

# Quantum Computing

## Summary

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# 1 Preliminaries

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## 1.1 Complex Numbers, Vectors, and Matrices

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## 1.2 Continued Fraction Expansion

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## 2 Postulates of Quantum Mechanics

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### 2.1 States

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### 2.2 Evolution

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#### 2.2.1 Gates

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### 2.3 Measurement

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### 2.4 Composite Systems and Tensor Products

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#### 2.4.1 Entanglement

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##### Multipartite

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##### Graph States

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#### 2.4.2 Multi-Qubit Gates

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### 2.5 Protocols

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#### 2.5.1 No-Cloning

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#### 2.5.2 Quantum Teleportation

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##### Concatenated Teleportation

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#### 2.5.3 Dense-Coding

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### 2.6 Why these postulates?

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## 3 Computational Complexity

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## 4 Universal Computation

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### 4.1 Classical Analogy

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### 4.2 Universal Quantum Gates

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#### 4.2.1 Proof

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##### Part 1/3: Unitaries as Two-Level Unitaries

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##### Part 2/3: Decomposition of Two-Level Unitaries

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##### Part 3/3: Approximation of Single-Qubit Gates

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#### 4.2.2 Final Gate Count

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## 5 Algorithms

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## 5.1 Quantum Parallelism

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Remarks

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## 6 Quantum Error Correction

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### 6.1 Tackling Bit-Flips

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### 6.2 Tackling Phase-Flips

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### 6.3 Shor's Code

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#### 6.3.1 Universal Error Correction

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### 6.4 Steane Code

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### 6.5 Fault-Tolerance and Transversality

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### 6.6 Threshold Theorem

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## 7 Quantum Nonlocality

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### 7.1 Elements of Reality

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### 7.2 CHSH Inequality

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### 7.3 Quantum Violation of the CHSH Inequality

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### 7.4 Tsirelson's Bound and Quantum Key Distribution

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## 8 Measurement-Based Quantum Computing

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### 8.1 Identity

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### 8.2 Arbitrary Rotations

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### 8.3 CNOT

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### 8.4 Cluster States

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### 8.5 Handling Errors

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### 8.6 Important Gates

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