



## A Simple Method for 3D Reconstruction from Two Views

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### Abstract

In this paper, we address the problem of 3D reconstruction from two views and a simple method is presented. This reconstruction process can be divided into three parts: first, auto-matching and estimate the fundamental matrix from point correspondence; second, compute the camera matrices; in the end, compute the points in space that project to these two image points. We have proposed a fast matching algorithm to extract a sequence of corresponding points from two views of an object. Also we have described a self-calibration technology to compute the camera matrix. The resulting sequence of points will be used later to reconstruct a set of points in space representing the object surfaces on the scene. And experimental results on real images show this method works well.

**Keywords:** 3D reconstruction, stereo matching, fundamental matrix, SVD, self-calibration, essential matrix

### 1. Introduction

3D reconstruction from two views is one of the fundamental problems in computer vision. Earliest work in this area concentrated on calibrated cameras, and many good algorithms are known [1,2]. With a shift of interest towards uncalibrated cameras, it was shown in [3] that given reasonable assumptions about the calibration, it is still possible to achieve 3D reconstruction. In this paper, we

present a simple method of reconstruction using self-calibration. Specifically, if the pixels are assumed to be square and the principal point known, then the focal lengths of the two cameras may be computed from the fundamental matrix. All camera parameters now being known, the problem is reduced to a calibrated reconstruction problem. And the experimental results show this method is effective.

Another important job is to compute the fundamental matrix, which the camera calibration is very sensitive to. It is often said that the computation of the fundamental matrix is unstable, and to an extent this is true. This is a characteristic of the problem itself, and not the algorithms used since many algorithms that obtain close to the Maximum Likelihood estimate are known. In this paper an effective matching algorithm was used and the fundamental matrix was computed from point correspondences by RANSAC.

### 2. Auto-matching and estimate fundamental matrix from point correspondences

As we know, stereo matching has traditionally been, and continues to be, one of the most heavily investigated topics in computer vision. A large number of algorithms for stereo matching have been developed [4,5]. We have presented a fast stereo matching algorithm by means of epipolar constraint and multi resolution approach



[6] which is applied in this paper. First, the interest points of the images are extracted using the Harris corner detector [7]. And initial point correspondence sets are estimated by intensity correlation. Then, a general and very successful robust estimator---the RANSAC (Random Sample Consensus) algorithm [8] was used as a “search engine” to the initial point correspondence sets to eliminate the mismatches and give the estimated fundamental matrix. Finally, in order to get more accurate matching points, applying the multiresolution algorithm [9], the intensity correlation and the epipolar constraints to correct the matching result for every interest point in the image.

With the final points correspondences, we can rectify the fundamental matrix with RANSAC again. So this process is iterative and the output is a more accurate fundamental matrix together with a set of interest points in correspondence. And we have developed a software for this work. So the whole work can be completed very well by computer.

### 3. Compute the camera matrices $P$

#### 3.1 Self-calibration of camera

In this paper, we use perspective projection as camera model, with the following intrinsic parameters: the focal length  $f$ , the aspect ratio  $\tau$  and the principal point  $(u_0, v_0)$ . A 3D point  $X$  is projected to an image point  $x$  via:

$$x \sim PX \sim KR[I | -t]X,$$

where the rotation matrix  $R$  and the vector  $t$  represent the camera's orientation and position, and the calibration matrix  $K$  is defined as:

$$K = \begin{pmatrix} \tau f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

and the fundamental matrix is given by:

$$F \sim K'^{-T} R[t]_x K^{-1}$$

It has been shown that camera self-calibration is possible by making rather simple assumptions on the internal parameters. In practice, however, one often tries to use the prior information of internal parameters as much as possible. Typically, it is possible to know the aspect ratio rather accurately, pixels are nearly perfectly square and the assumption of the principal point in the center of the image has been shown to be valid. Thus, it is possible to concentrate, in an initial stage, on the parameter most likely to be unknown, the focal length. So we suppose that the aspect ratio and the principal point is known for both images and that their focal lengths are identical. We can thus move from a completely uncalibrated space to a “semi-calibrated” one, by computing an intermediate matrix  $G$  between the fundamental matrix and the essential matrix ( $R[t]_x$  in the above equation):

$$G \sim \begin{pmatrix} \tau' & 0 & 0 \\ 0 & 1 & 0 \\ u_0' & v_0' & 1 \end{pmatrix} K'^{-T} R[t]_x K^{-1} \begin{pmatrix} \tau & 0 & u_0 \\ 0 & 1 & v_0 \\ 0 & 0 & 1 \end{pmatrix} \\ \sim \text{diag}(1, 1, f) R[t]_x \text{diag}(1, 1, f)$$

And the *SVD* of  $G$  is:  $G = UDV^T$ ,

with  $D = \text{diag}(a, b, 0)$  the diagonal matrix of singular values ( $a, b > 0$ ) and  $U$  and  $V$  orthogonal matrices. Let us denote by  $u_{ij}$  and  $v_{ij}$

the  $i$ th row and  $j$ th column of  $U$  and  $V$  respectively. With the Kruppa *equations* and other relationships exist that link the fundamental matrix and the epipole, we can obtain two linear equations and a quadratic one [10] which are as follows:

$$f^2 (au_{13}u_{23}(1-v_{13}^2) + bv_{13}v_{23}(1-u_{23}^2)) \\ + u_{23}v_{13}(au_{13}v_{13} + bu_{23}v_{23}) = 0 \quad (2)$$

$$f^2 (av_{13}v_{23}(1-u_{13}^2) + bu_{13}u_{23}(1-v_{23}^2))$$



$$+u_{13}v_{23}(au_{13}v_{13}+bu_{23}v_{23})=0 \quad (3)$$

$$\begin{aligned} & f^4(a^2(1-u_{13}^2)(1-v_{13}^2)-b^2(1-u_{23}^2)(1-v_{23}^2)) \\ & +f^2(a^2(u_{13}^2+v_{13}^2-2u_{13}^2v_{13}^2)-b^2(u_{23}^2+v_{23}^2-2u_{23}^2v_{23}^2)) \\ & + (a^2u_{13}^2v_{13}^2 - b^2u_{23}^2v_{23}^2) = 0 \end{aligned} \quad (4)$$

Thus  $f$  can be estimated by solving any of the equations (2) and (4). In practice, we only solve the quadratic equation (4). Now all internal parameters are known,  $K$  can be given by equation (1).

### 3.2 Extraction of cameras from the essential matrix

As we know the essential matrix can be given with the fundamental matrix  $F$  and the calibration matrix  $K$ :

$$E = K^T F K$$

We may assume that the first camera matrix is

$P = [I | 0]$ . Suppose that the  $SVD$  of  $E$  is:

$$E = U \text{diag}(1, 1, 0) V^T$$

There are four possible choices for the second camera matrix  $P'$ , namely [11]:

$$P' = [U W V^T | u_3] \text{ or}$$

$$[U W V^T | -u_3] \text{ or } [U W^T V^T | u_3] \text{ or}$$

$$[U W^T V^T | -u_3] \quad (5)$$

While  $u_3$  is the last column of  $U$ , and

$$W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The solutions are illustrated in (5). Where it is shown that a reconstructed point  $X$  will be in front of both cameras in one of these four solutions only. Thus, testing with some points to determine if it is in front of both cameras is sufficient to decide between the four different solutions for the camera

matrix  $P'$ .

### 4. Compute the points in space that project to these image points

The points in space can be computed by the Linear Triangulation method [11] which is the direct analogue of the DLT method.

In each image we have a measurement

$$x = PX, x' = P'X,$$

and these equations can be combined into a form  $AX = 0$ , which is a linear equation in  $X$ . It can be solved by  $SVD$ . Thus all points in space that project to these two image points can be computed.

### 5. Experimental results

The method which we have presented in the above works well both on simulated images and real images. In this section, we only represent the results of real images by performing our method. Figure 1 shows two views of a house, and Figure 2 shows the 3D reconstruction results of the house.

#### Experiment:



Figure 1: Two views of a house





**Figure 2: 3D reconstruction results of the house**

## 6. Conclusion

We presented a simple method for 3D reconstruction from two views. The procedure can be divided into three parts. And since the reconstruction is very sensitive to the computed fundamental matrix, the process should be interactive and iterative. The good experimental results on real images was produced according to this method. It also shows that given reasonable assumptions about the calibration, 3D reconstruction from two views is feasible. Future work will concern the investigation of improving its robustness.

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