Search via Parallel Lévy Walks on $\mathbb{Z}^2$

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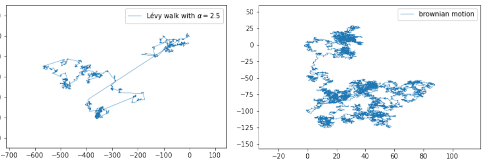
Lévy walk



Lévy walk (informal):
A Lévy walk is a random walk whose step-length density distribution is proportional to a power-law, namely, for each $d \in \mathbb{R}^+$, $f(d) \sim 1/d^{\alpha}$, for some $\alpha > 1$

Note: the speed of the walk is constant

Movement model

Lévy walks are used to model **movement patterns** [Reynolds, Biology Open 2018]

Examples:

- T cells within the brain
- swarming bacteria
- midge swarms
- termite broods
- schools of fish
- Australian desert ants
- a variety of molluscs

Widely employed in the **foraging theory**

Lévy walk optimality

[Viswanathan et al., Nature 1999]: the Lévy walk with $\alpha = 2$ is optimal for foraging in $\mathbb{R}^n$

[Guinard et Korman, Sciences Advances 2021]: the Lévy walk with $\alpha = 2$ is optimal for finding targets of all shapes in $\mathbb{T}^2$

Lévy flight foraging hypothesis

Since Lévy flights/walks optimize random searches, biological organisms must have therefore evolved to exploit Lévy flights/walks [Viswanathan et al., Physics of Life Reviews 2008]

Seems there is a special exponent $\alpha = 2$

We test this hypothesis by focusing on a distributed search problem:

- the ANTS (Ants Nearby Treasure Search) problem

Full version of the work available at: `https://arxiv.org/abs/2004.01562`

The ANTS problem

Introduced by [Feinerman et al., PODC 2012]:

Setting:

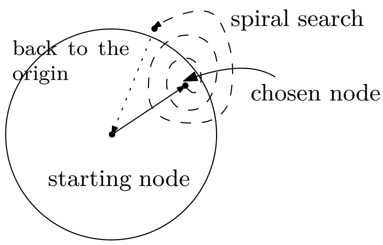
- k (mutually) independent agents start moving on $\mathbb{Z}^2$ from the origin
- time is synchronous and marked by a global clock
- one special node $\mathcal{P} \in \mathbb{Z}^2$, the *target*, placed by an adversary at unknown (Manhattan) distance ℓ from the origin

Task: find the target as fast as possible

Lower bound: for any $k \geq 1$, and for any search algorithm $\mathcal{A}$, the hitting time to find $\mathcal{P}$ is $\Omega\left(\ell^2/k + \ell\right)$ both with constant probability and in expectation

Proposed solution

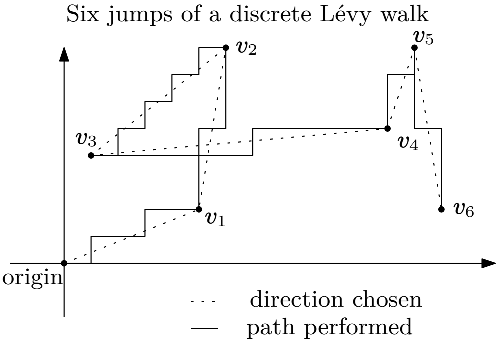
- no communication, no advice
- achieves expected hitting time $\mathcal{O}((\ell^2/k + \ell)\log^{1+\epsilon}\ell)$
- not natural (simple, lightweight)



i fix a ball of some radius ℓ_i
ii agents go to random nodes in the ball
iii agents perform a spiral search of length d_i around the chosen nodes
iv agents return to the source node
v increase ℓ_i and d_i, and repeat (i)-(v)

Discrete Lévy walk

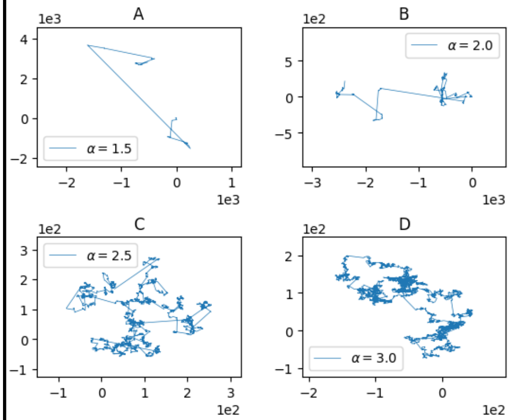
Six jumps of a discrete Lévy walk



Lévy walk: $\alpha > 1$, the agent

a) chooses a distance $d \in \mathbb{N}$ as follows: $d = 0$ w.p. $1/2$, and $d \geq 1$ w.p. c_α/d^α
b) chooses a destination u.a.r. among those at distance d
c) walks along an approximating path for d steps, one edge at a time, crossing d nodes
d) repeats the procedure

α-Behavior of Lévy walks



- $1 < \alpha \leq 2$ ballistic diffusion (fig.s A and B)
- $2 < \alpha < 3$ super diffusion (fig. C)
- $3 \leq \alpha$ normal diffusion (fig. D)

Input domain partition

Recall: ℓ target distance, k number of agents

Three different possible settings:

1. close target: $\ell \leq k/\text{polylog}(k)$
2. far target: $k/\text{polylog}(k) \leq \ell \leq \exp(k^{\Theta(1)})$
3. very far target: $\exp(k^{\Theta(1)}) \leq \ell$

Recall: an event E depending on a parameter ℓ holds with high probability in ℓ if $\mathbb{P}(E) \geq 1 - \ell^{-\Theta(1)}$

Hitting time

Close target

ballistic walks: any α in $(1, 2]$

hitting time $\mathcal{O}(\ell \text{polylog}(\ell))$ w.h.p.

Very far target

diffusive walks: any $\alpha \geq 3$ (brownian-like)

the whalks eventually find the target w.p. 1

Far target

best strategy: ... it depends!

$\alpha^\star = 3 - \log k / \log \ell$: super-diffusive range

The following holds w.h.p. in ℓ

- if $\alpha = \alpha^\star + \mathcal{O}(\log\log\ell/\log\ell)$, the hitting time is $\mathcal{O}((\ell^2/k + \ell)\text{polylog}(\ell))$
- if $\alpha = \alpha^\star + \epsilon$, the hitting time is $\Omega((\ell^2/k + \ell)\ell^{c})$, for some constant $c > 0$
- if $\alpha = \alpha^\star - \epsilon$ the hitting time is *infinite*

Search algorithm

How can we find $\alpha^\star$? **We don't have to!**

Algorithm: *each agent u samples u.a.r. a real number $\alpha_u \in (2, 3)$. Then, it performs a discrete Lévy walk with exponent α_u*

If $\ell \leq \exp(k^{\Theta(1)})$, the hitting time is $\mathcal{O}\left((\ell^2/k + \ell)\text{polylog}(\ell)\right)$ w.h.p.

Natural, time-homogeneus, almost-optimal solution for the ANTS problem