# **Probabilistic Graphical Models**

**Zusammenfassung** Fabian Damken 17. Februar 2022



#### **Inhaltsverzeichnis**

1	Intro	ductio			ö
	1.1	Examp	nples		8
	1.2	Funda	amental Questions		8
2	Four	ndation	ns	,	9
_			ability Theory		9
	2.1	2.1.1			9
			•		э 9
					9 9
	0.0		Potentials		
	2.2		nine Learning		9
		2.2.1	(Document) Classification	 •	9
3	Baye	esian N	Networks	10	0
	3.1	The N	Naive Bayes Model	 . 1	1
		3.1.1	Classification	 . 1	1
		3.1.2			
		3.1.3			
	3.2		ition and Independence Assumptions		
		3.2.1			
		3.2.2			
		3.2.3			
		3.2.4	•		
	3.3		ded Independencies		
	5.5	3.3.1	<del>-</del>		
		3.3.2	•		
		3.3.3	1		
		3.3.4			
		3.3.5	The Bayes' Ball Algorithm	 . 1	1
4	Infe	rence		1:	2
	4.1	Chain	n Models	 . 1	2
	4.2	Variab	ble Elimination	 . 1	2
		4.2.1	Evidence	 . 1	2
		4.2.2	Complexity	 . 1	2
					2
	4.3		ictive Inference		2
	• • •	4.3.1			
		4.3.2	•		
	4.4		plexity of Conditional Queries		
		Morali		 1	

	4.6	Variable	e Elimination in Moral Graphs	
		4.6.1 P	Perfect Elimination Sequences	
		4.6.2 C	Complexity	
		4.6.3 In	Induced Graph	
			Induced Treewidth	
			Elimination on Trees	
			General Networks	
		1.0.0	General Records	
5	Mar	kov Rand	dom Fields	13
	5.1	Bavesian	n Networks as MRFs	
	5.2	-	ılated Graphs	
	5.3	•	ees	
	5.4		n Trees	
	J.7		Collecting Evidence	
			Distributing Evidence	
			angulated Graphs	
	5.5	Non-111a	angulated Graphs	
6	Lear	rning		14
U		•	te and Incomplete Data Sets	
	0.1	_	Hidden Variables	
	6.2			
	0.2		ter Estimation	
			Known Structure, Complete Data	
			Known Structure, Incomplete Data (Expectation-M	
			Gradient Ascent	
			Bayesian Parameter Estimation	
			Summary	
	6.3		re Learning / Model Selection	
		6.3.1 N	Minimal I-Maps	
		6.3.2 P	Perfect Maps (P-Maps)	
		6.3.3 I-	I-Equivalence	
		6.3.4 C	Obtaining a P-Map	
		6.3.5 A	Accurate Structures	
			Learning	
			Structure Search as Optimization	
		6.3.8 S	Structural EM	
			Summary	
		0.0.7		
7	Dyna	amic Bay	yesian Networks	16
	7.1	-	Markov Models	
	7.2	Inference		
	,		Decoding	
			Best State Sequence	
			<del>-</del>	
	7 2			
	7.3		stimation (Kalman Filter)	
			Recursive Bayesian Updating	
			(Modeling) Actions	
			Bayes Filter	
		7.3.4 D	Discrete-Time Kalman Filter	

7.4	Genera	al Dynamic Bayesian Networks	17
	7.4.1	Exact Inference	17
	7.4.2	Tractable, Approximate Inference	17
			18
8.1	Messag	ge Passing	18
	8.1.1	Sum-Product Belief Propagation	18
	8.1.2	(Acyclic) Belief Propagation as Dynamic Programming	18
	8.1.3	Loopy Belief Propagation	18
8.2	Sampli	ing	18
	8.2.1	Forward Sampling (Without Evidence)	18
	8.2.2	Forward Sampling (With Evidence)	18
Trac	table P	robabilistic Models	19
	-	· ·	
,.0			
		<del>-</del>	
	7.0.0	inference on Devices	10
Deep	o Gener	rative Models 2	20
10.1	Likelih	ood-Based	20
10.2		<del>-</del>	
10.3			
	Apple 8.1 8.2  Trac 9.1 9.2 9.3  Deep 10.1	7.4.1 7.4.2  Approximat 8.1 Messag 8.1.1 8.1.2 8.1.3 8.2 Sampl 8.2.1 8.2.2 8.2.3 8.2.4  Tractable P 9.1 Deep I 9.2 Probat 9.3 Sum-P 9.3.1 9.3.2 9.3.3  Deep Gener 10.1 Likelih 10.1.1 10.1.2 10.1.3 10.2 Likelih 10.2.1	7.4.1 Exact Inference 7.4.2 Tractable, Approximate Inference  Approximate Inference  8.1 Message Passing 8.1.1 Sum-Product Belief Propagation 8.1.2 (Acyclic) Belief Propagation as Dynamic Programming 8.1.3 Loopy Belief Propagation  8.2 Sampling 8.2.1 Forward Sampling (Without Evidence) 8.2.2 Forward Sampling (With Evidence) 8.2.3 Gibbs Sampling 8.2.4 Likelihood Weighting  Tractable Probabilistic Models 9.1 Deep Learning 9.2 Probabilistic Circuits 9.3 Sum-Product Networks 9.3.1 Inference 9.3.2 Learning 9.3.3 Inference on Devices

# Abbildungsverzeichnis

### **Tabellenverzeichnis**

# Liste der Algorithmen

## 1 Introduction

- 1.1 Examples
- 1.2 Fundamental Questions

## 2 Foundations

2.1 Probability Theory	
2.1.1 (Conditional) Independence	
Monty Hall Problem	
2.1.2 Inference	
Information Theory	
2.1.3 Potentials	
2.2 Machine Learning	_
2.2.1 (Document) Classification	

## 3 Bayesian Networks

3.1 The Naive Bayes Model
3.1.1 Classification
3.1.2 Maximum Likelihood Parameter Estimation
2.1.2 Application
3.1.3 Application
3.2 Definition and Independence Assumptions
· · · · · · · · · · · · · · · · · · ·
3.2.1 Local Markov Assumption
3.2.2 "Explaining Away" / Berkson's Paradox
3.2.3 Representation Theorem
2.0.4 Duilding a Daysaian Naturals
3.2.4 Building a Bayesian Network
3.3 Encoded Independencies
<b>-</b>
3.3.1 Dependency Structures
3.3.2 d-Separation
(Active) Trails
Independencies
Soundness
Completeness
3.3.3 Faithful Distributions
3.3.4 Context-Specific Independence (CSI)
Tree CPD
Determinism
2.2.5. The Davies' Bell Algerithm
3.3.5 The Bayes' Ball Algorithm

## 4 Inference

4.1 Chain Models
4.2 Variable Elimination
4.2.1 Evidence
4.2.2 Complexity
4.2.3 VE for Potentials
4.3 Abductive Inference
4.3.1 Consistency
4.3.2 Finding Most Probable Explanations (MPEs)
440 1 1 60 101 10 1
4.4 Complexity of Conditional Queries
4.5 Moralizing
4.6 Variable Elimination in Moral Graphs
4.6.1 Perfect Elimination Sequences
4.6.2 Complexity
4.6.3 Induced Graph
4.6.4 Induced Treewidth
4.6.5 Elimination on Trees
Delutrose
Polytrees
4.6.6 General Networks
T.U.U GEHEIGH MELWUINS

### **5 Markov Random Fields**

E 1 Payasian Natworks as MPEs	
5.1 Bayesian Networks as MRFs	
5.2 Triangulated Graphs	
5.3 Join Trees	
5.4 Junction Trees	
5.4.1 Collecting Evidence	
5.4.2 Distributing Evidence	
5.5 Non-Triangulated Graphs	

## 6 Learning

6.1 Complete and Incomplete Data Sets
6.1.1 Hidden Variables
6.2 Parameter Estimation
6.2.1 Known Structure, Complete Data
Maximum Likelihood
Decomposability of the Likelihood
Likelihood for (Conditional) Bi- and Multinomials
COOK Otherstone Incomplete Date (Francische Marini-stick)
6.2.2 Known Structure, Incomplete Data (Expectation-Maximization)
EM Idea
Complete-Data Likelihood
- The state of the
EM for (Conditional) Multinomials
Monotonicity
6.2.3 Gradient Ascent
6.2.4 Bayesian Parameter Estimation
•
Laplace Estimation
Bayesian Prediction
Bayesian Frediction
Conjugate Priors
Binomial Prior
Dirichlet Prior

Bayesian Networks and Bayesian Prediction			
6.2.5 Summary			
6.3 Structure Learning / Model Selection			
6.3.1 Minimal I-Maps			
6.3.2 Perfect Maps (P-Maps)			
6.3.3 I-Equivalence			
Skeleton and Immoralities			
6.3.4 Obtaining a P-Map			
Identifying the Skeleton			
Identifying Immoralities			
From Immoralities to Structures			
6.3.5 Accurate Structures			
6.3.6 Learning			
Constrained-Based			
Score-Based			
Likelihood Score			
Bayesian Score and Bayesian Information Criterion			
6.3.7 Structure Search as Optimization			
Learning Trees (Complete Data)			
Heuristic (Local) Search			
6.3.8 Structural EM			
6.3.9 Summary			

## 7 Dynamic Bayesian Networks

7.1 Llidden Merkey Medele			
7.1 Hidden Markov Models			
7.2 Inference			
7.2.1 Decoding			
Forward Pass			
Backward Pass			
7.2.2 Best State Sequence			
Viterbi Algorithm			
7.2.3 Parameter Estimation			
7.3 State Estimation (Kalman Filter)			
7.3.1 Recursive Bayesian Updating			
7.3.2 (Modeling) Actions			
7.3.3 Bayes Filter			
7.3.4 Discrete-Time Kalman Filter			
Dynamics and Observations			
Belief Update: Prediction			
Belief Update: Correction			
7.4 General Dynamic Bayesian Networks			
7.4.1 Exact Inference			
7.4.2 Tractable, Approximate Inference			
Assumed Density Filtering			

## 8 Approximate Inference

8.1 Message Passing
8.1.1 Sum-Product Belief Propagation
0.4.0. /A !! \ D. !! . ( D !' D ! . D !
8.1.2 (Acyclic) Belief Propagation as Dynamic Programming
8.1.3 Loopy Belief Propagation
8.2 Sampling
8.2.1 Forward Sampling (Without Evidence)
8.2.2 Forward Sampling (With Evidence)
8.2.3 Gibbs Sampling
Burn-In
Irreducibility, Aperiodicity, and Ergodicity
Treducibility, Aperiodicity, and Ergodicity
Convergence
Performance
Speeding Convergence
Skipping Samples
Randomized Variable Order
Randonnized Variable Order
Blocking
Rao-Blackwellization
Multiple Chains
8.2.4 Likelihood Weighting

### 9 Tractable Probabilistic Models

9.1 Deep Learning	
9.2 Probabilistic Circuits	
9.3 Sum-Product Networks	
9.3.1 Inference	
9.3.2 Learning	
Directly Learning SPNs	
9.3.3 Inference on Devices	

# 10 Deep Generative Models

10.1 Likelihood-Based
10.1.1 Autoregressive Generatie Models
Learning and Inference
Parametrization
10.1.2 Variational Auto-Encoders
Inference as Optimization
Variational Bayes
Learning and Inference
Open Questions
10.1.3 Normalizing Flows
Learning and Inference
10.2 Likelihood-Free
10.2.1 Generative Adversarial Networks
Inference
10.3 Applications in Scientific Discovery