In these pseudo-codes, we refer to equation numbers introduced in our paper:

"Spectral image clustering on dual-energy CT scans using functional regression mixtures" (arxiv link).

Algorithm 1
Pseudo code of EM with SsMFR and SgMFR

Inputs: 4D-image (n curves $(\boldsymbol{v}_i, \boldsymbol{x}_i, \boldsymbol{y}_i)_{i=1}^n$), # clusters K, degree p (and # knots q)

Initialization: $\boldsymbol{\theta}^{(0)} = (\boldsymbol{\alpha}^{(0)}, \boldsymbol{\theta}_1^{(0)}, \ldots, \boldsymbol{\theta}_K^{(0)})$; set $t \leftarrow 0$
while increment in log-likelihood $> \epsilon$ (e.g. $1e^{-6}$) **do**
  E-Step: `% Conditional memberships :`
  for $k = 1, \ldots, K$ **do**
    compute $\tau_{ik}^{(t)}$ for $i = 1, \ldots, n$ using (12) for `SgMFR` or using an adaptation for `SsMFR`
  end for
  M-Step: `%a. Spatial Mixture Weights`
  if `SgMFR` model is used: **then**
    `%Update Spatial Gaussian-Gating Functions:`
    for $k = 1, \ldots, K$ **do**
      compute $w_k^{(t+1)}$ (14), $\boldsymbol{\mu}_k^{(t+1)}$ (15), and $\mathbf{R}_k^{(t+1)}$ (16)
    end for
  end if
  if `SsMFR` model is used: **then**
    `% Update Spatial Softmax-Gating Functions:`
    IRLS Algorithm:
    Initialize: $\boldsymbol{\alpha}^{(s)} \leftarrow \boldsymbol{\alpha}^{(t)}$ and $s \leftarrow 0$ (IRLS iteration)
    while increment in $Q_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}, \boldsymbol{\theta}^{(t)}) > \delta$ (eg. 1e-6) **do**
      compute $\boldsymbol{\alpha}^{(s+1)}$ using *the IRLS algorithm*
      $s \leftarrow s + 1$
    end while
    $\boldsymbol{\alpha}^{(t+1)} \leftarrow \boldsymbol{\alpha}^{(s)}$
  end if
  `%b. Update Functional Mixture Components:`
  for $k = 1, \ldots, K$ **do**
    compute $\boldsymbol{\beta}_k^{(t+1)}$ using (17) and $\sigma_k^{2(t+1)}$ using (18)
  end for
  `% Convergence test`
  Compute the current log-likelihood using (9) for `SgMFR` or an adaptation for `SsMFR`.
  $t \leftarrow t + 1$
end while
Outputs: $\widehat{\boldsymbol{\theta}} = (\boldsymbol{\alpha}^{(t)}, \boldsymbol{\theta}_1^{(t)}, \ldots \boldsymbol{\theta}_K^{(t)})$ the MLE of $\boldsymbol{\theta}$ and the conditional probabilities $\tau_{ik}^{(t)}$

Algorithm 2
Pseudo code of EM with alternative SgMVFR and SsMVFR

Inputs: 4D-image (n curves $(\boldsymbol{v}_i, \boldsymbol{x}_i, \boldsymbol{y}_i)_{i=1}^n$), # clusters K, degree p (and # knots q)

for $i = 1, \ldots, n$ **do**
  Compute the functional data representations $\widehat{\boldsymbol{\beta}}_i$ by (19)
end for
Initialization: $\boldsymbol{\theta}^{(0)} = (\boldsymbol{\alpha}^{(0)}, \boldsymbol{\theta}_1^{(0)}, \ldots, \boldsymbol{\theta}_K^{(0)})$; set $t \leftarrow 0$ (EM iteration)
while increment in log-likelihood $> \epsilon$ (eg. 1e-6) **do**
  E-Step: `% Conditional memberships :`
  for $k = 1, \ldots, K$ **do**
    compute $\tau_{ik}^{(t)}$ for $i = 1, \ldots, n$ using (21) for `SsMVFR` or using (22) for `SgMVFR`
  end for
  M-Step: `%(a) Update Spatial Weights:`
  if `SgMVFR` model is used: **then**
    `%Update Spatial Gaussian-Gating Functions:`
    for $k = 1, \ldots, K$ **do**
      compute $w_k^{(t+1)}$ using (14), $\boldsymbol{\mu}_k^{(t+1)}$ using (15), and $\mathbf{R}_k^{(t+1)}$ using (16)
    end for
  end if
  if `SsMVFR` model is used: **then**
    `% Update Spatial Softmax-Gating Functions:`
    $\boldsymbol{\alpha}^{(t+1)} \leftarrow$ is returned by the *IRLS algorithm*:
  end if
  `%(b) Update Multivariate Mixture Components:`
  for $k = 1, \ldots, K$ **do**
    compute $\boldsymbol{m}_k^{(t+1)}$ using (23) and $\mathbf{C}_k^{(t+1)}$ using (24)
  end for
  `% Convergence test`
  Compute the current log-likelihood using (9) for `SsMVFR` or an adaptation for `SgMVFR`.
  $t \leftarrow t + 1$
end while
Outputs: $\widehat{\boldsymbol{\theta}} = (\boldsymbol{\alpha}^{(t)}, \boldsymbol{\theta}_1^{(t)}, \ldots \boldsymbol{\theta}_K^{(t)})$ the MLE of $\boldsymbol{\theta}$ and the conditional probabilities $\tau_{ik}^{(t)}$