

Inference Rules for Propositional Logic

Rules for intuitionistic logic

Truth (\top)	Falsity (\perp)
$\frac{}{\top} \quad [\top I] \quad \text{In Lean: trivial}$ <p>No Truth-Elimination rule!</p>	$\frac{\perp}{A} \quad [\perp E] \quad \text{In Lean: false.elim}$ <p>No false introduction rule!</p>
AND/Conjunction (\wedge)	OR/Disjunction (\vee)
$\frac{A, B}{A \wedge B} \quad [\wedge I] \quad \text{In Lean: and.intro}$ <hr/> $\frac{A \wedge B}{B} \quad [\wedge E_{\text{right}}] \quad \text{In Lean: and.elim_right}$ <hr/> $\frac{A \wedge B}{A} \quad [\wedge E_{\text{left}}] \quad \text{In Lean: and.elim_left}$	$\frac{A}{A \vee B} \quad [\vee I_{\text{left}}] \quad \text{In Lean: or.intro_left}$ <hr/> $\frac{B}{A \vee B} \quad [\vee I_{\text{right}}] \quad \text{In Lean: or.intro_right}$ <hr/> $\frac{A \vee B, A \vdash C, B \vdash C}{C} \quad [\vee E] \quad \text{In Lean: or.elim}$
Implication (\rightarrow)	Negation (\neg)
$\frac{A \vdash B}{A \rightarrow B} \quad [\rightarrow I] \quad \text{In Lean: assume}$ <hr/> $\frac{A, A \rightarrow B}{B} \quad [\rightarrow E]$ <p>In Lean: juxtaposition: f : A -> B, a : A - f a : B</p>	$\frac{A \vdash \perp}{\neg A} \quad [\neg I] \quad \text{In Lean: assume}$ <hr/> $\frac{A, \neg A}{\perp} \quad [\neg E] \quad \text{In Lean: juxtaposition}$

Two additional (equivalent) rules for classical logic

In Lean, use open classical to make these rules available.

Law of Excluded Middle	Double Negation Elimination
$\frac{}{A \vee \neg A} \quad [EM] \quad \text{In Lean: em}$	$\frac{\neg \neg A}{A} \quad [DNE] \quad \text{In Lean: by_contradiction}$

(Four extra) Inference Rules for Predicate Logic

For-all (\forall)	Exists (\exists)
$\frac{\forall x P(x)}{P(b)} \quad [\forall E]$ <p>Condition: b must not clash with any bound variables of P</p> <hr/> $\frac{P(y)}{\forall x P(x)} \quad [\forall I]$ <p>Condition: y must not be free in any not-yet-discharged hypothesis</p>	$\frac{\begin{array}{c} P(y)^1 \\ \vdots \\ \vdots \\ \exists x, P(x) \quad B \end{array}}{B} \quad 1 \quad [\exists E]$ <p>Condition: y should not be free in B or any not-yet-discharged hypothesis</p> <hr/> $\frac{P(b)}{\exists x P(x)} \quad [\exists I]$ <p>Condition: b must not clash with any bound variables of P</p>