1_liquid_crystals

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1 Liquid Crystals

$$Q_{\alpha\beta}(r) = \frac{1}{N}\sum_i \left(u_\alpha^{(i)}u_\beta^{(i)} - \frac{1}{3}\delta_{\alpha\beta}\right)$$

- $Q_{\alpha\beta} = Q_{\beta\alpha}$ since $u_\alpha^{(i)}u_\beta^{(i)} = u_\beta^{(i)}u_\alpha^{(i)}$
- $$\begin{aligned}\operatorname{Tr} Q_{\alpha\beta} &= Q_{xx} + Q_{yy} + Q_{zz} = \\ &= \frac{1}{N}\sum_i \left[\left(u_x^{(i)}\right)^2 + \left(u_y^{(i)}\right)^2 + \left(u_z^{(i)}\right)^2 - 1\right] = 0\end{aligned}$$
- $$\begin{aligned}u_x &= \sin\theta\cos\phi \\ u_y &= \sin\theta\sin\phi \\ u_z &= \cos\theta\end{aligned}$$

$$Q_{\alpha\beta} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta f(\theta,\phi)\left(u_\alpha u_\beta - \frac{1}{3}\delta_{\alpha\beta}\right)$$

$$\begin{aligned}Q_{zz} &= \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \left(\cos^2\theta - \frac{1}{3}\right) = \\ &= \frac{1}{6}\left(x^3 - x\right)\Big|_{-1}^{1} = 0.\end{aligned}$$

prolate

$$\mathbf{Q}^{\text{prolate}} = \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix}$$

oblate

$$\mathbf{Q}^{\text{oblate}} = \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

$$Q_{\alpha\beta} = S\left(n_\alpha n_\beta - \frac{1}{3}\delta_{\alpha\beta}\right)$$

$$Q_{zz} = \frac{2}{3}S, \quad Q_{xx} = Q_{yy} = -\frac{1}{3}S$$

$$S = \int_0^{\pi} \left(1 - \frac{3}{2}\sin^2\theta\right) f(\theta)\sin\theta d\theta$$

$$f(\theta) = \int_0^{2\pi} f(\theta, \phi) d\phi$$