Advanced Machine Learning Assignment 2

Olteanu Fabian Cristian

FMI, AI Master, Year 1

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1. Exercise 1

We have the following hypothesis class:

$$\mathcal{H} = \{h_a : \mathbb{R} \to \{0,1\} \mid a > 0, a \in \mathbb{R}, \text{ where } h_a(x) = \mathbf{1}_{[-a,a]}(x) = \begin{cases} 1, & x \in [-a,a] \\ 0, & x \notin [-a,a] \end{cases}$$

1.a.

We can make the observation that $x \in [-a,a]$ is equivalent to $|x| \leq a, \forall a \in \mathbb{R}_+, x \in \mathbb{R}$. Based on this observation we can say that the hypothesis class is a threshold class. Thus, in order to compute its growth function, we can follow a very similar approach to the one illustrated in lecture 8 [1] and conclude that $\tau_{\mathcal{H}}(m) = m + 1$, where $C = \{c_1, c_2, ..., c_m\}$ and $m = |C|$.

1.b.

Since $\mathcal{H}$ is a threshold hypothesis class, we know that $VCDim(\mathcal{H}) = 1$. Sauer's lemma states that $\tau_{\mathcal{H}}(m) \leq \sum_{i=0}^{d} \binom{m}{i}$, where $d = VCdim(\mathcal{H})$.

In conclusion, $\tau_{\mathcal{H}}(m) \leq \binom{m}{0} + \binom{m}{1} = m + 1$, so $\tau_{\mathcal{H}}(m)$ is equal to the general upper bound given by Sauer's lemma.

References

[1] Bogdan Alexe, Advanced machine Learning, Lecture 8, 2023