Lab Assigment - Probabilistic Robotics Course
Kalman Filter

Samuel Cerqueira

ITAndroids

1 First Problem

Let's start simple. Imagine you have a One Dimensional Robot, with stochastic model:

$$x_{k+1} = x_k + u_k + w_k \tag{1}$$

$$z_k = x_k + v_k \tag{2}$$

Where $w_k \sim \mathcal{N}(0, \sigma_w^2)$, $v_k \sim \mathcal{N}(0, \sigma_v^2)$ and $x_0 \sim \mathcal{N}(m_0, \sigma_0^2)$. $\{\{w_k\}_{k\geq 0}, \{v_k\}_{k\geq 0}, x_0\}$ are assumed to be jointly Gaussian and mutually independent.

Obs.: the notation $\alpha \sim \mathcal{N}(m, \Sigma)$ means that a random variable α is Gaussian distributed with mean m and covariance matrix Σ.

1.1 Problem 1.1

From the multidimensional equations for the Kalman Filter, derive the one dimensional ones. Compute analytically the value σ_∞^2, i.e., the value of the covariance when $k \to \infty$.

1.2 Problem 1.2

In the filtering step, compute the limits for the case where $\sigma_v \to 0$ and the case where $\sigma_w \to 0$. Provide an intuitive meaning for these results, given that w_k is the noise associated to the movement model and v_k the one related to the measurement noise.

2 Second Problem

Let's move towards real implementation. Imagine you have a 2D ominidirectional robot (i.e. a robot that lives in the x-y plane and is able to walk in any direction in this plane). Its movement model is described by:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x,k} \\ u_{y,k} \end{bmatrix} + w_k \tag{3}$$

2.1 Problem 2.1

Assume the following observation model:

$$z_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix} + v_k \tag{4}$$

with $w_k \sim \mathcal{N}(0_{2\times 1}, Q_k)$, $v_k \sim \mathcal{N}(0_{2\times 1}, R_k)$ and

$$Q_k = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix}, \quad R_k = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix} \tag{5}$$

Implement the Kalman Filter model and run the simulation. For this, you will have to setup the model in the file *runKalmanFilter* and implement the prediction and update equations in the file *KalmanFilter*.

Run the simulation and check if the results seem reasonable.

2.2 Problem 2.2

Repeat the previous exercise but with

$$Q_k = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix}, \quad R_k = \begin{bmatrix} 10^{2} & 0 \\ 0 & 10^{2} \end{bmatrix} \tag{6}$$

explain what these new values of covariances matrices mean. Does the covariance of the estimate stop growing? Why does it happen? Explain it intuitively using the result from Problem 1.2.

2.3 Problem 2.3

Repeat the previous exercise but with

$$Q_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R_k = \begin{bmatrix} 10^{-8} & 0 \\ 0 & 10^{-8} \end{bmatrix} \tag{7}$$

Again, explain the meaning of these new values of covariances. In this case, does the estimate covariance increase indefinitively? Or is it upper bounded? Again, explain it intuitively using the result of problem 1.2.

2.4 Problem 2.4

Now, assume the following observation model:

$$z_k = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix} + v_k \tag{8}$$

with $v_k \sim \mathcal{N}(0, R_k)$ and

$$Q_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R_k = 10^{-4} \tag{9}$$

What happens with the covariance? Try to explain it.

3 Problem 3

In this problem, we are going to examine the robustness of Kalman Filters to model inaccuracy. More precisely, the filter will be designed using one model and the simulation will be conducted with another model.

For this, we will use the ball tracking model with a still observator, as shown in class. In the movement model, we have assumed that

$$s_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix}, \quad v_k = \begin{bmatrix} v_{x,k} \\ v_{y,k} \end{bmatrix} \tag{10}$$

$$s_{k+1} = s_k + v_k \Delta T \tag{11}$$

$$v_{k+1} = v_k + \alpha \tag{12}$$

where α is random, s_k represents the position and v_k the velocity of the robot. Assuming $\alpha \sim \mathcal{N}(0_{2\times 1}, Q_k)$ and

$$Q_k = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix} \tag{13}$$

and the following observation model:

$$z_k = s_k + \eta_k, \;\; \eta_k \sim \mathcal{N}(0_{2\times 1}, R_k), \;\; R_k = \begin{bmatrix} 10^{-2} & 0 \\ 0 & 10^{-2} \end{bmatrix} \tag{14}$$

3.1 Problem 3.1

Write this problem in the state-space format (i.e. obtain all the matrices necessary to the Kalman Filter Design). Notice that there is no input u_k. *Hint:* The state will be a 4×1 matrix.

The observation model observes only the positions. Will the filter be able to also track the velocity?

3.2 Problem 3.2

Now, assume that the ball is subject to viscous friction, which can be mathematically modeled as:

$$v_{k+1} = \gamma v_k + \alpha, 0 < \gamma \leq 1 \tag{15}$$

First, rewrite the model obtained in Problem 3.1 to take this effect into account.

Then, assume that when designing the filter you have neglected this viscous friction. To take this into account in the simulation, you will need to create two different instances of *LinearStochasticModel* and then use the actual model (with viscous friction) in the constructor of the *Simulation* object and the model that does not consider viscous friction in the *KalmanFilter* constructor.

Analyze the results for $\gamma = 0.9$, $\gamma = 0.5$ and $\gamma = 0.1$. Choose a nice initial value for the velocity. What happens as γ gets lower? Is the *Kalman Filter* still able to track the ball?

4 Problem 4 (Challenge!)

In the lecture, we saw how to model the ball tracking with a fixed observer and a moving ball and how to track the ball in local coordinates when the ball has null velocity but the observer moves. The challenge is to obtain the model (matrices A,B,C) for the tracking in local coordinates when both the observer (robot) and the ball move. *Hint:* This model is already implemented on the Humanoid Ball Tracker and Soccer 3D Ball Tracker.