1_ray_optics

April 2, 2024

1 General Optics

[1]:
```
%matplotlib widget
import ipywidgets as widgets
import matplotlib.pyplot as plt
import numpy as np
```

1.1 Ray Optics

Ray optics delivers the simplest mathematical description of light propagation, which is despite of its simplifications very powerful. However, it does not provide any insights into the physical mechanisms that govern the propagation of light.

Geometrical or ray optics is therefore based on a number of assumptions for light propagation, i.e.

- light travels in form of rays, which are emitted by light sources and detected by detectors
- the effect of a medium on the light propagation is described by the refractive index $n = c_0/c$
- light takes time to travels a distance d which is given by $t = d/c = nd/c_0$
- $n d $ therefore amounts to the optical path length

You see on these assumptions, that this is not really satisfying. We do neither really know what is propagation with the speed of light, nor where the refractive index comes from. This is something we need to address later, when we consider electromagnetic waves.

Based on this, we also can only infer from observations the laws that govern ray optics, i.e. the law of reflection and refraction.

1.1.1 Law of reflection

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Figure 1: Law of reflection

The law of reflection is the simplest one. It states, that the angle of incidence and the angle of reflection are always equal.

$$\theta_1 = \theta_3 \tag{1}$$

1.1.2 Law of refraction

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \tag{2}$$

The law of refraction is known as Snell's law and relates the incident angle θ_1 with the angle of the refracted ray θ_2.

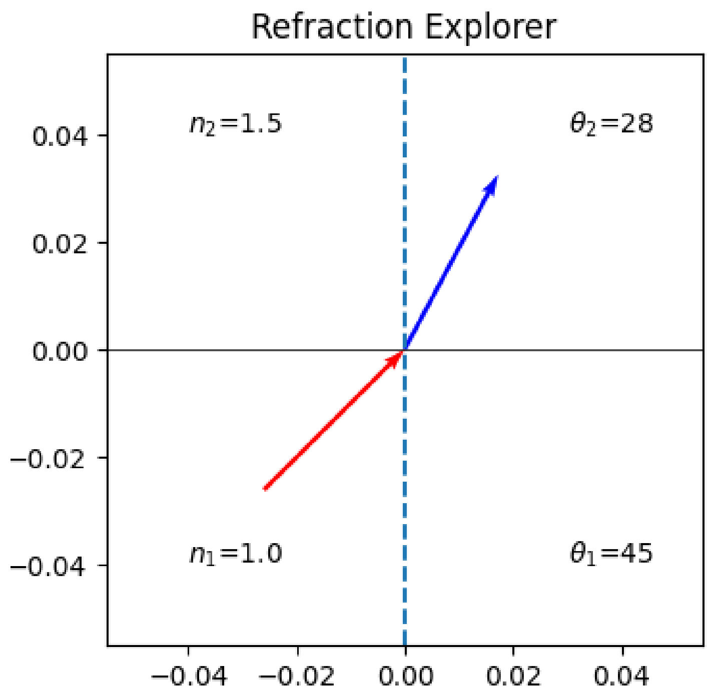
With this law of refraction, there are a number of special situations, that occur. This is for example the total internal refraction. You can explore different situations with the short program below which gives you control over the refractive indices and the incident angles.

[4]:
```
fig, ax = plt.subplots(figsize=(4, 4))
fig.canvas.header_visible = False

@widgets.interact(n1=(1,2,0.01),n2=(1,3,0.01), angle=(0, 90, 0.1))
def update(n1=1,n2=1.5,angle=45):
    """Remove old lines from plot and plot new one"""
    ax.cla()
    theta1=angle*np.pi/180
    if n1*np.sin(theta1)/n2<=1:
        theta2=np.arcsin(n1*np.sin(theta1)/n2)
    else:
        theta2=-theta1+np.pi
    ax.set_title("Refraction Explorer")
    ax.axvline(x=0,ls='--')
    ax.text(-0.04,0.04,r'$n_2$={}'.format(n2))
    ax.text(-0.04,-0.04,r'$n_1$={}'.format(n1))
    ax.text(0.03,0.04,r'$\theta_2$={}'.format(round(theta2*180/np.pi),1))
    ax.text(0.03,-0.04,r'$\theta_1$={}'.format(round(theta1*180/np.pi),1))


    ax.axhline(y=0,color='k',lw=0.5)
    ax.quiver(0,0,np.sin(theta1),np.cos(theta1),scale=3,pivot='tip'
 ↪,color='red')
    ax.quiver(0,0,np.sin(theta2),np.cos(theta2),scale=3,color='blue')
```

```
interactive(children=(FloatSlider(value=1.0, description='n1', max=2.0, min=1.0,
 ↪step=0.01), FloatSlider(value…
```



Refraction on spherical boundary Many of the interesting situations of refraction occur on interfaces, which are curved. Lenses, for example, have curved surfaces and the angles of refraction therefore change depending on the position. We can model in the simplest case by a spherical surface or in 2D just by a circular interface. The diagram below shows the definition of such a situation.

$$-\theta_2 + (\pi - \alpha) + \delta = \pi \tag{3}$$

$$-\theta_2 + \pi - \alpha + \delta = \pi \tag{4}$$

$$\delta = \alpha + \theta_2 \tag{5}$$

$$\sin(\alpha) = \frac{y}{R}, \tan(\theta_2) = \frac{y}{z_2}, \tan(\theta_1) = \frac{y}{z_1} \tag{6}$$

$$n_1 \sin(\alpha + \theta_1) = n_2 \sin(\alpha + \theta_2) \tag{7}$$

linearization: $\sin(\gamma) \approx \gamma$

$$n_1(\alpha + \theta_1) = n_2(\alpha + \theta_2) \tag{8}$$

$$n_1\alpha + n_1\theta_1 = n_2\alpha + n_2\theta_2 \tag{9}$$

$$n_2\theta_2 = (n_1 - n_2)\alpha + n_1\theta_1 \tag{10}$$

$$\theta_2 = -\frac{n_2 - n_1}{n_2}\alpha + \frac{n_1}{n_2}\theta_1 \tag{11}$$

replace angles by y

$$\theta_2 \approx \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y \tag{12}$$

conjugated planes

$$\frac{\Delta y}{z_2} = \frac{y}{z_1} \tag{13}$$

$$(1) \quad \theta_1 = 0 \quad \Rightarrow \quad \theta_2^1 = -\frac{n_2 - n_1}{n_2 R} y, \quad \tan\theta_2 = \frac{\Delta y + y}{z_2} \tag{14}$$

$$(2) \quad \theta_1 = \theta_2 \quad n_2\theta_2^2 = n_1\theta_1^2 \tag{15}$$

$$\frac{\Delta y + y}{z_2} = -\frac{n_2 - n_1}{n_2 R} y \tag{16}$$

$$\Delta y + y = -\frac{n_2 - n_1}{n_2 R} y z_2, \quad \frac{\Delta y}{z_2} = \frac{y}{z_1} \tag{17}$$

$$y\left(\frac{z_2}{z_1} + 1\right) = -\frac{n_2 - n_1}{n_2 R} z_2 y, \quad \Delta y = \frac{y}{z_1} z_2 \tag{18}$$

$$\left(\frac{z_2}{z_1} + 1\right) = -\frac{n_2 - n_1}{n_2 R} z_2 \tag{19}$$

$$\frac{1}{z_1} + \frac{1}{z_2} = -\frac{n_2 - n_1}{n_2 R} \tag{20}$$

independent of $y \Rightarrow$ all y at z_1 end up in z_2

conjugated planes means: - there is one point in z_1 which belongs to a point in z_2 - foundation of image formation

biconvex lens

apply the formula twice

$$\theta_2 = \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y \tag{21}$$

first refraction, $n_1 = 1, n_2 = n$

$$\theta_t = \frac{1}{n}\theta_1 - \frac{n-1}{nR_1} y \tag{22}$$

second refraction, $n_1 = n, n_2 = 1$

$$\theta_t \to \theta_1, y \to y \tag{23}$$

$$\theta_2 = \frac{n}{1}\theta_t - \frac{1-n}{R_2} y \tag{24}$$

$$\theta_2 = \frac{n}{1}\left(\frac{1}{n}\theta_1 - \frac{n-1}{nR_1} y\right) - \frac{1-n}{R_2} y \tag{25}$$

$$\theta_2 = \theta_1 - \frac{n-1}{R_1} y - \frac{1-n}{R_2} y = \theta_1 - \left(\frac{n-1}{R_1} + \frac{1-n}{R_2}\right) y \tag{26}$$

$$\frac{n-1}{R_1} + \frac{1-n}{R_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f} \tag{27}$$

lens maker equation, it tells what radii to use if you want to make a lens with f

$$\theta_2 = \theta_1 - \frac{y}{f} \tag{28}$$

$$\theta_2 = \frac{y}{z_2}, \quad \theta_1 = \frac{y}{z_1} \tag{29}$$

$$\frac{y}{z_2} = \frac{y}{z_1} - \frac{y}{f} \quad \Rightarrow \quad \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \tag{30}$$

flipped the sign of one z since it is going to the other direction

$\Rightarrow$ from the ??? rays

$$y_2 = -\frac{z_2}{z_1} y_1 \tag{31}$$

magnification

two equations which can be used to ??? images of refractive optics in the limit of paraxial approximation

$$\sin(x) \approx x - \frac{x^3}{6} + \frac{x^5}{120} + ... \tag{32}$$

that is

$$\frac{x^3}{6} \ll x \quad \text{or} \quad \frac{\mathrm{x}^2}{6} \ll 1 \quad \text{or} \quad \mathrm{x} \ll 0.4 \text{ in radians} \tag{33}$$

this is not true for microscopy lenses, for example, there we have angles up to 80°, but it is then still a first oder approximation

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