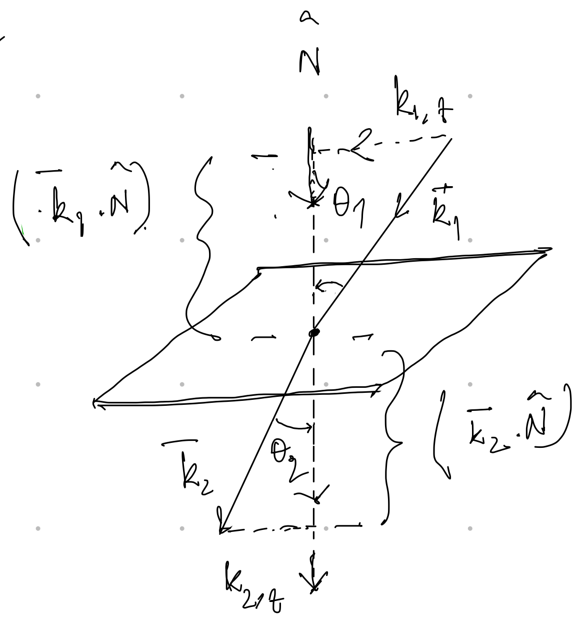
Equating the tangential components,



$$\bar{k}_{1,p} = (\bar{k}_1 \cdot \hat{N})\,\hat{N}$$

$$\bar{k}_{1,t} = \bar{k}_1 - \bar{k}_{1,p}$$

$$= \bar{k}_1 - (\bar{k}_1 \cdot \hat{N})\,\hat{N}$$

$$\bar{k}_{2,p} = (\bar{k}_2 \cdot \hat{N})\,\hat{N}$$

$$\bar{k}_{2,t} = \bar{k}_2 - (\bar{k}_2 \cdot \hat{N})\,\hat{N}$$

Now, $\bar{k}_{1,t} = \bar{k}_{2,t}$

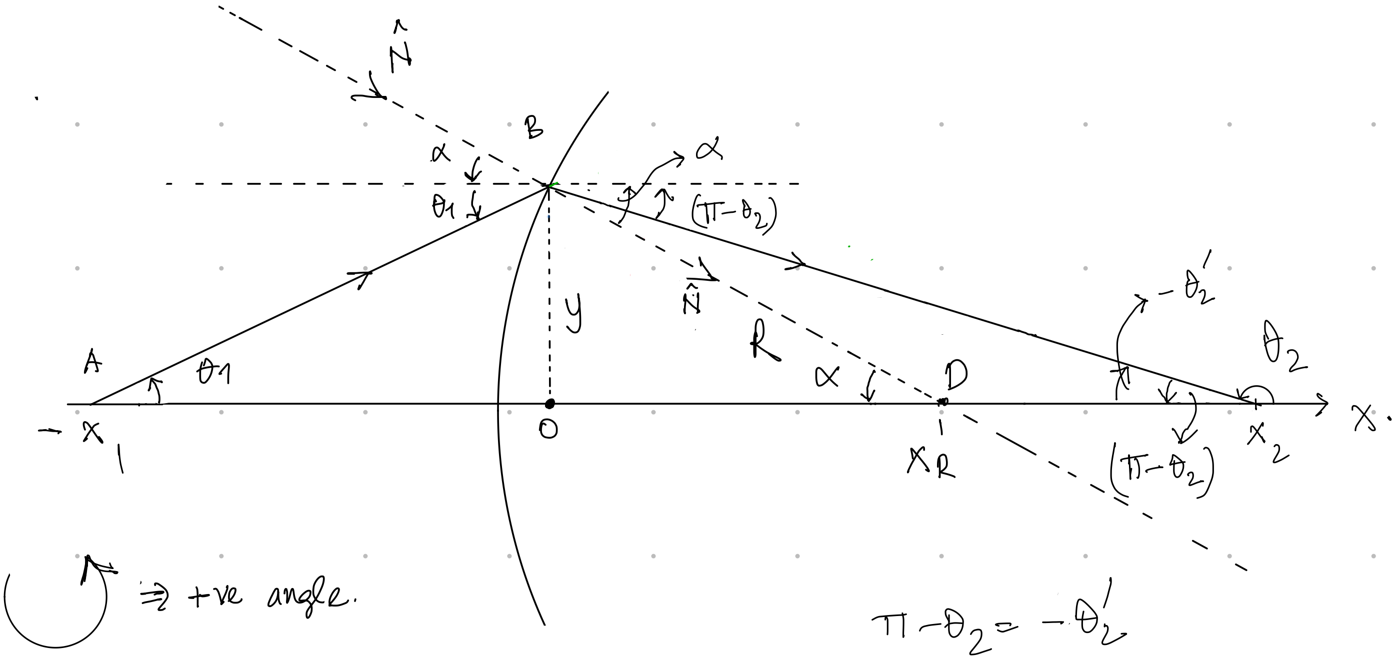
Now, $|\vec{k}_1| = (n_1 k_0)\,\hat{k}_1$ and $|\bar{k}_2| = (n_2 k_0)\,\hat{k}_2$

then, $\left[n_1 k_0 \hat{k}_1 - (\bar{k}_1 \cdot \hat{N})\hat{N}\right] \times \hat{N} = \left[n_2 k_0 \hat{k}_2 - (k_2 \cdot \hat{N})\hat{N}\right] \times \hat{N}$

$$n_1 k_0 (\hat{k}_1 \times \hat{N}) = n_2 k_0 (\hat{k}_2 \times \hat{N})$$

then,

$$\boxed{n_1 (\hat{k}_1 \times \hat{N}) = n_2 (\hat{k}_2 \times \hat{N})}$$



$$n_1 (\hat{k}_1 \times \hat{N}) = n_2 (\hat{k}_2 \times \hat{N})$$

$$n_1 \sin(\alpha+\theta_1) = n_2 \sin(\alpha - (\pi-\theta_2))$$

$$= n_2 \sin(\alpha + \theta_2')$$

$$n_1\alpha + n_1\theta_1 = n_2\alpha + n_2\theta_2'$$

$$\Rightarrow \theta_2' = \frac{n_1}{n_2}\theta_1 + \frac{n_1-n_2}{n_2}\frac{y}{R}$$

$$\hat{k}_1 = \frac{1}{\sqrt{y^2+x_1^2}}\begin{pmatrix} x_1 \\ y \end{pmatrix} \qquad \hat{N} = \frac{1}{R}\begin{pmatrix} x_R \\ -y \end{pmatrix}$$

$$\hat{k}_1 \times \hat{N} = \frac{1}{R\sqrt{y^2+x_1^2}}\begin{pmatrix} 0 \\ 0 \\ -x_1 y - y x_R \end{pmatrix}$$

$$\hat{k}_2 = \frac{1}{\sqrt{y^2+x_2^2}}\begin{pmatrix} x_2 \\ -y \end{pmatrix}$$

$$\hat{k}_2 \times \hat{N} = \frac{1}{R\sqrt{y^2+x_2^2}}\begin{pmatrix} 0 \\ 0 \\ -x_2 y + y x_R \end{pmatrix}$$

$$\frac{n_1}{\sqrt{y^2 + x_1^2}} \begin{pmatrix} 0 \\ 0 \\ -x_1 y - y x_R \end{pmatrix} = \frac{n_2}{\sqrt{y^2 + x_2^2}} \begin{pmatrix} 0 \\ 0 \\ -x_2 y + y x_R \end{pmatrix}$$

$$\Rightarrow \frac{n_1}{\sqrt{y^2 + x_1^2}} \left(-x_1 y - y x_R\right) = \frac{n_2}{\sqrt{y^2 + x_2^2}} \left(-x_2 y + y x_R\right)$$

$$\Rightarrow -n_1 \left(\frac{x_1}{\sqrt{y^2 + x_1^2}} + \frac{x_R}{y} \frac{y}{\sqrt{y^2 + x_1^2}} \right) = n_2 \left(-\frac{x_2}{\sqrt{y^2 + x_2^2}} + \frac{x_R}{y} \frac{y}{\sqrt{y^2 + x_2^2}} \right)$$

$$\Rightarrow -n_1 \left(\cos\theta_1 + \frac{\cos\alpha}{\sin\alpha} \sin\theta_1 \right) = -n_2 \left(\cos\theta_2' - \frac{\cos\alpha}{\sin\alpha} \sin(-\theta_2') \right)$$

$$\Rightarrow n_1 \left(\cos\theta_1 \sin\alpha + \cos\alpha \sin\theta_1 \right) = n_2 \left(\sin\alpha \cos\theta_2' + \cos\alpha \sin\theta_2' \right)$$

$$\Rightarrow n_1 \sin(\alpha + \theta_1) = n_2 \sin(\theta_2' + \alpha)$$

$$\Rightarrow n_1(\alpha + \theta_1) = n_2 (\theta_2' + \alpha)$$

$$\frac{n_1 - n_2}{n} \alpha + \frac{n}{n_2} \theta_1 = \theta_2'$$