

## ch1: Hypergraph

理解:

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

$\mathcal{G}$  : hypergraph

$\mathcal{V}$  : vertices

U: Vertex Weight Matrix

order:V

X: Vertex Feature Matrix

Y: Vertex Label Matrix

$\mathcal{E}$  : hyperedges

W: Hyperedge Weight Matrix

size:E

$$H \in |\mathcal{V}| * |\mathcal{E}| \quad H(v, e) = \begin{cases} 1 & \text{if } v \in e \\ 0 & \text{if } v \notin e \end{cases}$$

$$d(v) = \sum_{e \in \mathcal{E}} H(v, e) * w(e) \quad D_v$$

$$d(e) = \sum_{v \in \mathcal{V}} H(v, e) \quad D_e$$

*hyperpath* :  $v1 - e1 - v2 - e2 - v3 - \dots$

*connected*

$$A = HWD_e^{-1}H^T$$

$$\Delta = D_v - HWD_e^{-1}H^T$$

$$\Delta = I - D_v^{-1/2}HWD_e^{-1}H^TD_v^{-1/2}$$

## ch2: Spectral Hypergraph Clustering

理解:

1: 推导超图的拉普拉斯矩阵

$$\text{hypergraph} \quad (\text{cut/partition}) : V = S + S^c$$

$$\text{hyperedge} \quad (\text{cut/partition}) : (e \cap S) + (e \cap S^c)$$

$$\text{hypergraph} \quad \text{boundary} : \partial S = \{(e \cap S) + (e \cap S^c)\}$$

$$\text{vol}(S) = \sum_{v \in S} d(v)$$

$$\text{vol}(\partial S) = \sum_{e \in \partial S} |e \cap S| |e \cap S^c| \frac{w(e)}{\delta(e)}$$

$$\text{vol}(\partial S) = \text{vol}(\partial S^c)$$

$$g \star x = \phi((\phi^T g) \odot (\phi^T x)) = \phi g(\wedge)(\phi^T x)$$

$$g(\wedge) = \text{diag}(g(\lambda_1), \dots, g(\lambda_n))$$

$$g \star x \approx \sum_{k=0}^K \theta_k T_k(\tilde{\Delta}) x$$

$$\tilde{\Delta} = \frac{2}{\lambda_{max}} \Delta - I$$

$$K = 2 \qquad \lambda_{max} = 2$$

$$g \star x \approx \theta_0 x - \theta_1 D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x$$

$$\theta_0 = (1/2)\theta D_v^{-1/2} H D_e^{-1} H^T D_v^{-1/2} \qquad \theta_1 = (-1/2)\theta$$

$$g \star x \approx (1/2)\theta D_v^{-1/2} H (I + W) D_e^{-1} H^T D_v^{-1/2} x$$

$$\approx \theta D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} x$$

$$X^{t+1} = \sigma(D_v^{-1/2} H W D_e^{-1} H^T D_v^{-1/2} X^t \Theta)$$

## ch3: Hypergraph Generation and Transformation

理解:

隐式: 距离、特征

显式: 属性、网络

## ch4: Hypergraph Learning Architecture

理解:

超图游走

(1) Features

$$X \in R^{|V| \times d} \quad Y \in R^{|E| \times d'}$$

External + Internal(local+global) + Identity

(2) Transformation

Reductive Transformation

$$(E, X, Y) \Rightarrow A$$

hyperedges to edges  
clique expansion + adaptive expansion

Non-reductive Transformation

star/line/tensor expansion

(3) Message

whose: v-v v-e e-v

what : e-consistent + e-dependent

how : fixed-pooling + learnable-pooling

(4) Training