Geometria Analítica - Lista I ~~# 6, 8~~ 10

1\. Considere as seguintes matrizes: $A = \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix}_{2\times2}$, $B = \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix}_{2\times2}$,

$C = \begin{bmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{bmatrix}_{2\times3}$, $D = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix}_{3\times3}$, $E = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{bmatrix}_{3\times3}$, $F = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}_{3\times1}$

1.1)

a) $A + 2B$ $\quad \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 7 & 3 \end{bmatrix}_{2\times2}$

b) $AB - BA$

$$\cancel{AB = \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 15 & -14 \end{bmatrix} \quad BA = \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 0 & -14 \end{bmatrix}}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} (1\cdot0 + 0\cdot2) & (1\cdot5 + 0\cdot-2) \\ (3\cdot0 + 7\cdot2) & (3\cdot5 + 7\cdot-2) \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 14 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} (0\cdot1 + 5\cdot3) & (0\cdot0 + 5\cdot7) \\ (2\cdot1 + (-2)\cdot3) & (2\cdot0 + (-2)\cdot7) \end{bmatrix} = \begin{bmatrix} 15 & 35 \\ -4 & -14 \end{bmatrix}$$

$$AB - BA = \begin{bmatrix} 0 & 5 \\ 14 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 35 \\ -4 & -14 \end{bmatrix} = \begin{bmatrix} -15 & -30 \\ 18 & 15 \end{bmatrix} //$$

c) $2C - D = 2 \cdot \begin{bmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{bmatrix} - \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix}$ não é possível

d) $2D^t - 3E^t = 2 \cdot \begin{bmatrix} -3 & 1 & -2 \\ 2 & 1 & 0 \\ 0 & 4 & 2 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & -1 & -6 \\ 4 & 0 & 0 \\ -3 & -4 & -1 \end{bmatrix} =$

$$\begin{bmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{bmatrix} = \begin{bmatrix} -12 & 5 & 14 \\ -8 & 2 & 0 \\ 9 & 20 & 7 \end{bmatrix} //$$

e) $D^2 + DE =$

$$D \cdot D = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} (\overset{9}{-3\cdot(-3)} + \overset{2}{2\cdot 1} + \overset{0}{0\cdot(-2)}) & (\overset{-6}{-3\cdot 2} + \overset{2}{2\cdot 1} + \overset{0}{0\cdot 0}) & (\overset{0}{-3\cdot 0} + \overset{8}{2\cdot 4} + \overset{0}{0\cdot 2}) \\ (\overset{-3}{1\cdot(-3)} + \overset{1}{1\cdot 1} + \overset{-8}{4\cdot(-2)}) & (\overset{2}{1\cdot 2} + \overset{1}{1\cdot 1} + \overset{0}{4\cdot 0}) & (\overset{0}{1\cdot 0} + \overset{4}{1\cdot 4} + \overset{8}{4\cdot 2}) \\ (\overset{6}{-2\cdot(-3)} + \overset{0}{0\cdot 1} + \overset{-4}{2\cdot(-2)}) & (\overset{-4}{-2\cdot 2} + \overset{0}{0\cdot 1} + \overset{0}{2\cdot 0}) & (\overset{0}{-2\cdot 0} + \overset{0}{0\cdot 4} + 2\cdot 2) \end{bmatrix} =$$

$$D^2 = \begin{bmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{bmatrix}$$

$$DE = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} (\overset{-6}{-3\cdot 2} + \overset{-2}{2\cdot(-1)} + \overset{0}{0\cdot(-6)}) & (\overset{-12}{-3\cdot 4} + \overset{0}{2\cdot 0} + \overset{0}{0\cdot 0}) & (\overset{9}{-3\cdot(-3)} + \overset{-8}{2\cdot(-4)} + \overset{0}{0\cdot(-1)}) \\ (\overset{2}{1\cdot 2} + \overset{-1}{1\cdot(-1)} + \overset{-24}{4\cdot(-6)}) & (\overset{4}{1\cdot 4} + \overset{0}{0\cdot 0} + \overset{0}{4\cdot 0}) & (\overset{-3}{1\cdot(-3)} + \overset{-4}{1\cdot(-4)} + \overset{-4}{4\cdot(-1)}) \\ (\overset{-4}{-2\cdot 2} + \overset{0}{0\cdot(-1)} + \overset{-12}{2\cdot(-6)}) & (\overset{-8}{-2\cdot 4} + \overset{0}{0\cdot 0} + \overset{0}{2\cdot 0}) & (\overset{6}{-2\cdot(-3)} + \overset{0}{0\cdot(-4)} + \overset{-2}{2\cdot(-1)}) \end{bmatrix}$$

$$DE = \begin{bmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{bmatrix}$$

$$D^2 + DE = \begin{bmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{bmatrix} + \begin{bmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & -16 & 9 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{bmatrix}$$

f) $C^t A = \begin{bmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{bmatrix}_{3\times 2} \cdot \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix}_{2\times 2} = \begin{bmatrix} (\underbrace{-2\cdot 1}_{-2} + \underbrace{7\cdot 3}_{21}) & (\underbrace{-2\cdot 0}_{0} + \underbrace{7\cdot 7}_{49}) \\ (\underbrace{3\cdot 1}_{3} + \underbrace{(-3)\cdot 3}_{-9}) & (\underbrace{3\cdot 0}_{0} + \underbrace{(-3)\cdot 7}_{-21}) \\ (\underbrace{-7\cdot 1}_{-7} + \underbrace{(-2)\cdot 3}_{-6}) & (\underbrace{-7\cdot 0}_{0} + \underbrace{(-2)\cdot 7}_{-14}) \end{bmatrix} =$

$\begin{bmatrix} 19 & 49 \\ -6 & -21 \\ -13 & -14 \end{bmatrix}$ //

g) $[E]_{3\times 3} - [A]_{2\times 2} \cdot [C]_{2\times 3}$ → resulta em matriz 2×3
→ não é possível

h) $F^t E = [1\ -2\ 0]_{1\times 3} \cdot \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{bmatrix}_{3\times 3} =$ ~~1·2+(-2)·(-1)+0·-6~~

$[(\underbrace{1\cdot 2}_{2} + \underbrace{(-2)\cdot(-1)}_{2} + \underbrace{0\cdot(-6)}_{0})\ (1\cdot 4 + \underbrace{(-2)\cdot 0}_{0} + \underbrace{0\cdot 0}_{0})\ (\underbrace{1\cdot(-3)}_{-3} + \underbrace{(-2)\cdot(-4)}_{8} + 0)]$

$[4, 4, 5]$ //

i) $\underset{2\times 2}{B}\,\underset{2\times 3}{C}\,\underset{3\times 1}{F}$

$CF = \begin{bmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} (\underbrace{-2\cdot 1}_{-2} + \underbrace{3\cdot(-2)}_{-6} + \underbrace{(-7)\cdot 0}_{0}) \\ (\underbrace{7\cdot 1}_{7} + \underbrace{(-3)\cdot(-2)}_{6} + \underbrace{(-2\cdot 0)}_{0}) \end{bmatrix} =$

$\begin{bmatrix} -8 \\ 13 \end{bmatrix}$

$B(CF) = \begin{bmatrix} 0 & 5 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 13 \end{bmatrix} \quad \begin{bmatrix} (\underbrace{0\cdot(-8)}_{0} + \underbrace{5\cdot 13}_{65}) \\ (\underbrace{2\cdot(-8)}_{-16} + \underbrace{(-2)\cdot 13}_{26}) \end{bmatrix} = \begin{bmatrix} 65 \\ -42 \end{bmatrix}$ //

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2.

a) produto existe, ordem de $C = 2\times4$.

$A_{2\times3}\ B_{3\times4}$ BA não possui produto definido

$B_{3\times4}\ A_{2\times3}$

b) $A_{4\times1} \cdot B_{1\times2}$ produto existe, $C_{4\times2}$, BA não possui produto definido.

$B_{1\times2}\ A_{4\times1}$

c) $A_{1\times2}\ B_{3\times1}$ produto não existe, BA possui produto definido

$B_{3\times1}\ A_{1\times2}$

d) $A_{5\times2}\ B_{2\times3}$ produto existe; $C_{5\times3}$; BA não possui produto definido

$B_{2\times3}\ A_{5\times2}$

e) $A_{4\times4}\ B_{3\times3}$ produto não existe, BA não possui produto definido

f) $A_{4\times2}\ B_{2\times4}$ produto existe; $C_{4\times4}$; BA possui produto definido.

$B_{2\times4}\ A_{4\times2}$

g) $A_{2\times1}\ B_{1\times3}$ produto existe; $C_{2\times3}$; BA não possui produto definido

$B_{1\times3}\ A_{2\times1}$

h) $A_{2\times2}\ B_{2\times2}$ produto existe; $C_{2\times2}$; BA possui produto definido.

$B_{2\times2}\ A_{2\times2}$

3.

a) $A = (a_{ij})_{2\times3}$, onde $a_{ij} = 3i - 2j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} (3-2) & (3-4) & (3-6) \\ (6-2) & (6-4) & (6-6) \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{bmatrix} //$$

b) $B = (b_{ij})_{3\times3}$, onde $b_{ij} = \begin{cases} 2i + j, & \text{se } i = j \\ i^2 - j, & \text{se } i \neq j \end{cases}$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 3 & -1 & -2 \\ 3 & 6 & 1 \\ 8 & 7 & 9 \end{bmatrix} //$$

c) $C = (c_{ij})_{1\times4}$, onde $c_{ij} = j^i$

$$C = [c_{11} \; c_{12} \; c_{13} \; c_{14}] = [(1^1) \; (2^1) \; (3^1) \; (4^1)] = [1\,2\,3\,4] //$$

d) $D = (d_{ij})_{4\times4}$, onde $d_{ij} = \begin{cases} i^2 + j^2, & \text{se } i = j \\ 2ij, & \text{se } i \neq j \end{cases}$

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} (1+1) & (2\cdot1\cdot2) & (2\cdot1\cdot3) & (2\cdot1\cdot4) \\ (2\cdot2\cdot1) & (4+4) & (2\cdot2\cdot3) & (2\cdot2\cdot4) \\ (2\cdot3\cdot1) & (2\cdot3\cdot2) & (9+9) & (2\cdot3\cdot4) \\ (2\cdot4\cdot1) & (2\cdot4\cdot2 & (2\cdot4\cdot3) & (16+16) \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{bmatrix} //$$

4. Dadas as matrizes $A=\begin{bmatrix}1 & 2 & 1\\ -2 & -3 & 2\\ 1 & 4 & 5\end{bmatrix}$ e $B=\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & 4\\ -3 & -1 & -17\end{bmatrix}$, use apenas as linhas e colunas indicadas para encontrar:

a) $[BA]_{23}$ $[2\ -1\ 4]\cdot\begin{bmatrix}1\\2\\5\end{bmatrix} = (\overset{2}{2\cdot 1} + \overset{-2}{(-1)\cdot 2} + \overset{20}{4\cdot 5}) = 20$

b) $[AB]_{23}$ $[-2\ -3\ 2]\cdot\begin{bmatrix}3\\4\\-17\end{bmatrix} = [(\overset{-6}{-2\cdot 3} + \overset{-12}{(-3)\cdot 4} + \overset{-34}{2\cdot(-17)})] = -52$

c) $[B^2]_{31}$ $[-3\ -1\ -17]\cdot\begin{bmatrix}1\\2\\-3\end{bmatrix} = (\overset{-3}{-3\cdot 1} + \overset{-2}{(-1)\cdot 2} + \overset{51}{(-17)\cdot(-3)}) = 46$

d) $tr(A) = a_{11} + a_{22} + a_{33} = 1 + (-3) + 5 = 3$

e) $tr(B^t) = [B^t]_{11} + [B^t]_{22} + [B^t]_{33} = 1 + (-1) + (-17) = -17$

f) $tr(A-B) = [A+B]_{11} + [A-B]_{22} + [A-B]_{33} = (1-1) + (-3-1) + (5-(-17)) =$

$0 + \cancel{(-4)} + 22 = \cancel{18}$

(-2) 20

g) $tr(AB) = [AB]_{11} + [AB]_{22} + [AB]_{33} =$

$(1\cdot 1 + (-2)\cdot 2 + 1\cdot(-3)) + (-2\cdot 0 + (-3)\cdot(-1) + 2\cdot(-1)) + (1\cdot 3 + 4\cdot 4 + 5\cdot(-17)) =$

$1 + -4 - 3 + 0 + 3 - 2 + 3 + 16 - 85 =$

$-6 + \underbrace{0 + 3 - 2}_{} + 3 + 16 - 85 =$

$-6 + 1 \cancel{4} - 66 = -71$

5. Sendo $A = \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ e $C = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$, determine ~~se~~ X e Y em cada uma das expressões matriciais abaixo:

a) $2X + A = 3B + C$

$2X = 3B + C - A$

$X = \frac{3B}{2} + \frac{C}{2} - \frac{A}{2}$

$\rightarrow X = \frac{3}{2} \cdot \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} =$

$\begin{bmatrix} 3 & \frac{3}{2} \\ 6 & \frac{9}{2} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{bmatrix} - \begin{bmatrix} -\frac{1}{2} & \frac{7}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{bmatrix}$

b) $Y + A = \frac{1}{2}(B - C)^t$

$Y + A = (\frac{1}{2}B - \frac{1}{2}C)^t -$

$Y = (\frac{1}{2}B - \frac{1}{2}C)^t - A$

$= \left(\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{bmatrix} \right)^t - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & -\frac{1}{2} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix}^t - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{11}{2} \\ \frac{5}{2} & -\frac{9}{2} \end{bmatrix}$

c) $3X + A = B - X$

$4X = B - A$

$X = \frac{B}{4} - \frac{A}{4}$

$X = (B - A) \cdot \frac{1}{4}$

$\rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} \cdot \frac{1}{4} = \begin{bmatrix} 3 & -6 \\ 2 & -3 \end{bmatrix} \cdot \frac{1}{4} = \begin{bmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$ //

d) $\begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases}$ $+$ $2X = 3A + 2B + C$

$X = (3A + 2B + C) \cdot \frac{1}{2}$

$Y = 3A - X$

$\boxed{Y = \begin{bmatrix} -3 & 21 \\ 6 & 18 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{bmatrix}}$ //

$\left(\begin{bmatrix} -3 & 21 \\ 6 & 18 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \right) \frac{1}{2} =$

$\boxed{\begin{bmatrix} 1 & 25 \\ 15 & 24 \end{bmatrix} \cdot \frac{1}{2} = \begin{bmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{bmatrix}}$ //

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6.

$A = \begin{bmatrix} 1 & \frac{1}{x} \\ x & 1 \end{bmatrix}$ $x = 1, 2, 3, 4 \ldots$ $\boxed{x \in \mathbb{R} \quad x \neq 0}$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}^2 = 2 \cdot \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} (1\cdot 1 + \frac{1}{2}\cdot 2) & (\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 1) \\ (2\cdot 1 + 1\cdot 2) & (2\cdot\frac{1}{2} + 2\cdot 1) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}^1 = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}^3 = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} (1\cdot 2 + 1\cdot 2) & (2\cdot\frac{1}{2} + 1\cdot 1) \\ (4\cdot 1 + 2\cdot 2) & (4\cdot\frac{1}{2} + 2\cdot 1) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}^3 = 4 \cdot \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}}$$

equações:

$A^1 = 1 \cdot A$

$A^2 = 2 \cdot A$

$A^3 = 4 \cdot A$

$A^4 = 8 \cdot A$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}^4 = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} (4\cdot 1 + 2\cdot 2) & (4\cdot\frac{1}{2} + 2\cdot 1) \\ (8\cdot 1 + 4\cdot 2) & (8\cdot\frac{1}{2} + 4\cdot 1) \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 16 & 8 \end{bmatrix} = \boxed{8 \cdot A}$$

Equação geral p/ A^n: $\boxed{A^n = 2^{n-1} \cdot A}$

$A^1 = 2^0 \cdot A$

$A^2 = 2^1 \cdot A$

$A^3 = 2^2 \cdot A$

$A^4 = 2^3 \cdot A$

7. Sejam $X = AB$ e $Y = AC$ matrizes definidas a partir dos produtos AB e AC, respectivamente. Calcule as expressões abaixo em função de X e Y.

a) $A(B+C) = AB + AC = X + Y$ //

b) $B^t \cdot A^t = [AB]^t = [X]^t$ //

c) $C^t \cdot A^t = [AC]^t = [Y]^t$ //

d) $(ABA)C = (AB)(AC) = X \cdot Y$ //

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8.

a) Suponha que $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ seja uma matriz simétrica, isto é $A^t = A$. Determine x e A.

$$\begin{bmatrix} 4 & (x+2) \\ (2x-3) & (x+1) \end{bmatrix} = \begin{bmatrix} 4 & (2x-3) \\ (x+2) & (x+1) \end{bmatrix}$$

$x+2 = 2x-3$

$x = 5$

$$A = \begin{bmatrix} 4 & (5+2) \\ (2\cdot 5-3) & (5+1) \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 7 & 6 \end{bmatrix}_{//}$$

b) Determine x, y e z de modo que a matriz $B = \begin{bmatrix} 0 & -4 & 2 \\ x & 0 & 1-z \\ y & 2z & 0 \end{bmatrix}$ seja assimétrica, isto é, $B^t = -B$

$$\begin{bmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -2 \\ -x & 0 & -1+z \\ -y & -2z & 0 \end{bmatrix}$$

$-y = 2 \rightarrow y = -2_{//}$

$-x = -4 \rightarrow x = 4_{//}$

$2z = -1+z \rightarrow z = -1_{//}$

$1-z = -2z$

$1 = -z$

9. encontre x, y, z e t que satisfazem a equação matricial.

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & x+y+6 \\ z+t-1 & 2t+3 \end{bmatrix}$$

$3x = x+4 \rightarrow 2x = 4$ $\boxed{x = 4/2 = 2}$

$3y = x+y+6$ $\quad 2y = 2+6 \quad 2y = 8$ $\boxed{y = \frac{8}{2} = 4}$

$3t = 2t+3$ $\boxed{t = 3}$

$3z = z+t-1 \quad 2z = 3-1 \quad 2z = 2$ $\boxed{z = \frac{2}{2} = 1}$

10. Uma matriz A de ordem nxn é chamada matriz ortogonal se $A \cdot A^t = A^t \cdot A = I_n$, onde $I_n (\delta ij)_{n\times n}$ é a matriz identidade de ordem n.

a) Mostre que a matriz $R(\theta) = \begin{pmatrix} \cos\theta & \text{sen}\,\theta \\ -\text{sen}\,\theta & \cos\theta \end{pmatrix}$ é ortogonal.

I. $R \cdot R^t = R^t \cdot R \rightarrow \begin{bmatrix} \cos\theta & \text{sen}\theta \\ -\text{sen}\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\text{sen}\theta \\ \text{sen}\theta & \cos\theta \end{bmatrix}$

$$\begin{bmatrix} (\cos^2\theta + \text{sen}^2\theta) & ((\cos \cdot -\text{sen}\theta) + (\text{sen}\theta \cdot \cos\theta)) \\ (-\text{sen}\theta \cdot \cos\theta + \cos\theta \cdot \text{sen}\theta) & (\text{sen}^2\theta + \cos^2\theta) \end{bmatrix} =$$

$-\text{sen}\theta\cos\theta + \text{sen}\theta\cos\theta = 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \therefore R \cdot R^t = I_2$$

$$\begin{bmatrix} \cos\theta & -\text{sen}\theta \\ \text{sen}\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \text{sen}\theta \\ -\text{sen}\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} (\cos^2\theta + \text{sen}^2\theta) & (\cos\theta \cdot \text{sen}\theta + -\text{sen}\theta \cdot \cos\theta) \\ (\text{sen}\theta \cdot \cos\theta + \cos\theta \cdot -\text{sen}\theta) & (\text{sen}^2\theta + \cos^2\theta) \end{bmatrix} =$$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \therefore R^t \cdot R = I_2$, $\quad R \cdot R^t = R^t \cdot R$

$R(\theta)$ é ortogonal

b) Encontre os valores de x, y e z para os quais a matriz $A = \begin{bmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{bmatrix}$ é ortogonal

$$R \cdot R^t = I_3 \longrightarrow \begin{bmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} (1\cdot 1 + 0\cdot 0 + x^2) & (1\cdot 0 + 0\cdot\frac{1}{\sqrt{2}} + x\cdot y) & (1\cdot 0 + 0\cdot\frac{1}{\sqrt{2}} + x\cdot z) \\ (0\cdot 1 + \frac{1}{\sqrt{2}}\cdot 0 + y\cdot x) & (0\cdot 0 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + y^2) & (0\cdot 0 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + y\cdot z) \\ (0\cdot 1 + \frac{1}{\sqrt{2}}\cdot 0 + z\cdot x) & (0\cdot 0 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + z\cdot y) & (0\cdot 0 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + z^2) \end{bmatrix}$$

$(1\cdot 1 + 0\cdot 0 + x^2) = 1$	$(0\cdot 0 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + y^2) = 1$	$(0\cdot 0 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + z^2 = 1)$
$1 + 0 + x^2 = 1$	$0 + \frac{1}{2} + y^2 = 1$	$0 + \frac{1}{2} + z^2 = 1$
$x^2 = 0 \quad x = 0$	$y^2 = \frac{1}{2} \quad y = \frac{\pm 1}{\sqrt{2}}$	$z^2 = \frac{1}{2} \quad z = \frac{\pm 1}{\sqrt{2}}$

$z \cdot x = 0, \quad \frac{1}{2} + zy = 0 \quad y \cdot z = -\frac{1}{2}$