

# Higgs bundles

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## 1 Motivic classes of the moduli spaces of twisted Higgs bundles over curves

Based on [arXiv:1104.5698](#) and [arXiv:1901.02439](#)

Given a curve  $C$  of genus  $g$  and a line bundle  $L$  over  $C$  of degree  $\ell$ , we consider the moduli space  $M_L(r, d)$  of semistable twisted Higgs bundles  $(E, \phi)$ , where  $E$  is a vector bundle of rank  $r$  and degree  $d$  and  $\phi : E \rightarrow E \otimes L$  is a homomorphism.

If  $r$  and  $d$  are coprime, then the motivic class of  $M_L(r, d)$  is independent of  $d$ . In the examples below we compute this motivic class (actually just the virtual Poincare polynomial or the E-polynomial) for various values of  $g, \ell, r$ .

We always assume that  $\ell > 2g - 2$  or  $\ell = 2g - 2$  and  $L = K_C$  (the usual Higgs bundles).

```
[1]: import sys
      sys.path.append('.') #add path for import
      from msinvar.higgs_bundles import CurveAlgebra
      from msinvar.higgs_bundles import twisted_higgs_bundles_invariant as invar
```

### 1.1 Computations for curves of genus 0

```
[2]: C=CurveAlgebra(g=0)
      for l in range(1,4):
          for r in range(1,5):
              print('l='+str(l)+', r='+str(r)+':')
              show(invar(C,l,r).factor())
```

l=1, r=1:

$y^4$

l=1, r=2:

$y^{10}$

l=1, r=3:

$y^{18} \cdot (y^2 + 1)$

l=1, r=4:

$$y^{28} \cdot (y^6 + y^4 + 3y^2 + 2)$$

l=2, r=1:

$$y^6$$

l=2, r=2:

$$y^{16} \cdot (y^2 + 1)$$

l=2, r=3:

$$y^{30} \cdot (y^8 + y^6 + 3y^4 + 4y^2 + 3)$$

l=2, r=4:

$$y^{48} \cdot (y^2 + 1) \cdot (y^{16} + 3y^{12} + 2y^{10} + 7y^8 + 6y^6 + 12y^4 + 10y^2 + 10)$$

l=3, r=1:

$$y^8$$

l=3, r=2:

$$y^{22} \cdot (y^4 + y^2 + 2)$$

l=3, r=3:

$$y^{42} \cdot (y^2 + 1) \cdot (y^{12} + 3y^8 + y^6 + 6y^4 + 3y^2 + 6)$$

l=3, r=4:

$$y^{68} \cdot (y^{30} + y^{28} + 3y^{26} + 5y^{24} + 9y^{22} + 13y^{20} + 22y^{18} + 30y^{16} + 45y^{14} + 56y^{12} + 75y^{10} + 85y^8 + 97y^6 + 87y^4 + 63y^2 + 28)$$

```
[3]: C=CurveAlgebra(g=0,vars='u,v')
      for l in range(1,4):
          for r in range(1,5):
              print('l='+str(l)+' , r='+str(r)+' :')
              show(invar(C,l,r).factor())
```

l=1, r=1:

$$v^2 \cdot u^2$$

l=1, r=2:

$$v^5 \cdot u^5$$

l=1, r=3:

$$v^9 \cdot u^9 \cdot (uv + 1)$$

l=1, r=4:

$$v^{14} \cdot u^{14} \cdot (u^3v^3 + u^2v^2 + 3uv + 2)$$

l=2, r=1:

$$v^3 \cdot u^3$$

l=2, r=2:

$$v^8 \cdot u^8 \cdot (uv + 1)$$

l=2, r=3:

$$v^{15} \cdot u^{15} \cdot (u^4 v^4 + u^3 v^3 + 3u^2 v^2 + 4uv + 3)$$

l=2, r=4:

$$v^{24} \cdot u^{24} \cdot (uv + 1) \cdot (u^8 v^8 + 3u^6 v^6 + 2u^5 v^5 + 7u^4 v^4 + 6u^3 v^3 + 12u^2 v^2 + 10uv + 10)$$

l=3, r=1:

$$v^4 \cdot u^4$$

l=3, r=2:

$$v^{11} \cdot u^{11} \cdot (u^2 v^2 + uv + 2)$$

l=3, r=3:

$$v^{21} \cdot u^{21} \cdot (uv + 1) \cdot (u^6 v^6 + 3u^4 v^4 + u^3 v^3 + 6u^2 v^2 + 3uv + 6)$$

l=3, r=4:

$$v^{34} \cdot u^{34} \cdot (u^{15} v^{15} + u^{14} v^{14} + 3u^{13} v^{13} + 5u^{12} v^{12} + 9u^{11} v^{11} + 13u^{10} v^{10} + 22u^9 v^9 + 30u^8 v^8 + 45u^7 v^7 + 56u^6 v^6 + 75u^5 v^5 + 85u^4 v^4 + 97u^3 v^3 + 87u^2 v^2 + 63uv + 28)$$

## 1.2 Computations for curves of genus 1

```
[4]: C=CurveAlgebra(g=1)
for l in range(1,4):
    for r in range(1,5):
        print('l='+str(l)+' , r='+str(r)+' :')
        show(invar(C,l,r).factor())
```

l=1, r=1:

$$y^2 \cdot (y - 1)^2$$

l=1, r=2:

$$(y - 1)^2 \cdot y^6 \cdot (y^2 + 1)$$

l=1, r=3:

$$(y - 1)^2 \cdot y^{12} \cdot (y^6 + y^4 - 2y^3 + 2y^2 - 2y + 1)$$

l=1, r=4:

$$(y - 1)^2 \cdot y^{20} \cdot (y^{12} + y^{10} - 2y^9 + 3y^8 - 4y^7 + 5y^6 - 8y^5 + 10y^4 - 10y^3 + 9y^2 - 6y + 2)$$

l=2, r=1:

$$(y - 1)^2 \cdot y^4$$

l=2, r=2:

$$(y - 1)^2 \cdot y^{12} \cdot (y^4 + y^2 - 2y + 1)$$

l=2, r=3:

$$(y-1)^2 \cdot y^{24} \cdot (y^{12} + y^{10} - 2y^9 + 3y^8 - 4y^7 + 5y^6 - 8y^5 + 10y^4 - 10y^3 + 10y^2 - 8y + 3)$$

l=2, r=4:

$$(y-1)^2 \cdot y^{40} \cdot (y^{24} + y^{22} - 2y^{21} + 3y^{20} - 4y^{19} + 6y^{18} - 10y^{17} + 14y^{16} - 18y^{15} + 25y^{14} - 36y^{13} + 46y^{12} - 56y^{11} + 73y^{10} - 94y^9 + 108y^8 - 120y^7 + 137y^6 - 144y^5 + 131y^4 - 108y^3 + 78y^2 - 40y + 10)$$

l=3, r=1:

$$(y-1)^2 \cdot y^6$$

l=3, r=2:

$$(y-1)^2 \cdot y^{18} \cdot (y^2 + 1) \cdot (y^4 - 2y + 2)$$

l=3, r=3:

$$(y-1)^2 \cdot y^{36} \cdot (y^{18} + y^{16} - 2y^{15} + 3y^{14} - 4y^{13} + 5y^{12} - 8y^{11} + 12y^{10} - 14y^9 + 17y^8 - 24y^7 + 28y^6 - 30y^5 + 33y^4 - 32y^3 + 27y^2 - 18y + 6)$$

l=3, r=4:

$$(y-1)^2 \cdot y^{60} \cdot (y^{36} + y^{34} - 2y^{33} + 3y^{32} - 4y^{31} + 6y^{30} - 10y^{29} + 14y^{28} - 18y^{27} + 25y^{26} - 36y^{25} + 48y^{24} - 60y^{23} + 79y^{22} - 106y^{21} + 133y^{20} - 164y^{19} + 205y^{18} - 256y^{17} + 312y^{16} - 370y^{15} + 439y^{14} - 520y^{13} + 599y^{12} - 676y^{11} + 753y^{10} - 818y^9 + 862y^8 - 872y^7 + 838y^6 - 760y^5 + 632y^4 - 464y^3 + 285y^2 - 126y + 28)$$

```
[5]: C=CurveAlgebra(g=1,vars='u,v')
for l in range(1,4):
    for r in range(1,5):
        print('l='+str(l)+' , r='+str(r)+':')
        show(invar(C,l,r).factor())
```

l=1, r=1:

$$v \cdot (v-1) \cdot u \cdot (u-1)$$

l=1, r=2:

$$(v-1) \cdot (u-1) \cdot v^3 \cdot u^3 \cdot (uv+1)$$

l=1, r=3:

$$(v-1) \cdot (u-1) \cdot v^6 \cdot u^6 \cdot (u^3v^3 + u^2v^2 - u^2v - uv^2 + 2uv - u - v + 1)$$

l=1, r=4:

$$(v-1) \cdot (u-1) \cdot v^{10} \cdot u^{10} \cdot (u^6v^6 + u^5v^5 - u^5v^4 - u^4v^5 + 3u^4v^4 - 2u^4v^3 - 2u^3v^4 + 5u^3v^3 - 4u^3v^2 - 4u^2v^3 + u^3v + 8u^2v^2 + uv^3 - 5u^2v - 5uv^2 + u^2 + 7uv + v^2 - 3u - 3v + 2)$$

l=2, r=1:

$$(v-1) \cdot (u-1) \cdot v^2 \cdot u^2$$

l=2, r=2:

$$(v-1) \cdot (u-1) \cdot v^6 \cdot u^6 \cdot (u^2v^2 + uv - u - v + 1)$$

l=2, r=3:

$$(v-1) \cdot (u-1) \cdot v^{12} \cdot u^{12} \cdot (u^6v^6 + u^5v^5 - u^5v^4 - u^4v^5 + 3u^4v^4 - 2u^4v^3 - 2u^3v^4 + 5u^3v^3 - 4u^3v^2 - 4u^2v^3 + u^3v + 8u^2v^2 + uv^3 - 5u^2v - 5uv^2 + u^2 + 8uv + v^2 - 4u - 4v + 3)$$

l=2, r=4:

$$(v-1) \cdot (u-1) \cdot v^{20} \cdot u^{20} \cdot (u^{12}v^{12} + u^{11}v^{11} - u^{11}v^{10} - u^{10}v^{11} + 3u^{10}v^{10} - 2u^{10}v^9 - 2u^9v^{10} + 6u^9v^9 - 5u^9v^8 - 5u^8v^9 + u^9v^7 + 12u^8v^8 + u^7v^9 - 9u^8v^7 - 9u^7v^8 + 2u^8v^6 + 21u^7v^7 + 2u^6v^8 - 18u^7v^6 - 18u^6v^7 + 5u^7v^5 + 36u^6v^6 + 5u^5v^7 - 28u^6v^5 - 28u^5v^6 + 9u^6v^4 + 55u^5v^5 + 9u^4v^6 - u^6v^3 - 46u^5v^4 - 46u^4v^5 - u^3v^6 + 15u^5v^3 + 78u^4v^4 + 15u^3v^5 - u^5v^2 - 59u^4v^3 - 59u^3v^4 - u^2v^5 + 21u^4v^2 + 95u^3v^3 + 21u^2v^4 - 3u^4v - 69u^3v^2 - 69u^2v^3 - 3uv^4 + 21u^3v + 89u^2v^2 + 21uv^3 - 2u^3 - 52u^2v - 52uv^2 - 2v^3 + 12u^2 + 54uv + 12v^2 - 20u - 20v + 10)$$

l=3, r=1:

$$(v-1) \cdot (u-1) \cdot v^3 \cdot u^3$$

l=3, r=2:

$$(v-1) \cdot (u-1) \cdot v^9 \cdot u^9 \cdot (uv+1) \cdot (u^2v^2 - u - v + 2)$$

l=3, r=3:

$$(v-1) \cdot (u-1) \cdot v^{18} \cdot u^{18} \cdot (u^9v^9 + u^8v^8 - u^8v^7 - u^7v^8 + 3u^7v^7 - 2u^7v^6 - 2u^6v^7 + 5u^6v^6 - 4u^6v^5 - 4u^5v^6 + u^6v^4 + 10u^5v^5 + u^4v^6 - 7u^5v^4 - 7u^4v^5 + u^5v^3 + 15u^4v^4 + u^3v^5 - 12u^4v^3 - 12u^3v^4 + 3u^4v^2 + 22u^3v^3 + 3u^2v^4 - 15u^3v^2 - 15u^2v^3 + 4u^3v + 25u^2v^2 + 4uv^3 - 16u^2v - 16uv^2 + 3u^2 + 21uv + 3v^2 - 9u - 9v + 6)$$

l=3, r=4:

$$(v-1) \cdot (u-1) \cdot v^{30} \cdot u^{30} \cdot (u^{18}v^{18} + u^{17}v^{17} - u^{17}v^{16} - u^{16}v^{17} + 3u^{16}v^{16} - 2u^{16}v^{15} - 2u^{15}v^{16} + 6u^{15}v^{15} - 5u^{15}v^{14} - 5u^{14}v^{15} + u^{15}v^{13} + 12u^{14}v^{14} + u^{13}v^{15} - 9u^{14}v^{13} - 9u^{13}v^{14} + 2u^{14}v^{12} + 21u^{13}v^{13} + 2u^{12}v^{14} - 18u^{13}v^{12} - 18u^{12}v^{13} + 5u^{13}v^{11} + 38u^{12}v^{12} + 5u^{11}v^{13} - 30u^{12}v^{11} - 30u^{11}v^{12} + 9u^{12}v^{10} + 61u^{11}v^{11} + 9u^{10}v^{12} - u^{12}v^9 - 52u^{11}v^{10} - 52u^{10}v^{11} - u^9v^{12} + 17u^{11}v^9 + 99u^{10}v^{10} + 17u^9v^{11} - u^{11}v^8 - 81u^{10}v^9 - 81u^9v^{10} - u^8v^{11} + 28u^{10}v^8 + 149u^9v^9 + 28u^8v^{10} - 3u^{10}v^7 - 125u^9v^8 - 125u^8v^9 - 3u^7v^{10} + 46u^9v^7 + 220u^8v^8 + 46u^7v^9 - 5u^9v^6 - 180u^8v^7 - 180u^7v^8 - 5u^6v^9 + 67u^8v^6 + 305u^7v^7 + 67u^6v^8 - 9u^8v^5 - 251u^7v^6 - 251u^6v^7 - 9u^5v^8 + 96u^7v^5 + 407u^6v^6 + 96u^5v^7 - 13u^7v^4 - 325u^6v^5 - 325u^5v^6 - 13u^4v^7 + 125u^6v^4 + 503u^5v^5 + 125u^4v^6 - 18u^6v^3 - 391u^5v^4 - 391u^4v^5 - 18u^3v^6 + 148u^5v^3 + 566u^4v^4 + 148u^3v^5 - 22u^5v^2 - 414u^4v^3 - 414u^3v^4 - 22u^2v^5 + 147u^4v^2 + 544u^3v^3 + 147u^2v^4 - 20u^4v - 360u^3v^2 - 360u^2v^3 - 20uv^4 + 110u^3v + 412u^2v^2 + 110uv^3 - 10u^3 - 222u^2v - 222uv^2 - 10v^3 + 45u^2 + 195uv + 45v^2 - 63u - 63v + 28)$$

### 1.3 Computations for curves of genus 2

```
[6]: C=CurveAlgebra(g=2)
for l in range(2,4):
    for r in range(1,5):
        print('l='+str(l)+' , r='+str(r)+' :')
        show(invar(C,l,r).factor())
```

l=2, r=1:

$$y^4 \cdot (y-1)^4$$

l=2, r=2:

$$(y-1)^4 \cdot y^{10} \cdot (y^2+1) \cdot (y^4-4y+2)$$

l=2, r=3:

$$(y-1)^4 \cdot y^{20} \cdot (y^{16} + y^{14} - 4y^{13} + 3y^{12} - 8y^{11} + 10y^{10} - 16y^9 + 29y^8 - 32y^7 + 48y^6 - 64y^5 + 67y^4 - 68y^3 + 48y^2 - 24y + 6)$$

l=2, r=4:

$$(y-1)^6 \cdot y^{34} \cdot (y^{28} + 2y^{27} + 4y^{26} + 2y^{25} + 3y^{24} - 4y^{23} - 16y^{21} - y^{20} - 26y^{19} + 18y^{18} - 34y^{17} + 53y^{16} - 52y^{15} + 112y^{14} - 108y^{13} + 173y^{12} - 218y^{11} + 274y^{10} - 374y^9 + 399y^8 - 524y^7 + 544y^6 - 568y^5 + 504y^4 - 408y^3 + 234y^2 - 100y + 22)$$

l=3, r=1:

$$y^4 \cdot (y-1)^4$$

l=3, r=2:

$$(y-1)^4 \cdot y^{14} \cdot (y^8 + y^6 - 4y^5 + 2y^4 - 4y^3 + 8y^2 - 4y + 2)$$

l=3, r=3:

$$(y-1)^4 \cdot y^{30} \cdot (y^{22} + y^{20} - 4y^{19} + 3y^{18} - 8y^{17} + 10y^{16} - 16y^{15} + 29y^{14} - 32y^{13} + 49y^{12} - 72y^{11} + 92y^{10} - 120y^9 + 149y^8 - 184y^7 + 210y^6 - 216y^5 + 207y^4 - 168y^3 + 96y^2 - 36y + 6)$$

l=3, r=4:

$$(y-1)^6 \cdot y^{52} \cdot (y^{40} + 2y^{39} + 4y^{38} + 2y^{37} + 3y^{36} - 4y^{35} - 16y^{33} - y^{32} - 26y^{31} + 18y^{30} - 34y^{29} + 53y^{28} - 52y^{27} + 112y^{26} - 108y^{25} + 174y^{24} - 224y^{23} + 291y^{22} - 394y^{21} + 480y^{20} - 646y^{19} + 804y^{18} - 1002y^{17} + 1240y^{16} - 1530y^{15} + 1850y^{14} - 2186y^{13} + 2547y^{12} - 2976y^{11} + 3288y^{10} - 3556y^9 + 3719y^8 - 3726y^7 + 3413y^6 - 2900y^5 + 2198y^4 - 1368y^3 + 630y^2 - 196y + 28)$$

```
[7]: C=CurveAlgebra(g=2,vars='u,v')
for l in range(2,4):
    for r in range(1,5):
        print('l='+str(l)+' , r='+str(r)+' :')
        show(invar(C,l,r).factor())
```

l=2, r=1:

$$v^2 \cdot (v-1)^2 \cdot u^2 \cdot (u-1)^2$$

l=2, r=2:

$$(v-1)^2 \cdot (u-1)^2 \cdot v^5 \cdot u^5 \cdot (uv+1) \cdot (u^2v^2 - 2u - 2v + 2)$$

l=2, r=3:

$$(v-1)^2 \cdot (u-1)^2 \cdot v^{10} \cdot u^{10} \cdot (u^8v^8 + u^7v^7 - 2u^7v^6 - 2u^6v^7 + 3u^6v^6 - 4u^6v^5 - 4u^5v^6 + u^6v^4 + 8u^5v^5 + u^4v^6 - 8u^5v^4 - 8u^4v^5 + 5u^5v^3 + 19u^4v^4 + 5u^3v^5 - 16u^4v^3 - 16u^3v^4 + 8u^4v^2 + 32u^3v^3 + 8u^2v^4 - 2u^4v - 30u^3v^2 - 30u^2v^3 - 2uv^4 + 12u^3v + 43u^2v^2 + 12uv^3 - 2u^3 - 32u^2v - 32uv^2 - 2v^3 + 8u^2 + 32uv + 8v^2 - 12u - 12v + 6)$$

l=2, r=4:

$$(v-1)^2 \cdot (u-1)^2 \cdot v^{17} \cdot u^{17} \cdot (u^{15}v^{15} + u^{14}v^{14} - 2u^{14}v^{13} - 2u^{13}v^{14} + 3u^{13}v^{13} - 4u^{13}v^{12} - 4u^{12}v^{13} + u^{13}v^{11} + 9u^{12}v^{12} + u^{11}v^{13} - 10u^{12}v^{11} - 10u^{11}v^{12} + 5u^{12}v^{10} + 21u^{11}v^{11} + 5u^{10}v^{12} - 20u^{11}v^{10} - 20u^{10}v^{11} + 12u^{11}v^9 + 45u^{10}v^{10} + 12u^9v^{11} - 2u^{11}v^8 - 46u^{10}v^9 - 46u^9v^{10} - 2u^8v^{11} + 26u^{10}v^8 + 87u^9v^9 + 26u^8v^{10} - 6u^{10}v^7 - 90u^9v^8 - 90u^8v^9 - 6u^7v^{10} + 53u^9v^7 + 163u^8v^8 + 53u^7v^9 - 18u^9v^6 - 174u^8v^7 - 174u^7v^8 -$$

$$18u^6v^9 + u^9v^5 + 103u^8v^6 + 293u^7v^7 + 103u^6v^8 + u^5v^9 - 34u^8v^5 - 302u^7v^6 - 302u^6v^7 - 34u^5v^8 + 5u^8v^4 + 188u^7v^5 + 497u^6v^6 + 188u^5v^7 + 5u^4v^8 - 68u^7v^4 - 502u^6v^5 - 502u^5v^6 - 68u^4v^7 + 12u^7v^3 + 310u^6v^4 + 777u^5v^5 + 310u^4v^6 + 12u^3v^7 - 110u^6v^3 - 738u^5v^4 - 738u^4v^5 - 110u^3v^6 + 22u^6v^2 + 442u^5v^3 + 1063u^4v^4 + 442u^3v^5 + 22u^2v^6 - 2u^6v - 154u^5v^2 - 934u^4v^3 - 934u^3v^4 - 154u^2v^5 - 2uv^6 + 27u^5v + 488u^4v^2 + 1154u^3v^3 + 488u^2v^4 + 27uv^5 - 2u^5 - 140u^4v - 850u^3v^2 - 850u^2v^3 - 140uv^4 - 2v^5 + 17u^4 + 343u^3v + 834u^2v^2 + 343uv^3 + 17v^4 - 56u^3 - 432u^2v - 432uv^2 - 56v^3 + 91u^2 + 274uv + 91v^2 - 72u - 72v + 22)$$

l=3, r=1:

$$v^2 \cdot (v-1)^2 \cdot u^2 \cdot (u-1)^2$$

l=3, r=2:

$$(v-1)^2 \cdot (u-1)^2 \cdot v^7 \cdot u^7 \cdot (u^4v^4 + u^3v^3 - 2u^3v^2 - 2u^2v^3 + 2u^2v^2 - 2u^2v - 2uv^2 + u^2 + 6uv + v^2 - 2u - 2v + 2)$$

l=3, r=3:

$$(v-1)^2 \cdot (u-1)^2 \cdot v^{15} \cdot u^{15} \cdot (u^{11}v^{11} + u^{10}v^{10} - 2u^{10}v^9 - 2u^9v^{10} + 3u^9v^9 - 4u^9v^8 - 4u^8v^9 + u^9v^7 + 8u^8v^8 + u^7v^9 - 8u^8v^7 - 8u^7v^8 + 5u^8v^6 + 19u^7v^7 + 5u^6v^8 - 16u^7v^6 - 16u^6v^7 + 8u^7v^5 + 33u^6v^6 + 8u^5v^7 - 2u^7v^4 - 34u^6v^5 - 34u^5v^6 - 2u^4v^7 + 17u^6v^4 + 58u^5v^5 + 17u^4v^6 - 4u^6v^3 - 56u^5v^4 - 56u^4v^5 - 4u^3v^6 + 29u^5v^3 + 91u^4v^4 + 29u^3v^5 - 8u^5v^2 - 84u^4v^3 - 84u^3v^4 - 8u^2v^5 + u^5v + 42u^4v^2 + 124u^3v^3 + 42u^2v^4 + uv^5 - 10u^4v - 98u^3v^2 - 98u^2v^3 - 10uv^4 + u^4 + 42u^3v + 121u^2v^2 + 42uv^3 + v^4 - 8u^3 - 76u^2v - 76uv^2 - 8v^3 + 18u^2 + 60uv + 18v^2 - 18u - 18v + 6)$$

l=3, r=4:

$$(v-1)^2 \cdot (u-1)^2 \cdot v^{26} \cdot u^{26} \cdot (u^{21}v^{21} + u^{20}v^{20} - 2u^{20}v^{19} - 2u^{19}v^{20} + 3u^{19}v^{19} - 4u^{19}v^{18} - 4u^{18}v^{19} + u^{19}v^{17} + 9u^{18}v^{18} + u^{17}v^{19} - 10u^{18}v^{17} - 10u^{17}v^{18} + 5u^{18}v^{16} + 21u^{17}v^{17} + 5u^{16}v^{18} - 20u^{17}v^{16} - 20u^{16}v^{17} + 12u^{17}v^{15} + 45u^{16}v^{16} + 12u^{15}v^{17} - 2u^{17}v^{14} - 46u^{16}v^{15} - 46u^{15}v^{16} - 2u^{14}v^{17} + 26u^{16}v^{14} + 87u^{15}v^{15} + 26u^{14}v^{16} - 6u^{16}v^{13} - 90u^{15}v^{14} - 90u^{14}v^{15} - 6u^{13}v^{16} + 53u^{15}v^{13} + 163u^{14}v^{14} + 53u^{13}v^{15} - 18u^{15}v^{12} - 174u^{14}v^{13} - 174u^{13}v^{14} - 18u^{12}v^{15} + u^{15}v^{11} + 103u^{14}v^{12} + 294u^{13}v^{13} + 103u^{12}v^{14} + u^{11}v^{15} - 34u^{14}v^{11} - 306u^{13}v^{12} - 306u^{12}v^{13} - 34u^{11}v^{14} + 5u^{14}v^{10} + 194u^{13}v^{11} + 515u^{12}v^{12} + 194u^{11}v^{13} + 5u^{10}v^{14} - 70u^{13}v^{10} - 530u^{12}v^{11} - 530u^{11}v^{12} - 70u^{10}v^{13} + 12u^{13}v^9 + 339u^{12}v^{10} + 857u^{11}v^{11} + 339u^{10}v^{12} + 12u^9v^{13} - 128u^{12}v^9 - 872u^{11}v^{10} - 872u^{10}v^{11} - 128u^9v^{12} + 26u^{12}v^8 + 571u^{11}v^9 + 1382u^{10}v^{10} + 571u^9v^{11} + 26u^8v^{12} - 2u^{12}v^7 - 228u^{11}v^8 - 1398u^{10}v^9 - 1398u^9v^{10} - 228u^8v^{11} - 2u^7v^{12} + 49u^{11}v^7 + 910u^{10}v^8 + 2130u^9v^9 + 910u^8v^{10} + 49u^7v^{11} - 4u^{11}v^6 - 374u^{10}v^7 - 2128u^9v^8 - 2128u^8v^9 - 374u^7v^{10} - 4u^6v^{11} + 90u^{10}v^6 + 1401u^9v^7 + 3168u^8v^8 + 1401u^7v^9 + 90u^6v^{10} - 10u^{10}v^5 - 588u^9v^6 - 3110u^8v^7 - 3110u^7v^8 - 588u^6v^9 - 10u^5v^{10} + 144u^9v^5 + 2017u^8v^6 + 4447u^7v^7 + 2017u^6v^8 + 144u^5v^9 - 18u^9v^4 - 852u^8v^5 - 4258u^7v^6 - 4258u^6v^7 - 852u^5v^8 - 18u^4v^9 + u^9v^3 + 223u^8v^4 + 2735u^7v^5 + 5869u^6v^6 + 2735u^5v^7 + 223u^4v^8 + u^3v^9 - 30u^8v^3 - 1138u^7v^4 - 5386u^6v^5 - 5386u^5v^6 - 1138u^4v^7 - 30u^3v^8 + u^8v^2 + 289u^7v^3 + 3297u^6v^4 + 6945u^5v^5 + 3297u^4v^6 + 289u^3v^7 + u^2v^8 - 42u^7v^2 - 1320u^6v^3 - 5998u^5v^4 - 5998u^4v^5 - 1320u^3v^6 - 42u^2v^7 + 3u^7v + 325u^6v^2 + 3417u^5v^3 + 7094u^4v^4 + 3417u^3v^5 + 325u^2v^6 + 3uv^7 - 42u^6v - 1220u^5v^2 - 5464u^4v^3 - 5464u^3v^4 - 1220u^2v^5 - 42uv^6 + 2u^6 + 250u^5v + 2669u^4v^2 + 5569u^3v^3 + 2669u^2v^4 + 250uv^5 + 2v^6 - 24u^5 - 762u^4v - 3546u^3v^2 - 3546u^2v^3 - 762uv^4 - 24v^5 + 98u^4 + 1283u^3v + 2802u^2v^2 + 1283uv^3 + 98v^4 - 200u^3 - 1212u^2v - 1212uv^2 - 200v^3 + 222u^2 + 606uv + 222v^2 - 126u - 126v + 28)$$

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