Package ‘meteorits’

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Type Package

Title Mixtures-of-Experts Modeling for Complex and Non-Normal Distributions ('MEteorits')

Version 0.1.0

Description Provides several original and flexible mixtures-of-experts models to model, cluster and classify heterogeneous data in many complex situations where the data are distributed according to non-normal, possibly skewed distributions, and when they might be corrupted by atypical observations. Mixtures-of-Experts Models for Complex and Non-Normal Distributions ('MEteorits') are originally introduced and written in 'Matlab' by Faicel Chamroukhi. The references are mainly the following ones.
Chamroukhi F., Same A., Govaert, G. and Aknin P. (2009) <doi:10.1016/j.neunet.2009.06.040>.
Chamroukhi F. (2010) <https://chamroukhi.com/FChamroukhi-PhD.pdf>.
Chamroukhi F. (2015) <arXiv:1506.06707>.
Chamroukhi F. (2015) <https://chamroukhi.com/FChamroukhi-HDR.pdf>.
Chamroukhi F. (2016) <doi:10.1109/IJCNN.2016.7727580>.
Chamroukhi F. (2016) <doi:10.1016/j.neunet.2016.03.002>.
Chamroukhi F. (2017) <doi:10.1016/j.neucom.2017.05.044>.

URL https://github.com/fchamroukhi/MEteorits

BugReports https://github.com/fchamroukhi/MEteorits/issues

License GPL (>= 3)

Depends R (>= 2.10)

Imports pracma,
methods,
stats,
MASS,
Rcpp

Suggests knitr,
rmarkdown

LinkingTo Rcpp,
RcppArmadillo

Collate meteorits-package.R
RcppExports.R
logsumexp.R
utils.R
sampleUnivNMoE.R

sampleUnivSNMoE.R
sampleUnivSTMoE.R
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ParamSNMoE.R
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StatSNMoE.R
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StatTMoE.R
StatNMoE.R
ModelSNMoE.R
ModelStMoE.R
ModelTMoE.R
ModelNMoE.R
emSNMoE.R
emStMoE.R
emTMoE.R
emNMoE.R
data-tempanomalies.R

VignetteBuilder knitr

Encoding UTF-8

LazyData true

Roxygen list(markdown = TRUE)

RoxygenNote 6.1.1

R topics documented:

meteorits-package *MEteorits: Mixtures-of-ExperTs modEling for cOmplex and non-noRmal dIsTributions*

Description

`meteorits` is a package containing several original and flexible mixtures-of-experts models to model, cluster and classify heteregenous data in many complex situations where the data are distributed according to non-normal and possibly skewed distributions, and when they might be corrupted by atypical observations. The toolbox also contains sparse mixture-of-experts models for high-dimensional data.

`meteorits` contains the following Mixture-of-Experts models:

- NMoE (Normal Mixtures-of-Experts) provides a flexible framework for heterogenous data with Normal expert regressors network;
- SNMoE (Skew-Normal Mixtures-of-Experts) provides a flexible modeling framework for heterogenous data with possibly skewed distributions to generalize the standard Normal mixture of expert model;
- tMoE (t Mixtures-of-Experts) provides a flexible and robust modeling framework for heterogenous data with possibly heavy-tailed distributions and corrupted by atypical observations;
- StMoE (Skew t Mixtures-of-Experts) provides a flexible and robust modeling framework for heterogenous data with possibly skewed, heavy-tailed distributions and corrupted by atypical observations.

For the advantages/differences of each of them, the user is referred to our mentioned paper references.

To learn more about `meteorits`, start with the vignettes: `browseVignettes(package = "meteorits")`

Author(s)

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- Marius Bartcus <marius.bartcus@gmail.com> (R port) [translator]

References

Chamroukhi, F. 2017. *Skew-T Mixture of Experts.* Neurocomputing - Elsevier 266: 390–408. https://chamroukhi.com/papers/STMoE.pdf.

Chamroukhi, F. 2016a. *Robust Mixture of Experts Modeling Using the T-Distribution.* Neural Networks - Elsevier 79: 20–36. https://chamroukhi.com/papers/TMoE.pdf.

Chamroukhi, F. 2016b. *Skew-Normal Mixture of Experts.* In The International Joint Conference on Neural Networks (IJCNN). Vancouver, Canada. https://chamroukhi.com/papers/Chamroukhi-SNMoE-IJCNN2016.pdf.

Chamroukhi, F. 2015a. *Non-Normal Mixtures of Experts.* http://arxiv.org/pdf/1506.06707.pdf.

Chamroukhi, F. 2015b. *Statistical Learning of Latent Data Models for Complex Data Analysis.* Habilitation Thesis (HDR), Universite de Toulon. https://chamroukhi.com/FChamroukhi-HDR.pdf.

Chamroukhi, F. 2010. *Hidden Process Regression for Curve Modeling, Classification and Tracking.* Ph.D. Thesis, Universite de Technologie de Compiegne. https://chamroukhi.com/FChamroukhi-PhD.pdf.

Chamroukhi, F., A. Same, G. Govaert, and P. Aknin. 2009. *Time Series Modeling by a Regression Approach Based on a Latent Process.* Neural Networks 22 (5-6): 593–602. https://chamroukhi.com/papers/Chamroukhi_Neural_Networks_2009.pdf.

See Also

Useful links:

- https://github.com/fchamroukhi/MEteorits
- Report bugs at https://github.com/fchamroukhi/MEteorits/issues

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`emNMoE` *emNMoE implements the EM algorithm to fit a Normal Mixture of Experts (NMoE).*

Description

emNMoE implements the maximum-likelihood parameter estimation of a Normal Mixture of Experts (NMoE) model by the Expectation-Maximization (EM) algorithm.

Usage

```
emNMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
  threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

X	Numeric vector of length *n* representing the covariates/inputs $x_1, \ldots, x_n$.
Y	Numeric vector of length *n* representing the observed response/output $y_1, \ldots, y_n$.
K	The number of experts.
p	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the EM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the EM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the EM as stopping criteria.
verbose	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during EM iterations.
verbose_IRLS	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the EM algorithm.

Details

emNMoE function implements the EM algorithm for the NMoE model. This function starts with an initialization of the parameters done by the method `initParam` of the class ParamNMoE, then it alternates between the E-Step (method of the class StatNMoE) and the M-Step (method of the class ParamNMoE) until convergence (until the relative variation of log-likelihood between two steps of the EM algorithm is less than the `threshold` parameter).

Value

EM returns an object of class ModelNMoE.

See Also

ModelNMoE, ParamNMoE, StatNMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

nmoe <- emNMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

nmoe$plot()
```

`emSNMoE` *emSNMoE implements the ECM algorithm to fit a Skew-Normal Mixture of Experts (SNMoE).*

Description

emSNMoE implements the maximum-likelihood parameter estimation of a Skew-Normal Mixture of Experts (SNMoE) model by the Expectation Conditional Maximization (ECM) algorithm.

Usage

```
emSNMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
  threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

`X`	Numeric vector of length *n* representing the covariates/inputs $x_1, \ldots, x_n$.
`Y`	Numeric vector of length *n* representing the observed response/output $y_1, \ldots, y_n$.
`K`	The number of experts.
`p`	Optional. The order of the polynomial regression for the experts.
`q`	Optional. The order of the logistic regression for the gating network.
`n_tries`	Optional. Number of runs of the ECM algorithm. The solution providing the highest log-likelihood will be returned.
`max_iter`	Optional. The maximum number of iterations for the ECM algorithm.

`threshold`	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the ECM as stopping criteria.
`verbose`	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during ECM iterations.
`verbose_IRLS`	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the ECM algorithm.

Details

emSNMoE function implements the ECM algorithm for the SNMoE model. This function starts with an initialization of the parameters done by the method `initParam` of the class ParamSNMoE, then it alternates between the E-Step (method of the class StatSNMoE) and the M-Step (method of the class ParamSNMoE) until convergence (until the relative variation of log-likelihood between two steps of the ECM algorithm is less than the `threshold` parameter).

Value

ECM returns an object of class ModelSNMoE.

See Also

ModelSNMoE, ParamSNMoE, StatSNMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

snmoe <- emSNMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

snmoe$plot()
```

---

emStMoE — *emStMoE implements the ECM algorithm to fit a Skew-t Mixture of Experts (StMoE).*

Description

emStMoE implements the maximum-likelihood parameter estimation of a Skew-t Mixture of Experts (StMoE) model by the Expectation Conditional Maximization (ECM) algorithm.

Usage

```
emStMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
  threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

X	Numeric vector of length n representing the covariates/inputs $x_1, \ldots, x_n$.
Y	Numeric vector of length n representing the observed response/output $y_1, \ldots, y_n$.
K	The number of experts.
p	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the ECM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the ECM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the ECM as stopping criteria.
verbose	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during ECM iterations.
verbose_IRLS	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the ECM algorithm.

Details

emStMoE function implements the ECM algorithm for the StMoE model. This function starts with an initialization of the parameters done by the method `initParam` of the class ParamStMoE, then it alternates between the E-Step (method of the class StatStMoE) and the M-Step (method of the class ParamStMoE) until convergence (until the relative variation of log-likelihood between two steps of the ECM algorithm is less than the `threshold` parameter).

Value

ECM returns an object of class ModelStMoE.

See Also

ModelStMoE, ParamStMoE, StatStMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

stmoe <- emStMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

stmoe$plot()
```

emTMoE *emTMoE implements the ECM algorithm to fit a t Mixture of Experts (TMoE).*

Description

emTMoE implements the maximum-likelihood parameter estimation of a Student Mixture of Experts (TMoE) model by the Conditional Expectation Maximization (ECM) algorithm.

Usage

```
emTMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
  threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

X	Numeric vector of length *n* representing the covariates/inputs $x_1, \dots, x_n$.
Y	Numeric vector of length *n* representing the observed response/output $y_1, \dots, y_n$.
K	The number of experts.
p	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the ECM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the ECM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the ECM as stopping criteria.
verbose	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during ECM iterations.
verbose_IRLS	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the ECM algorithm.

Details

emTMoE function implements the ECM algorithm for the TMoE model. This function starts with an initialization of the parameters done by the method `initParam` of the class ParamTMoE, then it alternates between the E-Step (method of the class StatTMoE) and the M-Step (method of the class ParamTMoE) until convergence (until the relative variation of log-likelihood between two steps of the ECM algorithm is less than the `threshold` parameter).

Value

ECM returns an object of class ModelTMoE.

See Also

ModelTMoE, ParamTMoE, StatTMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

tmoe <- emTMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

tmoe$plot()
```

ModelNMoE-class *A Reference Class which represents a fitted NMoE model.*

Description

ModelNMoE represents an estimated NMoE model.

Fields

`param` A ParamNMoE object. It contains the estimated values of the parameters.

`stat` A StatNMoE object. It contains all the statistics associated to the NMoE model.

Methods

`plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...)` Plot method.

`what` The type of graph requested:

- `"meancurve" =` Estimated mean and estimated experts means given the input `X` (fields `Ey` and `Ey_k` of class StatNMoE).
- `"confregions" =` Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field `Vary` of class StatNMoE).
- `"clusters" =` Estimated experts means (field `Ey_k`) and hard partition (field `klas` of class StatNMoE).
- `"loglikelihood" =` Value of the log-likelihood for each iteration (field `stored_loglik` of class StatNMoE).

`...` Other graphics parameters.

By default, all the graphs mentioned above are produced.

See Also

ParamNMoE, StatNMoE

`ModelSNMoE-class` *A Reference Class which represents a fitted SNMoE model.*

Description

ModelSNMoE represents an estimated SNMoE model.

Fields

`param` A ParamSNMoE object. It contains the estimated values of the parameters.

`stat` A StatSNMoE object. It contains all the statistics associated to the SNMoE model.

Methods

`plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...)` Plot method.

`what` The type of graph requested:

- `"meancurve" =` Estimated mean and estimated experts means given the input `X` (fields `Ey` and `Ey_k` of class StatSNMoE).
- `"confregions" =` Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field `Vary` of class StatSNMoE).
- `"clusters" =` Estimated experts means (field `Ey_k`) and hard partition (field `klas` of class StatSNMoE).
- `"loglikelihood" =` Value of the log-likelihood for each iteration (field `stored_loglik` of class StatSNMoE).

`...` Other graphics parameters.

By default, all the graphs mentioned above are produced.

See Also

ParamSNMoE, StatSNMoE

`ModelStMoE-class` *A Reference Class which represents a fitted StMoE model.*

Description

ModelStMoE represents an estimated StMoE model.

Fields

`param` A ParamStMoE object. It contains the estimated values of the parameters.

`stat` A StatStMoE object. It contains all the statistics associated to the StMoE model.

Methods

`plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...)` Plot method.

`what` The type of graph requested:

- `"meancurve" =` Estimated mean and estimated experts means given the input `X` (fields `Ey` and `Ey_k` of class StatStMoE).
- `"confregions" =` Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field `Vary` of class StatStMoE).
- `"clusters" =` Estimated experts means (field `Ey_k`) and hard partition (field `klas` of class StatStMoE).
- `"loglikelihood" =` Value of the log-likelihood for each iteration (field `stored_loglik` of class StatStMoE).

`...` Other graphics parameters.

By default, all the graphs mentioned above are produced.

See Also

ParamStMoE, StatStMoE

`ModelTMoE-class` *A Reference Class which represents a fitted TMoE model.*

Description

ModelTMoE represents an estimated TMoE model.

Fields

`param` A ParamTMoE object. It contains the estimated values of the parameters.

`stat` A StatTMoE object. It contains all the statistics associated to the TMoE model.

Methods

`plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...)` Plot method.

`what` The type of graph requested:

- `"meancurve" =` Estimated mean and estimated experts means given the input `X` (fields `Ey` and `Ey_k` of class StatTMoE).
- `"confregions" =` Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field `Vary` of class StatTMoE).
- `"clusters" =` Estimated experts means (field `Ey_k`) and hard partition (field `klas` of class StatTMoE).
- `"loglikelihood" =` Value of the log-likelihood for each iteration (field `stored_loglik` of class StatTMoE).

`...` Other graphics parameters.

By default, all the graphs mentioned above are produced.

See Also

ParamTMoE, StatTMoE

ParamNMoE-class — *A Reference Class which contains parameters of a NMoE model.*

Description

ParamNMoE contains all the parameters of a NMoE model.

Fields

`X` Numeric vector of length *n* representing the covariates/inputs $x_1, \dots, x_n$.

`Y` Numeric vector of length *n* representing the observed response/output $y_1, \dots, y_n$.

`n` Numeric. Length of the response/output vector `Y`.

`K` The number of experts.

`p` The order of the polynomial regression for the experts.

`q` The order of the logistic regression for the gating network.

`alpha` Parameters of the gating network. $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_{K-1})$ is a matrix of dimension $(q + 1, K - 1)$, with `q` the order of the logistic regression for the gating network. `q` is fixed to 1 by default.

`beta` Polynomial regressions coefficients for each expert. $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K)$ is a matrix of dimension $(p + 1, K)$, with `p` the order of the polynomial regression. `p` is fixed to 3 by default.

`sigma2` The variances for the `K` mixture components (matrix of size $(1, K)$).

`df` The degree of freedom of the NMoE model representing the complexity of the model.

Methods

`initParam(segmental = FALSE)` Method to initialize parameters `alpha`, `beta` and `sigma2`.

If `segmental = TRUE` then `alpha`, `beta` and `sigma2` are initialized by clustering the response `Y` uniformly into `K` contiguous segments. Otherwise, `alpha`, `beta` and `sigma2` are initialized by clustering randomly the response `Y` into `K` segments.

ParamSNMoE-class — *A Reference Class which contains parameters of a SNMoE model.*

Description

ParamSNMoE contains all the parameters of a SNMoE model.

Fields

`X` Numeric vector of length *n* representing the covariates/inputs $x_1, \dots, x_n$.

`Y` Numeric vector of length *n* representing the observed response/output $y_1, \dots, y_n$.

`n` Numeric. Length of the response/output vector `Y`.

`K` The number of experts.

`p` The order of the polynomial regression for the experts.

`q` The order of the logistic regression for the gating network.

`alpha` Parameters of the gating network. $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_{K-1})$ is a matrix of dimension $(q+1, K-1)$, with `q` the order of the logistic regression for the gating network. `q` is fixed to 1 by default.

`beta` Polynomial regressions coefficients for each expert. $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K)$ is a matrix of dimension $(p+1, K)$, with `p` the order of the polynomial regression. `p` is fixed to 3 by default.

`sigma2` The variances for the `K` mixture components (matrix of size $(1, K)$).

`lambda` The skewness parameters for each experts (matrix of size $(1, K)$).

`delta` delta is equal to $\delta = \frac{\lambda}{\sqrt{1+\lambda^2}}$.

`df` The degree of freedom of the SNMoE model representing the complexity of the model.

Methods

`initParam(segmental = FALSE)` Method to initialize parameters `alpha`, `beta` and `sigma2`.

If `segmental = TRUE` then `alpha`, `beta` and `sigma2` are initialized by clustering the response `Y` uniformly into `K` contiguous segments. Otherwise, `alpha`, `beta` and `sigma2` are initialized by clustering randomly the response `Y` into `K` segments.

`MStep(statSNMoE, verbose_IRLS)` Method which implements the M-step of the EM algorithm to learn the parameters of the SNMoE model based on statistics provided by the object `statSNMoE` of class StatSNMoE (which contains the E-step).

`ParamStMoE-class` *A Reference Class which contains parameters of a StMoE model.*

Description

ParamStMoE contains all the parameters of a StMoE model.

Fields

`X` Numeric vector of length *n* representing the covariates/inputs $x_1, \dots, x_n$.

`Y` Numeric vector of length *n* representing the observed response/output $y_1, \dots, y_n$.

`n` Numeric. Length of the response/output vector `Y`.

`K` The number of experts.

`p` The order of the polynomial regression for the experts.

`q` The order of the logistic regression for the gating network.

`alpha` Parameters of the gating network. $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_{K-1})$ is a matrix of dimension $(q+1, K-1)$, with `q` the order of the logistic regression for the gating network. `q` is fixed to 1 by default.

`beta` Polynomial regressions coefficients for each expert. $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K)$ is a matrix of dimension $(p+1, K)$, with `p` the order of the polynomial regression. `p` is fixed to 3 by default.

`sigma2` The variances for the `K` mixture components (matrix of size $(1, K)$).

`lambda` The skewness parameters for each experts (matrix of size $(1, K)$).

`delta` delta is equal to $\delta = \frac{\lambda}{\sqrt{1+\lambda^2}}$.

`nu` The degree of freedom for the Student distribution for each experts (matrix of size $(1, K)$).

`df` The degree of freedom of the StMoE model representing the complexity of the model.

Methods

`initParam(segmental = FALSE)` Method to initialize parameters `alpha`, `beta` and `sigma2`.

If `segmental = TRUE` then `alpha`, `beta` and `sigma2` are initialized by clustering the response `Y` uniformly into `K` contiguous segments. Otherwise, `alpha`, `beta` and `sigma2` are initialized by clustering randomly the response `Y` into `K` segments.

`MStep(statStMoE, calcAlpha = FALSE, calcBeta = FALSE, calcSigma2 = FALSE, calcLambda = FALSE, calcNu =` Method which implements the M-step of the EM algorithm to learn the parameters of the StMoE model based on statistics provided by the object `statStMoE` of class StatStMoE (which contains the E-step).

`ParamTMoE-class` *A Reference Class which contains parameters of a TMoE model.*

Description

ParamTMoE contains all the parameters of a TMoE model.

Fields

`X` Numeric vector of length *n* representing the covariates/inputs $x_1, \dots, x_n$.

`Y` Numeric vector of length *n* representing the observed response/output $y_1, \dots, y_n$.

`n` Numeric. Length of the response/output vector `Y`.

`K` The number of experts.

`p` The order of the polynomial regression for the experts.

`q` The order of the logistic regression for the gating network.

`alpha` Parameters of the gating network. $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_{K-1})$ is a matrix of dimension $(q + 1, K - 1)$, with `q` the order of the logistic regression for the gating network. `q` is fixed to 1 by default.

`beta` Polynomial regressions coefficients for each expert. $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K)$ is a matrix of dimension $(p + 1, K)$, with `p` the order of the polynomial regression. `p` is fixed to 3 by default.

`sigma2` The variances for the `K` mixture components (matrix of size $(1, K)$).

`nu` The degree of freedom for the Student distribution for each experts (matrix of size $(1, K)$).

`df` The degree of freedom of the TMoE model representing the complexity of the model.

Methods

`initParam(segmental = FALSE)` Method to initialize parameters `alpha`, `beta` and `sigma2`.

If `segmental = TRUE` then `alpha`, `beta` and `sigma2` are initialized by clustering the response `Y` uniformly into `K` contiguous segments. Otherwise, `alpha`, `beta` and `sigma2` are initialized by clustering randomly the response `Y` into `K` segments.

`MStep(statTMoE, verbose_IRLS)` Method which implements the M-step of the EM algorithm to learn the parameters of the TMoE model based on statistics provided by the object `statTMoE` of class StatTMoE (which contains the E-step).

sampleUnivNMoE *Draw a sample from a normal mixture of linear experts model.*

Description

Draw a sample from a normal mixture of linear experts model.

Usage

```
sampleUnivNMoE(alphak, betak, sigmak, x)
```

Arguments

`alphak`	The parameters of the gating network. `alphak` is a matrix of size *(q + 1, K - 1)*, with *K - 1*, the number of regressors (experts) and *q* the order of the logistic regression
`betak`	Matrix of size *(p + 1, K)* representing the regression coefficients of the experts network.
`sigmak`	Vector of length *K* giving the standard deviations of the experts network.
`x`	A vector og length *n* representing the inputs (predictors).

Value

A list with the output variable `y` and statistics.

- `y` Vector of length *n* giving the output variable.
- `zi` A vector of size *n* giving the hidden label of the expert component generating the i-th observation. Its elements are $zi[i] = k$, if the i-th observation has been generated by the k-th expert.
- `z` A matrix of size *(n, K)* giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- `stats` A list whose elements are:
  - `Ey_k` Matrix of size *(n, K)* giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation *zi = k*, and the value of predictor *X = xi*.
  - `Ey` Vector of length *n* giving the conditional expectation of Yi given the value of predictor *X = xi*.
  - `Vary_k` Vector of length *k* representing the conditional variance of Yi given *zi = k*, and *X = xi*.
  - `Vary` Vector of length *n* giving the conditional expectation of Yi given *X = xi*.

Examples

```
n <- 500 # Size of the sample
alphak <- matrix(c(0, 8), ncol = 1) # Parameters of the gating network
betak <- matrix(c(0, -2.5, 0, 2.5), ncol = 2) # Regression coefficients of the experts
sigmak <- c(1, 1) # Standard deviations of the experts
x <- seq.int(from = -1, to = 1, length.out = n) # Inputs (predictors)

# Generate sample of size n
```

```
sample <- sampleUnivNMoE(alphak = alphak, betak = betak, sigmak = sigmak, x = x)

# Plot points and estimated means
plot(x, sample$y, pch = 4)
lines(x, sample$stats$Ey_k[, 1], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey_k[, 2], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey, col = "red", lwd = 1.5)
```

`sampleUnivSNMoE` *Draw a sample from a skew-normal mixture of linear experts model.*

Description

Draw a sample from a skew-normal mixture of linear experts model.

Usage

```
sampleUnivSNMoE(alphak, betak, sigmak, lambdak, x)
```

Arguments

`alphak`	The parameters of the gating network. `alphak` is a matrix of size *(q + 1, K - 1)*, with *K - 1*, the number of regressors (experts) and *q* the order of the logistic regression
`betak`	Matrix of size *(p + 1, K)* representing the regression coefficients of the experts network.
`sigmak`	Vector of length *K* giving the standard deviations of the experts network.
`lambdak`	Vector of length *K* giving the skewness parameter of each experts.
`x`	A vector og length *n* representing the inputs (predictors).

Value

A list with the output variable `y` and statistics.

- `y` Vector of length *n* giving the output variable.
- `zi` A vector of size *n* giving the hidden label of the expert component generating the i-th observation. Its elements are $zi[i] = k$, if the i-th observation has been generated by the k-th expert.
- z A matrix of size *(n, K)* giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- `stats` A list whose elements are:
  - `Ey_k` Matrix of size *(n, K)* giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation *zi = k*, and the value of predictor *X = xi*.
  - `Ey` Vector of length *n* giving the conditional expectation of Yi given the value of predictor *X = xi*.
  - `Vary_k` Vector of length *k* representing the conditional variance of Yi given *zi = k*, and *X = xi*.
  - `Vary` Vector of length *n* giving the conditional expectation of Yi given *X = xi*.

Examples

```
n <- 500 # Size of the sample
alphak <- matrix(c(0, 8), ncol = 1) # Parameters of the gating network
betak <- matrix(c(0, -2.5, 0, 2.5), ncol = 2) # Regression coefficients of the experts
lambdak <- c(3, 5) # Skewness parameters of the experts
sigmak <- c(1, 1) # Standard deviations of the experts
x <- seq.int(from = -1, to = 1, length.out = n) # Inputs (predictors)

# Generate sample of size n
sample <- sampleUnivSNMoE(alphak = alphak, betak = betak, sigmak = sigmak,
                          lambdak = lambdak, x = x)

# Plot points and estimated means
plot(x, sample$y, pch = 4)
lines(x, sample$stats$Ey_k[, 1], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey_k[, 2], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey, col = "red", lwd = 1.5)
```

---

sampleUnivSTMoE *Draw a sample from a univariate skew-t mixture.*

Description

Draw a sample from a univariate skew-t mixture.

Usage

```
sampleUnivSTMoE(alphak, betak, sigmak, lambdak, nuk, x)
```

Arguments

alphak	The parameters of the gating network. alphak is a matrix of size *(q + 1, K - 1)*, with *K - 1*, the number of regressors (experts) and *q* the order of the logistic regression
betak	Matrix of size *(p + 1, K)* representing the regression coefficients of the experts network.
sigmak	Vector of length *K* giving the standard deviations of the experts network.
lambdak	Vector of length *K* giving the skewness parameter of each experts.
nuk	Vector of length *K* giving the degrees of freedom of the experts network t densities.
x	A vector og length *n* representing the inputs (predictors).

Value

A list with the output variable y and statistics.

- y Vector of length *n* giving the output variable.
- zi A vector of size *n* giving the hidden label of the expert component generating the i-th observation. Its elements are $zi[i] = k$, if the i-th observation has been generated by the k-th expert.

- z A matrix of size *(n, K)* giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- stats A list whose elements are:
  - Ey_k Matrix of size *(n, K)* giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation *zi = k*, and the value of predictor *X = xi*.
  - Ey Vector of length *n* giving the conditional expectation of Yi given the value of predictor *X = xi*.
  - Vary_k Vector of length *k* representing the conditional variance of Yi given *zi = k*, and *X = xi*.
  - Vary Vector of length *n* giving the conditional expectation of Yi given *X = xi*.

Examples

```
n <- 500 # Size of the sample
alphak <- matrix(c(0, 8), ncol = 1) # Parameters of the gating network
betak <- matrix(c(0, -2.5, 0, 2.5), ncol = 2) # Regression coefficients of the experts
sigmak <- c(0.5, 0.5) # Standard deviations of the experts
lambdak <- c(3, 5) # Skewness parameters of the experts
nuk <- c(5, 7) # Degrees of freedom of the experts network t densities
x <- seq.int(from = -1, to = 1, length.out = n) # Inputs (predictors)

# Generate sample of size n
sample <- sampleUnivSTMoE(alphak = alphak, betak = betak, sigmak = sigmak,
                          lambdak = lambdak, nuk = nuk, x = x)

# Plot points and estimated means
plot(x, sample$y, pch = 4)
lines(x, sample$stats$Ey_k[, 1], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey_k[, 2], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey, col = "red", lwd = 1.5)
```

---

sampleUnivTMoE *Draw a sample from a univariate t mixture of experts (TMoE).*

Description

Draw a sample from a univariate t mixture of experts (TMoE).

Usage

```
sampleUnivTMoE(alphak, betak, sigmak, nuk, x)
```

Arguments

alphak	The parameters of the gating network. alphak is a matrix of size *(q + 1, K - 1)*, with *K - 1*, the number of regressors (experts) and *q* the order of the logistic regression
betak	Matrix of size *(p + 1, K)* representing the regression coefficients of the experts network.
sigmak	Vector of length *K* giving the standard deviations of the experts network.

nuk	Vector of length K giving the degrees of freedom of the experts network t densities.
x	A vector of length n representing the inputs (predictors).

Value

A list with the output variable y and statistics.

- y Vector of length n giving the output variable.
- zi A vector of size n giving the hidden label of the expert component generating the i-th observation. Its elements are $zi[i] = k$, if the i-th observation has been generated by the k-th expert.
- z A matrix of size *(n, K)* giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- stats A list whose elements are:
  - Ey_k Matrix of size *(n, K)* giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation *zi = k*, and the value of predictor *X = xi*.
  - Ey Vector of length n giving the conditional expectation of Yi given the value of predictor *X = xi*.
  - Vary_k Vector of length k representing the conditional variance of Yi given *zi = k*, and *X = xi*.
  - Vary Vector of length n giving the conditional expectation of Yi given *X = xi*.

Examples

```
n <- 500 # Size of the sample
alphak <- matrix(c(0, 8), ncol = 1) # Parameters of the gating network
betak <- matrix(c(0, -2.5, 0, 2.5), ncol = 2) # Regression coefficients of the experts
sigmak <- c(0.5, 0.5) # Standard deviations of the experts
nuk <- c(5, 7) # Degrees of freedom of the experts network t densities
x <- seq.int(from = -1, to = 1, length.out = n) # Inputs (predictors)

# Generate sample of size n
sample <- sampleUnivTMoE(alphak = alphak, betak = betak, sigmak = sigmak,
                         nuk = nuk, x = x)

# Plot points and estimated means
plot(x, sample$y, pch = 4)
lines(x, sample$stats$Ey_k[, 1], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey_k[, 2], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey, col = "red", lwd = 1.5)
```

StatNMoE-class *A Reference Class which contains statistics of a NMoE model.*

Description

StatNMoE contains all the statistics associated to a NMoE model. It mainly includes the E-Step of the EM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelihood.

Fields

`piik` Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \mathbf{\Psi}) = P(z_i = k|\boldsymbol{x}; \Psi)$ of the latent variable $z_i, i = 1, \ldots, n$.

`z_ik` Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z\_ik = 1 \text{ if } z\_ik = \arg\max_s \tau_{is};\ 0 \text{ otherwise}, k = 1, \ldots, K$.

`klas` Column matrix of the labels issued from `z_ik`. Its elements are $klas(i) = k$, $k = 1, \ldots, K$.

`tik` Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.

`Ey_k` Matrix of dimension *(n, K)* giving the estimated means of the experts.

`Ey` Column matrix of dimension *n* giving the estimated mean of the NMoE.

`Var_yk` Column matrix of dimension *K* giving the estimated means of the experts.

`Vary` Column matrix of dimension *n* giving the estimated variance of the response.

`loglik` Numeric. Observed-data log-likelihood of the NMoE model.

`com_loglik` Numeric. Complete-data log-likelihood of the NMoE model.

`stored_loglik` Numeric vector. Stored values of the log-likelihood at each EM iteration.

`BIC` Numeric. Value of BIC (Bayesian Information Criterion).

`ICL` Numeric. Value of ICL (Integrated Completed Likelihood).

`AIC` Numeric. Value of AIC (Akaike Information Criterion).

`log_piik_fik` Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i,\ z_i = k|\boldsymbol{x}, \mathbf{\Psi})$, $i = 1, \ldots, n$.

`log_sum_piik_fik` Column matrix of size *m* giving the values of $\log \sum_{k=1}^{K} P(y_i,\ z_i = k|\boldsymbol{x}, \mathbf{\Psi})$, $i = 1, \ldots, n$.

Methods

`computeLikelihood(reg_irls)` Method to compute the log-likelihood. `reg_irls` is the value of the regularization part in the IRLS algorithm.

`computeStats(paramNMoE)` Method used in the EM algorithm to compute statistics based on parameters provided by the object `paramNMoE` of class ParamNMoE.

`EStep(paramNMoE)` Method used in the EM algorithm to update statistics based on parameters provided by the object `paramNMoE` of class ParamNMoE (prior and posterior probabilities).

`MAP()` MAP calculates values of the fields `z_ik` and `klas` by applying the Maximum A Posteriori Bayes allocation rule.

$z_{ik} = 1 \text{ if } k = \arg\max_s \tau_{is};\ 0 \text{ otherwise}$

See Also

ParamNMoE

StatSNMoE-class *A Reference Class which contains statistics of a SNMoE model.*

Description

StatSNMoE contains all the statistics associated to a SNMoE model. It mainly includes the E-Step of the ECM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelihood.

Fields

piik Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \mathbf{\Psi}) = P(z_i = k|\boldsymbol{x}; \Psi)$ of the latent variable $z_i, i = 1, \ldots, n$.

z_ik Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z\_ik = 1$ if $z\_ik = \arg\max_s \tau_{is};\ 0$ otherwise, $k = 1, \ldots, K$.

klas Column matrix of the labels issued from z_ik. Its elements are $klas(i) = k$, $k = 1, \ldots, K$.

tik Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.

Ey_k Matrix of dimension *(n, K)* giving the estimated means of the experts.

Ey Column matrix of dimension *n* giving the estimated mean of the SNMoE.

Var_yk Column matrix of dimension *K* giving the estimated means of the experts.

Vary Column matrix of dimension *n* giving the estimated variance of the response.

loglik Numeric. Observed-data log-likelihood of the SNMoE model.

com_loglik Numeric. Complete-data log-likelihood of the SNMoE model.

stored_loglik Numeric vector. Stored values of the log-likelihood at each ECM iteration.

BIC Numeric. Value of BIC (Bayesian Information Criterion).

ICL Numeric. Value of ICL (Integrated Completed Likelihood).

AIC Numeric. Value of AIC (Akaike Information Criterion).

log_piik_fik Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i,\ z_i = k|\boldsymbol{x}, \mathbf{\Psi})$, $i = 1, \ldots, n$.

log_sum_piik_fik Column matrix of size *m* giving the values of $\log \sum_{k=1}^{K} P(y_i,\ z_i = k|\boldsymbol{x}, \mathbf{\Psi})$, $i = 1, \ldots, n$.

E1ik Conditional expectations of U_i (Matrix of size (n, K)).

E2ik Conditional expectations of U_i^2 (Matrix of size (n, K)).

Methods

computeLikelihood(reg_irls) Method to compute the log-likelihood. reg_irls is the value of the regularization part in the IRLS algorithm.

computeStats(paramSNMoE) Method used in the ECM algorithm to compute statistics based on parameters provided by the object paramSNMoE of class ParamSNMoE.

EStep(paramSNMoE) Method used in the ECM algorithm to update statistics based on parameters provided by the object paramSNMoE of class ParamSNMoE (prior and posterior probabilities).

MAP() MAP calculates values of the fields z_ik and klas by applying the Maximum A Posteriori Bayes allocation rule.

$z_{ik} = 1$ if $k = \arg\max_s \tau_{is};\ 0$ otherwise

See Also

ParamSNMoE

`StatStMoE-class` *A Reference Class which contains statistics of a StMoE model.*

Description

StatStMoE contains all the statistics associated to a StMoE model. It mainly includes the E-Step of the ECM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelhood.

Fields

`piik` Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \mathbf{\Psi}) = P(z_i = k|\boldsymbol{x}; \Psi)$ of the latent variable $z_i, i = 1, \ldots, n$.

`z_ik` Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z\_ik = 1$ if $z\_ik = \arg\max_s \tau_{is};\ 0$ otherwise, $k = 1, \ldots, K$.

`klas` Column matrix of the labels issued from `z_ik`. Its elements are $klas(i) = k$, $k = 1, \ldots, K$.

`tik` Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.

`Ey_k` Matrix of dimension *(n, K)* giving the estimated means of the experts.

`Ey` Column matrix of dimension *n* giving the estimated mean of the StMoE.

`Var_yk` Column matrix of dimension *K* giving the estimated means of the experts.

`Vary` Column matrix of dimension *n* giving the estimated variance of the response.

`loglik` Numeric. Observed-data log-likelihood of the StMoE model.

`com_loglik` Numeric. Complete-data log-likelihood of the StMoE model.

`stored_loglik` Numeric vector. Stored values of the log-likelihood at each ECM iteration.

`BIC` Numeric. Value of BIC (Bayesian Information Criterion).

`ICL` Numeric. Value of ICL (Integrated Completed Likelihood).

`AIC` Numeric. Value of AIC (Akaike Information Criterion).

`log_piik_fik` Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i,\ z_i = k|\boldsymbol{x}, \mathbf{\Psi})$, $i = 1, \ldots, n$.

`log_sum_piik_fik` Column matrix of size *m* giving the values of $\log \sum_{k=1}^{K} P(y_i,\ z_i = k|\boldsymbol{x}, \mathbf{\Psi})$, $i = 1, \ldots, n$.

`dik` It represents the value of d_{ik}.

`wik` Conditional expectations w_{ik}.

`E1ik` Conditional expectations $e_{1,ik}$.

`E2ik` Conditional expectations $e_{2,ik}$.

`E3ik` Conditional expectations $e_{3,ik}$.

`stme_pdf` Skew-t mixture of experts density.

Methods

`computeLikelihood(reg_irls)` Method to compute the log-likelihood. `reg_irls` is the value of the regularization part in the IRLS algorithm.

`computeStats(paramStMoE)` Method used in the ECM algorithm to compute statistics based on parameters provided by the object `paramStMoE` of class ParamStMoE.

`EStep(paramStMoE, calcTau = FALSE, calcE1 = FALSE, calcE2 = FALSE, calcE3 = FALSE)` Method used in the ECM algorithm to update statistics based on parameters provided by the object `paramStMoE` of class ParamStMoE (prior and posterior probabilities).

`MAP()` MAP calculates values of the fields `z_ik` and `klas` by applying the Maximum A Posteriori Bayes allocation rule.
$z_{ik} = 1$ if $k = \arg\max_s \tau_{is};\ 0$ otherwise

See Also

ParamStMoE

`StatTMoE-class` *A Reference Class which contains statistics of a TMoE model.*

Description

StatTMoE contains all the statistics associated to a TMoE model. It mainly includes the E-Step of the ECM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelihood.

Fields

`piik` Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \boldsymbol{\Psi}) = P(z_i = k|\boldsymbol{x}; \Psi)$ of the latent variable $z_i, i = 1, \dots, n$.

`z_ik` Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z\_ik = 1$ if $z\_ik = \arg\max_s \tau_{is};\ 0$ otherwise, $k = 1, \dots, K$.

`klas` Column matrix of the labels issued from `z_ik`. Its elements are $klas(i) = k$, $k = 1, \dots, K$.

`tik` Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.

`Ey_k` Matrix of dimension *(n, K)* giving the estimated means of the experts.

`Ey` Column matrix of dimension *n* giving the estimated mean of the TMoE.

`Var_yk` Column matrix of dimension *K* giving the estimated means of the experts.

`Vary` Column matrix of dimension *n* giving the estimated variance of the response.

`loglik` Numeric. Observed-data log-likelihood of the TMoE model.

`com_loglik` Numeric. Complete-data log-likelihood of the TMoE model.

`stored_loglik` Numeric vector. Stored values of the log-likelihood at each ECM iteration.

`BIC` Numeric. Value of BIC (Bayesian Information Criterion).

`ICL` Numeric. Value of ICL (Integrated Completed Likelihood).

`AIC` Numeric. Value of AIC (Akaike Information Criterion).

`log_piik_fik` Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i,\ z_i = k|\boldsymbol{x}, \boldsymbol{\Psi})$, $i = 1, \dots, n$.

`log_sum_piik_fik` Column matrix of size *m* giving the values of log $\sum_{k=1}^{K} P(y_i,\ z_i = k|\boldsymbol{x}, \boldsymbol{\Psi})$, $i = 1, \dots, n$.

`Wik` Conditional expectations w_{ik}.

Methods

`computeLikelihood(reg_irls)` Method to compute the log-likelihood. `reg_irls` is the value of the regularization part in the IRLS algorithm.

`computeStats(paramTMoE)` Method used in the ECM algorithm to compute statistics based on parameters provided by the object `paramTMoE` of class ParamTMoE.

`EStep(paramTMoE)` Method used in the ECM algorithm to update statistics based on parameters provided by the object `paramTMoE` of class ParamTMoE (prior and posterior probabilities).

`MAP()` MAP calculates values of the fields `z_ik` and `klas` by applying the Maximum A Posteriori Bayes allocation rule.

$z_{ik} = 1$ if $k = \arg\max_s \tau_{is};\ 0$ otherwise

See Also

ParamTMoE

`tempanomalies` *Global Annual Temperature Anomalies (Land Meteorological Stations) (1880-2015)*

Description

This dataset is from https://cdiac.ess-dive.lbl.gov/ftp/trends/temp/hansen/gl_land.txt.

Usage

```
tempanomalies
```

Format

A data frame with 136 rows and 3 columns:

Year Year of observation.

AnnualAnomaly Value in degrees C of the global annual temperature anomaly.

5-YearMean 5-Year mean of temperature anomalies.

Details

Global annual temperature anomalies (degrees C) computed using data from land meteorological stations, 1880-2015. Anomalies are relative to the 1951-1980 base period means.

Non-computed values are indicated by "-99.99".

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