#### **Problem 1: Basics of Neural Networks**

- Learning Objective: In this problem, you are asked to implement a basic multilayer fully connected neural network from scratch, including forward and backward passes of certain essential layers, to perform an image classification task on the CIFAR100 dataset. You need to implement essential functions in different indicated python files under directory lib.
- **Provided Code:** We provide the skeletons of classes you need to complete. Forward checking and gradient checkings are provided for verifying your implementation as well.
- **TODOs:** You are asked to implement the forward passes and backward passes for standard layers and loss functions, various widely-used optimizers, and part of the training procedure. And finally we want you to train a network from scratch on your own. Also, there are inline questions you need to answer. See README.md to set up your environment.

```
In [1]: from lib.mlp.fully_conn import *
        from lib.mlp.layer_utils import *
        from lib.datasets import *
        from lib.mlp.train import *
        from lib.grad_check import *
        from lib.optim import *
        import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
        plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'
        # for auto-reloading external modules
        # see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ip
        %load ext autoreload
        %autoreload 2
```

#### Loading the data (CIFAR-100 with 20 superclasses)

In this homework, we will be classifying images from the CIFAR-100 dataset into the 20 superclasses. More information about the CIFAR-100 dataset and the 20 superclasses can be found here.

Download the CIFAR-100 data files here, and save the .mat files to the data/cifar100 directory.

Load the dataset.

```
data = CIFAR100_data('data/cifar100/')
In [3]:
        for k, v in data.items():
            if type(v) == np.ndarray:
                print ("Name: {} Shape: {}, {}".format(k, v.shape, type(v)))
            else:
                print("{}: {}".format(k, v))
        label_names = data['label_names']
        mean_image = data['mean_image'][0]
        std_image = data['std_image'][0]
        Name: data_train Shape: (40000, 32, 32, 3), <class 'numpy.ndarray'>
        Name: labels_train Shape: (40000,), <class 'numpy.ndarray'>
        Name: data_val Shape: (10000, 32, 32, 3), <class 'numpy.ndarray'>
        Name: labels_val Shape: (10000,), <class 'numpy.ndarray'>
        Name: data_test Shape: (10000, 32, 32, 3), <class 'numpy.ndarray'>
        Name: labels_test Shape: (10000,), <class 'numpy.ndarray'>
        label_names: ['aquatic_mammals', 'fish', 'flowers', 'food_containers', 'fru
        it_and_vegetables', 'household_electrical_devices', 'household_furniture',
        'insects', 'large_carnivores', 'large_man-made_outdoor_things', 'large_natu
```

Name: mean\_image Shape: (1, 1, 1, 3), <class 'numpy.ndarray'>
Name: std\_image Shape: (1, 1, 1, 3), <class 'numpy.ndarray'>

#### Implement Standard Layers

vehicles\_1', 'vehicles\_2']

You will now implement all the following standard layers commonly seen in a fully connected neural network (aka multi-layer perceptron, MLP). Please refer to the file lib/mlp/layer\_utils.py . Take a look at each class skeleton, and we will walk you through the network layer by layer. We provide results of some examples we precomputed for you for checking the forward pass, and also the gradient checking for the backward pass.

ral\_outdoor\_scenes', 'large\_omnivores\_and\_herbivores', 'medium\_mammals', 'n
on-insect\_invertebrates', 'people', 'reptiles', 'small\_mammals', 'trees', '

#### FC Forward [2pt]

In the class skeleton flatten and fc in lib/mlp/layer\_utils.py, please complete the forward pass in function forward. The input to the fc layer may not be of dimension (batch size, features size), it could be an image or any higher dimensional data. We want to convert the input to have a shape of (batch size, features size). Make sure that you handle this dimensionality issue.

```
In [246... %reload ext autoreload
         # Test the fc forward function
         input bz = 3 # batch size
         input_dim = (7, 6, 4)
         output_dim = 4
         input_size = input_bz * np.prod(input_dim)
         weight_size = output_dim * np.prod(input_dim)
         flatten_layer = flatten(name="flatten_test")
         single_fc = fc(np.prod(input_dim), output_dim, init_scale=0.02, name="fc_tes"
         x = np.linspace(-0.1, 0.4, num=input_size).reshape(input_bz, *input_dim)
         w = np.linspace(-0.2, 0.2, num=weight_size).reshape(np.prod(input_dim), outp
         b = np.linspace(-0.3, 0.3, num=output_dim)
         single_fc.params[single_fc.w_name] = w
         single_fc.params[single_fc.b_name] = b
         out = single_fc.forward(flatten_layer.forward(x))
         correct_out = np.array([[0.63910291, 0.83740057, 1.03569824, 1.23399591],
                                  [0.61401587, 0.82903823, 1.04406058, 1.25908294],
                                  [0.58892884, 0.82067589, 1.05242293, 1.28416997]])
         # Compare your output with the above pre-computed ones.
         # The difference should not be larger than 1e-8
         print ("Difference: ", rel_error(out, correct_out))
```

Difference: 4.026016656214849e-09

#### FC Backward [2pt]

Please complete the function backward as the backward pass of the flatten and fc layers. Follow the instructions in the comments to store gradients into the predefined dictionaries in the attributes of the class. Parameters of the layer are also stored in the predefined dictionary.

```
In [312... %reload ext autoreload
         # Test the fc backward function
         inp = np.random.randn(15, 2, 2, 3)
         w = np.random.randn(12, 15)
         b = np.random.randn(15)
         dout = np.random.randn(15, 15)
         flatten_layer = flatten(name="flatten_test")
         x = flatten_layer.forward(inp)
         single_fc = fc(np.prod(x.shape[1:]), 15, init_scale=5e-2, name="fc_test")
         single_fc.params[single_fc.w_name] = w
         single_fc.params[single_fc.b_name] = b
         dx_num = eval_numerical_gradient_array(lambda x: single_fc.forward(x), x, do
         dw_num = eval_numerical_gradient_array(lambda w: single_fc.forward(x), w, do
         db_num = eval_numerical_gradient_array(lambda b: single_fc.forward(x), b, do
         out = single_fc.forward(x)
         dx = single_fc.backward(dout)
         dw = single_fc.grads[single_fc.w_name]
         db = single_fc.grads[single_fc.b_name]
         dinp = flatten_layer.backward(dx)
         # The error should be around 1e-9
         print("dx Error: ", rel_error(dx_num, dx))
         # The errors should be around 1e-10
         print("dw Error: ", rel_error(dw_num, dw))
         print("db Error: ", rel_error(db_num, db))
         # The shapes should be same
         print("dinp Shape: ", dinp.shape, inp.shape)
         dx Error: 1.9504093719045184e-09
         dw Error: 1.305653099063425e-09
         db Error: 3.4065475213175096e-10
         dinp Shape: (15, 2, 2, 3) (15, 2, 2, 3)
```

#### GeLU Forward [2pt]

In the class skeleton gelu in lib/mlp/layer\_utils.py , please complete the forward pass.

GeLU is a smooth version of ReLU and it's used in pre-training LLMs such as GPT-3 and BERT.

$${
m GeLU}(x) = x\Phi(x) pprox 0.5 x (1 + anh(\sqrt{2/\pi}(x + 0.044715 x^3)))$$

Where  $\Phi(x)$  is the CDF for standard Gaussian random variables. You should use the approximate version to compute forward and backward pass.

Difference: 1.8037541876132445e-08

#### GeLU Backward [2pt]

Please complete the backward pass of the class gelu.

```
In [131... %reload_ext autoreload

# Test the relu backward function
x = np.random.randn(15, 15)
dout = np.random.randn(*x.shape)
gelu_b = gelu(name="gelu_b")

dx_num = eval_numerical_gradient_array(lambda x: gelu_b.forward(x), x, dout)

out = gelu_b.forward(x)
dx = gelu_b.backward(dout)

# The error should not be larger than 1e-4, since we are using an approximat print ("dx Error: ", rel_error(dx_num, dx))
```

dx Error: 9.494869310277931e-06

## Dropout Forward [2pt]

In the class dropout in lib/mlp/layer\_utils.py , please complete the forward pass.

Remember that the dropout is **only applied during training phase**, you should pay attention to this while implementing the function.

Important Note1: The probability argument input to the function is the "keep probability": probability that each activation is kept.

Important Note2: If the keep\_prob is set to 1, make it as no dropout.

```
In [174... %reload ext autoreload
        x = np.random.randn(100, 100) + 5.0
        print ("-----")
        for p in [0, 0.25, 0.50, 0.75, 1]:
            dropout f = dropout(keep prob=p)
            out = dropout f.forward(x, True)
            out_test = dropout_f.forward(x, False)
            # Mean of output should be similar to mean of input
            # Means of output during training time and testing time should be simila
            print ("Dropout Keep Prob = ", p)
            print ("Mean of input: ", x.mean())
            print ("Mean of output during training time: ", out.mean())
            print ("Mean of output during testing time: ", out_test.mean())
            print ("Fraction of output set to zero during training time: ", (out ==
            print ("Fraction of output set to zero during testing time: ", (out_test
            print ("-----
        Dropout Keep Prob = 0
        Mean of input: 4.992070685477767
        Mean of output during training time: 4.992070685477767
```

```
Mean of output during testing time: 4.992070685477767
Fraction of output set to zero during training time: 0.0
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 0.25
Mean of input: 4.992070685477767
Mean of output during training time: 4.804921321508981
Mean of output during testing time: 4.992070685477767
Fraction of output set to zero during training time: 0.7594
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 0.5
Mean of input: 4.992070685477767
Mean of output during training time: 4.940950244843111
Mean of output during testing time: 4.992070685477767
Fraction of output set to zero during training time: 0.5047
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 0.75
Mean of input: 4.992070685477767
Mean of output during training time: 5.072558378654152
Mean of output during testing time: 4.992070685477767
Fraction of output set to zero during training time: 0.2378
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 1
Mean of input: 4.992070685477767
Mean of output during training time: 4.992070685477767
Mean of output during testing time: 4.992070685477767
Fraction of output set to zero during training time: 0.0
Fraction of output set to zero during testing time: 0.0
```

#### **Dropout Backward [2pt]**

Please complete the backward pass. Again remember that the dropout is only applied during training phase, handle this in the backward pass as well.

```
In [180... %reload_ext autoreload

x = np.random.randn(5, 5) + 5
dout = np.random.randn(*x.shape)

keep_prob = 0.75
dropout_b = dropout(keep_prob, seed=100)
out = dropout_b.forward(x, True, seed=1)
dx = dropout_b.backward(dout)
dx_num = eval_numerical_gradient_array(lambda xx: dropout_b.forward(xx, True)

# The error should not be larger than 1e-10
print ('dx relative error: ', rel_error(dx, dx_num))
```

dx relative error: 3.003117874135711e-11

#### Testing cascaded layers: FC + GeLU [2pt]

Please find the TestFCGeLU function in lib/mlp/fully\_conn.py.

You only need to complete a few lines of code in the TODO block.

Please design an Flatten  $\rightarrow$  FC  $\rightarrow$  GeLU network where the parameters of them match the given x, w, and b.

Please insert the corresponding names you defined for each layer to param\_name\_w, and param\_name\_b respectively. Here you only modify the param\_name part, the \_w , and \_b are automatically assigned during network setup

```
In [254... %reload ext autoreload
      x = np.random.randn(3, 5, 3) # the input features
      w = np.random.randn(15, 5) # the weight of fc layer
                         # the bias of fc layer
      b = np.random.randn(5)
      dout = np.random.randn(3, 5) # the gradients to the output, notice the shape
      tiny_net = TestFCGeLU()
      # TODO: param name should be replaced accordingly #
      tiny_net.net.assign("fc1_w", w)
      tiny net.net.assign("fc1 b", b)
      END OF YOUR CODE
      out = tiny_net.forward(x)
      dx = tiny_net.backward(dout)
      # TODO: param name should be replaced accordingly #
      dw = tiny_net.net.get_grads("fc1_w")
      db = tiny net.net.get grads("fc1 b")
      END OF YOUR CODE
      dx_num = eval_numerical_gradient_array(lambda x: tiny_net.forward(x), x, dou
      dw_num = eval_numerical_gradient_array(lambda w: tiny_net.forward(x), w, dou
      db_num = eval_numerical_gradient_array(lambda b: tiny_net.forward(x), b, dou
      # The errors should not be larger than 1e-7
      print ("dx error: ", rel_error(dx_num, dx))
      print ("dw error: ", rel_error(dw_num, dw))
      print ("db error: ", rel_error(db_num, db))
       (3, 15)
       (3, 5)
       (15,)
       (5,)
       (15, 5)
       dx error: 4.8513086385104e-06
       dw error: 3.8899350805416096e-05
       db error: 3.421977219350853e-05
```

#### SoftMax Function and Loss Layer [2pt]

In the lib/mlp/layer\_utils.py, please first complete the function softmax, which will be used in the function cross\_entropy. Then, implement corss\_entropy using softmax. Please refer to the lecture slides of the mathematical expressions of the cross entropy loss function, and complete its forward pass and backward pass. You should also take care of size\_average on whether or not to divide by the batch size.

```
In [238... %reload_ext autoreload

num_classes, num_inputs = 6, 100
x = 0.001 * np.random.randn(num_inputs, num_classes)
y = np.random.randint(num_classes, size=num_inputs)

test_loss = cross_entropy()

dx_num = eval_numerical_gradient(lambda x: test_loss.forward(x, y), x, verbout

loss = test_loss.forward(x, y)
dx = test_loss.backward()

# Test softmax_loss function. Loss should be around 1.792
# and dx error should be at the scale of 1e-8 (or smaller)
print ("Cross Entropy Loss: ", loss)
print ("dx error: ", rel_error(dx_num, dx))

Cross Entropy Loss: 1.7918038711988697
```

### Test a Small Fully Connected Network [2pt]

dx error: 8.063889982552038e-09

Please find the SmallFullyConnectedNetwork function in lib/mlp /fully\_conn.py .

Again you only need to complete few lines of code in the TODO block.

Please design an FC --> GeLU --> FC network where the shapes of parameters match the given shapes.

Please insert the corresponding names you defined for each layer to param\_name\_w, and param\_name\_b respectively.

Here you only modify the param\_name part, the \_w , and \_b are automatically assigned during network setup.

```
In [315... %reload_ext autoreload
       seed = 1234
       np.random.seed(seed=seed)
       model = SmallFullyConnectedNetwork()
       loss_func = cross_entropy()
       N, D, = 4, 4 # N: batch size, D: input dimension
       H, C = 30, 7 # H: hidden dimension, C: output dimension
       std = 0.02
       x = np.random.randn(N, D)
       y = np.random.randint(C, size=N)
       print ("Testing initialization ... ")
       # TODO: param_name should be replaced accordingly #
       w1_std = abs(model.net.get_params("fc1_w").std() - std)
       b1 = model.net.get_params("fc1_b").std()
       w2_std = abs(model.net.get_params("fc2_w").std() - std)
       b2 = model.net.get params("fc2 b").std()
       END OF YOUR CODE
       assert w1_std < std / 10, "First layer weights do not seem right"</pre>
       assert np.all(b1 == 0), "First layer biases do not seem right"
       assert w2_std < std / 10, "Second layer weights do not seem right"</pre>
       assert np.all(b2 == 0), "Second layer biases do not seem right"
       print ("Passed!")
       print ("Testing test-time forward pass ... ")
       w1 = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
       w2 = np.linspace(-0.2, 0.2, num=H*C).reshape(H, C)
       b1 = np.linspace(-0.6, 0.2, num=H)
       b2 = np.linspace(-0.9, 0.1, num=C)
       # TODO: param_name should be replaced accordingly #
       model.net.assign("fc1_w", w1)
       model.net.assign("fc1_b", b1)
       model.net.assign("fc2 w", w2)
       model.net.assign("fc2_b", b2)
       END OF YOUR CODE
       feats = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
       scores = model.forward(feats)
       correct_scores = np.asarray([-2.33881897, -1.92174121, -1.50466344, -1.0875]
                               [-1.57214916, -1.1857013, -0.79925345, -0.4128]
                               [-0.80178618, -0.44604469, -0.0903032 , 0.2654
                               [-0.00331319, 0.32124836, 0.64580991, 0.9703]
```

```
scores_diff = np.sum(np.abs(scores - correct_scores))
assert scores_diff < 1e-6, "Your implementation might be wrong!"</pre>
print ("Passed!")
print ("Testing the loss ...",)
y = np.asarray([0, 5, 1, 4])
loss = loss_func.forward(scores, y)
dLoss = loss func.backward()
correct_loss = 2.4248995879903195
assert abs(loss - correct_loss) < 1e-10, "Your implementation might be wrong</pre>
print ("Passed!")
print ("Testing the gradients (error should be no larger than 1e-6) ...")
din = model.backward(dLoss)
for layer in model.net.layers:
    if not layer.params:
        continue
    for name in sorted(layer.grads):
        f = lambda _: loss_func.forward(model.forward(feats), y)
        grad num = eval numerical gradient(f, layer.params[name], verbose=Fa
        print ('%s relative error: %.2e' % (name, rel_error(grad_num, layer.
Testing initialization ...
Passed!
Testing test-time forward pass ...
Passed!
Testing the loss ...
Passed!
Testing the gradients (error should be no larger than 1e-6) ...
fc1_b relative error: 1.31e-08
fc1_w relative error: 3.53e-08
fc2_b relative error: 4.01e-10
fc2_w relative error: 2.50e-08
```

# Test a Fully Connected Network regularized with Dropout [2pt]

Please find the DropoutNet function in fully\_conn.py under lib/mlp directory.

For this part you don't need to design a new network, just simply run the following test code.

If something goes wrong, you might want to double check your dropout implementation.

```
In [321... %reload_ext autoreload
         seed = 1234
         np.random.seed(seed=seed)
         N, D, C = 3, 15, 10
         X = np.random.randn(N, D)
         y = np.random.randint(C, size=(N,))
         for keep_prob in [0, 0.25, 0.5]:
             np.random.seed(seed=seed)
             print ("Dropout p =", keep_prob)
             model = DropoutNet(keep_prob=keep_prob, seed=seed)
             loss_func = cross_entropy()
             output = model.forward(X, True, seed=seed)
             loss = loss_func.forward(output, y)
             dLoss = loss func.backward()
             dX = model.backward(dLoss)
             grads = model.net.grads
             print ("Error of gradients should be around or less than 1e-3")
             for name in sorted(grads):
                 if name not in model.net.params.keys():
                     continue
                 f = lambda _: loss_func.forward(model.forward(X, True, seed=seed), y
                 grad_num = eval_numerical_gradient(f, model.net.params[name], verbos
                 print ("{} relative error: {}".format(name, rel error(grad num, grad
             print ()
         Dropout p = 0
         Error of gradients should be around or less than 1e-3
         fc1_b relative error: 2.8516549520063804e-07
         fc1_w relative error: 3.7626907492775348e-06
         fc2_b relative error: 1.3390330536574157e-08
         fc2_w relative error: 3.0874875391929026e-05
         fc3_b relative error: 3.171994103778982e-10
         fc3_w relative error: 2.0488862038376876e-06
         Dropout p = 0.25
         Error of gradients should be around or less than 1e-3
         fc1_b relative error: 3.2230322968934954e-07
         fc1_w relative error: 2.784401997035636e-06
         fc2_b relative error: 1.4909849738002271e-07
         fc2_w relative error: 3.137678111642197e-05
         fc3_b relative error: 6.679255248099083e-11
         fc3_w relative error: 4.7620800542288264e-07
         Dropout p = 0.5
         Error of gradients should be around or less than 1e-3
         fc1_b relative error: 9.415776886825045e-07
         fc1_w relative error: 1.0482378081948802e-06
         fc2_b relative error: 1.3366658240212987e-08
         fc2_w relative error: 8.895735135607248e-06
         fc3_b relative error: 4.395930640345582e-10
```

fc3\_w relative error: 4.179186917341344e-06

#### Training a Network

In this section, we defined a TinyNet class for you to fill in the TODO block in lib/mlp/fully\_conn.py .

- Here please design a two layer fully connected network with Leaky ReLU activation (Flatten --> FC --> GeLU --> FC).
- You can adjust the number of hidden neurons, batch\_size, epochs, and learning rate decay parameters.
- Please read the lib/train.py carefully and complete the TODO blocks in the train\_net function first. Codes in "Test a Small Fully Connected Network" can be helpful.
- Implement SGD in lib/optim.py , you will be asked to complete weight decay and Adam in the later sections.

```
In [317... # Arrange the data
data_dict = {
    "data_train": (data["data_train"], data["labels_train"]),
    "data_val": (data["data_val"], data["labels_val"]),
    "data_test": (data["data_test"], data["labels_test"])
}

In [318... print("Data shape:", data["data_train"].shape)
print("Flattened data input size:", np.prod(data["data_train"].shape[1:]))
print("Number of data classes:", max(data['labels_train']) + 1)

Data shape: (40000, 32, 32, 3)
Flattened data input size: 3072
Number of data classes: 20
```

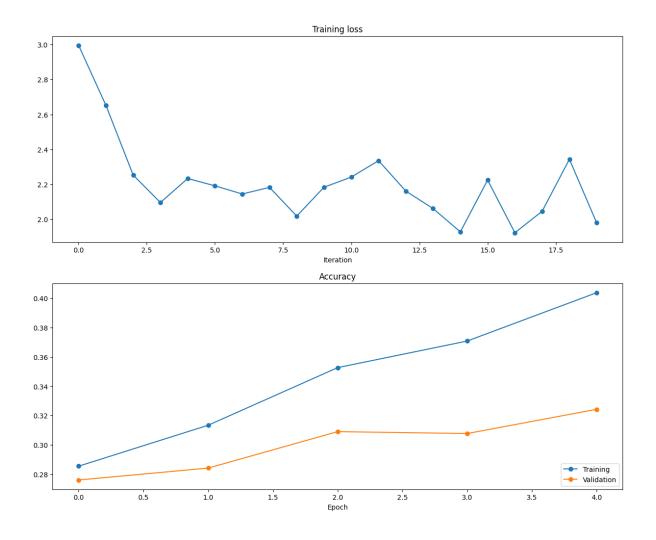
# Now train the network to achieve at least 30% validation accuracy [5pt]

You may only adjust the hyperparameters inside the TODO block

```
In [319... %autoreload
```

```
In [333...
       %reload ext autoreload
       seed = 123
       np.random.seed(seed=seed)
       model = TinyNet()
       loss_f = cross_entropy()
       optimizer = SGD(model.net, 0.1)
       results = None
       # TODO: Use the train_net function you completed to train a network
       batch size = 100
       epochs = 5
       lr decay = 0.99
       lr_decay_every = 100
       END OF YOUR CODE
       results = train_net(data_dict, model, loss_f, optimizer, batch_size, epochs,
                        lr_decay, lr_decay_every, show_every=10000, verbose=True
       opt_params, loss_hist, train_acc_hist, val_acc_hist = results
         2%|
        7/400 [00:00<00:11, 34.32it/s]
       (Iteration 1 / 2000) Average loss: 2.995905928014045
       100%
                                                       | || 400/400 [00:11
       <00:00, 35.98it/s]
       (Epoch 1 / 5) Training Accuracy: 0.2856, Validation Accuracy: 0.2762
       100%
                                                        || 400/400 [00:10
       <00:00, 37.39it/s]
       (Epoch 2 / 5) Training Accuracy: 0.31345, Validation Accuracy: 0.2843
       100%|
                                                       || 400/400 [00:11
       <00:00, 34.19it/s]
       (Epoch 3 / 5) Training Accuracy: 0.352625, Validation Accuracy: 0.3091
       100%
                                                        || 400/400 [00:10
       <00:00, 36.61it/s]
       (Epoch 4 / 5) Training Accuracy: 0.370725, Validation Accuracy: 0.3078
       100%|
                                                        || 400/400 [00:10
       <00:00, 39.43it/s]
       (Epoch 5 / 5) Training Accuracy: 0.403625, Validation Accuracy: 0.3243
       # Take a look at what names of params were stored
In [334...
       print (opt_params.keys())
       dict_keys(['fc1_w', 'fc1_b', 'fc2_w', 'fc2_b'])
```

```
In [335...
         # Demo: How to load the parameters to a newly defined network
         model = TinyNet()
         model.net.load(opt_params)
         val_acc = compute_acc(model, data["data_val"], data["labels_val"])
         print ("Validation Accuracy: {}%".format(val_acc*100))
         test_acc = compute_acc(model, data["data_test"], data["labels_test"])
         print ("Testing Accuracy: {}%".format(test_acc*100))
         Loading Params: fc1_w Shape: (3072, 300)
         Loading Params: fc1_b Shape: (300,)
         Loading Params: fc2_w Shape: (300, 20)
         Loading Params: fc2_b Shape: (20,)
         Validation Accuracy: 32.43%
         Testing Accuracy: 32.09%
In [336…  # Plot the learning curves
         plt.subplot(2, 1, 1)
         plt.title('Training loss')
         loss_hist_ = loss_hist[1::100] # sparse the curve a bit
         plt.plot(loss_hist_, '-o')
         plt.xlabel('Iteration')
         plt.subplot(2, 1, 2)
         plt.title('Accuracy')
         plt.plot(train_acc_hist, '-o', label='Training')
         plt.plot(val_acc_hist, '-o', label='Validation')
         plt.xlabel('Epoch')
         plt.legend(loc='lower right')
         plt.gcf().set_size_inches(15, 12)
         plt.show()
```



# Different Optimizers and Regularization Techniques

There are several more advanced optimizers than vanilla SGD, and there are many regularization tricks. You'll implement them in this section. Please complete the TODOs in the lib/optim.py.

# SGD + Weight Decay [2pt]

The update rule of SGD plus weigh decay is as shown below:

$$heta_{t+1} = heta_t - \eta 
abla_{ heta} J( heta_t) - \lambda heta_t$$

Update the SGD() function in lib/optim.py, and also incorporate weight decay options.

```
In [387... %reload ext autoreload
         # Test the implementation of SGD with Momentum
         seed = 1234
         np.random.seed(seed=seed)
         N, D = 4, 5
         test_sgd = sequential(fc(N, D, name="sgd_fc"))
         w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
         dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
         test_sgd.layers[0].params = {"sgd_fc_w": w}
         test_sgd.layers[0].grads = {"sgd_fc_w": dw}
         test\_sgd\_wd = SGD(test\_sgd, 1e-3, 1e-4)
         test_sgd_wd.step()
         updated w = test sqd.layers[0].params["sqd fc w"]
         expected_updated_w = np.asarray([
                [-0.39936, -0.34678632, -0.29421263, -0.24163895, -0.18906526],
                [-0.13649158, -0.08391789, -0.03134421, 0.02122947, 0.07380316],
                [ 0.12637684, 0.17895053, 0.23152421, 0.28409789, 0.33667158],
                [ 0.38924526, 0.44181895, 0.49439263, 0.54696632, 0.59954
         print ('The following errors should be around or less than 1e-6')
         print ('updated_w error: ', rel_error(updated_w, expected_updated_w))
```

The following errors should be around or less than 1e-6 updated\_w error: 8.677112905190533e-08

#### Comparing SGD and SGD with Weight Decay [2pt]

Run the following code block to train a multi-layer fully connected network with both SGD and SGD plus Weight Decay. You are expected to see Weight Decay have better validation accuracy than vinilla SGD.

```
In [388...] seed = 1234
         # Arrange a small data
         num train = 20000
         small data dict = {
             "data_train": (data["data_train"][:num_train], data["labels_train"][:num
             "data_val": (data["data_val"], data["labels_val"]),
             "data_test": (data["data_test"], data["labels_test"])
         }
         reset_seed(seed=seed)
         model_sgd = FullyConnectedNetwork()
         loss_f_sgd = cross_entropy()
         optimizer_sgd = SGD(model_sgd.net, 0.01)
         print ("Training with Vanilla SGD...")
         results_sgd = train_net(small_data_dict, model_sgd, loss_f_sgd, optimizer_sg
                                 max_epochs=50, show_every=10000, verbose=True)
         reset_seed(seed=seed)
         model_sgdw = FullyConnectedNetwork()
         loss_f_sgdw
                        = cross entropy()
         optimizer_sgdw = SGD(model_sgdw.net, 0.01, 1e-4)
         print ("\nTraining with SGD plus Weight Decay...")
         results_sgdw = train_net(small_data_dict, model_sgdw, loss_f_sgdw, optimizer
                                  max_epochs=50, show_every=10000, verbose=True)
         opt_params_sgd, loss_hist_sgd, train_acc_hist_sgd, val_acc_hist_sgd = re
         opt_params_sgdw, loss_hist_sgdw, train_acc_hist_sgdw, val_acc_hist_sgdw = re
         plt.subplot(3, 1, 1)
         plt.title('Training loss')
         plt.xlabel('Iteration')
         plt.subplot(3, 1, 2)
         plt.title('Training accuracy')
         plt.xlabel('Epoch')
         plt.subplot(3, 1, 3)
         plt.title('Validation accuracy')
         plt.xlabel('Epoch')
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_sgd, 'o', label="Vanilla SGD")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_sgd, '-o', label="Vanilla SGD")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_sgd, '-o', label="Vanilla SGD")
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_sgdw, 'o', label="SGD with Weight Decay")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
         for i in [1, 2, 3]:
```

```
plt.subplot(3, 1, i)
  plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
Training with Vanilla SGD...
  2%|
| 4/200 [00:00<00:05, 36.33it/s]
(Iteration 1 / 10000) Average loss: 3.3332154539088985
100%|
                                                          1 200/200 [00:05
<00:00, 38.18it/s]
(Epoch 1 / 50) Training Accuracy: 0.15095, Validation Accuracy: 0.1474
100%|
                                                           || 200/200 [00:05
<00:00, 38.79it/s]
(Epoch 2 / 50) Training Accuracy: 0.18815, Validation Accuracy: 0.1805
100%
                                                          || 200/200 [00:04
<00:00, 43.05it/s]
(Epoch 3 / 50) Training Accuracy: 0.2107, Validation Accuracy: 0.2029
100%
                                                          || 200/200 [00:04
<00:00, 43.72it/s]
(Epoch 4 / 50) Training Accuracy: 0.2314, Validation Accuracy: 0.212
100%
                                                           || 200/200 [00:04
<00:00, 40.66it/s]
(Epoch 5 / 50) Training Accuracy: 0.23915, Validation Accuracy: 0.2197
100%|
                                                          || 200/200 [00:04
<00:00, 40.94it/s]
(Epoch 6 / 50) Training Accuracy: 0.2552, Validation Accuracy: 0.2298
100%
                                                          || 200/200 [00:05
<00:00, 39.87it/s]
(Epoch 7 / 50) Training Accuracy: 0.26645, Validation Accuracy: 0.2403
100%
                                                           || 200/200 [00:05
<00:00, 38.36it/s]
(Epoch 8 / 50) Training Accuracy: 0.27555, Validation Accuracy: 0.2414
100%
                                                          1 200/200 [00:04
<00:00, 41.67it/s]
(Epoch 9 / 50) Training Accuracy: 0.28185, Validation Accuracy: 0.2413
100%
                                                           || 200/200 [00:05
<00:00, 38.99it/s]
(Epoch 10 / 50) Training Accuracy: 0.2944, Validation Accuracy: 0.252
100%
                                                           || 200/200 [00:05
<00:00, 38.98it/s]
(Epoch 11 / 50) Training Accuracy: 0.29735, Validation Accuracy: 0.2543
```

```
100%
                                                           || 200/200 [00:05
<00:00, 37.02it/s]
(Epoch 12 / 50) Training Accuracy: 0.3021, Validation Accuracy: 0.2587
100%
                                                           || 200/200 [00:05
<00:00, 39.30it/s]
(Epoch 13 / 50) Training Accuracy: 0.31105, Validation Accuracy: 0.2641
                                                          1 200/200 [00:05
<00:00, 39.42it/s]
(Epoch 14 / 50) Training Accuracy: 0.3168, Validation Accuracy: 0.2653
100%|
                                                           || 200/200 [00:04
<00:00, 40.24it/s]
(Epoch 15 / 50) Training Accuracy: 0.3217, Validation Accuracy: 0.2681
100%|
                                                          1 200/200 [00:05
<00:00, 36.73it/s]
(Epoch 16 / 50) Training Accuracy: 0.3307, Validation Accuracy: 0.2699
100%
                                                           I| 200/200 [00:04
<00:00, 42.35it/s]
(Epoch 17 / 50) Training Accuracy: 0.33835, Validation Accuracy: 0.2696
100%
                                                           || 200/200 [00:05
<00:00, 37.66it/s]
(Epoch 18 / 50) Training Accuracy: 0.34565, Validation Accuracy: 0.2737
100%
                                                          1 200/200 [00:05
<00:00, 39.06it/s]
(Epoch 19 / 50) Training Accuracy: 0.3495, Validation Accuracy: 0.2729
100%
                                                           || 200/200 [00:04
<00:00, 43.14it/s]
(Epoch 20 / 50) Training Accuracy: 0.35565, Validation Accuracy: 0.2758
100%
                                                          ■| 200/200 [00:04
<00:00, 40.17it/s]
(Epoch 21 / 50) Training Accuracy: 0.35825, Validation Accuracy: 0.2729
100%
                                                           I| 200/200 [00:04
<00:00, 43.70it/s]
(Epoch 22 / 50) Training Accuracy: 0.36895, Validation Accuracy: 0.278
100%
                                                          1 200/200 [00:04
<00:00, 44.87it/s]
(Epoch 23 / 50) Training Accuracy: 0.3734, Validation Accuracy: 0.2783
100%
                                                           1| 200/200 [00:04
<00:00, 44.39it/s]
(Epoch 24 / 50) Training Accuracy: 0.3756, Validation Accuracy: 0.2768
```

```
100%
                                                           || 200/200 [00:04
<00:00, 44.21it/s]
(Epoch 25 / 50) Training Accuracy: 0.38495, Validation Accuracy: 0.278
100%
                                                           || 200/200 [00:04
<00:00, 43.90it/s]
(Epoch 26 / 50) Training Accuracy: 0.38415, Validation Accuracy: 0.2757
                                                          1 200/200 [00:04
<00:00, 45.72it/s]
(Epoch 27 / 50) Training Accuracy: 0.40365, Validation Accuracy: 0.2804
100%|
                                                           || 200/200 [00:04
<00:00, 44.90it/s]
(Epoch 28 / 50) Training Accuracy: 0.40105, Validation Accuracy: 0.2812
100%|
                                                          1 200/200 [00:04
<00:00, 42.99it/s]
(Epoch 29 / 50) Training Accuracy: 0.40885, Validation Accuracy: 0.2773
100%
                                                           I| 200/200 [00:04
<00:00, 44.80it/s]
(Epoch 30 / 50) Training Accuracy: 0.4163, Validation Accuracy: 0.2803
100%
                                                           || 200/200 [00:04
<00:00, 43.43it/s]
(Epoch 31 / 50) Training Accuracy: 0.41745, Validation Accuracy: 0.2838
100%
                                                          1 200/200 [00:04
<00:00, 43.89it/s]
(Epoch 32 / 50) Training Accuracy: 0.42125, Validation Accuracy: 0.2758
100%
                                                           || 200/200 [00:04
<00:00, 45.21it/s]
(Epoch 33 / 50) Training Accuracy: 0.433, Validation Accuracy: 0.2777
100%
                                                          ■| 200/200 [00:04
<00:00, 44.35it/s]
(Epoch 34 / 50) Training Accuracy: 0.4322, Validation Accuracy: 0.2782
100%
                                                           || 200/200 [00:04
<00:00, 45.04it/s]
(Epoch 35 / 50) Training Accuracy: 0.44095, Validation Accuracy: 0.2753
100%
                                                          1 200/200 [00:05
<00:00, 36.53it/s]
(Epoch 36 / 50) Training Accuracy: 0.4517, Validation Accuracy: 0.2783
100%
                                                           I| 200/200 [00:05
<00:00, 38.36it/s]
(Epoch 37 / 50) Training Accuracy: 0.4583, Validation Accuracy: 0.2759
```

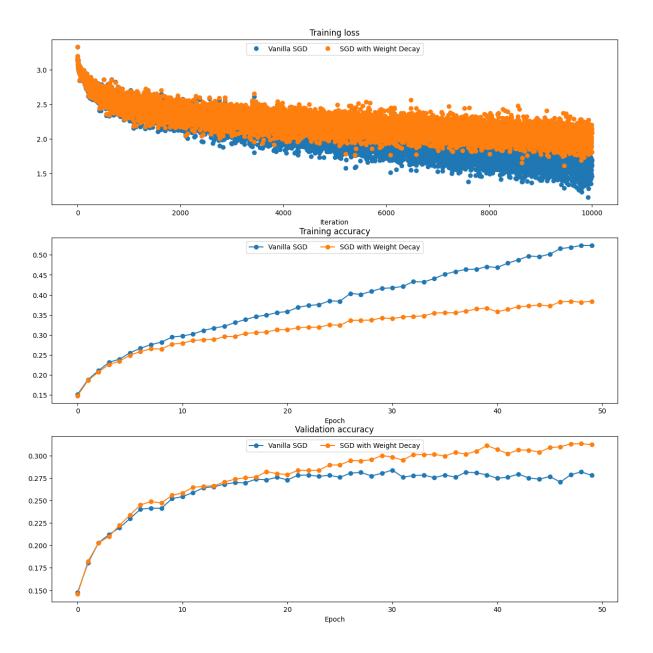
```
100%
                                                          1 200/200 [00:04
<00:00, 44.19it/s]
(Epoch 38 / 50) Training Accuracy: 0.4637, Validation Accuracy: 0.2815
100%
                                                           || 200/200 [00:04
<00:00, 43.82it/s]
(Epoch 39 / 50) Training Accuracy: 0.4642, Validation Accuracy: 0.2808
                                                          1 200/200 [00:04
<00:00, 43.95it/s]
(Epoch 40 / 50) Training Accuracy: 0.47055, Validation Accuracy: 0.2784
100%|
                                                           || 200/200 [00:04
<00:00, 45.51it/s]
(Epoch 41 / 50) Training Accuracy: 0.4684, Validation Accuracy: 0.2747
100%
                                                         1 200/200 [00:04
<00:00, 44.57it/s]
(Epoch 42 / 50) Training Accuracy: 0.4795, Validation Accuracy: 0.2758
100%
                                                          || 200/200 [00:04
<00:00, 44.58it/s]
(Epoch 43 / 50) Training Accuracy: 0.48745, Validation Accuracy: 0.2793
100%
                                                           || 200/200 [00:06
<00:00, 28.77it/s]
(Epoch 44 / 50) Training Accuracy: 0.49715, Validation Accuracy: 0.2751
100%
                                                         1 200/200 [00:04
<00:00, 40.55it/s]
(Epoch 45 / 50) Training Accuracy: 0.49545, Validation Accuracy: 0.2736
100%
                                                           1 200/200 [00:04
<00:00, 44.43it/s]
(Epoch 46 / 50) Training Accuracy: 0.50175, Validation Accuracy: 0.2767
100%
                                                          ■| 200/200 [00:04
<00:00, 41.30it/s]
(Epoch 47 / 50) Training Accuracy: 0.51565, Validation Accuracy: 0.2704
100%
                                                          || 200/200 [00:04
<00:00, 41.41it/s]
(Epoch 48 / 50) Training Accuracy: 0.51875, Validation Accuracy: 0.2786
100%
                                                          1 200/200 [00:04
<00:00, 44.06it/s]
(Epoch 49 / 50) Training Accuracy: 0.5235, Validation Accuracy: 0.2818
100%
                                                           || 200/200 [00:04
<00:00, 43.66it/s]
```

```
(Epoch 50 / 50) Training Accuracy: 0.52375, Validation Accuracy: 0.2779
Training with SGD plus Weight Decay...
  2%||
| 5/200 [00:00<00:04, 39.19it/s]
(Iteration 1 / 10000) Average loss: 3.3332154539088985
100%
                                                         ■| 200/200 [00:04
<00:00, 44.33it/s]
(Epoch 1 / 50) Training Accuracy: 0.148, Validation Accuracy: 0.1458
100%
                                                         1 200/200 [00:04
<00:00, 45.29it/s]
(Epoch 2 / 50) Training Accuracy: 0.186, Validation Accuracy: 0.1822
100%
                                                         1 200/200 [00:04
<00:00, 44.43it/s]
(Epoch 3 / 50) Training Accuracy: 0.2073, Validation Accuracy: 0.2027
100%
                                                          || 200/200 [00:04
<00:00, 44.87it/s]
(Epoch 4 / 50) Training Accuracy: 0.22575, Validation Accuracy: 0.2101
                                                          1| 200/200 [00:04
<00:00, 43.09it/s]
(Epoch 5 / 50) Training Accuracy: 0.2345, Validation Accuracy: 0.2223
100%
                                                          1| 200/200 [00:04
<00:00, 45.21it/s]
(Epoch 6 / 50) Training Accuracy: 0.24915, Validation Accuracy: 0.2338
100%|
                                                         1 200/200 [00:04
<00:00, 42.50it/s]
(Epoch 7 / 50) Training Accuracy: 0.2584, Validation Accuracy: 0.2451
100%
<00:00, 45.03it/s]
(Epoch 8 / 50) Training Accuracy: 0.2651, Validation Accuracy: 0.2488
100%
                                                          || 200/200 [00:04
<00:00, 45.10it/s]
(Epoch 9 / 50) Training Accuracy: 0.2648, Validation Accuracy: 0.2471
100%|
                                                200/200 [00:04
<00:00, 45.65it/s]
(Epoch 10 / 50) Training Accuracy: 0.27685, Validation Accuracy: 0.2558
100%
                                                          1 200/200 [00:04
<00:00, 44.38it/s]
(Epoch 11 / 50) Training Accuracy: 0.2792, Validation Accuracy: 0.2583
```

```
100%
                                                          1| 200/200 [00:04
<00:00, 44.89it/s]
(Epoch 12 / 50) Training Accuracy: 0.28575, Validation Accuracy: 0.2646
100%
                                                           || 200/200 [00:04
<00:00, 42.03it/s]
(Epoch 13 / 50) Training Accuracy: 0.2879, Validation Accuracy: 0.2657
                                                          1 200/200 [00:04
<00:00, 44.53it/s]
(Epoch 14 / 50) Training Accuracy: 0.28865, Validation Accuracy: 0.2664
100%|
                                                           || 200/200 [00:04
<00:00, 43.64it/s]
(Epoch 15 / 50) Training Accuracy: 0.29545, Validation Accuracy: 0.2705
100%|
                                                          1 200/200 [00:04
<00:00, 45.96it/s]
(Epoch 16 / 50) Training Accuracy: 0.2964, Validation Accuracy: 0.2737
100%
                                                           I| 200/200 [00:04
<00:00, 43.23it/s]
(Epoch 17 / 50) Training Accuracy: 0.30345, Validation Accuracy: 0.2752
100%
                                                           || 200/200 [00:04
<00:00, 44.38it/s]
(Epoch 18 / 50) Training Accuracy: 0.30555, Validation Accuracy: 0.276
100%
                                                          1 200/200 [00:05
<00:00, 39.58it/s]
(Epoch 19 / 50) Training Accuracy: 0.30715, Validation Accuracy: 0.2821
100%
                                                           || 200/200 [00:04
<00:00, 40.10it/s]
(Epoch 20 / 50) Training Accuracy: 0.31265, Validation Accuracy: 0.2799
100%
                                                           1 200/200 [00:05
<00:00, 37.24it/s]
(Epoch 21 / 50) Training Accuracy: 0.31315, Validation Accuracy: 0.2787
100%
                                                           I| 200/200 [00:04
<00:00, 45.16it/s]
(Epoch 22 / 50) Training Accuracy: 0.31755, Validation Accuracy: 0.2836
100%
                                                          1 200/200 [00:04
<00:00, 43.65it/s]
(Epoch 23 / 50) Training Accuracy: 0.3192, Validation Accuracy: 0.2833
100%
                                                           I| 200/200 [00:04
<00:00, 44.69it/s]
(Epoch 24 / 50) Training Accuracy: 0.31905, Validation Accuracy: 0.2837
```

```
100%
                                                          || 200/200 [00:04
<00:00, 45.24it/s]
(Epoch 25 / 50) Training Accuracy: 0.32525, Validation Accuracy: 0.2894
100%
                                                           | 200/200 [00:04
<00:00, 43.91it/s]
(Epoch 26 / 50) Training Accuracy: 0.3238, Validation Accuracy: 0.2895
                                                          1 200/200 [00:04
<00:00, 43.99it/s]
(Epoch 27 / 50) Training Accuracy: 0.33645, Validation Accuracy: 0.2944
100%|
                                                           || 200/200 [00:04
<00:00, 44.88it/s]
(Epoch 28 / 50) Training Accuracy: 0.33645, Validation Accuracy: 0.2941
100%
                                                          1 200/200 [00:05
<00:00, 38.44it/s]
(Epoch 29 / 50) Training Accuracy: 0.33695, Validation Accuracy: 0.2953
100%
                                                           I| 200/200 [00:04
<00:00, 40.46it/s]
(Epoch 30 / 50) Training Accuracy: 0.3425, Validation Accuracy: 0.3
100%
                                                           || 200/200 [00:04
<00:00, 43.56it/s]
(Epoch 31 / 50) Training Accuracy: 0.3406, Validation Accuracy: 0.2982
100%
                                                          1 200/200 [00:04
<00:00, 44.46it/s]
(Epoch 32 / 50) Training Accuracy: 0.34505, Validation Accuracy: 0.2949
100%
                                                           || 200/200 [00:04
<00:00, 44.13it/s]
(Epoch 33 / 50) Training Accuracy: 0.34595, Validation Accuracy: 0.3011
100%
                                                           1 200/200 [00:04
<00:00, 44.72it/s]
(Epoch 34 / 50) Training Accuracy: 0.34755, Validation Accuracy: 0.301
100%
                                                           || 200/200 [00:04
<00:00, 43.76it/s]
(Epoch 35 / 50) Training Accuracy: 0.3548, Validation Accuracy: 0.3012
100%
                                                          1 200/200 [00:04
<00:00, 48.02it/s]
(Epoch 36 / 50) Training Accuracy: 0.3552, Validation Accuracy: 0.2995
100%
                                                           I| 200/200 [00:05
<00:00, 39.97it/s]
(Epoch 37 / 50) Training Accuracy: 0.35525, Validation Accuracy: 0.3034
```

```
100%
                                                          1| 200/200 [00:04
<00:00, 44.58it/s]
(Epoch 38 / 50) Training Accuracy: 0.3593, Validation Accuracy: 0.3017
100%
                                                           || 200/200 [00:04
<00:00, 43.02it/s]
(Epoch 39 / 50) Training Accuracy: 0.3648, Validation Accuracy: 0.3048
                                                          1 200/200 [00:04
<00:00, 44.06it/s]
(Epoch 40 / 50) Training Accuracy: 0.36665, Validation Accuracy: 0.311
100%|
                                                           || 200/200 [00:04
<00:00, 45.64it/s]
(Epoch 41 / 50) Training Accuracy: 0.35765, Validation Accuracy: 0.3068
100%
                                                          1 200/200 [00:04
<00:00, 44.82it/s]
(Epoch 42 / 50) Training Accuracy: 0.36375, Validation Accuracy: 0.302
100%
                                                           || 200/200 [00:04
<00:00, 45.13it/s]
(Epoch 43 / 50) Training Accuracy: 0.3702, Validation Accuracy: 0.3062
100%
                                                           || 200/200 [00:04
<00:00, 44.98it/s]
(Epoch 44 / 50) Training Accuracy: 0.37215, Validation Accuracy: 0.306
100%
                                                          1 200/200 [00:04
<00:00, 45.08it/s]
(Epoch 45 / 50) Training Accuracy: 0.37475, Validation Accuracy: 0.3037
100%
                                                           || 200/200 [00:04
<00:00, 44.59it/s]
(Epoch 46 / 50) Training Accuracy: 0.37205, Validation Accuracy: 0.3089
100%
                                                          ■| 200/200 [00:04
<00:00, 42.72it/s]
(Epoch 47 / 50) Training Accuracy: 0.3827, Validation Accuracy: 0.3097
100%
                                                           || 200/200 [00:04
<00:00, 43.65it/s]
(Epoch 48 / 50) Training Accuracy: 0.38395, Validation Accuracy: 0.313
100%
                                                          1 200/200 [00:04
<00:00, 43.06it/s]
(Epoch 49 / 50) Training Accuracy: 0.38155, Validation Accuracy: 0.3131
100%
                                                           I| 200/200 [00:04
<00:00, 42.82it/s]
(Epoch 50 / 50) Training Accuracy: 0.38415, Validation Accuracy: 0.3121
```



# SGD with L1 Regularization [2pts]

With L1 Regularization, your regularized loss becomes  $ilde{J}_{\ell_1}( heta)$  and it's defined as

$${ ilde J}_{\ell_1}( heta) = J( heta) + \lambda \| heta\|_{\ell_1}$$

where

$$\| heta\|_{\ell_1} = \sum_{l=1}^n \sum_{k=1}^{n_l} | heta_{l,k}|$$

Please implmemt TODO block of apply\_l1\_regularization in lib/layer\_utils . Such regularization funcationality is called after gradient gathering in the backward process.

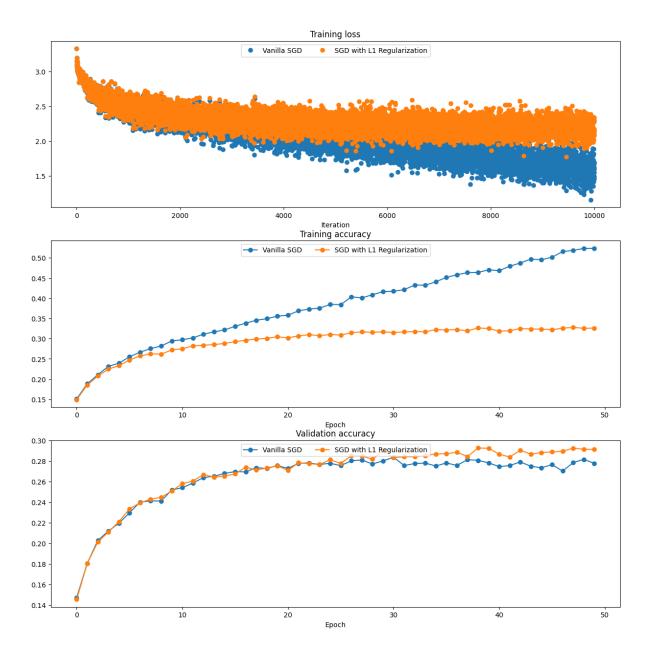
```
In [376... reset seed(seed=seed)
         model sqd l1
                       = FullyConnectedNetwork()
         loss_f_sgd_l1 = cross_entropy()
         optimizer sqd l1 = SGD(model sqd l1.net, 0.01)
         print ("\nTraining with SGD plus L1 Regularization...")
         results_sgd_l1 = train_net(small_data_dict, model_sgd_l1, loss_f_sgd_l1, opt
                                  max_epochs=50, show_every=10000, verbose=True, regu
         opt_params_sgd_l1, loss_hist_sgd_l1, train_acc_hist_sgd_l1, val_acc_hist_sgd
         plt.subplot(3, 1, 1)
         plt.title('Training loss')
         plt.xlabel('Iteration')
         plt.subplot(3, 1, 2)
         plt.title('Training accuracy')
         plt.xlabel('Epoch')
         plt.subplot(3, 1, 3)
         plt.title('Validation accuracy')
         plt.xlabel('Epoch')
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_sgd, 'o', label="Vanilla SGD")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_sgd, '-o', label="Vanilla SGD")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_sgd, '-o', label="Vanilla SGD")
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_sgd_l1, 'o', label="SGD with L1 Regularization")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
         for i in [1, 2, 3]:
           plt.subplot(3, 1, i)
           plt.legend(loc='upper center', ncol=4)
         plt.gcf().set_size_inches(15, 15)
         plt.show()
         Training with SGD plus L1 Regularization...
           0%|
         | 0/200 [00:00<?, ?it/s]/Users/chung/Documents/spring_2023/DeepLearningCS56
         6/csci566-assignment1/csci566-assignment1/lib/mlp/layer_utils.py:304: Runti
         meWarning: overflow encountered in cosh
           term3 = np.cosh(0.0356774*feat_3 + 0.797885*feat)**-2
           2%
         | 4/200 [00:00<00:05, 33.19it/s]
         (Iteration 1 / 10000) Average loss: 3.3332154539088985
         100%
                                                                    || 200/200 [00:04
         <00:00, 40.77it/s]
```

```
(Epoch 1 / 50) Training Accuracy: 0.1491, Validation Accuracy: 0.1457
100%
                                                           || 200/200 [00:05
<00:00. 37.20it/sl
(Epoch 2 / 50) Training Accuracy: 0.1854, Validation Accuracy: 0.1806
                                                         1 200/200 [00:06
<00:00, 30.69it/s]
(Epoch 3 / 50) Training Accuracy: 0.20755, Validation Accuracy: 0.2014
100%
                                                           || 200/200 [00:05
<00:00, 35.75it/s]
(Epoch 4 / 50) Training Accuracy: 0.22465, Validation Accuracy: 0.2111
100%
                                                     200/200 [00:05
<00:00, 38.44it/s]
(Epoch 5 / 50) Training Accuracy: 0.2331, Validation Accuracy: 0.2212
100%
<00:00, 38.27it/s]
(Epoch 6 / 50) Training Accuracy: 0.24735, Validation Accuracy: 0.2337
100%
                                                          1| 200/200 [00:05
<00:00, 37.20it/s]
(Epoch 7 / 50) Training Accuracy: 0.25725, Validation Accuracy: 0.2395
                                                         1 200/200 [00:05
<00:00, 34.28it/s]
(Epoch 8 / 50) Training Accuracy: 0.26245, Validation Accuracy: 0.2431
100%
                                                          1| 200/200 [00:07
<00:00, 25.69it/s]
(Epoch 9 / 50) Training Accuracy: 0.26185, Validation Accuracy: 0.2449
100%
                                                         1 200/200 [00:05
<00:00, 36.10it/s]
(Epoch 10 / 50) Training Accuracy: 0.27205, Validation Accuracy: 0.251
100% |
                                                         1 200/200 [00:05
<00:00, 39.47it/s]
(Epoch 11 / 50) Training Accuracy: 0.27515, Validation Accuracy: 0.2582
100%
                                                         1 200/200 [00:04
<00:00, 43.21it/s]
(Epoch 12 / 50) Training Accuracy: 0.28195, Validation Accuracy: 0.2606
100%
                                                         1 200/200 [00:05
<00:00, 39.19it/s]
(Epoch 13 / 50) Training Accuracy: 0.2838, Validation Accuracy: 0.267
100%|
                                                           | 200/200 [00:04
<00:00, 42.17it/s]
```

```
(Epoch 14 / 50) Training Accuracy: 0.28535, Validation Accuracy: 0.2645
100%
                                                           | 200/200 [00:05
<00:00. 33.87it/sl
(Epoch 15 / 50) Training Accuracy: 0.2883, Validation Accuracy: 0.2655
                                                         1 200/200 [00:04
<00:00, 41.03it/s]
(Epoch 16 / 50) Training Accuracy: 0.2926, Validation Accuracy: 0.2676
100%||
                                                           || 200/200 [00:05
<00:00, 39.06it/s]
(Epoch 17 / 50) Training Accuracy: 0.296, Validation Accuracy: 0.2742
                                                      200/200 [00:05
<00:00, 37.33it/s]
(Epoch 18 / 50) Training Accuracy: 0.2991, Validation Accuracy: 0.2715
100%
<00:00, 40.78it/s]
(Epoch 19 / 50) Training Accuracy: 0.30085, Validation Accuracy: 0.2734
100%
                                                          1| 200/200 [00:06
<00:00, 32.16it/s]
(Epoch 20 / 50) Training Accuracy: 0.30465, Validation Accuracy: 0.2756
                                                         1 200/200 [00:05
<00:00, 34.44it/s]
(Epoch 21 / 50) Training Accuracy: 0.30195, Validation Accuracy: 0.271
100%
                                                          ■| 200/200 [00:05
<00:00, 36.39it/s]
(Epoch 22 / 50) Training Accuracy: 0.3069, Validation Accuracy: 0.2785
100%
                                                          1 200/200 [00:05
<00:00, 39.00it/s]
(Epoch 23 / 50) Training Accuracy: 0.30985, Validation Accuracy: 0.2776
100% |
                                                          1 200/200 [00:05
<00:00, 37.38it/s]
(Epoch 24 / 50) Training Accuracy: 0.30745, Validation Accuracy: 0.2768
100%
                                                         1 200/200 [00:05
<00:00, 39.31it/s]
(Epoch 25 / 50) Training Accuracy: 0.3103, Validation Accuracy: 0.2814
100%
                                                         1 200/200 [04:46
<00:00, 1.43s/it]
(Epoch 26 / 50) Training Accuracy: 0.3091, Validation Accuracy: 0.2778
100%
                                                           | 200/200 [00:04
<00:00, 41.57it/s]
```

```
(Epoch 27 / 50) Training Accuracy: 0.31465, Validation Accuracy: 0.2853
100%
                                                           | 200/200 [00:05
<00:00. 35.12it/sl
(Epoch 28 / 50) Training Accuracy: 0.31695, Validation Accuracy: 0.2851
                                                         1 200/200 [00:05
<00:00, 37.88it/s]
(Epoch 29 / 50) Training Accuracy: 0.3157, Validation Accuracy: 0.2819
100%||
                                                           | 200/200 [00:05
<00:00, 38.20it/s]
(Epoch 30 / 50) Training Accuracy: 0.31705, Validation Accuracy: 0.2901
                                                     200/200 [00:05
<00:00, 37.45it/s]
(Epoch 31 / 50) Training Accuracy: 0.3152, Validation Accuracy: 0.2835
100%
<00:00, 38.64it/s]
(Epoch 32 / 50) Training Accuracy: 0.3168, Validation Accuracy: 0.2843
100%
                                                          1| 200/200 [00:05
<00:00, 38.88it/s]
(Epoch 33 / 50) Training Accuracy: 0.31745, Validation Accuracy: 0.2843
                                                         1 200/200 [00:04
<00:00, 40.17it/s]
(Epoch 34 / 50) Training Accuracy: 0.31705, Validation Accuracy: 0.2855
100%
                                                          ■| 200/200 [00:05
<00:00, 39.09it/s]
(Epoch 35 / 50) Training Accuracy: 0.32255, Validation Accuracy: 0.287
100%
                                                         1 200/200 [00:05
<00:00, 38.84it/s]
(Epoch 36 / 50) Training Accuracy: 0.3215, Validation Accuracy: 0.2873
100% |
                                                          1 200/200 [00:05
<00:00, 39.08it/s]
(Epoch 37 / 50) Training Accuracy: 0.3224, Validation Accuracy: 0.2887
100%
                                                         1 200/200 [00:05
<00:00, 38.59it/s]
(Epoch 38 / 50) Training Accuracy: 0.3196, Validation Accuracy: 0.2845
100%
                                                         1 200/200 [00:05
<00:00, 39.43it/s]
(Epoch 39 / 50) Training Accuracy: 0.32645, Validation Accuracy: 0.2928
100%|
                                                           | 200/200 [00:05
<00:00, 37.15it/s]
```

```
(Epoch 40 / 50) Training Accuracy: 0.3253, Validation Accuracy: 0.2926
100%
                                                          || 200/200 [00:05
<00:00. 38.90it/sl
(Epoch 41 / 50) Training Accuracy: 0.3185, Validation Accuracy: 0.2867
                                                         1 200/200 [00:05
<00:00, 38.52it/s]
(Epoch 42 / 50) Training Accuracy: 0.3197, Validation Accuracy: 0.2841
100%
                                                          || 200/200 [00:05
<00:00, 38.71it/s]
(Epoch 43 / 50) Training Accuracy: 0.32515, Validation Accuracy: 0.2906
100%
                                                    200/200 [00:05
<00:00, 38.77it/s]
(Epoch 44 / 50) Training Accuracy: 0.3239, Validation Accuracy: 0.2868
100%
                                                         1 200/200 [00:05
<00:00, 39.09it/s]
(Epoch 45 / 50) Training Accuracy: 0.3237, Validation Accuracy: 0.2884
100%
                                                          1 200/200 [00:05
<00:00, 39.20it/s]
(Epoch 46 / 50) Training Accuracy: 0.3223, Validation Accuracy: 0.289
                                                          1| 200/200 [00:05
<00:00, 38.34it/s]
(Epoch 47 / 50) Training Accuracy: 0.32585, Validation Accuracy: 0.2897
100%
                                                          ■| 200/200 [00:05
<00:00, 38.69it/s]
(Epoch 48 / 50) Training Accuracy: 0.32815, Validation Accuracy: 0.2927
100%
                                                         1 200/200 [00:05
<00:00, 39.20it/s]
(Epoch 49 / 50) Training Accuracy: 0.3257, Validation Accuracy: 0.2916
100%
                                                          ■| 200/200 [00:05
<00:00, 38.31it/s]
(Epoch 50 / 50) Training Accuracy: 0.32625, Validation Accuracy: 0.2915
```



# SGD with L2 Regularization [2pts]

With L2 Regularization, your regularized loss becomes  $ilde{J}_{\ell_2}( heta)$  and it's defined as

$${ ilde J}_{\ell_2}( heta) = J( heta) + \lambda \| heta\|_{\ell_2}^2$$

where

$$\| heta\|_{\ell_2}^2 = \sum_{l=1}^n \sum_{k=1}^{n_l} heta_{l,k}^2$$

Similarly, implmemt TODO block of apply\_l2\_regularization in lib/layer\_utils . For SGD, you're also asked to find the  $\lambda$  for L2 Regularization such that it achives the EXACTLY SAME effect as weight decay in the previous cells. As a reminder, learning rate is the same as previously, and the weight decay paramter was 1e-4.

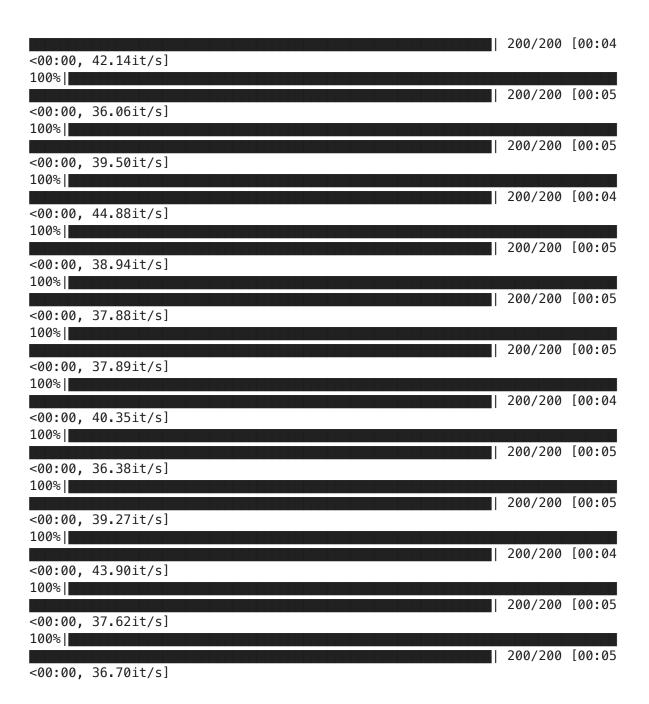
```
In [389... reset seed(seed=seed)
        model sqd l2 = FullyConnectedNetwork()
        loss_f_sgd_l2 = cross_entropy()
        optimizer sqd l2 = SGD(model sqd l2.net, 0.01)
        #### Find lambda for L2 regularization so that
                                                                     ####
        #### it achieves EXACTLY THE SAME learning curve as weight decay ####
        Since the gradient of updates for L2 is lr*(2*lam*w) and
        weight decay updates gradients by decay*w, we just need to find a lambda whe
        learning rate *2 * lambda = decay
        0.01*2*lam = 1e-4
        lam = 5e-3
        12 \quad lambda = 5e-3
        print ("\nTraining with SGD plus L2 Regularization...")
        results_sgd_l2 = train_net(small_data_dict, model_sgd_l2, loss_f_sgd_l2, opt
                                  max epochs=50, show every=10000, verbose=False, r
        opt_params_sgd_l2, loss_hist_sgd_l2, train_acc_hist_sgd_l2, val_acc_hist_sgd
        plt.subplot(3, 1, 1)
        plt.title('Training loss')
        plt.xlabel('Iteration')
        plt.subplot(3, 1, 2)
        plt.title('Training accuracy')
        plt.xlabel('Epoch')
        plt.subplot(3, 1, 3)
        plt.title('Validation accuracy')
        plt.xlabel('Epoch')
        plt.subplot(3, 1, 1)
        plt.plot(loss_hist_sgdw, 'o', label="SGD with Weight Decay")
        plt.subplot(3, 1, 2)
        plt.plot(train acc hist sqdw, '-o', label="SGD with Weight Decay")
        plt.subplot(3, 1, 3)
        plt.plot(val_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
        plt.subplot(3, 1, 1)
        plt.plot(loss_hist_sgd_l1, 'o', label="SGD with L1 Regularization")
        plt.subplot(3, 1, 2)
        plt.plot(train_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
        plt.subplot(3, 1, 3)
        plt.plot(val_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
        plt.subplot(3, 1, 1)
        plt.plot(loss_hist_sgd_l2, 'o', label="SGD with L2 Regularization")
        plt.subplot(3, 1, 2)
        plt.plot(train_acc_hist_sgd_l2, '-o', label="SGD with L2 Regularization")
        plt.subplot(3, 1, 3)
        plt.plot(val_acc_hist_sgd_l2, '-o', label="SGD with L2 Regularization")
```

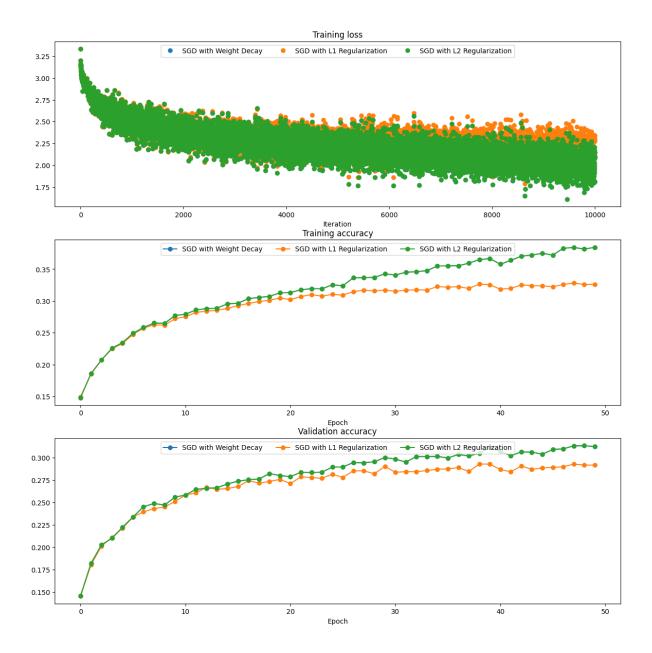
```
for i in [1, 2, 3]:
   plt.subplot(3, 1, i)
   plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
```

Training with SGD plus L2 Regularization...

100%		
<pre>&lt;00:00, 44.54it/s] 100% </pre>	200/200	[00:04
<00:00, 45.53it/s] 100%	200/200	[00:04
<00:00, 42.85it/s]	200/200	[00:04
100%  <	200/200	[00:04
100%		

<00:00,	46.00it/s]		
<00:00,	44.90it/s]	200/200	[00:04
<00:00,	25.59it/s]	200/200	[00:07
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100%		200/200	[00:04
100%		200/200	[00:04
<00:00, 100%	44.77it/s]	200/200	[00:04
-	46.06it/s]	200/200	
	43.20it/s]		
<00:00, 100%	43.61it/s]	200/200	[00:04
	43.74it/s]	200/200	[00:04
<00:00,	43.50it/s]	200/200	[00:04
100%	45.34it/s]	200/200	[00:04
100%	46.35it/s]	200/200	[00:04
100%		200/200	[00:04
100%		200/200	[00:04
<00:00, 100%	43.57it/s]	200/200	[00:04
<00:00, 100%	43.59it/s]	200/200	[00:05
<00:00, 100%	38.96it/s]		
<00:00, 100%	42.96it/s]	200/200	[UU:U4
<00:00, 100%	40.33it/s]	200/200	[00:04





# Adam [2pt]

The update rule of Adam is as shown below:

$$t=t+1 \ g_t: ext{gradients at update step } t \ m_t=eta_1m_{t-1}+(1-eta_1)g_t \ v_t=eta_2v_{t-1}+(1-eta_2)g_t^2 \ \hat{m_t}=m_t/(1-eta_1^t) \ \hat{v_t}=v_t/(1-eta_2^t) \ heta_{t+1}= heta_t-rac{\eta \ \hat{m_t}}{\sqrt{\hat{v_t}}+\epsilon}$$

Complete the Adam() function in lib/optim.py Important Notes:

- 1. t must be updated before everything else
- 2.  $eta_1^t$  is  $eta_1$  exponentiated to the t'th power
- 3. You should also enable weight decay in Adam, similar to what you did in SGD

```
In [382... %reload ext autoreload
         seed = 1234
         np.random.seed(seed=seed)
         # Test Adam implementation; you should see errors around 1e-7 or less
         N, D = 4, 5
         test adam = sequential(fc(N, D, name="adam fc"))
         w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
         dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
         m = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)
         v = np.linspace(0.7, 0.5, num=N*D).reshape(N, D)
         test adam.layers[0].params = {"adam fc w": w}
         test_adam.layers[0].grads = {"adam_fc_w": dw}
         opt_adam = Adam(test_adam, 1e-2, 0.9, 0.999, t=5)
         opt_adam.mt = {"adam_fc_w": m}
         opt adam.vt = {"adam fc w": v}
         opt adam.step()
         updated w = test adam.layers[0].params["adam fc w"]
         mt = opt_adam.mt["adam_fc_w"]
         vt = opt_adam.vt["adam_fc_w"]
         expected updated w = np.asarray([
           [-0.40094747, -0.34836187, -0.29577703, -0.24319299, -0.19060977],
           [-0.1380274, -0.08544591, -0.03286534, 0.01971428, 0.0722929],
           [ 0.1248705,
                        0.17744702, 0.23002243, 0.28259667, 0.33516969],
           [ 0.38774145, 0.44031188, 0.49288093, 0.54544852, 0.59801459]])
         expected_v = np.asarray([
                          0.68908382, 0.67851319, 0.66794809, 0.65738853,],
           [ 0.69966,
                                                   0.61520571, 0.60467385,],
           [ 0.64683452, 0.63628604, 0.6257431,
           [ 0.59414753, 0.58362676, 0.57311152, 0.56260183, 0.55209767,],
           [ 0.54159906, 0.53110598, 0.52061845, 0.51013645, 0.49966, ]])
         expected_m = np.asarray([
                          0.49947368, 0.51894737, 0.53842105, 0.55789474],
           [ 0.48,
           [0.57736842, 0.59684211, 0.61631579, 0.63578947, 0.65526316],
           [ 0.67473684, 0.69421053, 0.71368421, 0.73315789, 0.75263158],
           [ 0.77210526, 0.79157895, 0.81105263, 0.83052632,
                                                                          ]])
                                                                0.85
         print ('The following errors should be around or less than 1e-7')
         print ('updated_w error: ', rel_error(expected_updated_w, updated_w))
         print ('mt error: ', rel_error(expected_m, mt))
         print ('vt error: ', rel_error(expected_v, vt))
         The following errors should be around or less than 1e-7
```

```
updated_w error: 1.1395691798535431e-07
mt error: 4.214963193114416e-09
vt error: 4.208314038113071e-09
```

# Comparing the Weight Decay v.s. L2 Regularization in Adam [5pt]

Run the following code block to compare the plotted results between effects of weight decay and L2 regularization on Adam. Are they still the same? (we can make them the same as in SGD, can we also do it in Adam?)

```
In [383...] seed = 1234
         reset seed(seed)
         model_adam_wd
                           = FullyConnectedNetwork()
                          = cross_entropy()
         loss f adam wd
         optimizer_adam_wd = Adam(model_adam_wd.net, lr=1e-4, weight_decay=1e-6)
         print ("Training with AdamW...")
         results_adam_wd = train_net(small_data_dict, model_adam_wd, loss_f_adam_wd,
                                 max_epochs=50, show_every=10000, verbose=False)
         reset_seed(seed)
                            = FullyConnectedNetwork()
         model_adam_l2
         loss_f_adam_l2
                            = cross entropy()
         optimizer_adam_l2 = Adam(model_adam_l2.net, lr=1e-4)
         reg_lambda_l2 = 1e-4
         print ("\nTraining with Adam + L2...")
         results_adam_l2 = train_net(small_data_dict, model_adam_l2, loss_f_adam_l2,
                                  max_epochs=50, show_every=10000, verbose=False, reg
         opt_params_adam_wd, loss_hist_adam_wd, train_acc_hist_adam_wd, val_acc_hist_
         opt_params_adam_l2, loss_hist_adam_l2, train_acc_hist_adam_l2, val_acc_hist_
         plt.subplot(3, 1, 1)
         plt.title('Training loss')
         plt.xlabel('Iteration')
         plt.subplot(3, 1, 2)
         plt.title('Training accuracy')
         plt.xlabel('Epoch')
         plt.subplot(3, 1, 3)
         plt.title('Validation accuracy')
         plt.xlabel('Epoch')
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_sgd, 'o', label="Vanilla SGD")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_sgd, '-o', label="Vanilla SGD")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_sgd, '-o', label="Vanilla SGD")
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_sgdw, 'o', label="SGD with Weight Decay")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
         plt.subplot(3, 1, 1)
         plt.plot(loss_hist_adam_wd, 'o', label="Adam with Weight Decay")
         plt.subplot(3, 1, 2)
         plt.plot(train_acc_hist_adam_wd, '-o', label="Adam with Weight Decay")
         plt.subplot(3, 1, 3)
         plt.plot(val_acc_hist_adam_wd, '-o', label="Adam with Weight Decay")
```

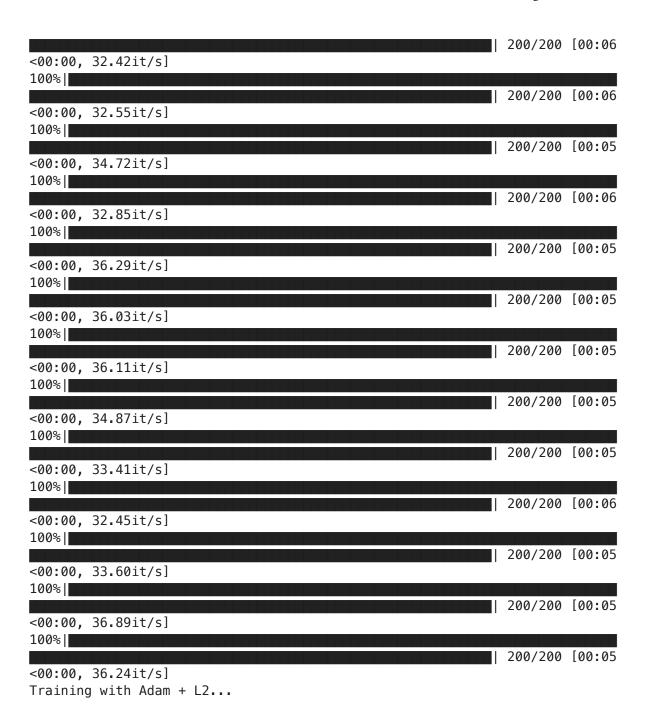
```
plt.subplot(3, 1, 1)
plt.plot(loss_hist_adam_l2, 'o', label="Adam with L2")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_adam_l2, '-o', label="Adam with L2")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_adam_l2, '-o', label="Adam with L2")

for i in [1, 2, 3]:
   plt.subplot(3, 1, i)
   plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
```

Training with AdamW...

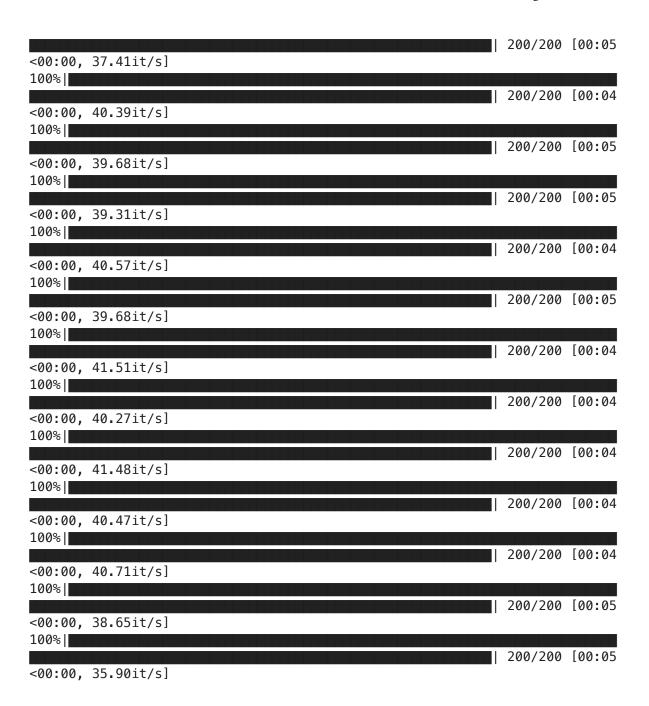
100%		
<00:00, 26.30it/s] 100%	200/200	[00:07
<00:00, 37.82it/s] 100%	200/200	[00:05
<00:00, 36.28it/s]	200/200	[00:05
100%  <	200/200	[00:05
100%		

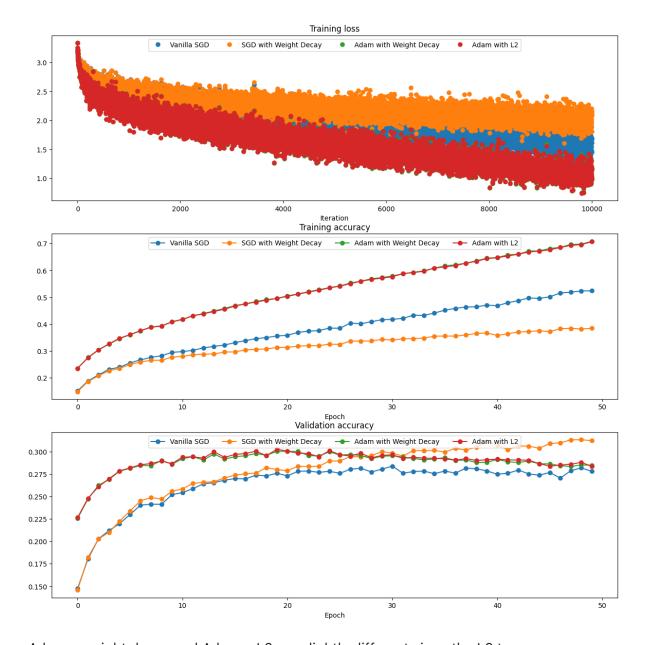
<00:00, 39.48it/s] 100%		
<00:00, 34.63it/s] 100%	200/200	[00:05
<00:00, 37.64it/s] 100%	200/200	[00:05
<00:00, 39.51it/s]	200/200	[00:05
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100%  <00:00, 41.64it/s]	200/200	[00:04
100%	200/200	[00:05
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<00:00, 39.25it/s] 100%	200/200	[00:05
<00:00, 41.89it/s]	200/200	[00:04
100%  <00:00, 38.80it/s]	200/200	[00:05
100%		

<00:00, 38.84it/s]		
<00:00, 39.18it/s]	200/200	[00:05
<00:00, 40.09it/s]	200/200	[00:04
100%  <00:00, 41.46it/s]	200/200	[00:04
100%  <00:00, 38.99it/s]	200/200	[00:05
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<00:00, 38.14it/s] 100%	200/200	[00:04
<00:00, 40.11it/s] 100%	200/200	[00:04
<00:00, 40.71it/s]		[00:05
<00:00, 38.25it/s] 100%		[00:04
<00:00, 40.21it/s] 100%		[00:05
<00:00, 39.06it/s]		
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<00:00, 38.88it/s] 100%	200/200	[00:05
<00:00, 39.01it/s] 100%	200/200	[00:05
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Adam + weight decay and Adam + L2 are slightly different since the L2 terms are affected by Adam's momentum whereas regular weight decay is not affected by momentum terms. (L2 updates by Ir*lambda*params whereas weight decay updates by decay\*params) They can be modified to become them same by adjusting L2's lambda term to account for m\_hat and v\_hat.

# **Submission**

Please prepare a PDF document problem\_1\_solution.pdf in the root directory of this repository with all plots and inline answers of your solution. Concretely, the document should contain the following items in strict order:

- 1. Training loss / accuracy curves for the simple neural network training with > 30% validation accuracy
- 2. Plots for comparing vanilla SGD to SGD + Weight Decay, SGD + L1 and SGD + L2
- 3. "Comparing different Regularizations with Adam" plots

Note that you still need to submit the jupyter notebook with all generated solutions. We will randomly pick submissions and check that the plots in the PDF and in the notebook are equivalent.

# **Problem 2: Incorporating CNNs**

- Learning Objective: In this problem, you will learn how to deeply understand how Convolutional Neural Networks work by implementing one.
- Provided Code: We provide the skeletons of classes you need to complete.
   Forward checking and gradient checkings are provided for verifying your implementation as well.
- TODOs: you will implement a Convolutional Layer and a MaxPooling Layer to improve on your classification results in part 1.

```
In [1]: from lib.mlp.fully_conn import *
        from lib.mlp.layer_utils import *
        from lib.mlp.train import *
        from lib.cnn.layer_utils import *
        from lib.cnn.cnn_models import *
        from lib.datasets import *
        from lib.grad_check import *
        from lib.optim import *
        import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
        plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'
        # for auto-reloading external modules
        # see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ip
        %load_ext autoreload
        %autoreload 2
```

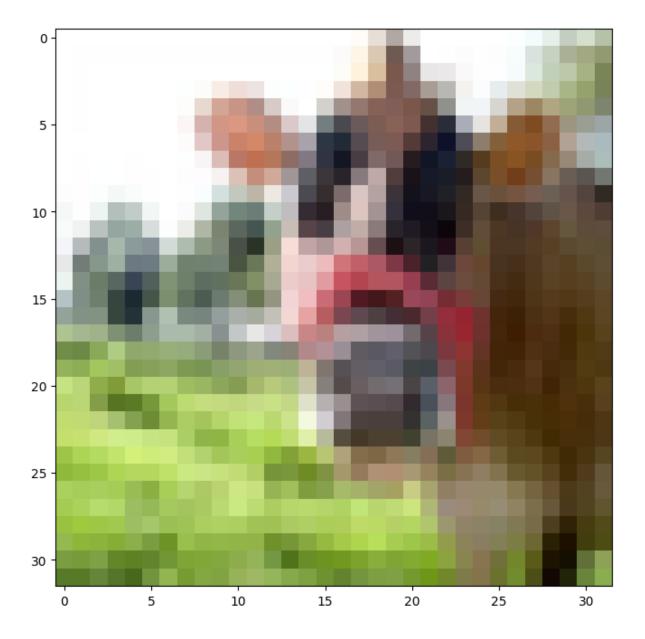
## Loading the data (CIFAR-100 with 20 superclasses)

In this homework, we will be classifying images from the CIFAR-100 dataset into the 20 superclasses. More information about the CIFAR-100 dataset and the 20 superclasses can be found here.

Download the CIFAR-100 data files here, and save the .mat files to the data/cifar100 directory.

```
In [2]: data = CIFAR100 data('data/cifar100/')
        for k, v in data.items():
            if type(v) == np.ndarray:
                print ("Name: {} Shape: {}, {}".format(k, v.shape, type(v)))
                print("{}: {}".format(k, v))
        label_names = data['label_names']
        mean image = data['mean image'][0]
        std_image = data['std_image'][0]
        Name: data_train Shape: (40000, 32, 32, 3), <class 'numpy.ndarray'>
        Name: labels_train Shape: (40000,), <class 'numpy.ndarray'>
        Name: data_val Shape: (10000, 32, 32, 3), <class 'numpy.ndarray'>
        Name: labels_val Shape: (10000,), <class 'numpy.ndarray'>
        Name: data_test Shape: (10000, 32, 32, 3), <class 'numpy.ndarray'>
        Name: labels_test Shape: (10000,), <class 'numpy.ndarray'>
        label_names: ['aquatic_mammals', 'fish', 'flowers', 'food_containers', 'fru
        it_and_vegetables', 'household_electrical_devices', 'household_furniture',
        'insects', 'large_carnivores', 'large_man-made_outdoor_things', 'large_natu
        ral_outdoor_scenes', 'large_omnivores_and_herbivores', 'medium_mammals', 'n
        on-insect_invertebrates', 'people', 'reptiles', 'small_mammals', 'trees', '
        vehicles_1', 'vehicles_2']
        Name: mean_image Shape: (1, 1, 1, 3), <class 'numpy.ndarray'>
        Name: std_image Shape: (1, 1, 1, 3), <class 'numpy.ndarray'>
In [3]: idx = 0
        image_data = data['data_train'][idx]
        image_data = ((image_data*std_image + mean_image) * 255).astype(np.int32)
        plt.imshow(image_data)
        label = label names[data['labels train'][idx]]
        print("Label:", label)
```

Label: large omnivores and herbivores



#### **Convolutional Neural Networks**

We will use convolutional neural networks to try to improve on the results from Problem 1. Convolutional layers make the assumption that local pixels are more important for prediction than far-away pixels. This allows us to form networks that are robust to small changes in positioning in images.

#### Convolutional Layer Output size calculation [2pts]

As you have learned, two important parameters of a convolutional layer are its stride and padding. To warm up, we will need to calculate the output size of a convolutional layer given its stride and padding. To do this, open the <code>lib/cnn/layer\_utils.py</code> file and fill out the TODO section in the <code>get\_output\_size</code> function in the ConvLayer2D class.

Implement your function so that it returns the correct size as indicated by the block below.

```
In [50]: %reload_ext autoreload
    input_image = np.zeros([32, 28, 28, 3]) # a stack of 32 28 by 28 rgb images
    in_channels = input_image.shape[-1] #must agree with the last dimension of t
    k_size = 4
    n_filt = 16

    conv_layer = ConvLayer2D(in_channels, k_size, n_filt, stride=2, padding=3)
    output_size = conv_layer.get_output_size(input_image.shape)

print("Received {} and expected [32, 16, 16, 16]".format(output_size))

Received (32, 16, 16, 16) and expected [32, 16, 16, 16]
```

#### Convolutional Layer Forward Pass [5pts]

Now, we will implement the forward pass of a convolutional layer. Fill in the TODO block in the forward function of the ConvLayer2D class.

```
In [257... %reload ext autoreload
         # Test the convolutional forward function
         input image = np.linspace(-0.1, 0.4, num=1*8*8*1).reshape([1, 8, 8, 1]) # a
         in_channels, k_size, n_filt = 1, 5, 2
         weight_size = k_size*k_size*in_channels*n_filt
         bias size = n filt
         single_conv = ConvLayer2D(in_channels, k_size, n_filt, stride=1, padding=0,
         w = np.linspace(-0.2, 0.2, num=weight_size).reshape(k_size, k_size, in_chann)
         b = np.linspace(-0.3, 0.3, num=bias_size)
         single_conv.params[single_conv.w_name] = w
         single_conv.params[single_conv.b_name] = b
         out = single conv.forward(input image)
         print("Received output shape: {}, Expected output shape: (1, 4, 4, 2)".forma
         correct_out = np.array([[
            [[-0.03874312, 0.57000324],
             [-0.03955296, 0.57081309],
             [-0.04036281, 0.57162293],
             [-0.04117266, 0.57243278]],
            [[-0.0452219, 0.57648202],
            [-0.04603175, 0.57729187],
            [-0.04684159, 0.57810172],
             [-0.04765144, 0.57891156]],
            [[-0.05170068, 0.5829608],
            [-0.05251053, 0.58377065],
             [-0.05332038, 0.5845805],
             [-0.05413022, 0.58539035]],
            [[-0.05817946, 0.58943959],
            [-0.05898931, 0.59024943],
            [-0.05979916, 0.59105928],
             [-0.06060901, 0.59186913]]])
         # Compare your output with the above pre-computed ones.
         # The difference should not be larger than 1e-7
         print ("Difference: ", rel_error(out, correct_out))
```

Received output shape: (1, 4, 4, 2), Expected output shape: (1, 4, 4, 2) Difference: 5.110565335399418e-08

#### Conv Layer Backward [5pts]

Now complete the backward pass of a convolutional layer. Fill in the TODO block in the backward function of the ConvLayer2D class. Check you results with this code and expect differences of less than 1e-6.

```
%reload_ext autoreload
In [273...
         # Test the conv backward function
         img = np.random.randn(15, 8, 8, 3)
         w = np.random.randn(4, 4, 3, 12)
         b = np.random.randn(12)
         dout = np.random.randn(15, 4, 4, 12)
         single_conv = ConvLayer2D(input_channels=3, kernel_size=4, number_filters=12
         single_conv.params[single_conv.w_name] = w
         single_conv.params[single_conv.b_name] = b
         dimg_num = eval_numerical_gradient_array(lambda x: single_conv.forward(img),
         dw_num = eval_numerical_gradient_array(lambda w: single_conv.forward(img), w
         db_num = eval_numerical_gradient_array(lambda b: single_conv.forward(img), b
         out = single_conv.forward(img)
         dimg = single_conv.backward(dout)
         dw = single_conv.grads[single_conv.w_name]
         db = single_conv.grads[single_conv.b_name]
         # The error should be around 1e-6
         print("dimg Error: ", rel_error(dimg_num, dimg))
         # The errors should be around 1e-8
         print("dw Error: ", rel_error(dw_num, dw))
         print("db Error: ", rel_error(db_num, db))
         # The shapes should be same
         print("dimg Shape: ", dimg.shape, img.shape)
         dimg Error: 4.644590234305012e-08
         dw Error: 3.127392012612529e-09
         db Error: 3.7320812919197414e-10
         dimg Shape: (15, 8, 8, 3) (15, 8, 8, 3)
```

#### Max pooling Layer

Now we will implement maxpooling layers, which can help to reduce the image size while preserving the overall structure of the image.

#### Forward Pass max pooling [5pts]

Fill out the TODO block in the forward function of the MaxPoolingLayer class.

```
In [274...
         # Test the convolutional forward function
         input_image = np.linspace(-0.1, 0.4, num=64).reshape([1, 8, 8, 1]) # a singl
         maxpool= MaxPoolingLayer(pool size=4, stride=2, name="maxpool test")
         out = maxpool.forward(input_image)
         print("Received output shape: {}, Expected output shape: (1, 3, 3, 1)".forma
         correct_out = np.array([[
            [[0.11428571],
            [0.13015873],
            [0.14603175],
            [[0.24126984],
            [0.25714286],
            [0.27301587],
            [[0.36825397],
            [0.38412698],
                        ]]]])
            [0.4
         # Compare your output with the above pre-computed ones.
         # The difference should not be larger than 1e-7
         print ("Difference: ", rel_error(out, correct_out))
```

Received output shape: (1, 3, 3, 1), Expected output shape: (1, 3, 3, 1) Difference: 1.8750000280978013e-08

#### Backward Pass Max pooling [5pts]

Fill out the backward function in the MaxPoolingLayer class.

```
img = np.random.randn(15, 8, 8, 3)

dout = np.random.randn(15, 3, 3, 3)

maxpool= MaxPoolingLayer(pool_size=4, stride=2, name="maxpool_test")

dimg_num = eval_numerical_gradient_array(lambda x: maxpool.forward(img), img

out = maxpool.forward(img)
    dimg = maxpool.backward(dout)

# The error should be around 1e-8
    print("dimg Error: ", rel_error(dimg_num, dimg))
# The shapes should be same
    print("dimg Shape: ", dimg.shape, img.shape)

dimg Error: 3.279406847766653e-12
    dimg Shape: (15, 8, 8, 3) (15, 8, 8, 3)
```

## Test a Small Convolutional Neural Network [3pts]

Please find the TestCNN class in lib/cnn/cnn\_models.py . Again you only need to complete few lines of code in the TODO block. Please design a Convolutional --> Maxpool --> flatten --> fc network where the shapes of parameters match the given shapes. Please insert the corresponding names you defined for each layer to param\_name\_w, and param\_name\_b respectively. Here you only modify the param\_name part, the \_w, and \_b are automatically assigned during network setup.

```
In [287... %reload_ext autoreload
       seed = 1234
       np.random.seed(seed=seed)
       model = TestCNN()
       loss_func = cross_entropy()
       B, H, W, iC = 4, 8, 8, 3 #batch, height, width, in_channels
       k = 3 #kernel size
       oC, Hi, 0 = 3, 27, 5 # out channels, Hidden Layer input, Output size
       std = 0.02
       x = np.random.randn(B,H,W,iC)
       y = np.random.randint(0, size=B)
       print ("Testing initialization ... ")
       # TODO: param_name should be replaced accordingly #
       w1 std = abs(model.net.get params("conv1 w").std() - std)
       b1 = model.net.get_params("conv1_b").std()
       w2 std = abs(model.net.get params("fc1 w").std() - std)
       b2 = model.net.get_params("fc1_b").std()
       END OF YOUR CODE
       assert w1_std < std / 10, "First layer weights do not seem right"</pre>
       assert np.all(b1 == 0), "First layer biases do not seem right"
assert w2_std < std / 10, "Second layer weights do not seem right"</pre>
       assert np.all(b2 == 0), "Second layer biases do not seem right"
       print ("Passed!")
       print ("Testing test-time forward pass ... ")
       w1 = np.linspace(-0.7, 0.3, num=k*k*iC*oC).reshape(k,k,iC,oC)
       w2 = np.linspace(-0.2, 0.2, num=Hi*0).reshape(Hi, 0)
       b1 = np.linspace(-0.6, 0.2, num=oC)
       b2 = np.linspace(-0.9, 0.1, num=0)
       # TODO: param name should be replaced accordingly #
       model.net.assign("conv1_w", w1)
       model.net.assign("conv1_b", b1)
       model.net.assign("fc1_w", w2)
       model.net.assign("fc1_b", b2)
       END OF YOUR CODE
       feats = np.linspace(-5.5, 4.5, num=B*H*W*iC).reshape(B,H,W,iC)
       scores = model.forward(feats)
       print(scores)
       correct_scores = np.asarray([-13.85107294, -11.52845818, -9.20584342, -6.
        [-11.44514171, -10.21200524, -8.97886878, -7.74573231, -6.51259584],
```

```
[-9.03921048, -8.89555231, -8.75189413, -8.60823596, -8.46457778],
 [-6.63327925, -7.57909937, -8.52491949, -9.4707396, -10.41655972]])
scores_diff = np.sum(np.abs(scores - correct_scores))
assert scores_diff < 1e-6, "Your implementation might be wrong!"</pre>
print ("Passed!")
print ("Testing the loss ...",)
y = np.asarray([0, 2, 1, 4])
loss = loss_func.forward(scores, y)
dLoss = loss_func.backward()
correct loss = 4.56046848799693
assert abs(loss - correct_loss) < 1e-10, "Your implementation might be wrong</pre>
print ("Passed!")
print ("Testing the gradients (error should be no larger than 1e-6) ...")
din = model.backward(dLoss)
for layer in model.net.layers:
    if not layer.params:
        continue
    for name in sorted(layer.grads):
        f = lambda _: loss_func.forward(model.forward(feats), y)
        grad_num = eval_numerical_gradient(f, layer.params[name], verbose=Fa
        print ('%s relative error: %.2e' % (name, rel_error(grad_num, layer.
Testing initialization ...
Passed!
Testing test-time forward pass ...
[[-13.85107294 -11.52845818 -9.20584342 -6.88322866 -4.5606139]
 [-11.44514171 - 10.21200524 - 8.97886878 - 7.74573231 - 6.51259584]
 [-9.03921048 -8.89555231 -8.75189413 -8.60823596 -8.46457778]
 [-6.63327925 -7.57909937 -8.52491949 -9.4707396 -10.41655972]]
5.304109507164867e-08
Passed!
Testing the loss ...
Passed!
Testing the gradients (error should be no larger than 1e-6) ...
conv1_b relative error: 4.88e-09
conv1_w relative error: 9.26e-10
fc1_b relative error: 8.77e-11
fc1_w relative error: 3.89e-07
```

#### Training the Network [25pts]

In this section, we defined a SmallConvolutionalNetwork class for you to fill in the TODO block in lib/cnn/cnn\_models.py .

Here please design a network with at most two convolutions and two maxpooling layers (you may use less). You can adjust the parameters for any layer, and include layers other than those listed above that you have implemented (such as fully-connected layers and non-linearities). You are also free to select any optimizer you have implemented (with any learning rate).

You will train your network on CIFAR-100 20-way superclass classification. Try to find a combination that is able to achieve 40% validation accuracy.

Since the CNN takes significantly longer to train than the fully connected network, it is suggested to start off with fewer filters in your Conv layers and fewer intermediate fully-connected layers so as to get faster initial results.

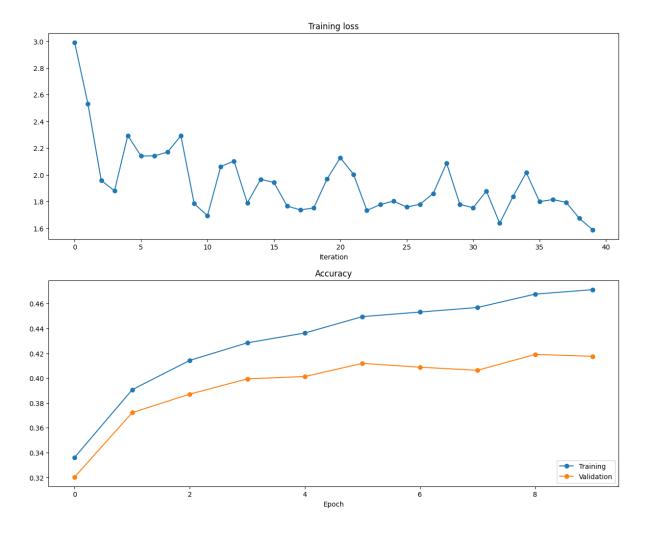
```
In [288... # Arrange the data
data_dict = {
    "data_train": (data["data_train"], data["labels_train"]),
    "data_val": (data["data_val"], data["labels_val"]),
    "data_test": (data["data_test"], data["labels_test"])
}
In [289... print("Data shape:", data_dict["data_train"][0].shape)
print("Flattened data input size:", np.prod(data["data_train"].shape[1:]))
print("Number of data classes:", max(data['labels_train']) + 1)

Data shape: (40000, 32, 32, 3)
Flattened data input size: 3072
Number of data classes: 20
```

```
In [371...
       %reload ext autoreload
       seed = 123
       np.random.seed(seed=seed)
       model = SmallConvolutionalNetwork()
       loss_f = cross_entropy()
       results = None
       # TODO: Use the train_net function you completed to train a network
       # You may only adjust the hyperparameters within this block
       optimizer = Adam(model.net, 1e-3)
       batch size = 100#50#10
       epochs = 10#5
       lr_{decay} = 1.0 \#.999
       lr_decay_every = 30#10
       regularization = "none"
       reg_lambda = 0.01
       END OF YOUR CODE
       results = train_net(data_dict, model, loss_f, optimizer, batch_size, epochs,
                        lr_decay, lr_decay_every, show_every=4000, verbose=True,
       opt_params, loss_hist, train_acc_hist, val_acc_hist = results
         0%||
        | 1/400 [00:01<10:58, 1.65s/it]
       (Iteration 1 / 4000) Average loss: 2.996304593414753
       100%
                                                        | 400/400 [10:30
       <00:00, 1.58s/it]
       (Epoch 1 / 10) Training Accuracy: 0.336125, Validation Accuracy: 0.3203
       100%
                                                       || 400/400 [10:28
       <00:00, 1.57s/it]
       (Epoch 2 / 10) Training Accuracy: 0.390675, Validation Accuracy: 0.3722
       100%
                                                       || 400/400 [10:28
       <00:00, 1.57s/it]
       (Epoch 3 / 10) Training Accuracy: 0.4143, Validation Accuracy: 0.3871
       100%
                                                       || 400/400 [10:29
       <00:00, 1.57s/it]
       (Epoch 4 / 10) Training Accuracy: 0.428475, Validation Accuracy: 0.3994
       100%
                                                       || 400/400 [10:28
       <00:00, 1.57s/it]
       (Epoch 5 / 10) Training Accuracy: 0.4364, Validation Accuracy: 0.4013
```

```
100%
                                                           ■| 400/400 [10:27
<00:00, 1.57s/it]
(Epoch 6 / 10) Training Accuracy: 0.4496, Validation Accuracy: 0.4119
100%
                                                            || 400/400 [10:23
<00:00, 1.56s/it]
(Epoch 7 / 10) Training Accuracy: 0.453275, Validation Accuracy: 0.4088
                                                           || 400/400 [10:23
<00:00, 1.56s/it]
(Epoch 8 / 10) Training Accuracy: 0.45695, Validation Accuracy: 0.4063
100%
                                                            || 400/400 [10:27
<00:00, 1.57s/it]
(Epoch 9 / 10) Training Accuracy: 0.467725, Validation Accuracy: 0.4191
100%
                                                           | 400/400 [10:25
<00:00, 1.56s/it]
(Epoch 10 / 10) Training Accuracy: 0.4713, Validation Accuracy: 0.4176
Run the code below to generate the training plots.
```

```
In [372...
         %reload_ext autoreload
         opt_params, loss_hist, train_acc_hist, val_acc_hist = results
         # Plot the learning curves
         plt.subplot(2, 1, 1)
         plt.title('Training loss')
         loss_hist_ = loss_hist[1::100] # sparse the curve a bit
         plt.plot(loss_hist_, '-o')
         plt.xlabel('Iteration')
         plt.subplot(2, 1, 2)
         plt.title('Accuracy')
         plt.plot(train_acc_hist, '-o', label='Training')
         plt.plot(val_acc_hist, '-o', label='Validation')
         plt.xlabel('Epoch')
         plt.legend(loc='lower right')
         plt.gcf().set size inches(15, 12)
         plt.show()
```

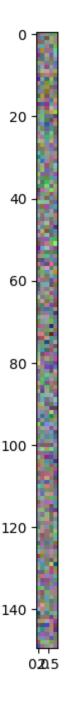


## Visualizing Layers [5pts]

An interesting finding from early research in convolutional networks was that the learned convolutions resembled filters used for things like edge detection. Complete the code below to visualize the filters in the first convolutional layer of your best model.

```
In [373... im_array = None
       nrows, ncols = None, None
       # TODO: read the weights in the convolutional
       # layer and reshape them to a grid of images to
       # view with matplotlib.
       layer1_params = model.net.get_params('conv1_w')
       im_array = np.zeros(shape=(layer1_params.shape[:-1]))
       for i in range(0, layer1_params.shape[3]):
          if i == 0:
             im_array = layer1_params[:,:,:,i]
             im_array = np.append(im_array, layer1_params[:,:,:,i],axis=0)
       # https://stackoverflow.com/questions/49922460/scale-a-numpy-array-with-from
       im_array = ((im_array - im_array.min()) * (1/(im_array.max() - im_array.min())
       END OF YOUR CODE
       plt.imshow(im_array)
```

Out[373]: <matplotlib.image.AxesImage at 0x7ff68ab091f0>



Inline Question: Comment below on what kinds of filters you see. Include your response in your submission [5pts]

The RGB weights learned usually correspond to important local components in the original image. It isnt

clear from above since the a larger kernel size could have been used for a 5x5 kernel convolved over a 32x32 image

### Extra-Credit: Analysis on Trained Model [5pts]

For extra credit, you can perform some additional analysis of your trained model. Some suggested analyses are:

- Plot the confusion matrix of your model's predictions on the test set. Look for trends to see which classes are frequently misclassified as other classes (e.g. are the two vehicle superclasses frequently confused with each other?).
- 2. Implement BatchNorm and analyze how the models train with and without BatchNorm.
- 3. Introduce some small noise in the labels, and investigate how that affects training and validation accuracy.

You are free to choose any analysis question of interest to you. We will not be providing any starter code for the extra credit. Include your extra-credit analysis as the final section of your report pdf, titled "Extra Credit".

## **Submission**

Please prepare a PDF document problem\_2\_solution.pdf in the root directory of this repository with all plots and inline answers of your solution. Concretely, the document should contain the following items in strict order:

- 1. Training loss / accuracy curves for CNN training
- 2. Visualization of convolutional filters
- 3. Answers to inline questions about convolutional filters

Note that you still need to submit the jupyter notebook with all generated solutions. We will randomly pick submissions and check that the plots in the PDF and in the notebook are equivalent.

In [ ]: