No distributed quantum advantage for approximate graph coloring

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Approximate graph coloring

Input:
- parameters $2 \leq \chi \leq c \in \{2, 3, 4, \dots\}$
- χ-chromatic graph G

Output:
- a c-coloring of G

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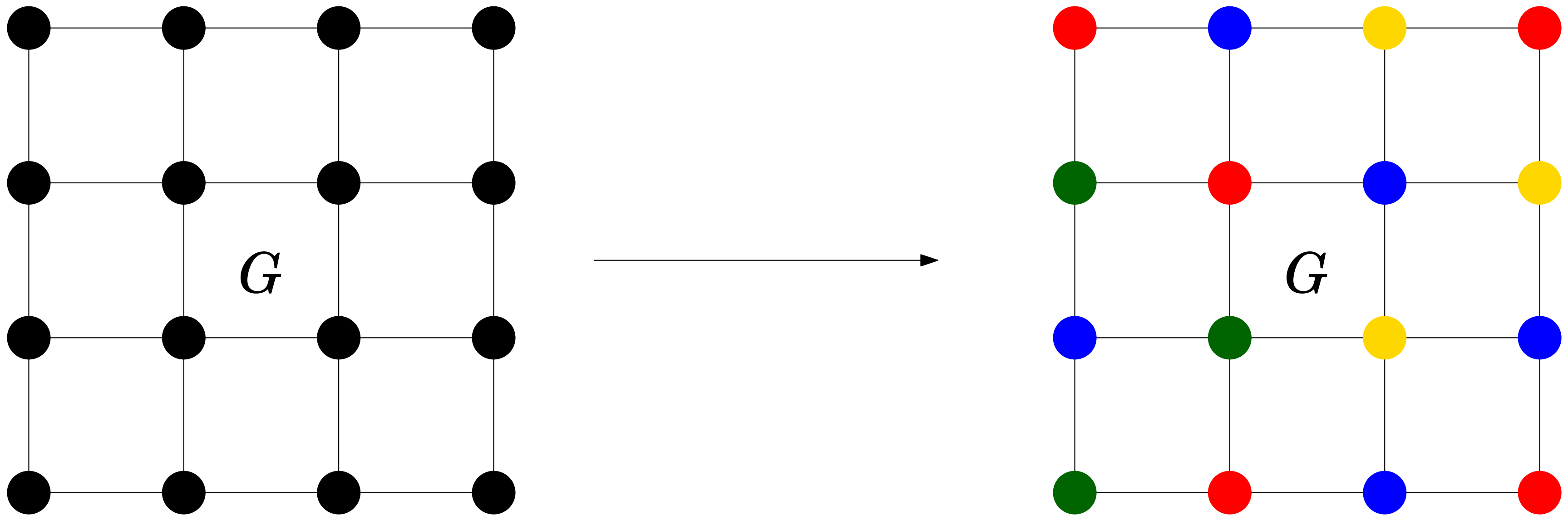
Examples:

• parameters $\chi = 2$, $c = 4$



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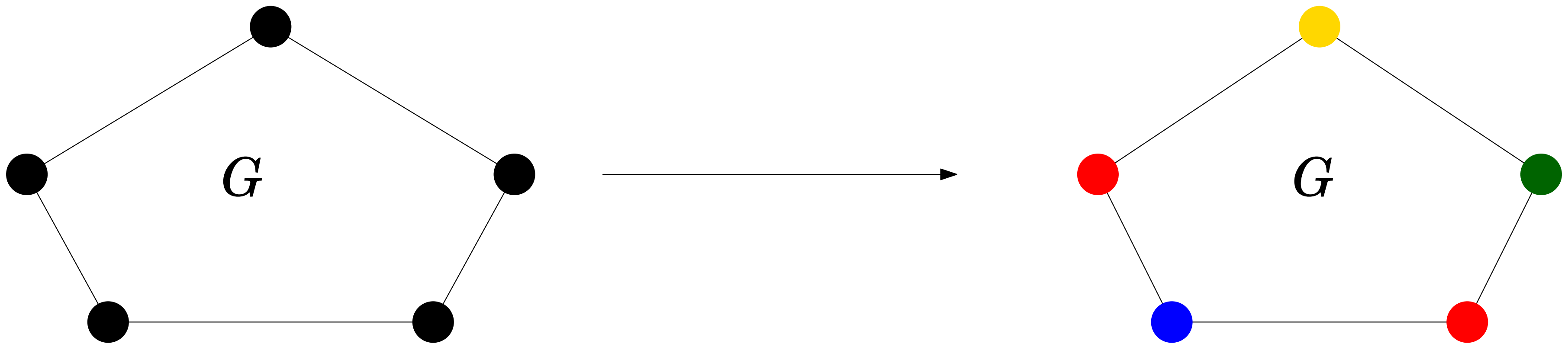
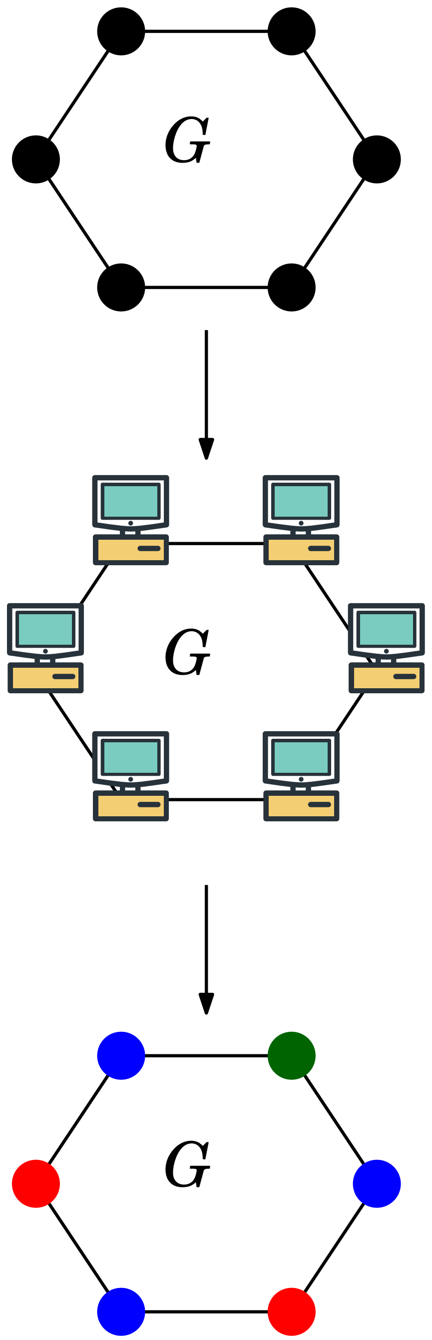


Model of distributed computation [Linial '87]:

- the input graph is a distributed system



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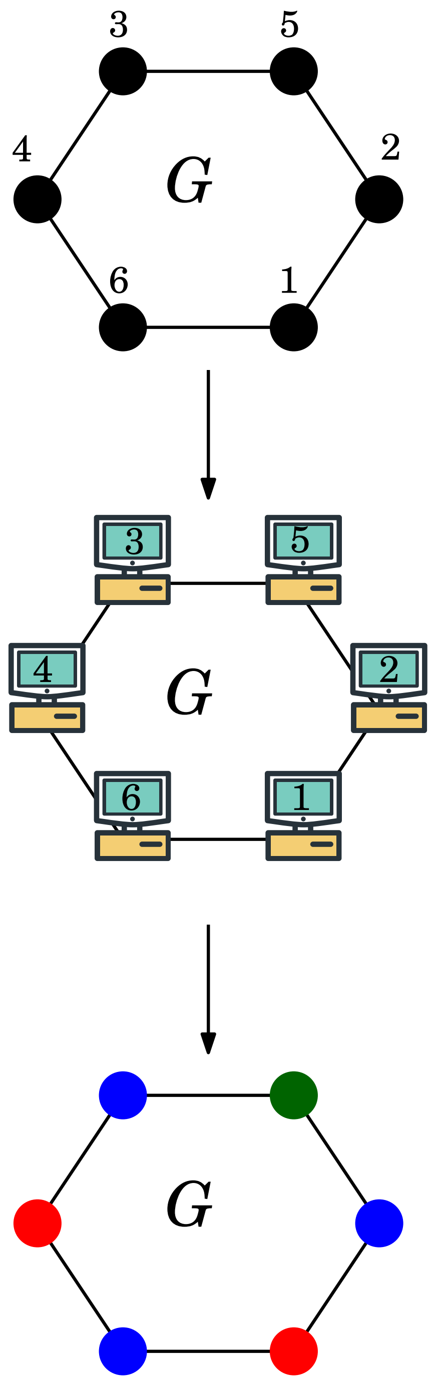

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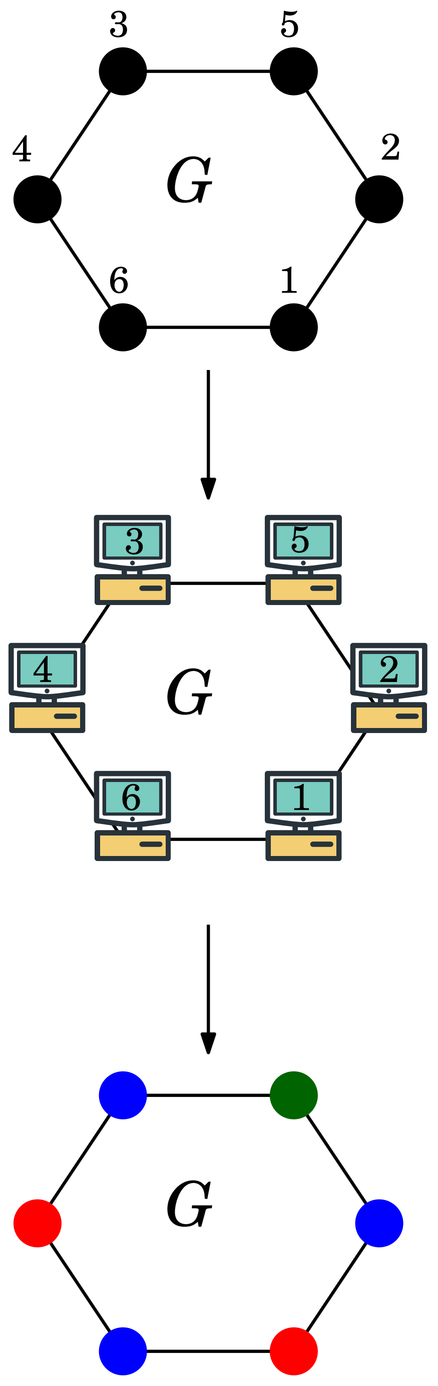

The LOCAL model

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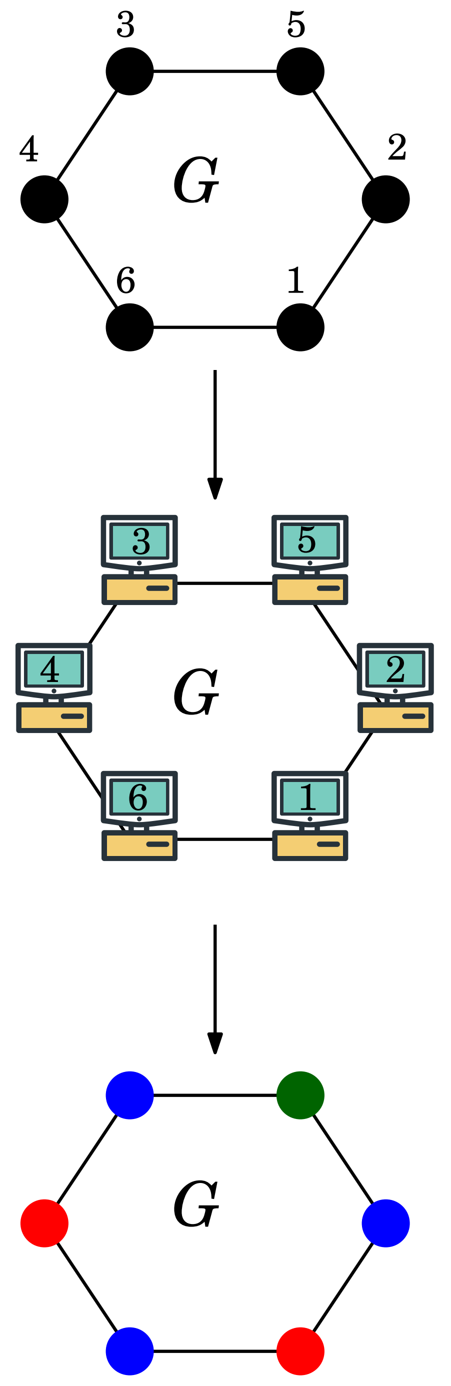

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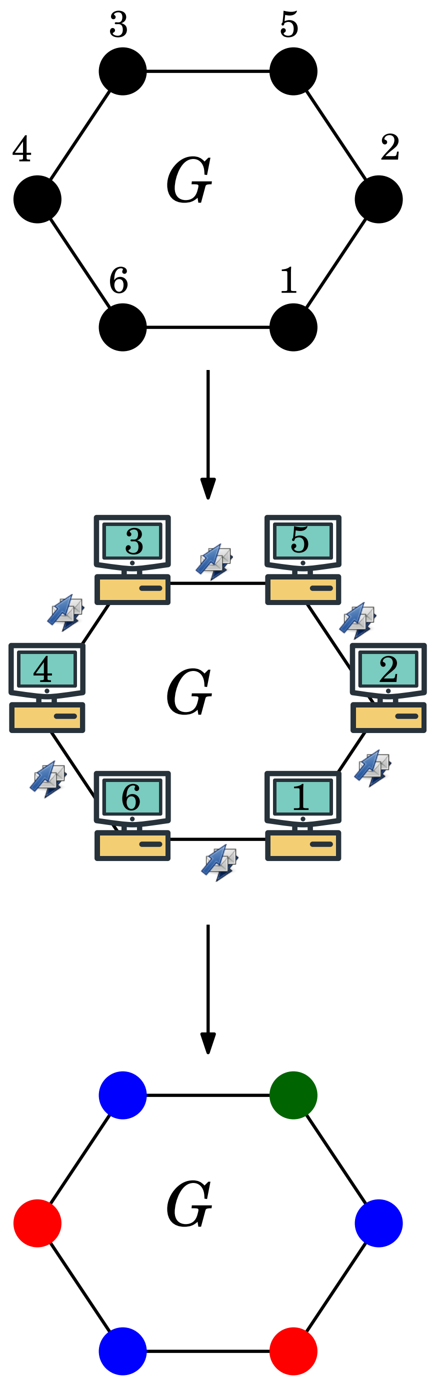

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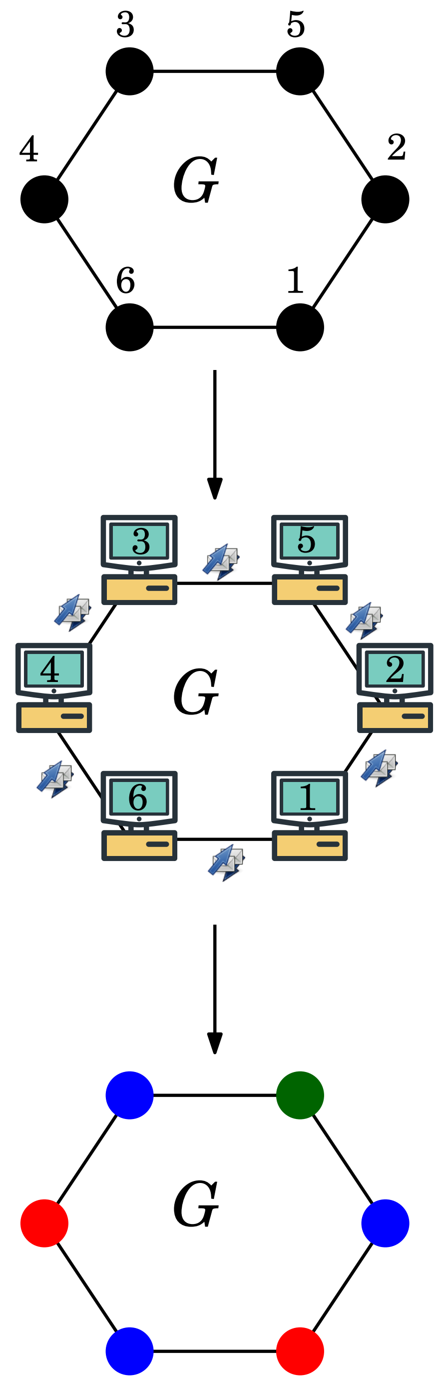
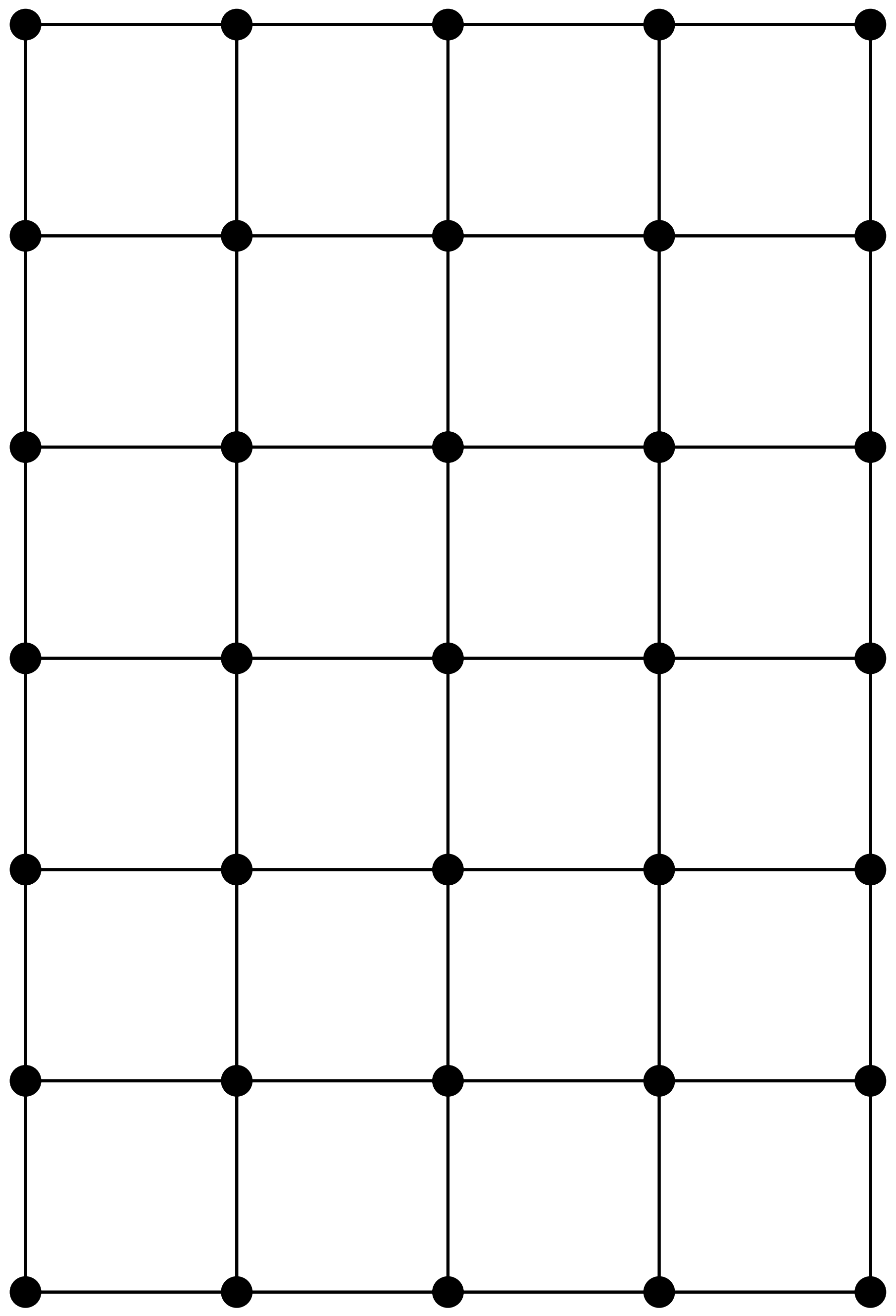
  - sends messages to and receives messages from all neighbors
- unbounded computational power & no bandwith limitations



Complexity measure

Complexity: minimum number of communication rounds



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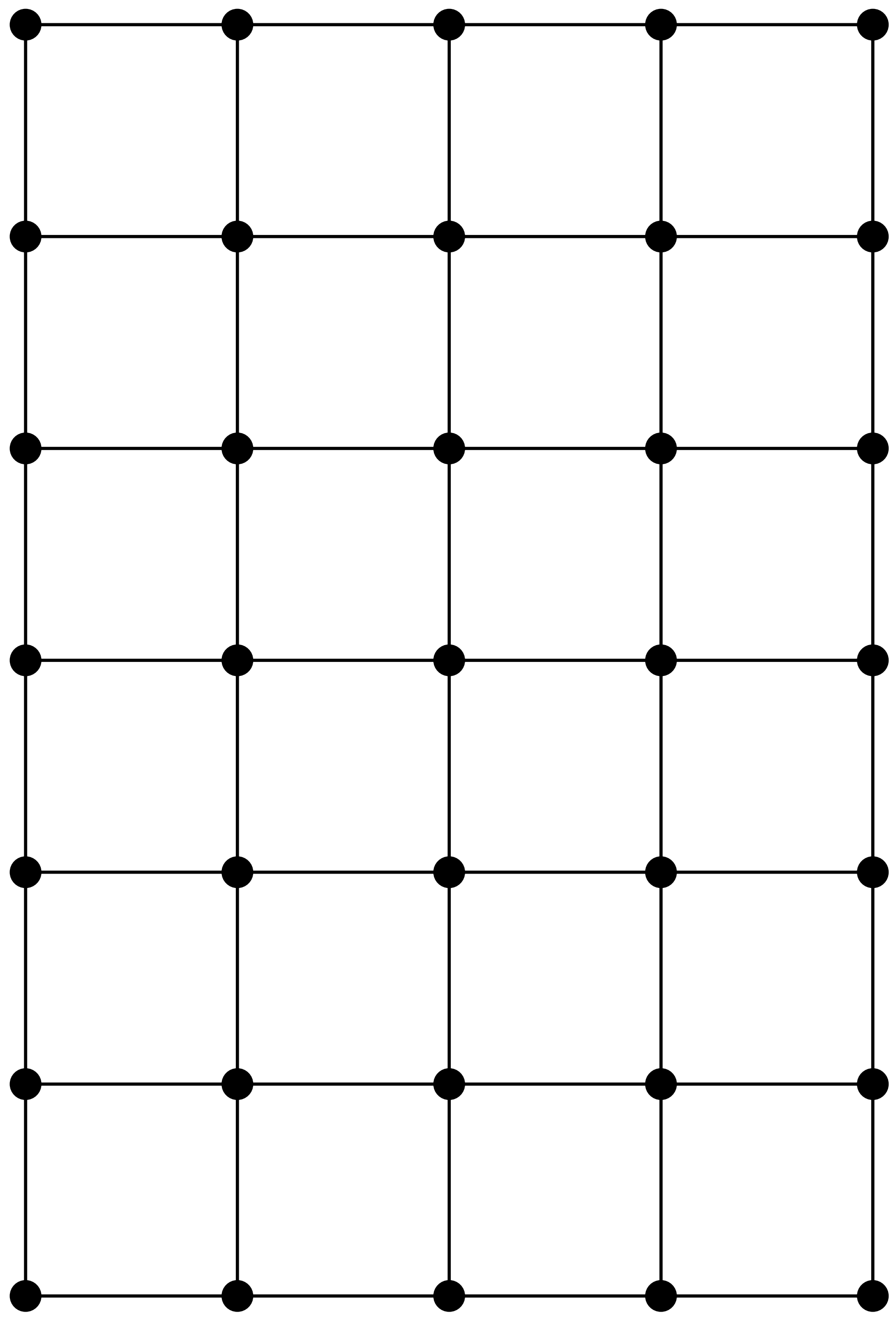

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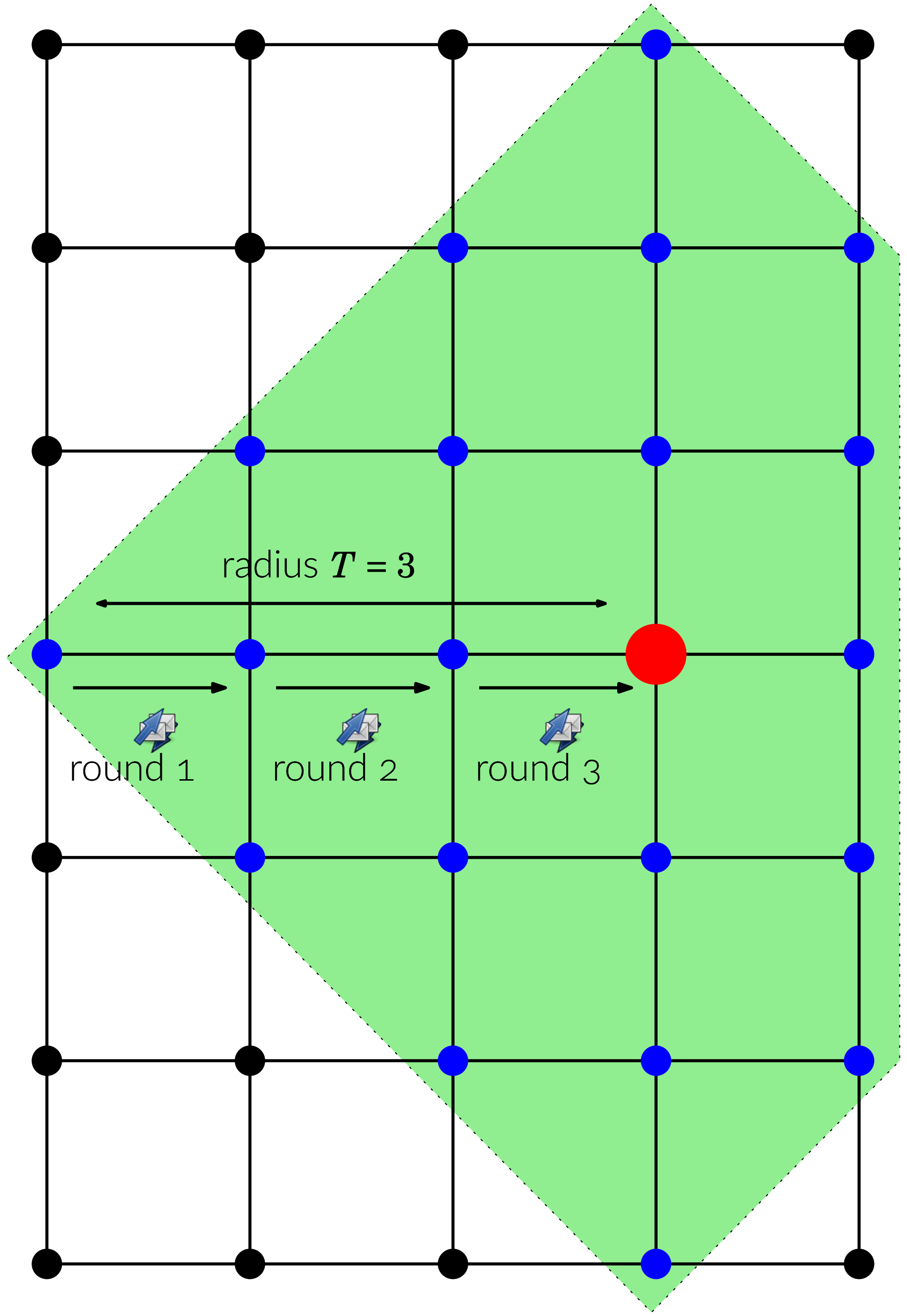

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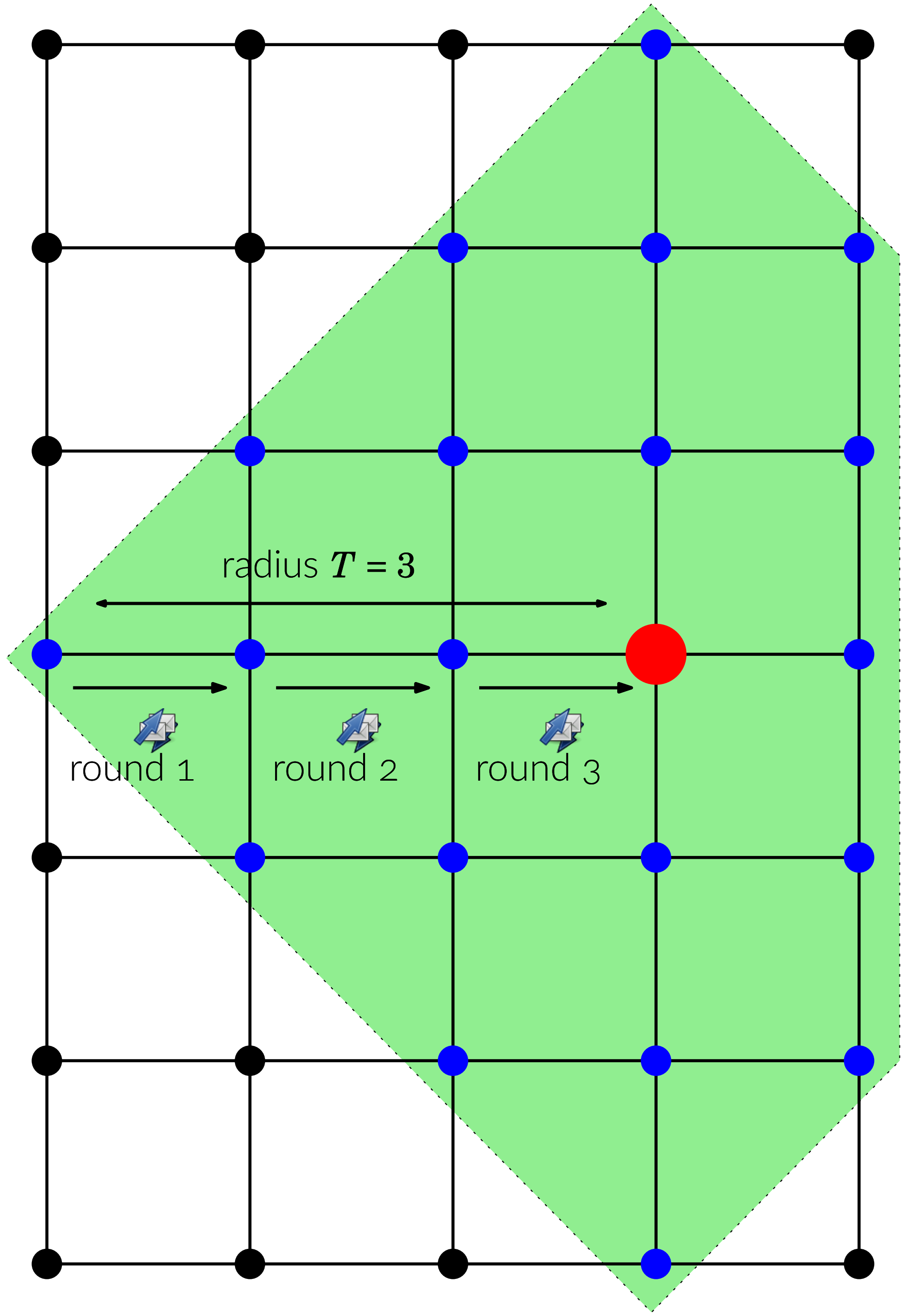
Complexity: minimum locality

- Locality $\mathrm{diam}(G) + 1$ solves every problem



Complexity of approximate graph coloring

Problem: c-coloring χ-chromatic graphs

		upper bound		lower bound		
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2	2	$O(n)$	$O(n)$	$\Omega(n)$	$\Omega(n)$	trivial
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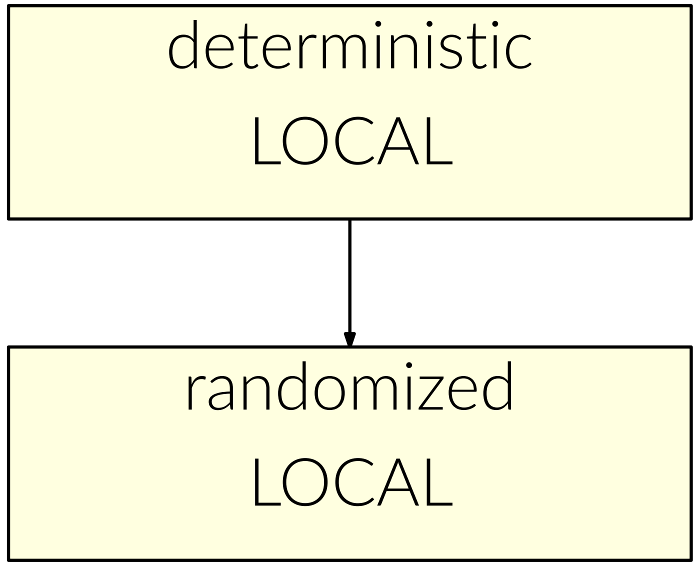
χ	$c > \chi$	$O(n)$	$\tilde{O}(n^{1/\lfloor\frac{c-1}{\chi-1}\rfloor})$	$\Omega(\log n)$	$\Omega(n^{1/\lfloor\frac{c-1}{\chi-1}\rfloor})$	[Linial '92]

- $\alpha = \lfloor\frac{c-1}{\chi-1}\rfloor$ approximation ratio

Beyond the LOCAL model

Surprise: the result holds for a wide range of distributed models



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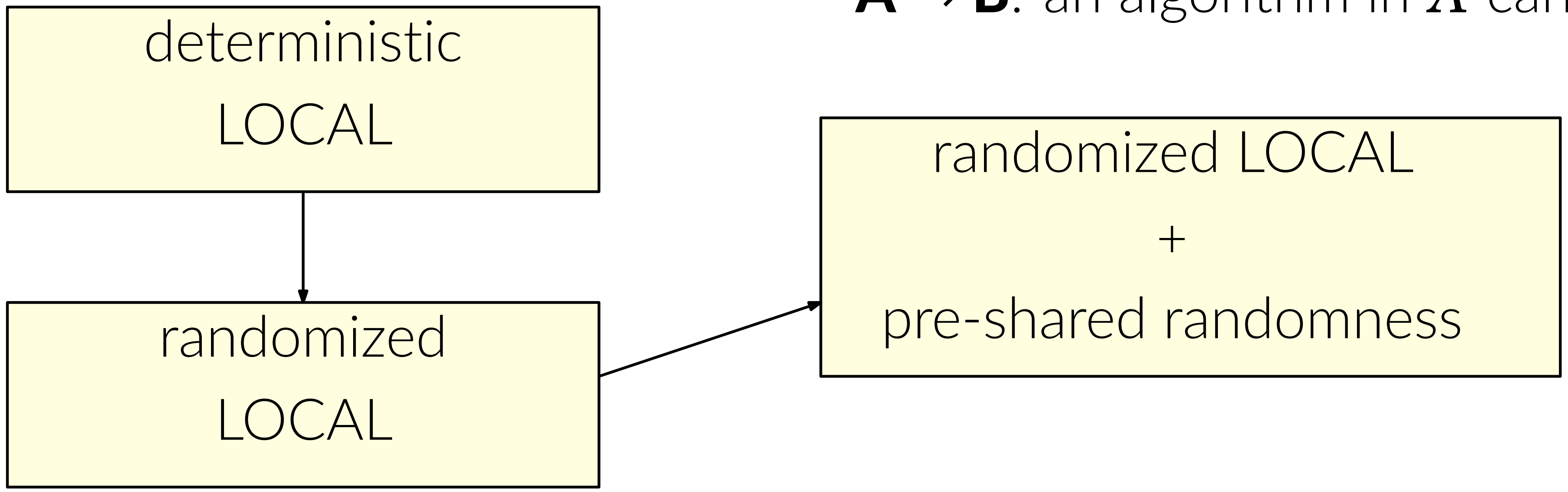

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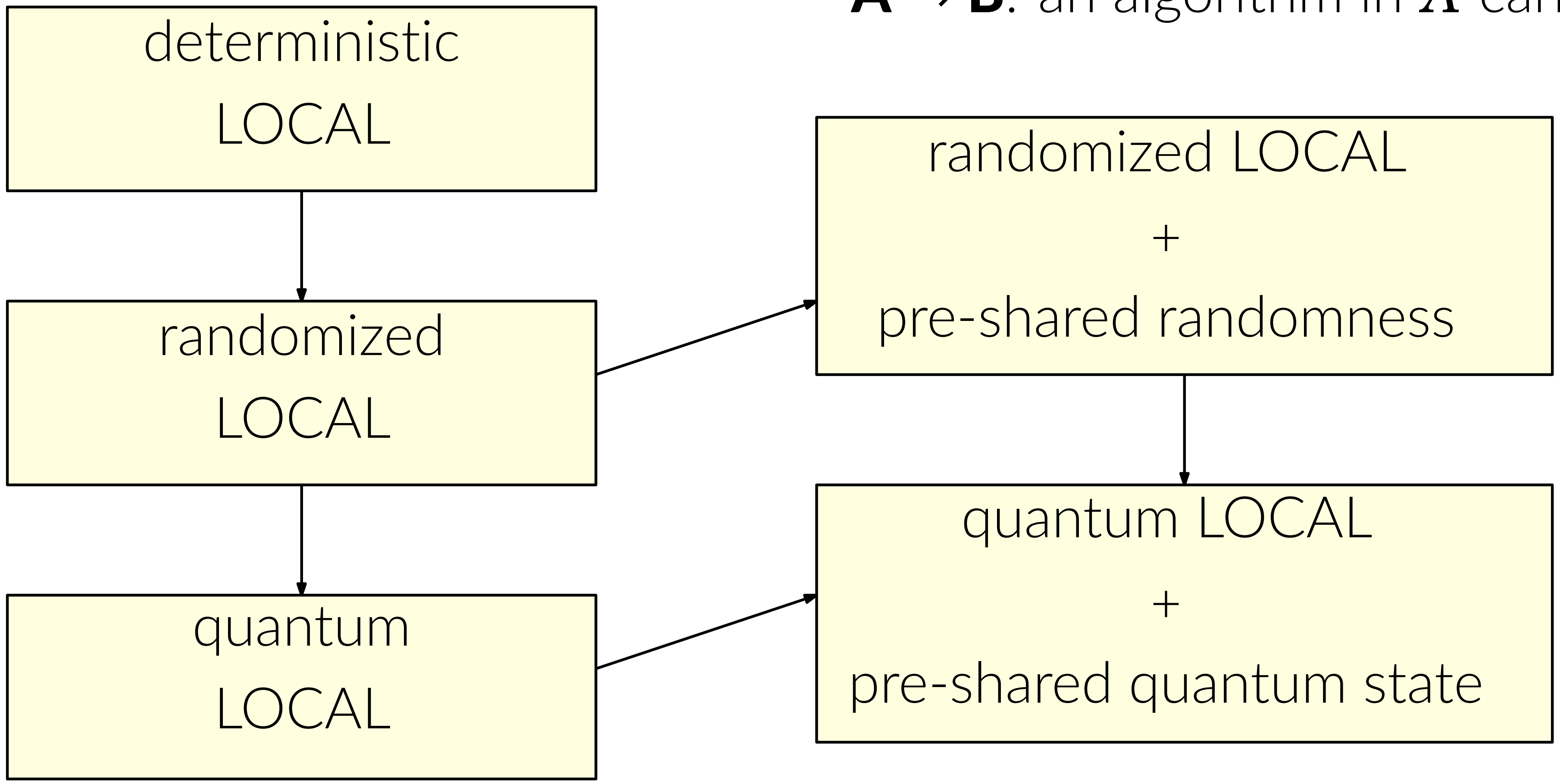

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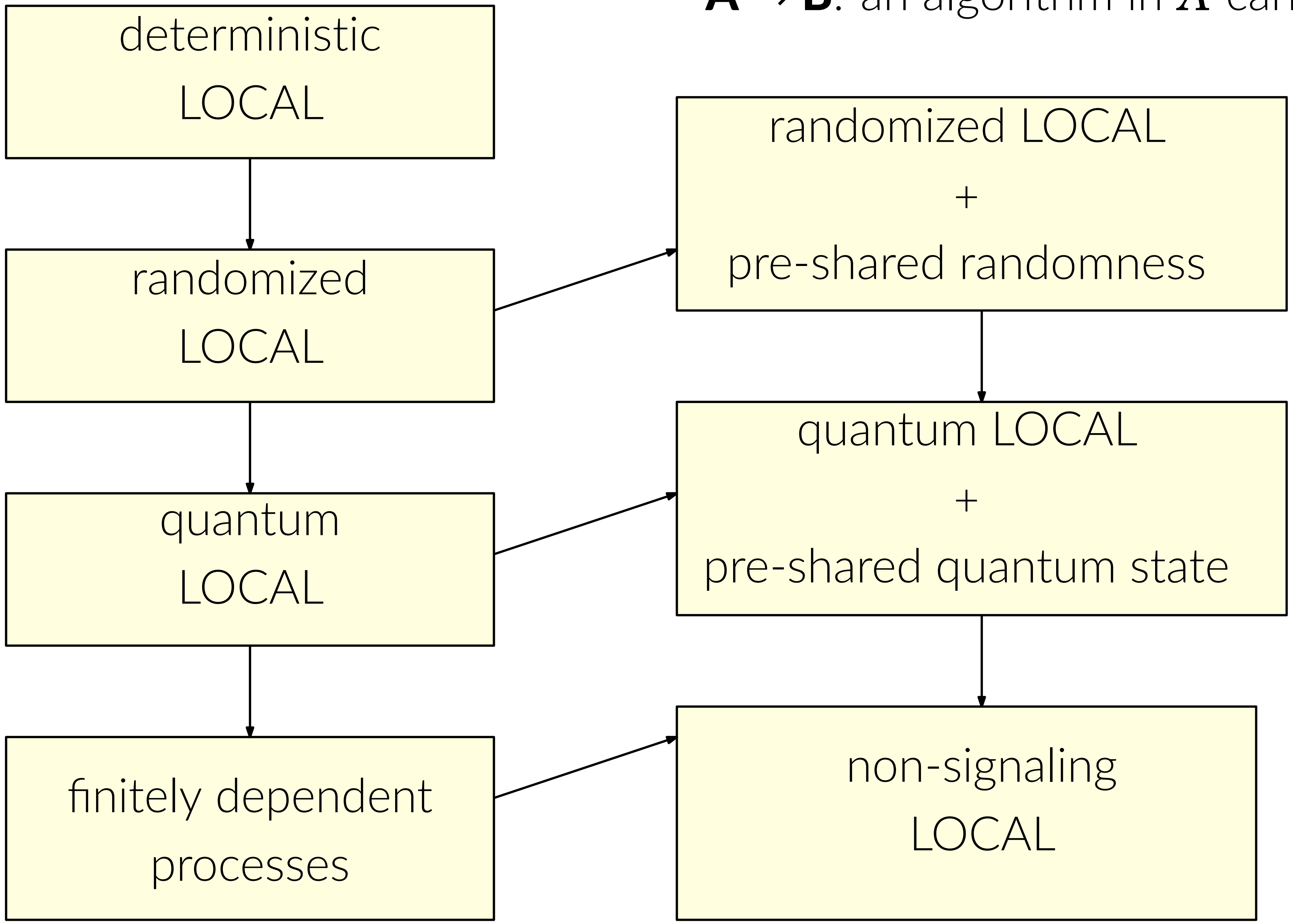

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almost matching
upper bound

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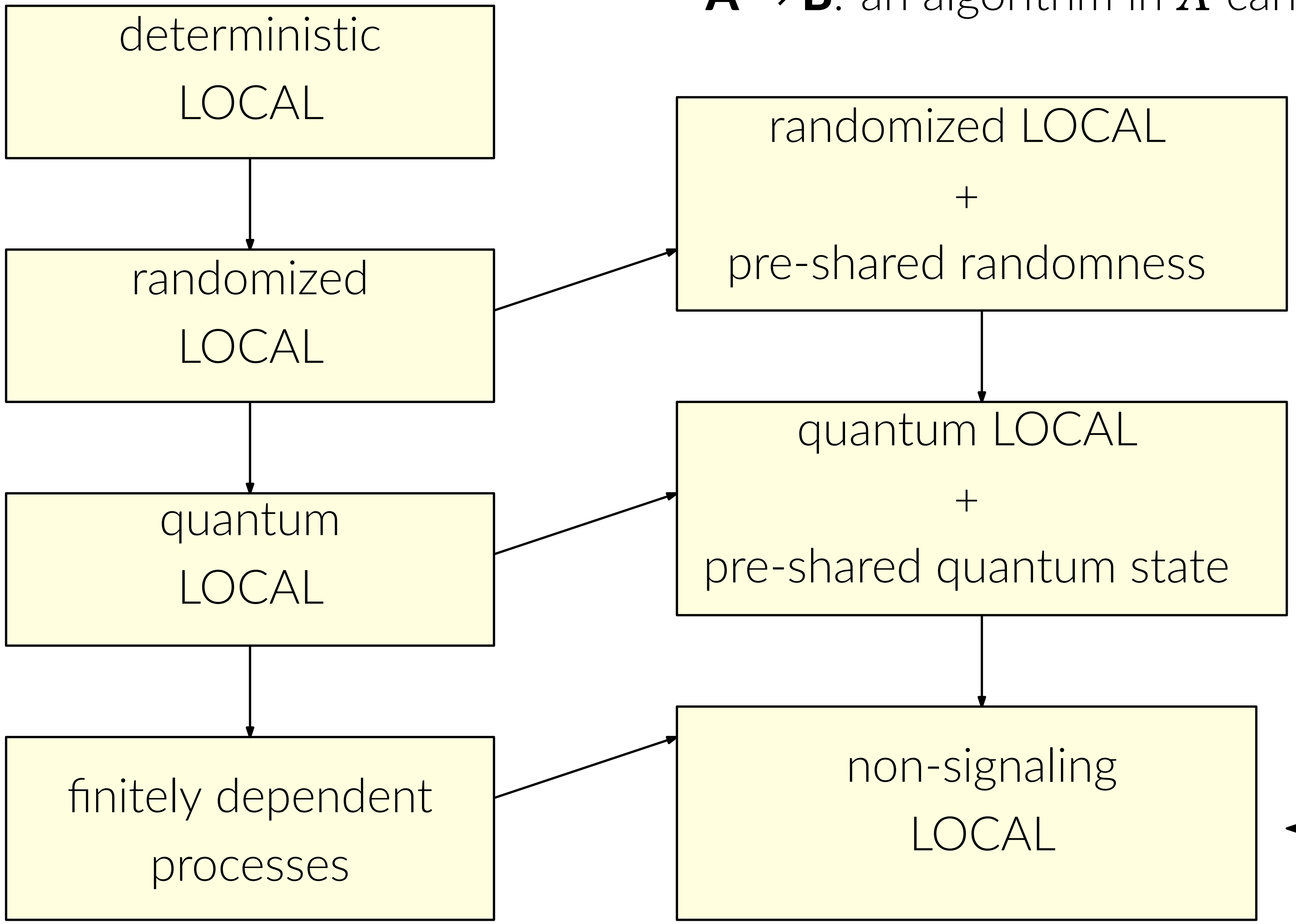

- no quantum advantage!

lower bound

The non-signaling LOCAL model

The lower bound holds in the **non-signaling LOCAL** model [Gavoille et al. '09] [Arfaoui and Fraigniaud '14]

- stronger than any *physical* synchronous distributed model
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No-signaling principle (informal): *no signal can be sent from the future to the past*

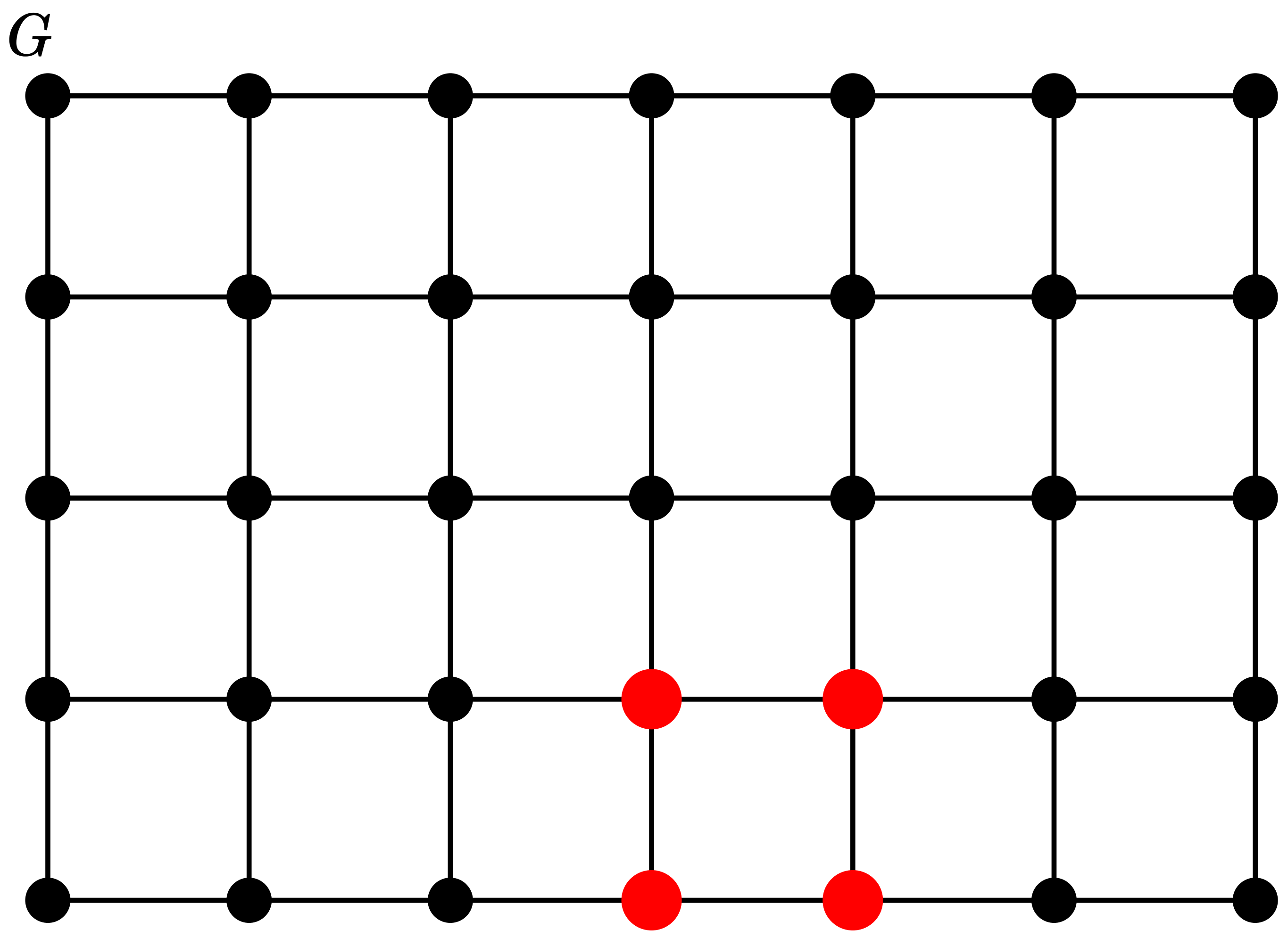
- causality

No-signaling property

Idea (in algorithms)

- fix a distributed system with $|V| = n$ nodes

- fix $S \subseteq V$



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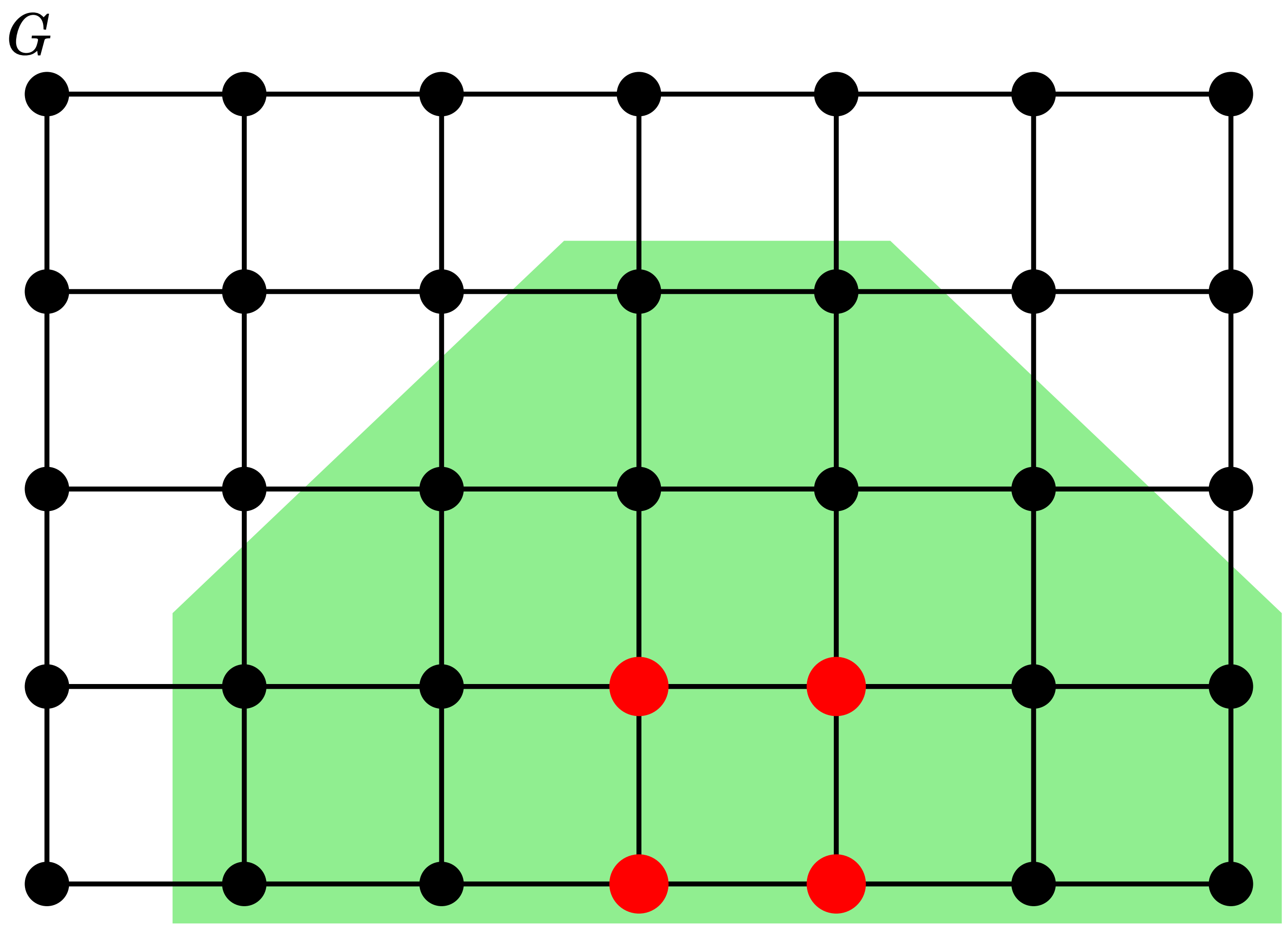

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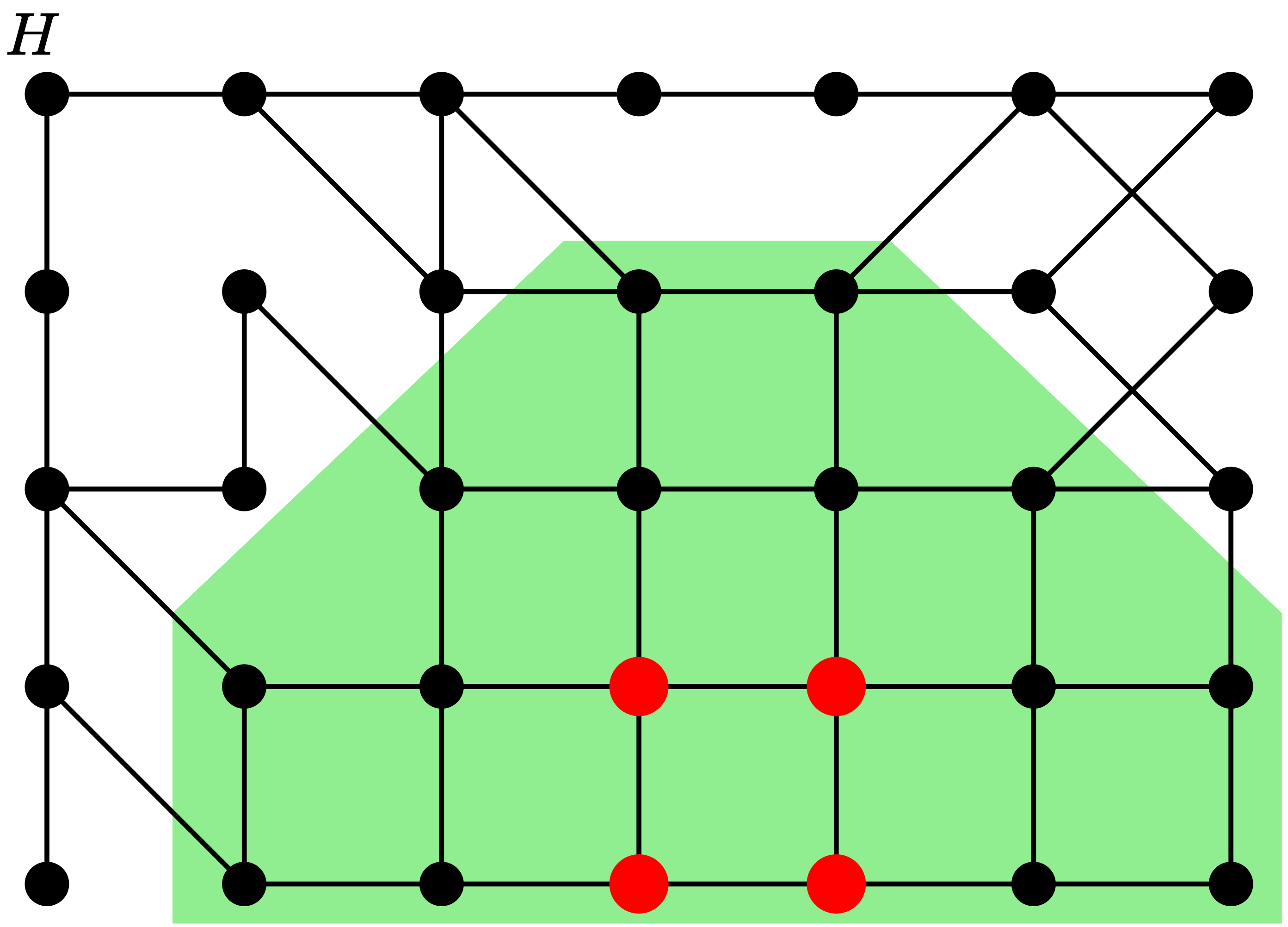

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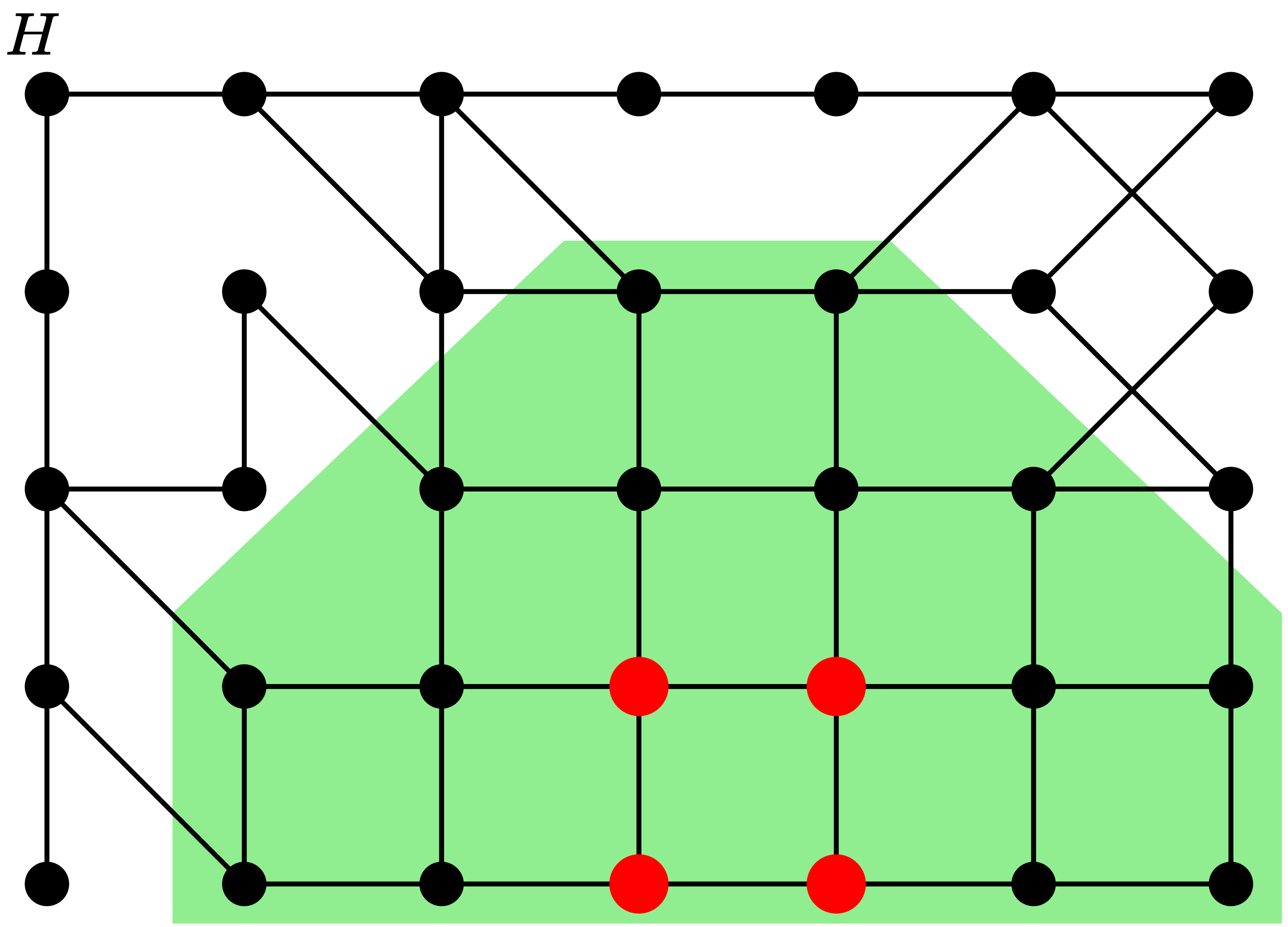

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- A non-signaling outcome abstracts this idea and gives only the **output distribution**

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Problem: 3-coloring 2-chromatic graphs

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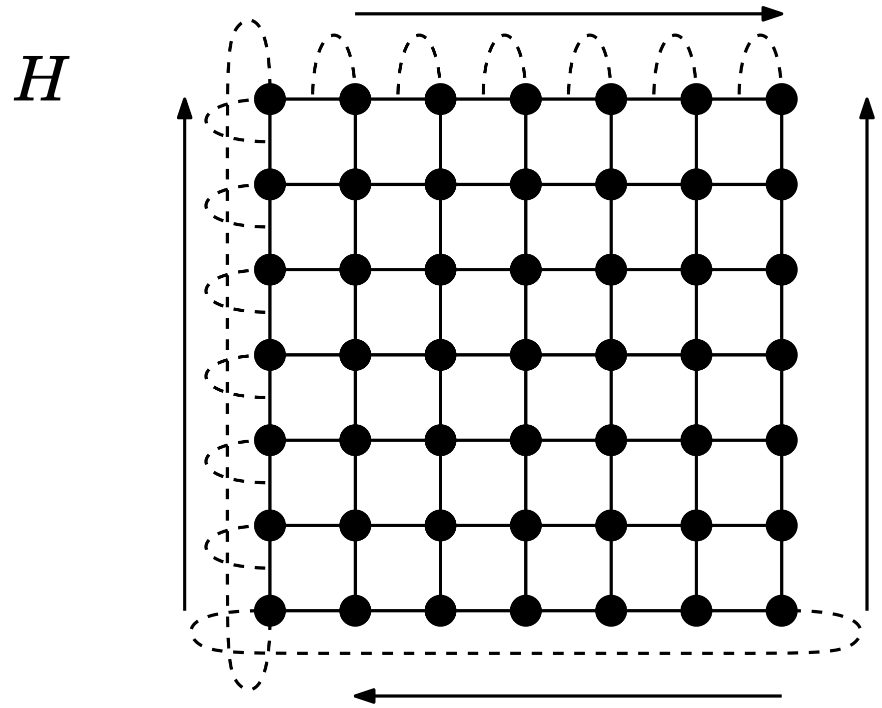

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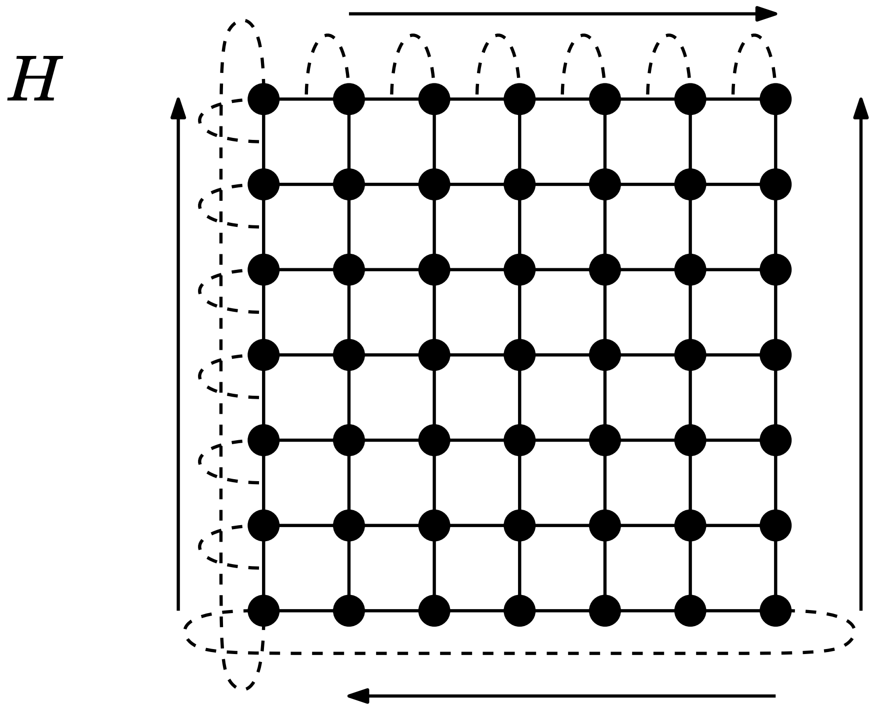

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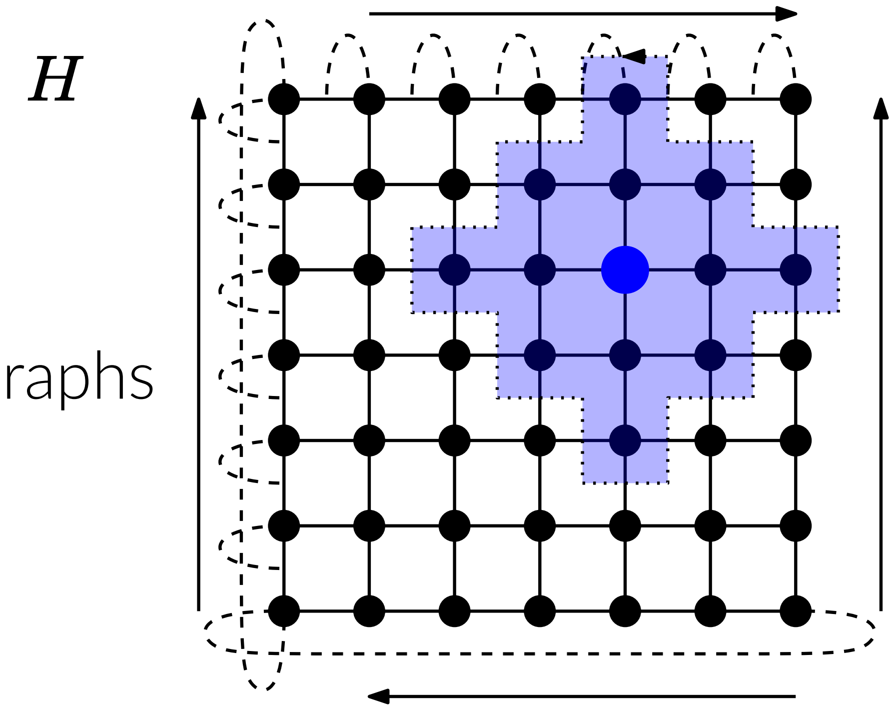

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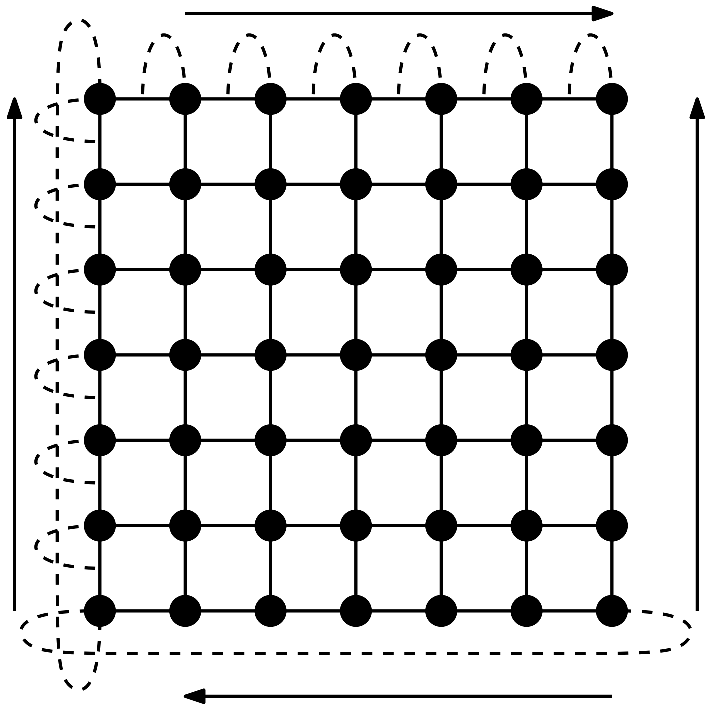

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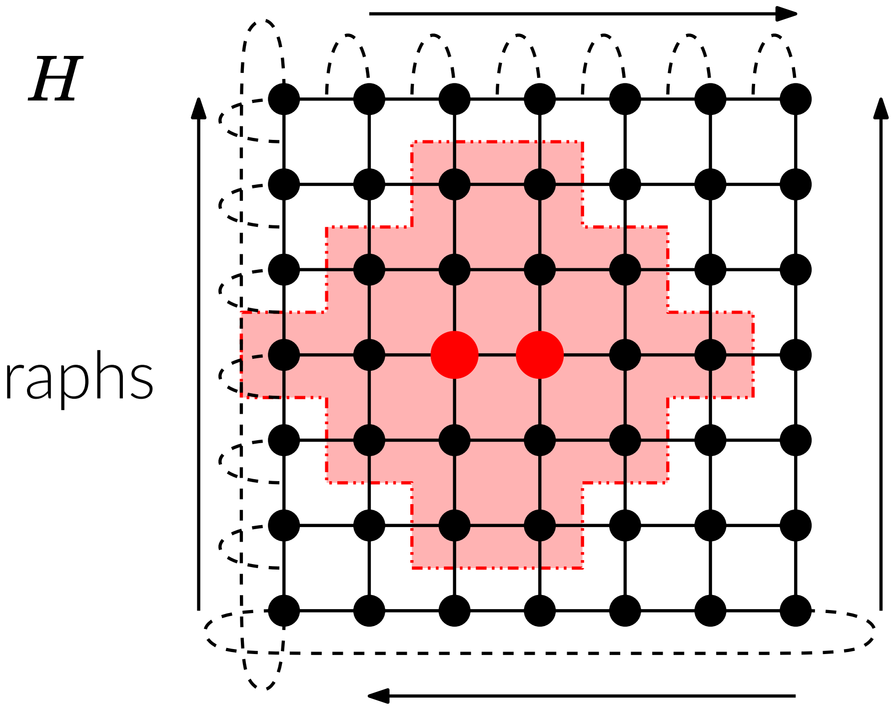

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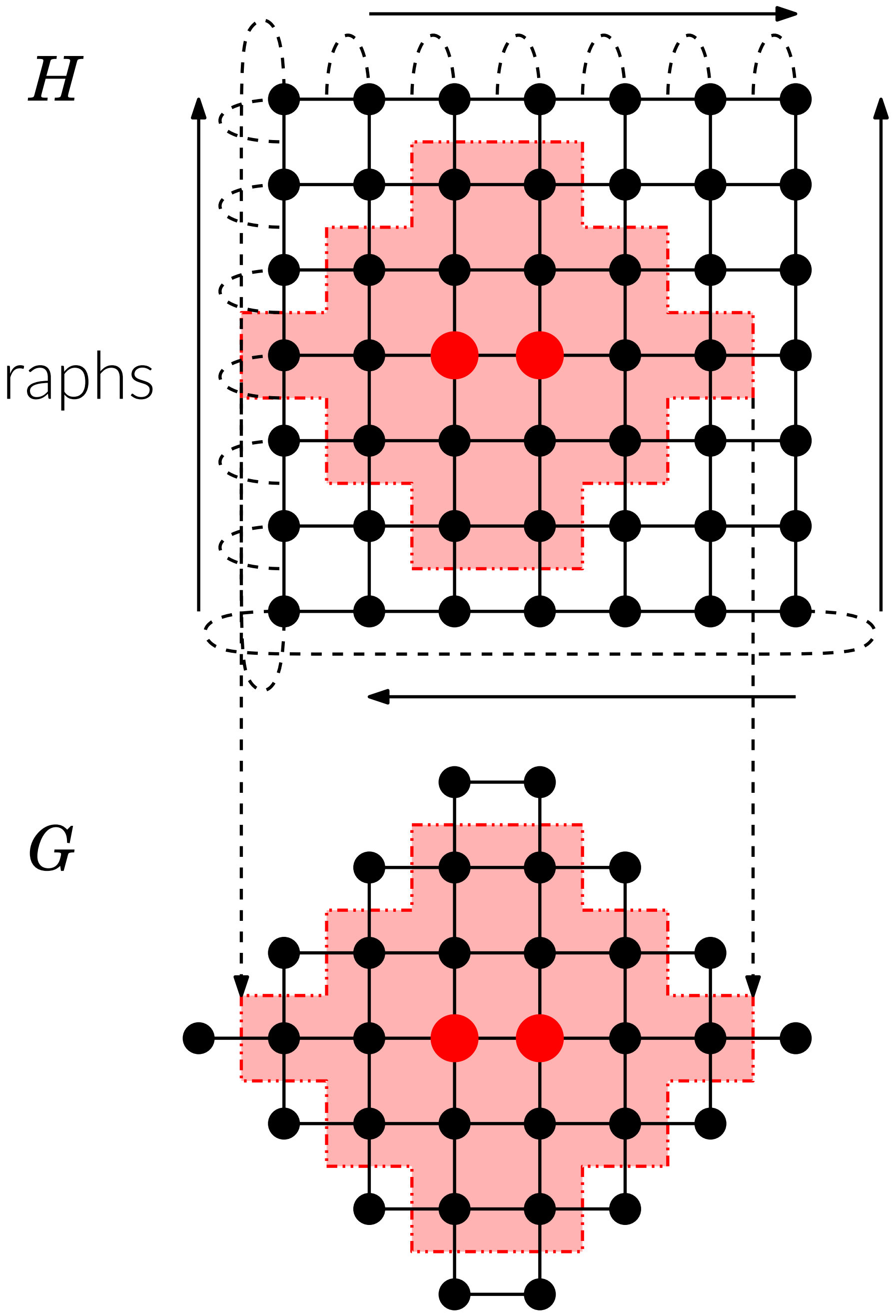

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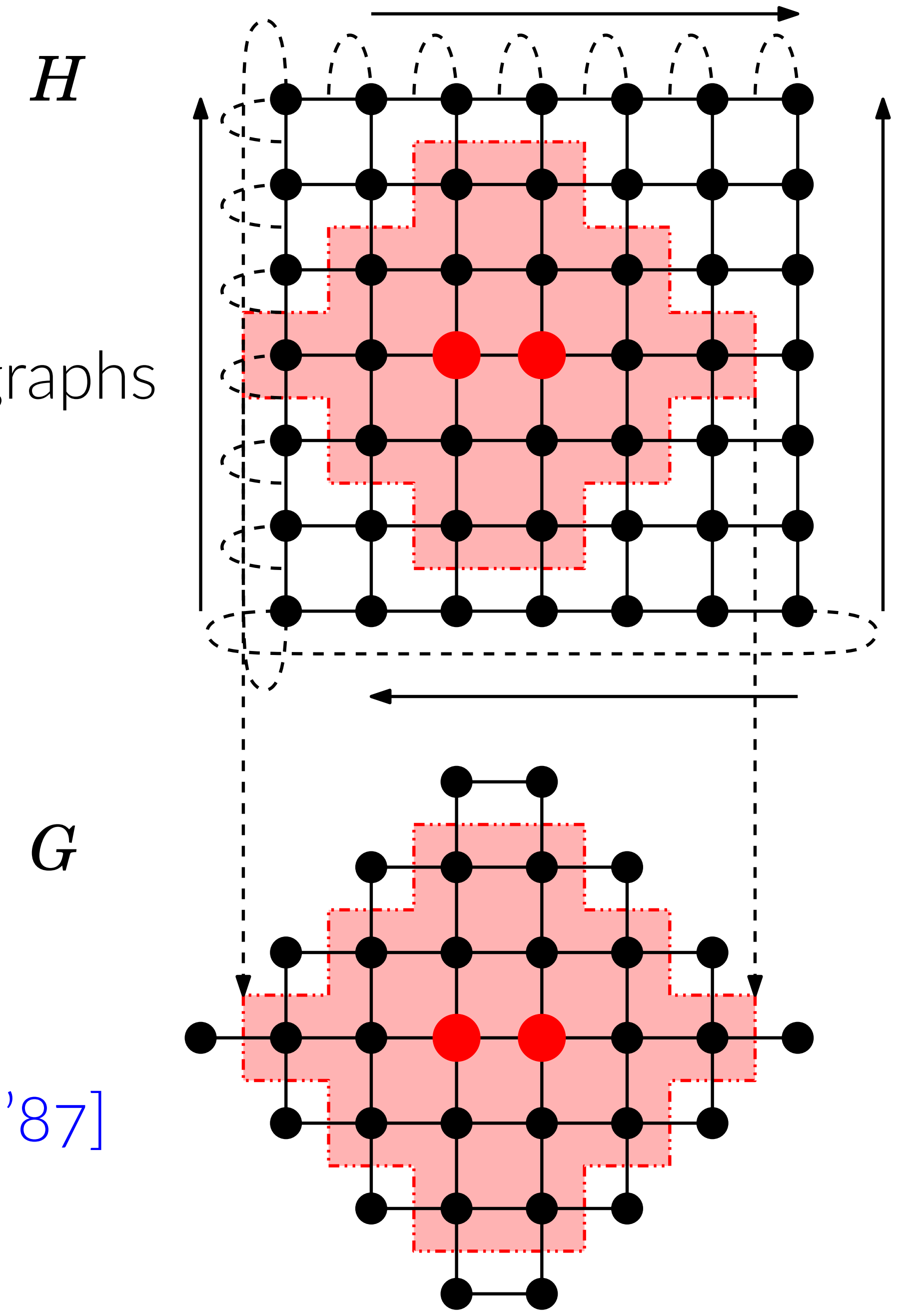

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- **Graph-existential indistinguishability argument**: lower bound technique [Linial '87]
  - main contribution: we extend it all the way up to non-signaling LOCAL

Upper bound: network decomposition

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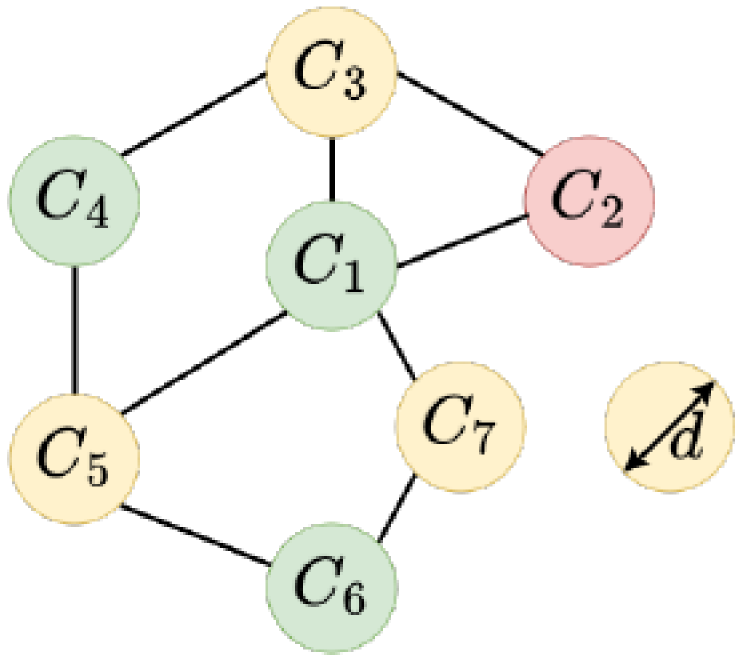

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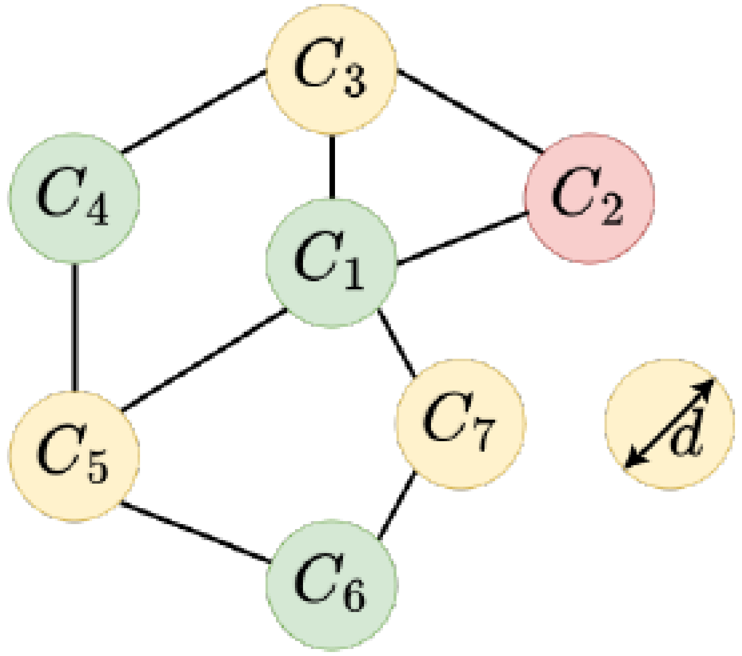

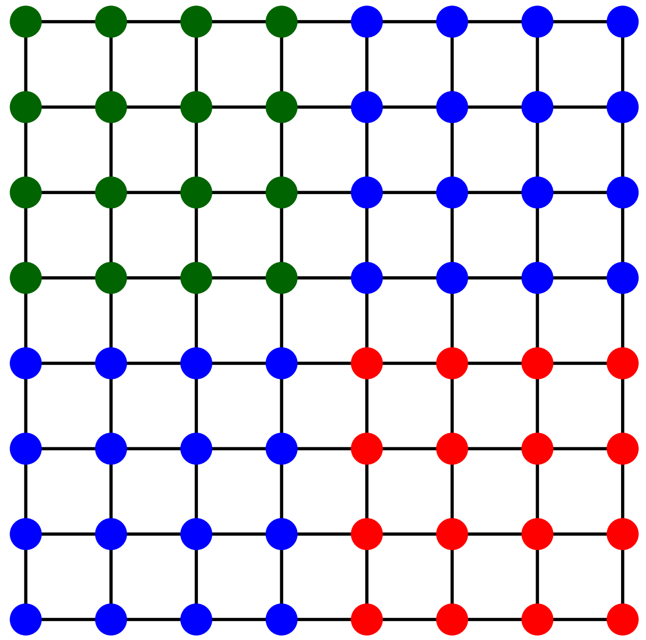

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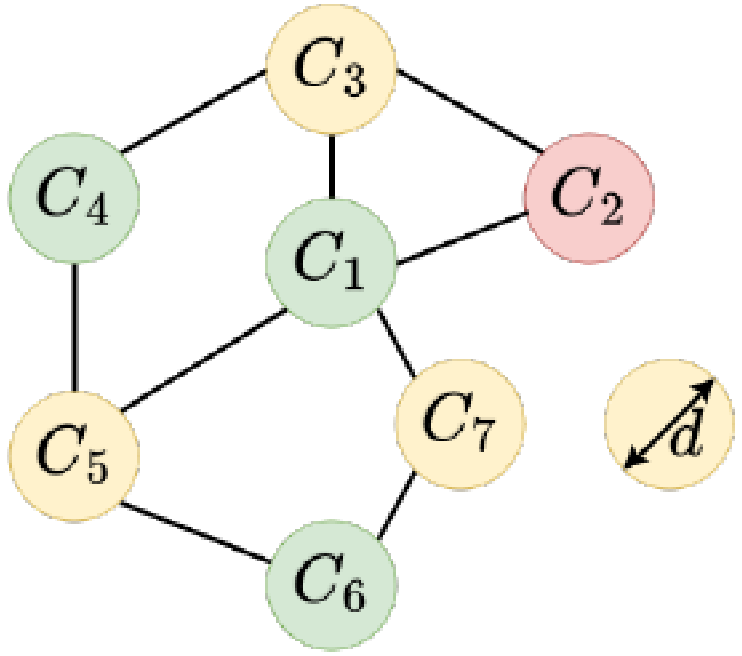
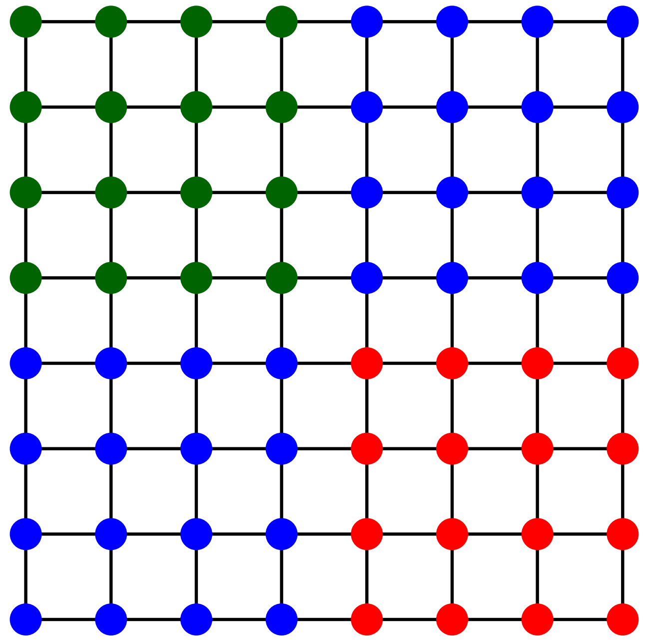


- We develope (α, d)-network decomposition for small α and $d = \Theta(n^{1/\alpha})$ in time $\tilde{O}(d)$ using [Ghaffari et al. '23 ; Chang and Li '23]
  - reminder: lower bound $\Omega(n^{1/\lfloor \frac{c-1}{\chi-1} \rfloor})$



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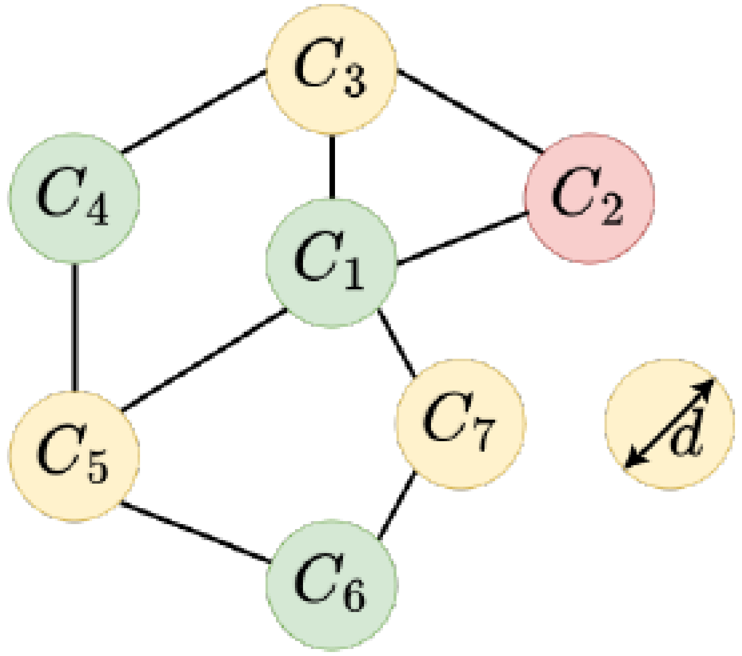
  - C_i has weak diamater d
  - clusters are monochromatic with colors in $\{1, \ldots, \alpha\}$
  - adjacent clusters have different colors



- We develope (α, d)-network decomposition for small α and $d = \Theta(n^{1/\alpha})$ in time $\tilde{O}(d)$ using [Ghaffari et al. '23 ; Chang and Li '23]
  - reminder: lower bound $\Omega(n^{1/\lfloor \frac{c-1}{\chi-1} \rfloor})$

$\implies$ $\alpha\chi$-coloring in time $\Theta(d)$ [Barenboim '13]
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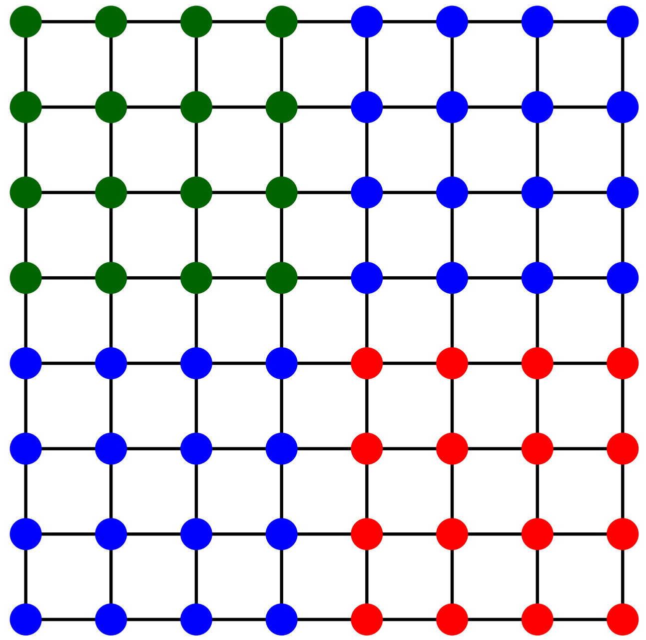
- total time $\tilde{O}(n^{1/\alpha})$, $\alpha = \lfloor \frac{c-1}{\chi-1} \rfloor$



$(3, 6)$-network decomposition
$\implies$ 6-coloring

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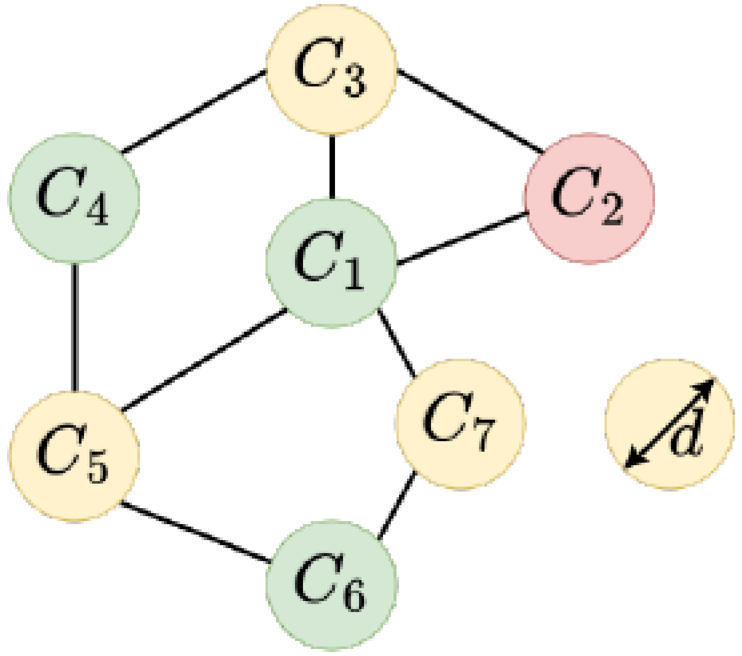

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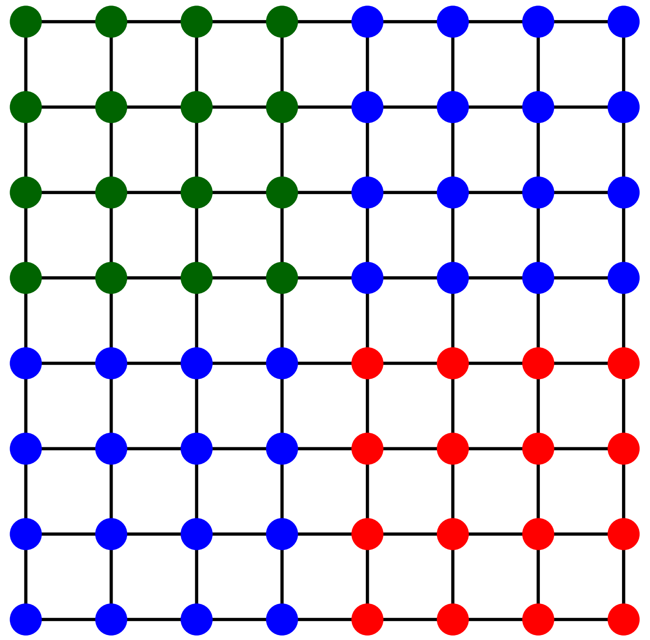

$(3,6)$-network decomposition
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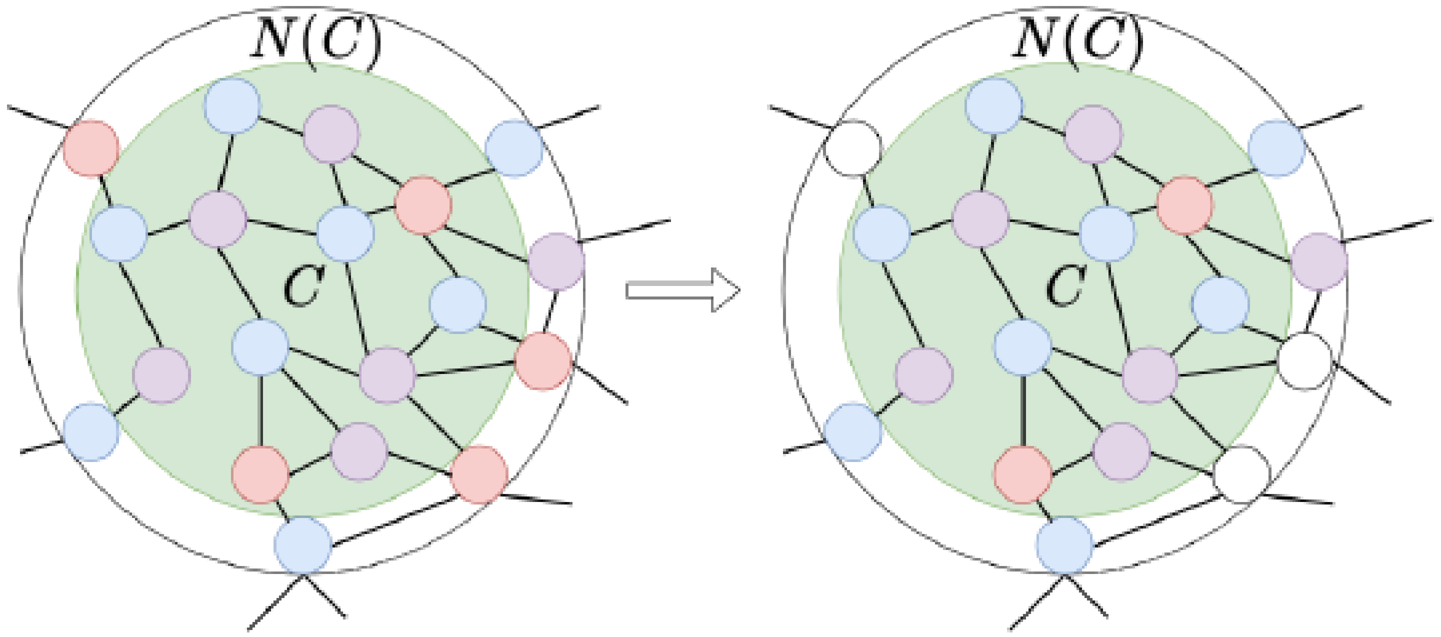
- **Not enough!**: $\alpha\chi > c$. We can actually do better (next slide)

Upper bound: the hiding trick

Given (α, d)-network decomposition

- "Hide" one color (e.g., color 1) from "neighborhood" of clusters
  - "reuse" color 1 instead of i in cluster of color i
  - save $\alpha - 1$ colors



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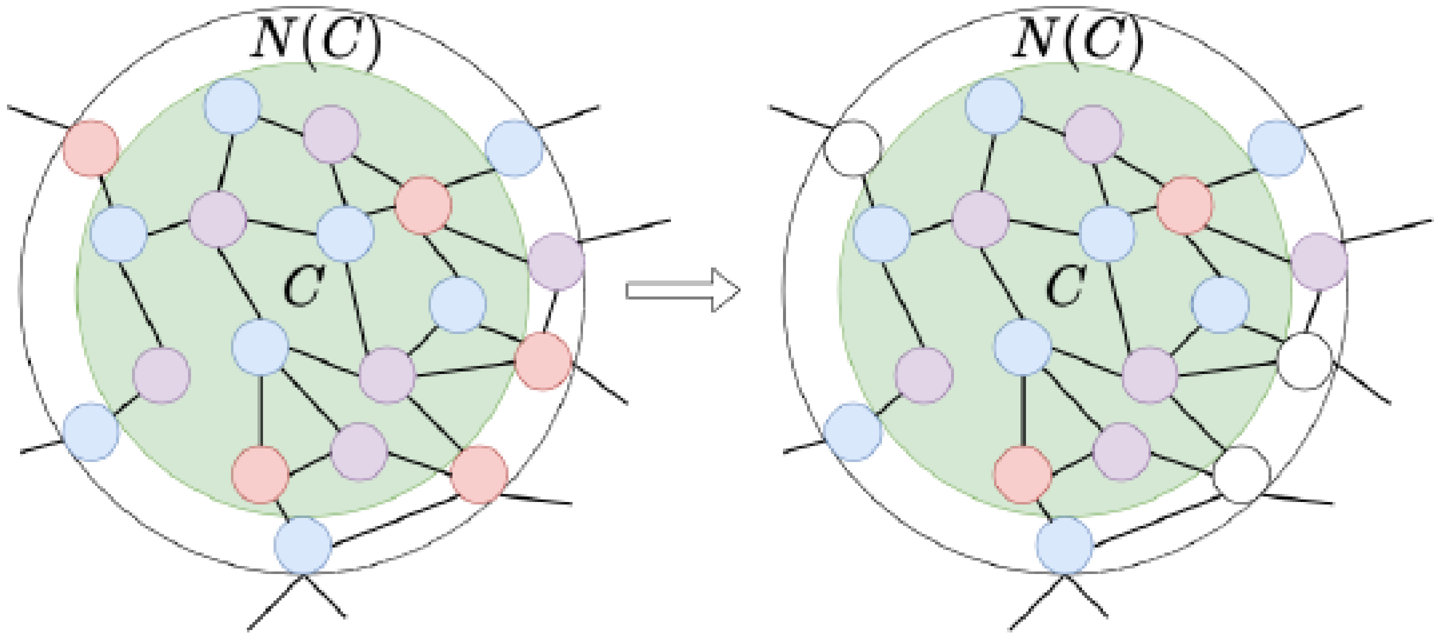

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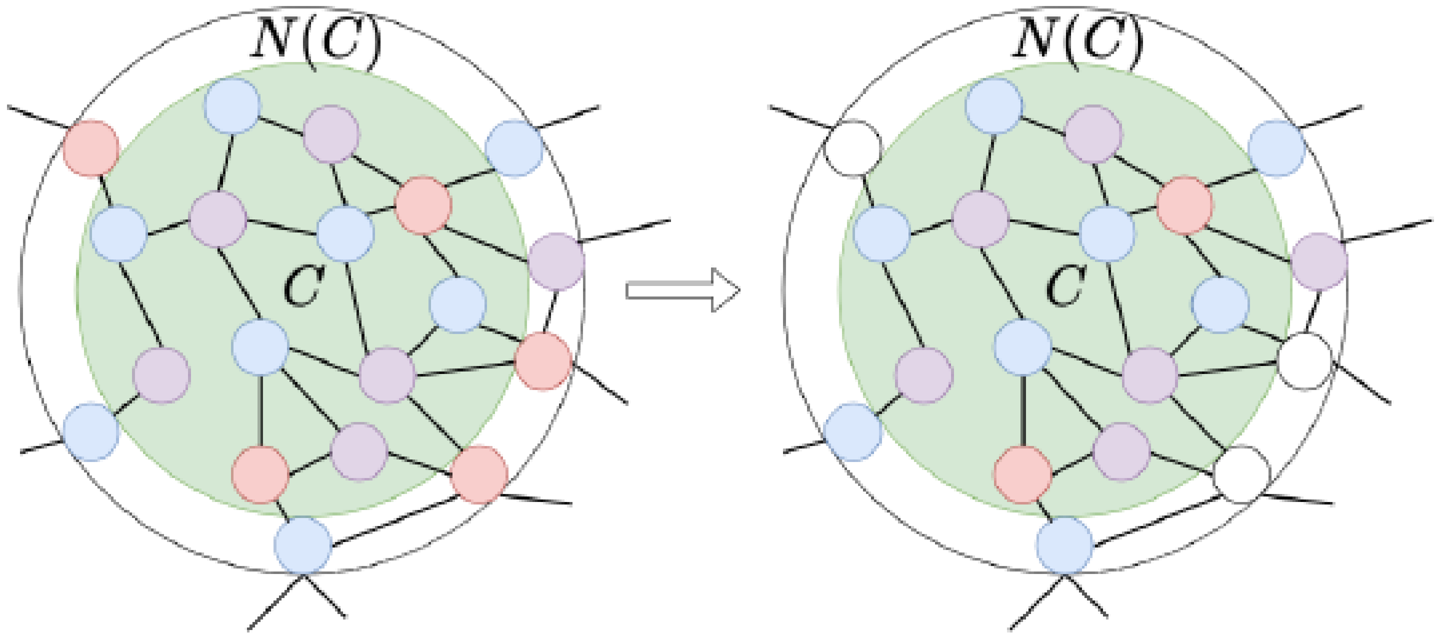

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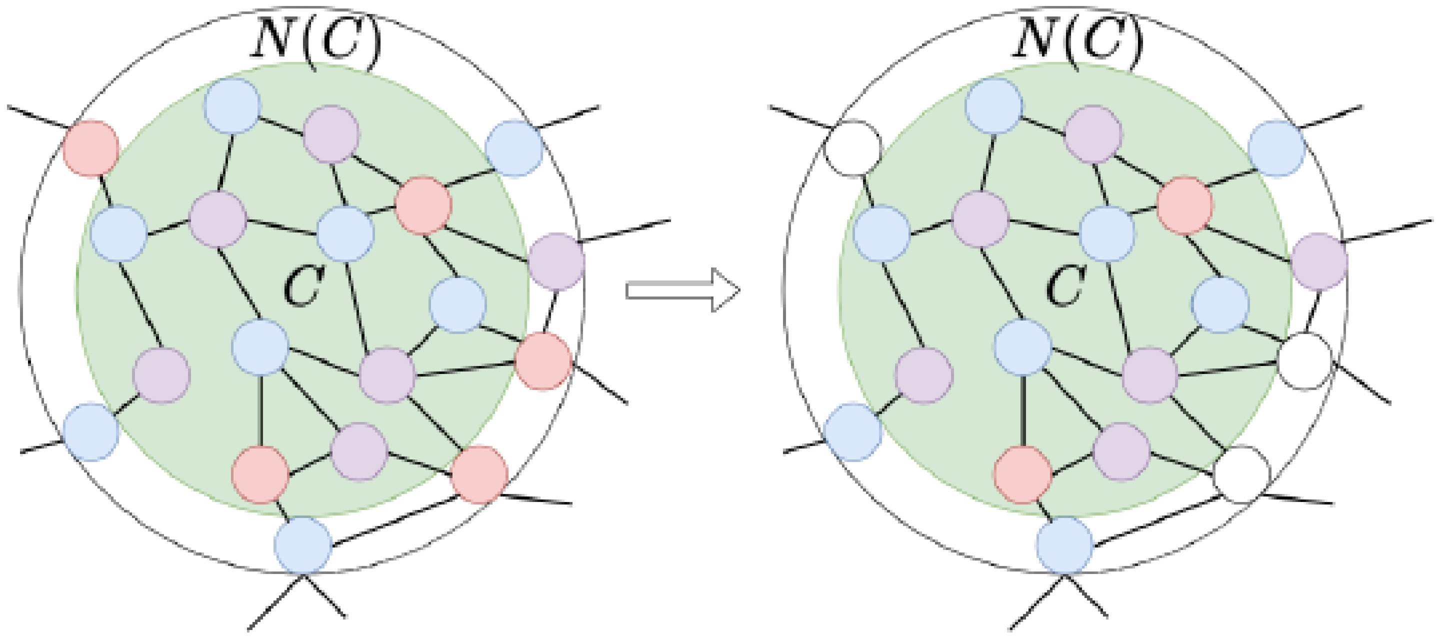

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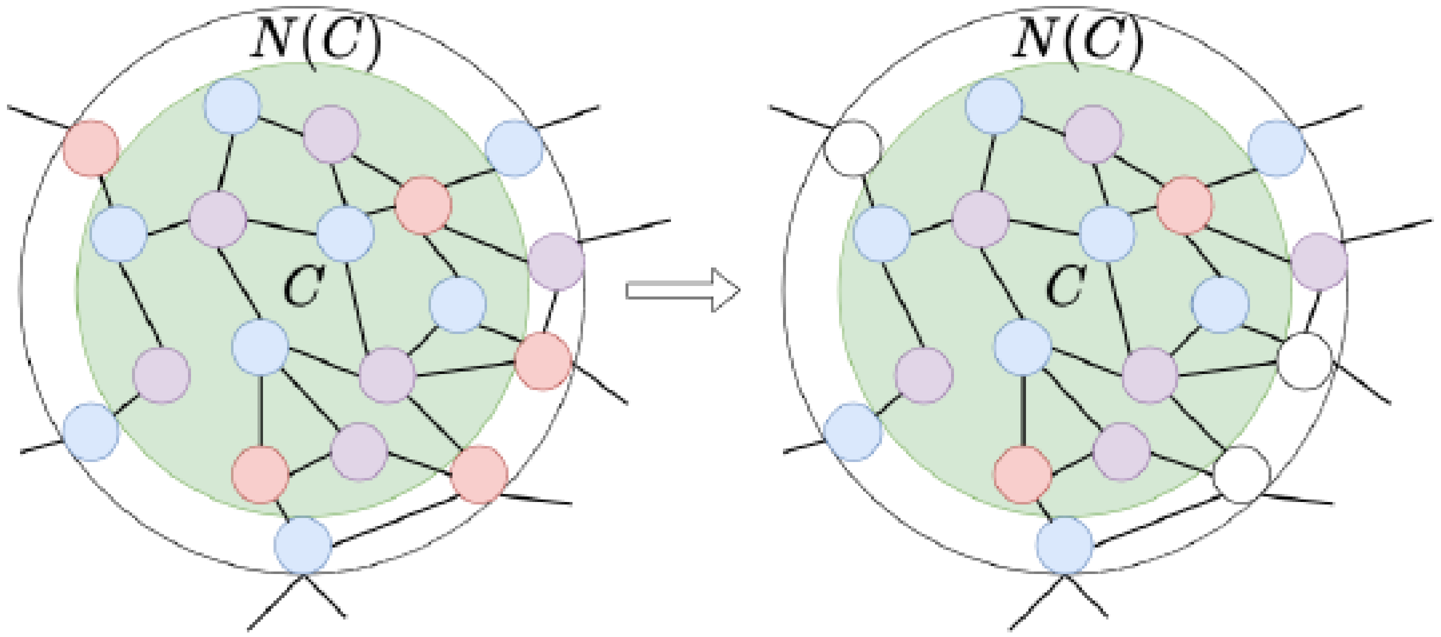

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  - complexity increases with multiples of $\chi - 1$

Summary

Problem: c-coloring χ-chromatic graphs

- LOCAL model of computation: previously
  - case $\chi = c$ (trivial)
  - case $\chi = 2$, $c = 3$ only lower bound [Brandt et al. '17]

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THANKS!

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