

1_liquid_crystals

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1 Liquid Crystals

$$Q_{\alpha\beta}(r) = \frac{1}{N} \sum_i \left(u_{\alpha}^{(i)} u_{\beta}^{(i)} - \frac{1}{3} \delta_{\alpha\beta} \right)$$

- $Q_{\alpha\beta} = Q_{\beta\alpha}$ since $u_{\alpha}^{(i)} u_{\beta}^{(i)} = u_{\beta}^{(i)} u_{\alpha}^{(i)}$

$$\text{Tr } Q_{\alpha\beta} = Q_{xx} + Q_{yy} + Q_{zz} =$$

- $$= \frac{1}{N} \sum_i \left[\left(u_x^{(i)} \right)^2 + \left(u_y^{(i)} \right)^2 + \left(u_z^{(i)} \right)^2 - 1 \right] = 0$$

$$u_x = \sin \theta \cos \phi$$

- $u_y = \sin \theta \sin \phi$

$$u_z = \cos \theta$$

$$Q_{\alpha\beta} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta f(\theta, \phi) \left(u_{\alpha} u_{\beta} - \frac{1}{3} \delta_{\alpha\beta} \right)$$

$$\begin{aligned} Q_{zz} &= \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \left(\cos^2 \theta - \frac{1}{3} \right) = \\ &= \frac{1}{6} (x^3 - x) \Big|_{-1}^1 = 0. \end{aligned}$$

prolate

$$\mathbf{Q}^{\text{prolate}} = \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix}$$

oblate

$$\mathbf{Q}^{\text{oblate}} = \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

$$Q_{\alpha\beta} = S \left(n_{\alpha} n_{\beta} - \frac{1}{3} \delta_{\alpha\beta} \right)$$

$$Q_{zz} = \frac{2}{3}S, \quad Q_{xx} = Q_{yy} = -\frac{1}{3}S$$

$$S = \int_0^\pi \left(1 - \frac{3}{2} \sin^2 \theta\right) f(\theta) \sin \theta d\theta$$

$$f(\theta) = \int_0^{2\pi} f(\theta, \phi) d\phi$$