Phase margin vs. damping ratio (non-unity feedback)

Asked 2 years, 3 months ago Modified 2 years, 3 months ago Viewed 557 times

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So there is a well known condition which relates the phase margin with the damping ratio for a **unity feedback** system:

$$\Phi_m = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

This equation assumes that the closed-loop transfer function is a damped second order function:

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

where L(s) is the open-loop transfer function of a unity-feedback system (with L(s) = G(s)). The derivation for the phase margin above is done by assuming that

$$L(s) = G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

My question is, how would the phase margin equation above change if the system was non-unity feedback (i.e., if the open-loop transfer function was L(s) = G(s)H(s), where G(s) is in the feedforward path, and H(s) is in the feedback path)? I could not find any book or paper that provides this derivation.

Your help is much appreciated!

control control-system transfer-function feedback phase-margin

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edited May 10, 2020 at 14:01

asked May 10, 2020 at 12:45

Johnny Que
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Please provide a link to a site/paper/document that defines that *well known condition*. – Andy aka May 10, 2020 at 13:21

Control Systems Engineering (Norman Nise, 6th edition) – Johnny Que May 10, 2020 at 13:57

I don't think that your 2nd formula is correct - shouldn't it be $2\omega_n^2$ in the denominator? – Andy aka May 10, 2020 at 13:58

The was a square in the formula for G(s), which I corrected (should not have been a square) – Johnny Que May 10, 2020 at 14:03

I still don't think your 2nd formula is correct - maybe you can photograph the page of the book and post to your question. – Andy aka May 10, 2020 at 14:46

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Johnny Que - may I ask you WHY do you think that the shown relation between the damping factor and the phase margin (for a second-order sysytem) would be valid for unity feedback only?

I rather think - better: I am convinced - that it applies to all 2nd-order systems.

That means: For the **loop gain expression L(s)=G(s)H(s)** .

Reference: Robert C. Dorf: "Modern Control Systems", 6th Edition, Addison Wesley, 1992

Comment: The formula is derived in the referenced book.

More than that, it can be shown that for PM < 65 deg and damping < 0.707 this expression can be approximated with good accuracy by PM=damping/0.01.

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edited May 10, 2020 at 13:53

answered May 10, 2020 at 13:36

LvW

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So the derivation of the phase margin assumes that

$$L(s) = G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

...With this expression (and with unity feedback), you can get the typical second order response T(s) that I wrote in my original post. However, with non-unity feedback, you can't assume

$$L(s) = G(s)H(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

because then you won't get the second order function T(s), since in the non-unity feedback case,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

– Johnny Que May 10, 2020 at 14:01

@JohnnyQue You seem to be missing the point that phase margin is an open-loop phenomenon so you just calculate PM for the open loop case. – Andy aka May 10, 2020 at 14:14

@Andy aka The point of the phase margin equation I posted in my original post is to relate the phase margin with the damping of the closed-loop response. So the PM (which is calculated using open-loop data) can be used to relate the closed-loop behavior...it's not just an "open-loop phenomenon."
– Johnny Que May 10, 2020 at 14:22

@JohnnyQue - Phase margin is defined as the amount of change in open-loop phase needed to make a closed-loop system unstable. In other words it's an open-loop phenomenon that can be used to predict closed-loop behaviour. – Andy aka May 10, 2020 at 14:43

I think - The phase margin is basically a "closed-loop phenomenon". It is true that it is defined - very often - for the open loop (loop gain) because this allows a simple measurement technique. But it originates from the closed-loop system: The phase margin is the additional phase which must be inserted into the closed loop in order to shift the closed-loop poles to the imaginary axis (oscillation). Hence, the formula under discussion involves closed-loop parameters – LvW May 10, 2020 at 14:59

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Taking a 2-pole amplifier, the open loop frequency response of which is shown below.

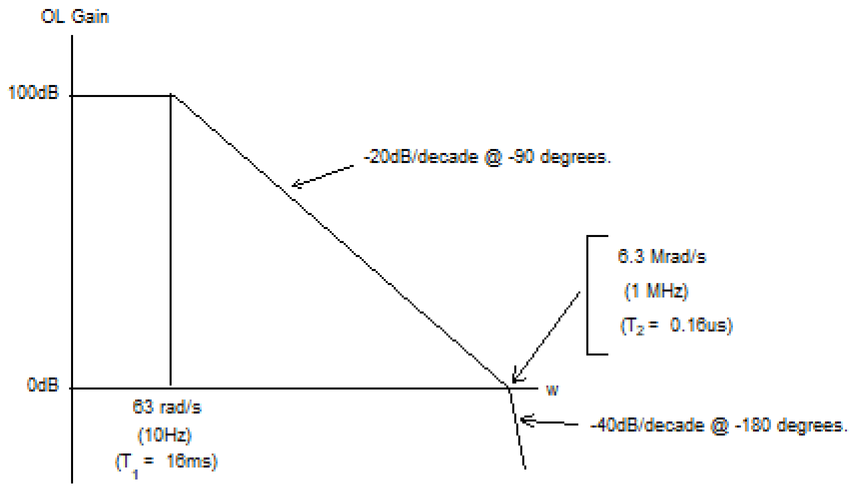

Fig.1. Open loop gain vs frequency plot of example op amp.

From this open loop frequency response, and assuming a H(s) = 1/2, I generated the loop transfer function. Then substituting s=jw I entered various values of w into the transfer function until I converged upon the value of w which resulted in a loop gain of 1, thereby also obtaining the phase at this frequency which reveals the phase margin.

Next I generated the closed loop transfer function, G(s)/(1+GH(s)), from the denominator of which I obtained the value of zeta.

I found that for a dc closed loop gain of 2 the value of zeta is 0.707 and the phase margin is 65.5 degrees.

I repeated the above analysis for a closed loop gain of 4, H(s)=1/4, and found the value of zeta to be 1 and the phase margin to be 76.3 degrees.

These results for dc closed loop gains of 2 & 4 (zeta = 0.707 & 1 respectively) agree almost exactly with the results given by your phase margin equation in your question.

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edited May 11, 2020 at 8:28

answered May 10, 2020 at 14:58

James

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