

0 1 3 6 2 7  
: 13  
: 20  
23 12  
10 22 11 21

# THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES<sup>®</sup>

founded in 1964 by N. J. A. Sloane

2,5,10,17,26,37,50

Search

[Hints](#)(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))Search: **seq:2,5,10,17,26,37,50**

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page 1 [2](#)Sort: relevance | [references](#) | [number](#) | [modified](#) | [created](#)Format: long | [short](#) | [data](#)[A002522](#) $a(n) = n^2 + 1$ .+30  
415

1, **2**, **5**, **10**, **17**, **26**, **37**, **50**, 65, 82, 101, 122, 145, 170, 197, 226, 257, 290, 325, 362, 401, 442, 485, 530, 577, 626, 677, 730, 785, 842, 901, 962, 1025, 1090, 1157, 1226, 1297, 1370, 1445, 1522, 1601, 1682, 1765, 1850, 1937, 2026, 2117, 2210, 2305, 2402, 2501  
([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,2

COMMENTS

An  $n \times n$  nonnegative matrix  $A$  is primitive (see [A070322](#)) iff every element of  $A^k$  is  $> 0$  for some power  $k$ . If  $A$  is primitive then the power which should have all positive entries is  $\leq n^2 - 2n + 2$  (Wielandt).

$a(n) = \Phi_4(n)$ , where  $\Phi_k$  is the  $k$ -th cyclotomic polynomial.

As the positive solution to  $x=2n+1/x$  is  $x=n+\sqrt{a(n)}$ , the continued fraction expansion of  $\sqrt{a(n)}$  is  $\{n; 2n, 2n, 2n, 2n, \dots\}$ . - [Benoit Cloitre](#), Dec 07 2001

$a(n)$  is one less than the arithmetic mean of its neighbors:  $a(n) = (a(n-1) + a(n+1))/2 - 1$ . E.g.,  $2 = (1+5)/2 - 1$ ,  $5 = (2+10)/2 - 1$ . - [Amarnath Murthy](#), Jul 29 2003

Equivalently, the continued fraction expansion of  $\sqrt{a(n)}$  is  $(n; 2n, 2n, 2n, \dots)$ . - [Franz Vrabec](#), Jan 23 2006

Number of  $\{12, 1^*2^*, 21\}$ -avoiding signed permutations in the hyperoctahedral group.

The number of squares of side 1 which can be drawn without lifting the pencil, starting at one corner of an  $n \times n$  grid and never visiting an edge twice is  $n^2 - 2n + 2$ . - [Sébastien Dumortier](#), Jun 16 2005

From [Cino Hilliard](#), Feb 21 2006: (Start)

Also, except for the first term, numbers that cannot be expressed as a perfect power, i.e.,  $x^2 + 1 \neq y^n$  for all  $x, y, n > 1$ . Proof: We assume the truth of the following theorem. Proofs can be found in elementary texts on number theory and online. Theorem I: A number  $N$  is a sum of two squares if and only if all prime factors of  $N$  of the form  $4m+3$  have even exponents.

We are now ready to prove  $x^2 + 1 \neq y^n$  for all  $x, y, n > 1$ . We assume equality and seek a contradiction for  $n$  even and  $n$  odd. If  $n$  is even  $= 2k$ ,  $x^2 + 1 = y^{2k} = (y^k)^2$  and  $(y^k - x)(y^k + x) = 1$ . This implies  $y^k - x = y^k + x = 1$  or  $2x = 0$  contrary to  $x > 1$ . So  $n$  must be odd for equality to hold.

Then  $x^2 + 1 = y^{(2k+1)}$  implies all prime factors of  $y$ , including those of the form  $4m+3$  are raised to an odd exponent contrary to Theorem I. So we have shown  $x^2 + 1 = y^n$  is false for  $n$  even or  $n$  odd. Therefore  $x^2 + 1 \neq y^n$  as was desired. (End)

Note that in the above proof,  $y$  doesn't necessarily have any prime factors of the form  $4m+3$ . - [Jon Perry](#), Aug 06 2012

Also, numbers  $m$  such that  $m^3 - m^2$  is a square,  $(n*(1 + n^2))^2$ . - [Zak Seidov](#)

$1 + 2/2 + 2/5 + 2/10 + \dots = \pi \coth \pi$  [Jolley], see [A113319](#). - [Gary W. Adamson](#), Dec 21 2006

For  $n \geq 1$ ,  $a(n-1)$  is the minimal number of choices from an  $n$ -set such that at least one particular element has been chosen at least  $n$  times or each of the  $n$  elements has been chosen at least once. Some games define "matches" this way; e.g., in the classic Parker Brothers, now Hasbro, board game Risk,  $a(2)=5$  is the number of cards of three available types (suits) required to guarantee at least one match of three different types or of three of the same type (ignoring any jokers or wildcards). - [Rick L. Shepherd](#), Nov 18 2007

Positive  $X$  values of solutions to the equation  $X^3 + (X - 1)^2 + X - 2 = Y^2$ . To prove that  $X = n^2 + 1$ :  $Y^2 = X^3 + (X - 1)^2 + X - 2 = X^3 + X^2 - X - 1 = (X - 1)(X^2 + 2X + 1) = (X - 1)(X + 1)^2$  it means:  $(X - 1)$  must be a perfect square, so  $X = n^2 + 1$  and  $Y = n(n^2 + 2)$ . - [Mohamed Bouhamida](#), Nov 29 2007

$\{a(k): 0 \leq k < 4\}$  = divisors of 10. - [Reinhard Zumkeller](#), Jun 17 2009

Number of units of  $a(n)$  belongs to a periodic sequence: 1, 2, 5, 0, 7, 6, 7, 0, 5, 2. - [Mohamed Bouhamida](#), Sep 04 2009

Appears in [A054413](#) and [A086902](#) in relation to sequences related to the numerators and denominators of continued fractions convergents to  $\sqrt{(2^n)^2/4 + 1}$ ,  $n=1, 2, 3, \dots$ . - [Johannes W. Meijer](#), Jun 12 2010

For  $n>0$ , continued fraction  $[n,n] = n/a(n)$ ; e.g.,  $[5,5] = 5/26$ . - [Gary W. Adamson](#), Jul 15 2010

The only real solution of the form  $f(x) = A \cdot x^p$  with negative  $p$  which satisfies  $f^{(m)}(x) = f^{[-1]}(x)$ ,  $x \geq 0$ ,  $m \geq 1$ , with  $f^{(m)}$  the  $m$ -th derivative and  $f^{[-1]}$  the compositional inverse of  $f$ , is obtained for  $m=2^n$ ,  $p=p(n) = -(\sqrt{a(n)} - n)$  and  $A=A(n) = (\text{fallfac}(p(n), 2^n))^{\frac{1}{p(n)/(p(n)+1)}}$ , with  $\text{fallfac}(x,k) := \text{Product}_{j=0..k-1} (x-j)$  (falling factorials). See the T. Koshy reference, pp. 263-4 (there are also two solutions for positive  $p$ , see the corresponding comment in [A087475](#)). - [Wolfdieter Lang](#), Oct 21 2010

$n + \sqrt{a(n)} = [2^n; 2^n, 2^n, \dots]$  with the regular continued fraction with period 1. This is the even case. For the general case see [A087475](#) with the Schroeder reference and comments. For the odd case see [A078370](#).

$a(n-1)$  counts configurations of non-attacking bishops on a  $2 \times n$  strip [Chaiken et al., Ann. Combin. 14 (2010) 419]. - [R. J. Mathar](#), Jun 16 2011

Also numbers  $n$  such that  $4^n - 4$  is a square. Hence this sequence is the union of [A053755](#) and [A069894](#). - [Arkadiusz Wesolowski](#), Aug 02 2011

$a(n)$  is also the Moore lower bound on the order, [A191595](#)( $n$ ), of an  $(n,5)$ -cage. - [Jason Kimberley](#), Oct 17 2011

Left edge of the triangle in [A195437](#):  $a(n+1) = \text{A195437}(n,0)$ . - [Reinhard Zumkeller](#), Nov 23 2011

If  $h(5,17,37,65,101,\dots)$  is prime is relatively prime to 6, then  $h^2-1$  is divisible by 24. - [Vincenzo Librandi](#), Apr 14 2014

The identity  $(4^{n^2+2})^2 - (n^2+1) \cdot (4^n)^2 = 4$  can be written as [A005899](#)( $n$ )<sup>2</sup> -  $a(n) \cdot \text{A008586}(n)^2 = 4$ . - [Vincenzo Librandi](#), Jun 15 2014

$a(n)$  is also the number of permutations simultaneously avoiding 213 and 321 in the classical sense which can be realized as labels on an increasing strict binary tree with  $2n-1$  nodes. See [A245904](#) for more information on increasing strict binary trees. - [Manda Riehl](#), Aug 07 2014

$\sum_{n \geq 0} (-1)^n / a(n) = (1 + \pi / \sinh(\pi)) / 2 = 0.636014527491\dots$ . - [Vaclav Kotesovec](#), Feb 14 2015

$a(n-1)$  is the maximum number of stages in the Gale-Shapley algorithm for finding a stable matching between two sets of  $n$  elements given an ordering of preferences for each element (see Gura et al.). - [Melvin Peralta](#), Feb 07 2016

Because of Fermat's little theorem,  $a(n)$  is never divisible by 3. - [Altug Alkan](#), Apr 08 2016

For  $n > 0$ , if  $a(n)$  points are placed inside an  $n \times n$  square, it will always be the case that at least two of the points will be a distance of  $\sqrt{2}$  units apart or less. - [Melvin Peralta](#), Jan 21 2017

Also the limit as  $q \rightarrow 1^-$  of the unimodal polynomial  $(1 - q^{(n \cdot k + 1)}) / (1 - q)$  after making the simplification  $k=n$ . The unimodal polynomial is from O'Hara's proof of unimodality of  $q$ -binomials after making the restriction to partitions of size  $\leq 1$ . See  $G_1(n,k)$  from arXiv:1711.11252. As the size restriction  $s$  increases,  $G_s \rightarrow G_{\infty} = G$ : the  $q$ -binomials. Then substituting  $k=n$  and  $q=1$  yields the central binomial coefficients: [A000984](#). - [Bryan T. Ek](#), Apr 11 2018

$a(n)$  is the smallest number congruent to both 1 (mod  $n$ ), and 2 (mod  $n+1$ ). - [David James Sycamore](#), Apr 04 2019

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[Index to values of cyclotomic polynomials of integer argument](#)

[Index entries for linear recurrences with constant coefficients](#), signature (3, -3, 1).

FORMULA

O.g.f.:  $(1-x+2x^2)/((1-x)^3)$ . - [Eric Werley](#), Jun 27 2011

Sequences of the form  $a(n) = n^2 + K$  with offset 0 have o.g.f.  $(K - 2Kx + Kx^2 + x + x^2)/(1-x)^3$  and recurrence  $a(n) = 3a(n-1) - 3a(n-2) + a(n-3)$ . - [R. J. Mathar](#), Apr 28 2008

For  $n > 0$ :  $a(n-1) = \text{A143053}(\text{A000290}(n)) - 1$ . - [Reinhard Zumkeller](#), Jul 20 2008

$\text{A143053}(a(n)) = \text{A000290}(n+1)$ . - [Reinhard Zumkeller](#), Jul 20 2008

$a(n)*a(n-2) = (n-1)^4 + 4$ . - [Reinhard Zumkeller](#), Feb 12 2009

$a(n) = \text{A156798}(n)/\text{A087475}(n)$ . - [Reinhard Zumkeller](#), Feb 16 2009

From [Reinhard Zumkeller](#), Mar 08 2010: (Start)

$a(n) = \text{A170949}(\text{A002061}(n+1))$ ;

$\text{A170949}(a(n)) = \text{A132411}(n+1)$ ;

$\text{A170950}(a(n)) = \text{A002061}(n+1)$ . (End)

For  $n > 1$ ,  $a(n)^2 + (a(n) + 1)^2 + \dots + (a(n) + n - 2)^2 + (a(n) + n - 1 + a(n) + n)^2 = (n+1) * (6n^4 + 18n^3 + 26n^2 + 19n + 6) / 6 = (a(n) + n)^2 + \dots + (a(n) + 2n)^2$ . - [Charlie Marion](#), Jan 10 2011

From [Eric Werley](#), Jun 27 2011: (Start)

$a(n) = 2*a(n-1) - a(n-2) + 2$ .

$a(n) = a(n-1) + 2*n - 1$ . (End)

$a(n) = (n-1)^2 + 2(n-1) + 2 = 122$  read in base  $n-1$  (for  $n > 3$ ). - [Jason Kimberley](#), Oct 20 2011

$a(n)*a(n+1) = a(n*(n+1) + 1)$  so  $a(1)*a(2) = a(3)$ . More generally,  $a(n)*a(n+k) = a(n*(n+k) + 1) + k^2 - 1$ . - [Jon Perry](#), Aug 01 2012

$a(n) = (n!)^2 * [x^n] \text{BesselI}(0, 2*\sqrt{x}) * (1+x)$ . - [Peter Luschny](#), Aug 25 2012

$a(n) = \text{A070216}(n, 1)$  for  $n > 0$ . - [Reinhard Zumkeller](#), Nov 11 2012

E.g.f.:  $\exp(x) * (1 + x + x^2)$ . - [Geoffrey Critzer](#), Aug 30 2013

$a(n) = \text{A254858}(n-2, 3)$  for  $n > 2$ . - [Reinhard Zumkeller](#), Feb 09 2015

$\text{Sum}_{n \geq 0} 1/a(n) = (1 + \text{Pi} * \coth(\text{Pi}))/2 = 2.076674\dots = \text{A113319}$ . - [Vaclav Kotesovec](#), Apr 10 2016

$4*a(n) = \text{A001105}(n-1) + \text{A001105}(n+1)$ . - [Bruno Berselli](#), Jul 03 2017

From [Amiram Eldar](#), Jan 20 2021: (Start)

$\text{Product}_{n \geq 0} (1 + 1/a(n)) = \sqrt{2} * \text{csch}(\text{Pi}) * \sinh(\sqrt{2} * \text{Pi})$ .

$\text{Product}_{n \geq 1} (1 - 1/a(n)) = \text{Pi} * \text{csch}(\text{Pi})$ . (End)

EXAMPLE

G.f. =  $1 + 2x + 5x^2 + 10x^3 + 17x^4 + 26x^5 + 37x^6 + 50x^7 + 65x^8 + \dots$

MAPLE

```
A002522 := proc(n)
    numtheory[cyclotomic](4, n) ;
end proc;
```

```
seq(A002522(n), n=0..20) ; # R. J. Mathar, Feb 07 2014
```

MATHEMATICA

```
Table[n^2 + 1, {n, 0, 50}]; (* Vladimir Joseph Stephan Orlovsky, Dec 15 2008 *)
```

PROG

```
(Magma) [n^2 + 1: n in [0..50]]; // Vincenzo Librandi, May 01 2011
(PARI) a(n)=n^2+1 \\ Charles R Greathouse IV, Jun 10 2011
(Haskell)
```

$a002522 = (+ 1) . (^ 2)$   
 $a002522\_list = scanl (+) 1 [1, 3..]$   
 -- [Reinhard Zumkeller](#), Apr 06 2012  
 (Maxima) [A002522](#)(n):=n^2+1\$ makelist([A002522](#)(n), n, 0, 30); /\* [Martin Ettl](#),  
 Nov 07 2012 \*/

CROSSREFS  
 Left edge of [A055096](#).  
 Cf. [A059100](#), [A117950](#), [A087475](#), [A117951](#), [A114949](#), [A117619](#) (sequences of form  $n^2 + K$ ).  
 $a(n+1) = \text{A101220}(n, n+1, 3)$ .  
 Cf. [A059592](#), [A124808](#), [A132411](#), [A132414](#), [A028872](#), [A005408](#), [A000124](#), [A016813](#),  
[A086514](#), [A000125](#), [A058331](#), [A080856](#), [A000127](#), [A161701](#)-[A161704](#), [A161706](#),  
[A161707](#), [A161708](#), [A161710](#)-[A161713](#), [A161715](#), [A006261](#).  
 Moore lower bound on the order of a (k,g) cage: [A198300](#) (square); rows:  
[A000027](#) (k=2), [A027383](#) (k=3), [A062318](#) (k=4), [A061547](#) (k=5), [A198306](#) (k=6),  
[A198307](#) (k=7), [A198308](#) (k=8), [A198309](#) (k=9), [A198310](#) (k=10), [A094626](#)  
 (k=11); columns: [A020725](#) (g=3), [A005843](#) (g=4), this sequence (g=5),  
[A051890](#) (g=6), [A188377](#) (g=7). - [Jason Kimberley](#), Oct 30 2011  
 Cf. [A002496](#) (primes).  
 Cf. [A254858](#).  
 Cf. [A302612](#), [A302644](#), [A302645](#), [A302646](#).

KEYWORD  
 nonn,easy,changed  
 AUTHOR  
[N. J. A. Sloane](#)  
 EXTENSIONS  
 Partially edited by [Joerg Arndt](#), Mar 11 2010  
 STATUS  
 approved

## [A003132](#)

Sum of squares of digits of n.  
 (Formerly M3355)

+30  
 106

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 1, **2, 5, 10, 17, 26, 37, 50**, 65, 82, 4, 5, 8, 13,  
 20, 29, 40, 53, 68, 85, 9, 10, 13, 18, 25, 34, 45, 58, 73, 90, 16, 17, 20, 25, 32, 41, 52,  
 65, 80, 97, 25, 26, 29, 34, 41, 50, 61, 74, 89, 106, 36, 37, 40, 45, 52, 61, 72, 85, 100,  
 117, 49 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET  
 0,3

COMMENTS  
 It is easy to show that  $a(n) < 81 \cdot (\log_{10}(n)+1)$ . - [Stefan Steinerberger](#), Mar 25 2006  
 It is known that  $a(0)=0$  and  $a(1)=1$  are the only fixed points of this map.  
 For more information about iterations of this map, see [A007770](#), [A099645](#)  
 and [A000216](#) ff. - [M. F. Hasler](#), May 24 2009

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LINKS  
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 Arthur Porges, [A set of eight numbers](#), Amer. Math. Monthly 52 (1945), 379-382.  
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[Index entries for Colombian or self numbers and related sequences](#)

FORMULA  

$$a(n) = n^2 - 20 \cdot n \cdot \text{floor}(n/10) + 81 \cdot (\text{Sum}_{\{k>0\}} \text{floor}(n/10^k)^2) + 20 \cdot \text{Sum}_{\{k>0\}} \text{floor}(n/10^k) \cdot (\text{floor}(n/10^k) - \text{floor}(n/10^{k+1})).$$
 - [Hieronymus Fischer](#), Jun 17 2007

$a(10n+k) = a(n)+k^2$ ,  $0 \leq k < 10$ . - [Hieronymus Fischer](#), Jun 17 2007

$a(n) = \text{A007953}(\text{A048377}(n)) - \text{A007953}(n)$ . - [Reinhard Zumkeller](#), Jul 10 2011

MAPLE  
[A003132](#) := proc(n) local d; add(d^2, d=convert(n, base, 10)); end proc; #  
[R. J. Mathar](#), Oct 16 2010

MATHEMATICA  
 Table[Sum[DigitCount[n][[i]]\*i^2, {i, 1, 9}], {n, 0, 40}] (\* [Stefan Steinerberger](#), Mar 25 2006 \*)

Total/@(IntegerDigits[Range[0, 80]]^2) (\* [Harvey P. Dale](#), Jun 20 2011 \*)

PROG  
 (PARI) [A003132](#)(n)=norml2(digits(n)) \\ [M. F. Hasler](#), May 24 2009, updated Apr 12 2015

(Haskell)

[a003132](#) 0 = 0

[a003132](#) x = d ^ 2 + [a003132](#) x' where (x', d) = divMod x 10

-- [Reinhard Zumkeller](#), May 10 2015, Aug 07 2012, Jul 10 2011

(Magma) [0] cat [&+[d^2: d in Intseq(n)]: n in [1..80]]; // [Bruno Berselli](#), Feb 01 2013  
 (Python)  
 def [A003132](#)(n): return sum(int(d)\*\*2 for d in str(n)) # [Chai Wah Wu](#), Apr 02 2021

CROSSREFS Cf. [A052034](#), [A052035](#).  
 Cf. [A007953](#), [A055017](#), [A076313](#), [A076314](#).  
 Concerning iterations of this map, see [A003621](#), [A039943](#), [A099645](#), [A031176](#), [A007770](#), [A000216](#) (starting with 2), [A000218](#) (starting with 3), [A080709](#) (starting with 4, this is the only nontrivial limit cycle), [A000221](#) (starting with 5), [A008460](#) (starting with 6), [A008462](#) (starting with 8), [A008463](#) (starting with 9), [A139566](#) (starting with 15), [A122065](#) (starting with 74169). - [M. F. Hasler](#), May 24 2009  
 Cf. [A080151](#), [A051885](#) (record values and where they occur).  
 Cf. [A257588](#), [A332919](#).

KEYWORD nonn,easy,[look](#),base,nice

AUTHOR [N. J. A. Sloane](#)

EXTENSIONS More terms from [Stefan Steinerberger](#), Mar 25 2006  
 Terms checked using the given PARI code, [M. F. Hasler](#), May 24 2009  
 Replaced the Maple program with a version which works also for arguments with >2 digits, [R. J. Mathar](#), Oct 16 2010  
 Added ref to Porges. Steinhaus also treated iterations of this function in his Polish book Sto zadań, but I don't have access to it. - [Don Knuth](#), Sep 07 2015

STATUS approved

[A101337](#) Sum of (each digit of n raised to the power (number of digits in n)).

+30  
19

1, 2, 3, 4, 5, 6, 7, 8, 9, 1, **2, 5, 10, 17, 26, 37, 50**, 65, 82, 4, 5, 8, 13, 20, 29, 40, 53, 68, 85, 9, 10, 13, 18, 25, 34, 45, 58, 73, 90, 16, 17, 20, 25, 32, 41, 52, 65, 80, 97, 25, 26, 29, 34, 41, 50, 61, 74, 89, 106, 36, 37, 40, 45, 52, 61, 72, 85, 100, 117, 49, 50, 53, 58, 65 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS Sometimes referred to as "narcissistic function" (in base 10). Fixed points are the narcissistic (or Armstrong, or plus perfect) numbers [A005188](#). - [M. F. Hasler](#), Nov 17 2019

LINKS Michael De Vlieger, [Table of n, a\(n\) for n = 1..10000](#)  
 Wikipedia, [Narcissistic number](#), as of Nov 18 2019.

FORMULA  $a(n) \leq \text{A055642}(n) \cdot 9^{\text{A055642}(n)}$  with equality for all  $n = 10^k - 1$ . Write  $n = 10^x$  to get  $a(n) < n$  when  $1 + \log_{10}(x+1) < (x+1)(1 - \log_{10}(9)) \Leftrightarrow x > 59.85$ . It appears that  $a(n) < n$  already for all  $n > 1.02 \cdot 10^{59}$ . - [M. F. Hasler](#), Nov 17 2019

EXAMPLE  $a(75) = 7^2 + 5^2 = 74$  and  $a(705) = 7^3 + 0^3 + 5^3 = 468$ .  
 $a(1.02e59 - 1) = 102429587095122578993551250282047487264694110769657513064859 \sim 1.024e59$  is an example of n close to the limit beyond which  $a(n) < n$  for all n. - [M. F. Hasler](#), Nov 17 2019

MATHEMATICA Array[Total[IntegerDigits[#]^IntegerLength[#]]&, 80] (\* [Harvey P. Dale](#), Aug 27 2011 \*)

PROG (PARI) a(n)=my(d=digits(n)); sum(i=1, #d, d[i]^#d) \\ [Charles R Greathouse IV](#), Aug 10 2017  
 (PARI) apply( [A101337](#)(n)=vecsum([d^#n|d<-n=digits(n)]), [0..99]) \\ [M. F. Hasler](#), Nov 17 2019  
 (Python)  
 def [A101337](#)(n):  
     s = str(n)  
     l = len(s)  
     return sum(int(d)\*\*l for d in s) # [Chai Wah Wu](#), Feb 26 2019  
 (Magma) f:=func<n|&+[Intseq(n)[i]^#Intseq(n):i in [1..#Intseq(n)]]>; [f(n):n in [1..75]]; // [Marius A. Burtea](#), Nov 18 2019

CROSSREFS Cf. [A055642](#), [A179239](#), [A306360](#).

KEYWORD base,easy,nonn

AUTHOR [Gordon Hamilton](#), Dec 24 2004

EXTENSIONS Name changed by [Axel Harvey](#), Dec 26 2011

Edited by [M. F. Hasler](#), Nov 17 2019

STATUS approved

[A303372](#) Numbers of the form  $a^2 + b^6$ , with integers a, b > 0.

+30  
8



**2, 5, 10, 17, 26, 37, 50**, 65, 68, 73, 80, 82, 89, 100, 101, 113, 122, 128, 145, 164, 170, 185, 197, 208, 226, 233, 257, 260, 289, 290, 320, 325, 353, 362, 388, 401, 425, 442, 464, 485, 505, 530, 548, 577, 593, 626, 640, 677, 689, 730, 733, 738, 740, 745, 754, 765, 778 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 1,1

COMMENTS A subsequence of [A055394](#), the numbers of the form  $a^2 + b^3$ .  
Although it is easy to produce many terms of this sequence, it is nontrivial to check whether a very large number is of this form.

LINKS [Table of n, a\(n\) for n=1..57.](#)

EXAMPLE The first terms are  $1^2 + 1^6 = 2$ ,  $2^2 + 1^6 = 5$ ,  $3^2 + 1^6 = 10$ ,  $4^2 + 1^6 = 17$ ,  $5^2 + 1^6 = 26$ , ...,  $8^2 + 1^6 = 1^2 + 2^6 = 65$ ,  $2^2 + 2^6 = 68$ ,  $3^2 + 2^6 = 73$ , ...

PROG (PARI) is(n, k=2, m=6)=for(b=1, sqrtint(n-1, m), ispower(n-b^m, k)&&return(b)) \\ Returns b > 0 if n is in the sequence, else 0.  
[A303372](#) vec(L=10^5, k=2, m=6, S=List())={for(a=1, sqrtint(L-1, m), for(b=1, sqrtint(L-a^m, k), listput(S, a^m+b^k))); Set(S)} \\ List of all terms up to limit L

CROSSREFS Cf. [A055394](#) ( $a^2 + b^3$ ), [A111925](#) ( $a^2 + b^4$ ), [A100291](#) ( $a^4 + b^3$ ), [A100292](#) ( $a^5 + b^2$ ), [A100293](#) ( $a^5 + b^3$ ), [A100294](#) ( $a^5 + b^4$ ).  
Cf. [A303373](#) ( $a^3 + b^6$ ), [A303374](#) ( $a^4 + b^6$ ), [A303375](#) ( $a^5 + b^6$ ).

KEYWORD nonn,easy

AUTHOR [M. F. Hasler](#), Apr 22 2018

STATUS approved

[A082607](#)  $a(0)=1$ ; for  $n > 0$ ,  $a(n)$  = least  $k$  not included earlier such that  $k*a(n-1) - 1$  is a square.

+30  
3

**1, 2, 5, 10, 17, 26, 37, 50**, 65, 34, 13, 25, 41, 61, 85, 113, 145, 122, 101, 82, 293, 634, 1105, 53, 109, 185, 74, 149, 250, 377, 205, 146, 97, 58, 29, 73, 137, 221, 181, 650, 541, 442, 353, 274, 953, 2042, 3541, 5450, 409, 173, 370, 289, 218, 157, 106, 337, 698 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,2

LINKS [Table of n, a\(n\) for n=0..56.](#)

MATHEMATICA  $l = \{1\}$ ; Do[k = 1; While[MemberQ[l, k] || !IntegerQ[Sqrt[k\*Last[l]-1]], k++]; AppendTo[l, k], {n, 50}]; l (\* [Ryan Propper](#), Jun 13 2006 \*)

PROG (PARI) a=[1]; print1(1, " "); for(n=2, 100, k=1; f=1; while(f, if(issquare(k\*a[n-1]-1), f=0; for(i=1, n-1, if(a[i]==k, f=1))); k++); a=concat(a, k-1); print1(k-1, " ")) \\ Herman Jamke (hermanjamke(AT)fastmail.fm), May 01 2007

CROSSREFS Cf. [A082608](#), [A082609](#), [A082610](#), [A082611](#), [A082612](#).

KEYWORD nonn

AUTHOR [Amarnath Murthy](#), Apr 28 2003

EXTENSIONS Corrected and extended by [Ryan Propper](#), Jun 13 2006

Definition corrected by [R. J. Mathar](#), Nov 12 2006

More terms from Herman Jamke (hermanjamke(AT)fastmail.fm), May 01 2007

STATUS approved

[A160457](#)  $a(n) = n^2 - 2*n + 2$ .

+30  
3

**2, 1, 2, 5, 10, 17, 26, 37, 50**, 65, 82, 101, 122, 145, 170, 197, 226, 257, 290, 325, 362, 401, 442, 485, 530, 577, 626, 677, 730, 785, 842, 901, 962, 1025, 1090, 1157, 1226, 1297, 1370, 1445, 1522, 1601, 1682, 1765, 1850, 1937, 2026, 2117, 2210, 2305, 2402, 2501, 2602, 2705, 2810 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,1

COMMENTS Competition number of the complete bipartite graph  $K_{n,n}$ .  
Formula given on p. 3 of Sano.

LINKS [Table of n, a\(n\) for n=0..54.](#)

Yoshio Sano, [The competition numbers of regular polyhedra](#), arXiv:0905.1763 [math.CO], 2009.

[Index entries for linear recurrences with constant coefficients](#), signature (3, -3, 1).

FORMULA  $a(n) = a(n-1) + 2*n - 3$  (with  $a(0)=2$ ). - [Vincenzo Librandi](#), Dec 03 2010

$a(n) = +3*a(n-1) - 3*a(n-2) + a(n-3)$ .

G.f.:  $-(2-5*x+5*x^2)/(x-1)^3$ .

$a(n) = \text{A002522}(n-1)$ . - [Michel Marcus](#), Feb 03 2016

MATHEMATICA Table[n^2-2\*n+2, {n, 0, 5!}] (\* [Vladimir Joseph Stephan Orlovsky](#), Dec 29 2010 \*)

LinearRecurrence[{3, -3, 1}, {2, 1, 2}, 60] (\* [Harvey P. Dale](#), Mar 29 2015 \*)

PROG (PARI) vector(100, n, n--; n^2 - 2\*n + 2)

CROSSREFS Cf. [A002522](#), [A160450](#).

KEYWORD easy,nonn

AUTHOR [Jonathan Vos Post](#), May 14 2009

EXTENSIONS More terms from [Vincenzo Librandi](#), Nov 08 2009  
Sequence corrected by [Joerg Arndt](#), Dec 03 2010

STATUS approved

[A300164](#) Numbers of the form  $n^2+1$  not expressible as  $j^2+k^2$  with  $j>k>1$ .

+30  
3

**2, 5, 10, 17, 26, 37, 50,** 82, 101, 122, 197, 226, 257, 362, 401, 577, 626, 677, 842, 1226, 1297, 1522, 1601, 1682, 2026, 2402, 2602, 2917, 3137, 3482, 3722, 4226, 4357, 4762, 5042, 5477, 6242, 7057, 7226, 8101, 8837, 9026, 10202, 12101, 13457, 14401 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 1,1

LINKS Hugo Pfoertner, [Table of n, a\(n\) for n = 1..10000](#)

EXAMPLE The first numbers of the form  $n^2 + 1$  not in the sequence are:  
65 =  $8^2 + 1$  because it can be expressed as  $65 = 7^2 + 4^2$ ,  
145 =  $12^2 + 1 = 9^2 + 8^2$ ,  
170 =  $13^2 + 1 = 11^2 + 7^2$ .

CROSSREFS Cf. [A050796](#), [A065876](#), [A071557](#), [A299708](#).

KEYWORD nonn

AUTHOR [Hugo Pfoertner](#), Feb 27 2018

STATUS approved

[A322008](#)  $1/(1 - \text{Integral}_{x=0..1} x^x(x^n) dx)$ , rounded to the nearest integer.

+30  
3

**2, 5, 10, 17, 26, 37, 50,** 65, 82, 101, 123, 146, 171, 198, 227, 258, 291, 326, 364, 403, 444, 487, 532, 579, 628, 679, 733, 788, 845, 904, 965, 1028, 1093, 1160, 1230, 1301, 1374, 1449, 1526, 1605, 1686, 1769, 1855, 1942, 2031, 2122, 2215, 2310, 2407, 2506, 2608 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,1

COMMENTS Linked to the problem of sorting parenthesized expressions  $(x^x \dots x)$  (cf. [A000081](#) and [A222379](#), [A222380](#)) according to the value of their integral from 0 to 1: This value is maximal, for a given number  $n$  of  $x$ 's, for  $F[n](x) := ((x^x)^x \dots)^x = x^{(x^{(n-1)})}$ , which converges pointwise to  $x^0 = 1$  for all  $x < 1$ , as  $n \rightarrow \infty$ . The corresponding integrals therefore tend to 1 as  $n \rightarrow \infty$ . This sequence is a convenient measure of the distance of these integrals from 1.  
See [A322009](#) for the minimal values of such integrals.

LINKS [Table of n, a\(n\) for n=0..50](#).  
Vladimir Reshetnikov, [Integrals of power towers](#), on MathOverflow.net, Feb. 26, 2019.

FORMULA Conjectures from [Colin Barker](#), Mar 07 2019: (Start)  
G.f.:  $(2 + x + 2x^2 + 2x^3 + 2x^4 + 2x^5 + 2x^6 + 2x^7 + x^9 + x^{10} - x^{11}) / ((1 - x)^3(1 + x)(1 + x^2)(1 + x^4))$ .  
 $a(n) = 2a(n-1) - a(n-2) + a(n-8) - 2a(n-9) + a(n-10)$  for  $n > 11$ .  
(End)

EXAMPLE For  $n=0$ ,  $\text{Integral}_{x=0..1} x^x(x^0) dx = \text{Integral}_{x=0..1} x^1 dx = 1/2$ , so  $a(0) = 1/(1 - 1/2) = 1 / 0.5 = 2$ .  
For  $n=1$ ,  $\text{Integral}_{x=0..1} x^x(x^1) dx = \text{Integral}_{x=0..1} x^x dx = \text{A083648} = 0.78343\dots$ , so  $a(1) = \text{round}(1 / (1 - 0.78343\dots)) = \text{round}(1 / 0.21656\dots) = 5$ .

MAPLE  $a := n \rightarrow \text{round}(\text{evalf}(1/(1 - (\text{int}(x^x(x^n), x=0..1))))):$   
 $\text{seq}(a(n), n=0..50);$  # [Alois P. Heinz](#), Mar 01 2019

MATHEMATICA  $f[n_] := \text{Round}[1/(1 - \text{NIntegrate}[x^x(x^n), \{x, 0, 1\}])]; \text{Array}[f, 51, 0] (*$   
[Robert G. Wilson v](#), Mar 01 2019 \*)

PROG (PARI) apply( [A322008](#)(n)=1\intnum(x=0, 1, 1-x^x^n), [0..50])

CROSSREFS Cf. [A322009](#); [A000081](#), [A222379](#), [A222380](#), [A306679](#); [A083648](#).

KEYWORD nonn

AUTHOR [M. F. Hasler](#), Mar 01 2019

STATUS approved

[A327194](#) For any  $n \geq 0$ : consider the different ways to split the binary representation of  $n$  into two

+30  
3

(possibly empty) parts, say with value  $x$  and  $y$ ;  $a(n)$  is the least possible value of  $x^2 + y^2$ .

0, 1, 1, 2, 1, 2, 5, 10, 1, 2, 5, 10, 9, 10, 13, 18, 1, 2, 5, 10, 17, 26, 29, 34, 9, 10, 13, 18, 25, 34, 45, 58, 1, **2, 5, 10, 17, 26, 37, 50**, 25, 26, 29, 34, 41, 50, 61, 74, 9, 10, 13, 18, 25, 34, 45, 58, 49, 50, 53, 58, 65, 74, 85, 98, 1, 2, 5, 10, 17, 26, 37 ([list](#);  
[graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,4

COMMENTS This sequence shares graphical features with [A286327](#).

LINKS Rémy Sigrist, [Table of  \$n\$ ,  \$a\(n\)\$  for  \$n = 0..8192\$](#)

FORMULA  $a(n) = 1$  iff  $n$  is a power of 2.

EXAMPLE For  $n=42$ :

- the binary representation of 42 is "101010",
- there are 7 ways to split it:
  - "" and "101010":  $x=0$  and  $y=42$ :  $0^2 + 42^2 = 1764$ ,
  - "1" and "01010":  $x=1$  and  $y=10$ :  $1^2 + 10^2 = 101$ ,
  - "10" and "1010":  $x=2$  and  $y=10$ :  $2^2 + 10^2 = 104$ ,
  - "101" and "010":  $x=5$  and  $y=2$ :  $5^2 + 2^2 = 29$ ,
  - "1010" and "10":  $x=10$  and  $y=2$ :  $10^2 + 2^2 = 104$ ,
  - "10101" and "0":  $x=21$  and  $y=0$ :  $21^2 + 0^2 = 441$ ,
  - "101010" and "":  $x=42$  and  $y=0$ :  $42^2 + 0^2 = 1764$ ,
- hence  $a(42) = 29$ .

PROG (PARI) a(n) = my (v=oo, b=binary(n)); for (w=0, #b, v=min(v, fromdigits(b[1..w], 2)^2 + fromdigits(b[w+1..#b], 2)^2)); v

CROSSREFS See [A327186](#) for other variants.  
Cf. [A286327](#).

KEYWORD nonn,[look](#),base

AUTHOR [Rémy Sigrist](#), Aug 25 2019

STATUS approved

[A159547](#) Smallest number  $b$  such that the number whose digits are  $n$  in base  $b$  is a skinny number.

+30  
0

**2, 5, 10, 17, 26, 37, 50**, 65, 82, 2, 3, 5, 10, 17, 26, 37, 50, 65, 82, 5, 5, 9, 13, 17, 26, 37, 50, 65, 82, 10, 10, 13, 19, 25, 31, 37, 50, 65, 82, 17, 17, 17, 25, 33, 41, 49, 57, 65, 82, 26, 26, 26, 31, 41, 51, 61, 71, 81, 91, 37, 37, 37, 37, 49, 61, 73, 85, 97, 109 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 1,1

COMMENTS I assume that "the number whose digits are  $n$  in base  $b$ " means the number  $\sum c_i b^i$ , where the decimal expansion of  $n$  is  $\sum c_i 10^i$ . - [N. J. A. Sloane](#), Jun 19 2021

LINKS [Table of  \$n\$ ,  \$a\(n\)\$  for  \$n=1..69\$](#) .

FORMULA  $a(n) \leq 10$  iff  $n$  is in [A061909](#).

EXAMPLE  $a(10) = 2$  because  $10^2 = 100$  in all bases  $\geq 2$ .

$a(14) = 17$  because  $14_{16} = 20_{10}$ , so the square is  $400_{10} = (1,9,0)_{16}$ , but  $\text{digitsum}((1,9,0)_{16}) = 10 \neq \text{digitsum}((1,4)_{16})^2$ ; while in base 17,  $14_{17} = 21_{10}$ , so the square is  $441_{10} = (1,8,16)_{17}$  and  $\text{digitsum}((1,8,16)_{17}) = 25 = \text{digitsum}((1,4)_{17})^2$ .

PROG (PARI) a(n) = my(d=digits(n), s); s=vecsum(d); for(b=1+vecmax(d), oo, if(s^2==sumdigits(fromdigits(d, b)^2, b), return(b))); \ [Jinyuan Wang](#), Jun 19 2021

CROSSREFS Cf. [A061909](#).

KEYWORD nonn,base

AUTHOR [J. Lowell](#), Apr 14 2009

EXTENSIONS More terms from [Jinyuan Wang](#), Jun 19 2021

STATUS approved

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