```
<< GA30`;
```

Dipole calculation

```
ClearAll[A, ϕ, a, F, r, x, y, z, μ, ϵ, c, R, θ, Φ]
(* These assumptions aren't for the calculation,
but just for the final spherical substitution after the
  field is computed to simplify expressions like x^2 + y^2 + z^2 *)
$Assumptions = μ > 0 && ϵ > 0 && x > 0 && y > 0 && z > 0 && R > 0 && θ > 0 && ϕ > 0;
r[x_, y_, z_] := Norm[{x, y, z}]

c := 1 / Sqrt[ϵ μ];
η := Sqrt[ϵ / μ];
A := Vector[μ i dl E^(-I k r[x, y, z]) / (4 Pi r[x, y, z]), 3];
Φ := Scalar[-c / (I k)] div[A, {x, y, z}] ;
a := c A - Φ ;
spherical = {x → R Sin[θ] Cos[ϕ], y → R Sin[θ] Sin[ϕ], z → R Cos[θ]};
F := (((grad[a, {x, y, z}] - I k a ) ) /. spherical ) // Simplify;
```

```
ClearAll[bold, prettyPrint]
bold := Style[#, Bold] &;
prettyPrint = {i → "I"};
({# // First, " = ", (((# // Last ) /. prettyPrint) // TraditionalForm) } &/@
  {{bold["A"], A /. spherical},
   {"Φ", Φ /. spherical // Simplify},
   {Row[{"A = c ", bold["A"] , " - Φ"}], a /. spherical},
   {Row[{"F = (", bold["∇"], " - j k) A"}], F},
   {bold["E"], F // VectorSelection},
   {bold["H"], -Trivector[1] (F // BivectorSelection) / η}
  }) // Grid
```

$$\mathbf{A} = \frac{\mathrm{dl}\,\mathbf{e}_3\,\mathrm{I}\,\mu\,e^{-i k R}}{4 \pi R}$$

$$\Phi = \frac{\mathrm{dl}\,\mathrm{I}\,\mu\cos(\theta)\,e^{-i k R}\,(k R - i)}{4 \pi k R^2 \sqrt{\mu \epsilon}}$$

$$A = c\,\mathbf{A} - \Phi = \frac{\mathrm{dl}\,\mathbf{e}_3\,\mathrm{I}\,\mu\,e^{-i k R}}{4 \pi R \sqrt{\mu \epsilon}} - \frac{\mathrm{dl}\,\mathrm{I}\,\mu\cos(\theta)\,e^{-i k R}\,(k R - i)}{4 \pi k R^2 \sqrt{\mu \epsilon}}$$

$$\begin{aligned} F = (\nabla - j\,k)\,A = & -\frac{i\,\mathrm{dl}\,\mathrm{I}\,\mu\,\mathbf{e}_{23}\sin(\theta)\,e^{-i k R}\,(k R - i)\sin(\phi)}{4 \pi R^2 \sqrt{\mu \epsilon}} + \frac{\mathrm{dl}\,\mathrm{I}\,\mu\,\mathbf{e}_{31}\sin(\theta)\,e^{-i k R}\,(1 + i k R)\cos(\phi)}{4 \pi R^2 \sqrt{\mu \epsilon}} + \\ & \frac{\mathrm{dl}\,\mathbf{e}_2\,\mathrm{I}\,\mu\sin(2\theta)\,e^{-i k R}\left(i k^2 R^2 + 3 k R - 3 i\right)\sin(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\ & \frac{\mathrm{dl}\,\mathbf{e}_1\,\mathrm{I}\,\mu\sin(2\theta)\,e^{-i k R}\left(i k^2 R^2 + 3 k R - 3 i\right)\cos(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\ & \frac{\mathrm{dl}\,\mathbf{e}_3\,\mathrm{I}\,\mu\,e^{-i k R}\left(\cos(2\theta)\left(i k^2 R^2 + 3 k R - 3 i\right) - i k^2 R^2 + k R - i\right)}{8 \pi k R^3 \sqrt{\mu \epsilon}} \end{aligned}$$

$$\begin{aligned} \mathbf{E} = & \frac{\mathrm{dl}\,\mathbf{e}_2\,\mathrm{I}\,\mu\sin(2\theta)\,e^{-i k R}\left(i k^2 R^2 + 3 k R - 3 i\right)\sin(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\ & \frac{\mathrm{dl}\,\mathbf{e}_1\,\mathrm{I}\,\mu\sin(2\theta)\,e^{-i k R}\left(i k^2 R^2 + 3 k R - 3 i\right)\cos(\phi)}{8 \pi k R^3 \sqrt{\mu \epsilon}} + \\ & \frac{\mathrm{dl}\,\mathbf{e}_3\,\mathrm{I}\,\mu\,e^{-i k R}\left(\cos(2\theta)\left(i k^2 R^2 + 3 k R - 3 i\right) - i k^2 R^2 + k R - i\right)}{8 \pi k R^3 \sqrt{\mu \epsilon}} \end{aligned}$$

$$\mathbf{H} = \frac{\mathrm{dl}\,\mathbf{e}_2\,\mathrm{I}\,\mu\sin(\theta)\,e^{-i k R}\,(1 + i k R)\cos(\phi)}{4 \pi R^2 \epsilon} - \frac{i\,\mathrm{dl}\,\mathbf{e}_1\,\mathrm{I}\,\mu\sin(\theta)\,e^{-i k R}\,(k R - i)\sin(\phi)}{4 \pi R^2 \epsilon}$$