ch1: Hypergraph

理解:

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$

$\mathcal{G}$: hypergraph

$\mathcal{V}$: vertices U: Vertex Weight Matrix order:V

X: Vertex Feature Matrix Y: Vertex Label Matrix

$\mathcal{E}$: hyperedges W: Hyperedge Weight Matrix size:E

$$H \in |\mathcal{V}| * |\mathcal{E}| \qquad H(v,e) = \begin{cases} 1 & \text{if } v \in e \\ 0 & \text{if } v \notin e \end{cases}$$

$$d(v) = \sum_{e \in \mathcal{E}} H(v,e) * w(e) \qquad D_v$$

$$d(e) = \sum_{v \in \mathcal{V}} H(v,e) \qquad D_e$$

$$hyperpath : v1 - e1 - v2 - e2 - v3 - ... \qquad connected$$

$$A = HWD_e^{-1}H^T$$

$$\Delta = D_v - HWD_e^{-1}H^T$$

$$\Delta = I - D_v^{-1/2}HWD_e^{-1}H^TD_v^{-1/2}$$

ch2: Hypergraph Random Walk

$$u - e/w(e) - v/uni - e/w(e) - v/uni - ...$$

$$p(u,v) = \sum_{e \in E} h(u,e) \frac{w(e)}{d(u)} h(v,e) \frac{1}{d(e)}$$

$$P = D_v^{-1} H W D_e^{-1} H^T$$

$$\pi(v) = \frac{d(v)}{vol(V)}$$

one-step wlak to v from random start

$$\sum_{u \in V} \pi(u) p(u,v) = \sum_{u \in V} \frac{d(u)}{vol(V)} \sum_{e \in E} h(u,e) \frac{w(e)}{d(u)} h(v,e) \frac{1}{d(e)}$$

$$= \frac{1}{vol(V)} \sum_{u \in V} \sum_{e \in E} \frac{h(u,e) w(e) h(v,e)}{d(e)}$$

$$= \frac{1}{vol(V)} \sum_{e \in E} w(e) \sum_{u \in V} h(u,e) \frac{h(v,e)}{d(e)}$$

$$= \frac{1}{vol(V)} \sum_{e \in E} w(e) h(v,e) = \frac{d(v)}{vol(V)}$$

ch3: Hypergraph Cut/Partition

$$\begin{aligned}
&hypergraph \quad (cut/partition) : V = S + S^c \qquad \text{k-way partition}\\
&hyperedge \quad (cut/partition) : (e \cap S) + (e \cap S^c)\\
&hyperedge \quad bountry : \partial S = \{(e \cap S) + (e \cap S^c)\}
\end{aligned}$$

$$\begin{aligned}
\mathrm{vol}(S) \quad &= \sum_{v \in S} d(v)\\
\mathrm{vol}(\partial S) &= \sum_{e \in \partial S} |e \cap S||e \cap S^c| \frac{w(e)}{d(e)} \qquad C^2_{d(e)}\\
\mathrm{vol}(\partial S) &= \mathrm{vol}(\partial S^c)
\end{aligned}$$

$$\begin{aligned}
&argmin\ c(S) := vol(\partial S)(\frac{1}{volS} + \frac{1}{volS^c})\\
&argmin\ c(S) := \frac{vol(\partial S)}{vol(V)}(\frac{1}{\frac{vol(S)}{vol(V)}} + \frac{1}{\frac{vol(S^c)}{vol(V)}})\\
&\frac{vol(S)}{vol(V)} = \sum_{v \in S} \frac{d(v)}{vol(V)} = \sum_{v \in S} \pi(v)\\
&\frac{vol(\partial S)}{vol(V)} = \sum_{e \in \partial S} |e \cap S||e \cap S^c| \frac{w(e)}{d(e)} \frac{1}{vol(V)}\\
&= \sum_{e \in \partial S} \sum_{u \in S} \sum_{v \in S^c} h(u,e) \frac{w(e)}{d(u)} h(v,e) \frac{1}{d(e)} \frac{d(u)}{vol(V)}\\
&= \sum_{u \in S} \sum_{v \in S^c} \frac{d(u)}{vol(V)} \sum_{e \in \partial S} h(u,e) \frac{w(e)}{d(u)} h(v,e) \frac{1}{d(e)}\\
&= \sum_{u \in S} \sum_{v \in S^c} \frac{d(u)}{vol(V)} p(u,v) = \sum_{u \in S} \sum_{v \in S^c} \pi(u) p(u,v)
\end{aligned}$$

ch4: Hypergraph Laplacian

$$argmin\ c(S) := vol(\partial S)(\frac{1}{volS} + \frac{1}{volS^c})$$

$$argmin_{f\in R^{|V|}} \frac{1}{2}\sum_{e\in E}\sum_{\{u,v\}\in e}\frac{w(e)}{d(e)}\left(\frac{f(u)}{\sqrt{d(u)}} - \frac{f(v)}{\sqrt{d(v)}}\right)^2 \qquad R^{|V|} = \{1,-1\}^{|V|}$$

subject to: $\sum_{v\in V} f^2(v) = 1 \qquad \sum_{v\in V} f(v)\sqrt{d(v)} = 0$

$$\Theta = D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2}$$

$$\Delta = I - \Theta$$

$$\frac{1}{2}\sum_{e\in E}\sum_{\{u,v\}\in e}\frac{w(e)}{d(e)}\left(\frac{f(u)}{\sqrt{d(u)}} - \frac{f(v)}{\sqrt{d(v)}}\right)^2 = f^T\Delta f$$

solution: $\Phi\ of\ \Delta$ (eigenvector of smallest nonzero eigenvalue)

ch4: Hypergraph Convolution

$$g \star x = \phi((\phi^T g) \odot (\phi^T x)) = \phi g(\wedge)(\phi^T x)$$
$$g(\wedge) = diag(g(\lambda_1), ..., g(\lambda_n))$$

$$g \star x \approx \sum_{k=0}^{K} \theta_k T_k(\tilde{\Delta})x$$
$$\tilde{\Delta} = \frac{2}{\lambda_{max}}\Delta - I$$

$$K = 2 \qquad \lambda_{max} = 2$$
$$g \star x \approx \theta_0 x - \theta_1 D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x$$

$$\theta_0 = (1/2)\theta D_v^{-1/2} H D_e^{-1} H^T D_v^{-1/2} \qquad \theta_1 = (-1/2)\theta$$
$$g \star x \approx (1/2)\theta D_v^{-1/2} H(I + W) D_e^{-1} H^T D_v^{-1/2} x$$
$$\approx \theta D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x$$

$$X^{t+1} = \sigma(D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} X^t \Theta)$$

ch5: Hypergraph Generation and Transformation

理解：
隐式：距离、特征
显式：属性、网络

ch6: Hypergraph Learning Architecture

理解:
超图游走

(1) Features

$X \in R^{|V| \times d}$ $\quad\quad$ $Y \in R^{|E| \times d'}$

Externel + Internal(local+global) + Identity

(2) Transformation

Reductive Transformation

$(E, X, Y) \Rightarrow A$ $\quad\quad$ hyperedges to edges

clique expansion + adaptive expansion

Non-reductive Transformation

star/line/tensor expansion

(3) Message

whose: v-v v-e e-v

what : e-consistent + e-dependent

how : fixed-pooling + learnable-pooling

(4) Training