Actividad 1

Ejercicio CEF.

a) Si $e = Xu$, donde X y u son independientes $N(0,1)$. Demuestre que, condicional en X, el error tiene distribución $N(0, X^2)$
b) Lo anterior qué significa en terminos de la independencia entre e y X (vea BH, p.24-25). Relacione esto con el concepto de homocedasticidad.
c) Pruebe que: $var[Y] \geq var[Y - [E|X]] \geq var[Y - [E|X_1, X_2]]$ (vea BH 2.33)
d) Pruebe que Q_{XX} es una matriz positiva semidefinida (vea BH p.39)
e) Consider the linear projection:

$$\mathcal{P}[log(wage)|experience] = 0.046experience - 0.07experience^2 + 0.11education + 2.3$$

Formally (using derivatives), what is the effect of one extra year of experience on log(wage)?
f) Consider the linear projection

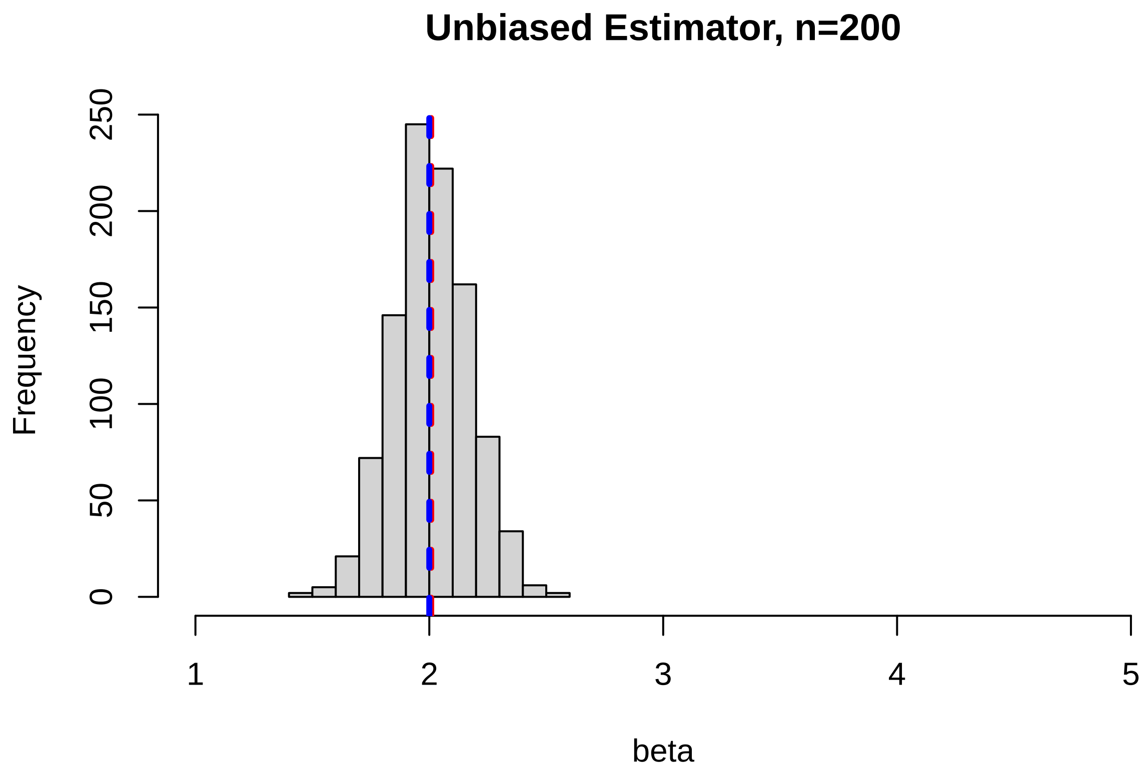
$$\mathcal{P}[log(wage)|experience] = 0.046experience - 0.07experience^2 - 0.09female + 0.11education + 0.07education * female + 1.$$

Formally (using derivatives), what is the return to one extra year of education on log(wage) for males? and for females?
g) Draw a two-dimensional graph showing the linear projections onto education by gender.

h) This is a piece of code simulating an unbiased estimator β_1 and β_2.

```
repet <- 1000
n <- 200
beta <- NULL

for (i in 1:repet){
  x <- rnorm(n)
  x2 <- rnorm(n)
  u <- rnorm(n,0,1)
  y=2+2*x+2*x2+u
  beta[i] <- lm(y~x)$coef[2] #
}
hist(beta, main="Unbiased Estimator, n=200", xlim = c(1,5) )
abline(v = mean(beta), col="red", lwd=3, lty=2,)
abline(v = 2, col="blue", lwd=3, lty=2)
```



- Modify the code so that it shows a **biased** estimator of β_1.
- While keeping this modification make the bias go (see BH 2.24)

i) Give a concrete example of a linear projection on which we most likely suffer from OVB.