

ch1: Hypergraph

理解:

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

\mathcal{G} : hypergraph

\mathcal{V} : vertices

U: Vertex Weight Matrix

order:V

X: Vertex Feature Matrix

Y: Vertex Label Matrix

\mathcal{E} : hyperedges

W: Hyperedge Weight Matrix

size:E

$$H \in |\mathcal{V}| * |\mathcal{E}| \quad H(v, e) = \begin{cases} 1 & \text{if } v \in e \\ 0 & \text{if } v \notin e \end{cases}$$

$$d(v) = \sum_{e \in \mathcal{E}} H(v, e) * w(e) \quad D_v$$

$$d(e) = \sum_{v \in \mathcal{V}} H(v, e) \quad D_e$$

$$hyperpath : v1 - e1 - v2 - e2 - v3 - \dots$$

connected

$$A = HWD_e^{-1}H^T$$

$$\Delta = D_v - HWD_e^{-1}H^T$$

$$\Delta = I - D_v^{-1/2}HWD_e^{-1}H^TD_v^{-1/2}$$

ch2: Hypergraph Random Walk

$$u - e/w(e) - v/uni - e/w(e) - v/uni - \dots$$

$$p(u, v) = \sum_{e \in E} h(u, e) \frac{w(e)}{d(u)} h(v, e) \frac{1}{d(e)}$$

$$P = D_v^{-1} H W D_e^{-1} H^T$$

$$\pi(v) = \frac{d(v)}{\text{vol}(V)} \quad \text{one-step walk to } v \text{ from random start}$$

$$\begin{aligned} \sum_{u \in V} \pi(u) p(u, v) &= \sum_{u \in V} \frac{d(u)}{\text{vol}(V)} \sum_{e \in E} h(u, e) \frac{w(e)}{d(u)} h(v, e) \frac{1}{d(e)} \\ &= \frac{1}{\text{vol}(V)} \sum_{u \in V} \sum_{e \in E} \frac{h(u, e) w(e) h(v, e)}{d(e)} \\ &= \frac{1}{\text{vol}(V)} \sum_{e \in E} w(e) \sum_{u \in V} h(u, e) \frac{h(v, e)}{d(e)} \\ &= \frac{1}{\text{vol}(V)} \sum_{e \in E} w(e) h(v, e) = \frac{d(v)}{\text{vol}(V)} \end{aligned}$$

ch3: Hypergraph Cut/Partition

$$\text{hypergraph} \quad (\text{cut/partition}) : V = S + S^c$$

$$\text{hyperedge} \quad (\text{cut/partition}) : (e \cap S) + (e \cap S^c)$$

$$\text{hyperedge} \quad \text{bountry} : \partial S = \{(e \cap S) + (e \cap S^c)\}$$

$$\text{vol}(S) = \sum_{v \in S} d(v)$$

$$\text{vol}(\partial S) = \sum_{e \in \partial S} |e \cap S| |e \cap S^c| \frac{w(e)}{d(e)} \quad C_{d(e)}^2$$

$$\text{vol}(\partial S) = \text{vol}(\partial S^c)$$

$$\text{argmin } c(S) := \text{vol}(\partial S) \left(\frac{1}{\text{vol} S} + \frac{1}{\text{vol} S^c} \right)$$

$$\text{argmin } c(S) := \frac{\text{vol}(\partial S)}{\text{vol}(V)} \left(\frac{1}{\frac{\text{vol}(S)}{\text{vol}(V)}} + \frac{1}{\frac{\text{vol}(S^c)}{\text{vol}(V)}} \right)$$

$$\frac{\text{vol}(S)}{\text{vol}(V)} = \sum_{v \in S} \frac{d(v)}{\text{vol}(V)} = \sum_{v \in S} \pi(v)$$

$$\frac{\text{vol}(\partial S)}{\text{vol}(V)} = \sum_{e \in \partial S} |e \cap S| |e \cap S^c| \frac{w(e)}{d(e)} \frac{1}{\text{vol}(V)}$$

$$= \sum_{e \in \partial S} \sum_{u \in S} \sum_{v \in S^c} h(u, e) \frac{w(e)}{d(u)} h(v, e) \frac{1}{d(e)} \frac{d(u)}{\text{vol}(V)}$$

$$= \sum_{u \in S} \sum_{v \in S^c} \frac{d(u)}{\text{vol}(V)} \sum_{e \in \partial S} h(u, e) \frac{w(e)}{d(u)} h(v, e) \frac{1}{d(e)}$$

$$= \sum_{u \in S} \sum_{v \in S^c} \frac{d(u)}{\text{vol}(V)} p(u, v) = \sum_{u \in S} \sum_{v \in S^c} \pi(u) p(u, v)$$

ch4: Hypergraph Laplacian

$$\operatorname{argmin}_S c(S) := \operatorname{vol}(\partial S) \left(\frac{1}{\operatorname{vol} S} + \frac{1}{\operatorname{vol} S^c} \right)$$

$$\operatorname{argmin}_{f \in \mathbb{R}^{|V|}} \frac{1}{2} \sum_{e \in E} \sum_{\{u,v\} \in e} \frac{w(e)}{d(e)} \left(\frac{f(u)}{\sqrt{d(u)}} - \frac{f(v)}{\sqrt{d(v)}} \right)^2$$

$$\text{subject to:} \quad \sum_{v \in V} f^2(v) = 1 \quad \sum_{v \in V} f(v) \sqrt{d(v)} = 0$$

$$\Theta = D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2}$$

$$\Delta = I - \Theta$$

$$\frac{1}{2} \sum_{e \in E} \sum_{\{u,v\} \in e} \frac{w(e)}{d(e)} \left(\frac{f(u)}{\sqrt{d(u)}} - \frac{f(v)}{\sqrt{d(v)}} \right)^2 = f^T \Delta f$$

$$\text{solution:} \quad \Phi \text{ of } \Delta \quad (\text{eigenvector of smallest nonzero eigenvalue})$$

ch4: Hypergraph Convolution

$$g \star x = \phi((\phi^T g) \odot (\phi^T x)) = \phi g(\wedge)(\phi^T x)$$

$$g(\wedge) = \text{diag}(g(\lambda_1), \dots, g(\lambda_n))$$

$$g \star x \approx \sum_{k=0}^K \theta_k T_k(\tilde{\Delta}) x$$

$$\tilde{\Delta} = \frac{2}{\lambda_{max}} \Delta - I$$

$$K = 2 \quad \lambda_{max} = 2$$

$$g \star x \approx \theta_0 x - \theta_1 D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x$$

$$\theta_0 = (1/2) \theta D_v^{-1/2} H D_e^{-1} H^T D_v^{-1/2} \quad \theta_1 = (-1/2) \theta$$

$$g \star x \approx (1/2) \theta D_v^{-1/2} H (I + W) D_e^{-1} H^T D_v^{-1/2} x$$

$$\approx \theta D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x$$

$$X^{t+1} = \sigma(D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} X^t \Theta)$$

ch5: Hypergraph Generation and Transformation

理解:

隐式: 距离、特征

显式: 属性、网络

ch6: Hypergraph Learning Architecture

理解:

超图游走

(1) Features

$$X \in R^{|V| \times d} \quad Y \in R^{|E| \times d'}$$

External + Internal(local+global) + Identity

(2) Transformation

Reductive Transformation

$(E, X, Y) \Rightarrow A$ hyperedges to edges

clique expansion + adaptive expansion

Non-reductive Transformation

star/line/tensor expansion

(3) Message

whose: v-v v-e e-v

what : e-consistent + e-dependent

how : fixed-pooling + learnable-pooling

(4) Training