

```
In[62]:= << GA30`;
```

Dipole calculation

```
In[63]:= ClearAll[A,  $\phi$ , a, F, r, x, y, z,  $\mu$ ,  $\epsilon$ , c, R,  $\theta$ ,  $\mathfrak{E}$ ]
(* These assumptions aren't for the calculation,
but just for the final spherical substitution after the
field is computed to simplify expressions like  $x^2 + y^2 + z^2$  *)
$Assumptions =  $\mu > 0 \ \&\& \ \epsilon > 0 \ \&\& \ x > 0 \ \&\& \ y > 0 \ \&\& \ z > 0 \ \&\& \ R > 0 \ \&\& \ \theta > 0 \ \&\& \ \phi > 0$ ;
r[x_, y_, z_] := Norm[{x, y, z}]

c := 1 / Sqrt[ $\epsilon \mu$ ];
 $\eta$  := Sqrt[ $\epsilon / \mu$ ];
A := Vector[ $\mu$  i d l E-I k r[x, y, z] / (4 Pi r[x, y, z]), 3];
 $\mathfrak{E}$  := Scalar[-c / (I k)] Div[A, {x, y, z}];
a := c A -  $\mathfrak{E}$ ;
spherical = {x → R Sin[ $\theta$ ] Cos[ $\phi$ ], y → R Sin[ $\theta$ ] Sin[ $\phi$ ], z → R Cos[ $\theta$ ]};
F := (((Grad[a, {x, y, z}] - I k a) ) /. spherical) // Simplify;
```

In[73]:=

```

ClearAll[bold, prettyPrint]
bold := Style[#, Bold] &;
prettyPrint = {i → "I"};
({# // First, " = ", ((# // Last) /. prettyPrint) // TraditionalForm) } & /@
{{bold["A"], A /. spherical},
 {"Φ", Φ /. spherical // Simplify},
 {Row[{"A = c ", bold["A"], " - Φ"}], a /. spherical},
 {Row[{"F = (", bold["∇"], " - j k) A"}], F},
 {bold["E"], F // VectorSelection},
 {bold["H"], -Trivector[1] (F // BivectorSelection) / η}
}) // Grid

```

$$\begin{aligned}
 \mathbf{A} &= \frac{d\mathbf{e}_3 I \mu e^{-i k R}}{4 \pi R} \\
 \Phi &= \frac{dI \mu \cos(\theta) e^{-i k R} (k R - i)}{4 \pi k R^2 \sqrt{\mu \epsilon}} \\
 \mathbf{A} = c \mathbf{A} - \Phi &= \frac{d\mathbf{e}_3 I \mu e^{-i k R}}{4 \pi R \sqrt{\mu \epsilon}} - \frac{dI \mu \cos(\theta) e^{-i k R} (k R - i)}{4 \pi k R^2 \sqrt{\mu \epsilon}} \\
 \mathbf{F} = (\nabla - j k) \mathbf{A} &= -\frac{i dI \mu \mathbf{e}_{23} \sin(\theta) e^{-i k R} (k R - i) \sin(\phi)}{4 \pi R^2 \sqrt{\mu \epsilon}} + \frac{dI \mu \mathbf{e}_{31} \sin(\theta) e^{-i k R} (1 + i k R) \cos(\phi)}{4 \pi R^2 \sqrt{\mu \epsilon}} \\
 &\quad + \frac{d\mathbf{e}_2 I \mu \sin(2\theta) e^{-i k R} (i k^2 R^2 + 3 k R - 3 i) \sin(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\
 &\quad + \frac{d\mathbf{e}_1 I \mu \sin(2\theta) e^{-i k R} (i k^2 R^2 + 3 k R - 3 i) \cos(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\
 &\quad + \frac{d\mathbf{e}_3 I \mu e^{-i k R} (\cos(2\theta) (i k^2 R^2 + 3 k R - 3 i) - i k^2 R^2 + k R - i)}{8 \pi k R^3 \sqrt{\mu \epsilon}} \\
 \mathbf{E} &= \frac{d\mathbf{e}_2 I \mu \sin(2\theta) e^{-i k R} (i k^2 R^2 + 3 k R - 3 i) \sin(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\
 &\quad + \frac{d\mathbf{e}_1 I \mu \sin(2\theta) e^{-i k R} (i k^2 R^2 + 3 k R - 3 i) \cos(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\
 &\quad + \frac{d\mathbf{e}_3 I \mu e^{-i k R} (\cos(2\theta) (i k^2 R^2 + 3 k R - 3 i) - i k^2 R^2 + k R - i)}{8 \pi k R^3 \sqrt{\mu \epsilon}} \\
 \mathbf{H} &= \frac{d\mathbf{e}_2 I \mu \sin(\theta) e^{-i k R} (1 + i k R) \cos(\phi)}{4 \pi R^2 \epsilon} - \frac{i d\mathbf{e}_1 I \mu \sin(\theta) e^{-i k R} (k R - i) \sin(\phi)}{4 \pi R^2 \epsilon}
 \end{aligned}$$

Out[76]=