

Assignment 1

Deadline: Wednesday, 26th of April 2023

Upload your solutions at: <https://tinyurl.com/AML-2023-ASSIGNMENT1>

1. **(0.75 points)** Give examples for:

- a) a finite hypothesis class \mathcal{H} with $\text{VCdim}(\mathcal{H}) = 2023$. Justify your choice. **(0.25 points)**
- b) an infinite hypothesis class \mathcal{H} with $\text{VCdim}(\mathcal{H}) = 2023$. Justify your choice. **(0.25 points)**
- c) an infinite hypothesis class \mathcal{H} with $\text{VCdim}(\mathcal{H}) = \infty$. Justify your choice. **(0.25 points)**

2. **(0.75 points)** Consider \mathcal{H} to be the following hypothesis class:

$$\mathcal{H} = \{h_a : \mathbb{R}^3 \rightarrow \{0, 1\} \mid h_a(\mathbf{x}) = \mathbf{1}_{[\|\mathbf{x}\|_2 \leq a]}(\mathbf{x}), \mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3, \|\mathbf{x}\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2}\}$$

Consider the realizability assumption.

- a) show that \mathcal{H} can be (ϵ, δ) -PAC learned by giving an algorithm A and determining the sample complexity $m_{\mathcal{H}}(\epsilon, \delta)$ such that the definition of PAC learnability is satisfied. **(0.5 points)**
- b) compute $\text{VCdim}(\mathcal{H})$. **(0.25 points)**

3. **(1 point)** Compute $\text{VCdim}(\mathcal{H})$ where \mathcal{H} is the following hypothesis class:

$$\mathcal{H} = \{h_{\theta_1, \theta_2} : \mathbb{R}^2 \rightarrow \{0, 1\} \mid h_{\theta_1, \theta_2}(\mathbf{x}) = h_{\theta_1, \theta_2}(x_1, x_2) = \mathbf{1}_{[\theta_1 + x_1 \cdot \sin(\theta_2) + x_2 \cdot \cos(\theta_2) > 0]}, \theta_1, \theta_2 \in \mathbb{R}, \mathbf{x} \in \mathbb{R}^2\}.$$

4. **(1.5 points)** Consider \mathcal{H}_α to be the class of axis aligned rectangles with fixed aspect-ratio α , where $\alpha \in \mathbb{R}$ and $\alpha > 0$:

$$\mathcal{H}_\alpha = \{h_{a,b,c,d} : \mathbb{R}^2 \rightarrow \{0, 1\} \mid h_{a,b,c,d}(\mathbf{x}) = \mathbf{1}_{[a,b] \times [c,d]}(\mathbf{x}), a, b, c, d \in \mathbb{R}, a < b, c < d, \frac{d-c}{b-a} = \alpha\}$$

Consider the realizability assumption.

- a) give a learning algorithm A that returns a hypothesis h_S from \mathcal{H}_α , $h_S = A(S)$ consistent with the training set S (h_S has empirical risk 0 on S). **(0.75 points)**
- b) find the sample complexity $m_{\mathcal{H}_\alpha}(\epsilon, \delta)$ in order to show that \mathcal{H}_α is PAC – learnable. **(0.75 points)**

5. **(1 point)** Compute $\text{VCdim}(\mathcal{H})$ where \mathcal{H} is the following hypothesis class:

$$\mathcal{H} = \{h_\theta : \mathbb{R} \rightarrow \{0, 1\} \mid h_\theta(x) = \mathbf{1}_{[\theta, \theta+1] \cup [\theta+2, \theta+4] \cup [\theta+6, \theta+9]}(x), \theta \in \mathbb{R}\}.$$

Ex-officio: 0.5 points