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Developing coal pillar stability chart using logistic regression

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ABSTRACT

Logistic regression was utilised to calculate the probability that a particular pillar of a given geometry (width to height ratio) and a known stress condition (strength to stress ratio) will be stable. The stable-failure boundary was also determined. The logistic regression was also used to calculate and draw isoprobability contours. These contours represent the probability of stability of coal pillars based on the probability function for each stability class and are a valuable design tool in quantifying the instability probability of coal pillars.

1. Introduction

Pillars in underground coal mines are mainly designed to ensure the protection of roadways and entries. Pillars stability is consequently the most important factor that must be guaranteed through the entire life of mine that can be years or even decades long. Pillar stability can be analysed by a number of methods that are generally based on the ratio between the pillar strength and pillar load that is expressed in Factor of Safety (*FoS*).

The strength characteristics of coal pillars has been studied by many researchers and the subject has been well discussed in the literature, for examples [1–5]. A number of fairly intensive new developments relating to coal pillar strength estimation methods have also been carried out.

Two coal pillar strength approaches based on a tentative empirical failure criterion for coal seams were proposed [6]. The first, progressive failure type approach, gave incorrect estimates of pillar strength for low width-height ratios while the second pillar strength approach performed satisfactorily for both slender and fiat pillars. The performance of these new equations has been compared with some of the more popular strength formulas and tested against 16 failed and 27 stable pillar case studies.

A new coal pillar strength equation was developed and tested, along with existing equations, against 23 failed and 20 stable case studies [7]. The results revealed that in situ strength is more affected by depth of cover than indicated by laboratory tests, and a new safety factor based on depth and width/height ratio was proposed.

Alternative methods of coal pillar strength estimation using numerical modelling with strain softening constitutive behaviour of coal were also provided. The numerical models implemented in the researches were three dimensional finite difference [8] and finite element [9] methods. They might give decent results and better than the previous pillar strength estimation method.

Furthermore, it was found that the *FoS* calculated using deterministic approach had some intrinsic limitations in handling uncertainties in material properties, non-regular geometries and different mining operations [10]. A probabilistic expression for *FoS* was then suggested, which provided a confidence interval to express the reliability of coal pillar stability.

In this paper, a statistical approach based on real stable and failed cases of coal pillars was suggested. As pillar stability was only defined as stable or failed, logistic regression was utilised because the method is suitable for categorical dependent variables.

2. Logistic regression analysis

2.1. Logistic regression model

Logistic regression is a statistical modelling technique where the dependent variable (Y) has only two possible values and it is a useful tool for analysing data that includes categorical response variables, such as yes/no or live/die or stable/failed, as compared to the regression of numerical values. Logistic regression does not model the dependent variable directly but it is rather based on the probabilities associated with the values of Y. For simplicity, and because it is the case most commonly encountered in practice, Y can be coded as 1 in the case positive outcome or

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success (i.e. yes or live or stable) and coded as 0 in the case of negative outcome (i.e. no or die or failed). If there is a collection of p independent variables denoted by the vector $\boldsymbol{x}'=(x_1, x_2,\ldots, x_p)$, the hypothetical population proportion of cases for which $Y=1$ is defined as [11]

$$P(Y=1/x)=p(x) \tag{1}$$

and the specific form of the logistic regression model is

$$p(x)=\frac{e^{\beta_0+\beta_1x_1+\beta_2x_2+\ldots+\beta_px_p}}{1+e^{\beta_0+\beta_1x_1+\beta_2x_2+\ldots+\beta_px_p}} \tag{2}$$

where β_0, β_1,..., β_p are the logistic regression model parameters.

For theoretical mathematical reasons, logistic regression is based on a linear model for the natural logarithm of the odds in favour of $Y=1$, which are simply the ratio of the proportions for the two possible outcomes [12], written as

$$Odds=\frac{\pi(x)}{1-\pi(x)} \tag{3}$$

and the general form of the logistic regression model can now be written as:

$$\ln\left[\frac{\pi(x)}{1-\pi(x)}\right]=\beta_0+\beta_1x_1+\beta_2x_2+\ldots+\beta_px_p \tag{4}$$

The log-odds, as defined above is also known as the logit transformation of $\pi(x)$ and the related analysis is sometimes known as logit analysis.

2.2. Logistic regression model fitting

If there is a sample of n independent observations (x_i,y_i), $i=1$, 2,..., n, the model fitting requires the estimates of the vector $\beta'=(\beta_0, \beta_1, \beta_2,\ldots, \beta_p)$ and the general method of estimation is called maximum likelihood. In a very general sense, the method of maximum likelihood yields values for the unknown parameters which maximise the probability of obtaining the observed data [11]. In order to utilise the method, a likelihood function must be first constructed, in the form of

$$l(\boldsymbol{\beta})=\prod_{i=1}^{n}\pi(x_i)^{Y_i}[1-\pi(x_i)]^{1-Y_i} \tag{5}$$

It is mathematically easier to work with the log of Eq. (5), and this expression, the log likelihood, is defined as

$$L(\boldsymbol{\beta})=\ln[l(\boldsymbol{\beta})]=\sum_{i=1}^{n}\{Y_i\ln[\pi(x_i)]+(1-Y_i)\ln[1-\pi(x_i)]\} \tag{6}$$

To find the values of $\boldsymbol{\beta}$ that maximise $L(\boldsymbol{\beta})$, Eq. (6) is differentiated with respect to β_0, β_1, β_2,..., β_p and resulting equations, known as the likelihood equations, are set to zero, as follows:

$$\sum_{i=1}^{n}[Y_i-\pi(x_i)]=0 \tag{7}$$

and

$$\sum_{i=1}^{n}x_{ij}[Y_i-\pi(x_i)]=0 \tag{8}$$

for $j=1, 2,\ldots, p$.

In linear regression, the likelihood equations are linear in the unknown parameters and easily solved. In logistic regression, the expressions in Eq. (7) and Eq. (8) are nonlinear in β_0, β_1, β_2,..., β_p and special methods are required for their solutions. These methods are iterative in nature and have been programmed into available logistic regression software. A general discussion of the methods used by most programs may be seen in [13] where it is shown that, in particular, the solution to Eqs. (7) and (8) may be obtained using an iterative weighted least squares procedure [11].

3. Coal pillar case histories

3.1. Coal pillar stability data

Coal pillar case histories have been compiled by [14], as listed in Table 1. The pillars are grouped into stable (denoted by 1) and failed (denoted by 0) pillars for the purpose of logistic regression analysis.

As pillar stability is theoretically governed by the width to height ratio and strength to stress ratios, the logistic regression analysis conducted in this paper was based on these two ratios. The strengths of pillars in Table 1 were estimated using the equations proposed by [15] as follows:

$$S=0.27\sigma_ch^{-0.36}+\left(\frac{H}{250}+1\right)\left(\frac{w}{h}-1\right)(\text{MPa}) \tag{9}$$

where S is pillar strength in MPa, σ_c is uniaxial compressive strength of coal in MPa, H is pillar location depth in m, w is pillar width in m, and h is pillar height in m.

Pillar stresses in Table 1 were estimated simply by using the tributary area theory where the unit weight of coal bearing strata overburden was taken as 0.025 MN/m^3 as follows:

$$P=0.025H\frac{(w+B)^2}{w^2}(\text{MPa}) \tag{10}$$

where P is pillar stress in MPa and B is roadway width in m.

Theoretically, a pillar with S/P ratio, which is basically the *FoS*, greater than 1.0 would be stable. However, as can be seen in Table 1, the stable pillars have S/P ratios in the range of 0.84–5.55, whereas the failed pillars have S/P ratios in the range of 0.78–1.37. It is then obvious, that there is still a possibility of failure for pillar with a S/P ratio greater than 1.0.

3.2. Logistic regression model from coal pillar stability data

The logistic regression model from pillar stability data was developed with the independent variables of w/h $(=x_1)$ and S/P $(=x_2)$ and the dependent variable (Y) is stability that coded as 1 for stable pillar and 0 for failed pillar. The resulted model is

$$P\left(\text{pillar is stable}\,|\left(\frac{w}{h}\right),\left(\frac{S}{P}\right)\right)=\frac{\exp\left[-13.146+2.774\left(\frac{w}{h}\right)+5.668\left(\frac{S}{P}\right)\right]}{1+\exp\left[-13.146+2.774\left(\frac{w}{h}\right)+5.668\left(\frac{S}{P}\right)\right]} \tag{11}$$

The probability of stability for each pillar can be calculated and compared to the actual stability, as depicted in Table 2. The table reveals that the logistic regression model estimated that the probability of a stable pillars is stable is in the range of 29.53%–100%, whereas that of a failed pillars is in the range of 0.19%–87.04%. Table 2 also shows that the proposed logistic regression model predict low probability of stability (i.e. 29.53%) for one stable pillar and on the other hand predict high probability of stability (i.e. 87.04%) for one failed pillar. These two cases are indicated with (*) in Table 2. It can also be observed that based on the estimated probability of stability, the accuracy of the proposed logistic regression model is acceptable, as given in Table 3 that shows the comparison between actual and predicted stability.

4. Coal pillar stability chart

4.1. Stable-failure boundary line

The logistic regression determines the orientation of the stable-failure boundary line that can be used to separate the different stability categories. Determination of the position of this boundary was conducted following the similar approach to that was used in determining the stable-failure and failure-caving boundaries for block

Table 1
Coal pillar case histories [14].

Pillar ID	Mine (Seam)	Depth H (m)	Pillar height h (m)	Pillar width w (m)	w/h	Roadway width B (m)	Uniaxial compressive strength (MPa)	Pillar strength S (MPa)	Pillar stress P (MPa)	S/P	Stability
1	Bellampalli (Ross)	36	3.0	5.4	1.80	6.0	48	9.64	4.01	2.40	1
2	Nimcha (Nega)	48	6.0	9.9	1.65	6.0	50	7.85	3.09	2.54	1
3	Morganpit (Salarjung)	270	3.0	8.1	2.70	3.6	46	11.89	14.08	0.84	1
4	Ramnagar (Ramnagar)	75	2.7	9.9	3.67	6.6	28	8.75	5.20	1.68	1
5	Lachhipur (Lower Kajora)	38	5.1	7.2	1.41	3.9	33	5.43	2.25	2.41	1
6	N. Salanpur (X)	30	5.1	9.0	1.76	6.0	21	4.01	2.08	1.93	1
7	Bankola (Jambad Top)	102	4.8	10.1	2.10	2.4	35	6.92	3.90	1.77	1
8	Bankola (Jambad Top)	75	3.0	6.3	2.10	4.2	35	7.79	5.20	1.50	1
9	Surakacchar (G–I)	106	3.5	16.0	4.57	4.0	29	10.07	4.14	2.43	1
10	Lachhipur (Lower Kajora)	38	5.1	18.3	3.59	4.2	33	7.93	1.43	5.55	1
11	Sripur (Koithee)	266	4.8	40.0	8.33	5.0	43	21.73	8.41	2.58	1
12	E. Angarapatra (XII)	30	2.1	6.0	2.86	6.0	19	6.00	3.00	2.00	1
13	Kargali Incline (Kathara)	36	3.6	9.3	2.58	7.2	40	8.62	2.34	3.68	1
14	Jamadoba 6 and 7 Pits (XVI)	80	2.0	5.8	2.90	5.5	29	8.60	7.59	1.13	1
15	Topsi (Singharan)	85	1.8	7.0	3.89	3.9	41	12.83	5.15	2.49	1
16	Amritnagar (Nega Jamehari)	30	4.5	3.6	0.80	5.7	45	6.85	5.01	1.37	0
17	Amritnagar (Nega Jamehari)	30	6.0	3.6	0.60	5.4	45	5.93	4.68	1.27	0
18	Begonia (Begonia)	36	3.0	3.9	1.30	6.0	26	5.07	5.80	0.87	0
19	Amlai (Burhar)	30	5.4	4.5	0.83	4.5	25	3.49	3.61	0.97	0
20	Sendra Bansjora (X)	23	8.1	4.7	0.57	5.6	24	2.59	2.77	0.94	0
21	W. Chirimiri (Main)	90	3.8	5.4	1.44	6.0	45	8.15	8.12	1.00	0
22	Birsingpur (Johilla Top)	129	3.6	7.5	2.08	6.0	38	8.11	10.45	0.78	0
23	Pure Kajora (Lower Kajora)	54	3.6	5.4	1.50	6.0	33	6.23	6.02	1.03	0
24	Pure Kajora (Lower Kajora)	56	3.6	5.0	1.38	6.5	33	6.08	7.43	0.82	0
25	Shankarpur (Jambad Bottom)	42	4.8	4.5	0.94	4.5	47	5.46	4.20	1.30	0
26	Ramnagar (Begunia)	70	1.8	2.9	1.58	3.2	26	6.43	7.76	0.83	0
27	Ramnagar (Begunia)	51	1.8	3.0	1.67	3.6	26	6.48	6.17	1.05	0
28	Kankanee (XIII)	160	6.6	19.8	3.00	4.2	27	6.98	5.88	1.19	0
29	Kankanee (XIV)	140	8.4	18.6	2.21	5.4	25	5.03	5.83	0.86	0

Table 2
Calculated probability of stability and actual stability of coal pillars.

Pillar ID	Mine (Seam)	w/h	S/P	Probability of stability (%)	Actual stability
1	Bellampalli (Ross)	1.80	2.40	99.58	1
2	Nimcha (Nega)	1.65	2.54	99.71	1
3(*)	Morganpit (Salarjung)	2.70	0.84	29.53	1
4	Ramnagar (Ramnagar)	3.67	1.68	99.86	1
5	Lachhipur (Lower Kajora)	1.41	2.41	98.84	1
6	N. Salanpur (X)	1.76	1.93	93.56	1
7	Bankola (Jambad Top)	2.10	1.77	93.98	1
8	Bankola (Jambad Top)	2.10	1.50	76.32	1
9	Surakacchar (G–I)	4.57	2.43	100.00	1
10	Lachhipur (Lower Kajora)	3.59	5.55	100.00	1
11	Sripur (Koithee)	8.33	2.58	100.00	1
12	E. Angarapatra (XII)	2.86	2.00	99.78	1
13	Kargali Incline (Kathara)	2.58	3.68	100.00	1
14	Jamadoba 6 and 7 Pits (XVI)	2.90	1.13	78.93	1
15	Topsi (Singharan)	3.89	2.49	100.00	1
16	Amritnagar (Nega Jamehari)	0.80	1.37	4.00	0
17	Amritnagar (Nega Jamehari)	0.60	1.27	1.34	0
18	Begonia (Begonia)	1.30	0.87	1.01	0
19	Amlai (Burhar)	0.83	0.97	0.47	0
20	Sendra Bansjora (X)	0.57	0.94	0.19	0
21	W. Chirimiri (Main)	1.44	1.00	3.04	0
22	Birsingpur (Johilla Top)	2.08	0.78	4.89	0
23	Pure Kajora (Lower Kajora)	1.50	1.03	4.23	0
24	Pure Kajora (Lower Kajora)	1.38	0.82	0.91	0
25	Shankarpur (Jambad Bottom)	0.94	1.30	4.00	0
26	Ramnagar (Begunia)	1.58	0.83	1.70	0
27	Ramnagar (Begunia)	1.67	1.05	7.11	0
28(*)	Kankanee (XIII)	3.00	1.19	87.04	0
29	Kankanee (XIV)	2.21	0.86	10.78	0

caving mines [15,16]. In the approach, the cumulative probabilities of stability were calculated using a particular band width or bin size. The percentage of a given stability class in each bin was determined and summed to determine the cumulative distribution graph for each stability class. The cross-over points on the cumulative distribution graph represent the probability of stability value that will define the separation line that will have the same proportion of mismatched points either side of the line. The cumulative distribution graph also indicates the percentage of points lying above and below any line, defined by a particular probability of stability, on the stability graph. In this paper, a bin size of 0.1 was chosen and the cross-over point was at probability of stability value of 0.215, as depicted in Fig. 1.

Table 3
Comparison between actual stability and predicted stability of coal pillars.

Stability	Actual	Predicted		Percent correct
		Stable	Failed	
Stable	15	14	1	93.33
Failed	14	1	13	92.86
Overall per cent correct				93.10

Using the probability of probability value 0.215, the stable-failure boundary line was then determined. Referring to the logistic parameters given in Eq. (11), the relationship for the equation of the line is

$$0.215 = -13.146 + 2.774\left(\frac{w}{h}\right) + 5.668\left(\frac{S}{P}\right) \tag{12}$$

that can be rewritten as

$$\frac{S}{P} = \frac{13.361 - 2.774(w/h)}{5.668} \tag{13}$$

It should be noted that Eq. (13) is only the equation of the line separating the stable and failure zones based on the pillar case

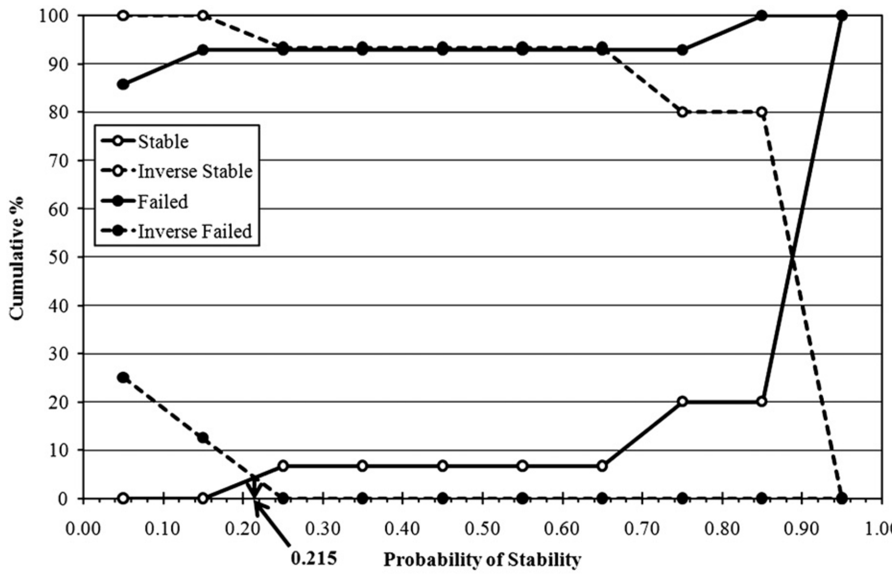

Fig. 1. Cumulative distribution graphs for the probability of stability for each stability class.

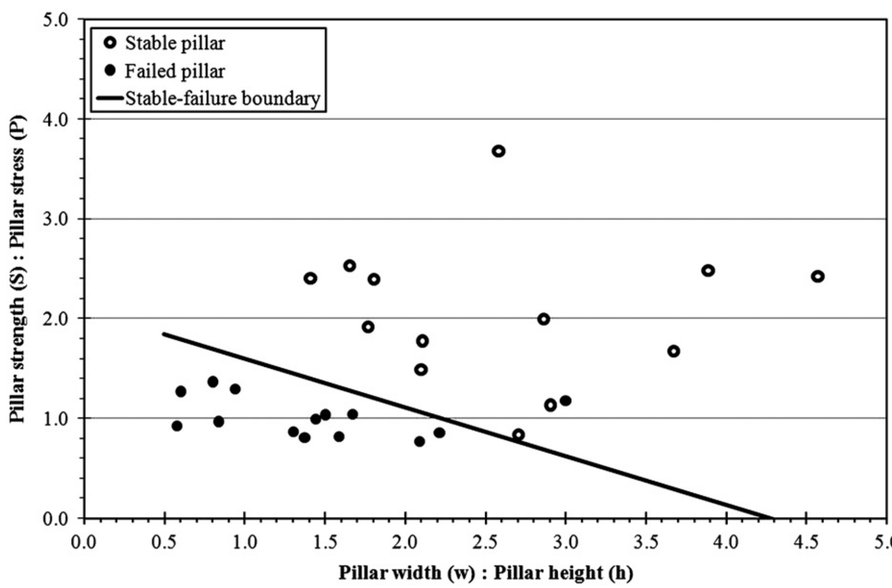

Fig. 2. Coal pillar stability graph.

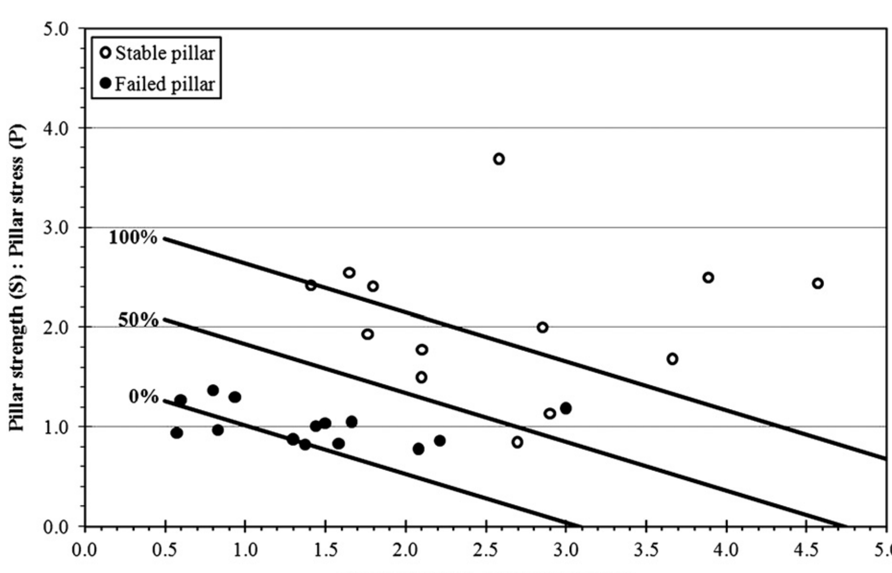

Fig. 3. Isoprobability contours of 100%, 50%, and 0% for stable pillars.

histories and it does not mean that S/P ($=FoS$) is solely a function of w/h. The stable-failure boundary line is plotted in Fig. 2 where area above the line is the stable zone and area below the line is the failure zone. For graphical purposes Fig. 2 is limited to w/h and S/P of 5.0. It can also be observed in Fig. 2 that only one pillar is reported in the wrong zone (i.e. failed pillar reported in the stable zone). This shows that the accuracy is improved from 93.10% based on probability of stability in Table 3 to 96.55% by using the stable-failure boundary line.

4.2. Isoprobability contours

Although stability zones can be defined statistically, a number of case histories might be reported to wrong zones. This is to be expected given the inherent variability of rock masses, data that can be somewhat subjective, and measurement and observational errors. Isoprobability contours were proposed to account for the uncertainties inherent within the design limits of the Mathew Stability Graph [17].

Isoprobability contours are lines of equal probability of a given stability outcome that can be drawn onto the stability graph [15]. The contours allow the probabilities of stability determined from the probability functions to be presented directly on the stability graph. Contours of equal probabilities indicate the stability associated with an underground structure and are a valuable measure of the reliability of the stability zone boundary and allow the statistical meaning of the stability boundaries to be quantified in terms of probable stability outcomes.

For a desired probability value, the probability of stability value is determined from the graph in Fig. 1. The cumulative per cent probability on the y-axis of the graph is read across to intercept the stable probability curve and then read down to the x-axis to obtain the corresponding probability of stability value. By following the same steps in determining Eq. (13), the equation of isoprobability contour for particular probability can then be defined. The isoprobability contours of 100%, 50%, and 0% for stable pillars are given in Fig. 3 and equations for the complete isoprobablity contours are presented in Table 4.

Fig. 4 illustrates an example of using the isoprobability contours. Suppose a FoS of 1.5 is required for a coal pillar. The pillar with w/h of 1.7 would have a probability of stability of 50% whereas the pillar with w/h of 3.3 would have a probability of stability of 100%.

Table 4
Equations of isoprobability contours for stable pillars.

Probability of stability (%)	Constant A in equation $\left(\frac{S}{P}\right)=A-0.489\left(\frac{w}{h}\right)$
100	3.130
90	2.707
80	2.564
70	2.469
60	2.391
50	2.319
40	2.248
30	2.170
20	2.075
10	1.932
0	1.509

5. Conclusions

The logistic regression model for predicting the probability of stability of a coal pillar for given geometry (width to height ratio) and stress condition (strength to stress ratio) has been developed. The model prediction is close to the actual stability data, where the model predicts all stable cases correctly and 13 out of 14 failure cases.

A coal pillar stability chart has been constructed. A stable-failure boundary line, which will have the same proportion of mismatched points either side of the line, has also been defined and plotted on the stability chart.

Isoprobability contours for stable pillars have been developed, which allow the probabilities of risk associated with pillar stability to be determined and presented directly on the stability graph. The contours are a valuable measure of the reliability of the stability zone boundary and allow the statistical meaning of the stability boundaries to be quantified in terms of probable stability outcomes. The use of the contours must be in conjunction with different conditions experienced at mine sites, which

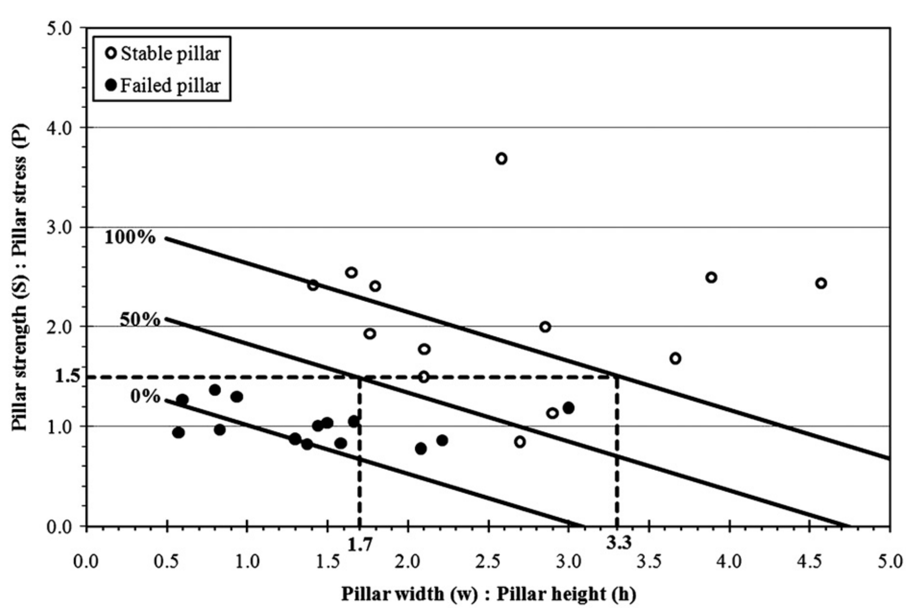

Fig. 4. An example of the use of isoprobability contours.

define the risk level of acceptance and consequently the choice of isoprobability contour.

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