Design Optimisation of the IC Racing Green Shell Eco Marathon Vehicle

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Abstract

An optimisation study was conducted to minimise the mass of the chassis and the gears in the drivetrain and maximise the efficiency of the chain. The models used consisted of metamodeling and mathematical approaches and a weighted system level optimisation was conducted to optimise the entire vehicle for fuel economy.

1. Introduction

We contacted the chief engineer of Imperial College Racing Green, offering our services to optimise certain aspects of the Shell Eco Marathon vehicle. He pointed us in the direction of a study that was conducted last year to better understand the vehicles fuel economy. The main areas analysed were: supercapacitor size, powertrain inefficiencies, and race strategies. Little work was done to understand lower level efficiencies beyond those such as losses due to the drivetrain or free-rolling friction. Therefore, there was room for further exploration within the vehicle itself to optimise various parameters, such as mass, efficiency, etc. within the subsystems; Drivetrain, and Structure. In the same study, it was found that rolling resistance and drivetrain losses account for nearly 35% of the total energy loses in the vehicle. It is for this reason we chose to optimise these two subsystems.

One beam in the chassis located near the drivetrain was chosen to simplify the subsystem. It is joined to another beam which is the support for the motor and one gear of the drivetrain. By optimising the beam for mass, certain geometric changes may occur, that would lead to a modification in the centre distance between the two gears in the drivetrain subsystem. Thus, we expect certain geometric trade-offs between the two subsystems, that would lead to the conflict at a system level. Figure 1 shows this potential conflict.

System Level Optimisation Goal:

"To optimise the system for fuel economy, by optimising the subsystems; Chassis & drivetrain for minimum mass and maximum efficiency. "

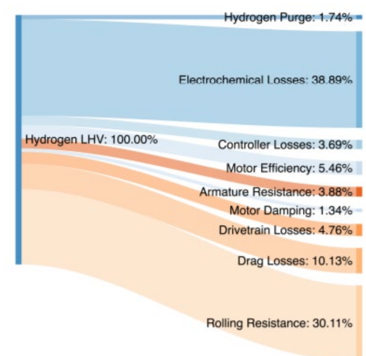

Figure 1: Sankey diagram of energy losses in the vehicle [1]

2. System-level problem and Subsystem breakdown

Maximise:

$$Efficiency(R_d, C) = \frac{P_i - (N_t \cdot \omega_o \cdot W_{rk})}{P_i}$$

Therefore Minimise:

$$Efficiency(R_d, C) = \frac{P_i}{P_i - (N_t \cdot \omega_o \cdot W_{rk})}$$

& Minimise:

$$M_{chassisbeams} = P \times \pi\,(R2 - r2) \times B/sin(a)$$

Therefore:

$$F_{Rd} = w_1 Efficiency(R_d, C) + w_2 M(P, R2, r2, a)$$

For definitions of variables, please see either the table of nomenclature, or the sub-system optimisation sections.

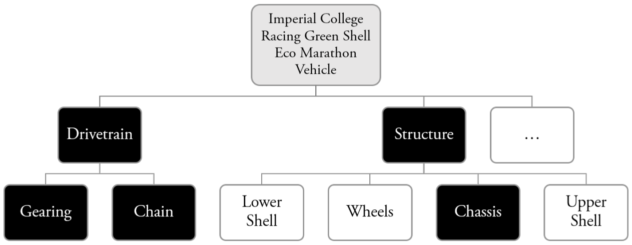

Figure 2: Component partitioning

Figure 2 shows the component partitioning of the system. The two key areas we chose to focus on are the drivetrain and chassis as mentioned. Within the drivetrain we are looking into both the gearing, aiming to minimise the mass of the gears, as well as the chain, to maximise its efficiency.

There are interdependencies between the two subsystems, and these can be seen in Figure 3.

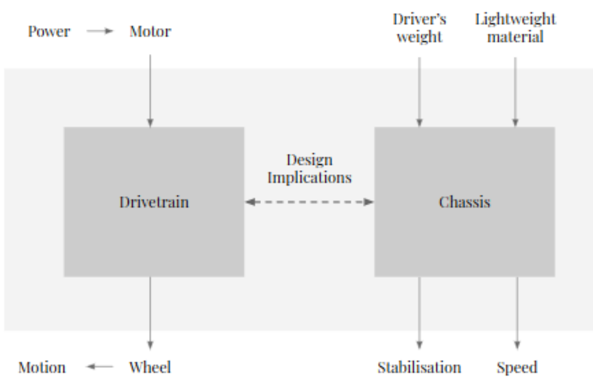

Figure 3: System level interdependencies

3. Subsystem 1: Vehicle Chassis

Rolling resistance loses account for nearly a third of the total of energy loses in the shell eco marathon. [1] It is a function of the mass of the vehicle and the coefficient of rolling friction [2]. Hence to reduce these losses, the overall mass of the vehicle must be minimised, by focussing on chassis, the heaviest component [1].

To ease on the complexity of the system, we chose to focus on just one beam in the structure that could have conflict or trade-offs with the second subsystem, i.e., drivetrain. This beam XZ is shown in Figure 4.

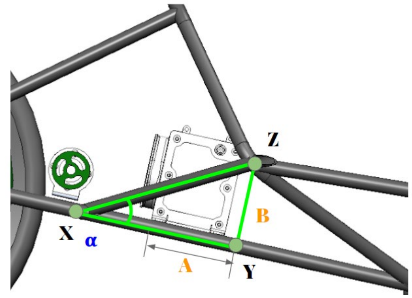

Figure 4: Chassis subsystem overview

Modelling of the objective function and constraints utilise a mathematical as well as a metamodeling approach. Data for the metamodeling was obtained by carrying out static simulations on a Solidworks model of the vehicle chassis.

3.1 Optimisation formulation and Modelling Approach

Min $\quad M = P \times \pi (R^2 - r^2) \times B/sin(\alpha)$

where $\quad B = 0.11\ m$

$A = 0.127\ m$

$SF = 10$

Subject to:

$g_1(\alpha):\ 1 - cos(\alpha) - sin(\alpha) \leqslant 0$

$g_2(\alpha): Atan(\alpha) - B \leqslant 0$

$g_3(r, R, \alpha): (\beta_0 + \beta_1 * \alpha + \beta_2 * \alpha^2)/(pi * (R^2 - r^2)) - YS/SF$

$g_4(r, R, \alpha, P): (\beta_0 + \beta_1 * \alpha + \beta_2 * \alpha^2)/(pi * (R^2 - r^2)) - pi^2 * E * (sin(\alpha))^2 * (R^2 - r^2)/(SF * B^2)$

$g_5(r, R):\ r - R = 0.0015$

$g_6(R):\ 0 \leqslant R \leqslant 0.04$

$g_7(r): 0 \leqslant r \leqslant 0.04$

$g_8(\alpha): 0 \leqslant \alpha \leqslant pi/2$

$g_9(P):\ P = [2700;2810;7200;8027]$

Objective Function:

Minimise mass - This equation is of the form of first principle equation:

Mass (M) = Density (P) x Volume

Volume in turn is the product of cross sectional area and length of the beam, which is a function of B and variable α.

Variables:

The optimisation problem has 4 variables. These include: r, Inner radius of the beam, R, Outer radius of the beam P, Density, a material property of the beam, α, angle; formed between beam XZ and beam XY in Figure 4.

r, R and α, are continuous variables, whereas, P is a discrete variable and takes 4 values. The follow materials (Table 1) were considered beam materials commonly used in go kart and bicycle chassis':

Table 1: Material choices

Material	Density kg/m^3	Young's Modulus Pa	Yield Stress Pa
6061 Al Alloy	2700	6.9*e10	55148500
7075 (T6) Al Alloy	2810	7.2*e10	505000000
Plain Carbon Steel	7200	2.1*e11	220594000
Stainless Steel	8027	2.0*e11	170000000

Constraints:

g1: Triangle inequality **-** The theorem states that "the sum of the lengths of any two sides of a triangle is greater than the length of the third side." [3] This constraint ensures that the triangle XYZ, in Figure 4, remains.

g2: Lateral movement of joint P **-** This is a geometric constraint. Due to the positioning of the fan in the eco marathon assembly, vertex X in triangle XYZ (Figure 4) is restricted in terms of lateral movement. Hence, the minimum length of side XY is A.

g3: Stress in beam **-** This constraint examines the failure criteria of the beam. To establish a relationship between r, R, α and stress, a metamodel was created. It is not possible to introduce α into the first principle equation of stress analytically. However, we know that it is related to force.

Steps taken for metamodeling:

Step 1 - Optimal Latin Hypercube Sampling: This method was utilised to draw a representative sample from the parameter space of R, r and α.

Step 2 - Simulations and data collection: Simulation were conducted on the Solidworks part of the chassis of the vehicle. 40 different variations of this model were created, based on r, R and α, and these were used to obtain axial stress in the beam.

Step 3 - Force for all values of Stress, r and R: The first principle equation of Stress was used to calculate Force. [5]

$$Force \;=\; Stress * pi * (R^2 - r^2)$$

Step 4 - Split data: Data was then split into test and train data, in the ratio of 75:25.

Step 5 - Polynomial linear regression between alpha and Force: Force vs. α was plotted and engineering intuition was used to fit the data with a polynomial. Adjusted R^2 values (0.938) were calculated and plots generated (Figure 5).

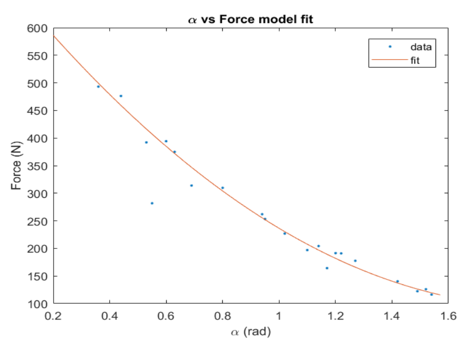

Figure 5: Alpha vs. Force model fit

Step 6 - Substitute Force as a function of alpha in the stress equation:

$$Stress \;=\; \frac{\beta_0 + \beta_1\alpha + \beta_2\alpha}{pi * (R^2 - r^2)}$$

Step 7 -Stress less than yield stress: The equation is written in negative null form with the yield stress of the material dependant on the material choice P. A safety factor of 10 is applied to the equation, accounting for the stress due to acceleration and minor impact forces.

g4: Critical buckling stress in beam - Another failure criterion is buckling. This constraint was introduced to find out the true failure criterion of the beam. The equation for critical buckling stress is [4]:

$$Stress = pi^2 * E * ((R - r)/l)^2$$

g5: Beam wall thickness - This equation defines that the pipe must have a wall thickness of 1 mm as this is the minimum wall thickness for manufacturing [6].

g 6,7,8: R, r,α Positivity constraints

g9: Discrete material choice - A set of materials were chosen and is described in Table 1.

Parameters:

B = 0.11 m; Height of the triangle, (Figure 4)

A = 0.127 m; The minimum length of base, (Figure 4)

SF = 10; Accounts for any forces due to drag or acceleration as well as minor crash forces.

Assumptions:

B cannot change - Height of the point at which the beam connect to frame is fixed. This was done to simplify the design space.

Connecting beam moves along with joint - The beam that supports the motor and drivetrain, and provides the conflict between the 2 subsystems, moves along with point P when alpha changes. This is for equal distribution of load and added support.

Thickness of the beam - Fixed to 1mm to obtain minimum values for variables R and r. Further discussion in section 3.3.

3.2 Explore the problem space

A monotonicity analysis was performed prior to running the simulations. Refer to interim presentation for the table.

It was concluded that by solving g1, g2 and g8 for alpha, the dominant constraint was g2. Therefore, g2 is an active constraint. Additionally, g6 and g7 are dominated by g3, g4 and g5 as they set the limit for the upper and lower bound of r and R.

To find out which of g3 and g4 are the dominant constraint, buckling stress and yield stress were calculated at the minimum point obtained after optimisation. It was found that critical buckling stress was always less than Yield stress, proving that it is the first failure criterion, therefore being the active constraint in the problem. This was verified with constraint activity.

The simplified problem now contains only constraints g2, g4, g5 and g9.

3.3 Optimise

As the problem was a nonlinear, constrained optimisation problem, the first algorithm tested was fmincon. For computational efficiency, the solver was run 4 times for the 4 different materials. At the start, the constraint g5 was set us an inequality:

$$r - R - 0.0015 \leq 0$$

This was done to allow some freedom for the algorithm to choose values for r and R. However, this resulted in different values for r and R at different starting points, and hence resulted in different mass outputs. To solve this issue, genetic algorithm (GA) was used which not only allows discrete variables as inputs, but also provides the global minimum within the range of input values. The output for this is shown in Table 2.

Table 2: GA outputs

Iteration	R	r	α	P	M
1	0.0333	0.0332	0.7138	2810	0.0096
2	0.0054	0.0048	0.7138	2810	0.0096
3	0.0158	0.0156	0.7138	2810	0.0096

Iteration 1, 2 and 3 all have the same optimum mass and optimiser values for alpha and density. However, the values of R and r change for every iteration. This is because there are many combinations of R and r for a given same mass. Hence, g5 was set as an equality constraint in fmincon (not possible in GA), with the starting point of alpha set as the output from genetic algorithm. This provided the results shown in Table 3.

Table 3: Comparison of results before and after optimisation

	R (m)	r (m)	α (rads)	$P\,(\frac{kg}{m3})$	M (kg)
Starting	0.01	0.0085	0.506	7200	0.15
Optimum	0.0038	0.0028	0.7138	2810	0.0096

The hessian matrix and respective eigenvalues were then found to be all positive definite, which indicated a local minimum was found within the range of values. This was then verified, by plotting a combination of variables against mass (Figure 6), and the result was convex which indicated a true local minimum.

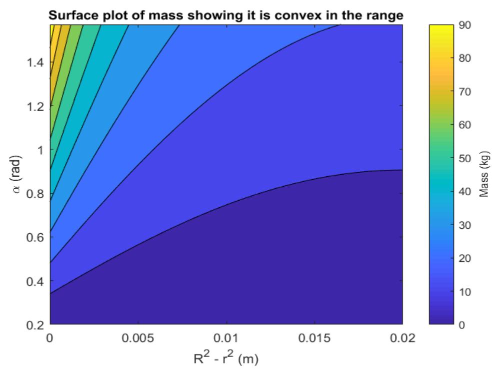

Figure 6: Surface plot showing convex nature

3.4 Discussion

The objective of this subsystem optimisation was to minimise the mass of a beam, hence reducing the overall mass of the chassis, leading to a decrease in rolling resistance losses.

Genetic algorithm helped with identifying the global minimum of mass as well as 2 of the variables, α and P, within the limits of the problem space. These outputs were then used to find the minimum of r and R in fmincon.

The overall mass has reduced to 9.7 g from an initial 150g. This is a substantial reduction in the mass of the chassis, leading to a decrease in rolling resistance.

The key challenge faced was finding the optimum values for R and r, before it was set to an equality constraint. In the future, safety factor of the stress in the beam can be incorporated as a constraint. Additionally, there is scope for accounting for crash conditions.

4. Subsystem 2: Drivetrain

The second subsystem was the drivetrain, which was to be optimised for maximum efficiency, while also minimising the mass of the driven gear (Figure 4). As shown in figure [1], losses due to the efficiency of the drivetrain accounted for 4.76% of all losses, whereas mass accounted for over 30% of loss. Thus, optimising both systems of the drivetrain resulted in an even better optimisation of the entire system.

For the efficiency, a purely mathematical formula was modelled, and optimised using gradient descent methods. The mass model was partly meta-modelled, optimised with a gradient descent method, while approximating discrete variables as continuous. Finally, the multi objective problem was combined utilising a genetic multi-objective problem solver to achieve a pareto front, and derive a solution based on the 4.76% - 30.11%weightings.

4.1 Optimisation formulation

Maximise:

$$Efficiency(R_d, C) = \frac{P_i - (N_t \cdot \omega_o \cdot W_{rk})}{P_i}$$

Therefore Minimise:

$$Efficiency(R_d, C) = \frac{P_i}{P_i - (N_t \cdot \omega_o \cdot W_{rk})}$$

& Minimise:

$$Mass(N_b, N_t, W, B, R_d) = \varphi(N_b, N_t, W, B, R_d)$$

Where:

$$W_{rk} = \frac{\frac{\tau}{R_d} + mg\sqrt{C^2 + \frac{21}{11}{R_d}^2} + m \cdot {R_d}^2 \cdot {\omega_d}^2}{\sqrt{1 + {\mu_p}^2}} \cdot {\mu_p}^2 \cdot r_b \cdot \frac{2\pi}{N_t}$$

$$p_1 = \rho,\ p_2 = P,\ p_3 = Dr,\ p_4 = \beta,\ p_5 = r_{di},$$

$$p_6 = \tau,\ p_7 = P_i,\ p_8 = \omega_o,\ p_9 = \omega_d,\ p_{10} = m,$$

$$p_{11} = \mu_p,\ p_{12} = r_b,\ p_{13} = \alpha_m,\ p_{14} = N_d$$

$$p_{15} = R_o, \qquad p_{16} = \frac{32}{11} = \frac{R_o}{R_d} = \frac{N_t}{N_d}$$

Subject to:

$$g_1(N_t, R_d) = \frac{N_t \cdot P}{2000\pi} \cdot \frac{11}{32} - R_d = 0$$

$$g_2(R_d) = \frac{P}{1000} - R_d \cdot \alpha_m \leq 0$$

$$g_3(R_d) = R_d - 0.05 \leq 0$$

$$g_4(R_d, C) = 0.254 + R_d - C \leq 0$$

$$g_5(R_d, C) = R_d + C - 0.379 \leq 0$$

$$g_6(N_b, W) = W \cdot N_b - 58.3\pi \leq 0$$

$$g_7(W) = W - 2 \cdot r_{di} \leq 0$$

$$g_8(W, R_d) = W - 2\sqrt{8.93 \cdot \frac{32}{11} \cdot R_d - 69.35} \leq 0$$

$$g_9(W) = 2 - W \leq 0$$

$$g_{10}(R_d, C) = 17.14 + 1.275C - 2.78 \times 10^8 R_d \leq 0$$

$$g_{11}(C) = C - 0.28 \leq 0$$

$$g_{12}(N_b, N_t, W, B) = \emptyset(N_b, N_t, W, B) - 3 \leq 0$$

The Efficiency model was developed by C J Lodge and Burgess [7] and is based only on frictional losses within the roller chain pin and bush. It has been slightly adapted to include the mass of the chain which is assumed as an extra force in the chain tension. W_{rk} C and R_d are the variables. Efficiency is calculated based on power in minus power losses due to friction.

The other objective function, *Mass*, is unknown due the gears complex geometry. The variables are shown in Figure 7. Various parameters were assumed, such as the constant maximum torque of the Maxon DC 200W motor, the pitch of the chain, various gear dimensions, and chain mass per length, as these values were less significant in altering the objective functions. The ratio between driven and driving gear was assumed to be constant at 32:11, as this was pre-calculated by ICRG.

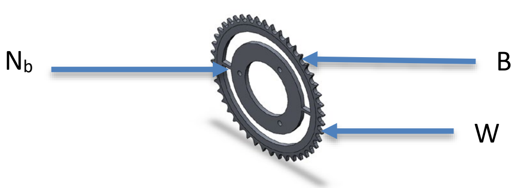

Figure 7: Rear Driven Gear

4.2 Modelling approach

The aim was to maximise the efficiency equation; thus, the inverse was taken to minimise. This equation is linked to subsystem 1: Vehicle Chassis, via variable C, as the driving motor is mounted to a beam connected to the section optimised. As the mass of subsystem 1 decreases, so does the efficiency of the drivetrain. The mass model was found through meta-modelling and linear regression but could not be calculated from first principles due to its complex geometry. It was possible to treat the beams as a separate section and calculate their mass assuming they were rectangular. The remaining mass could then be linearly meta-modelled. FEA was used to model the gear, although it should be noted that Solidworks does not consider non-linearity of material deformation. While the stresses in this case did not reach critical levels, it is always worth physically testing predictions.

g1 is the geometric relationship between N_t and R_d based on the gear module and pitch equation.
g2 finds the minimum wrapping angle of the driving gear, thus defines the lower bound for R_d.
g3 is a geometric constraint on R_d to prevent interference with the beam below.
g4, g5 are to ensure the centre distance is kept within bounds between the wheel and fan.
g6 prevents the gear beams overlapping and increasing mass unnecessarily, as it was proven through FEA in Solidworks that the minimum safety factor was achievable without this added material.
g7 ensures the beam width does not exceed the inner circle radius, which would result in 2 beams.
g8 is an upper bound on gear beam width to prevent interference with the teeth and is based on Pythagorean principles.
g9 relates to the manufacturing process of sprockets. Laser cutting machines possess a kerf width of up to 1 mm [8]. Thus, as a safety precaution, the beam width was limited to 2mm.
g10 is derived from the regular gear stress equation, to ensure the maximum driving gear stress was not exceeded.
g11 is a geometric constrain to prevent the motor encountering the wheel.
g12 is the equation for the safety factor of the driven sprocket, which must be greater than 3 for gears [9]. As the gear being optimised, with 5 variables, the equation is irregular and was meta-modelled using non-linear regression alongside the mass objective function.

4.3 Explore the problem space

A monotonicity analysis was conducted for the Efficiency model. The graphical plot of these 2 functions separately w.r.t 1/efficiency, found they were increasing either side of y, and since Rd and C > 0, and < 1, they can therefore be assumed to be increasing. It was also found that the acceptable lower bound for Nt is 12, to avoid severe chordal action [9], thus Nt was substituted into G2, which evaluated as a lower bound for Rd = 0.01212. Please refer to interim presentation for the monotonicity table.

G10 was active with respect to Rd, and used to eliminate Rd, leaving Efficiency(C). Leaving G4, G5, and G11, a constraint activity was conducted to find the dominant constraints, eliminate G4, which output 0.266 as a lower bound, compared to 0.28 from G11. The solution space of C was evaluated to be 0.28 ≤ C ≤ 0.367, for the updated model where:

$$R_d = \frac{17.14 + 1.275C}{2.78 \times 10^8}$$

R_d was used to find the upper bound of N_t, by evaluating G3 as 143. The updated equation was ready for optimisation.

To obtain an equation for the mass, the gear was split into the beams, and the rest. The beams mass could be calculated by modelling as cuboids as functions of *W* & N_b, due to their small size, such that:

$$Mass_{beams} = \frac{W}{1000} \cdot N_b \cdot \left(0.625P - 0.5Dr + \frac{0.5P}{\sin\frac{\pi}{N_t}} - \beta - r_{di}\right) \cdot \frac{B}{1000} \cdot \rho$$

A linear regression was performed to derive the formula of the rest of the gear. A Latin hypercube sample of 20 solutions was generated between 0 – 1, then scaled to the upper and lower bounds of N_t & *B.* A 75:25 train test data set was used, and a 3D plot of the 2 independent and 1 dependant variables found a strong linear correlation. Normalisation of the data further improved the fit, with R^2 = 0.9930 and adjusted R^2 = 0.9947 on the test set. Thus, the final mass equation was as follows:

$$Mass_{gear} = Mass_{beams} + unnorm(1.0109N_{tnorm} + 0.3407B_{norm})$$

Where 'unnorm' is an unpacking function to transpose the data set back to the solution space.

Regarding the calculation of the safety factor constraint (G12), monotonicity was not possible. G1 was used to eliminate R_d, reducing to only 4 variables, where a Latin hypercube sample of 40 (for 4 variables) generated scaled solutions for inputting into Solidworks [10]. A nonlinear regression was trialled but returned R^2 = 0.03 on the test data set. Thus, the function polyfitn was used to construct a model of differing polynomial combinations, of order 2. Higher orders caused overfitting. It should be mentioned that due to the nature of the solver, the inputs had to be greater than 0, thus normalisation was not possible. A 15-term equation was generated in the form of G12, with adjusted R^2 = 0.96 and RMSE = 34.6 on the test set. The equation for the stress of a sprocket tooth (different to that of a gear tooth) is not available, thus the equation could not be derived from first principles. The model was trained with solutions closer towards lower values of mass, thus performs better in this region.

With a metamodel for the mass, and now Safety Factor constraint, this model too was ready for optimisation.

4.4 Optimisation

To optimise the efficiency of the gear train, fmincon was used, as it is a nonlinear gradient based solver that converts a constrained problem into an unconstrained problem, using the Newton-Raphson method to solve it. It also generates a Hessian matrix, helping identify global solutions more easily.

Patternsearch (a global optimisation technique) was used as a pre-optimisation technique to find the optimum starting point to avoid local minima. It is understood that a smaller C results in less mass, while a larger R results in a smaller wrapping angle, thus and ideal solution has a larger R and smaller C. These extremes were input as general starting points for the patternsearch, which found a more optimum starting point for fmincon, which returned optimum values of Rd = 0.05, and C = 0.304, which resulted in an efficiency increase from 97% to 98.7%. The eigenvalues of the Hessian were evaluated > 0, thus it was assumed the global minimum was found. A variety of starting points were checked, yet all converged to the same value, likely due to the small solution space.

Likewise, the same methods were applied to the optimisation of the Mass of the gear. Although some variables were discrete the range of the Nt meant it could be treated continuously and rounded up. Leaving beams as the only discrete variable, it was rounded up also, which simplified the problem hugely. Intuitively, one would understand that a smaller, thinner gear with thinner and fewer beams would weigh less. Thus, a range of starting points were testing from high to low, which fed into patternsearch, which an ideal starting point into fmincon, which output a combination of values leading to Mass = 17.035 grams; a reduction from 94.04 grams. The values were input to Solidworks for a sanity check, and found to be satisfactory, with Mass = 19.3g, and SF = 122.6 Again, the hessian eigenvalues > 0, ensuring a minimum point. GA was also tested, but was unable to converge to the global minimum, and is unfit for this type of non-discrete problem.

4.5 Multi-Objective Optimisation

The gamultiobj solver was used to generate a pareto front as shown in figure (8)

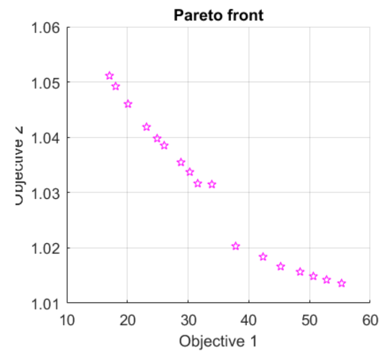

Figure 8: Obj 1 – Mass, Obj 2 - Efficiency

Since there is a 30.11%:4.76% = ~1:6 weighting for mass to drivetrain efficiency, the mass was assigned 6/7 importance, and the efficiency only 1/7. Scaling this step to the graph in figure the ideal solution is therefore at 23.2 grams and 1/1.041855 = 96% efficiency. While there is a 1% reduction in gear train efficiency, the mass is reduced by 388%.

4.6 Discussion

This optimisation was to maximise drivetrain efficiency by reducing frictional losses, while minimising gear mass, reducing losses in the drivetrain and to rolling resistance. Both mathematical and metamodeling approaches helped formulate the problem which solved using 3 optimizers. Patternsearch identified a good starting point for the global minimum of both problems, which was iterated over and check with the Hessian. These values were fed into fmincon which gave a similar but slightly more optimal answer.

The main challenge was metamodeling the G12 stress constraint. Due to the fact each variable was interlinked, a model fit type was impossible to find, thus I had to rely on analytical methods and statistics to assess fit. To improve this, a greater sample could be input into the metamodel, as well as further model types explored, such as exponential.

The final multi objective optimisation found a pareto frontier, and the solution was picked from it according to the weight. Overall a reduction in mass by 388% from 90.4g to 23.2g was achieved, while the drivetrain suffered a 1% decrease. Final variable values were: N_B = 33, N_t = 45, W = 2, B = 2.27, C = 0.3462. No constraints are violated with these solutions. Further work would explore extra variables in the gear, and deduce extra frictional losses in the chain, such as rubbing friction between the chain coming into contact with the gear tooth.

5. System-level optimisation

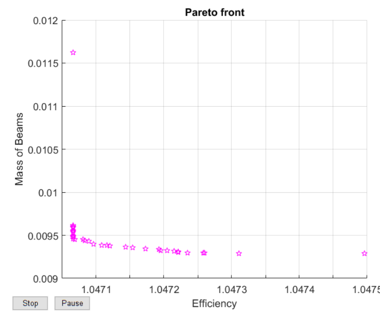

Figure 9: Pareto set for system level design optimisation

The change in values observed is small, due to the fact that centre distance does not heavily impact the efficiency. Although subsystem 2 used a multi objective problem, because the only shared variable is centre distance, the results from the other multi objective analysis were used as constraints, constraining R_d < 0.0138.

Again, assigning a weight of 1:6, the optimal solution can be found, efficiency = 1/1.0471 = 95.5%, and mass = 9.4 grams.

6. Conclusion

An optimisation study was conducted of the Imperial College Racing Green Shell Eco Marathon vehicle. Two separate subsystems were analysed; the chassis, which was optimised to reduce mass, and the drivetrain, which was optimised to reduce mass and improve efficiency. All goals were met adequately. During separate optimisations, the mass of beams of the chassis were reduced from 187 grams to 9 grams, and the mass of the driven gear reduced from 90.4 to 23.2 grams. However, the drivetrain efficiency suffered as a result, reducing from 97% to 95.5%. The greatest challenges in the system design were finding subsystem that interlinked, while ensuring there would be enough dependence to result in a significant change. If the study were to be conducted again, it would be in our interests to attempt to find subsystems with greater overall system impact.

References

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Appendix A. Nomenclature

M - Mass

P – Density

R – Outer radius of beam

r- Inner radius of beam

α- Angle between two beams

A – Minimum length of base

B- Fixed height of triangle

SF - Safety Factor

E – Youngs Modulus

YS – Yield Stress

F- Force

P (in 2nd formulation) - chain pitch

β - Distance from sprocket OD to next internal surface in mm.

r_{di} - Outer radius of inner sprocket ring in mm.

τ - Max motor torque = 0.4285

$P_i - Max\ power\ in = 200W$

ω_o - angular velocity out = 6.097

$\omega_d - angular\ velocity\ in = 17.737$

m - mass per unit length of chain = kg/m

$\mu_p\ coefficient\ of\ friction\ =\ 0.11$

$r_b -\ radius\ of\ bush$

$\alpha_m -\ wrapping\ angle$

$N_d - number\ of\ teeth\ on\ driving\ gear$

$R_o - radius\ of\ driven\ gear$