ch1: Hypergraph

理解:

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$

$\mathcal{G}$: hypergraph

$\mathcal{V}$: vertices　　U: Vertex Weight Matrix　　order:V

X: Vertex Feature Matrix　　Y: Vertex Label Matrix

$\mathcal{E}$: hyperedges　　W: Hyperedge Weight Matrix　　size:E

$$H \in |\mathcal{V}| * |\mathcal{E}| \qquad H(v,e) = \begin{cases} 1 & \text{if } v \in e \\ 0 & \text{if } v \notin e \end{cases}$$

$$d(v) = \sum_{e \in \mathcal{E}} H(v,e) * w(e) \qquad D_v$$

$$d(e) = \sum_{v \in \mathcal{V}} H(v,e) \qquad D_e$$

$$hyperpath : v1 - e1 - v2 - e2 - v3 - ... \qquad connected$$

$$A = HWD_e^{-1}H^T$$

$$\Delta = D_v - HWD_e^{-1}H^T$$

$$\Delta = I - D_v^{-1/2}HWD_e^{-1}H^TD_v^{-1/2}$$

ch2: Spectral Hypergraph Clustering

理解:

1: 推导超图的拉普拉斯矩阵

$$
\begin{aligned}
g \star x &= \phi((\phi^T g) \odot (\phi^T x)) = \phi g(\wedge)(\phi^T x) \\
g(\wedge) &= diag(g(\lambda_1), ..., g(\lambda_n))
\end{aligned}
$$

$$
\begin{aligned}
g \star x &\approx \sum_{k=0}^{K} \theta_k T_k(\tilde{\Delta}) x \\
\tilde{\Delta} &= \frac{2}{\lambda_{max}} \Delta - I
\end{aligned}
$$

$$
\begin{aligned}
K &= 2 \qquad\qquad \lambda_{max} = 2 \\
g \star x &\approx \theta_0 x - \theta_1 D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x
\end{aligned}
$$

$$
\begin{aligned}
\theta_0 &= (1/2)\theta D_v^{-1/2} H D_e^{-1} H^T D_v^{-1/2} \qquad\qquad \theta_1 = (-1/2)\theta \\
g \star x &\approx (1/2)\theta D_v^{-1/2} H(I + W) D_e^{-1} H^T D_v^{-1/2} x \\
&\approx \theta D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x
\end{aligned}
$$

$$
X^{t+1} = \sigma(D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} X^t \Theta)
$$

ch3: Hypergraph Generation and Transformation

理解：
隐式：距离、特征
显式：属性、网络

ch4: Hypergraph Learning Architecture

理解:
超图游走

(1) Features

$X \in R^{|V| \times d}$ $\quad\quad$ $Y \in R^{|E| \times d'}$

Externel + Internal(local+global) + Identity

(2) Transformation

Reductive Transformation

$(E, X, Y) \Rightarrow A$ $\quad\quad$ hyperedges to edges

clique expansion + adaptive expansion

Non-reductive Transformation

star/line/tensor expansion

(3) Message

whose: v-v v-e e-v

what : e-consistent + e-dependent

how : fixed-pooling + learnable-pooling

(4) Training