# Lista 2 - GEOMETRIA ANALÍTICA

1. Calcule os determinantes abaixo usando o método de Laplace.

a) $\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$

$\det A = a_{11} \cdot \tilde{a}_{11} + a_{12} \cdot \hat{a}_{12} = 2 \cdot (-1)^2 \cdot \det[3] + 1 \cdot (-1)^3 \cdot \det[-4] =$

$2 \cdot 1 \cdot 3 + 1 \cdot (-1) \cdot (-4) = 6 + 4 = 10$

b) $B = \begin{bmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{bmatrix}$ $\det B = b_{11} \cdot \hat{b}_{11} + b_{21} + \tilde{b}_{21} = \sqrt{2} \cdot (-1)^2 \cdot \sqrt{3} + 2 \cdot (-1)^3 \cdot 3\sqrt{6} =$

$\sqrt{6} + (-6\sqrt{6}) = \sqrt{6} - 6\sqrt{6} = -5\sqrt{6}$

c) $C = \begin{bmatrix} \pi & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $\det C = c_{31} \cdot \tilde{c}_{31} + \overset{\to 0}{c_{32}} \cdot \tilde{c}_{32} + \overset{\to 0}{c_{33}} \cdot \tilde{c}_{33} =$

$1 \cdot (-1)^4 \cdot \det\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix} + 0 + 0$

$1 \cdot (-1)^4 \cdot 2 = 2$

$\to 0 \cdot (-1)^2 \cdot \det[1] + (-1) \cdot (-1)^3 \cdot 2 =$

$0 + 2 = 2$

d) $D = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{bmatrix}$ $\det(D) = d_{11} \cdot \tilde{d}_{11} + d_{12} \cdot \hat{d}_{12} + d_{13} \cdot \tilde{d}_{13} =$

$-2 \cdot (-1)^2 \cdot \det\begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix} + 1 \cdot (-1)^3 \det\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix} + (-1) \cdot (-1)^4 \cdot \det\begin{bmatrix} 1 & 5 \\ -3 & 4 \end{bmatrix}$

$-2 \cdot \det\begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix} - 1 \cdot \det\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix} - 1 \cdot \det\begin{bmatrix} 1 & 5 \\ -3 & 4 \end{bmatrix}$

$-2 \cdot (-6) - 1 \cdot 14 - 1 \cdot 19$

$12 - 14 - 19 = -21$

$\det\begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix} = a_{12} \cdot \tilde{a}_{12} + a_{22} \tilde{a}_{22}$

$4 \cdot (-1)^3 \cdot 4 + 2 \cdot (-1)^4 \cdot 5$

$-16 + 10 = -6$

$\det\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix} = a_{11} \cdot \hat{a}_{11} + a_{21} \tilde{a}_{21}$

$1 \cdot (-1)^2 \cdot 2 + (-3) \cdot (-1)^3 \cdot 4$

$2 + 12 = 14$

$\begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix} = a_{11} \cdot \hat{a}_{11} + a_{21} \cdot \tilde{a}_{21}$

$1 \cdot (-1)^2 \cdot 4 + (-3) \cdot (-1)^3 \cdot 5$

$4 + 15 = 19$

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e) $|E| = \begin{vmatrix} \boxed{0 \quad 2 \quad 0} \\ 1 \quad 3 \quad 5 \\ 2 \quad -1 \quad 2 \end{vmatrix} = e_{11} \cdot \hat{e}_{11} + e_{12} \cdot \hat{e}_{12} + e_{13} \cdot \hat{e}_{13} = 0 \cdot \hat{e}_{11} + 2 \cdot (-1)^3 \cdot \det\begin{bmatrix} 1 & 2 \\ 5 & 2 \end{bmatrix} +$

$2 \cdot (-1)^3 \cdot \begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix}$

$2 \cdot (-1) \cdot (-8) = 16$ //

$1 \cdot (-1)^2 \cdot 2 + 2 \cdot (-1)^3 \cdot 5$

$2 + (-10) = -8$

f) $|F| = \begin{vmatrix} \boxed{3} & -1 & 1 & 1 \\ \boxed{0} & 1 & 0 & 1 \\ \boxed{0} & -1 & 1 & 0 \\ \boxed{0} & 0 & 1 & -1 \end{vmatrix} = f_{11} \cdot \hat{f}_{11} + f_{21} \cdot \hat{f}_{21} + f_{31} \cdot \hat{f}_{31} + f_{41} \cdot \hat{f}_{41}$

$3 \cdot (-1)^2 \cdot \begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} + 0 + 0 + 0$

$3 \cdot 1 \cdot (-2) = -6$

$\begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 1 \end{vmatrix}$ $-1-1 = -2$

g) $\begin{vmatrix} \boxed{1 \quad 0 \quad 0 \quad 0 \quad 0} \\ 5 \quad 1 \quad 2 \quad 5 \quad 3 \\ 7 \quad 2 \quad \sqrt{3} \quad 0 \quad 0 \\ 10 \quad -3 \quad 6 \quad 1 \quad 0 \\ -2 \quad -3 \quad 0 \quad 0 \quad 0 \end{vmatrix} = 1 \cdot (-1)^2 \cdot \begin{vmatrix} 1 & 2 & 5 & \boxed{3} \\ 2 & \sqrt{3} & 0 & \boxed{0} \\ -3 & 6 & 1 & \boxed{0} \\ -3 & 0 & 0 & \boxed{0} \end{vmatrix}$

$\to 3 \cdot (-1)^5 \cdot \begin{vmatrix} 2 & \sqrt{3} & 0 \\ -3 & 6 & 1 \\ \boxed{-3 \quad 0 \quad 0} \end{vmatrix}$

$3 \cdot (-1) \cdot -3\sqrt{3} =$

$+9\sqrt{3}$

$-3 \cdot (-1)^4 \cdot \begin{vmatrix} \sqrt{3} & 0 \\ 6 & 1 \end{vmatrix}$

$-3 \cdot 1 \cdot \sqrt{3} = -3\sqrt{3}$

$\sqrt{3} \cdot 1 - 0 \cdot 6$

$\sqrt{3}$

$1 \cdot (1) \cdot (+9\sqrt{3}) = +9\sqrt{3}$ //

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h) $|H| = \begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}$

$3 \cdot (-1)^2 \cdot \begin{vmatrix} 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \end{vmatrix}$

$3 \cdot 1 \cdot (-8) = -24$

$-2 \cdot (-1)^5 \cdot \begin{vmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & 0 & 0 \end{vmatrix}$

$-2 \cdot (-1) \cdot (-4) = -8$

$1 \cdot (-1)^5 \cdot \begin{vmatrix} 0 & 2 \\ -2 & 0 \end{vmatrix}$

$1 \cdot (-1) \cdot 4 = -4$

$= 0 \cdot 0 - 2 \cdot (-2) = +4$

2. Dadas as matrizes $A = \begin{bmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{bmatrix}$

calcule os seguintes determinantes:

a) det (A+B)

$\begin{bmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{bmatrix} \begin{matrix} 7 & -2 \\ 3 & 2 \\ 4 & -8 \end{matrix}$

$28 \cdot 4 =$ 80, 32, 112; $42 \cdot 8 = 112$; 320 + 16 = 336

$28 - 80 - 336 + 12 + 560 - 112$

$\underbrace{28 + 12}_{40} + 560 - 80 - 336 - 112$

$600 - 528 = 72$

b) det(AB) = det(A) · det(B)

det (A) = $\begin{matrix} 3 & -5 & 7 & 3 & -5 \\ 4 & 2 & 8 & 4 & 2 \\ 1 & -9 & 6 & 1 & -9 \end{matrix}$

$36 - 40 - 252 + 120 + 216 - 14 = \underbrace{\underbrace{-40 - 252}_{-292} - 14}_{-306} + \underbrace{\underbrace{36 + 120}_{156} + 216}_{372} = 372 - 306 =$

$72 - 6 = 66$

det (B) = $\begin{matrix} 4 & 3 & 7 & 4 & 3 \\ -1 & 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & 3 & 1 \end{matrix}$

$18 - 7 - 12 - 8 = 18 - 27 = -9$

$66 \cdot (-9) = $ 660 − 66 = 594; $-594$

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c) $\det(B^t A^t)$

$\det(AB)^t = \det(AB) = -594$

d) $\det(2A-3C+B) = 2 \cdot \begin{bmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 6 & -10 & 14 \\ 8 & 4 & 16 \\ 1 & -9 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 9 & -3 \\ 18 & 27 & -6 \\ 24 & 36 & -9 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -19 & 17 \\ -10 & -23 & 22 \\ -23 & -45 & 15 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -16 & 24 \\ -11 & -23 & 24 \\ -20 & -44 & 11 \end{bmatrix}$$

$$\cancel{\left| \begin{array}{ccc|cc} 4 & -16 & 24 & 4 & -16 \\ -11 & -23 & 24 & -11 & -23 \\ -20 & -44 & 11 & -20 & -44 \end{array} \right|}$$

$$4 \cdot (-1)^2 \cdot \overbrace{\begin{vmatrix} -23 & 24 \\ -44 & 11 \end{vmatrix}}^{-253+} + (-16) \cdot (-1)^3 \cdot \overbrace{\begin{vmatrix} -11 & 24 \\ -20 & 11 \end{vmatrix}}^{= -121+} +$$

$$24 \cdot (-1)^4 \cdot \overbrace{\begin{vmatrix} -11 & -23 \\ -20 & -44 \end{vmatrix}}^{= 484 - 460}$$

$4 \cdot 1 \cdot 803 + 16 \cdot 359 + 24 \cdot 24 = 9532$

e) $\det(AC^t) = \det(A) \cdot \det(C^t) = \det(A) \cdot \det(C)$

$$\underset{66}{\det(A)} \cdot \underset{0}{\det(C^t)} = 0$$

$$\left|\begin{matrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{matrix}\right| \begin{matrix} 2 & 3 \\ 6 & 9 \\ 8 & 12 \end{matrix}$$

$\cancel{-54} - \cancel{48} - \cancel{72} + \cancel{54} + \cancel{48} + \cancel{72} = 0$

3. Sabendo que $\det(A) = -2$, onde $A$ é uma matriz de ordem 4, encontre os determinantes

a) $\det(A^t) = \det(A) = -2$ //

b) $\det(5A) = 5 \cdot \det A = -10$ //

c) $\det(A^6) = -2^6 = 64$ //

d) $\det(A^{-1}) = (\det(A))^{-1} = -2^{-1} = \frac{1}{-2}$ //

4. Sabendo que $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$, calcule:

a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix} = 5 \cdot -3 = -15$ //

b) $\begin{vmatrix} a & b-2c \\ 3d & 3e-6f \\ g & h-2i \end{vmatrix} = 3 \cdot \begin{vmatrix} a & b & -2c \\ d & e & -2f \\ g & h & -2i \end{vmatrix} \; 3 \cdot (-2) \cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3 \cdot (-2) \cdot (-3) = 18$ //

c) $\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} \xrightarrow{L_2 \leftrightarrow L_3} \begin{vmatrix} -a & -b & -c \\ -d & -e & -f \\ g & h & i \end{vmatrix} = (-1) \cdot (-1) \cdot (-3) = -3$

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d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = +3$

e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} \xrightarrow{l_2 - l_1} \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} \; 2 \cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2 \cdot 3 = 6$

f) $\begin{vmatrix} ka+a & kb+b & kc+c \\ d & e & f \\ g & h & i \end{vmatrix} \longrightarrow k \cdot (-3)$

5. Calcule o determinante da matriz A:

$$\begin{bmatrix} 5 & 4 & 20 & 1 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{bmatrix} \quad 10 \cdot \begin{vmatrix} 5 & 4 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -5 & -7 & -3 & 9 \\ 3 & -6 & -3 & 12 \end{vmatrix} = 10 \cdot 2 \cdot 3 \begin{vmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 12 \end{vmatrix}$$

$$-60 \cdot \begin{vmatrix} 1 & -2 & -1 & 12 \\ 2 & 3 & 1 & -2 \\ -5 & -1 & -3 & 9 \\ 5 & 4 & 2 & 1 \end{vmatrix}$$

$l_2 = l_2 - 2l_1$
$l_3 = l_3 + 5l_1$
$l_4 = l_4 - 5l_1$

$$\begin{vmatrix} 1 & -2 & -1 & 12 \\ 0 & 7 & 3 & -26 \\ 0 & -11 & -8 & 69 \\ 0 & -6 & 7 & -59 \end{vmatrix}$$

$$1 \cdot (-1)^2 \cdot \begin{vmatrix} 7 & 3 & -26 \\ -11 & -8 & 69 \\ -6 & 7 & -59 \end{vmatrix} \begin{matrix} 7 & 3 \\ -11 & -8 \\ -6 & 7 \end{matrix} = -16$$

$-16 \cdot -60 = 960$

$$\begin{array}{r} 60 \\ 16 \\ \hline 360 \\ 600 \\ \hline 960 \end{array}$$

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6. Encontre os valores de x:

a) $\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128 \qquad x \cdot \begin{vmatrix} 4 & 6 & 1 \\ 7 & 4 & 2 \\ 5 & 2 & -1 \end{vmatrix} = -128$

$$\begin{array}{r} \overset{11}{\cancel{1}16} \\ -52 \\ \hline 64 \end{array}$$

$$\begin{array}{ccc|cc} 4 & 6 & 1 & 4 & 6 \\ 7 & 4 & 2 & 7 & 4 \\ 5 & 2 & -1 & 5 & 2 \end{array} = -16 + 60 + 14 + 42 - 16 - 20 = -52 + 116 = 64$$

74

116

$x \cdot 64 = -128 \quad x = -2$ //

b) $\begin{vmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{vmatrix} = 39 \qquad x \cdot \begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 4 & 6 & 7 \end{vmatrix} = 39$

$x \cdot 13 = 39$

$x = 3$ //

$$\begin{array}{ccc|cc} 3 & 5 & 7 & 3 & 5 \\ 2 & 1 & 3 & 2 & 1 \\ 4 & 6 & 7 & 4 & 6 \end{array}$$

$21 + 60 + 84 - 70 - 54 - 28 =$

$-7 + 6 + 14 = -7 + 20 = 13$

c) $\begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7$

$$\begin{array}{ccc|cc} (x+3) & (x+1) & (x+4) & (x+3) & (x+1) \\ 4 & 5 & 3 & 4 & 5 \\ 9 & 10 & 7 & 9 & 10 \end{array}$$

$(x+3) \cdot \overset{35}{5 \cdot 7} + (x+1) \cdot \overset{27}{3 \cdot 9} + (x+4) \cdot \overset{40}{4 \cdot 10} - (x+1) \cdot \overset{28}{4 \cdot 7} - (x+3) \cdot 3 \cdot 10 - (x+4) \cdot \overset{45}{5 \cdot 9} = -7$

62x   102x   -58x   -103x

$35x + 105 + 27x + 27 + 40x + 160 - 28x - 28 - 30x - 90 - 45x - 180 = -7$

132   292   -118   -298

$102x - 103x + 292 - 298 = -x - 6 = -7$

$-x = -1$

$x = 1$ //

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d) $\begin{bmatrix} x & x+2 \\ 1 & x \end{bmatrix}$ é singular

$x^2 - (x+2) = 0$

$x^2 - x - 2 = 0$

$a = 1 \quad b = -1 \quad c = -2$

$\dfrac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \dfrac{1 \pm \sqrt{1+8}}{2} = \dfrac{1 \pm \sqrt{9}}{2} = \dfrac{1 \pm 3}{2} = \{-1, 2\}$ //

e) $\begin{bmatrix} (x-4) & 0 & 3 \\ 2 & 0 & (x-9) \\ 0 & 3 & 0 \end{bmatrix}$ é inversível

$\left[\begin{array}{ccc} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{array}\right] \begin{array}{cc} x-4 & 0 \\ 2 & 0 \\ 0 & 3 \end{array}$

$0 \quad 0$

$18 - ((x-4)(x-9) \cdot 3) \neq 0$

$x^2 - 9x - 4x + 36 = (x^2 - 13x + 36) \cdot 3$

$18 - (3x^2 - 39x + \cancel{36}\ 108) \neq 0$

$18 - 3x^2 + 39x - 108$

$-3x + 39x - 90 \neq 0$

$\boxed{-x + 13x - 30 \neq 0}$

todos os casos exceto os que det = 0

$-x + 13x - 30 = 0$

$a = -1 \quad b = 13 \quad c = -30$

$\dfrac{-13 \pm \sqrt{169 - 120}}{-2} \qquad \dfrac{-13 \pm \sqrt{49}}{-2} = \dfrac{-13 \pm 7}{-2}$ $\begin{cases} \dfrac{-20}{-2} = 10 \\ \dfrac{-6}{-2} = 3 \end{cases}$

A é ~~in~~ reversível se $x \neq 10$ e $x \neq 3$

A é inversível $\forall x \mid x \neq 10, x \neq 3$

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7. Considere a matriz $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, com entradas a, b, c e d reais.

a) Encontre uma fórmula para $A^{-1}$, tal que $AA^{-1} = A^{-1}A = I_2$ onde $I_2$ é a matriz identidade de ordem 2, e indique a condição de existência da matriz inversa

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{Adj(A)}{det(A)} \quad \rightarrow Adj(A) = (cof(A))^t$$

$$cof(A) = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$$

$$(cof(A))^t = \begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{det(A)} \cdot \begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \text{ e } B = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$$

b) $A^{-1} = \frac{1}{1} \cdot \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{1} \cdot \begin{bmatrix} 2 & -7 \\ -4 & 2 \end{bmatrix}$

~~ante da matriz~~

8. Calcule a matriz dos cofatores

a) $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$ $\quad cof(A) = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

b) $B = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $\quad cof(B) = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$

$B_{11} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2-1 = -3$

$B_{12} = \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -1-0 = -1$

$B_{13} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1-0 = 1$

$B_{21} = \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} = 2-0 = 2$

$B_{22} = \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2-0 = -2$

$B_{23} = \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} \; 2-0 = 2$

$B_{31} = \begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} = -2-0 = -2$

$B_{32} = \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2-0 = 2$

$B_{33} = \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 4+2 = 6$ $\quad cof(B) = \begin{bmatrix} -3 & -1 & 1 \\ 2 & -2 & 2 \\ -2 & 2 & 6 \end{bmatrix}$

9. Use a matriz adjunta para encontrar a inversa das matrizes abaixo:

$A^{-1} = \frac{1}{det(A)} \cdot adj(A) \quad adj(A) = (cof(A))^{t}$

a) $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$ $\quad det(A) = 2 \cdot 1 - (-2 \cdot 3) \;\; 2+6 = 8$

$cof(A) = \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix}$ $\quad A^{-1} = \frac{1}{8} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix} = \cancel{\begin{bmatrix} 8 & 16 \\ 24 & 16 \end{bmatrix}} \begin{bmatrix} \frac{1}{8} & \frac{2}{8} \\ \frac{-3}{8} & \frac{2}{8} \end{bmatrix}$ //

b) $B = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $\quad cof(B) = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \tilde{b}_{13} \\ \tilde{b}_{21} & \tilde{b}_{22} & \tilde{b}_{23} \\ \tilde{b}_{31} & \tilde{b}_{32} & \tilde{b}_{33} \end{bmatrix}$

$\tilde{b}_{11} = (-1)^2 \cdot \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$

$\tilde{b}_{21} = (-1)^3 \cdot \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} = -2$

$\tilde{b}_{31} = (-1)^4 \cdot \begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} = -2$

$\tilde{b}_{12} = (-1)^3 \cdot \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = 1$

$\tilde{b}_{22} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2$

$\tilde{b}_{23} = (-1)^5 \cdot \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -2$

$\tilde{b}_{13} = (-1)^4 \cdot \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

$\tilde{b}_{23} = (-1)^5 \cdot \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = -2$

$\tilde{b}_{33} = (-1)^6 \cdot \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 6$

$det(B) = \begin{matrix} 2 & -2 & 0 & 2 & -2 \\ 1 & 2 & 1 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 \end{matrix} \quad -4 - 2 - 2 = -8$

$cof(B) = \begin{bmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$ $\quad (cof(B))^t = \begin{bmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{bmatrix} = adj(B)$

$adj(B) \cdot \frac{1}{det(B)} = \begin{bmatrix} -3 & 2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{bmatrix} \cdot \frac{1}{-8} = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$

//

c) $C = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ $\quad cof(C) = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \tilde{c}_{13} \\ \tilde{c}_{21} & \tilde{c}_{22} & \tilde{c}_{33} \\ \tilde{c}_{31} & \tilde{c}_{32} & \tilde{c}_{33} \end{bmatrix}$

$\tilde{c}_{11} = (-1)^2 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$\tilde{c}_{12} = (-1)^3 \cdot \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1$

$\tilde{c}_{13} = (-1)^4 \cdot \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$

$\tilde{c}_{21} = (-1)^3 \cdot \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$\tilde{c}_{22} = (-1)^4 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$\tilde{c}_{23} = (-1)^5 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = -1$

$\tilde{c}_{31} = (-1)^4 \cdot \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = 1$

$\tilde{c}_{32} = (-1)^5 \cdot \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = 2$

$\tilde{c}_{33} = (-1)^6 \cdot \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} = 2$

$det(C) = \begin{matrix} 0 & -1 & 1 & 0 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & \end{matrix} = 1 + 2 = 3$

$cof(C) = \begin{bmatrix} 1 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$ $\quad (cof(C))^t = adj(C) = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{bmatrix}$

$adj(C) \cdot \frac{1}{det(C)} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

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d)

$$D = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\det(D) = 1 + (-1)^4 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \qquad -2+3=1$$

$$\det(D) = 1 \cdot 1 = 1$$

$$\mathrm{cof}(D) = \begin{bmatrix} \tilde d_{11} & \tilde d_{12} & \tilde d_{13} & \tilde d_{14} \\ \tilde d_{21} & \tilde d_{22} & \tilde d_{23} & \tilde d_{24} \\ \tilde d_{31} & \tilde d_{32} & \tilde d_{33} & \tilde d_{34} \\ \tilde d_{41} & \tilde d_{42} & \tilde d_{43} & \tilde d_{44} \end{bmatrix}$$

$$\tilde d_{11} = (-1)^2 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 3$$

$$\tilde d_{12} = (-1)^3 \cdot \begin{vmatrix} 0 & 0 & 0 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

$$\tilde d_{13} = (-1)^4 \cdot \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\tilde d_{14} = (-1)^5 \cdot \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = -2$$

$$\tilde d_{21} = (-1)^3 \cdot \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

$$\tilde d_{22} = (-1)^4 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = -2+3 = 1$$

$$\tilde d_{23} = (-1)^5 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\tilde d_{24} = (-1)^6 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = 0$$

$$\tilde d_{31} = (-1)^4 \cdot \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -1$$

$$\tilde d_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

$$\tilde d_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\tilde d_{34} = (-1)^7 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = +1$$

$$\tilde d_{41} = (-1)^5 \cdot \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{vmatrix} = -(2)$$

$$\tilde d_{42} = (-1)^6 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 3 \end{vmatrix} = 0$$

$$\tilde d_{43} = (-1)^7 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = (-1)$$

$$\tilde d_{44} = (-1)^8 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{vmatrix} = 2$$

$$\mathrm{cof}(D) = \begin{bmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{bmatrix}$$

$$\mathrm{adj}(D) = (\mathrm{cof}(D))^t = \begin{bmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{bmatrix}$$

$$D^{-1} = \frac{1}{\det(D)} \cdot \mathrm{adj}(D) = 1 \cdot \mathrm{adj}(D) =$$

$$\begin{bmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{bmatrix}$$

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10. Considere a equação matricial $A^t - BX = 2C$ onde todas as matrizes são quadradas e de ordem 3x3.

a) Determine X em função das matrizes A, B e C e comente se é necessária alguma imposição à matriz B.

$A^t - BX = 2C \qquad \exists \det(B) \neq 0$

$-BX = 2C \cancel{-} - A^t$

$(\cdot B^{-1})\, BX = A^t - 2C\ (\cdot B^{-1}) \rightarrow (B B^{-1}) \cdot X = B^{-1}(A^t - 2C)$

$I_3 \cdot X = B^{-1}(A^t - 2C)$

$\boxed{X = B^{-1}(A^t - 2C)}$

b) Sendo $A = \begin{bmatrix} -1 & 2 & 6 \\ 2 & -1 & 4 \\ 6 & 4 & -1 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ e $C = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix}$, determine a matriz X.

$X = B^{-1}(A^t - 2C)$

$\rightarrow A^t - 2C = \begin{bmatrix} -1 & 2 & 6 \\ 2 & -1 & 4 \\ 6 & 4 & -1 \end{bmatrix} - 2C: \begin{bmatrix} 0 & 2 & 6 \\ 2 & 0 & 4 \\ 6 & 4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

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$$B^{-1} = \frac{1}{\det(B)} \cdot \operatorname{Adj}(B)$$

$$\left|\begin{matrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{matrix}\right. \begin{matrix} 1 & 0 \\ 2 & -1 \\ 4 & 1 \end{matrix} \quad \cancel{-8} + 0 + 4 - 0 - 3 + \cancel{8} = 1$$

$$\operatorname{cof}(B) = \begin{bmatrix} -11 & -4 & 6 \\ 2 & 0 & -1 \\ 2 & 1 & -1 \end{bmatrix} \quad \operatorname{adj}(B) = (\operatorname{cof}(B))^{t} = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & -1 \\ 6 & -1 & -1 \end{bmatrix} \cdot \cancel{\det} = B^{-1}$$

$$X = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & -1 \\ 6 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 11 & -2 & -2 \\ 4 & 0 & 1 \\ -6 & 1 & 1 \end{bmatrix}$$

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