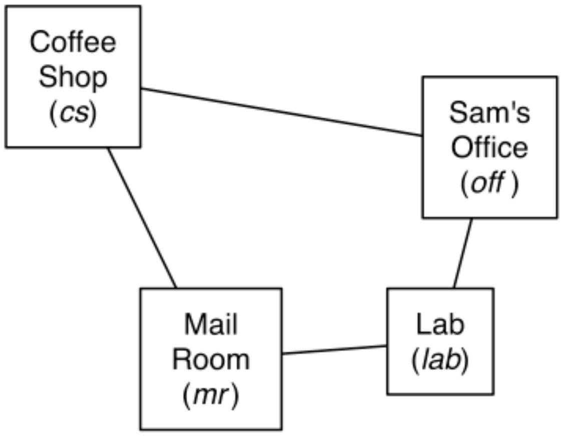
WEEK 3 TUTORIAL SOLUTIONS

PART I

Activity 1. Consider the planning problem from the lectures.
(a) Give the STRIPS representations for the pick up mail (pum) and deliver mail (dm) actions.
(b) Give the feature-based representation of the MW and RHM features.

Features:
RLoc – Rob's location
RHC – Rob has coffee
SWC – Sam wants coffee
MW – Mail is waiting
RHM – Rob has mail

Actions:
mc – move clockwise
mcc – move counterclockwise
puc – pickup coffee
dc – deliver coffee
pum – pickup mail
dm – deliver mail



(a) The pickup mail action (pum) is defined using STRIPS by: *Preconditions: RLoc = mr ∧ mw*
Effects: [¬mw, rhm]
The deliver mail action (dm) is defined using STRIPS by:
Preconditions: RLoc = off ∧ rhm Effects: [¬rhm]
(b) The MW feature can be axiomatised by defining when *MW = true (written as mw): mw' ← mw ∧ Action ≠ pum*
The RHM feature can be axiomatised by defining when *RHM = true (written as rhm): rhm' ← Action = pum*
rhm' ← rhm ∧ Action ≠ dm

Activity 2. Formulate the blocks world using STRIPS planning operators. The actions are stack (move one block to the top of another) and unstack (move one block to the table). The robot can hold only one block at a time.

To simplify the world, assume the only objects are the blocks and the table, and that the only relations are the on relation between (table and) blocks and the clear predicate on table and blocks. Also assume that it is not possible for more than one block to directly support another block (and vice versa).

stack(A, B):
Preconditions: clear(A) ∧ clear(B)
Effects: on(A, B) ∧ ¬ clear(B) unstack(A):
Preconditions: clear(A) ∧ on(A, B)
Effects: on(A, Table) ∧ clear(B) ∧ ¬on(A, B)

Note: The Effects could also be written as an addles contain only the positive literals and the delete list, containing only the negative literals.

PART II

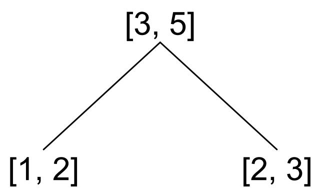
Activit 1. Consider the task of predicting whether children are likely to be hired to play members of the Von Trapp Family in a production of The Sound of Music, based on these data:

height	**hair**	**eyes**	**hired**
short	blond	blue	+
tall	red	blue	+
tall	blond	blue	+
tall	blond	brown	–
short	dark	blue	–
tall	dark	blue	–
tall	dark	brown	–
short	blond	brown	–

(1) Compute the information (entropy) gain for each of the three attributes (height, hair, eyes) in terms of classifying objects as belonging to the class, + or – .

(2) Construct a decision tree based on the minimum entropy principle.

(1) There are 3 objects in class '+' and 5 in '-', so the entropy is:
Entropy(parent) = $\Sigma_i P_i \log_2 P_i$ = -(3/8)log(3/8) - (5/8)log(5/8) = 0.954
Suppose we split on height:



Of the 3 'short' items, 1 is '+' and 2 are '-', so Entropy(short) = -(1/3)log(1/3) - (2/3)log(2/3) = 0.918
Of the 5 'tall' items, 2 are '+' and 3 are '-', so Entropy(tall) = -(2/5)log(2/5) - (3/5)log(3/5) = 0.971
The average entropy after splitting on 'height' is Entropy(height) = (3/8)(0.918) + (5/8)(0.971) = 0.951
The information gained by testing this attribute is: 0.954 - 0.951 = 0.003 (i.e. very little)

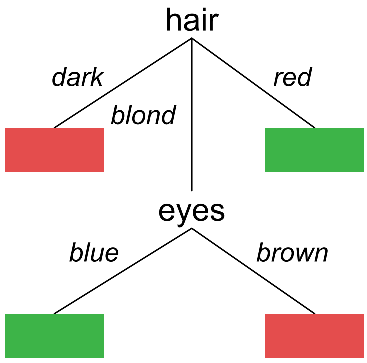
If we try splitting on 'hair' we find that the branch for 'dark' has 3 items, all '-' and the branch for 'red' has 1 item, in '+'. Thus, these branches require no further information to make a decision. The branch for 'blond' has 2 '+' and 2 '-' items and so requires 1 bit. That is,

Entropy(hair) = (3/8)(0) + (1/8)(0) + (4/8)(1) = 0.5
and the information gained by testing hair is 0.954 - 0.5 = 0.454 bits.

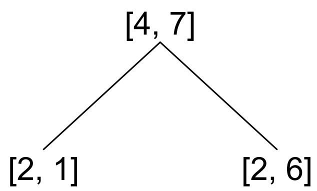
By a similar calculation, the entropy for testing 'eyes' is (5/8)(0.971) + (3/8)(0) = 0.607, so the information gained is 0.954 - 0.607 = 0.347 bits.

Thus 'hair' gives us the maximum information gain.

(2) Since the 'blond' branch for hair still contains a mixed population, we need to apply the procedure recursively to these four items. Note that we now only need to test 'height' and 'eyes' since the 'hair' attribute has already been used. If we split on 'height', the branch for 'tall' and 'short' will each contain one '+' and one '-', so the entropy gain is zero. If we split on 'eyes', the 'blue' brach contains two '+'s and the 'brown' branch two '-'s, so the tree is complete:



Activity 2. The Laplace error estimate for pruning a node in a Decision Tree is given by:



where is the total number of items, is the number of items in the majority class and is the number of classes. Given the following sub-tree, should the children be pruned or not? Show your calculations.

Error(Parent) = 1 - (7+1)/(11+2) = 1 - 8/13 = 5/13 = 0.385
Error(Left) = 1 - (2+1)/(3+2) = 1 - 3/5 = 2/5 = 0.4
Error(Right) = 1 - (6+1)/(8+2) = 1 - 7/10 = 3/10 = 0.3
Backed Up Error = (3/11)*(0.4) + (8/11)*(0.3) = 0.327 < 0.385
Since Error of Parent is larger than Backed Up Error ⇒ Don't Prune

Activity 3. Construct a Decision Tree for the following set of examples.

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

What class is assigned to the instance {D15, Sunny, Hot, High, Weak}?

1. (i) $Values(Outlook) = \{sunny, overcast, rain\}$

$$S = [9+, 5-]$$
$$S_{sunny} \leftarrow [2+, 3-]$$
$$S_{overcast} \leftarrow [4+, 0-]$$
$$S_{rain} \leftarrow [3+, 2-]$$
$$Gain(S, Outlook) = Entropy(S) - \sum_{v=\{sunny,overcast,rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$
$$= Entropy(S) - \frac{5}{14} Entropy(S_{sunny}) - \frac{4}{14} Entropy(S_{overcast}) - \frac{5}{14} Entropy(S_{rain})$$
$$= 0.940 - \frac{5}{14} \times 0.971 - \frac{4}{14} \times 0 - \frac{5}{14} \times 0.971$$
$$= 0.247$$
$$Entropy(S) = Entropy([9+, 5-]) = -\frac{9}{14} log_2 \frac{9}{14} - \frac{5}{14} log_2 \frac{5}{14}$$
$$= 0.940$$
$$Entropy(S_{sunny}) = Entropy([2+, 3-]) = -\frac{2}{5} log_2 \frac{2}{5} - \frac{3}{5} log_2 \frac{3}{5} = 0.971$$
$$Entropy(S_{overcast}) = Entropy([4+, 0-]) = -\frac{4}{4} log_2 \frac{4}{4} - \frac{0}{4} log_2 \frac{0}{4} = 0$$
$$Entropy(S_{rain}) = Entropy([3+, 2-]) = -\frac{3}{5} log_2 \frac{3}{5} - \frac{2}{5} log_2 \frac{2}{5} = 0.971$$

(ii) $Values(Temperature) = \{hot, mild, cool\}$
$S = [9+, 5-]$
$S_{hot} \leftarrow [2+, 2-]$
$S_{mild} \leftarrow [4+, 2-]$
$S_{cool} \leftarrow [3+, 1-]$
$Gain(S, Temperature) = Entropy(S) - \sum_{v=\{hot,mild,cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$
$= Entropy(S) - \frac{4}{14} Entropy(S_{hot}) - \frac{6}{14} Entropy(S_{mild}) - \frac{4}{14} Entropy(S_{cool})$
$= 0.940 - \frac{4}{14} \times 1.00 - \frac{6}{14} \times 0.918 - \frac{4}{14} \times 0.232$
$= 0.029$
$Entropy(S_{hot}) = Entropy([2+, 2-]) = -\frac{2}{4} log_2 \frac{2}{4} - \frac{2}{4} log_2 \frac{2}{4} = 1.00$
$Entropy(S_{mild}) = Entropy([4+, 2-]) = -\frac{4}{6} log_2 \frac{4}{6} - \frac{2}{6} log_2 \frac{2}{6} = 0.918$
$Entropy(S_{cool}) = Entropy([3+, 1-]) = -\frac{3}{4} log_2 \frac{3}{4} - \frac{1}{4} log_2 \frac{1}{4} = 0.232$

(iii) $Values(Humidity) = \{high, normal\}$
$S = [9+, 5-]$
$S_{high} \leftarrow [3+, 4-]$
$S_{normal} \leftarrow [6+, 1-]$
$Gain(S, Humidity) = Entropy(S) - \sum_{v=\{high,normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$
$= Entropy(S) - \frac{7}{14} Entropy(S_{high}) - \frac{7}{14} Entropy(S_{normal})$
$= 0.940 - \frac{7}{14} \times 0.985 - \frac{7}{14} \times 0.592$
$= 0.152$
$Entropy(S_{high}) = Entropy([3+, 4-]) = -\frac{3}{7} log_2 \frac{3}{7} - \frac{4}{7} log_2 \frac{4}{7} = 0.985$
$Entropy(S_{mild}) = Entropy([6+, 1-]) = -\frac{6}{7} log_2 \frac{6}{7} - \frac{1}{7} log_2 \frac{1}{7} = 0.592$

(iv) $Values(Wind) = \{weak, strong\}$
$S = [9+, 5-]$
$S_{weak} \leftarrow [6+, 2-]$
$S_{strong} \leftarrow [3+, 3-]$
$Gain(S, Wind) = Entropy(S) - \sum_{v=\{weak,strong\}} \frac{|S_v|}{|S|} Entropy(S_v)$
$= Entropy(S) - \frac{8}{14} Entropy(S_{weak}) - \frac{6}{14} Entropy(S_{strong})$
$= 0.940 - \frac{8}{14} \times 0.811 - \frac{6}{14} \times 1.00$
$= 0.048$
$Entropy(S_{weak}) = Entropy([6+, 2-]) = -\frac{6}{8} log_2 \frac{6}{8} - \frac{6}{8} log_2 \frac{6}{8} = 0.811$
$Entropy(S_{strong}) = Entropy([3+, 3-]) = -\frac{3}{6} log_2 \frac{3}{6} - \frac{3}{6} log_2 \frac{3}{6} = 1.00$

2. (i) $Values(Temperature) = \{hot, mild, cool\}$

$S_{sunny} = [2+, 3-]$
$S_{sunny,hot} \leftarrow [0+, 2-]$
$S_{sunny,mild} \leftarrow [1+, 1-]$
$S_{sunny,cool} \leftarrow [1+, 0-]$

$Gain(S_{sunny}, Temperature) = Entropy(S_{sunny}) - \sum_{v=\{hot,mild,cool\}} \frac{|S_{sunny,v}|}{|S_{sunny}|} Entropy(S_{sunny,v})$
$= Entropy(S) - \frac{2}{5}Entropy(S_{sunny,hot}) - \frac{2}{5}Entropy(S_{sunny,mild}) - \frac{1}{5}Entropy(S_{sunny,cool})$
$= 0.971 - \frac{2}{5} \times 0.00 - \frac{2}{5} \times 1.00 - \frac{1}{5} \times 0.00$
$= 0.571$

$Entropy(S_{sunny}) = Entropy([2+, 3-]) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.971$
$Entropy(S_{sunny,hot}) = Entropy([0+, 2-]) = -\frac{0}{2}log_2\frac{0}{2} - \frac{2}{2}log_2\frac{2}{2} = 0.00$
$Entropy(S_{sunny,mild}) = Entropy([1+, 1-]) = -\frac{1}{2}log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2} = 1.00$
$Entropy(S_{sunny,cool}) = Entropy([1+, 0-]) = -\frac{1}{1}log_2\frac{1}{1} - \frac{0}{1}log_2\frac{0}{1} = 0.00$

(ii) $Values(Humidity) = \{high, normal\}$

$S_{sunny} = [2+, 3-]$
$S_{sunny,high} \leftarrow [0+, 3-]$
$S_{sunny,normal} \leftarrow [2+, 0-]$

$Gain(S, Humidity) = Entropy(S_{sunny}) - \sum_{v=\{high,normal\}} \frac{|S_{sunny,v}|}{|S_{sunny}|} Entropy(S_{sunny,v})$
$= Entropy(S_{sunny}) - \frac{3}{5}Entropy(S_{sunny,high}) - \frac{2}{5}Entropy(S_{sunny,normal})$
$= 0.971 - \frac{3}{5} \times 0.00 - \frac{2}{5} \times 0.00$
$= 0.971$

$Entropy(S_{sunny,high}) = Entropy([0+, 3-]) = -\frac{0}{3}log_2\frac{0}{3} - \frac{3}{3}log_2\frac{3}{3} = 0.00$
$Entropy(S_{sunny,mild}) = Entropy([2+, 0-]) = -\frac{2}{2}log_2\frac{2}{2} - \frac{0}{2}log_2\frac{0}{2} = 0.00$

(iii) $Values(Wind) = \{weak, strong\}$

$S_{sunny} = [2+, 3-]$
$S_{weak} \leftarrow [1+, 2-]$
$S_{strong} \leftarrow [1+, 1-]$

$Gain(S, Wind) = Entropy(S_{sunny}) - \sum_{v=\{weak,strong\}} \frac{|S_{sunny,v}|}{|S_{sunny}|} Entropy(S_{sunny,v})$
$= Entropy(S) - \frac{3}{5}Entropy(S_{sunny,weak}) - \frac{2}{5}Entropy(S_{strong})$
$= 0.971 - \frac{3}{5} \times 0.918 - \frac{2}{5} \times 1.00$
$= 0.020$

$Entropy(S_{weak}) = Entropy([1+, 2-]) = -\frac{1}{3}log_2\frac{1}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.918$
$Entropy(S_{strong}) = Entropy([1+, 1-]) = -\frac{1}{2}log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2} = 1.00$

3. (i) $Values(Temperature) = \{hot, mild, cool\}$

$S_{rain} = [2+, 3-]$
$S_{rain,hot} \leftarrow [0+, 0-]$
$S_{rain,mild} \leftarrow [2+, 1-]$
$S_{rain,cool} \leftarrow [1+, 1-]$

$Gain(S_{rain}, Temperature) = Entropy(S_{rain}) - \sum_{v=\{hot,mild,cool\}} \frac{|S_{rain,v}|}{|S_{rain}|} Entropy(S_{rain,v})$
$= Entropy(S) - \frac{0}{5}Entropy(S_{rain,hot}) - \frac{3}{5}Entropy(S_{rain,mild}) - \frac{2}{5}Entropy(S_{rain,cool})$
$= 0.971 - \frac{0}{5} \times 0.00 - \frac{3}{5} \times 0.918 - \frac{2}{5} \times 1.00$
$= 0.020$

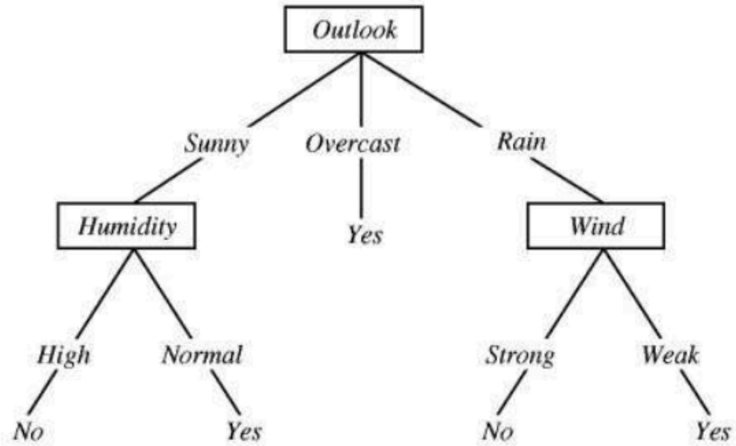
$Entropy(S_{rain}) = Entropy([3+, 2-]) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.971$
$Entropy(S_{rain,hot}) = Entropy([0+, 0-]) = -\frac{0}{0}log_2\frac{0}{0} - \frac{0}{0}log_2\frac{0}{0} = 0.00$
$Entropy(S_{rain,mild}) = Entropy([2+, 1-]) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.918$
$Entropy(S_{rain,cool}) = Entropy([1+, 1-]) = -\frac{1}{2}log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2} = 1.00$

(ii) $Values(Humidity) = \{high, normal\}$

$S_{rain} = [3+, 2-]$

$S_{rain,high} \leftarrow [1+, 1-]$
$S_{rain,normal} \leftarrow [1+, 1-]$
$Gain(S, Humidity) = Entropy(S_{rain}) - \sum_{v=\{high,normal\}} \frac{|S_{rain,v}|}{|S_{rain}|} Entropy(S_{rain,v})$
$= Entropy(S_{rain}) - \frac{2}{5} Entropy(S_{rain,high}) - \frac{3}{5} Entropy(S_{rain,normal})$
$= 0.971 - \frac{2}{5} \times 1.00 - \frac{3}{5} \times 0.551$
$= 0.020$
$Entropy(S_{rain,high}) = Entropy([1+, 1-]) = -\frac{1}{2} log_2 \frac{1}{2} - \frac{1}{2} log_2 \frac{1}{2} = 1.00$
$Entropy(S_{rain,mild}) = Entropy([2+, 1-]) = -\frac{2}{3} log_2 \frac{2}{3} - \frac{1}{3} log_2 \frac{1}{3} = 0.551$

(iii) $Values(Wind) = \{weak, strong\}$
$S_{rain} = [3+, 2-]$
$S_{weak} \leftarrow [3+, 0-]$
$S_{strong} \leftarrow [0+, 2-]$
$Gain(S, Wind) = Entropy(S_{rain}) - \sum_{v=\{weak,strong\}} \frac{|S_{rain,v}|}{|S_{rain}|} Entropy(S_{rain,v})$
$= Entropy(S) - \frac{3}{5} Entropy(S_{rain,weak}) - \frac{2}{5} Entropy(S_{strong})$
$= 0.971 - \frac{3}{5} \times 0.00 - \frac{2}{5} \times 0.00$
$= 0.971$
$Entropy(S_{weak}) = Entropy([3+, 0-]) = -\frac{3}{3} log_2 \frac{3}{3} - \frac{0}{3} log_2 \frac{0}{3} = 0.00$
$Entropy(S_{strong}) = Entropy([0+, 2-]) = -\frac{0}{2} log_2 \frac{0}{2} - \frac{2}{2} log_2 \frac{2}{2} = 0.00$



So the example is assigned the *No* class.