SIR Models

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Alternative formulations for SIR Models

Assuming the typical SIR formulation as 3 ordinary differential equations on a closed population: $S(t) + I(t) + R(t) = N$, $\forall t \geq 0$. We have that for this formulation the basic reproduction number $\mathcal{R}_0 = \frac{\beta}{\gamma}$.

If we take the ratio:

$$\frac{dS}{dR} = \frac{-\beta SI}{\gamma I} = -\mathcal{R}_0 S \tag{1}$$

It can be integrated to $S(t) = S_0 e^{-\mathcal{R}_0 R}$. We can then substitute this definition of $S(t)$ into the standard $\frac{dR}{dt} = \gamma I$, and obtain

$$\frac{dR}{dt} = \gamma \left(N - R - S_0 e^{-\mathcal{R}_0 R} \right) \tag{2}$$

With this we have reduce a 3-dimensional system to a single equation embodying the full SIR dynamics.