Exercise 2.3 Simplifying a Formula ($*$) — GRADED (8 points)

Please upload your solution by 02/10/2025

Consider the propositional formula

$$F = (B \to A) \land \neg\big((\neg A \land \neg C) \land (\neg C \lor B)\big)$$

Give a formula G that is equivalent to F, but in which each atomic formula A, B, and C appears at most once. Prove that $F \equiv G$ by providing a sequence of equivalence transformations with at most 7 steps.

Expectation. Your proof should be in the form of a sequence of steps, where each step consists of applying the definition of $\to$ (that is, $F \to G \equiv \neg F \lor G$), one of the rules given in Lemma 2.1 of the lecture notes, or one of the following rules:

$$F \lor \bot \equiv F, \quad F \lor \neg F \equiv \top, \quad F \land \top \equiv F, \quad F \lor \top \equiv \top.$$

For this exercise, associativity is to be applied as in Lemma 2.1(3). Each step of your proof should apply a single rule once and state *which* rule was applied.

Solution:

$$G \equiv A \lor (\neg B \land C)$$

$$\begin{aligned}
F =& (B \to A) \land \neg\big((\neg A \land \neg C) \land (\neg C \lor B)\big) && \\
\equiv& (B \to A) \land \neg\big(\neg A \land (\neg C \land (\neg C \lor B))\big) && \text{(associativity)}\\
\equiv& (B \to A) \land \neg\big(\neg A \land \neg C\big) && \text{(absorption)}\\
\equiv& (B \to A) \land \neg\neg(A \lor C) && \text{(de Morgan)}\\
\equiv& (B \to A) \land (A \lor C) && \text{(double negation)}\\
\equiv& (\neg B \lor A) \land (A \lor C) && \text{(definition of } \to \text{)}\\
\equiv& (A \lor \neg B) \land (A \lor C) && \text{(commutativity)}\\
\equiv& A \lor (\neg B \land C) && \text{(second distributive law)}\\
\equiv& G &&
\end{aligned}$$