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		FECHA

9)

a.

rec2: $T(n)\begin{cases} \overset{\nearrow C_1}{1} & , n \leq 1 \\ C_2 + T(n-1) & , n > 1 \end{cases}$

$\triangleright C_2$ es el costo de multiplicar por 2 el valor que retorna de las recursiones (todas)

Paso 1: $C_2 + T(n-1)$

Paso 2: $C_2 + [C_2\ T(n-2)] \rightarrow 2C_2 + (T(n-2))$

Paso 3: $C_2 + [2C_2\ T(n-3)] \rightarrow 3C_2 + (T(n-3))$

Paso i: $i C_2 + T(n-i)$

$T(n) = 1$ cuándo...

$n - i = 1$

$n = i + 1$

$\Rightarrow T(n) = (n-1)C_2 + T(n-(n-1))$

$= (n-1)C_2 + T(1)$

$= \cancel{(n-1)C_2} + \cancel{C_1}$

$\Rightarrow O(n)$

rec1: $T(n)\begin{cases} \overset{\nearrow C_1}{1} & , n \leq 1 \\ \underbrace{2T(n-1)}_{\triangleright \text{ son dos llamadas}} & , n > 1 \end{cases}$

Paso 1: $T(n) = 2T(n-1)$

Paso 2: $T(n) = 2[2T(n-2)] \rightarrow 4T(n-2)$

Paso 3: $T(n) = 2[4T(n-3)] \rightarrow 8T(n-3)$

Paso k: $T(n) = 2^k \cdot T(n-k)$

$T(n) = 1$ cuándo...

$n - k = 1$

$n = k + 1$

$\Rightarrow T(n) = 2^{n-1} \cdot T(n-(n-1)) \rightarrow T(n) = 2^{n-1} \cdot T(1) = 2^{n-1} \cdot C_1$

NOTA

$= \cancel{\frac{1}{2}} \cdot 2^n \cdot \cancel{C_1} \Longrightarrow O(2^n)$

rec3: $T(n) \begin{cases} 0 \rightarrow C_1 \rightarrow C_0, & n == 0 \\ 1, & n == 1 \\ 2T(n-2) + C_2, & n > 1 \end{cases} = T(n) \begin{cases} C_1, & n \leq 1 \\ 2T(n-2) + C_2, & n > 1 \end{cases}$

Paso 1: $T(n) = 2T(n-2) + C$

Paso 2: $T(n) = 2[2T((n-2)-2) + C] + C$

$= 4T(n-4) + 2C$

Paso 3: $T(n) = 2[4T((n-4)-2) + C] + 2C$

$= 8T(n-6) + 3C$

Paso i: $T(n) = 2^i \cdot T(n-2i) + iC$

$T(n) = C_1$ cuando...

$n - 2i = C_1$

$n = C_1 + 2i$

$\Longrightarrow T(n) = 2^{\frac{n-C_1}{2}} \cdot T(n-(n-C_1)) + \frac{n-C_1}{2} C$

$= 2^{\frac{n-C_1}{2}} \cdot T(n-n+C_1) + \frac{1}{2}(n-C_1) \cdot C$

$= 2^{\frac{n-C_1}{2}} \cdot T(C_1) + \frac{1}{2}(Cn - C_1)$

$= 2^{\frac{n-C_1}{2}} + \frac{Cn}{2} - \frac{C_1}{2} = 2^{\frac{n}{2} - \frac{C_1}{2}} + \frac{Cn}{2} - \frac{C_1}{2}$

$= 2^{n/2} \cdot \cancel{\frac{1}{2^{C_1/2}}} + \cancel{\frac{Cn}{2}} - \frac{C_1}{2}$

$\Longrightarrow O(2^{n/2})$

HOJA N°

FECHA

Potencia_iter: $T(n) = C_1 + \sum_{i=2}^{n+1} (C_2)$ (Potencia $= x$)

$= C_1 + \sum_{i=0}^{n-1} C_2 = C_1 + (n-1)C_2$

$= \cancel{C_1} + \cancel{C_2} n - \cancel{C_2}$

$\Longrightarrow O(n)$

Potencia_rec: $T(n) = \begin{cases} C_1 \ (1, x), & n \leq 1 \\ T\left(\frac{n}{2}\right) + C_2 \end{cases}$

Paso 1: $T\left(\frac{n}{2}\right) + C_2$

Paso 2: $\left(T\left(\frac{n}{2}/2\right) + C_2\right) + C_2 \rightarrow T\left(\frac{n}{4}\right) + 2C_2$

Paso 3: $\left(T\left(\frac{n}{4}/2\right) + C_2\right) + 2C_2 \rightarrow T\left(\frac{n}{8}\right) + 3C_2$

Paso k: $T\left(\frac{n}{2^k}\right) + kC_2$

$T(n) = C_1$ cuando ...

$\frac{n}{2^k} = C_1$

$2^k = n \cdot C_1$

$\log_2(nC_1) = k$

$\Longrightarrow T(n) = T\left(\frac{n}{2^{\log_2(nC_1)}}\right) + \log_2(nC_1)C_2$

$= T\left(\frac{n}{\cancel{nC_1}}\right) + \log_2(nC_1)C_2$

$= T(1) + \log_{\cancel{2}}(n\cancel{C_1})\cancel{C_2}$

$\Longrightarrow O(\log n)$

NOTA