Lista 4 - Geometria Analítica

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1. Em cada item, suponha que a matriz aumentada de um sistema foi transformada usando operações elementares na matriz escalonada reduzida dada. Resolva o sistema correspondente.

a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$ $\begin{cases} x = 2 \\ y = -1 \end{cases}$

b) $\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ $\begin{cases} x = 4 \\ y = 3 \\ z = 2 \\ w = 1 \end{cases}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

$z + w = 2$
$z = 2 - w$
$w = \alpha$

$\begin{cases} x = 6 \\ y = 3 \\ z = 2 - \alpha \\ w = \alpha \end{cases}, \alpha \in \mathbb{R}$

d) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$ $x + 3z = 1$, $y - z = 2$

$x = 1 - 3z$
$y = 2 + z$
$z = \alpha$

$\begin{cases} x = 1 - 3\alpha \\ y = 2 + \alpha \\ z = \alpha \end{cases}, \alpha \in \mathbb{R}$

e) $\begin{bmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{bmatrix}$

$x - 7w = 8$
$y + 3w = 2$
$z + w = -5$
$w = \alpha$

$\begin{cases} x = 8 + 7\alpha \\ y = 2 - 3\alpha \\ z = -5 - \alpha \\ w = \alpha \end{cases}, \alpha \in \mathbb{R}$

f) $\begin{array}{cccccc} x & y & z & w & t & \\ \end{array}$
$\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x - 6y + 3t = -2$
$z + 4t = 7$
$w + 5t = 8$
$t = \alpha$

$x = 6y - 3t - 2$
$z = 7 - 4t$
$w = 8 - 5t$
$t = \alpha$
$y = \beta$

$\begin{cases} x = 6\beta - 3\alpha - 2 \\ z = 7 - 4\alpha \\ w = 8 - 5\alpha \\ t = \alpha \\ y = \beta \end{cases}, \alpha, \beta \in \mathbb{R}$

2. Resolva os sistemas lineares usando o método de Gauss-Jordan.

a) $\begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases}$ $\begin{bmatrix} 3 & -4 & 1 \\ 1 & 3 & 9 \end{bmatrix} \overset{l_1 \leftrightarrow l_2}{\sim} \begin{bmatrix} 1 & 3 & 9 \\ 3 & -4 & 1 \end{bmatrix} \overset{l_2 \leftarrow l_2 - 3l_1}{\sim} \begin{bmatrix} 1 & 3 & 9 \\ 0 & -13 & -26 \end{bmatrix} \overset{l_2 \leftarrow l_2 : -13}{\sim}$

$\begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 2 \end{bmatrix} \overset{l_1 \leftarrow l_1 - 3l_2}{\sim} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $\begin{cases} x = 3 \\ y = 2 \end{cases}$

b) $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$ $[A|B] = \begin{bmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - 2l_1} \begin{bmatrix} 5 & 8 & 34 \\ 0 & 0 & -18 \end{bmatrix} \underset{l_2 : -18}{\overset{l_1 : 5}{\sim}}$

$\begin{bmatrix} 1 & \frac{8}{5} & \frac{34}{5} \\ 0 & 0 & 1 \end{bmatrix}$ $x + \frac{8}{5}y = \frac{34}{5}$ [SI] Sistema Impossível

$0 = 1$

c) $\begin{cases} x + 2y = 5 \\ 2x - 3y = 4 \end{cases}$ $[A|B]$ $\begin{bmatrix} 1 & 2 & 5 \\ 2 & -3 & 4 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - 2l_1} \begin{bmatrix} 1 & 2 & 5 \\ 0 & -7 & -6 \end{bmatrix} \underset{\sim}{l_2 : (-7)} \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & \frac{6}{7} \end{bmatrix}$

$\underset{\sim}{l_1 \leftarrow l_1 - 2l_2} \begin{bmatrix} 1 & 0 & \frac{23}{7} \\ 0 & 1 & \frac{6}{7} \end{bmatrix}$ $\begin{cases} x = \frac{23}{7} \\ y = \frac{6}{7} \end{cases}$ Conferindo:

$\frac{23}{7} + 2 \cdot \frac{6}{7} = 5$ $\qquad \frac{35}{7} - \frac{12}{7} = \frac{23}{7}$

$\frac{23}{7} + \frac{12}{7} = 5 \quad \frac{35}{7} = 5$ ok :)

d) $\begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$ $[A|B] = \begin{bmatrix} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{bmatrix} \underset{\sim}{l_3 \leftrightarrow l_1} \begin{bmatrix} 1 & -2 & -3 & -4 \\ 2 & -4 & -2 & -4 \\ 3 & 2 & -5 & 8 \end{bmatrix} \underset{l_3 \leftarrow l_3 - 3l_1}{\overset{l_2 \leftarrow l_2 - 2l_1}{\sim}}$

$\begin{bmatrix} 1 & -2 & -3 & -4 \\ 0 & 0 & 4 & 4 \\ 0 & 8 & 4 & 20 \end{bmatrix} \underset{l_2 : 4,\ l_3 : 8}{\overset{l_2 \leftrightarrow l_3}{\sim}} \begin{bmatrix} 1 & -2 & -3 & -4 \\ 0 & 1 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & 1 \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 + 2l_2} \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & 1 \end{bmatrix} \underset{l_2 \leftarrow l_2 - \frac{1}{2}l_3}{\overset{l_1 \leftarrow l_1 + 2l_3}{\sim}}$

$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $\begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases}$

e) $\begin{cases} 2x - 6y = -4 \\ x + 3y = 1 \\ 4 + 12y = 2 \end{cases}$ $[A|B]$ $\begin{bmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{bmatrix} \underset{\sim}{l_2 \leftrightarrow l_1} \begin{bmatrix} 1 & 3 & 1 \\ 2 & -6 & -4 \\ 4 & 12 & 2 \end{bmatrix} \underset{l_3 \leftarrow l_3 - 4l_1}{\overset{l_2 \leftarrow l_2 - 2l_1}{\sim}}$

$\begin{bmatrix} 1 & 3 & 1 \\ 0 & -12 & -6 \\ 0 & 0 & -2 \end{bmatrix} \underset{l_3 : (-2)}{\overset{l_2 : (-12)}{\sim}} \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 - 3l_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{cases} x = -\frac{1}{2} \\ y = \frac{1}{2} \\ 0 = 1 \end{cases}$ [SI]

Sistema Impossív

f) $x + 2y - z = 2$
$2x - y + 3z = 9$
$3x + 3y - 2z = 3$

$[A|B] = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 9 \\ 3 & 3 & -2 & 3 \end{bmatrix} \underset{l_3 \leftarrow l_3 - 3l_1}{\overset{l_2 \leftarrow l_2 - 2l_1}{\sim}} \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & -3 \end{bmatrix}$

$\overset{l_2 : (-5)}{\sim} \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -3 & 1 & -3 \end{bmatrix} \underset{l_3 \leftarrow l_3 + 3l_2}{\overset{l_1 \leftarrow l_1 - 2l_2}{\sim}} \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -6 \end{bmatrix} \overset{l_3 : (-2)}{\sim} \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\underset{l_2 \leftarrow l_2 + l_3}{\overset{l_1 \leftarrow l_1 - l_3}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$

g) $\begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$

$\begin{bmatrix} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{bmatrix} \underset{l_3 \leftarrow l_3 - 3l_1}{\overset{l_2 \leftarrow l_2 - 2l_1}{\sim}} \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 12 \\ 0 & -2 & -14 & 50 \end{bmatrix}$

$\overset{l_2 : (-4)}{\sim} \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 1 & \frac{3}{2} & -3 \\ 0 & -2 & -14 & 50 \end{bmatrix} \overset{l_3 \leftarrow l_3 + 2l_2}{\sim} \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 1 & \frac{3}{2} & -3 \\ 0 & 0 & -11 & 44 \end{bmatrix} \overset{l_3 : (-11)}{\sim} \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 1 & \frac{3}{2} & -3 \\ 0 & 0 & 1 & -4 \end{bmatrix}$

$\underset{l_2 \leftarrow l_2 - \frac{3}{2}l_3}{\overset{l_1 \leftarrow l_1 - 3l_3}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{bmatrix} \begin{cases} x = 4 \\ y = 3 \\ z = -4 \end{cases}$

h) $\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 26 \\ 2x + 2y + 3z = 13 \end{cases}$

$[A|B] = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 26 \\ 2 & 2 & 3 & 13 \end{bmatrix} \underset{l_3 \leftarrow l_3 - 2l_1}{\overset{l_2 \leftarrow l_2 - 3l_1}{\sim}} \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -4 \\ 0 & -2 & -3 & -7 \end{bmatrix}$

$\overset{l_2 \leftarrow l_2 : (-2)}{\sim} \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & 1 & \frac{3}{2} & 2 \\ 0 & -2 & -3 & -7 \end{bmatrix} \underset{l_3 \leftarrow l_3 + 2l_2}{\overset{l_1 \leftarrow l_1 - 2l_2}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & \frac{3}{2} & 2 \\ 0 & 0 & 0 & -3 \end{bmatrix} \underset{l_2 \leftarrow l_2 - 2l_3}{\overset{l_1 \leftarrow l_1 + l_3}{\sim}}$

[SI] Sistema Impossível

i) $\begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases}$ $[A|B] = \begin{bmatrix} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - 2l_1}$

$\begin{bmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 : (5)} \begin{bmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 + 3l_2} \begin{array}{c} \begin{matrix} x & y & z & w & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & -1 & 11 \\ 0 & 1 & -1 & 0 & 3 \end{bmatrix} \end{array}$

$x + z - w = 11$ $\quad x = w - z + 11$ $\quad \begin{cases} x = \alpha - \beta + 11 \\ y = 3 + \beta \end{cases}, \alpha, \beta \in \mathbb{R}$

$y - z = 3$ $\quad y = 3 + z$

$w = \alpha$ $\quad \alpha, \beta \in \mathbb{R}$

$z = \beta$

3. Resolva os sistemas lineares cujas matrizes aumentadas são:

a) $\begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix} \underset{l_3 \leftarrow l_3 - l_1}{\overset{l_2 \leftarrow l_2 - l_1}{\sim}} \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{bmatrix} \underset{l_3 \leftarrow l_3 + 2l_2}{\overset{l_1 \leftarrow l_1 - 2l_2}{\sim}}$

$\begin{bmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{bmatrix} \underset{\sim}{l_3 \leftarrow l_3 : (-7)} \begin{bmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \underset{l_2 \leftarrow l_2 + 3l_3}{\overset{l_1 \leftarrow l_1 - 9l_3}{\sim}}$

$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$ $\quad \begin{matrix} x + w = 1 \\ y = 2 \\ z = 1 \\ w = \alpha \end{matrix}$ $\quad \begin{cases} x = 1 - \alpha \\ y = 2 \\ z = 1 \\ w = \alpha \end{cases}, \alpha \in \mathbb{R}$

b) $\begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{bmatrix} \xrightarrow{l_3 \leftarrow l_3 - l_1} \sim \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} \underset{l_3 \leftrightarrow l_2}{\overset{l_3 \cdot (-1)}{\sim}} \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -3 & 3 \end{bmatrix}$

$\underset{l_3 \leftarrow l_3 - 2l_2}{\overset{l_1 \leftarrow l_1 - l_2}{\sim}} \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \overset{l_3 \leftarrow l_3 \cdot (-1)}{\sim} \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \underset{l_2 \leftarrow l_2 - l_3}{\overset{l_1 \leftarrow l_1 - 2l_3}{\sim}} \begin{array}{c} \begin{matrix} x & y & z & w & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \end{array}$

$x + w = 1$
$y - w = 2$
$z - w = -1$
$w = \alpha$

$\begin{cases} x = 1 - \alpha \\ y = 2 + \alpha \\ z = \alpha - 1 \\ w = \alpha \end{cases}, \alpha \in \mathbb{R}.$

c) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \underset{l_4 \leftarrow l_4 - l_1}{\overset{\substack{l_2 \leftarrow l_2 - l_1 \\ l_3 \leftarrow l_3 - l_1}}{\sim}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \overset{l_2 \leftarrow l_2 \cdot (-1)}{\sim} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

$\overset{\substack{l_1 \leftarrow l_1 - 2l_2 \\ l_3 \leftarrow l_3 + l_2 \\ l_4 \leftarrow l_4 - l_2}}{\sim} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 \end{bmatrix} \overset{\substack{l_1 \leftarrow l_1 + l_3 \\ l_2 \leftarrow l_2 - 2l_3 \\ l_4 \leftarrow l_4 + 2l_3}}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$

4. Os sistemas lineares seguintes possuem a mesma matriz A de coeficientes. Resolva-os simultaneamente usando o método de Gauss-Jordan.

a) $\begin{cases} x - 2y + z = 1 \\ 2x - 5y + z = -2 \\ 3x - 7y + 2z = -1 \end{cases}$

b) $\begin{cases} x - 2y + z = 2 \\ 2x - 5y + z = -1 \\ 3x - 7y + 2z = 2 \end{cases}$

$$[A|B_1|B_2] = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \\ 3 & -7 & 2 & -1 & 2 \end{bmatrix} \begin{array}{l} \ell_2 \leftarrow \ell_2 - 2\ell_1 \\ \ell_3 \leftarrow \ell_3 - 3\ell_1 \end{array} \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{bmatrix} \ell_2 \leftarrow \ell_2 \cdot (-1$$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 1 & 1 & 4 & 4 \end{bmatrix} \begin{array}{l} \ell_1 \leftarrow \ell_1 + 2\ell_2 \\ \ell_3 \leftarrow \ell_3 - \ell_2 \end{array} \begin{bmatrix} 1 & 0 & 3 & 9 & 12 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[A|B_1] = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad \begin{array}{l} x + 3z = 9 \\ y + z = 4 \\ z = \alpha \end{array} \quad \begin{cases} x = 9 - 3\alpha \\ y = 4 - \alpha \\ z = \alpha \end{cases}, \quad \alpha \in \mathbb{R}$$

$$[A|B_2] = \begin{bmatrix} 1 & 0 & 3 & 12 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} x + 3z = 12 \\ y + z = 5 \\ \boxed{0 = 1} \longrightarrow SI \end{array}$$

5. Sejam $A = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix}$, O uma matriz nula de ordem 3×1 e X uma matriz 3×1.

a) Encontre a solução geral do sistema $(A + 4I_3)X = O$, onde $I_3 = (\delta_{ij})_{3\times3}$

$$A + 4I_3 = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} + 4 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B} \qquad [A|B] = \begin{bmatrix} 5 & 0 & 5 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \ell_1 \leftarrow \ell_1 : 5 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - l_1} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 : 5} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \underset{\sim}{l_3 \leftarrow l_3 - l_2}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 - l_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \quad X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

b) Encontre a solução geral do sistema $AX = 2X$

$$AX - 2X = \bar{0} \rightarrow (A - 2I_3)X = \bar{0}$$

$$A \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} - 2 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{l_1 \cdot (-1)} \begin{bmatrix} 1 & 0 & -5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - l_1} \begin{bmatrix} 1 & 0 & -5 \\ 0 & -1 & 6 \\ 0 & 1 & -6 \end{bmatrix} \underset{\sim}{l_2 \cdot (-1)} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 1 & -6 \end{bmatrix} \underset{\sim}{l_3 \leftarrow l_3 - l_2}$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x - 5z = 0 \\ y - 6z = 0 \\ z = \alpha \end{matrix} \quad \begin{cases} x = 5\alpha \\ y = 6\alpha \\ z = \alpha \end{cases}, \ \alpha \in \mathbb{R}$$

6. Para cada sistema linear dado, encontre todos os valores de a para os quais o sistema não tem solução, tem solução única e tem infinitas soluções.

a) $\begin{cases} x + y + z = 2 \\ 2x + 3y + 2z = 5 \\ 2x + 3y + (a^2-1)z = a+1 \end{cases}$ $[A|B]$ $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & (a^2-1) & (a+1) \end{bmatrix}$ $\begin{matrix} l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 2l_1 \end{matrix}$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & (a^2-3) & (a-3) \end{bmatrix}$ $\begin{matrix} l_1 \leftarrow l_1 - l_2 \\ l_3 \leftarrow l_3 - l_2 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & (a^2-3) & (a-4) \end{array}\right]$

[SI] $\to (a^2-3) = 0 \quad a^2 = 3 \quad a = \pm\sqrt{3}$

~~[SPI]~~ $(a^2-3) = 0 \quad a = \pm\sqrt{3}, \quad a - 4 = 0 \quad \boxed{\pm\sqrt{3} - 4 = 0} \to$ impossível.

[SPD] $a \neq \pm\sqrt{3} \quad a - 4 \neq 0 \quad a \neq 4$

$S \mid \forall a \{a \mid a \in \mathbb{R}, a \neq \pm\sqrt{3}, a \neq 4\}$

b) $x + 2y - 3z = 4$
$3x - y + 5z = 2$
$4x + y + (a^2-14)z = a+2$

$[A|B]$ $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & (a+2) \end{bmatrix}$

$\begin{matrix} l_2 \leftarrow l_2 - 3l_1 \\ l_3 \leftarrow l_3 - 4l_1 \end{matrix}$ $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & (a^2-2) & (a-14) \end{bmatrix}$ $l_2 \leftarrow l_2 : (-7)$ $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & -7 & (a^2-2) & (a-14) \end{bmatrix}$ $\begin{matrix} l_1 \leftarrow l_1 - 2l \\ l_3 \leftarrow l_3 + 7 \end{matrix}$

$4 - \frac{20}{7} = \frac{28}{7} - \frac{20}{7} = \frac{8}{7}$

$\begin{matrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & (a^2-16) & (a-4) \end{matrix}$

[SI] $(a^2-16) = 0 \quad a = \pm 4$

[SPD] $a \neq \pm 4, \quad a - 4 \neq 0 \quad a \neq 4 \to \boxed{a \neq \pm 4}$

[SPI] $\boxed{a = 4}$

Mit CamScanner gescannt

7. Use a matriz identidade e as operações elementares entre linhas para encontrar a matriz inversa de:

a) $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$ $[A|I] = \begin{bmatrix} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 : 2} \begin{bmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - 3l_1}$

$$\begin{bmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 4 & -\frac{3}{2} & 1 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 : 4} \begin{bmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 + l_2} \begin{bmatrix} 1 & 0 & \frac{1}{8} & \frac{1}{4} \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{bmatrix} \quad A^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$$

b) $B = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $[B|I] = \begin{bmatrix} 2 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \underset{\sim}{l_1 \leftrightarrow l_2}$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - 2l_1} \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -6 & -2 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \underset{\sim}{l_2 \leftrightarrow l_3}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & -6 & -2 & 1 & -2 & 0 \end{bmatrix} \underset{l_3 \leftarrow l_3 + 6l_2}{\overset{l_1 \leftarrow l_1 - 2l_2}{\sim}} \begin{bmatrix} 1 & 0 & 3 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & -8 & 1 & -2 & 6 \end{bmatrix} \underset{\sim}{l_3 \leftarrow l_3 \div (-8)}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix} \underset{l_2 \leftarrow l_2 + l_3}{\overset{l_1 \leftarrow l_1 - 3l_3}{\sim}} \begin{bmatrix} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$$

c) $C = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ $[C|1] = \left[\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \underset{\sim}{l_2 \leftrightarrow l_1} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 - 3l_1}$

$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -3 \end{bmatrix} \underset{\sim}{l_2 \cdot (-1)} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & -1 & 3 \end{bmatrix} \underset{\sim}{l_1 \leftarrow l_1 - 2l_2} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

d) $D = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ $[D|1_3] = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \underset{l_1 \leftrightarrow l_2}{\overset{l_1 \leftrightarrow l_3}{\sim}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$

$\underset{\sim}{l_3 \leftarrow l_3 - 2l_1} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 & 1 & -2 \end{bmatrix} \underset{\sim}{l_2 \leftarrow l_2 \cdot (-1)} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & -2 & -1 & 0 & 1 & -2 \end{bmatrix} \underset{l_3 \leftarrow l_3 + 2l_2}{\overset{l_1 \leftarrow l_1 - l_2}{\sim}}$

$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 1 & -2 \end{bmatrix} \underset{\sim}{l_3 \leftarrow l_3 \cdot (-\frac{1}{3})} \begin{bmatrix} 1 & 0 & 1 & 1 & \frac{1}{3} & 1 \\ 0 & 1 & -1 & -1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \underset{l_2 \leftarrow l_2 + l_3}{\overset{l_1 \leftarrow l_1 - l_3}{\sim}}$

$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ A^{-1}

e) $\left[\begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{l_1 \leftrightarrow l_3} \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} l_2 \leftarrow l_2 - 2l$

$\begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -4 & -5 & 1 & 0 & -2 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \overset{l_3 \leftrightarrow l_2}{} \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \underset{\sim}{l_2 \cdot (-1)} \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$l_3 \leftarrow l_3 - 2l_2$

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & -8 & -11 & 2 & 1 & -4 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} l_3/-8 \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & \frac{11}{8} & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$l_1 \leftarrow l_1 - 2l_3$

$l_2 \leftarrow l_2 - 4l_3$

$l_4 \leftarrow l_4 + 2l_3$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{8} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{8} & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{11}{4} & -\frac{1}{2} & -\frac{1}{4} & 1 & 1 \end{bmatrix} \frac{24}{8} \quad l_4 / \frac{11}{4} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{8} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{8} & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \boxed{-\frac{2}{11}} & \boxed{-\frac{1}{11}} & \boxed{\frac{4}{11}} & \boxed{\frac{4}{11}} \end{bmatrix}$$

$l_1 \leftarrow l_1 - \frac{1}{4} l_4$

$l_2 \leftarrow l_2 + \frac{1}{2} l_4$

$l_3 \leftarrow l_3 - \frac{11}{8} l_4$

$-\frac{2}{11} \cdot \frac{1}{4} = -\frac{1}{22} \quad \frac{11}{22} + \frac{1}{22} = \frac{12}{22} = \frac{6}{11}$

$-\frac{1}{11} \cdot \frac{1}{4} - \frac{1}{44} \quad \frac{11}{44} + \frac{1}{44} = \frac{12}{44} = \frac{6}{22} = \frac{3}{11}$

$\frac{4}{11} \cdot \frac{1}{4} = \frac{4}{44} = \frac{1}{11}$

$-\frac{2}{11} \cdot \frac{1}{2} = \frac{2}{22} = \frac{1}{11}$

$-\frac{1}{11} \cdot \frac{1}{2} = -\frac{1}{22} + \frac{11}{22} = \frac{10}{22} = \frac{5}{11}$

$\frac{\cancel{4}\,2}{11} \cdot \frac{1}{\cancel{2}} = \frac{2}{11}$

$-\frac{2}{\cancel{11}} \cdot \frac{\cancel{11}}{8} = -\frac{2}{8} \quad -\frac{2}{8} + \frac{2}{8} = 0$

$-\frac{1}{11} \cdot \frac{11}{8} = -\frac{1}{8}$

$\frac{4}{11} \cdot \frac{11}{8} = \frac{4}{8} = \frac{1}{2} \quad \frac{1}{2} - \frac{1}{2}$

$\frac{4}{4}$

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \begin{bmatrix} \frac{6}{11} & \frac{3}{11} & -\frac{1}{11} & -\frac{1}{11} \\ -\frac{1}{11} & \frac{5}{11} & \frac{2}{11} & \frac{2}{11} \\ 0 & 0 & 0 & -\frac{1}{2} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{bmatrix} \quad A^{-1}$$

8.

· 3 pacotes diferentes

1 c 2 s 3 b → 26

2c 5s 6b → 60

2c 3s 4b → 40

$$1x + 2y + 3z = 26$$
$$2x + 5y + 6z = 60$$
$$2x + 3y + 4z = 40$$

$$\begin{cases} x = 4 \\ y = 8 \\ z = 2 \end{cases}$$

$$2x + 4y + 6z = 52$$
$$\cancel{-2x} - 4y - \cancel{6z} = -52$$
$$\cancel{2x} + 5y + \cancel{6z} = 60$$
$$y = 8$$

$$x + 16 + 6 = 26$$
$$x = 4$$

$$\cancel{2x} + 5y + 6z = 60$$
$$\cancel{-2x} - 3y - 4z = -40$$
$$2y + 2z = 20$$
$$y + z = 10$$
$$8 + z = 10$$
$$z = 2$$

9.

· lucro 2200

sundae 5

casquinha 2

banana split 6

triplo caoq b. s.

$$5x + 2y + 6z = 2200$$

sundá casquinha banana split

$$y = 3z \qquad 3z = x + z$$
$$y = x + z \qquad 2z = x$$

$$\begin{cases} 5x + 2y + 6z = 2200 \\ y = 3z \\ x = 2z \end{cases}$$

$$10z + 6z + 6z = 2200$$
$$22z = 2200$$
$$z = \frac{2200}{22} = 100$$

$$x = 2 \cdot 100 = 200$$
$$y = 3 \cdot 100 = 300$$

Conferindo:

$$\underbrace{5 \cdot 200}_{1000} + \underbrace{2 \cdot 300}_{600} + \underbrace{6 \cdot 100}_{600} = 2200$$

10

$\cancel{40}t$, $\cancel{30}p$, $\cancel{10}$ piz, total $\cancel{7000}$

$\cancel{20}t$, $\cancel{40}p$, $\cancel{30}$ piz, total $\cancel{6000}$

$\cancel{10}t$, $\cancel{20}p$, $\cancel{40}$ piz, total $\cancel{5000}$

$$4x+3y+z=700$$
$$2x+4y+3z=600$$
$$x+2y+4z=500$$

$$\cancel{2x}+\cancel{4y}+8z=1000$$
$$-\cancel{2x}-\cancel{4y}-3z=-600$$
$$5z=400 \quad z=\frac{400}{5}=80$$

$$\cancel{4x}+3y+z=700$$
$$-\cancel{4x}-8y-6z=-1200$$
$$-5y-5z=-500$$
$$5y+5z=500$$
$$5y+400=500$$
$$5y=100 \quad y=\frac{100}{5}=20$$
$$4x+60+80=700$$
$$4x=560$$
$$x=\frac{560}{4}=140$$

$$\underbrace{4\cdot 140}+3\cdot 20+80$$
$$560+60+80=700$$
$$\underbrace{2\cdot 140}_{280}+\underbrace{4\cdot 20}_{80}+\underbrace{3\cdot 80}_{240}=600$$
$$360 \qquad 600$$

ok

$$\begin{cases} x=140 \\ y=20 \\ z=80 \end{cases}$$

11.

a, b, c

$2a \quad 3b \quad 1c \rightarrow 8420$

$1a \quad 2b \quad 2c \rightarrow 7940$

$4a \quad 3b \quad 0c \rightarrow 8110$

$$2x+3y+z=8420$$
$$x+2y+2z=7940$$
$$4x+3y \quad =8110$$

$$A=\left[\begin{array}{ccc|c} 2&3&1&8420\\ 1&2&2&7940\\ 4&3&0&8110 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1&2&2&7940\\ 2&3&1&8420\\ 4&3&0&8110 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1&2&2&7940\\ 0&-1&-3&-7460\\ 0&-5&-8&-23650 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1&2&2&\text{[illegible]}\\ 0&1&3&\text{[illegible]}\\ 0&5&8&\text{[illegible]} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1&0&-4&-6980\\ 0&1&3&7460\\ 0&0&-7&-13650 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1&0&-4&-6980\\ 0&1&3&7460\\ 0&0&7&13650 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1&0&-4&-6980\\ 0&1&3&7460\\ 0&0&1&1950 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1&0&0&820\\ 0&1&0&1610\\ 0&0&1&1950 \end{array}\right]$$

$$\begin{cases} x=820 \\ y=1610 \\ z=1950 \end{cases}$$