WEEK 4 TUTORIAL SOLUTIONS

PART I

Activity 1: Only 4% of the population are colour blind, but 7% of men are colour blind. What percentage of colour blind people are men?

If we assume 50% of the population are men, then the fraction of "colour blind men" is 0.5 * 0.07 = 0.035
This means the fraction of "colour blind women" is 0.04 - 0.035 = 0.005
Therefore, the fraction of colour blind people who are men is 0.035 / 0.04 = 87.5%.

Activity 2:

	toothache		**¬toothache**	
	catch	**¬catch**	**catch**	**¬catch**
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

1. Given the full joint distribution shown in Figure 13.3 (also on page 17 of the Uncertainty lecture slides), calculate the following:

- P(toothache ∧ ¬ catch) = 0.012 + 0.064 = 0.076
- P(catch) = 0.108 + 0.016 + 0.072 + 0.144 = 0.34
- P(cavity I catch) = P(cavity ∧ catch) / P(catch)
- = (0.108 + 0.072)/(0.108 + 0.072 + 0.016 + 0.144) = 0.18 / 0.34 = 0.53
- P(cavity I toothache ∨ catch) =
- P(cavity ∧ (toothache ∨ catch)) / P(toothache ∨ catch)
- = (0.108 + 0.012 + 0.072)/(0.108 + 0.012 + 0.072 + 0.016 + 0.064 + 0.144)
- = 0.192 / 0.416 = 0.46

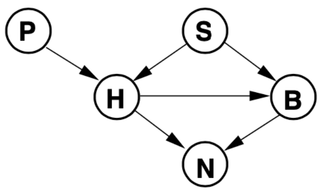
2. Verify the conditional independence claimed in the lecture slides by showing that P(catch I toothache, cavity) = P(catch I cavity)

P(catch I toothache ∧ cavity) =
P(catch ∧ toothache ∧ cavity) / P(toothache ∧ cavity)
= 0.108 /(0.108 + 0.012) = 0.108 / 0.12 = 0.9
P(catch I cavity) = P(catch ∧ cavity) / P(cavity)
= (0.108 + 0.072)/(0.108 + 0.012 + 0.072 + 0.008) = 0.18 / 0.2 = 0.9

Activity 3: Consider the following statements: Headaches and blurred vision may be the result of sitting to close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(i) Represent the causal links in a Bayesian network. Let H stan for "headache", B for "blurred vision"' S for "sitting too close to a monitor", P for "bad posture" and N for "nausea". In terms of conditional probabilities, write a formula for the event that all five variables are true, i.e. P(H Λ B Λ S Λ P Λ N).

(i)



(ii) Suppose the following probabilities are given

P(H|S,P) = 0.8 P(H|¬S,P) = 0.4
P(H|S,¬P) = 0.6 P(H|¬S,¬P) = 0.02
P(B|S,H) = 0.4 P(B|¬S,H) = 0.3
P(B|S,¬H) = 0.2 P(B|¬S,¬H) = 0.01
P(S) = 0.1
P(P) = 0.2
P(N|H,B) = 0.9 P(N|¬H,B) = 0.3
P(|H,¬B) = 0.5 P(N|¬H,¬B) = 0.7

Furthermore, assume that some patient is suffering from headaches but not from nausea. Calculate joint probabilities fro the 8 remaining possibilities (that is, according to whether S, B, P are true or false).

(ii) $P(H \wedge B \wedge S \wedge P \wedge \neg N) = P(H|S \wedge P).P(B|H \wedge S).P(S).P(P).P(\neg N|H \wedge B)$
$= 0.8 \times 0.4 \times 0.1 \times 0.2 \times 0.1$
$= 0.00064$
$P(H \wedge \neg B \wedge S \wedge P \wedge \neg N) = P(H|S \wedge P).P(\neg B|H \wedge S).P(S).P(P).P(\neg N|H \wedge \neg B)$
$= 0.8 \times 0.6 \times 0.1 \times 0.2 \times 0.5$
$= 0.00480$
$P(H \wedge B \wedge \neg S \wedge P \wedge \neg N) = P(H|\neg S \wedge P).P(B|H \wedge \neg S).P(\neg S).P(P).P(\neg N|H \wedge B)$
$= 0.4 \times 0.3 \times 0.9 \times 0.2 \times 0.1$
$= 0.00216$
$P(H \wedge \neg B \wedge \neg S \wedge P \wedge \neg N) = P(H|\neg S \wedge P).P(\neg B|H \wedge \neg S).P(\neg S).P(P).P(\neg N|H \wedge \neg B)$
$= 0.4 \times 0.7 \times 0.9 \times 0.2 \times 0.5$
$= 0.02520$
$P(H \wedge B \wedge S \wedge \neg P \wedge \neg N) = P(H|S \wedge \neg P).P(B|H \wedge S).P(S).P(\neg P).P(\neg N|H \wedge B)$
$= 0.6 \times 0.4 \times 0.1 \times 0.8 \times 0.1$
$= 0.00192$
$P(H \wedge \neg B \wedge S \wedge \neg P \wedge \neg N) = P(H|S \wedge \neg P).P(\neg B|H \wedge S).P(S).P(\neg P).P(\neg N|H \wedge \neg B)$
$= 0.6 \times 0.6 \times 0.1 \times 0.8 \times 0.5$
$= 0.0144$
$P(H \wedge B \wedge \neg S \wedge \neg P \wedge \neg N) = P(H|\neg S \wedge \neg P).P(B|H \wedge \neg S).P(\neg S).P(\neg P).P(\neg N|H \wedge B)$
$= 0.02 \times 0.3 \times 0.9 \times 0.8 \times 0.1$
$= 0.000432$
$P(H \wedge \neg B \wedge \neg S \wedge \neg P \wedge \neg N) = P(H|\neg S \wedge \neg P).P(\neg B|H \wedge \neg S).P(\neg S).P(\neg P).P(\neg N|H \wedge \neg B)$
$= 0.02 \times 0.7 \times 0.9 \times 0.8 \times 0.5$
$= 0.00504$

(iii) What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

(iii) $P(P|H \wedge \neg N) = \frac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)} = \frac{0.0328}{0.054592} = 0.60082$

Note:

$P(P \wedge H \wedge \neg N) = \Sigma_{b,s} P(H \wedge b \wedge s \wedge P \wedge \neg N) = 0.00064 + 0.00480 + 0.00216 + 0.02520$

$P(H \wedge \neg N) = \Sigma_{b,s,p} P(H \wedge b \wedge s \wedge p \wedge \neg N) = 0.00064 + 0.00480 + 0.00216 + 0.02520 + 0.00192 + 0.0144 + 0.000432 + 0.00504 = 0.05452$

We could also use direct inference, though this provides no advantage in this example. To do this, we need a conditional version of Bayes' Rule: $P(B|A,C) = \frac{P(A|B,C)P(B|C)}{P(A|C)}$

Then $P(P|H,\neg N) = \frac{P(\neg N|H,P).P(P|H)}{P(\neg N|H)}$

Now $P(\neg N|H,P) = P(\neg N|H,B,P).P(B|H,P) + P(\neg N|H,\neg B,P).P(\neg B|H,P)$
$= P(\neg N|H,B).P(B|H,P) + P(\neg N|H,\neg B).P(\neg B|H,P)$
$= P(\neg N|H,B).(P(B|H,S,P).P(S|H,P) + P(B|H,\neg S,P).P(\neg S|H,P)) +$
$P(\neg N|H,\neg B).(P(\neg B|H,S,P).P(S|H,P) + P(\neg B|H,\neg S,P).P(\neg S|H,P))$
$= [P(\neg N|H,B).(P(B|H,S).P(H|S,P).P(S|P)+P(B|H,\neg S).P(H|\neg S,P).P(\neg S|P))+$
$P(\neg N|H,\neg B).(P(\neg B|H,S).P(H|S,P).P(S|P)+P(\neg B|H,\neg S).P(H|\neg S,P).P(\neg S|P))]/P(H|P)$
$= [P(\neg N|H,B).(P(B|H,S).P(H|S,P).P(S) + P(B|H,\neg S).P(H|\neg S,P).P(\neg S)) +$
$P(\neg N|H,\neg B).(P(\neg B|H,S).P(H|S,P).P(S)+P(\neg B|H,\neg S).P(H|\neg S,P).P(\neg S))]/P(H|P)$

Also $P(P|H) = P(H|P).P(P)/P(H)$, so cancelling $P(H|P)$
$P(\neg N|H,P).P(P|H)$
$= [P(\neg N|H,B).(P(B|H,S).P(H|S,P).P(S) + P(B|H,\neg S).P(H|\neg S,P).P(\neg S)) +$
$P(\neg N|H,\neg B).(P(\neg B|H,S).P(H|S,P).P(S)+P(\neg B|H,\neg S).P(H|\neg S,P).P(\neg S))].P(P)/P(H)$

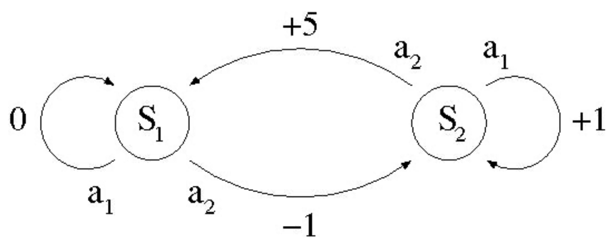
This generates four terms exactly as above, and similarly $P(\neg N|H)$ generates eight terms as above. The extra $P(H)$ cancel each other out.

PART II

Activity 1: Consider a world with two states and two actions , where the transitions and reward for each state and action are as follows:

$\delta(S_1, a_1) = S_1,$	$r(S_1, a_1) = 0$
$\delta(S_1, a_2) = S_2,$	$r(S_1, a_2) = -1$
$\delta(S_2, a_1) = S_2,$	$r(S_2, a_1) = 1$
$\delta(S_2, a_2) = S_1,$	$r(S_2, a_2) = 5$

(i) Draw a picture of this world, using circles for the states and arrows for the transitions.



(ii) Assuming a discount factor of γ = 0.9, determine:

- the optimal policy $\pi^* : S \rightarrow A$ $\pi^*(S_1) = a_2$
  $\pi^*(S_2) = a_2$

- the value function $V^*: S \to R$ $V(S_1) = -1 + \gamma V(S_2)$
  $V(S_2) = 5 + \gamma V(S_1)$ So $V(S_1) = -1 + 5\gamma + \gamma^2 V(S_1)$
- i.e. $V(S_1) = (-1 + 5\gamma) / (1 - \gamma^2) = 3.5 / 0.19 = 18.42$
  $V(S_2) = 5 + \gamma V(S_1) = 5 + 0.9 * 3.5 / 0.19 = 21.58$

- the "Q" function $Q: S \times A \to R$

• $Q(S_1, a_1) = \gamma V(S_1) = 16.58$
• $Q(S_1, a_2) = V(S_1) = 18.42$
• $Q(S_2, a_1) = 1 + \gamma V(S_2) = 20.42$
• $Q(S_2, a_2) = V(S_2) = 21.58$

(iii) Write the Q values in a matrix:

Q	**a_1**	**a_2**
S_1	16.58	18.42
S_2	20.42	21.58

(iv) Trace through the first few steps of the Q-learning algorithm, with all Q values initially set to zero. Explain why it is necessary to force exploration through probabilistic choice of actions, in order to ensure convergence to the true Q values.

current state	**chosen action**	**new Q value**
S_1	a_1	$0 + \gamma*0 = 0$
S_1	a_2	$-1 + \gamma*0 = -1$
S_2	a_1	$1 + \gamma*0 = +1$

At this point, the table looks like this:

Q	**a_1**	**a_2**
S_1	0	-1
S_2	1	0

If we do not force exploration, the agent will always prefer action a_1 in state S_2, so it will never explore action a_2. This means that $Q(S_2, a_2)$ will remain zero forever, instead of converging to the true value of 21.58 . If we force exploration, the next few steps might look like this:

current state	chosen action	new Q value
S_2	a_2	$5 + \gamma*0 = 5$
S_1	a_1	$0 + \gamma*0 = 0$
S_1	a_2	$-1 + \gamma*5 = 3.5$
S_2	a_1	$1 + \gamma*5 = 5.5$
S_2	a_2	$5 + \gamma*3.5 = 8.15$

Now we have this table:

Q	**a_1**	**a_2**
S_1	0	3.5
S_2	5.5	8.15

From this point on, the agent will prefer action a_2 both in state S_1 and in state S_2. Further steps refine the Q value estimates, and, in the limit, they will converge to their true values.

current state	chosen action	new Q value
S_1	a_1	0 + γ*3.5 = 3.15
S_1	a_2	-1 + γ*8.15 = 6.335
S_2	a_1	1 + γ*8.15 = 8.335
S_2	a_2	5 + γ*6.34 = 10.70

PART III

Activity 1:

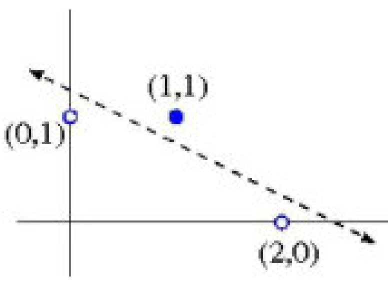
(a) Construct by hand a Perceptron which correctly classifies the following data; use your knowledge of plane geometry to choose appropriate values for the weights w_0, w_1 and w_2.

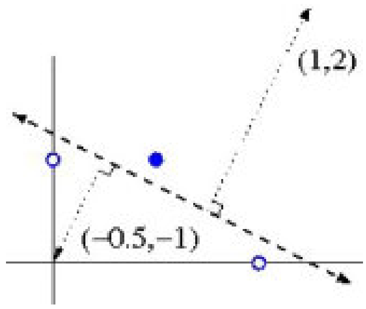
The first step is to plot the data on a 2-D graph, and draw a line which separates the positive from the negative data points:

This line has slope -1/2 and x_2-intersect 5/4, so its equation is:

$$x_2 = \frac{5}{4} - \frac{x_1}{2}$$

$$\text{i.e. } 2x_1 + 4x_2 - 5 = 0$$





Taking account of which side is positive, this corresponds to these weights:

$w_0 = -5$

$w_1 = 2$

$w_2 = 4$

Alternatively, we can derive weights w_1=1 and w_2=2 by drawing a vector normal to the separating line, in the direction pointing towards the positive data points:

The bias weight w_0 can then be found by computing the dot product of the normal vector with a perpendicular

vector from the separating line to the origin. In this case $w_0 = 1(-0.5) + 2(-1) = -2.5$

Note: these weights differ from the previous ones by a normalising constant, which is fine for a perceptron.

b) Demonstrate the Perceptron Learning Algorithm on the above data, using a learning rate of 1.0 and initial weight values of

$w_0 = -0.5$

$w_1 = 0$

$w_2 = 1$

Iteration	w_0	w_1	w_2	Example	x_1	x_2	Class	$w_0 + w_1x_1 + w_2x_2$	Action
1	-0.5	0	1	a	0	1	$-$	$+0.5$	Subtract
2	-1.5	0	0	b	2	0	$-$	-1.5	None
3	-1.5	0	0	c	1	1	$+$	-1.5	Add
4	-0.5	1	1	a	0	1	$-$	$+0.5$	Subtract
5	-1.5	1	0	b	2	0	$-$	$+0.5$	Subtract
6	-2.5	-1	0	c	1	1	$+$	-3.5	Add
7	-1.5	0	1	a	0	1	$-$	-0.5	None
8	-1.5	0	1	b	2	0	$-$	-1.5	None
9	-1.5	0	1	c	1	1	$+$	-0.5	Add
10	-0.5	1	2	a	0	1	$-$	$+1.5$	Subtract
11	-1.5	1	1	b	2	0	$-$	$+0.5$	Subtract
12	-2.5	-1	1	c	1	1	$+$	-2.5	Add
13	-1.5	0	2	a	0	1	$-$	$+0.5$	Subtract
14	-2.5	0	1	b	2	0	$-$	-2.5	None
15	-2.5	0	1	c	1	1	$+$	-1.5	Add
16	-1.5	1	2	a	0	1	$-$	$+0.5$	Subtract
17	-2.5	1	1	b	2	0	$-$	-0.5	None
18	-2.5	1	1	c	1	1	$+$	-0.5	Add
19	-1.5	2	2	a	0	1	$-$	$+0.5$	Subtract
20	-2.5	2	1	b	2	0	$-$	$+1.5$	Subtract
21	-3.5	0	1	c	1	1	$+$	-2.5	Add
22	-2.5	1	2	a	0	1	$-$	-0.5	None
23	-2.5	1	2	b	2	0	$-$	-0.5	None
24	-2.5	1	2	c	1	1	$+$	$+0.5$	None

Note that we only have three training examples (a, b, c), but we keep iterating over them until no corrections are required for the network to produce the correct output.