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EXERCISE 0.1

Q#1:

a) $-2.9, -2, 2.3, 2.9$

b) None

c) $y=0$

d) $y=0$ at $x=-3$ and $x=3$

$y<0$ at $-1.7 \leq x \leq 2.1$

e) Maximum:

$y = 2.8$ at $x = -2.6$

Minimum:

$y = -2.2$ at $x = 1.2$

Q#2:

a) $x = -1, 4$ b) None c) $y = -1$ d) $0, 3, 5$

e) Maximum:

$y = 9$ at $x = 6$

Minimum:

$y = -2$ at $x = 0$

Q#3:

a) Yes (vertical line test passes)

b) Yes (vertical line test passes)

c) No (vertical line touches twice on the graph)

d) No (vertical line touches twice on the graph)

Q#4:

a) The natural domain of the function $f(x) = \frac{x^2+x}{x+1}$ is the set of all x except -1 as it makes the ~~denom~~ denominator 0, which results as an undefined function.

The natural domain of $g(x) = x$ is the set of all x. $f(x) = g(x)$ on the intersection of their domains

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b)

The domain of $f(x) = \frac{x\sqrt{x}+\sqrt{x}}{x+1}$ is the set of all $x \geq 0$ as negative values inside the radical make the function undefined.

The natual domain of $g(x) = \sqrt{x}$ is the same as of $f(x)$ i.e. $x \geq 0$. $f(x) = g(x)$.

Q # 5

a) The median income was at its maximum in the year 1999 and the income was \$47,800.

b) The median income was at its minimum in the year 1993 and the income was \$41,600

c) The median income was declining in the first year of the 2-year period rapidly. It fell from approximately \$47,600 to \$46,600 in between 2000 and 2001. The slope between the first two years is steeper than the slope between 2001 and 2002.

Q # 6

a) $\text{Average} = \frac{(\text{Max} - \text{Min})\,\text{income}}{\text{years}} = \frac{47{,}700 - 41{,}600}{6} = \frac{6100}{6}$

$= 1016.66$ dollars.

The average yearly growth between 1993 and 1999 is approximately \$1017 per year.

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b) The median income was increasing more rapidly during the last three years as the slope between 1996 and 1999 is steeper than the slope between 1993 and 1996

c) 1994 and 2005.

Q #7

a) $f(x) = 3x^2 - 2$

$f(0) = 3(0)^2 - 2 = 0 - 2 = \boxed{-2}$

$f(2) = 3(2)^2 - 2 = 12 - 2 = \boxed{10}$

$f(-2) = 3(-2)^2 - 2 = 12 - 2 = \boxed{10}$

$f(3) = 3(3)^2 - 2 = 27 - 2 = \boxed{25}$

$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 6 - 2 = \boxed{4}$

$f(3t) = 3(3t)^2 - 2 = \boxed{27t^2 - 2}$

b) $f(x) = \begin{cases} 1/x, & x > 3 \\ 2x, & x \leq 3 \end{cases}$

$f(0) = 2(0) = 0$

$f(2) = 2(2) = 4$

$f(-2) = 2(-2) = -4$

$f(3) = 2(3) = 6$

$f(\sqrt{2}) = 2(\sqrt{2}) = 2\sqrt{2}$

$f(3t) = 1/3t$ (if $t > 1$)

$f(3t) = 6t$ (if $t \leq 1$)

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Q # 8

a) $g(x) = \frac{x+1}{x-1}$

$g(3) = \frac{3+1}{3-1} = \frac{4}{2} = \boxed{2}$

$g(-1) = \frac{-1+1}{-1-1} = \frac{0}{-2} = \boxed{0}$

$g(\pi) = \frac{\pi+1}{\pi-1} = \frac{3.14+1}{3.14-1} = \frac{4.14}{2.14} = \boxed{1.933}$

$g(-1.1) = \frac{-1.1+1}{-1.1-1} = \frac{-0.1}{-2.1} = \boxed{0.047}$

$g(t^2-1) = \frac{t^2-1+1}{t^2-1-1} = \boxed{\frac{t^2}{t^2-2}}$

b) $g(x) = \begin{cases} \sqrt{x+1}, & x \geq 1 \\ 3, & x < 1 \end{cases}$

$g(3) = \sqrt{3+1} = \sqrt{4} = \boxed{2}$

$g(-1) = \sqrt{-1+1} = \sqrt{0} = \boxed{0}$

$g(-1) = 3$

$g(\pi) = \sqrt{\pi+1}$

$g(-1.1) = 3$

$g(t^2-1) = \sqrt{t^2-1+1} = \sqrt{t^2} = \boxed{t}$ (if $t^2 \geq 2$)

$g(t^2-1) = 3$ (if $t^2 < 2$)

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Q # 9

a) $f(x) = \frac{1}{x-3}$

Natual Domain : $x \neq 3$ because that will make it undefined

Range : $y \neq 0$.

b) $F(x) = \frac{x}{|x|}$

Natual Domain : all x except $x = 0$.

Range : $\{-1, 1\}$.

c) $g(x) = \sqrt{x^2 - 3}$

Natual Domain : $\sqrt{3} \leq x \leq -\sqrt{3}$ (as it cannot contain -ve numbers inside radical)

Range : $y \geq 0$.

d) $G(x) = \sqrt{x^2 - 2x + 5}$

Domain : $x^2 - 2x + 5 \geq 0 \Rightarrow x^2 - 2x + 1 - 1 + 5 \geq 0$

$(x^2 - 2x + 1) + 4 \geq 0$

$(x-1)^2 + 4 \geq 0$

so, the domain of $G(x)$ is all x.

Range:

$y \geq \sqrt{4}$ or $y \geq 2$.

e) $h(x) = \frac{1}{1 - \sin x}$

Domain : $x \in \mathbb{R}$

Range :

Range of $\sin x = -1 \leq \sin x \leq 1$

also, $1 - \sin x \neq 0$ so $\sin x \neq 1$

Range of $1 - \sin x = 0 \leq 1 - \sin x \leq 2$

the range of $h(x)$ is

$h(x) \geq \frac{1}{2}$

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f) $H(x) = \sqrt{\dfrac{x^2-4}{x-2}}$

Domain: $\dfrac{x^2-4}{x-2} \geq 0 \Rightarrow \dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} \Rightarrow x+2 \geq 0.$

$x+2 \geq 0$

Domain is: $\boxed{x \geq -2}$

~~Range~~: So the domain of $H(x)$ is: $[-2, 2) \cup (2, +\infty)$

Range: The range of $\sqrt{x+2}$ is all $x \geq 0$, But we exclude $x=2$ because then $\sqrt{x+2} = 2$.. So the range is: $[0, 2) \cup (2, +\infty)$

Q # 10

a) $f(x) = \sqrt{3-x}$

Natural Domain : $x \leq 3$.

Range : $y \geq 0$

b) $F(x) = \sqrt{4-x^2}$

Natural Domain : $x \geq -2$ and $x \leq 2$

Range: $0 \leq y \leq 2$

c) $g(x) = 3+\sqrt{x}$

Natural Domain : $x \geq 0$

Range : $y \geq 3$

d) $G(x) = x^3+2$.

Natural Domain : set of all x

Range : set of all y

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e) $h(x) = 3\sin x$.

Natural Domain : set of all x

Range : $-3 \le y \le 3$

(f) Domain of $\sqrt{x}$: $\sqrt{x} \geq 0$.

Domain of $\sin\sqrt{x}$: $\sin\sqrt{x} \geq 0$.

$(\sin\sqrt{x})^{-2} = \frac{1}{(\sin\sqrt{x})^2}$ so, $\sin\sqrt{x} \neq 0$.

$\sqrt{x} = n\pi$ $(n = 0, 1, 2, 3 \ldots)$

$x = (n\pi)^2$ $(n = 0, 1, 2, 3 \ldots)$

So, Domain of $f(x)$ is : $x > 0$ &

$x \neq n\pi^2$

Range:

$0 < |\sin\sqrt{x}| \leq 1 \Rightarrow 0 < |\sin\sqrt{x}|^2 \leq 1$

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Range.

Q # 11

a) No, I won't expect it to be unbroken. The curve may break when somebody is born or somebody dies as it increases 1 unit.

b) The concentration will increase rapidly when antibiotic is injected, then it decreases for 8 hours and then repeats.

Q # 12

a) Yes, the temperature may change quickly under some conditions, but it will change continuously.

b) No, the graph will not be continuous as the number of boxes is always an integer and the graph will jump 1 unit atleast.

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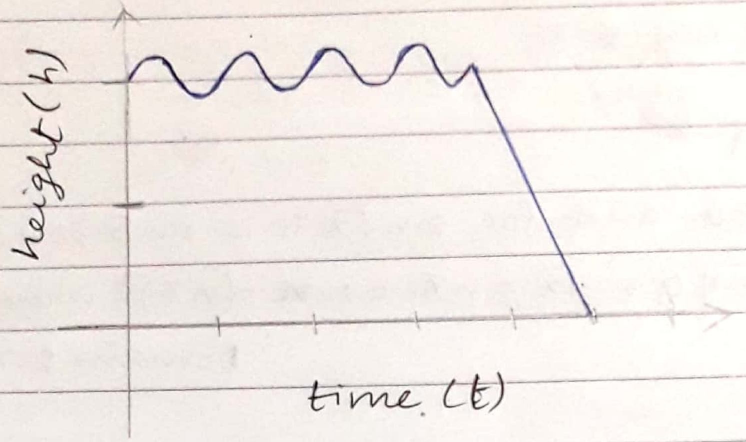
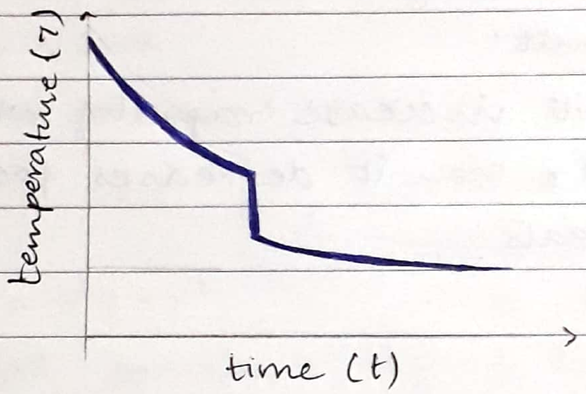
Q # 13



Q # 14 :



Q # 15 :

Yes. Formula: $x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x^2}$

Q16 Yes. Formula: $x^2 - y^2 = 25 \Rightarrow -y^2 = 25 - x^2$

$$y = -\sqrt{25 - x^2}$$

Q17 Yes.

- When $x \leq 0$ and $x \geq -5$,

Formula: $y^2 + x^2 = 25$

$$\boxed{y = \sqrt{25 - x^2}}$$

- When $x > 0$ and $x \leq 5$

$$-y^2 + x^2 = 25$$

$$\boxed{y = -\sqrt{25 - x^2}}$$

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$y = -\sqrt{25-x^2}$

Q # 18

No, the vertical line $x=0$ or y-axis meets the graph twice.

Q #19 — Q # 22

19) ~~Cxxx~~ False, the graph of a semi circle $y=\sqrt{25-x^2}$ meets the x-axis twice, but it is a function.

20) True, by definition 0.1.5.

21) False, as the ~~domain~~ range of absolute function also includes 0.

22) False, the domain needs to be > 0. for a radical function.

Q 23

~~a) $x = 2, 4$~~

b) None

a) $x^2 - 6x + 8 = 0$ $\therefore$ y is 0 at $x = 2, 4$.

$x^2 - 4x - 2x + 8 = 0$

$(x-4)(x-2) = 0$.

$\boxed{x=4}$ $\boxed{x=2}$

c) $x^2 - 6x + 8 \geq 0$

$x^2 - 2x - 4x + 8 \geq 0$

$(x-4)(x-2) \geq 0$

$(x \geq 4)$ $(x \leq 2)$

d) $y = x^2 - 6x + 8$ —(i)

differentiating w.r.t. x

$dy/dx = 2x - 6$

replacing dy/dx by 0.

$0 = 2x - 6$

$2x = 6 \Rightarrow \boxed{x = 3}$

Put $x = 3$ in eq —(i)

$y = (3)^2 - 6(3) + 8$

$= 9 - 18 + 8$

$= -9 + 8$

$\boxed{y = -1}$ y has only minimum value at $x = 3$.

The minimum value is $y = -1$.

Q # 24:

a) $1 + \sqrt{x} = 4$.

$\sqrt{x} = 3$

$\boxed{x = 9}$

b) $1 + \sqrt{x} = 0$.

The range includes $[1, +\infty)$, so y cannot be zero on any value of x.

c) $1+\sqrt{x} \geq 6$

$\sqrt{x} \geq 5$

$\boxed{x \geq 25}$

d) $1+\sqrt{x} = y$ —— (i)

differentiating w.r.t. x.

$dy/dx = \frac{1}{2} x^{-1/2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

replacing dy/dx by 0.

$0 = \frac{1}{2\sqrt{x}}$

$0 = x^{-1/2}$

$\boxed{0 = x}$

Put $x = 0$ in eq (i)

$1+\sqrt{0} = y$

$\boxed{1 = y}$

so y has a minimum value at $x=0$ and it has no maximum value.

Q # 29

a) By the figure, we have:

$l = 15-2x$, $w = (8-2x)$, $h = x$.

$V = l \times w \times h$

$= (15-2x)(8-2x)(x)$

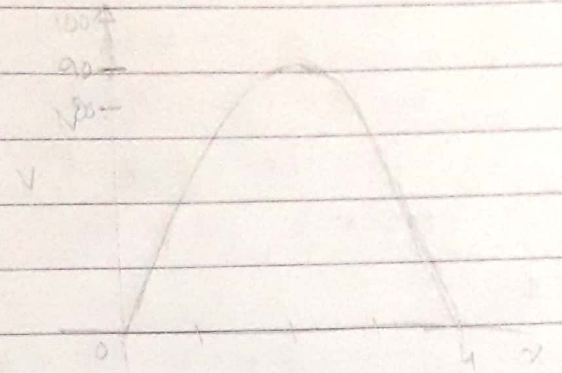
$= (120 - 30x - 16x + 4x^2)x$

$= 120x - 30x^2 - 16x^2 + 4x^3$

$V = 120x - 46x^2 + 4x^3 \Rightarrow \boxed{V = 4x^3 - 46x^2 + 120x}$

b) $0 < x < 4$.

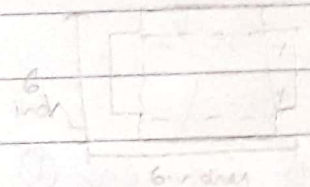
c)



Range = $0 < V < 91$, approx

d) x is independent here, so by putting ~~maximum~~ values of x we can construct boxes of maximum volume.

Q # 30



a) From the figure, we have,

length $= (6-2x)$, width $= (6-2x)$,

height $= x$.

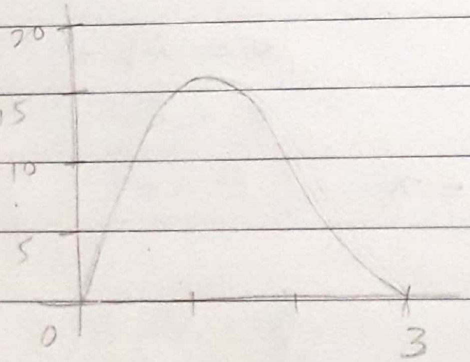
$\because V = l \times w \times h$

$V = (6-2x)(6-2x)(x)$

$V = (6-2x)^2(x)$.

b) Domain of V: $0 < x < 3$.

c)



Range: $0 < V \leq 16$.

d) As x is independent, when x increases, V also increases and then decreases. The maximum value occurs when $x = 1$, $y = 16$

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Q # 33

a) $c = f(r)$

$V = 500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$ — (i)

$C = (0.02)(2)\pi r^2 + (0.01) 2\pi r h$

$= 0.04\pi r^2 + 0.02 \cancel{\pi} \cancel{r} (500/\cancel{\pi} r^2)$

$C = 0.04\pi r^2 + \frac{10}{r}$ — (ii)

Derivating w.r.t r.

$\frac{dC}{dr} = 2(0.04)\pi r + (-1)(10) r^{-2}$

$= 0.08\pi r - 10/r^2$

$0 = 0.08\pi r - (10/r^2)$ [replacing $\frac{dC}{dr}$ by 0].

$r^3 = \frac{10}{0.08\pi}$

$r^3 = 39.788$

$r = 3.4139$ cm (taking cube root)

put $r = 3.413$ in eq (i)

$h = \frac{500}{\pi (3.413)^2} = 13.6870 \Rightarrow \boxed{13.7 = h}$

put $r = 3.413$ in eq (ii)

$C = 0.04\pi (3.413)^2 + \frac{10}{(3.413)}$

$\boxed{C = 4.39 \text{ cents}}$

b) Area of square $= 2r \times 2r = (2r)^2$.

$C = (0.02)(2)(2r)^2 + (0.01) 2\pi r h$.

$C = (0.04) 4r^2 + (0.02)\pi r \left(\frac{500}{\pi r^2}\right)$

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$C = 0.16r^2 + \frac{10}{r}$ —— (iv)

comparing eq — (ii) and eq — (iii)
we get:

$0.04\pi < 0.16$

$\boxed{0.12 < 0.16}$

The top and the bottom of the can now get more weight and more area.
since, they cost more, therefore, we can count the extra cost in the solution we conclude that cans become taller.

Eq iii) ⟹

c) $C = 0.16r^2 + \frac{10}{r}$

$\frac{dC}{dr} = 2(0.16)r + (-1)10r^{-2}$

$= 0.32r - 10r^{-2}$

$= 0.32r - \frac{10}{r^2}$

$\frac{dC}{dr} = \frac{0.32r^3 - 10}{r^2}$

replacing $\frac{dC}{dr}$ by 0.

$0 = 0.32r^3 - 10$

$r^3 = \frac{10}{0.32}$

$r^3 = 31.25$

$\boxed{r = 3.149 \text{ cm}}$

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$$h = \frac{500}{\pi r^2} = \frac{500}{\pi(3.149)^2} = 16.04 \text{ cm}$$

$$\boxed{h = 16.04 \text{ cm}}$$

$$C = (0.16)(3.14)^2 + \frac{10}{3.14} = 1.577 + 3.184.$$

$$\boxed{C = 4.761 \text{ cents}}$$

Q # 35:

i) The domain of the function f is:

$(x+2)(x-1) = 0.$

$x+2 = 0 \qquad x-1 = 0$

$x = -2 \qquad x = 1$

On $x=-2$ and $x=1$, the function becomes undefined, so -2 and 1 are excluded from the domain and the graph has two holes at the value $x=1$ and $x=-2$.

ii) $$f(x) = \frac{(\cancel{x+2})(x^2-1)}{(\cancel{x+2})(x-1)} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = x+1$$

$g(x) = x+1$

The new function is $\boxed{g(x) = x+1}$

Q # 36

$$f(x) = \frac{x^2+|x|}{|x|}$$

i) Domain: $|x| \neq 0 \Rightarrow \boxed{x \neq 0.}$

At $x=0$, the function becomes undefined, so we exclude 0 from the domain.

The graph has one hole at $x=0$.

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ii) $f(x) = \dfrac{x^2+|x|}{|x|} = \dfrac{x^2}{|x|} + \dfrac{\cancel{|x|}}{\cancel{|x|}} = \dfrac{x^2}{|x|} + 1$

$f(x) = \dfrac{|x|\cancel{|x|}}{\cancel{|x|}} + 1 = |x| + 1$

The new function is $g(x) = |x|+1$

Q # 37

a) $25°F$

b) $35.74 + 0.6215(25) - 35.75(15)^{0.16} + 0.4275(25)(15)^{0.16}$
$= 12.603 = 13°F$ approximately.

c) $35.74 + 0.6215(25) - 35.75(46)^{0.16} + 0.4275(25)(46)^{0.16}$
$= 5.032°F = 5°F$ approximately.

Q # 38.

$WCT = 35.74 + 0.6215\,T - 35.75\,V^{0.16} + 0.4275\,TV^{0.16}$ (as $V > 3$)

$-60 = 35.74 + 0.6215T - 35.75(48)^{0.16} + 0.4275\,T(48)^{0.16}$

$-60 + 30.676 = 0.6215T + 0.7942T$

$-29.323 = 1.4157T$

$T = -20.712$

$\boxed{T \approx 21°F}$

Q # 39

$WCT = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275\,T\,V^{0.16}$

$-10 = -30.676 + 1.4157T$

$\dfrac{-10 + 30.676}{1.4157} = T$

$T = 14.607 \Rightarrow \boxed{T \approx 15°F}$

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Q #40

The first formula, $WCT = T$ doesnot work for the given data as $5°F \neq 20°F$

So, trying the second formula:

$$WCT = 35.74 + 0.6215T - 35.75\,v^{0.16} + 0.4275\,Tv^{0.16}$$

$$5 = 48.17 - 35.75\,v^{0.16} + 8.55\,v^{0.16}$$

$$\cancel{\frac{5-48.17}{48.17} =}\quad -27.2\,v^{0.16} = 5 - 48.17$$

$$v^{0.16} = \frac{-43.17}{-27.2}$$

$$\cancel{\frac{0.1037}{-27.2} = v^{0.16}}$$

$$\cancel{-3.816\times10^{-3} = v^{\frac{4}{25}}}$$

$$v^{\frac{4}{25}} = 1.587$$

Taking power $25/4$ on both sides.

$$v = 17.9404.$$

$$v \approx 18\ mi/h.$$

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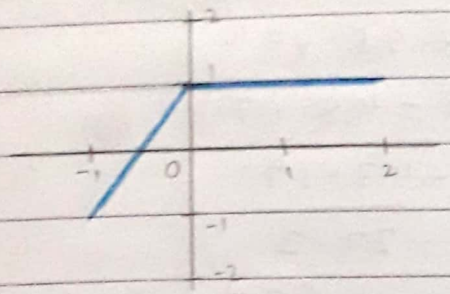
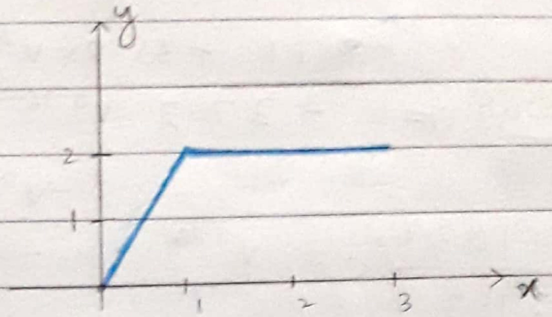
EXERCISE 0.2

Q# 1

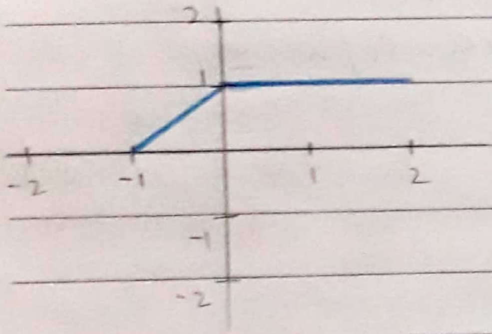
a) $y = f(x) - 1$



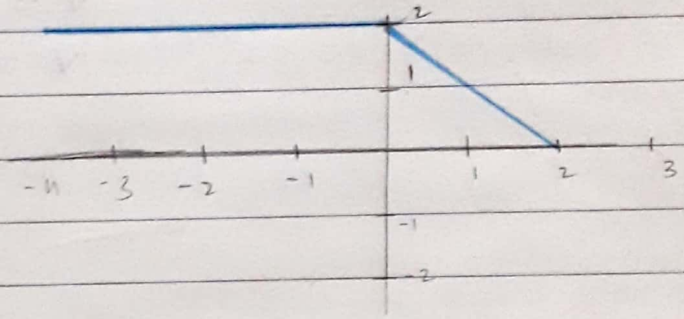
b) $y = f(x-1)$



c) $y = \frac{1}{2} f(x)$

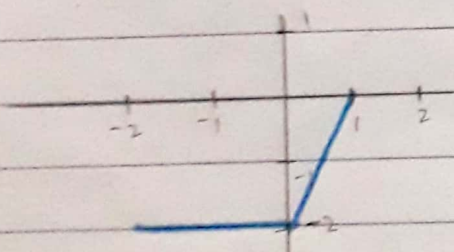


d) $y = f(-\frac{1}{2} x)$

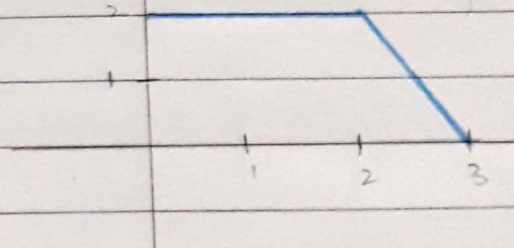


Q# 2

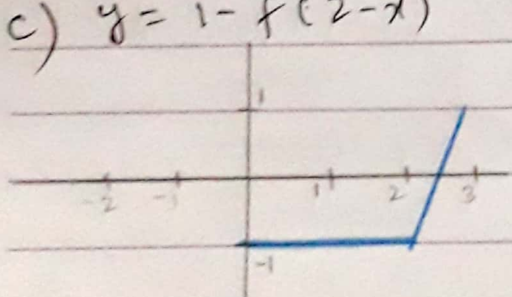
a) $y = -f(-x)$



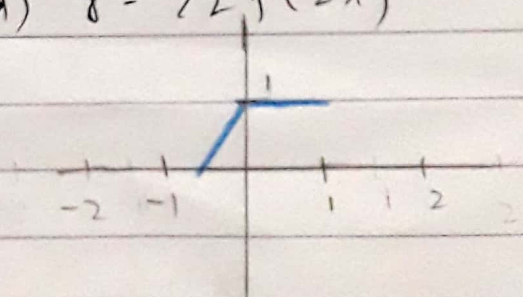
b) $y = f(2-x)$ $f(-x)$
$f(-(x-2))$



c) $y = 1 - f(2-x)$

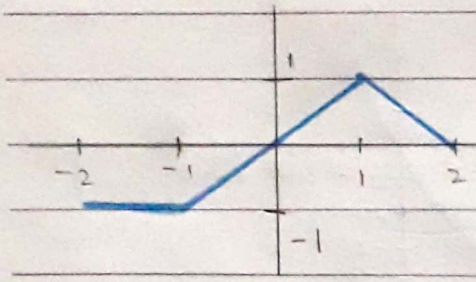


d) $y = \frac{1}{2} f(2x)$

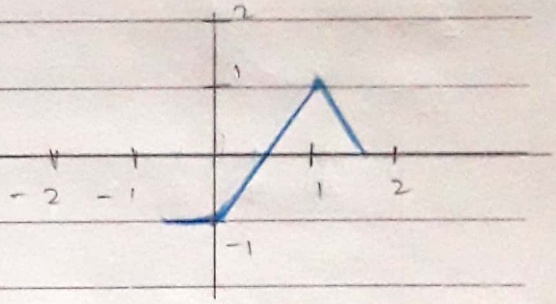


Q#3

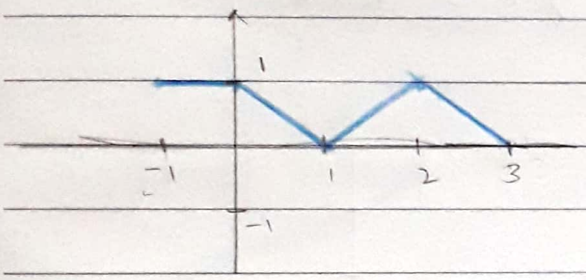
a) $y = f(x+1)$



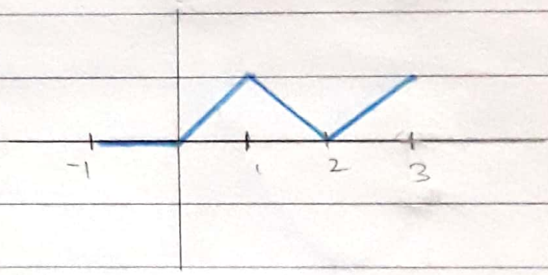
b) $y = f(2x)$



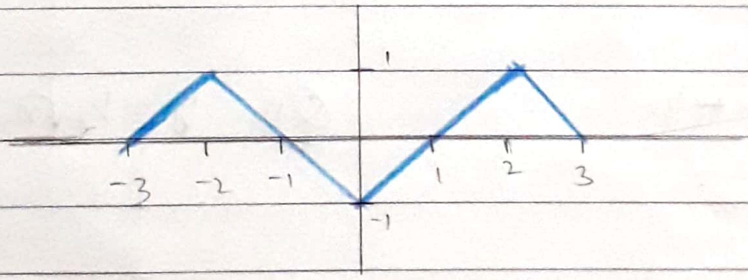
c) $y = |f(x)|$



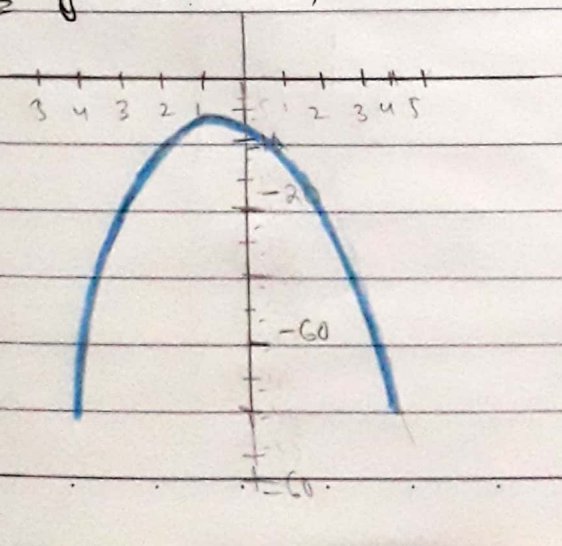
d) $y = 1 - |f(x)|$



Q #4:



Q5 $y = -2(x+1)^2 - 3$



[move left 1 unit, stretch vertically by a factor of 2, reflect about x axis, move down 3 units]

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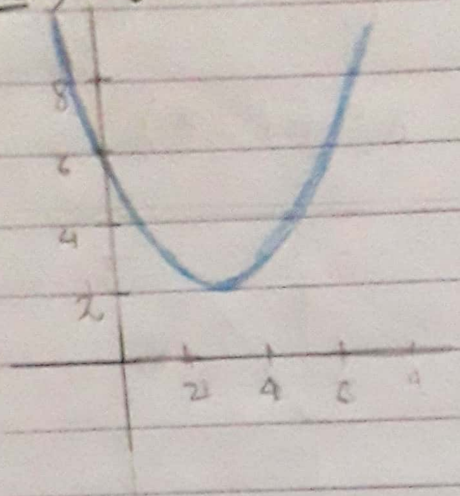
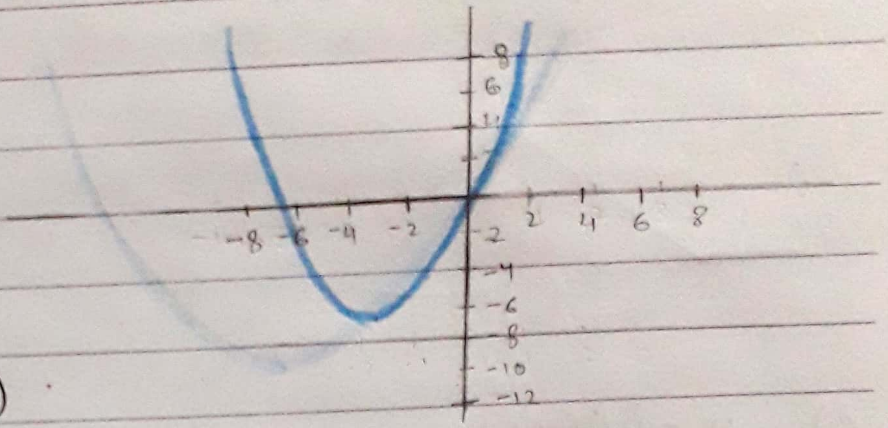
Q6) $y = \frac{1}{2}(x-3)^2+2$



Q7 $y = x^2+6x$

$y = x^2+6x+9-9$

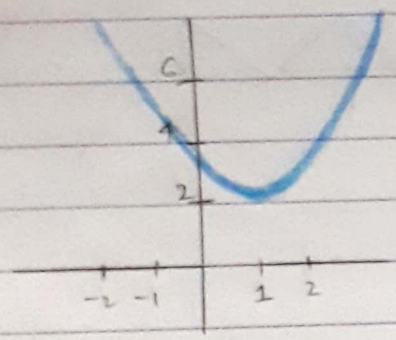
$y = (x+3)^2-9$.



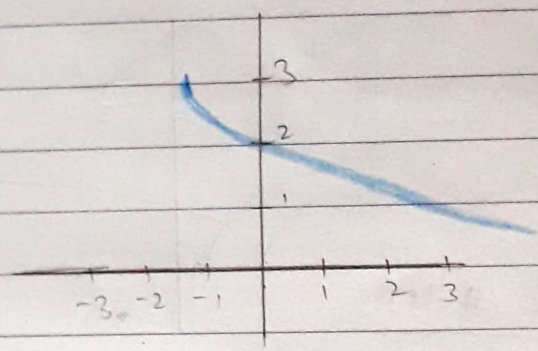
Q8 $y = \frac{1}{2}(x^2-2x+3)$

$y = \frac{1}{2}(x^2-2x+1+2)$

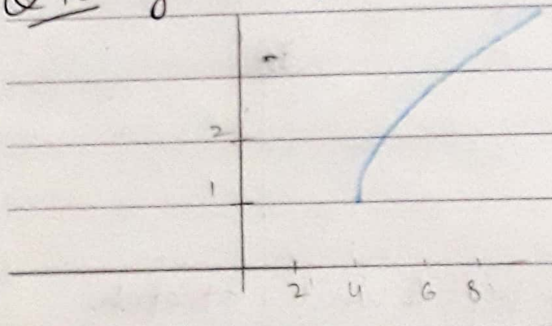
$= \frac{1}{2}\{(x-1)^2+2\}$.



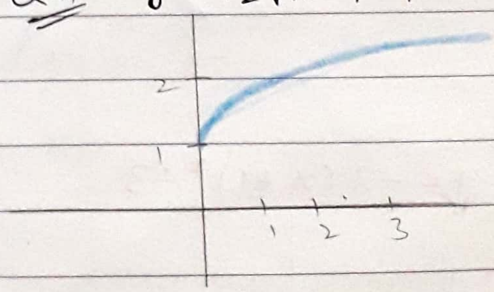
Q9 $y = 3-\sqrt{x+1}$



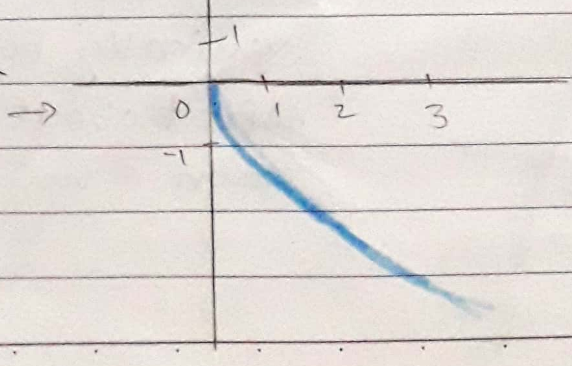
Q10 $y = 1+\sqrt{x+4}$



Q11 $y = \frac{1}{2}\sqrt{x}+1$



Q12 $y = -\sqrt{3x}$



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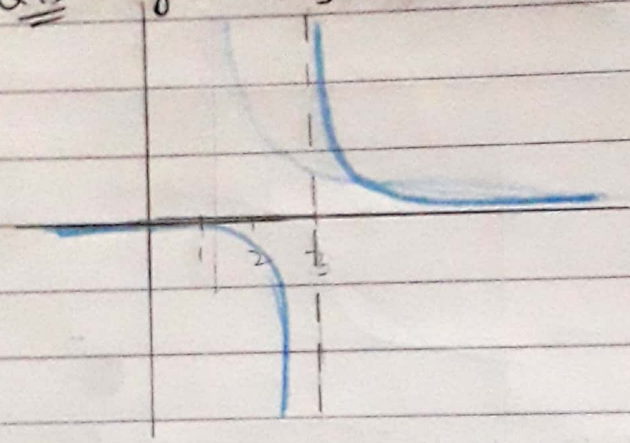
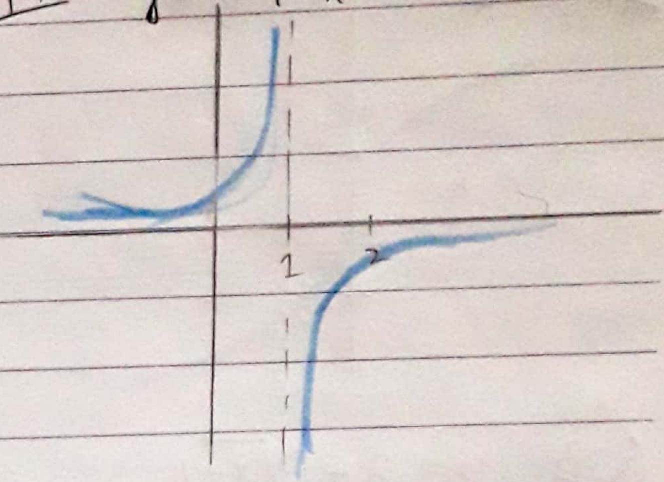
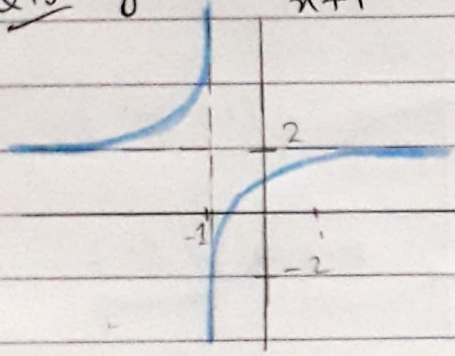
Q13 $y = \frac{1}{x-3}$



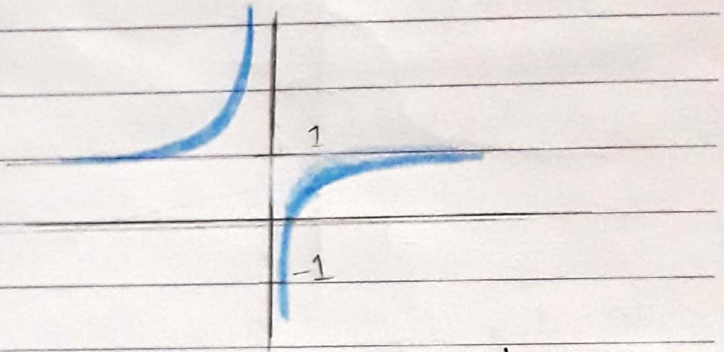
Q14 $y = \frac{1}{1-x}$



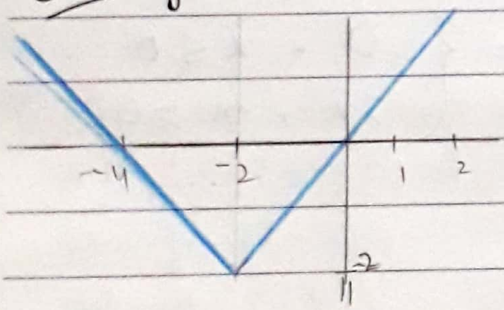
Q15 $y = 2 - \frac{1}{x+1}$



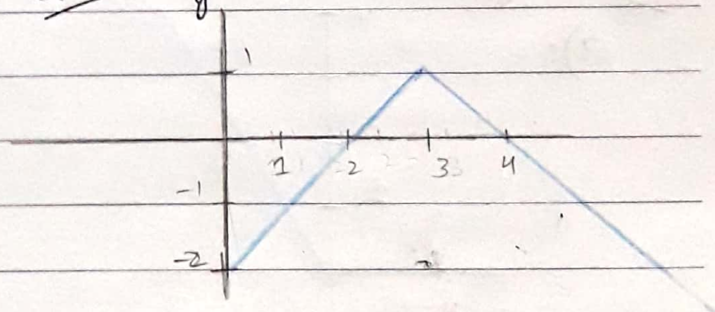
Q16 $y = \frac{x-1}{x} = \frac{x}{x} - \frac{1}{x} = 1 - \frac{1}{x}$



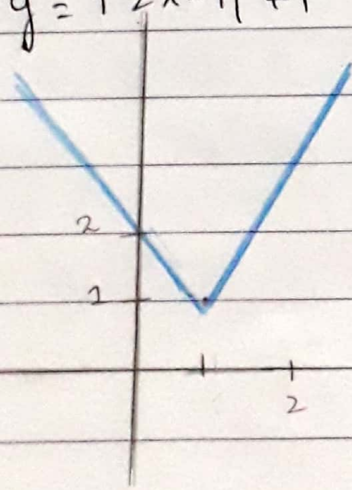
Q17 $y = |x+2| - 2$



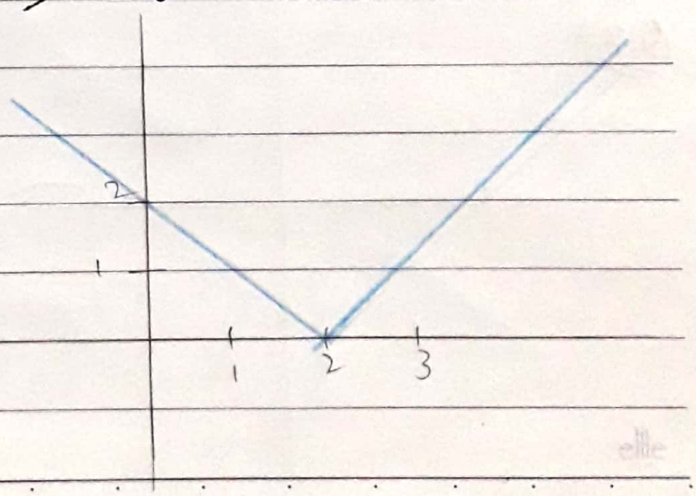
Q18 $y = 1 - |x-3|$



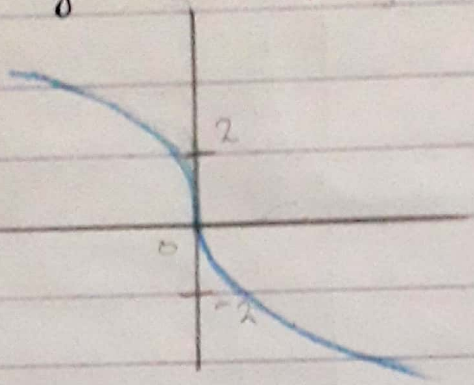
Q19 $y = |2x-1| + 1$



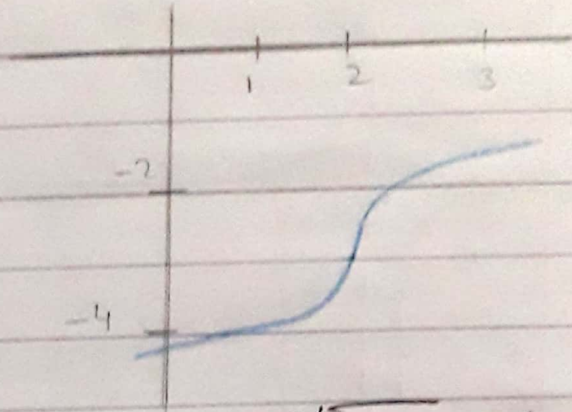
Q20 $y = \sqrt{x^2 - 4x + 4}$



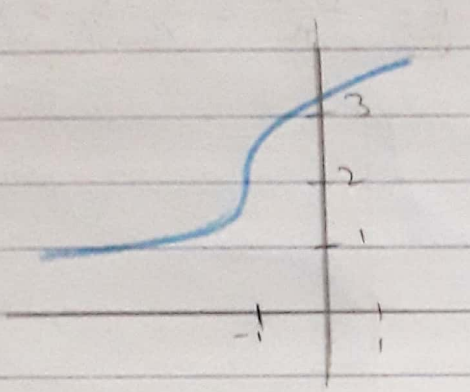
Q21 $y = 1 - 2\sqrt[3]{x}$



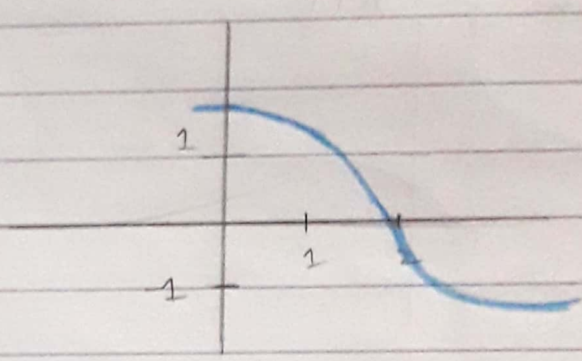
Q22 $y = \sqrt[3]{x-2} + 3$



Q23 $y = 2 + \sqrt[3]{x+1}$

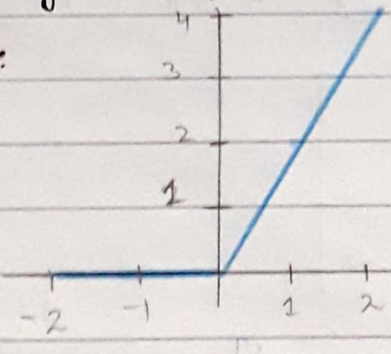


Q24 $y + \sqrt[3]{x-2} = 0.$



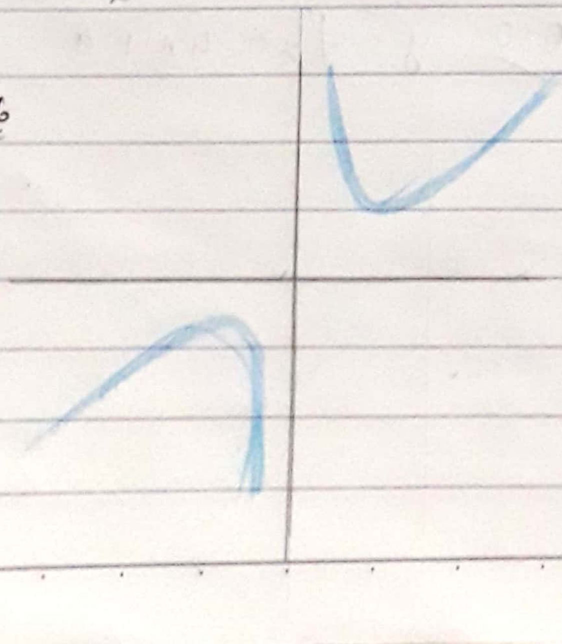
Q25 $y = x + |x|$

a):



b) $y = \begin{cases} 0, & x \le 0 \\ 2x, & x > 0 \end{cases}$

Q26



Q27:

$f+g(x) = 2\sqrt{x-1} + \sqrt{x-1} = 3\sqrt{x-1}$, $D: x \geq 1$

$f-g(x) = 2\sqrt{x-1} - \sqrt{x-1} = \sqrt{x-1}$, $D: x \geq 1$

$f.g(x) = (2\sqrt{x-1})(\sqrt{x-1}) = 2(x-1) = 2x-2$, $D = x \geq 1$

$f/g(x) = \dfrac{2\sqrt{x-1}}{\sqrt{x-1}} = 2$, $x > 1$

Q28 $f+g(x) = \dfrac{x}{1+x^2} + \dfrac{1}{x} = \dfrac{x^2+1+x^2}{x(1+x^2)} = \dfrac{2x^2+1}{x(1+x^2)} =$

Domain: all x except $x = 0$.

$f-g(x) = \dfrac{x}{1+x^2} - \dfrac{1}{x} = \dfrac{x^2-1-x^2}{x(1+x^2)} = \dfrac{-1}{x(1+x^2)}$, $D = x \neq 0$

$f \cdot g(x) = \left(\dfrac{\cancel{x}}{1+x^2}\right)\left(\dfrac{1}{\cancel{x}}\right) = \dfrac{1}{1+x^2}$, $D =$ all x

$(f/g)(x) = \left(\dfrac{x}{1+x^2}\right) \div \left(\dfrac{1}{x}\right) = \dfrac{x}{1+x^2} \times x = \dfrac{x^2}{1+x^2}$, $D:$ all $x \neq 0$

Q29 $f(x) = \sqrt{x}$ and $g(x) = x^3+1$.

a) $f(g(2)) = \sqrt{(2)^3+1} = \sqrt{9} = 3$

~~b) $g(\sqrt{x}+2) = (\sqrt{x}+2)^3+1 = 3\sqrt{x}+8+3(\sqrt{x})(2)(\sqrt{x}+2)$~~

b) $g(f(4)) = (\sqrt{4})^3+1 = 8+1 = 9$

c) $f(f(16)) = \sqrt{\sqrt{16}} = \sqrt{4} = 2$.

d) $g(g(0))$: $g(0) = 0+1 = 1$.

$g(g(0)) = (1)^3+1 = 2$.

e) $f(2+h) = \sqrt{2+h}$

f) $g(3+h) = (3+h)^3+1 = 27+h^3+3(3)(h)(3+h)$

$= 27+h^3+27h+9h^2$

$= h^3+9h^2+27h+27$

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Q 30. $g(x) = \sqrt{x}$.

a) $g(5s+2) = \sqrt{5s+2}$

b) $g(\sqrt{x}+2) = \sqrt{\sqrt{x}+2}$

c) $3g(5x) = 3\sqrt{5x}$

d) $1/g(x) = 1/\sqrt{x}$

h) $g((x-1)^2) = \sqrt{(x-1)^2} = x-1$

f) $= (g(x))^2 - g(x^2) = (\sqrt{x})^2 - x = x - x = 0$.

g) $g(1/\sqrt{x}) = \dfrac{1}{\sqrt{\sqrt{x}}} = \dfrac{1}{\sqrt[4]{x}}$.

~~h) $g((x-1)^2) =$~~

e) $g(g(x)) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$

i) $g(x+h) = \sqrt{x+h}$

Q 31: $f(x) = x^2$, $g(x) = \sqrt{1-x}$

$fog(x) = (\sqrt{1-x})^2 = 1-x$,

Domain: $x \leq 1$.

$gof(x) = \sqrt{1-x^2}$; $|x| \leq 1$

Q 32: $f(x) = \sqrt{x-3}$, $g(x) = \sqrt{x^2+3}$

$fog(x) = \sqrt{\sqrt{x^2+3} - 3}$; $|x| \geq \sqrt{6}$.

$gof(x) = \sqrt{(\sqrt{x-3})^2 + 3} = \sqrt{x-3+3} = \sqrt{x}$; $|x| \geq 3$.

Q 33: $f(x) = \dfrac{1+x}{1-x}$, $g(x) = \dfrac{x}{1-x}$

$fog(x) = \dfrac{1 + \frac{x}{1-x}}{1 - \frac{x}{1-x}} = \dfrac{\frac{1-x+x}{\cancel{1-x}}}{\frac{1-x-x}{\cancel{1-x}}} = \dfrac{1}{1-2x}$

Domain: $x \neq 1/2, 1$

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$$gof(x) = \frac{\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}} = \frac{\frac{1+x}{\cancel{1-x}}}{\frac{1-x-1-x}{\cancel{1-x}}} = \frac{1+x}{-2x} = -\frac{1}{2x} - \frac{\cancel{x}}{2\cancel{x}}$$

$$= -\frac{1}{2x} - \frac{1}{2}.$$ Domain: $x \neq 0, 1$

Q34 $f(x) = \frac{x}{1+x^2}$, $g(x) = \frac{1}{x}$.

$$fog(x) = \frac{\frac{1}{x}}{1+(\frac{1}{x})^2} = \frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{\frac{1}{\cancel{x}}}{\frac{x^2+1}{\cancel{x^2}}} = \frac{x}{x^2+1}$$

$=$ Domain: $x \neq 0$.

$$gof(x) = \frac{1}{\frac{x}{1+x^2}} = \frac{1+x^2}{x}$$, Domain: $x \neq 0$

Q35: $f(x) = x^2+1$, $g(x) = 1/x$, $h(x) = x^3$

$goh(x) = 1/x^3$

$fogoh(x) = (\frac{1}{x^3})^2 + 1 = \frac{1}{x^6} + 1 = x^{-6} + 1$

Q36 $f(x) = \frac{1}{1+x}$, $g(x) = \sqrt[3]{x}$, $h(x) = 1/x^3$

$goh(x) = \sqrt[3]{1/x^3} = (\frac{1}{x^3})^{1/3} = 1/x$

$fogoh(x) = \frac{1}{1+\frac{1}{x}} = \frac{x}{x+1}.$

Q37 a) $f(x) = \sqrt{x+2}$

$h(x) = x+2$

$g(x) = \sqrt{x}$

b) $f(x) = |x^2 - 3x + 5|$

$h(x) = x^2 - 3x.$

$g(x) = |x+5|$

Q38 a) $f(x) = x^2+1$

$h(x) = x^2$

$g(x) = x+1$

b) $f(x) = \frac{1}{x-3}$

$h(x) = x-3$

$g(x) = \frac{1}{x}$

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Q39 a) $f(x) = \sin^2 x$

$h(x) = \sin x$.

$g(x) = x^2$

b) $f(x) = \dfrac{3}{5+\cos x}$

$h(x) = 5+\cos x$

$g(x) = 3/x$

Q40 a) $f(x) = 3\sin(x^2)$

$h(x) = \sin x^2$

$g(x) = 3x$.

b) $3\sin^2 x + 4\sin x$

$h(x) = \sin x$

$g(x) = 3x^2 + 4x$.

Q41 a) $f(x) = (1+\sin(x^2))^3$.

$h(x) = \sin x^2$

$g(x) = (1+x)^3$

b) $f(x) = \sqrt{1-\sqrt[3]{x}}$

$h(x) = \sqrt[3]{x}$

$g(x) = \sqrt{1-x}$

Q42 a) $f(x) = \dfrac{1}{1-x^2}$

$h(x) = 1-x^2$

$g(x) = \dfrac{1}{x}$

b) $f(x) = |5+2x|$

$h(x) = 5+2x$

$g(x) = |x|$

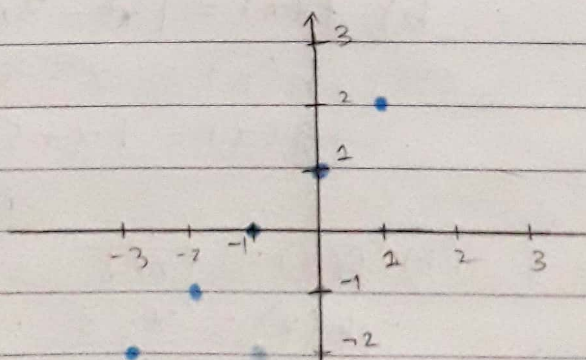
Q43 True. By definition 0.2.1.

Q44 False, the domain of g is the set of all x for which $g(x)$ is in the domain of f.

Q45 True. By definition 0.2.3(a).

Q46 False. The graph of $y = f(x+2)+3$ is obtained by translating left 2 units and moving up 3 units.

Q47



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Q48 $-3 < x < 4$

Q49 $x = -1, -2, -3, 0, 1, 2, 3$

$g(x)$: $g(0) = 0$

$g(-1) = 1.4$ $g(1) = -14$

$g(-2) = 2.4$ $g(2) = -2.4$

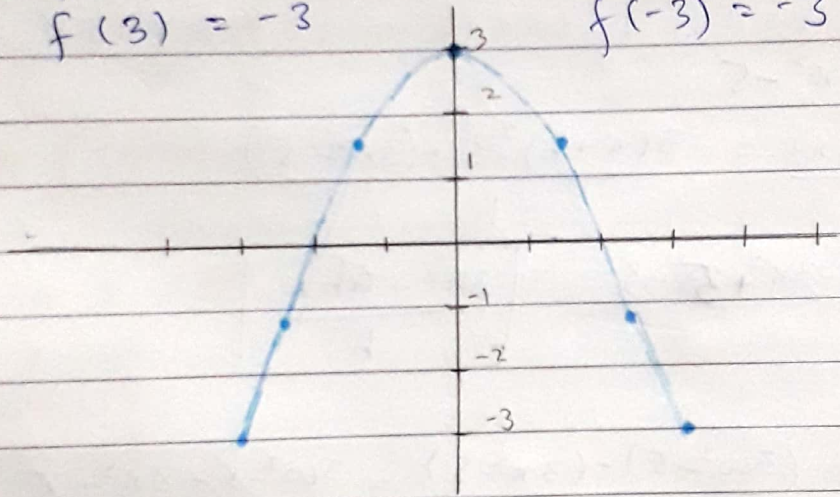
$g(-3) = 3$ $g(3) = -3$

$f(g(x))$: $f(0) = 3$

$f(1.4) = 1.5$ $f(-1.4) = 1.5$

$f(2.4) = -1.2$ $f(-2.4) = -1.2$

$f(3) = -3$ $f(-3) = -3$



Q50 $f(x)$:

$f(0) = 0$

$f(1) = 2$ $f(-1) = 2$

$f(2) = 0$ $f(-2) = 0$

$f(3) = -3$ $f(-3) = -3$

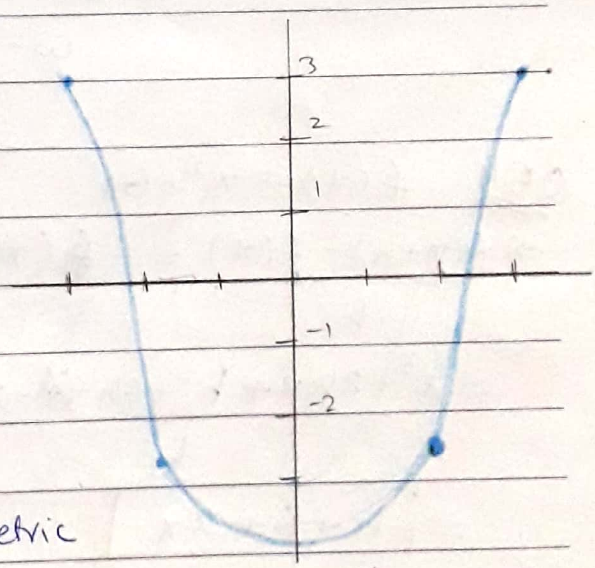
$g(f(x))$:

$g(2) = -2.8$ ~~g(2)=~~

$g(0) = 0$

$g(-3) = 3$

$g(f(x))$ is ~~odd~~ even so it is symmetric about y-axis.



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~~Q51~~ 51) From the graph of ex 49,

$f(g(x)) = 0$ when $g(x) = \pm 2$ and $g(x) = \pm 2$ at $\boxed{x = \pm 1.5}$

$g(f(x)) = 0$ when $f(x) = 0$ and $f(x) = 0$ at $\boxed{x = \pm 2}$

Q52 → $f(g(x)) = 0$ when $g(x) = 1$ and $g(x)$ is 1 at $\boxed{x = -1}$

→ $g(f(x)) = 0$ when $f(x) = -2$ and $f(x)$ is -2 at $\boxed{x = -1}$

Q53 $f(x) = 3x^2 - 5$

→ $\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$

$= \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{3x^2} + \cancel{5} - \cancel{5}}{h} = \boxed{\frac{3h^2 + 6hx}{h}}$

→ $\frac{f(\omega) - f(x)}{\omega - x} = \frac{(3\omega^2 - 5) - (3x^2 - 5)}{\omega - x} = \frac{3\omega^2 - \cancel{5} - 3x^2 + \cancel{5}}{\omega - x}$

$= \frac{3\omega^2 - 3x^2}{\omega - x} = \frac{3\cancel{(\omega - x)}(\omega + x)}{\cancel{(\omega - x)}} = 3(\omega + x)$

$= \boxed{3\omega + 3x}$

Q54 $f(x) = x^2 + 6x$

→ $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 6(h+x) - (x^2 + 6x)}{h}$

$= \frac{\cancel{x^2} + 2hx + h^2 + 6h + \cancel{6x} - \cancel{x^2} - \cancel{6x}}{h} = \frac{h^2 + 6h + 2hx}{h} = \frac{\cancel{h}(h + 6 + 2x)}{\cancel{h}}$

$= \boxed{h + 6 + 2x}$

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$$\rightarrow \frac{f(w)-f(x)}{w-x} = \frac{w^2+6w-(x^2+6x)}{w-x} = \frac{w^2+6w-x^2-6x}{w-x}.$$

$$= \frac{w^2-x^2+6(w-x)}{w-x} = \frac{(w+x)(w-x)+6(w-x)}{w-x}.$$

$$= \frac{\cancel{(w-x)}\{w+x+6\}}{\cancel{(w-x)}} = \boxed{w+x+6}$$

Q55 $\Rightarrow \dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{1}{(x+h)} - \frac{1}{x}}{h} = \dfrac{\frac{x-(x+h)}{x(x+h)}}{h} = \dfrac{x-(x+h)}{xh(x+h)}$

$= \dfrac{x - x - h}{xh(x+h)} = \boxed{\dfrac{-1}{x(x+h)}}$

$\rightarrow \dfrac{f(w)-f(x)}{w-x} = \dfrac{\frac{1}{w} - \frac{1}{x}}{w-x} = \dfrac{\frac{x-w}{wx}}{w-x} = \dfrac{-\cancel{(w-x)}}{wx\cancel{(w-x)}} = \boxed{\dfrac{-1}{wx}}$

Q56 $f(x) = 1/x^2$

$\rightarrow \dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \dfrac{x^2-(x+h)^2}{x^2(x+h)^2 h}$

$= \dfrac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{x^2(x+h)^2 h} = \dfrac{-\cancel{h}(2x+h)}{x^2(x+h)^2\cancel{h}} = \boxed{\dfrac{2x+h}{x^2(x+h)^2}}$

$\rightarrow \dfrac{f(w)-f(x)}{w-x} = \dfrac{\frac{1}{w^2} - \frac{1}{x^2}}{w-x} = \dfrac{x^2-w^2}{w^2x^2(w-x)} = \dfrac{(x+w)(x-w)}{w^2x^2(w-x)}$

$= -\dfrac{\cancel{(w-x)}(x+w)}{x^2w^2\cancel{(w-x)}} = \boxed{-\dfrac{x+w}{x^2w^2}}$

Q57

$f(x) =$ Neither
$g(x) =$ odd
$h(x) =$ even

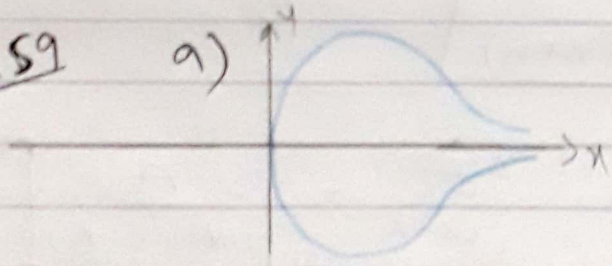
Q58 a) The graph is symmetric about y-axis when function is even, so:

x	-3	-2	-1	0	1	2	3
$f(x)$	1	-5	-1	0	-1	-5	1

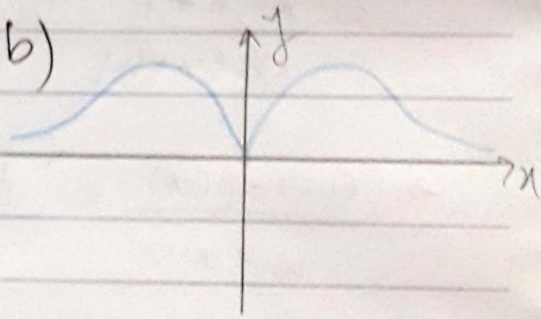
b) The graph is symmetric about origin when function is odd, so:

x	-3	-2	-1	0	1	2	3
$f(x)$	1	5	-1	0	1	-5	-1

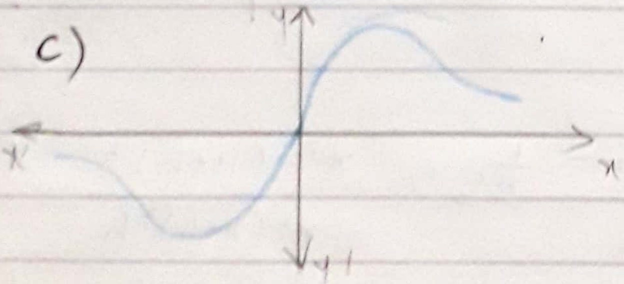
Q59 a)



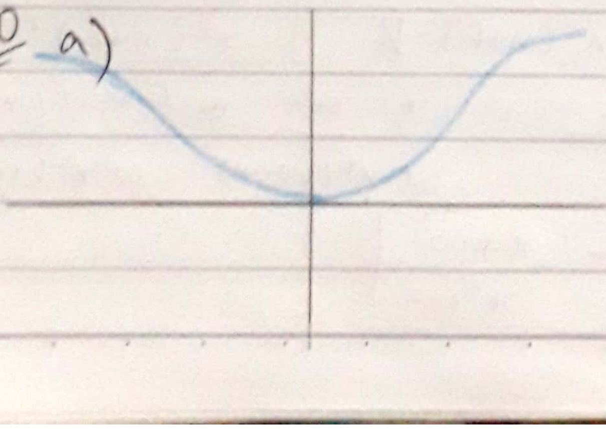
b)



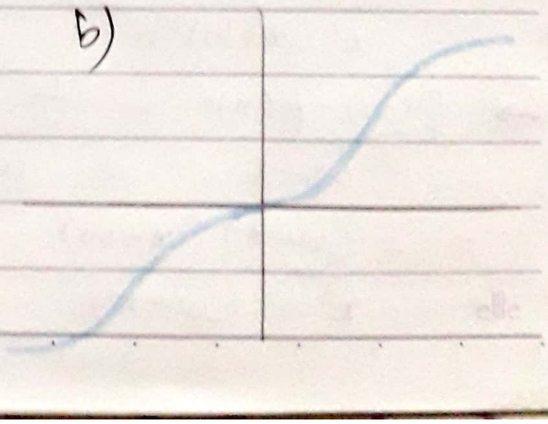
c)



Q60 a)



b)



Q61 a) even
b) odd

Q62 a) odd
b) Neither

Q63 a) $f(x) = x^2$.
$f(-x) = x^2$, function is even.

b) $f(x) = x^3$
$f(-x) = -(x^3)$ function is odd

c) $f(x) = |x|$.
$f(-x) = |x|$ function is even

d) $f(x) = x+1$.
$f(-x) = -x+1$ function is neither

e) $f(x) = \frac{x^5 - x}{1+x^2}$

$f(-x) = \frac{-x^5 + x}{1+x^2} \rightarrow = -\frac{(x^5 - x)}{1-x^2}$. function is ~~neither~~ odd.

f) $f(x) = 2$
$f(-x) = 2$ function is even

Q64 a) $g(-x) = \frac{f(-x)+f(x)}{2} = \frac{f(x)+f(-x)}{2}$, $g(-x)$ is even

b) $h(-x) = \frac{f(-x) - f(x)}{2} = -\frac{(f(x) - f(-x))}{2} = -h(x) = h(-x)$ is odd

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Q65: we found out in ex: 64 that $g(x)$ is even and $h(x)$ is odd.

$$f(x) = g(x) + h(x) = \frac{f(x)+f(-x)}{2} + \frac{f(x)-f(-x)}{2}$$

$$= \frac{f(x) + \cancel{f(-x)} + f(x) - \cancel{f(-x)}}{2} = \frac{\cancel{2}f(x)}{\cancel{2}} = f(x)$$

so, $f(x)$ can be written as a sum of odd and even functions for all x.

$x = -f(x)$
$y = f(-x)$

Q66 a) $x = 5y^2 + 9$.

replace y by $-y$: $x = 5y^2+9$, about x-axis

b) $x^2 - 2y^2 = 3 \Rightarrow x^2 = 3 + 2y^2$

replace y by $-y$: $x^2 = 3+2y^2$

replace x by $-x$: $x^2 = 3+2y^2$

replace x and y both by $-x$ and $-y$: $x^2 = 3+2y^2$

so the graph reflects about x-axis, y-axis and origin

c) $xy = 5$

replace both x and y by $-x$ and $-y$: $xy = 5$

so the graph reflects about origin

~~d)~~

Q67

a) $x^4 = 2y^3 + y$

replace x by $-x$: $x^4 = 2y^3+y$; about y-axis

b) $y = \frac{x}{3+x^2}$

replace x by $-x$: $y = -\left(\frac{x}{3+x^2}\right)$; about origin.

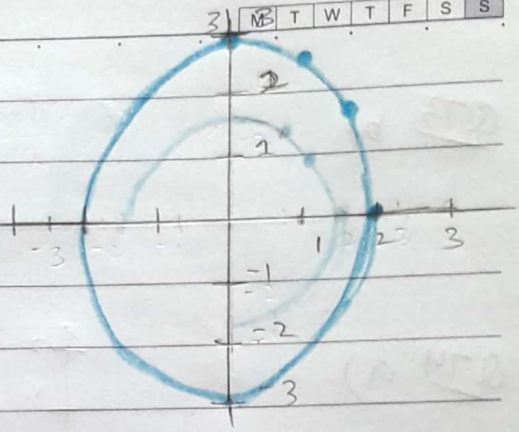
c) $y^2 = |x| - 5$

replacing y by $-y$, x by $-x$, and both x and y by $-x$ and $-y$ gives us: $y^2 = |x| - 5$

so it symmetries about x-axis, y-axis and origin.

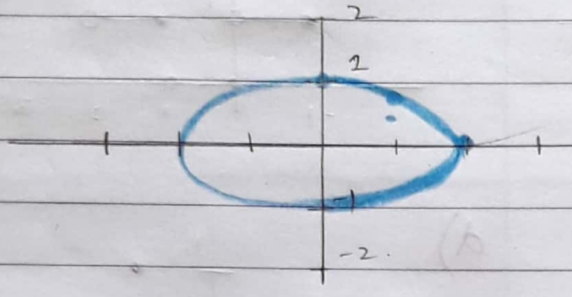
Q68 $9x^2+4y^2=36$.

$$y=\sqrt{\frac{36-9x^2}{4}}=\frac{\sqrt{36-9x^2}}{2}$$



Q69 $4x^2+16y^2=16$.

$$y=\sqrt{\frac{16-4x^2}{16}}=\frac{\sqrt{16-4x^2}}{4}.$$

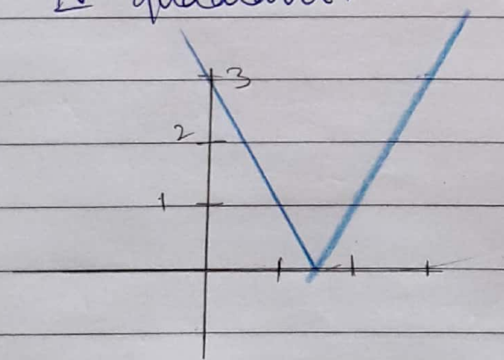


Q70 a) When we replace x with $-x$ & y with $-y$ and we get the same equation, the graph is said to be symmetric about x-axis, y-axis and the origin.

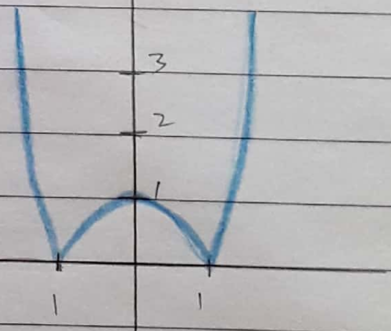
b) $y=(1-x^{2/3})^{3/2}$

c) For quadrant II, the same; for the III and IV quadrant, we use $y=-(1-x^{2/3})^{3/2}$.

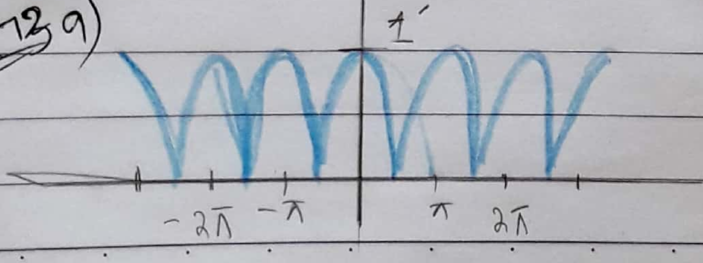
Q71



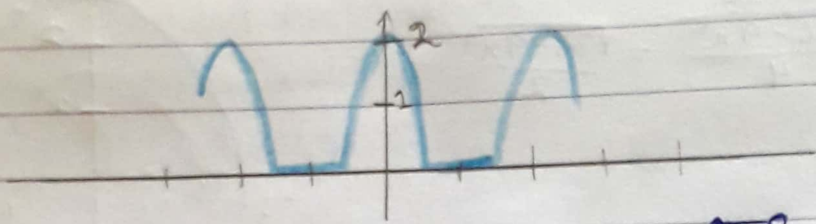
Q72



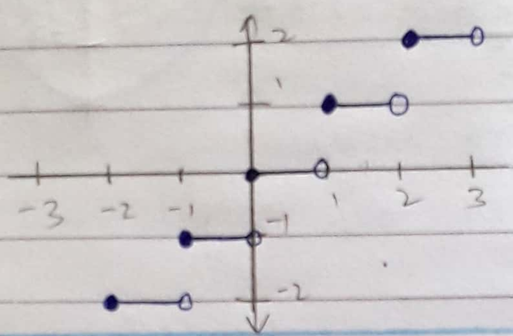
Q73 a)



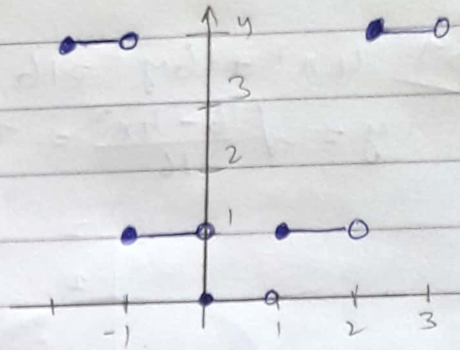
Q73 b)



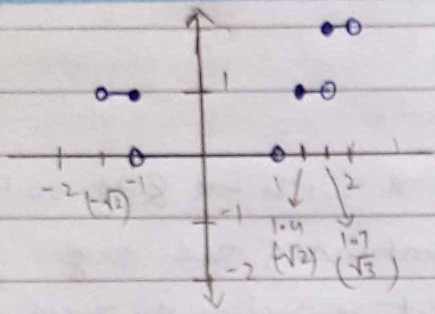
Q74 a)



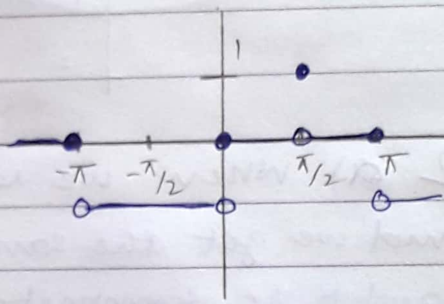
c)



b)



d)



Q75 Yes, it is true when $f(x) = x^k$ and $g(x) = x^n$.
$\therefore$ k and n are integers.

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