_s) + X(\omega + n\omega_s)}{\cancel{2}}\right.$$

$$\boxed{Y(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)}$$ A

$$Y(\omega) = \int_{-\infty}^{\infty} x(t)\, I_1(t)\, e^{-i\omega t}\, dt$$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} x(nT)\, e^{-i\omega nT}$$

$$\boxed{Y(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-i\omega nT}}$$

DISCRETE TIME FOURIER TRANSFORM DTFT

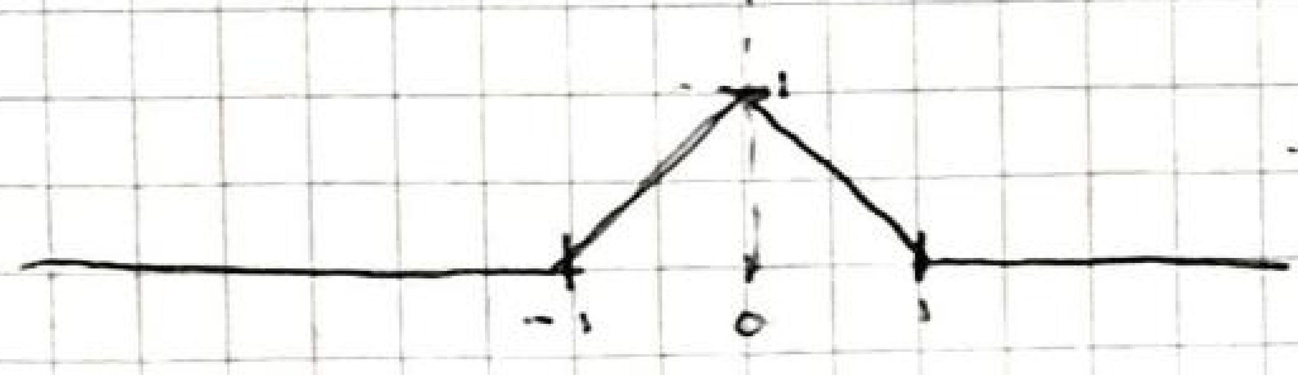
RECOVERING MEANS:

1) SAMPLE

2) DTFT A

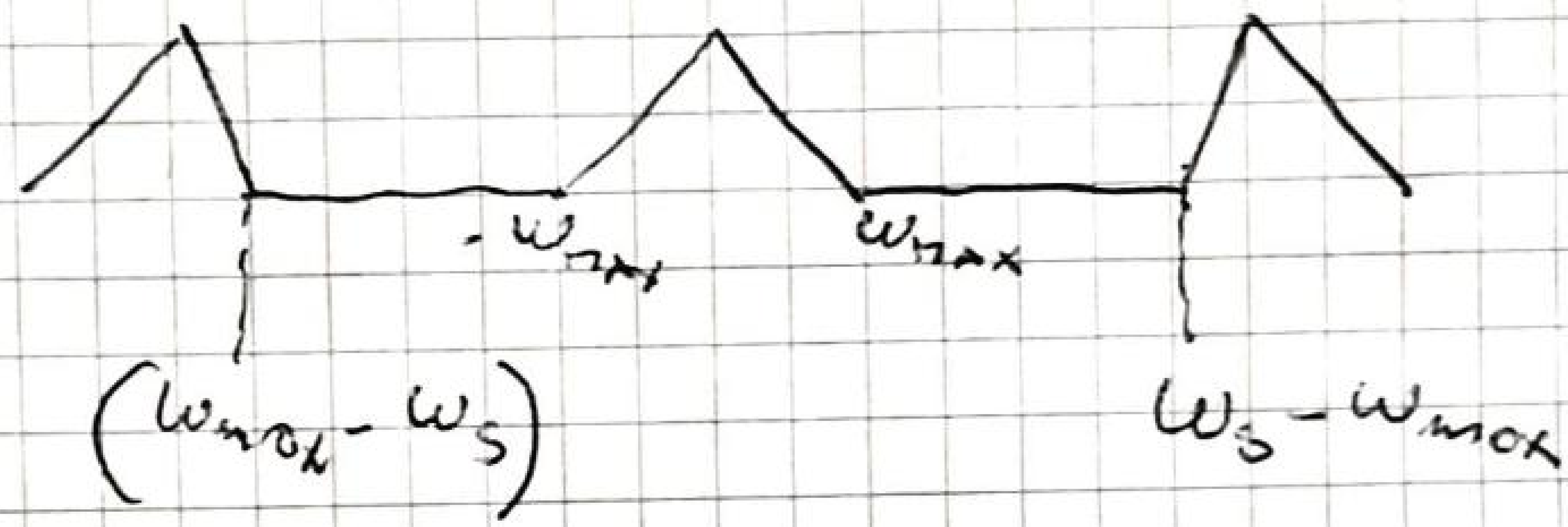
BAND LIMIT:

IF $|\omega| > \omega_{max}$ THEN $\hat{F}(\omega) = 0$



→ BAND LIMITED SIGNALS

THIS MAKES ANTI-ALIASING POSSIBLE



$$\boxed{\omega_s > 2\omega_{max}}$$

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1) $F.T\left[x[n]\right] = \sum_{n=-\infty}^{\infty} x[n] e^{-i\omega n T}$

2) MULTIPLY BY A WINDOW FUNCTION

$$X(\omega) = \begin{cases} \sum_{n=-\infty}^{\infty} T x[n] e^{-i\omega n T} & |\omega| < \frac{\omega_s}{2} \\ 0 & |\omega| \geq \frac{\omega_s}{2} \end{cases}$$

3) NOW WE USE THE INVERSE F.T.

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} \, d\omega}$$

Asamblea

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4) TO GET THE ORIGINAL SIGNAL

$$X(t) = \frac{1}{2\pi} \int_{-\frac{\omega_s}{2}}^{\frac{\omega_s}{2}} \sum_{n=-\infty}^{\infty} T \cdot x[n] e^{-i\omega nT} \cdot e^{i\omega t} d\omega$$

$$\boxed{\omega_s = \frac{2\pi}{T_s}}$$

$$\downarrow$$

$$X(t) = \frac{T}{2\pi} \int_{-\frac{\omega_s}{2}}^{\frac{\omega_s}{2}} \sum_{n=-\infty}^{\infty} x[n] e^{i\omega(t - nT_s)} d\omega$$

$$X(t) = \frac{1}{\omega_s} \sum_{n=-\infty}^{\infty} \int_{-\frac{\omega_s}{2}}^{\frac{\omega_s}{2}} x[n] e^{i\omega(t-nT_s)} d\omega$$

$$\downarrow$$

$$\boxed{X(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \text{sinc}\left(\frac{\omega_s}{2}(t - nT_s)\right)}$$

↳ RECOVERED SIGNAL