Lista 5 - Geometria Analítica

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2024.1.08.009

1. Sendo ABCDEFGH o paralelogramo abaixo, expresse os seguintes vetores em função de $\vec{AB} = \vec{b}$, $\vec{AC} = \vec{c}$ e $\vec{AF} = \vec{f}$

a) $\vec{BF} = \vec{BA} + \vec{AF} = -\vec{AB} + \vec{AF} = -\vec{b} + \vec{f} = \vec{f} - \vec{b}$

b) $\vec{AG} = \vec{AF} + \vec{FG} = \vec{AF} + \vec{BC} = \vec{AF} + \vec{BA} + \vec{AC} = \vec{f} - \vec{b} + \vec{c}$

c) $\vec{AE} = \vec{AF} + \vec{FE} = \vec{AF} + \vec{BA} = \vec{f} - \vec{b}$

d) $\vec{BG} = \vec{BF} + \vec{FG} = \vec{BF} + \vec{BC} = \vec{BF} + \vec{BA} + \vec{AC} = \vec{f} - \vec{b} - \vec{b} + \vec{c} = \vec{f} + \vec{c} - 2\vec{b}$

e) $\vec{HB} = \vec{HE} + \vec{EA} + \vec{AB} = \vec{CB} + \vec{FB} + \vec{AB}$ $-\vec{AC} + \vec{AB} - \vec{AF} + \vec{AB} + \vec{AB} = 3\vec{b} - \vec{c} - \vec{f}$

f) $\vec{AB} + \vec{FG} = \vec{AB} + \vec{BC} = \vec{AB} - \vec{AB} + \vec{AC} = \vec{AC} = \vec{c}$

g) $\vec{AD} + \vec{HG} = \vec{BC} + \vec{AB} = \vec{AB} - AB + \vec{AC} = \vec{AC} = \vec{c}$

h) $\vec{HF} + \vec{AG} - \vec{EF} = \cancel{\vec{AB}} + \vec{AB} - \vec{AC} + \vec{AG} - \cancel{\vec{EF}} = \vec{b} - \vec{e}$

i) $2(\vec{c} - \vec{b}) - (\vec{c} - \vec{b}) -$

2. Seja ABCDEF um hexágono regular, como abaixo. Expresse os seguintes vetores em função de $\vec{DC}$ e $\vec{DE}$

a) $\vec{DF} = \vec{DE} + \vec{DC} + \vec{DE} = 2\vec{DE} + \vec{DC}$

b) $\vec{DA} = \vec{DC} + \vec{DE} + \vec{DC} + \vec{DE} = 2\vec{DC} + 2\vec{DE}$

c) $\vec{DB} = \vec{DC} + \vec{DE} + \vec{DC} = 2\vec{DC} + \vec{DE}$

d) $\vec{DO} = \vec{DC} + \vec{DE}$

e) $\vec{EC} = \vec{DC} - \vec{DE}$

f) $\vec{EB} = 2\vec{DC}$

g) $\vec{OB} = \vec{DC}$

h) $\vec{AF} = -\vec{DC}$

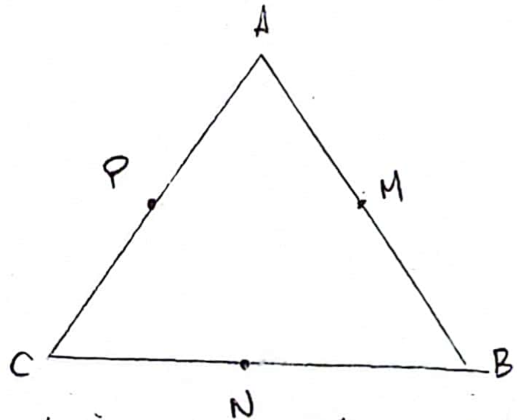
3. Dado um triângulo ΔABC, sejam M, N e P os pontos médios dos segmentos AB, BC e CA, respectivamente. Exprima os vetores $\vec{BP}$, $\vec{AN}$ e $\vec{CM}$ em função dos vetores $\vec{AB}$ e $\vec{AC}$.

$\vec{BP} = -\vec{AB} + \vec{AP} = -\vec{AB} + \frac{1}{2}\vec{AC}$

① $\vec{AP} = \frac{1}{2}\vec{AC}$

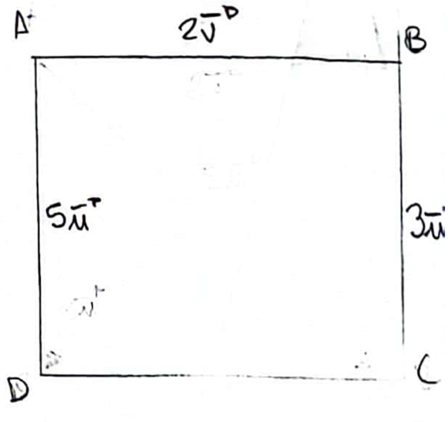
② $\vec{AN} = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{AB}$

③ $\vec{CM} = -\vec{AC} + \frac{1}{2}\vec{AB}$



4. Considere um quadrilátero ABCD, tal que $\vec{AD} = 5\vec{u}$, $\vec{BC} = 3\vec{u}$ e tal que $\vec{AB} = 2\vec{v}$

a) Determine o lado $\vec{CD}$ e as diagonais $\vec{BD}$ e $\vec{CA}$ em função de $\vec{u}$ e $\vec{v}$.



$\vec{CD} = \vec{CB} + \vec{BA} + \vec{AD} = -3\vec{u} - 2\vec{v} + 5\vec{u} = 2\vec{u} - 2\vec{v}$

$\vec{BD} = \vec{BA} + \vec{AD} = 5\vec{u} - 2\vec{v}$

$\vec{CA} = \vec{CB} + \vec{BA} = -3\vec{u} - 2\vec{v}$

b) Prove que ABCD é um trapézio usando vetores.

$\vec{AB} = 2\vec{v}$ $\quad \vec{AD} = 5\vec{u}$ $\quad \vec{BC} = 3\vec{u}$ $\quad \vec{CD} = 2\vec{u} - 2\vec{v}$

- Para ser trapézio, dois lados apenas podem ser paralelos

$\vec{AD} = \lambda \vec{BC}$ $\quad 5\vec{u} = \lambda 3\vec{u}$ $\quad \lambda = \frac{5}{3}$; AD e BC são paralelos.

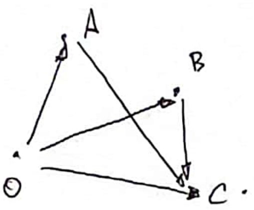
$2\vec{v} = \lambda(2\vec{u} - 2\vec{v})$ $\quad 2\vec{v} = \lambda 2\vec{u} - \lambda 2\vec{v}$ $\quad$ não são paralelos.

5. Considere os vetores $\vec{a} = \vec{OA}$, $\vec{b} = \vec{OB}$, $\vec{c} = \vec{OC}$ e sejam $\vec{AD} = \frac{1}{4}\vec{c}$ e $\vec{BE} = \frac{5}{6}\vec{a}$. Escreva o vetor DE em termos de $\vec{a}, \vec{b}, \vec{c}$.

$\vec{AD} - \vec{OA} + \vec{BE} - \vec{OB} = \vec{DE}$ $\qquad \vec{DE} = \frac{1}{4}\vec{c} - \frac{1}{6}\vec{a} - \vec{b}$

$\frac{1}{4}\vec{c} - \vec{a} + \frac{5}{6}\vec{a} - \vec{b} = \vec{DE}$

6. Dados $\vec{a}, \vec{b}$ vetores LI, sejam $\vec{OA} = \vec{a} + 2\vec{b}$, $\vec{OB} = 3\vec{a} + 2\vec{b}$ e $OC = 5\vec{a} + xb$, determine x de modo que os vetores $\vec{AC}$ e $\vec{BC}$ sejam <u>linearmente</u> <u>dependentes</u>.



$\Rightarrow$ determinante $= 0$

$\vec{AC} = \vec{OC} - \vec{OA} =$

$(5a + xb) - (a + 2b) = 4a + (x-2)b$

$\vec{BC} = \vec{OC} - \vec{OB} = (5a + xb) - (3a + 2b) = 2a + (x-2)b$

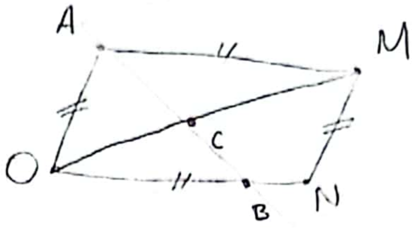
$$\begin{bmatrix} 4 & (x-2) \\ 2 & (x-2) \end{bmatrix} = (4x-8) - (2x-4) = 2x - 4$$

$2x - 4 = 0$

$2x = 4$

$x = \frac{4}{2} = 2$

7- Sejam B um ponto no lado ON do paralelogramo AMNO e C um ponto na diagonal OM tais que $\vec{OB} = \frac{1}{n}\vec{ON}$; $\vec{OC} = \frac{1}{1+n}\vec{OM}$. Prove que os pontos A, B e C são colineares.



$\vec{AC}$

$\vec{AC} = \vec{AO} + \frac{1}{1+n}\vec{OM}$

$\vec{AC} = \vec{MN} + \frac{1}{1+n}\vec{OM}$

$\vec{AC} = -\vec{OM} + ON + \frac{1}{1+n}\vec{OM}$

$\vec{AB}$ $\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AB} = \vec{AO} + \frac{1}{n}\vec{ON}$

$\vec{AB} = \vec{MN} + \frac{1}{n}\vec{ON}$

$\vec{AB} = -\vec{OM} + \vec{ON} + \frac{1}{n}\vec{ON}$

$\vec{BC}$ $\vec{BC} = - \quad \vec{ON} = -\vec{OB} + \vec{OC} = -\frac{1}{n}\vec{ON} + \frac{1}{1+n}\vec{OM}$

$\vec{AC} = \vec{ON} - \vec{OM} + \frac{1}{1+n}\vec{OM} = ON - \left(\frac{1}{1+n} - 1\right)\vec{OM}$

$\vec{AB} = \vec{ON} - \vec{OM} + \frac{1}{n}\vec{ON} = \left(\frac{1}{n} + 1\right)\vec{ON} - \vec{OM}$

$\vec{BC} = \frac{1}{1+n}OM - \frac{1}{n}\vec{ON}$.

$\lambda\left(\vec{ON} - \left(\frac{1}{1+n} - 1\right)\vec{OM}\right) = \left(\frac{1}{n} + 1\right)\vec{ON} - \vec{OM}$

$\lambda\vec{ON} - \lambda\vec{OM}\left(\frac{1}{1+n} - 1\right) = \left(\frac{1}{n} + 1\right)\vec{ON} - \vec{OM}$

$\lambda \cdot 1 = \frac{1}{n} + 1$

$\lambda\left(\frac{1}{1+n} - 1\right) = 1$.

$\lambda = \dfrac{1}{\frac{1}{1+n} - 1} = \dfrac{1}{\frac{1-1+n}{1+n}} = \dfrac{1}{\frac{n}{1+n}} = \frac{1}{1} \cdot \frac{1+n}{n} = \frac{1+n}{n}$

$$\begin{cases} \lambda = \frac{1+n}{n} \\ \lambda = \frac{1}{n} + 1 = \frac{1}{n} + \frac{n}{n} = \frac{1+n}{n} \end{cases}$$

$\rightarrow$ $\vec{AC}$ e $\vec{AB}$ são colinea

caso $\lambda \neq 0$

e $n \neq 0$

$\lambda \vec{AC} = \vec{BE}$

$\lambda(\vec{ON} - (\frac{1}{1+n} - 1)\vec{OM} = \frac{1}{1+n}\vec{OM} - \frac{1}{n}\vec{ON}$

$\lambda \vec{ON} - \lambda(\frac{1}{1+n} - 1)\vec{OM} = \frac{1}{1+n}\vec{OM} - \frac{1}{n}\vec{ON}$

$\lambda \cdot 1 = \frac{-1}{n}$

$(1+n) - (1+n)$

$1+n-1-n = \cancel{0}$

$-\lambda \cdot (\frac{1}{1+n} - 1) = (\frac{1}{1+n})$

$\lambda = \frac{\frac{-1}{1+n}}{\frac{1}{1+n}} = \frac{\frac{-1}{1+n}}{\frac{1 \cdot 1+n}{1+n-1+n}}$

8\. Mostre que o conjunto de vetores $\{\vec{u}, \vec{v}\}$ é uma base para o plano, então o conjunto $\{2\vec{u} + \vec{v}, \vec{u} - 2\vec{v}\}$

$\vec{u}, \vec{v}$ são LI, $2\vec{u} + \vec{v}, u - 2\vec{v}$ também são LI

$\lambda(2\vec{u} + \vec{v}) = u - 2\vec{v}$

$2\lambda\vec{u} + \lambda\vec{v} = u - 2\vec{v}$

$2\lambda = 1$ $\boxed{\frac{1}{2} = -2}$ → é LI e forma base

$\lambda = -2$

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combinação linear:

$\alpha_1(2\vec{u} + \vec{v}) + \alpha_2(\vec{u} - 2\vec{v}) = 0$

$2\vec{u}\alpha_1 + \alpha_1\vec{v} + \alpha_2\vec{u} - 2\alpha_2\vec{v} = 0$

$2\vec{u}\alpha_1 + \alpha_2\vec{u} + \alpha_1\vec{v} - 2\alpha_2\vec{v} = 0$

$\underbrace{\vec{u}(2\alpha_1 + \alpha_2)}_{=0} + \underbrace{\vec{v}(\alpha_1 - 2\alpha_2)}_{=0} = 0$

$\begin{cases} 2\alpha_1 + \alpha_2 = 0 \\ \alpha_1 - 2\alpha_2 = 0 \end{cases}$

$4\alpha_1 + 2\alpha_2 = 0$

$\alpha_1 - 2\alpha_2 = 0$

$5\alpha_1 = 0$

$\alpha_1 = 0$

$0 + \alpha_2 = 0$

$\alpha_2 = 0$

é LI e forma base.

9. Suponha que $\vec{u}, \vec{v}, \vec{w}$ formam um conjunto LI.

a) Mostre que os vetores $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}+\vec{w}$ e $\vec{u}+\vec{v}+\vec{w}$ também são LI.

$$\alpha_1(\vec{u}+\vec{v}) + \alpha_2(\vec{u}-\vec{v}+\vec{w}) + \alpha_3(\vec{u}+\vec{v}+\vec{w}) = 0$$

$$\alpha_1\vec{u} + \alpha_1\vec{v} + \alpha_2\vec{u} - \alpha_2\vec{v} + \alpha_2\vec{w} + \alpha_3\vec{u} + \alpha_3\vec{v} + \alpha_3\vec{w} = 0$$

$$(\alpha_1\vec{u} + \alpha_2\vec{u} + \alpha_3\vec{u}) + (\alpha_1\vec{v} - \alpha_2\vec{v} + \alpha_3\vec{v}) + (\alpha_2\vec{w} + \alpha_3\vec{w}) = 0$$

$$\vec{u}\underbrace{(\alpha_1+\alpha_2+\alpha_3)}_{=0} + \vec{v}\underbrace{(\alpha_1-\alpha_2+\alpha_3)}_{=0} + \vec{w}\underbrace{(\alpha_2+\alpha_3)}_{=0} = 0$$

$$\left.\begin{aligned}
&\cancel{\alpha_1}+\alpha_2+\alpha_3 = 0 \\
&\cancel{\alpha_1}-\alpha_2+\alpha_3 = 0 \\
&\alpha_2+\alpha_3 = 0 \\
&2\alpha_1 + 2\alpha_3 = 0 \\
&\alpha_1 + 2\alpha_3 = 0 \\
&-\alpha_1 - 2\alpha_3 = 0 \\
&\alpha_1 = 0 \\
&\alpha_3 = 0 \\
&\alpha_2 + 0 = 0 \\
&\alpha_2 = 0
\end{aligned}\right\} \boxed{LI}$$

b) Seja $\vec{t} = a\vec{u} + b\vec{v} + c\vec{w}$. Mostre que os vetores $\vec{u}+\vec{t}$, $\vec{v}+\vec{t}$ e $\vec{w}+\vec{t}$ são LI se, e somente se, $a+b+c \neq -1$

$$\alpha_1(\vec{u}+\vec{t}) + \alpha_2(\vec{v}+\vec{t}) + \alpha_3(\vec{w}+\vec{t}) = 0$$

$$\alpha_1(\vec{u} + a\vec{u} + b\vec{v} + c\vec{w}) + \alpha_2(\vec{v} + a\vec{u} + b\vec{v} + c\vec{w}) + \alpha_3(\vec{w} + a\vec{u} + b\vec{v} + c\vec{w})$$

$$\alpha_1\underbrace{\boxed{\vec{u}(1+a+b+c)}}_{=0} + \alpha_2\underbrace{\boxed{\vec{v}(\alpha_1+a+b+c)}}_{=0} + \alpha_3\underbrace{\boxed{\vec{w}(1+a+b+c)}}_{=0}$$

$1 + a+b+c \neq 0$ ($\alpha_1, \alpha_2, \alpha_3$ têm que ser zero,

$1 + (a+b+c) \neq 0$, se $a, b, c = 1$ eles admitem qualquer valor).

$$\boxed{a+b+c \neq -1}$$

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10 Dados os pontos $A=(1,3,2)$, $B=(1,0,-1)$, $C=(1,1,0)$, determine as coordenadas:

a) dos vetores $\vec{AB}, \vec{BC}$ e $\vec{CA}$.

$$\vec{AB} = \begin{bmatrix}1\\0\\-1\end{bmatrix} - \begin{bmatrix}1\\3\\2\end{bmatrix} = \begin{bmatrix}0\\-3\\-3\end{bmatrix} \quad \vec{BC} = C - B = \begin{bmatrix}1\\1\\0\end{bmatrix} - \begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix} \quad \vec{CA} = \begin{bmatrix}1\\3\\2\end{bmatrix} - \begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}0\\2\\2\end{bmatrix}$$

b) do vetor $\vec{AB} + \frac{2}{3}\vec{BC}$

$$\begin{bmatrix}0\\-3\\-3\end{bmatrix} + \frac{2}{3}\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\-3\\-3\end{bmatrix} + \begin{bmatrix}0\\\frac{2}{3}\\\frac{2}{3}\end{bmatrix} = \begin{bmatrix}0\\-\frac{7}{3}\\-\frac{7}{3}\end{bmatrix}$$

c) do ponto $C + \frac{1}{2}\vec{AB}$

$$\begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}0\\-\frac{3}{2}\\-\frac{3}{2}\end{bmatrix} = \begin{bmatrix}1\\\frac{1}{2}\\-\frac{3}{2}\end{bmatrix}$$

d) do ponto $A - 2\vec{BC}$

$$\begin{bmatrix}1\\3\\2\end{bmatrix} - 2\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}1\\3\\2\end{bmatrix} - \begin{bmatrix}0\\2\\2\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

11. determine quais dos conjuntos de vetores abaixo são LI

a) $\{(2,3), (0,2)\}$

$$\alpha_1(2,3) + \alpha_2(0,2) = \begin{cases}\alpha_1 \cdot 2 + \alpha_2 \cdot 0\\ \alpha_1 \cdot 3 + \alpha_2 \cdot 2\end{cases}$$

$2\alpha_1 = 0$
$3\alpha_1 + 2\alpha_2 = 0$

$\alpha_1 = 0$
$0 + 2\alpha_2 = 0$
$\alpha_2 = 0$

$\boxed{LI}$

b) $\{(3,0)(-2,0)$

$\begin{vmatrix}3 & 0\\-2 & 0\end{vmatrix} = 0$, então $\boxed{LD}$

$\alpha_1(3,0) + \alpha_2 \cdot (-2,0) = 0$
$3\alpha_1 - 2\alpha_2 = 0$

α_1 e α_2 admitem infinitos valores.

c) $\{(2,3,4),(0,3,3)\}$

$\alpha_1(2,3,4)+\alpha_2(0,3,3)=0$

$2\alpha_1+0\alpha_2=0 \rightarrow \alpha_1=0$

$3\alpha_1+3\alpha_2=0 \rightarrow \alpha_2=0$

$4\alpha_1+3\alpha_2=0$

$\boxed{LI}$

d) $\{(1,-1,2),(1,1,0),(1,-1,1)\}$

$\alpha_1(1,-1,2)+\alpha_2(1,1,0)+\alpha_3(1,-1,1)=0$

$\left.\begin{array}{l}\alpha_1+\alpha_2+\alpha_3=0\\ -\alpha_1+\alpha_2-\alpha_3=0\end{array}\right\}\ 2\alpha_2=0 \quad \alpha_2=0$

$\left.\begin{array}{l}2\alpha_1+0\alpha_2+\alpha_3=0\\ -\alpha_1-\alpha_2-\alpha_3=0\end{array}\right\}\ \alpha_1-\alpha_2=0$

$\alpha_1-0=0$

$\alpha_1=0$

$0+0+\alpha_3=0 \quad \alpha_3=0$

$\boxed{LI}$

e) $\{(1,-1,1),(-1,2,1),(-1,2,2)\}$

$\alpha_1(1,-1,1)+\alpha_2(-1,2,1)+\alpha_3(-1,2,2)=0$

$\left.\begin{array}{l}\alpha_1-\alpha_2-\alpha_3=0 \quad \cdot 2 \rightarrow 2\alpha_1-2\alpha_2-2\alpha_3\\ -\alpha_1+2\alpha_2+2\alpha_3=0\end{array}\right\}\ \alpha_1+0+0=0$

$\alpha_1=0$

$\alpha_1+\alpha_2+2\alpha_3=0$

$2\alpha_1+\alpha_3=0 \quad \alpha_3=0$

$\alpha_2=0$

$\boxed{LI}$

f) $\{(1,0,1),(0,0,1),(2,0,5)\}$

$\alpha_1(1,0,1)+\alpha_2(0,0,1)+\alpha_3(2,0,5)$

$\begin{vmatrix}1&0&1\\0&0&1\\2&0&5\end{vmatrix}=0$

$\boxed{LD}$

$\alpha_1+0+2\alpha_3=0$

$0+0+0=0$

$\alpha_1+\alpha_2+5\alpha_3=0$

$\alpha_2+3\alpha_3=0$

α_2 e α_3 admitem infinitos valores

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12. Faça a decomposição do vetor na base indicada:

a) exprima o vetor $\vec{w}=(1,1)$ como combinação linear de $\vec{u}=(2,-1)$ e $\vec{v}=(1,-1)$.

$$\vec{w} = 2\vec{u} + 3\vec{v}$$

$$\alpha_1(2,-1) + \alpha_2(1,-1) = (1,1)$$

$$+\begin{cases} 2\alpha_1 + \alpha_2 = 1 \\ -\alpha_1 - \alpha_2 = 1 \end{cases}$$

$\rightarrow \alpha_1 = 2$

$-2 - \alpha_2 = 1$

$-\alpha_2 = 3$

$\alpha_2 = -3$

b) Encontre as componentes do vetor $\vec{z}=(1,2,3)$ na base formada por $\vec{a}=(1,1,1)$, $\vec{b}=(0,1,1)$ e $\vec{c}=(1,1,0)$

$$\alpha_1(1,1,1) + \alpha_2(0,1,1) + \alpha_3(1,1,0) = (1,2,3)$$

$$\vec{z} = 2\vec{a} + \vec{b} - \vec{c}$$

$\alpha_1 + 0 + \alpha_3 = 1 \rightarrow \alpha_1 - 1 = 1$

$\alpha_1 + \alpha_2 + \alpha_3 = 2 \qquad \alpha_1 = 2$

$\alpha_1 + \alpha_2 + 0 = 3$

$\rightarrow -\alpha_1 - \alpha_2 = -3 \qquad 2 + \alpha_2 - 1 = 2$

$+ \rightarrow \alpha_3 = -1 \qquad \alpha_2 = 1$

13. Determine m e n de modo que os vetores $\vec{u}$ e $\vec{v}$ sejam linearmente dependentes.

a) $\vec{u} = (1, m-1, m)$, $\vec{v} = (m, 2n, 4)$

$\vec{u} = \lambda \vec{v}$

$(1, m-1, m) = \lambda(m, 2n, 4)$

$(1, m-1, m) = (\lambda m, \lambda 2n, \lambda 4)$

$1 = \lambda m \qquad m = \frac{1}{\lambda}$

$m-1 = \lambda 2n$

$m = \lambda 4 \qquad \frac{1}{\lambda} = 4\lambda \qquad 4\lambda^2 = 1 \qquad \lambda^2 = \frac{1}{4} \qquad \lambda = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$\rightarrow \pm 2 - 1 = \pm \frac{1}{2} 2n$

$\pm 2 - 1 = \pm n$

$2 - 1 = n$

$n = \pm 1, \pm 3$

$\pm\frac{1}{2} \cdot m = 1$

$m = \left\{-\frac{2}{1}, \frac{2}{1}\right\} = \{-2; 2\}$

$m = \pm 2$

b) $\vec{u} = (1, m, n+1)$, $\vec{v} = (m, n+1, 8)$

$\vec{u} = \lambda \vec{v}$

$(1, m, n+1) = (\lambda m, \lambda(n+1), \lambda 8)$

$\lambda m = 1 \rightarrow \lambda = \frac{1}{m}$

$m = \lambda(n+1)$

$n+1 = 8\lambda$

$n+1 = 8 \frac{1}{m} = \frac{8}{m}$

$m = \frac{1}{m} \cdot \frac{8}{m} = \frac{8}{m^2}$

$m \cdot m^2 = 8$

$m^3 = 8 \quad m = \sqrt[3]{8} = 2$

$\lambda = \frac{1}{2}$

$2 = \frac{1}{2}(n+1) = \frac{2}{\frac{1}{2}} = n+1 \qquad n+1 = 4$

$n = 3$

14. Sejam $\vec{u} = (m, -1, m^2+1)$, $\vec{v} = (m^2+1, m, 0)$ e $\vec{w} = (m, 1, 1)$. Mostre que os vetores $\vec{u}$, $\vec{v}$ e $\vec{w}$ ~~são~~ formam uma base para o espaço independente do valor de m.

$\alpha_1 (m, -1, m^2+1) + \alpha_2 (m^2+1, m, 0) + \alpha_3 (m, 1, 1) = 0$

$\alpha_1 m + \alpha_2 (m^2+1) + \alpha_3 m = 0$

$-\alpha_1 + \alpha_2 m + \alpha_3 = 0$

$\alpha_1 (m^2+1) + 0 + \alpha_3 = 0$

$$\left|\begin{array}{ccc} m & -1 & m^2+1 \\ m^2+1 & m & 0 \\ m & 1 & 1 \end{array}\right| \begin{array}{cc} m & -1 \\ m^2+1 & m \\ m & 1 \end{array}$$

$m^2 + 0 + (m^2+1)(m^2+1) + (m^2+1)$

$-0 - (m^2+1) \cdot (m) \cdot (m)$

$m^2 + \cancel{(m^4 + m^2)} + m^2 + 1 + m^2 + 1 -$

$\cancel{(m^4 + m^2)}$

$m^2 + m^2 + 1 + m^2 + 1 = 0$

$3m^2 + 2 = 0$

se $m \in \mathbb{R}$, é impossível que $3m^2+2$ seja igual a zero e portanto é impossível ser LD

$a = 3 \quad b = 0 \quad c = 2$

$0 \pm \sqrt{0 - 4 \cdot 3 \cdot 2}$

não tem resposta $m \in \mathbb{R}$

15. Considere fixada uma base $B = (\vec{e_1}, \vec{e_2}, \vec{e_3})$. Sejam $f_1 = (1,1,0)_B$, $f_2 = (1,0,1)_B$ e $f_3 (\phi, 1, -1)_B$.

a) Mostre que $C = (\vec{f_1}, \vec{f_2}, \vec{f_3})$ é uma base de V^3

$\alpha_1(1,1,0) + \alpha_2(1,0,1) + \alpha_3(1,1,-1) = 0$

$$\begin{cases} \alpha_1 + \alpha_2 + \alpha_3 = 0 \\ \alpha_1 + \alpha_3 = 0 \\ \alpha_2 - \alpha_3 = 0 \end{cases}$$

$\alpha_2 = \alpha_3$

$\alpha_1 = -\alpha_3$

$\alpha_2 = 0$

$0 - \alpha_3 = 0$

$\alpha_3 = 0$

$\alpha_1 + 0 + 0 = 0$

$\alpha_1 = 0$

portanto $\boxed{LI}$, forma base!

c) $\alpha_1(1,1,0) + \alpha_2(1,0,1) + \alpha_3(1,1,-1) = (2,3,7)$

$$\begin{cases} \alpha_1 + \alpha_2 + \alpha_3 = 2 \\ \alpha_1 + \alpha_3 = 3 \\ \alpha_2 - \alpha_3 = 7 \end{cases}$$

$\alpha_2 = -1$

$-1 - \alpha_3 = 7$

$-\alpha_3 = 6$

$\alpha_3 = -6$

$\alpha_1 - 6 = 3$

$\alpha_1 = 9$

$\alpha_1 = 9 \quad \alpha_2 = -1 \quad \alpha_3 = -6$

$(2,3,7)_B = (9_{e_1}, -1_{e_2}, -6_{e_3})_C$

b) $(2,3,7) \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} =$