ch1: Hypergraph

理解:

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$

$\quad \mathcal{G} \quad$: hypergraph

$\quad \mathcal{V} \quad$: vertices $\qquad$ U: Vertex Weight Matrix $\qquad$ order:V

$\qquad\qquad\qquad\qquad$ X: Vertex Feature Matrix $\qquad$ Y: Vertex Label Matrix

$\quad \mathcal{E} \quad$: hyperedges $\qquad$ W: Hyperedge Weight Matrix $\qquad$ size:E

$H \in |\mathcal{V}| * |\mathcal{E}| \qquad\qquad H(v,e) = \begin{cases} 1 & \text{if } v \in e \\ 0 & \text{if } v \notin e \end{cases}$

$d(v) = \displaystyle\sum_{e \in \mathcal{E}} H(v,e) * w(e) \qquad D_v$

$d(e) = \displaystyle\sum_{v \in \mathcal{V}} H(v,e) \qquad\qquad D_e$

$hyperpath : v1 - e1 - v2 - e2 - v3 - ... \qquad\qquad\qquad connected$

$A = HWD_e^{-1}H^T$

$\Delta = D_v - HWD_e^{-1}H^T$

$\Delta = I - D_v^{-1/2}HWD_e^{-1}H^T D_v^{-1/2}$

ch2: Spectral Hypergraph Clustering

理解:
1: 推导超图的拉普拉斯矩阵

$$hypergraph \quad (cut/partition) : V = S + S^c$$

$$hyperedge \quad (cut/partition) : (e \cap S) + (e \cap S^c)$$

$$hypergraph \quad bountry : \partial S = \{(e \cap S) + (e \cap S^c)\}$$

$$\text{vol}(S) \quad = \sum_{v \in S} d(v)$$

$$\text{vol}(\partial S) = \sum_{e \in \partial S} |e \cap S||e \cap S^c| \frac{w(e)}{\delta(e)} \qquad\qquad C^2_{d(e)}$$

$$\text{vol}(\partial S) = \text{vol}(\partial S^c)$$

$$g \star x = \phi((\phi^T g) \odot (\phi^T x)) = \phi g(\wedge)(\phi^T x)$$

$$g(\wedge) = diag(g(\lambda_1), ..., g(\lambda_n))$$

$$g \star x \approx \sum_{k=0}^{K} \theta_k T_k(\tilde{\Delta}) x$$

$$\tilde{\Delta} = \frac{2}{\lambda_{max}} \Delta - I$$

$$K = 2 \qquad \lambda_{max} = 2$$

$$g \star x \approx \theta_0 x - \theta_1 D_v^{-1/2} HW D_e^{-1} H^T D_v^{-1/2} x$$

$$\theta_0 = (1/2)\theta D_v^{-1/2} H D_e^{-1} H^T D_v^{-1/2} \qquad \theta_1 = (-1/2)\theta$$

$$g \star x \approx (1/2)\theta D_v^{-1/2} H(I+W) D_e^{-1} H^T D_v^{-1/2} x$$

$$\approx \theta D_v^{-1/2} HW D_e^{-1} H^T D_v^{-1/2} x$$

$$X^{t+1} = \sigma(D_v^{-1/2} HW D_e^{-1} H^T D_v^{-1/2} X^t \Theta)$$

ch3: Hypergraph Generation and Transformation

理解:

隐式: 距离、特征

显式: 属性、网络

ch4: Hypergraph Learning Architecture

理解:
超图游走

(1) Features

$$X \in R^{|V| \times d} \qquad Y \in R^{|E| \times d'}$$

Externel + Internal(local+global) + Identity

(2) Transformation

Reductive Transformation

$$(E, X, Y) \Rightarrow A \qquad \text{hyperedges to edges}$$

clique expansion + adaptive expansion

Non-reductive Transformation

star/line/tensor expansion

(3) Message

whose: v-v v-e e-v

what : e-consistent + e-dependent

how : fixed-pooling + learnable-pooling

(4) Training