

# Lista 3 - Geometria Analítica

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1. considere a matriz  $A = (a_{ij})_{3 \times 3}$ , tal que

$$a_{ij} = \begin{cases} i+j, & i < j \\ 2i-j, & i = j \\ j-i, & i > j \end{cases} \quad \text{Determine } X \text{ na equação matricial } AX = B, \text{ onde } B = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 5 \\ -2 & -1 & 3 \end{bmatrix} \quad \det(A) = \begin{vmatrix} 1 & 3 & 4 & 1 & 3 \\ -1 & 2 & 5 & -1 & 2 \\ -2 & -1 & 3 & -2 & -1 \end{vmatrix} = 6 - 30 + 4 + 9 + 5 + 16 = 10$$

$\det(A_1)$

$$\begin{vmatrix} -1 & 3 & 4 & -1 & 3 \\ 2 & 2 & 5 & 2 & 2 \\ -3 & -1 & 3 & -3 & -1 \end{vmatrix}$$

$$= -6 - 45 - 8 - 18 - 5 + 24 = -53$$

$$X_1 = -\frac{58}{10}$$

$\det(A_2)$

$$\begin{vmatrix} 1 & -1 & 4 & 1 & -1 \\ -1 & 2 & 5 & -1 & 2 \\ -2 & -3 & 3 & -2 & -3 \end{vmatrix}$$

$$= 6 + 10 + 12 - 3 + 15 + 116 = 56$$

$$X_2 = \frac{56}{10}$$

$\det(A_3)$

$$\begin{vmatrix} 1 & 3 & -1 & 1 & 3 \\ -1 & 2 & 2 & -1 & 2 \\ -2 & -1 & 3 & -2 & -1 \end{vmatrix}$$

$$= -6 - 12 - 1 - 9 + 2 - 4 = -30$$

$$X_3 = -\frac{30}{10} = -3$$

$$X = \begin{bmatrix} -\frac{58}{10} \\ \frac{56}{10} \\ -3 \end{bmatrix}$$

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2. Resolva as equações matriciais usando inversão de matrizes.

$$a) \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} X = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad X = \begin{bmatrix} 13 & -4 \\ -2 & 1 \end{bmatrix} \cdot \frac{1}{5} \cdot \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} (\frac{3}{5} - \frac{4}{5}) \\ (-\frac{2}{5} + \frac{1}{5}) \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} \\ -\frac{1}{5} \end{bmatrix}$$

$$b) A + BY = C \rightarrow BY = C - A \quad Y = B^{-1} \cdot (C - A)$$

$$Y = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} \cdot \left( \begin{bmatrix} 1 & 7 \\ 2 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 5 & 5 \end{bmatrix} \right) = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 4 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -16 \\ 1 & 6 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-1} \begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

$$c) \begin{matrix} \textcircled{A} \\ \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \end{matrix} W = \begin{matrix} \textcircled{B} \\ \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix} \end{matrix} \quad W = A^{-1} \cdot B$$

$$\det(A) = 1 \cdot 1 \cdot 1 = 1$$

$$\text{cof}(A) = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix} \quad \begin{aligned} \tilde{a}_{11} &= (-1)^2 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 & \tilde{a}_{21} &= (-1)^3 \cdot \begin{vmatrix} 0 & 0 \\ 3 & 1 \end{vmatrix} = 0 \\ \tilde{a}_{12} &= (-1)^3 \cdot \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = -2 & \tilde{a}_{22} &= (-1)^4 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 \\ \tilde{a}_{13} &= (-1)^4 \cdot \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 4 & \tilde{a}_{23} &= (-1)^5 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = -3 \end{aligned}$$

$$\tilde{a}_{31} = (-1)^4 \cdot \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad \tilde{a}_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0 \quad \tilde{a}_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$$

$$\text{cof}(A) = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}^t = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

3. Sejam  $A, B$  e  $C$  matrizes inversíveis de mesma ordem,  
 resolva as equações em  $X$ .

a)  $AXB = C \rightarrow (AB)X = C \quad X = (AB)^{-1} \cdot C \quad X = A^{-1} \cdot B^{-1} \cdot C$

b)  $A(B+X) = A \rightarrow B+X = A \cdot A^{-1} \rightarrow X = -B$

c)  $ACXB = C \rightarrow (ABC)X = C \quad X = (ABC)^{-1} \cdot C \rightarrow X = A^{-1} \cdot B^{-1} \cdot \cancel{C^{-1} \cdot C}$

d)  $\underbrace{(AB)^{-1} \cdot (AX)}_{\vec{0}} = \underbrace{CC^{-1}}_{\vec{0}} \rightarrow \underbrace{(A^{-1} \cdot A)}_{\vec{0}} (B^{-1} \cdot X) = \vec{0} \quad X = B$

e)  $AB^+XB^{-1} = A^+$   
 $B^+XB^{-1} = A^+A^{-1}$   
 $B^+X = A^+A^{-1} \cdot B \quad X = A^+A^{-1}B[B^+]^{-1}$

4. Resolva os sistemas lineares usando a regra de Cramer.

a)  $\begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases} \quad \textcircled{A} \quad \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \end{bmatrix} \quad \textcircled{B}$

$\det(A) = 13$

$\det(A_x) = \begin{vmatrix} 1 & -4 \\ 9 & 3 \end{vmatrix} = 39 \quad x = \frac{39}{13} = 3$

$\det(A_y) = \begin{vmatrix} 3 & 1 \\ 1 & 9 \end{vmatrix} = 26 \quad y = \frac{26}{13} = 2$

$$b) \begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$$

$$\det(A) = 80 - 80 = 0$$

$$\begin{bmatrix} 5 & 8 \\ 10 & 16 \end{bmatrix} \begin{bmatrix} 34 \\ 50 \end{bmatrix}$$

$$\det(Ax) = \begin{vmatrix} 34 & 8 \\ 50 & 16 \end{vmatrix} = 544 - 400 = 144$$

$$\begin{array}{r} 34 \\ 16 \\ \hline 204 \\ 340 \\ \hline 544 \end{array}$$

Sistema Impossível (SI)

$$c) \begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

$$\det(A) = -7$$

$$\det(Ax) = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = -15 + 8 = -7 \quad x = \frac{-7}{-7} = 1 //$$

$$\det(Ay) = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = -4 - 10 = -14 \quad y = \frac{-14}{-7} = 2 //$$

$$d) 3x + 2y - 5z = 8$$

$$2x - 4y - 2z = -4$$

$$x - 2y - 3z = -4$$

$$A = \begin{bmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ -4 \\ -4 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 3 & 2 & -5 & 3 & 2 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -2 & -3 & 1 & -2 \end{vmatrix} = 36 - 4 + 20 + 12 - 12 - 20 = 32$$

$$\det(Ax) = \begin{vmatrix} 8 & 2 & -5 & 8 & 2 \\ -4 & -4 & -2 & -4 & -4 \\ -4 & -2 & -3 & -4 & -2 \end{vmatrix}$$

$$96 + 16 - 40 - 24 - 32 + 80 = 96$$

$$x = \frac{96}{32} = \boxed{3}$$

$$192 - 96 = 96$$



$$\det(A_1) = \begin{vmatrix} 3 & 8 & -5 & 3 & 8 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -1 & -3 & 1 & -1 \end{vmatrix}$$

$$36 - 16 + 40 + 48 - 24 - 20 = 64$$

$$y = \frac{64}{32} = 2$$

$$\det(A_2) = \begin{vmatrix} 3 & 2 & 8 & 3 & 2 \\ 2 & -4 & -4 & 2 & -4 \\ 1 & -2 & -4 & 1 & -2 \end{vmatrix}$$

$$= 48 - 8 - 32 + 16 - 24 + 32 = 32$$

$$z = \frac{32}{32} = 1$$

$$e) \begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases}$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 9 \\ 3 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix}$$

$$2 + 18 - 6 + 8 - 9 - 3 = 10$$

$$\det(A_x) = \begin{vmatrix} 2 & 2 & -1 & 2 & 2 \\ 9 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix}$$

$$4 + 18 - 27 + 36 - 18 - 3 = 10 \quad x = \frac{10}{10} = 1$$

$$\det(A_y) = \begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 9 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix}$$

$$-18 + 18 - 6 + 8 - 9 + 27 = 20 \quad y = \frac{20}{10} = 2$$

$$\det(A_z) = \begin{vmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{vmatrix}$$

$$-3 + 54 + 12 - 12 - 27 + 6 = 30 \quad z = \frac{30}{10} = 3$$

$$f) \begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} -8 \\ -4 \\ 26 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 0 & 3 & 1 & 0 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & -2 & -5 & 3 & -2 \end{vmatrix}$$

$$20 + 0 - 12 + 0 + 0 + 36 = 44$$

$$\det(A_x) = \begin{vmatrix} -8 & 0 & 3 & | & -8 & 0 \\ -4 & -4 & 0 & | & -4 & -4 \\ 26 & -2 & -5 & | & 26 & -2 \end{vmatrix}$$

$$= -160 + 0 + 24 - 0 - 0 + 312 = 176$$

$$\det(A_y) = \begin{vmatrix} 1 & -8 & 3 & | & 1 & -8 \\ 2 & -4 & 0 & | & 2 & -4 \\ 3 & 26 & -5 & | & 3 & 26 \end{vmatrix}$$

$$= 20 - 0 + 156 - 80 - 0 + 36 = 132$$

$$y = \frac{132}{44} = 3$$

$$\det(A_z) = \begin{vmatrix} 1 & 0 & -8 & | & 1 & 0 \\ 2 & -4 & 0 & | & 2 & -4 \\ 3 & 26 & -5 & | & 3 & 26 \end{vmatrix}$$

$$= 104 + 0 + 32 - 0 - 8 + 96$$

$$z = \frac{234}{44}$$

confirma

$$g) \begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 3x + 2y + 3z = 10 \end{cases}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 23 \\ 10 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 & | & 1 & 2 \\ 3 & 4 & 6 & | & 3 & 4 \\ 3 & 2 & 3 & | & 3 & 2 \end{vmatrix}$$

$$= 12 + 36 + 18 - 18 - 12 - 36 = 0$$

SI ou SI

$$\det(A_x) = \begin{vmatrix} 10 & 2 & 3 & | & 10 & 2 \\ 23 & 4 & 6 & | & 23 & 4 \\ 10 & 2 & 3 & | & 10 & 2 \end{vmatrix}$$

$$120 + 120 + 138 - 138 - 120 - 120 = 0$$

$$\det(A_y) = \begin{vmatrix} 1 & 10 & 3 & | & 1 & 10 \\ 3 & 23 & 6 & | & 3 & 23 \\ 3 & 10 & 3 & | & 3 & 10 \end{vmatrix}$$

$$= 69 + 180 + 90 - 90 - 60 - 207 = -18$$

SI não

tem resposta

5. Classifique, quanto ao número de soluções, os seguintes sistemas homogêneos.

$$a) \begin{cases} 3x_1 - 4x_2 = 0 \\ -6x_1 + 8x_2 = 0 \end{cases}$$

$$A = \begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det(A) = 24 - 24 = 0$$

$$\det(A_{x_1}) = \begin{vmatrix} 0 & -4 \\ 0 & 8 \end{vmatrix} = 0$$

$$\det(A_{x_2}) = \begin{vmatrix} 3 & 0 \\ -6 & 0 \end{vmatrix} = 0$$

Sistema Possível Indeterminado (SPI), infinitas soluções.

$$b) \begin{cases} x + y + z = 0 \\ 2x + 2y + 4z = 0 \\ x + y + 3z = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{vmatrix} = 6 + 4 + 2 - 6 - 4 - 2 = 0$$

$$\det(A_x) = 0 \quad \det(A_y) = 0 \quad \det(A_z) = 0$$

SPI

$$c) \begin{cases} x + y + 2z = 0 \\ x - y - 3z = 0 \\ x + 4y = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -3 \\ 1 & 4 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & -3 \\ 1 & 4 & 0 \end{vmatrix} = 0 - 3 + 8 - 0 + 12 + 2 = 19$$

ISPD

$$x = 0 \quad y = 0 \quad z = 0$$

6. Verifique se existem valores de  $m$  para os quais os sistemas abaixo são possíveis e determináveis.

a) 
$$\begin{cases} 3x + my = 2 \\ x - y = 1 \end{cases}$$

$$A = \begin{bmatrix} 3 & m \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\det(A) = 3 - m$$

$$\boxed{\text{SPD se } m \neq -3}$$

$$\det(Ax) = \begin{vmatrix} 2 & m \\ 1 & -1 \end{vmatrix} = -2 - m$$

$$\boxed{1}$$

$$\det(Ay) = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1$$

b) 
$$\begin{cases} 3x + 2(m-1)y = 1 \neq 0 \\ mx - 4y = 0 \end{cases} \quad \begin{matrix} m \neq 3 \\ m \neq -3 \end{matrix}$$

$$A = \begin{bmatrix} 3 & 2m-2 \\ m & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

c)

$$\frac{-1 \pm \sqrt{12 - 4 \cdot 1 \cdot -6}}{2 \cdot 1}$$

$$\Delta = \frac{-1 \pm \sqrt{1-24}}{2}$$

não possui

$$-12 - (m \cdot (2m-2)) \quad -12 - (2m^2 - 2m)$$

$$-12 - 2m^2 + 2m$$

$$\downarrow \quad -m^2 + m - 6 \quad a=1 \quad b=1 \quad c=-6$$

raízes reais  $m | m \in \mathbb{R} \nexists$  p/ SPD

c) 
$$\begin{cases} x - y = 2 \\ x + my = -2 \rightarrow x + my + z = 0 \\ -x + y - z = 4 \end{cases}$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{vmatrix}$$

$$-m + 1 + 0 - 1 + 0$$

$$-m - 1 \neq 0$$

$$-m \neq 1 \quad \boxed{m \neq -1} \quad \text{APD se } m \neq -1$$



7.

6 por peça produzida corretamente  $\rightarrow x$   
 -2 por peça com defeito  $\rightarrow y$

225 peças no total

750 reais

$$\boxed{x = 150}$$

$$\begin{cases} x + y = 225 \\ 6x - 2y = 750 \end{cases}$$

$$\begin{array}{r} 2x + 2y = 450 \\ + 6x - 2y = 750 \\ \hline \end{array}$$

$$8x = 1200$$

$$x = \frac{1200}{8} = 150$$

8.

- 540 km por mês
- 60 c. / km no automóvel
- 20 c. / km na motocicleta
- custo mensal 300 reais

$$0,6x + 0,2y = 300$$

$$x + y = 540$$

$$3x + y = 1500$$

$$x + y = 540$$

$$2x = 960$$

$$\boxed{\begin{array}{l} x = 480 \\ y = 60 \end{array}}$$

9.

total 500 reais

cédulas de 2, 5 e 10

total de cédulas 92

qta. de 2 e 10 igual

20 de 2

20 de 10

52 de 5

$$2x + y = 92$$

$$2x + 10x + 5y = 500$$

$$\begin{array}{r} 12x + 5y = 500 \\ - 10x + 5y = 460 \\ \hline \end{array}$$

$$2x = 40$$

$$\boxed{\begin{array}{l} x = 20 \\ y = 52 \end{array}}$$

10.

$$x + y = 109$$

$$x + z = 142$$

$$z + y = 97$$

$$\begin{array}{r} x + y = 109 \\ - \quad x + z = 142 \\ \hline \end{array}$$

$$\begin{array}{r} y - z = -33 \\ + \quad z + y = 97 \\ \hline \end{array}$$

$$2y = 64 \quad y = 32 //$$

$$x + 32 = 109 \quad x = 109 - 32 = 77$$

$$77 + z = 142 \quad z = 142 - 77 = 65$$

$$\begin{cases} x = 77 \\ y = 32 \\ z = 65 \end{cases}$$

6.

d)  $mx + y - z = 4$

$$x + my + z = 0$$

$$x - y = 2$$

$$A = \begin{bmatrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} m & 1 & -1 & m \\ 1 & m & 1 & m \\ 1 & -1 & 0 & -1 \end{vmatrix}$$

$$0 + 1 + 1 - 0 + m + m \neq 0$$

$$2m + 2 \neq 0$$

$$2m \neq -2$$

$$\boxed{m \neq -1}$$