28

Laws of Similitude

Need for model testing

Even with cutting-edge CFD tools naval architects cannot reliably compute the resistance of a ship for daily design purposes. Although accuracy and reliability of CFD results have steadily improved, model tests will be used to confirm results for the foreseeable future. Since most ships are one-of-a-kind designs, prototypes cannot be used to improve the design which further increases the importance of model tests.

However, model tests are useful only if they are accompanied by methods which transfer physical quantities measured at model scale to the full scale design. This chapter explains the fundamental concepts of similarity and points out the difficulties that arise if only partial dynamic similarity is possible.

Learning Objectives

At the end of this chapter students will be able to:

- understand the concepts of geometric, kinematic, and dynamic similarity
- interpret Froude number, Reynolds number, and Euler number in the context of dynamic similarity
- identify the limitations of similarity in ship model tests
- discuss Froude's hypothesis and the ITTC form factor method

28.1 Similarities

Required similarities

Three basic conditions regulate the conversion of model test results into full scale results:

1. geometric similarity,
2. kinematic similarity, and
3. dynamic similarity.

Fundamentals of Ship Hydrodynamics: Fluid Mechanics, Ship Resistance and Propulsion, First Edition.
Lothar Birk.

Companion website: www.wiley.com/go/birk/hydrodynamics

The individual similarities will be discussed below. Unfortunately, we often cannot maintain all three similarities. Reliable approximations have to be introduced which provide corrections for the condition of partial similarity in the model test.

28.1.1 Geometric similarity

Geometric similarity

The necessity for geometric similarity is the most obvious. If you want to know the resistance of a sphere, you most likely will not get usable results by testing a box. We require that our ship is converted into a model by applying a fixed scale factor λ to all linear dimensions. Any length L_S at the ship will be λ times the corresponding length on the model L_M or

Scale of length, area and volume

$$\lambda = \frac{L_S}{L_M} \qquad \text{scaling of lengths} \tag{28.1}$$

Areas are of dimension (length unit) × (length unit) and volumes are of dimension (length unit)^3. Consequently, areas and volumes will scale according to

$$\lambda^2 = \frac{S_S}{S_M} \qquad \text{scaling of areas} \tag{28.2}$$

$$\lambda^3 = \frac{V_S}{V_M} \qquad \text{scaling of volumes} \tag{28.3}$$

As a result of the constant scale in length, angles are the same for full scale ship and model.

The recommended ITTC procedure on model ships (ITTC, 2011) states that the model should be accurate within ±1 mm in linear dimensions for all surfaces that get in contact with the water. The surfaces should be smoothly sanded with 300 to 400 grit sand paper. Small appendages, like struts or bilge keels, may not be reproduced in many cases, because members may become too thin at model scale. Details of the surface like weld seems or rivets are not reproduced at model scale.

Tank dimensions

In the context of geometric similarity, it is easily forgotten that we should scale the ocean environment as well. Due to space limitations, however, depth and especially width of our model oceans are limited. A careful assessment has to be performed to determine whether restricted water depth and tank width influence the model test results or not. If you are a stickler for detail, water particles should be scaled too, but let us file this under impossible.

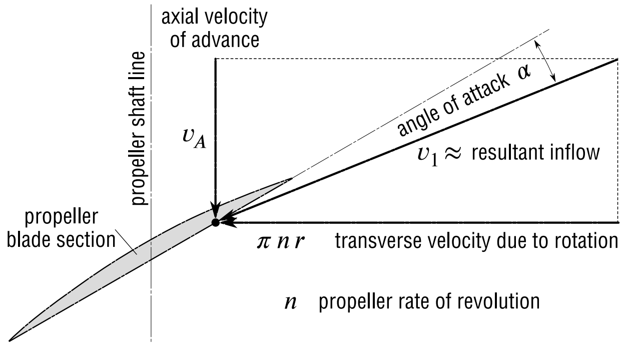
28.1.2 Kinematic similarity

Time scale

Kinematic similarity in fluid mechanics is often referred to as the similarity of streamline patterns. Fluid particles should not only take geometric similar paths, but, in addition, should maintain a constant time scale τ at full scale and model scale.

$$\tau = \frac{\Delta T_S}{\Delta T_M} \tag{28.4}$$

Figure 28.1 Simplified flow pattern at a propeller blade section. Kinematic similarity requires that the angle of attack remains the same for full scale and model propeller blade section



As a consequence, ratios of velocities are constant between full and model scale.

Similarity of velocities

A section of a rotating propeller blade may serve as an example (Figure 28.1). In order to have the same flow pattern, the angle of attack α between blade section and inflow must be equal in full scale and model scale. This is achieved if the ratio of axial flow velocity (speed of advance v_A) and tangential velocity (transverse speed due to propeller rotation, $1/2\pi\, n\, D$ for blade tip) is constant. For convenience we omit the factor π.

$$\left(\frac{v_{AM}}{n_M\, D_M}\right) = \left(\frac{v_{AS}}{n_S\, D_S}\right) = J \tag{28.5}$$

The fraction $v_A/(n\, D)$ is called advance coefficient J. Obviously, kinematic similarity cannot be achieved without geometric similarity.

Scale of velocities and acceleration

Together the scales for geometry λ and time τ define the scales of velocities and accelerations.

$$\text{velocity scale} \qquad \frac{v_S}{v_M} = \frac{\dfrac{L_S}{\Delta T_S}}{\dfrac{L_M}{\Delta T_M}} = \frac{\dfrac{L_S}{L_M}}{\dfrac{\Delta T_S}{\Delta T_M}} = \frac{\lambda}{\tau} \tag{28.6}$$

$$\text{acceleration scale} \qquad \frac{a_S}{a_M} = \frac{\dfrac{L_S}{\Delta T_S^2}}{\dfrac{L_M}{\Delta T_M^2}} = \frac{\dfrac{L_S}{L_M}}{\dfrac{\Delta T_S^2}{\Delta T_M^2}} = \frac{\lambda}{\tau^2} \tag{28.7}$$

28.1.3 Dynamic similarity

Types of forces

Dynamic similarity requires that forces in full and model scale have the same direction and the ratio κ of magnitudes full scale to model scale is constant. According to the Navier-Stokes equations for incompressible flow, four classes of forces act on our ship:

1. inertia force
2. gravity force (volume force)

3. friction force (surface force, shear stress)
4. pressure force (surface force, normal stress)

Below we determine the force ratios and the requirements for dynamic similarity.

Inertia forces

Inertia forces

Inertia forces may be expressed as mass times acceleration: $F_i = m\,a$. Mass is usually replaced by volume and density $m = \rho\,V$. As a first part of dynamic similarity, the ratio of inertia forces must be constant.

$$\kappa_i = \frac{F_{iS}}{F_{iM}} = \frac{\rho_S\,V_S\,a_S}{\rho_M\,V_M\,a_M} = \frac{\rho_S}{\rho_M}\,\lambda^3\,\frac{\lambda}{\tau^2} = \frac{\rho_S}{\rho_M}\,\frac{\lambda^4}{\tau^2} \tag{28.8}$$

Whatever geometric scale and time scale have been chosen, if only inertia forces would act on a system, they would scale by the ratio of the fluid densities times λ^4/τ^2. Of course, whenever there are inertia forces, at least one other class of force will act too. Otherwise no accelerations could occur according to Newton's second law.

Inertia and gravity forces

Gravity forces

The ratio of gravity forces $F_g = m\,g$ would be

$$\kappa_g = \frac{F_{gS}}{F_{gM}} = \frac{\rho_S\,V_S\,g_S}{\rho_M\,V_M\,g_M} = \frac{\rho_S}{\rho_M}\,\frac{g_S}{g_M}\,\lambda^3 \tag{28.9}$$

In most practical cases, gravitational acceleration at full and model scale will be the same: $g = g_S = g_M$. However, if you some day conduct experiments on the moon ...

Inertia and gravity forces

If *both* inertia and gravity forces act, dynamic similarity requires that the force ratios κ_i and κ_g are equal. There can be only one force ratio. If $\kappa_i = \kappa_g$ then

$$\frac{\rho_S}{\rho_M}\,\frac{\lambda^4}{\tau^2} = \frac{\rho_S}{\rho_M}\,\frac{g_S}{g_M}\,\lambda^3$$

or

$$\tau^2 = \frac{g_M}{g_S}\,\lambda. \tag{28.10}$$

If $g = g_S = g_M$, time scale must be equal to the square root of the geometric scale

$$\tau = \sqrt{\lambda} \tag{28.11}$$

Enforcing dynamic similarity of both inertia and gravity forces makes the time scale dependent on the geometric scale.

The scale of velocities (28.6) changes to

$$\frac{v_S}{v_M} = \frac{\lambda}{\tau} = \frac{\lambda}{\sqrt{\lambda}} = \sqrt{\lambda} = \sqrt{\frac{L_S}{L_M}} \tag{28.12}$$

Considering the first and last terms in Equation (28.12), we find the relation of *corresponding velocities*

$$\frac{v_S}{\sqrt{L_S}} = \frac{v_M}{\sqrt{L_M}} \tag{28.13}$$

Froude number This relation was published by the French marine engineer Ferdinand Reech. However, Froude (1868) was the first to put it to practical use in ship model testing. If we include the ratio of gravitational acceleration in Equation (28.10), we get

$$\frac{v_S}{v_M} = \frac{\lambda}{\tau} = \frac{\lambda}{\sqrt{\frac{g_M}{g_S}\lambda}} = \sqrt{\frac{g_S\, L_S}{g_M\, L_M}} \tag{28.14}$$

or again combining the first and last terms

$$\frac{v_S}{\sqrt{g_S\, L_S}} = \frac{v_M}{\sqrt{g_M\, L_M}} \tag{28.15}$$

This is obviously the definition of the Froude number Fr. Consequently, conducting model tests where the Froude number is equal for full scale ship and model ensures dynamic similarity of inertia and gravity forces (Froude similarity).

In the case of Froude similarity, the velocity at model scale is obtained by dividing full scale velocity by the square root of the geometric scale (assuming equal $g = g_S = g_M$).

$$v_M = \frac{v_S}{\sqrt{\lambda}} \qquad \text{and} \qquad v_S = v_M \sqrt{\lambda} \tag{28.16}$$

This velocity ratio is very practical. For example, with a typical geometric scale of $\lambda = 25$, a full scale ship speed of 10 m/s (almost 20 kn) translates into a model speed of 2 m/s. *In cases where only inertia and gravity forces are relevant*, full scale forces F_S are obtained from model scale forces F_M by the following simple conversion:

$$F_S = \frac{\rho_S}{\rho_M}\frac{g_S}{g_M}\lambda^3 F_M \qquad \text{for purely inertia and gravity forces} \tag{28.17}$$

Inertia and friction forces

Friction forces In real fluids a more or less noticeable friction force exists. Friction forces can be modeled as shear stress times wetted surface: $F_v = \mu \frac{\partial u}{\partial y} S$. Here, μ is the dynamic viscosity of the fluid and $\frac{\partial u}{\partial y}$ is the change in velocity perpendicular to the flow direction. Thus, the friction forces for ship and model form the ratio

$$\kappa_v = \frac{F_{vS}}{F_{vM}} = \frac{\mu_S \frac{\partial u_S}{\partial y_S} S_S}{\mu_M \frac{\partial u_M}{\partial y_M} S_M} = \frac{\mu_S}{\mu_M}\frac{\frac{\lambda}{\tau}}{\lambda}\lambda^2 = \frac{\mu_S}{\mu_M}\frac{\lambda^2}{\tau} \tag{28.18}$$

Inertia and friction forces Assuming inertia and friction forces are present, their respective force ratios must be equal to maintain dynamic similarity. For $\kappa_i = \kappa_\nu$ we get

$$\frac{\rho_S}{\rho_M}\,\frac{\lambda^4}{\tau^2} = \frac{\mu_S}{\mu_M}\,\frac{\lambda^2}{\tau} \tag{28.19}$$

or by introducing the kinematic viscosity $\nu = \mu/\rho$

$$\frac{\lambda^2}{\tau} = \frac{\mu_S}{\rho_S}\,\frac{\rho_M}{\mu_M} = \frac{\dfrac{\mu_S}{\rho_S}}{\dfrac{\mu_M}{\rho_M}} = \frac{\nu_S}{\nu_M}$$

Thus the necessary time scale is equal to

$$\tau = \frac{\nu_M}{\nu_S}\lambda^2 \tag{28.20}$$

In contrast to the similarity of combined inertia and gravity forces, the time scale is now proportional to the squared geometric scale.

$$\frac{\upsilon_S}{\upsilon_M} = \frac{\lambda}{\tau} = \frac{\lambda}{\dfrac{\nu_M}{\nu_S}\,\lambda^2} = \frac{\nu_S}{\nu_M}\,\frac{1}{\lambda} = \frac{\nu_S}{\nu_M}\,\frac{L_M}{L_S} \tag{28.21}$$

This yields

$$\frac{\upsilon_M\,L_M}{\nu_M} = \frac{\upsilon_S\,L_S}{\nu_S} \tag{28.22}$$

The Reynolds number $Re = \upsilon L/\nu$ governs the dynamic similarity of inertia and friction forces. Equal Reynolds number results in a very impractical velocity ratio. If model tests are conducted in the same fluid, i.e. $\nu_S = \nu_M$, model speed is equal to full scale speed times geometric scale

$$\upsilon_M = \lambda\,\upsilon_S \tag{28.23}$$

For a geometric scale of $\lambda = 25$ and a ship speed of 10 m/s, a model speed of $\upsilon_M =$ 250 m/s would be necessary to maintain dynamic similarity of inertia and friction forces. This is in the range of the cruising speed of a commercial airliner and very unlikely to be realized in a towing tank!

Inertia, gravity, and friction forces

Inertia, gravitational and friction forces

Our ship resistance problem includes inertia, gravity, *and* friction forces. In order to maintain dynamic similarity, the scales of all three classes of forces have to be equal:

$$\kappa_i = \kappa_g = \kappa_\nu \tag{28.24}$$

The second part yields with $\kappa_g = \kappa_\nu$

$$\begin{aligned}\frac{\rho_S}{\rho_M}\,\frac{g_S}{g_M}\,\lambda^3 &= \frac{\mu_S}{\mu_M}\,\frac{\lambda^2}{\tau}\\ \frac{g_S}{g_M}\,\lambda &= \frac{\nu_S}{\nu_M}\,\frac{1}{\tau}\end{aligned} \tag{28.25}$$

We enforce similarity of inertia and gravitational forces by selecting the time scale proportional to the square root of the geometric scale – see Equation (28.10). This also leads to more practical model speeds. Substituting $\tau = \sqrt{\lambda\, g_M / g_S}$ in (28.25) yields the condition

$$\lambda \sqrt{\lambda} \;=\; \frac{\nu_S}{\nu_M}\,\frac{g_S}{g_M} \tag{28.26}$$

With $g_S = g_M$, kinematic viscosity of the model fluid must be

$$\nu_M \;=\; \frac{\nu_S}{\lambda^{3/2}} \tag{28.27}$$

in order to maintain similarity of inertia, gravity, and friction forces. For a model scale of $\lambda = 25$, kinematic viscosity ν_M in the model test would have to be as small as $0.008\,\nu_S$, i.e. eight thousandths of the kinematic viscosity of seawater.

A fluid with a significant smaller kinematic viscosity than water is mercury. Not that anyone would seriously consider filling a towing tank with this extremely dangerous substance, but let us play with some numbers. The ratio of kinematic viscosities of water H_2O and mercury Hg is $\dfrac{\nu_S}{\nu_M} = \dfrac{\nu_{H_2O}}{\nu_{Hg}} \approx 8$. The resultant model scale would be as small as $\sqrt{\lambda^3} = 8$ or $\lambda = 4$. Unfortunately, the models would still be way too large for common ship sizes and available test facilities.

Partial dynamic similarity

In summary, we will not be able to conduct our ship resistance model test with complete dynamic similarity. We will have only *partial dynamic similarity* for inertia and gravity forces. All friction force components measured in the model test will have to be carefully corrected for scale effects before they can be applied to the full scale ship. For this we make use of the ITTC 1957 model–ship correlation line, which allows as to *compute* the frictional part of ship resistance.

$$C_F \;=\; \frac{0.075}{\left[\log_{10}(Re) - 2\right]^2} \tag{2.18}$$

We apply this equation to the model as well as to the ship. Of course, the Reynolds numbers will be quite different.

Inertia and pressure forces

Pressure forces

Remaining are the pressure forces which can be modeled as pressure times area $F_p = p\,S$. The ratio of pressure forces is

$$\kappa_p \;=\; \frac{p_S\,S_S}{p_M\,S_M} \;=\; \frac{p_S}{p_M}\,\lambda^2 \tag{28.28}$$

Inertia and pressure forces

Enforcing dynamic similarity for inertia and pressure forces requires $\kappa_i = \kappa_p$. Then

$$\frac{\rho_S}{\rho_M}\,\frac{\lambda^4}{\tau^2} \;=\; \frac{p_S}{p_M}\,\lambda^2$$

$$\frac{p_S}{p_M} \;=\; \frac{\rho_S}{\rho_M}\left(\frac{\lambda}{\tau}\right)^2 \tag{28.29}$$

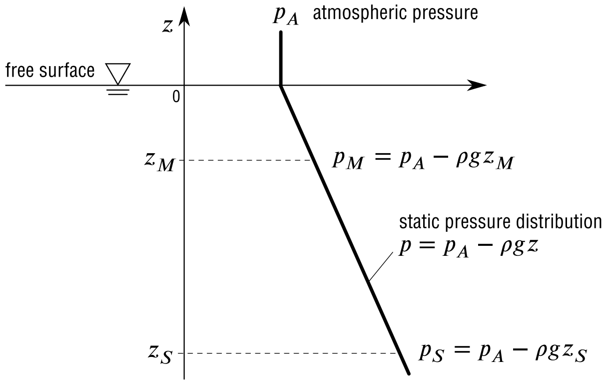


Figure 28.2 Relationship of total static pressures for model and full scale ship

The ratio λ/τ is the the scale of velocities, i.e.

$$\frac{p_S}{p_M} = \frac{\rho_S}{\rho_M} \frac{v_S^2}{v_M^2} \tag{28.30}$$

$$\frac{p_S}{\frac{1}{2}\rho_S\, v_S^2} = \frac{p_M}{\frac{1}{2}\rho_M\, v_M^2} \tag{28.31}$$

An Euler number or a dimensionless pressure coefficient $\overline{C}_P = 2p/(\rho v^2)$ can be used to characterize dynamic similarity for systems subject to inertia and pressure forces.

Inertia, gravity and pressure forces

We already concluded that we cannot simultaneously attain dynamic similarity for inertia, gravity, and friction forces. What about the combination of inertia, gravity, and pressure forces? Again we choose the time scale according to Froude's law of similarity, i.e. $\tau = \sqrt{\lambda\, g_M/g_S}$. The required ratio of pressures becomes

$$\frac{p_S}{p_M} = \frac{\rho_S}{\rho_M}\, \lambda \tag{28.32}$$

Is this possible? Figure 28.2 compares the total static pressure for a depth z_M related to a model and the corresponding depth z_S of the full scale ship. Certainly $z_S/z_M = \lambda$. In this case the ratio of total static pressures yields

$$\frac{p_S}{p_M} = \frac{p_A - \rho_S\, g\, z_S}{p_A - \rho_M\, g\, z_M} \neq \frac{\rho_S}{\rho_M}\, \lambda \tag{28.33}$$

Since the atmospheric pressure is equal for full scale ship and model, we do *not* have the required dynamic similarity for the total pressures.

However, pressure forces are the result of pressure differences. The atmospheric pressure acts equally on all parts of the ship surface above and below the waterline. Hence, its resultant force vanishes. Comparing only the hydrostatic pressures for model and full scale reveals that the necessary similarity for pressure differences is satisfied.

$$\frac{\Delta p_S}{\Delta p_M} = \frac{-\rho_S\, g\, z_S}{-\rho_M\, g\, z_M} = \frac{\rho_S}{\rho_M}\, \lambda \tag{28.34}$$

As a consequence, we are able to maintain simultaneous dynamic similarity for inertia, gravity, and pressure forces.

Cavitation For some flow problems, similarity of absolute pressure is needed. *Cavitation* is one of these flow phenomena. Cavitation occurs when the pressure in the fluid reaches the vapor pressure level. Water changes from liquid into gaseous phase forming bubbles with low pressure gas in the flow. The bubbles are transported with the flow and collapse when they reach areas of higher pressure. This process can destroy propellers, rudders, or any other material surface nearby.

The inception of cavitation depends on the absolute pressure level. It is usually higher in the model than it is in the full scale ship, because we do not normally scale back the atmospheric pressure (equal for both). Therefore models appear cavitation free whereas the full scale vessel will show cavitation which can have significant effects on performance. To investigate this phenomenon, special towing tanks or circulating water tunnels are used that allow the atmospheric pressure above the water surface to be changed.

28.1.4 Summary

Summary The discussion of the requirements for dynamic similarity discloses one of the greatest difficulties of marine model testing: *We are unable to conduct our model tests under complete dynamic similarity.* Given the available conditions, we are not able to maintain similarity of friction forces. Elaborate procedures have been developed over the past 140 years to account for this deficiency. Although practical solutions have been found, you should always be aware of their limitations. Inertia, gravity, and pressure forces are modeled correctly – within the limitations regarding geometric and kinematic similarity – if the time scale is selected based on equal Froude numbers for full scale ship and model $Fr_M = Fr_S$.

28.2 Partial Dynamic Similarity

As we have learned in the previous section, full dynamic similarity is *not* achievable in practical ship model testing. We are unable to satisfy the requirements of equal Froude number and equal Reynolds number at the same time. The lack of full dynamic similarity has severe consequences on our model testing procedures. To emphasize the problem we first look at the *hypothetical* case of full dynamic similarity.

28.2.1 Hypothetical case: full dynamic similarity

Hypothetical full dynamic similarity Assuming the requirements of geometric similarity and kinematic similarity are satisfied, a hypothetical full dynamic similarity is achieved when all force scales are equal:

$$\kappa_i = \kappa_g = \kappa_f = \kappa_p \tag{28.35}$$

Putting Equation (28.35) into words, the ratios of full scale to model scale forces have to be the same for all type of forces important to the problem at hand.

$$\frac{F_{iS}}{F_{iM}} = \frac{F_{gS}}{F_{gM}} = \frac{F_{fS}}{F_{fM}} = \frac{F_{pS}}{F_{pM}} \tag{28.36}$$

Here, scales would have to be equal for inertia, gravity (representing body forces), frictional (representing surface forces due to viscous stress), and pressure force (representing surface forces due to normal pressure).

Hypothetical full scale predictions

Making full scale predictions from model test results would be easy, *if* we could have full dynamic similarity. Since all forces would scale the same way, the actual force scale could be derived from the physical force unit.

$$\mathrm{N} = \mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}^2} \qquad \longrightarrow \qquad \kappa = \frac{\rho_S}{\rho_M}\lambda^3\,\frac{\lambda}{\tau^2} \tag{28.37}$$

Applying – for practical reasons – the time scale that follows from similarity of inertia and gravity forces $\tau = \sqrt{\lambda}$ (follows from $Fr_S = Fr_M$) yields:

$$\kappa = \frac{\rho_S}{\rho_M}\lambda^3 \tag{28.38}$$

Therefore, all model scale forces would scale up to full scale like volumes (λ^3) multiplied by the scale of densities.

$$\kappa_i = \kappa_g = \kappa_f = \kappa_p = \frac{\rho_S}{\rho_M}\lambda^3 \qquad \text{hypothetical} \tag{28.39}$$

Hypothetical full dynamic similarity example

Assume that we are testing a model of geometric scale $\lambda = 40$ in fresh water of density $\rho_M = 1000.0\,\mathrm{kg/m^3}$ under the condition that $Fr_M = Fr_S$. In the hypothetical case of full dynamics similarity, a drag force of $F_{DM} = 25.21\,\mathrm{N}$ measured at model scale would be equivalent to a full scale drag force of

$$\begin{aligned}
\frac{F_{DS}}{F_{DM}} &= \frac{\rho_S}{\rho_M}\lambda^3 \\
F_{DS} &= \frac{\rho_S}{\rho_M}\lambda^3 \cdot F_{DM} \\
F_{DS} &= \frac{1026.0}{1000.0}\,40^3 \cdot 25.21\,\mathrm{N} \\
&= 1\,655\,389.44\,\mathrm{N} = 1655.4\,\mathrm{kN}
\end{aligned}$$

in salt water of density $\rho_S = 1026.0\,\mathrm{kg/m^3}$.

Dimensionless coefficients

It is common practice to make experimental results dimensionless. The selection of an appropriate normalizing force is up to the user. In ship hydrodynamics forces are mostly normalized by dynamic pressure $1/2\rho v^2$ multiplied with the wetted surface at rest S. The result is a dimensionless drag coefficient:

$$C_D = \frac{F_{DM}}{\frac{1}{2}\rho_M v_M^2 S_M} \tag{28.40}$$

The coefficient would be the same for model and full scale vessel because with

$$F_{DM} = C_D \frac{1}{2}\rho_M v_M^2 S_M \qquad \text{and} \qquad F_{DS} = C_D \frac{1}{2}\rho_S v_S^2 S_S \tag{28.41}$$

we get

$$\begin{aligned} \frac{F_{DS}}{F_{DM}} &= \frac{C_D \frac{1}{2}\rho_S v_S^2 S_S}{C_D \frac{1}{2}\rho_M v_M^2 S_M} \\ &= \frac{\rho_S}{\rho_M} \frac{v_S^2}{v_M^2} \frac{S_S}{S_M} \\ &= \frac{\rho_S}{\rho_M} \frac{\lambda^4}{\tau^2} \end{aligned} \tag{28.42}$$

Again, assuming we performed the model tests under the condition of equal Froude number, the time scale is fixed to $\tau = \sqrt{\lambda}$. Substituting the time scale τ into Equation (28.42) results in

$$\frac{F_{DS}}{F_{DM}} = \frac{\rho_S}{\rho_M} \lambda^3 \tag{28.43}$$

which is the force scale factor (28.39) we derived above.

Consequently, dimensionless coefficients would apply to both model and full scale, if the quantities (forces) could satisfy the conditions of full dynamic similarity.

28.2.2 Real world: partial dynamic similarity

As we have pointed out in the previous section, we cannot achieve full dynamic similarity. Conducting experiments under the requirement of equal Froude number achieves dynamic similarity for inertia, gravity, and pressure forces, but not for friction forces.

$$\kappa = \kappa_i = \kappa_g = \kappa_p \tag{28.44}$$

$$\kappa \neq \kappa_f \tag{28.45}$$

This complicates ship model testing tremendously. The dimensionless total calm water resistance coefficient

$$C_{TM} = \frac{R_{TM}}{\frac{1}{2}\rho_M v_M^2 S_M} \tag{28.46}$$

cannot be applied to the full scale vessel without further corrections.

$$C_{TS} \neq C_{TM} \tag{28.47}$$

28.2.3 Froude's hypothesis revisited

Froude's hypothesis

William Froude was the first to come up with a practical solution to the scaling problem (Froude, 1872). He postulated that the resistance has to be divided into two parts:

$$R_T = R_R + R_F \tag{28.48}$$

or in dimensionless form

$$C_T = C_R + C_F \tag{28.49}$$

Residuary resistance

1. The first part contains the forces with full dynamic similarity if the tests are carried out for corresponding speeds which is equivalent to equal Froude number with $g_S = g_M$.

$$\frac{v_S}{\sqrt{L_S}} = \frac{v_M}{\sqrt{L_M}} \tag{28.50}$$

   This resistance part became known as the residuary resistance R_R. Its dimensionless coefficient applies to both model and full scale vessel:

$$C_R = \frac{R_{RM}}{\frac{1}{2}\rho_M v_M^2 S_M} = C_{RM} = C_{RS} \tag{28.51}$$

Frictional resistance

2. The second part is the frictional resistance R_F which is a function of the Reynolds number. Since Reynolds numbers for ship and model are different $Re_S \neq Re_M$, viscous forces are not dynamically similar. The frictional resistance coefficient will differ for model and full scale vessel. Consequently, a method is needed to scale the model test results.

At model scale the total resistance R_{TM} is measured and used to find the residuary resistance coefficient C_R which applies to both the model and the full scale vessel:

$$C_R = C_{TM} - C_{FM} \tag{28.52}$$

The full scale resistance coefficient is derived by summing up the residuary resistance, the frictional resistance (full scale), and some more or less empirical corrections:

$$C_{TS} = C_R + C_{FS} + \text{corrections} \tag{28.53}$$

For this procedure to work, we need a method to derive frictional resistance coefficients C_F for the model and the full scale vessel. They cannot be directly measured with the ship model. Froude determined a curve for C_F experimentally by testing flat plates of different sizes and roughness. Nowadays we employ the ITTC 1957 model–ship correlation line (2.18).

Improving dynamic similarity

Froude knew that his hypothesis was flawed because the residuary resistance still contains the viscous pressure resistance R_{VP}. Since R_{VP} is influenced by the viscosity of the fluid it depends at least somewhat on the Reynolds number. Therefore, the residuary resistance C_R coefficient does not reflect full dynamic similarity.

The ITTC introduced the form factor k to eliminate most of the viscous effects from the force determined under Froude similarity. The dynamically similar force is the wave resistance.

$$C_W = C_{TM} - (1+k)C_{FM} \tag{28.54}$$

This is better in theory but still hard to accomplish in practice, because accurate determination of the form factor is difficult. We will study the procedures recommended by the ITTC in the following chapter.

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Self Study Problems

1. Provide definitions of Froude and Reynolds number (according to the ITTC).
2. What is the greatest difficulty in marine model testing?
3. Under the assumption of equal Froude number for model and full scale vessel, how do the following quantities scale if the scale of the model is $\lambda = L_S/L_M = 50$?
   (a) velocity $v_S/v_M =$?
   (b) acceleration $a_S/a_M =$?
   (c) force $F_S/F_M =$?
   (d) torque $Q_S/Q_M =$?
   (e) wave period $T_S/T_M =$?
   (f) wave frequency $\omega_S/\omega_M =$?