WAND 2023

The Self-stabilizing Bit-Dissemination Problem

Robin Vacus

Joint works with Luca Becchetti, Andrea Clementi, Amos Korman, Francesco Pasquale, Luca Trevisan, Isabella Ziccardi

Introduction: The “computational lens”

The “computational lens” (personal view)

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Biological scenario

- foraging
- navigation
- house hunting...

Image credit: birdfact.com

The “computational lens” (personal view)

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Gathering, analysing data...

↓

Observed behavior

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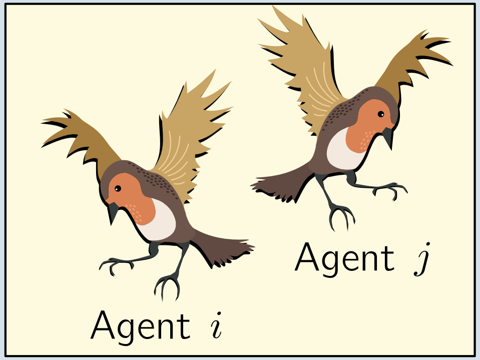

Idealised algorithmic problem

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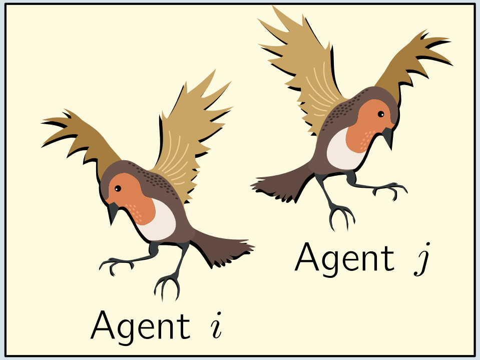

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Family of optimal algorithms

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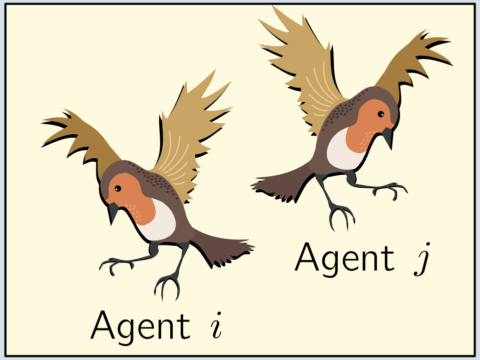

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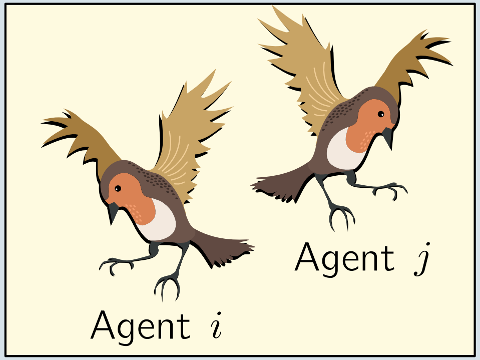
reflect on the relevance of the model



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Wednesday, October 11, 9am
Keynote talk by Amos Korman

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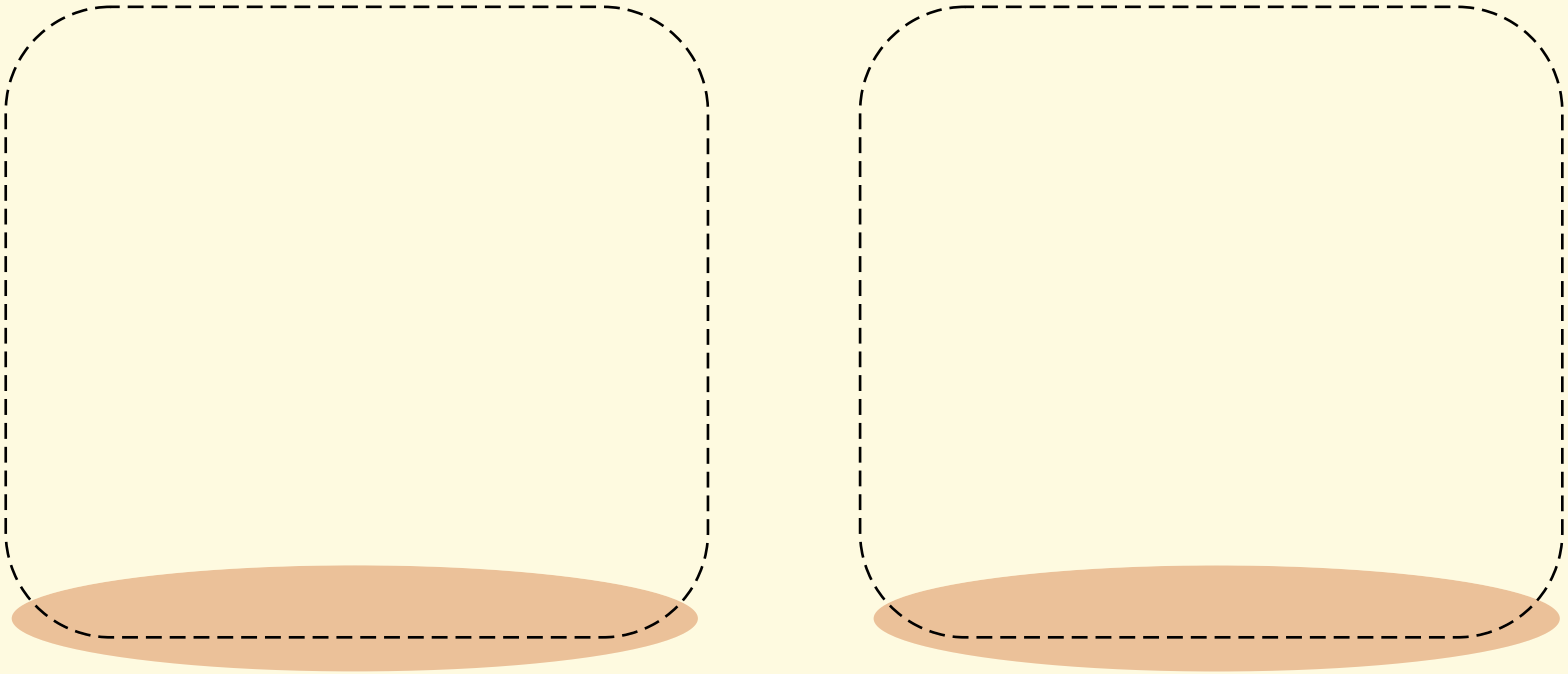
The bit-dissemination problem

Introduced in

Boczkowski, Korman & Emanuele Natale.
Minimizing message size in stochastic communication patterns: Fast self-stabilizing protocols with 3 bits.
SODA, 2017.

Biological Motivation



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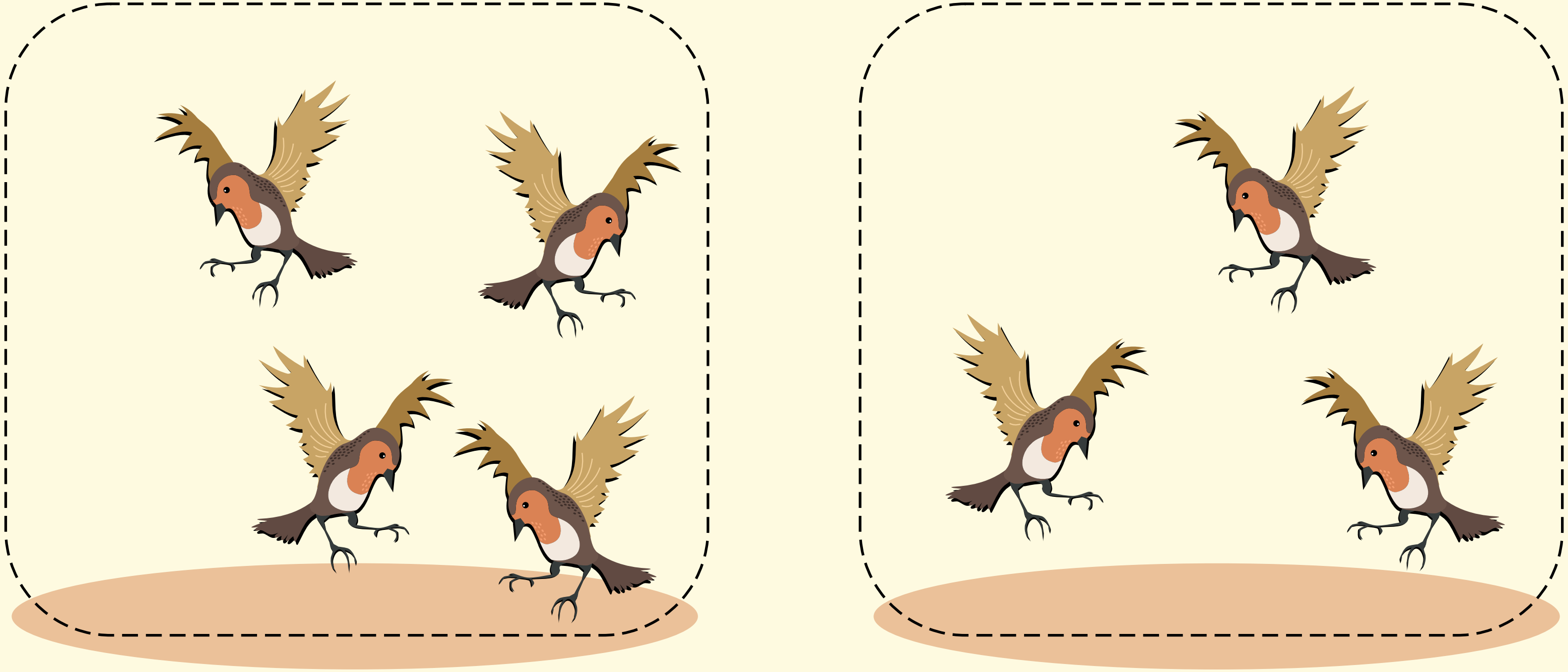

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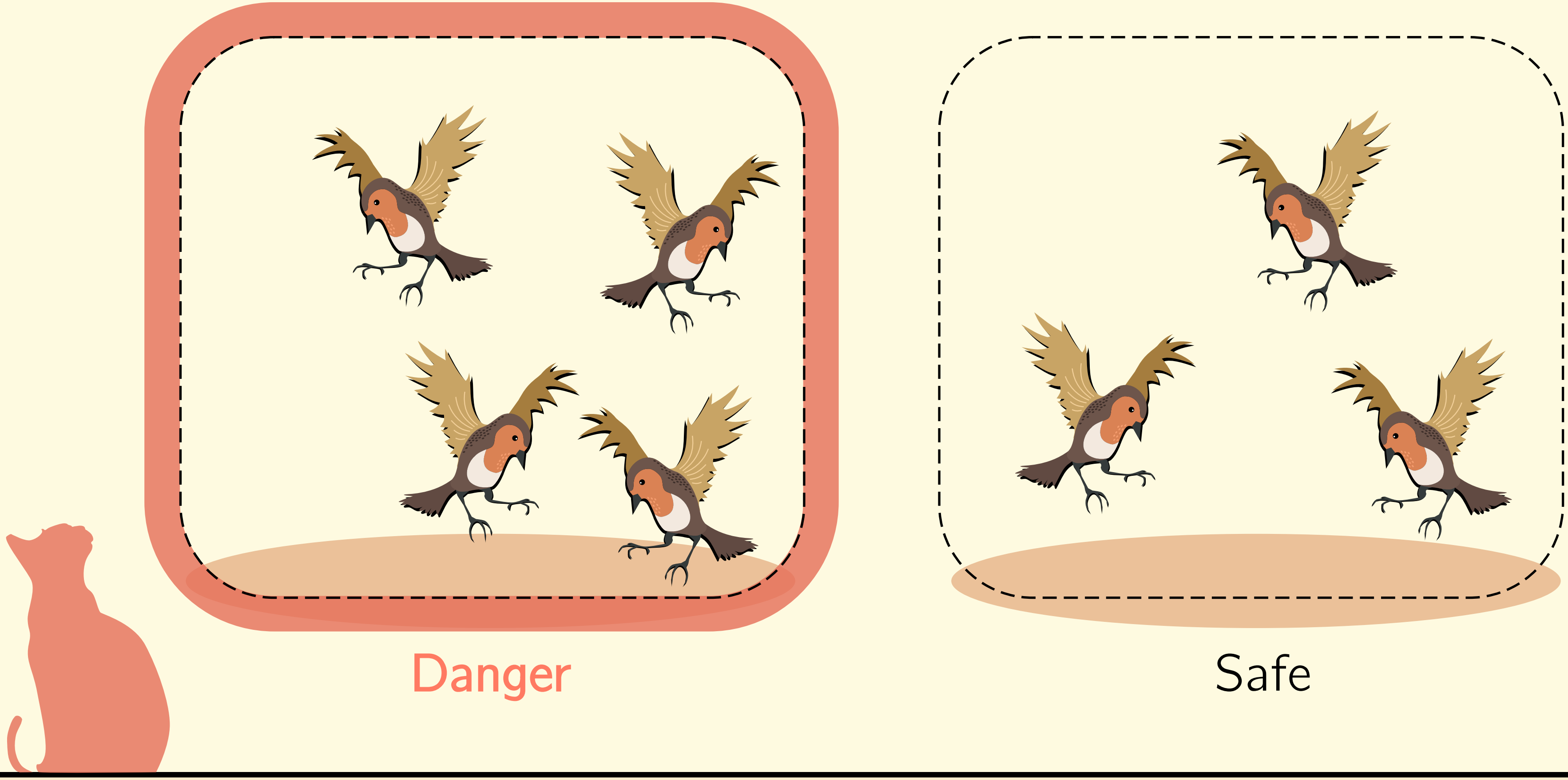

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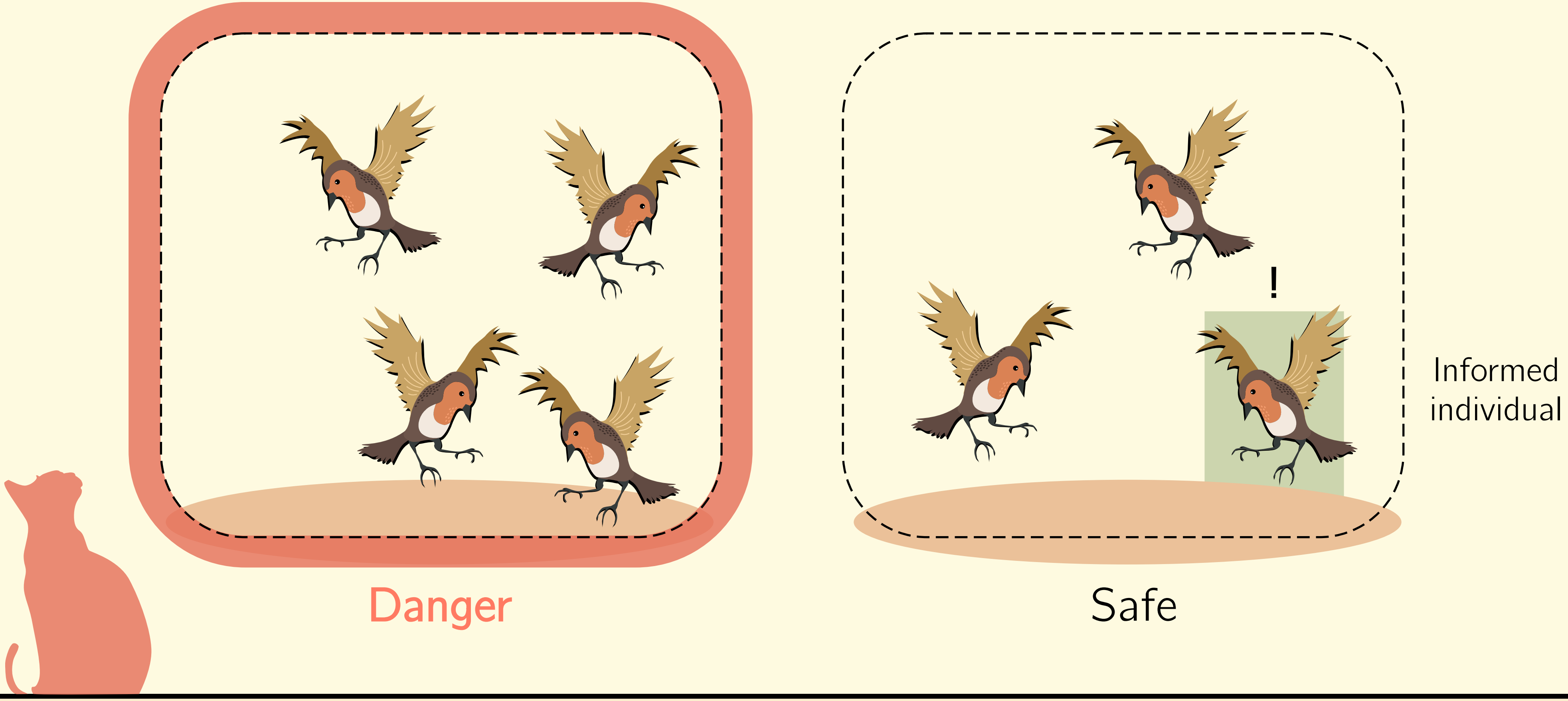

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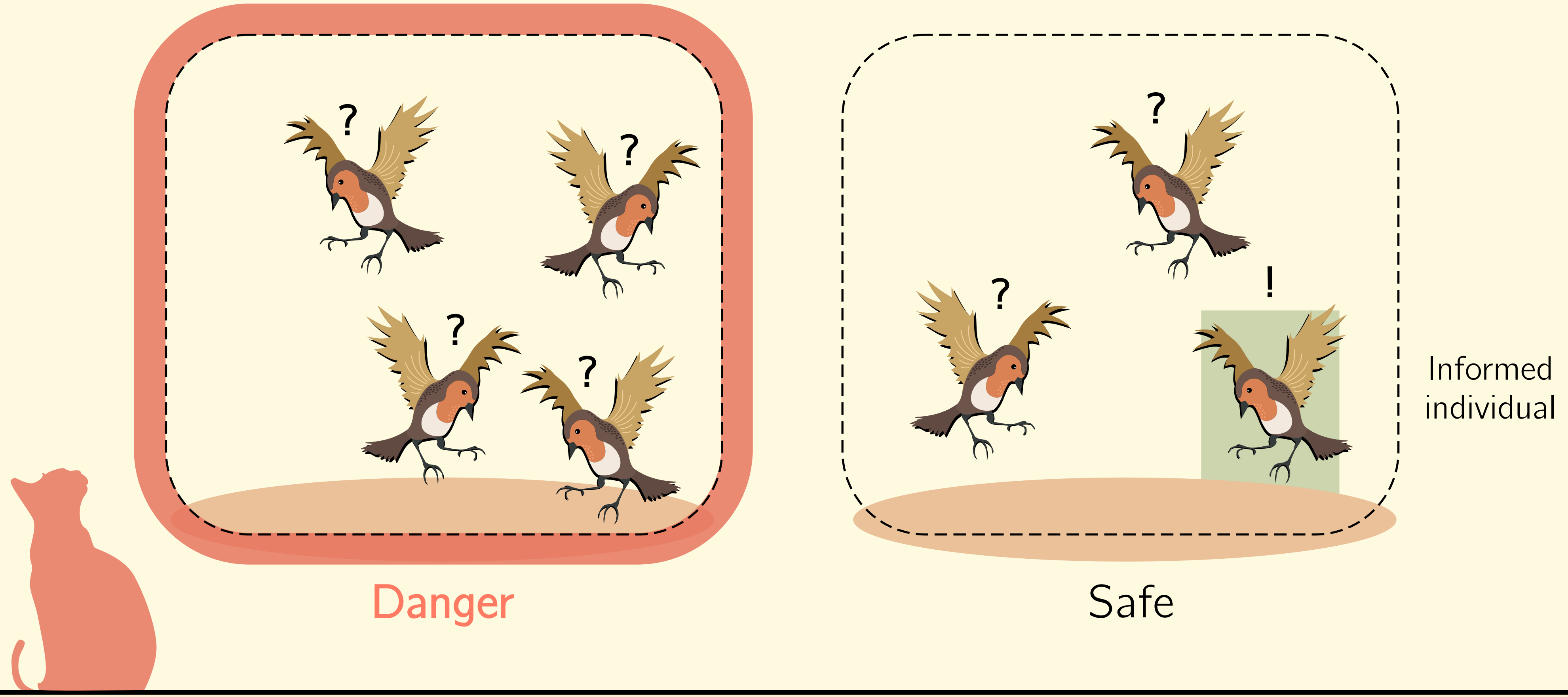

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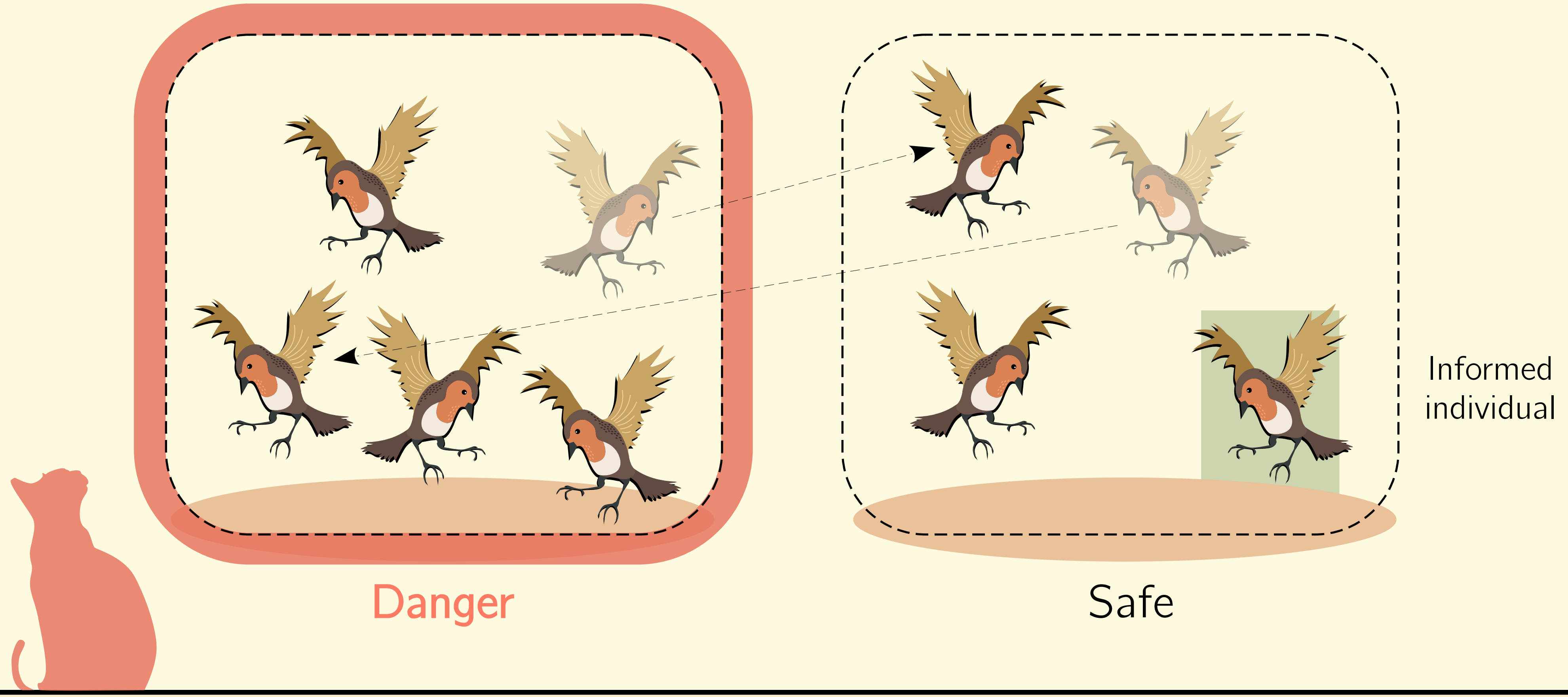

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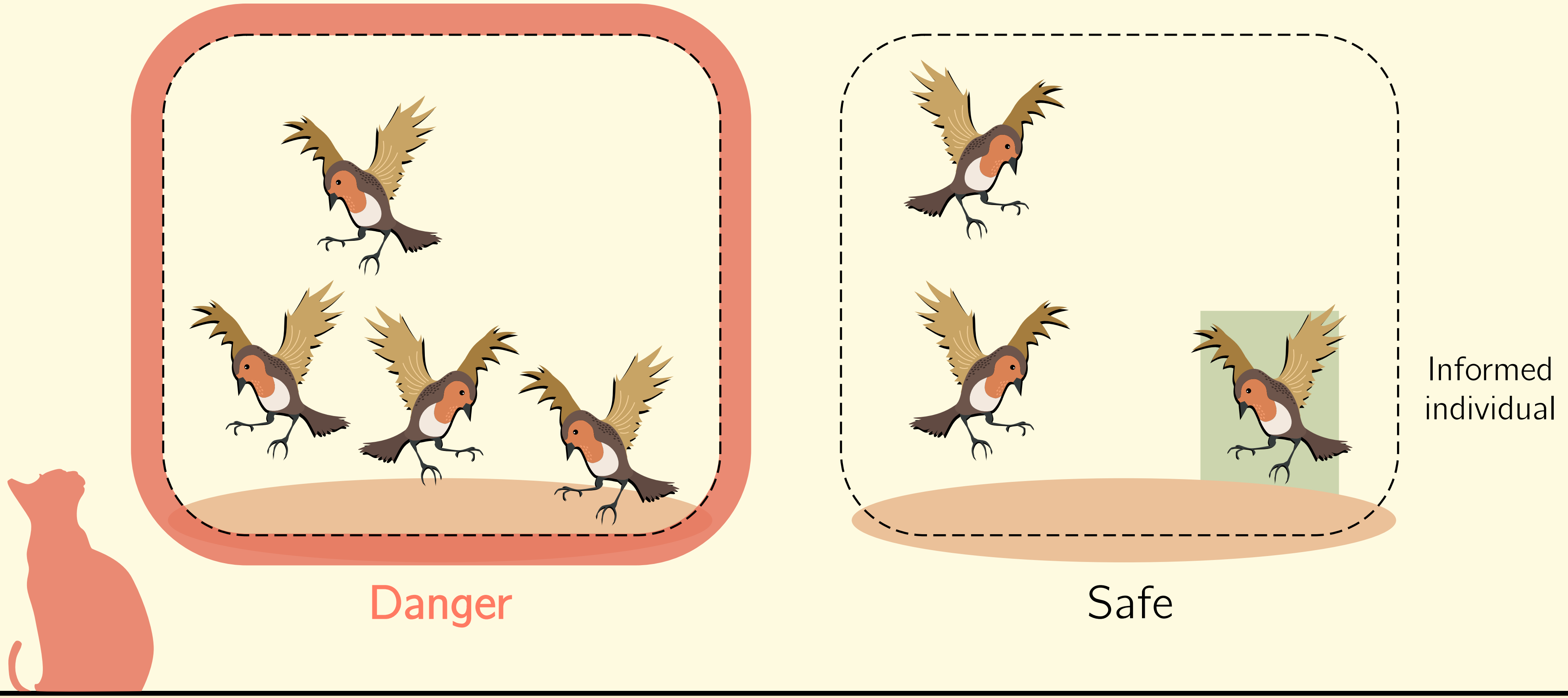
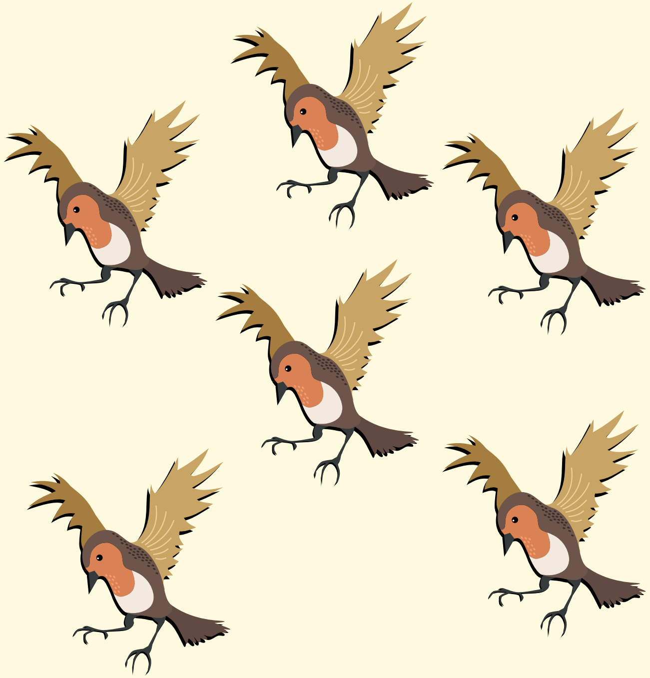


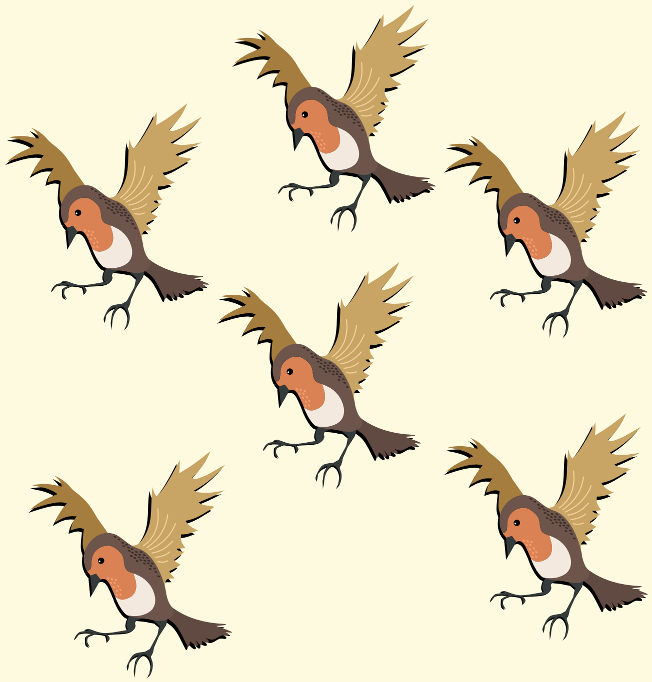
Self-stabilizing Bit-Dissemination

- Agents $i = 1, \ldots, n$



Self-stabilizing Bit-Dissemination

- Agents $i = 1, \ldots, n$
- Discrete time $t = 1, 2, 3, \ldots$



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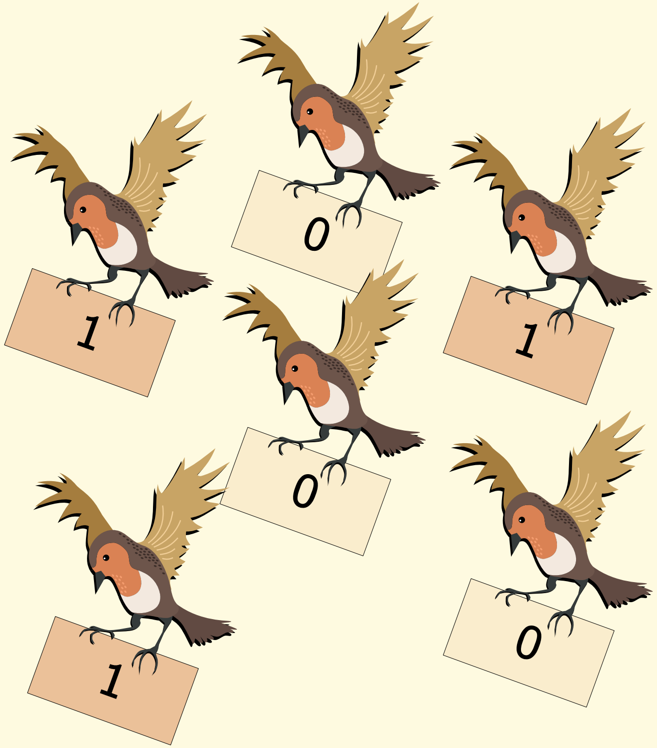

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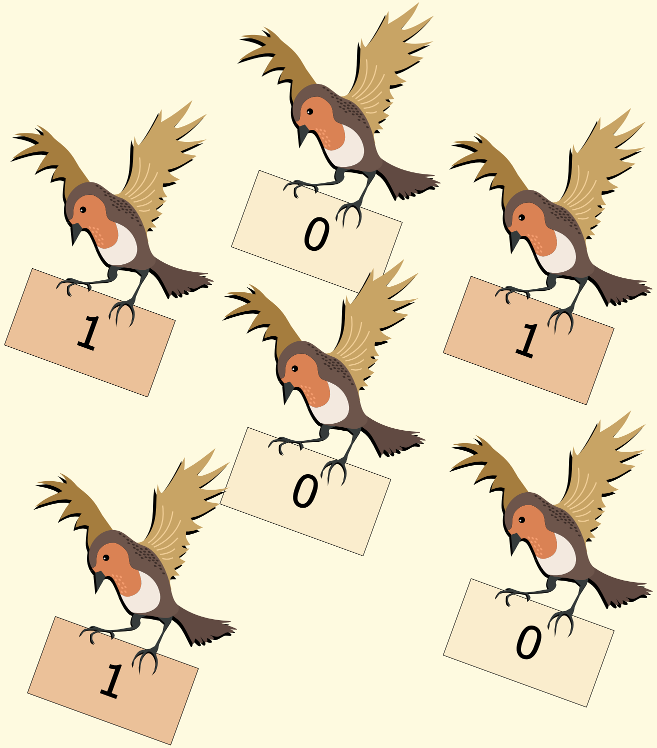

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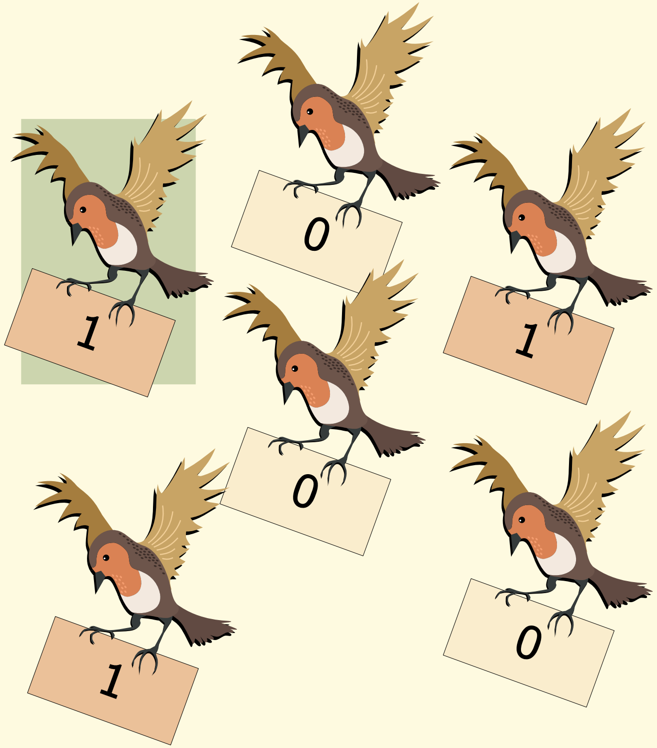
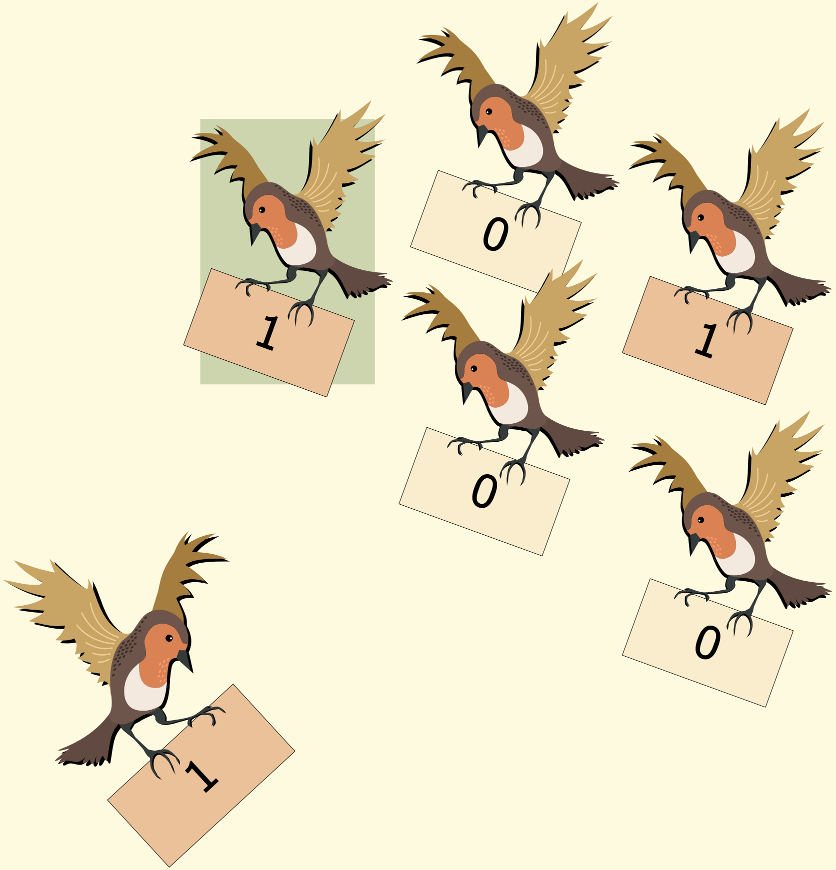
- One agent is the source (knows the **correct** opinion and keep it)



Self-stabilizing Bit-Dissemination

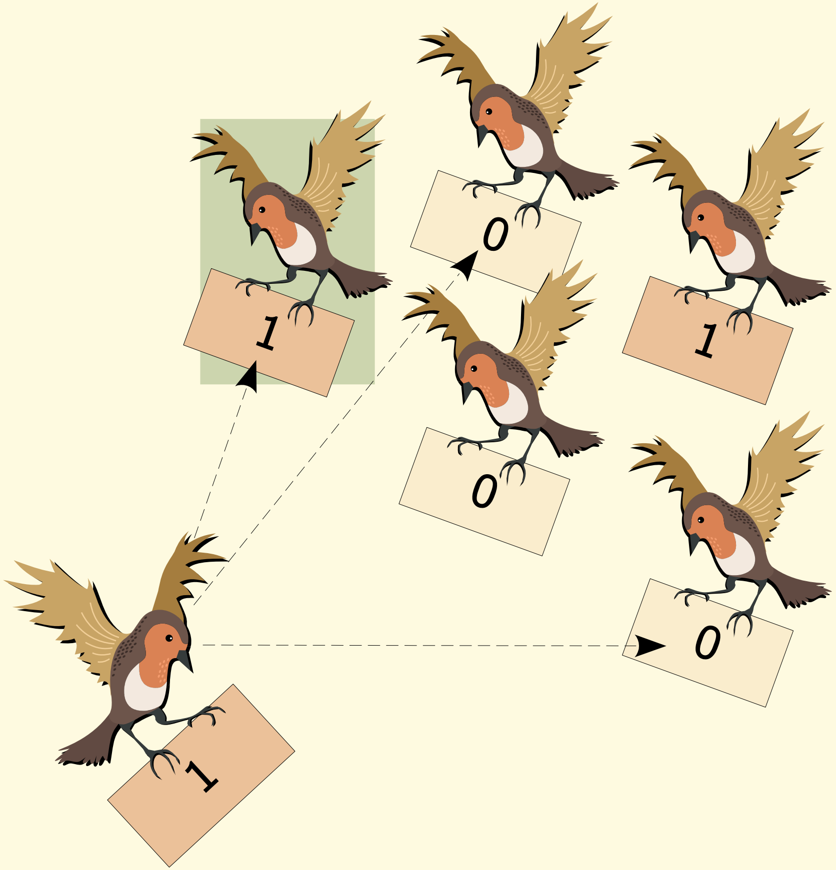
- Whenever an agent is activated:



Self-stabilizing Bit-Dissemination

- Whenever an agent is activated:

(1) it samples ℓ other agents u.a.r.



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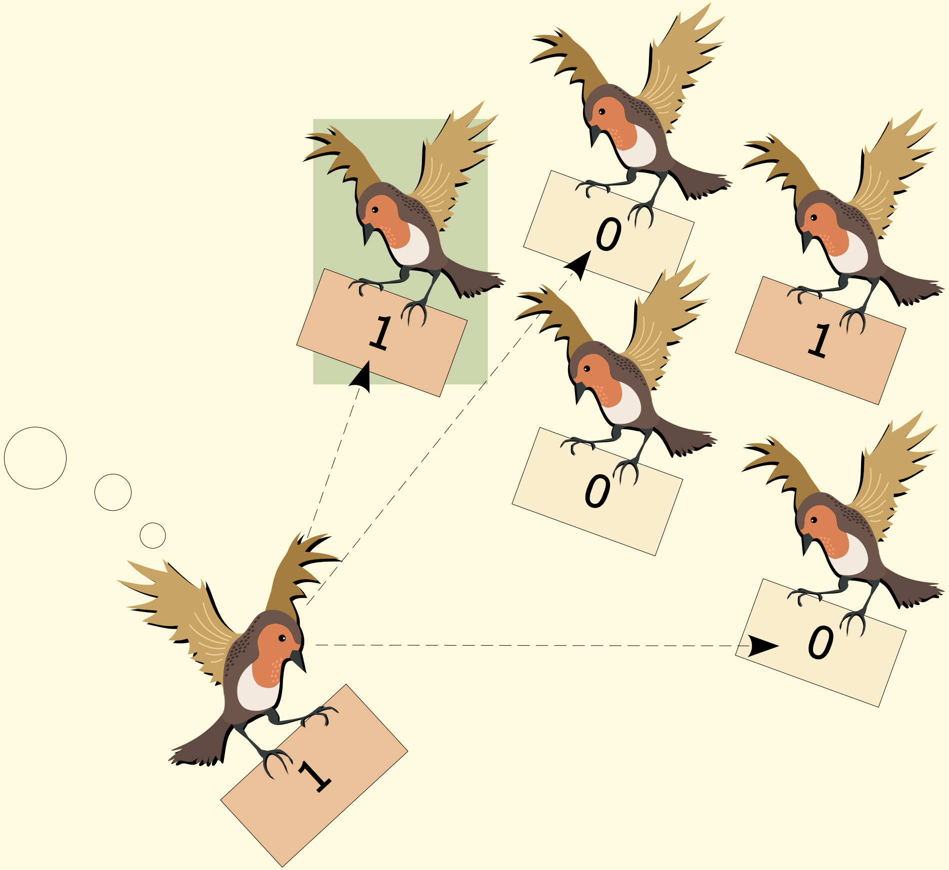
Passive communication

1	0	0



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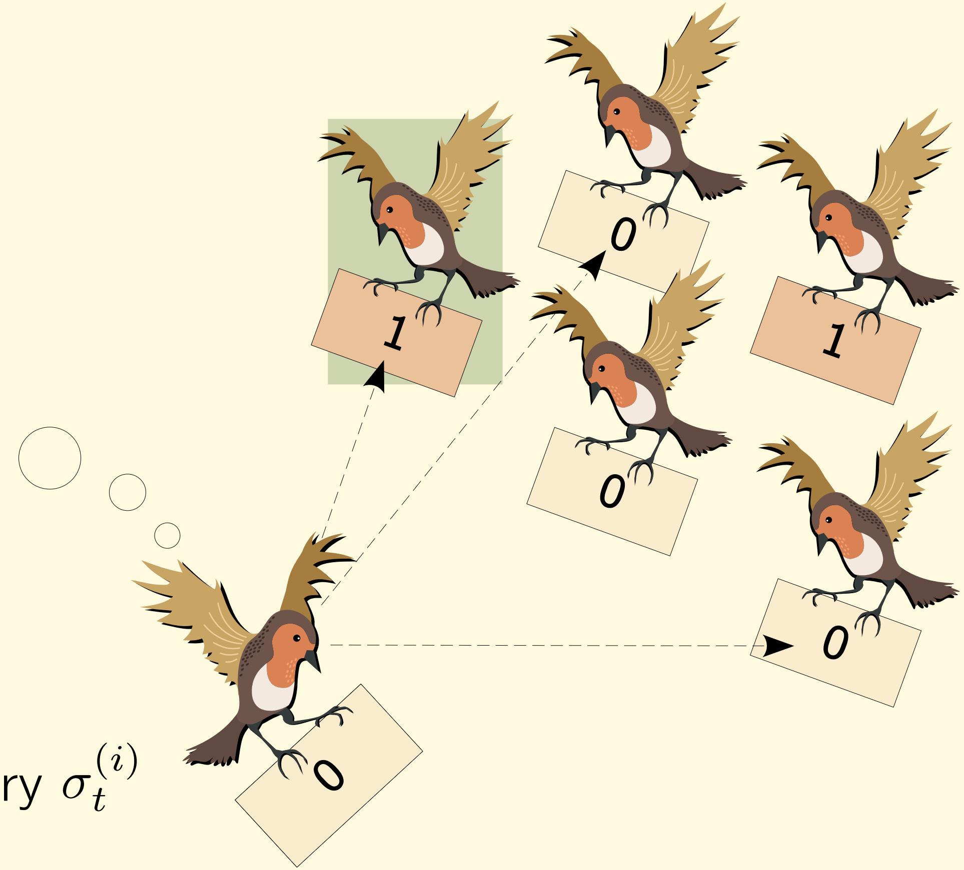

(2) it can update its state:

opinion $Y_t^{(i)}$ **and** internal memory $\sigma_t^{(i)}$

Self-stabilizing Bit-Dissemination

- **Goal:**
  Reach a consensus on the correct opinion as fast as possible and remain with it

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  - … of the opinions
  - … of the internal memory

Overview

Overview

	$\ell = O(\log n)$	$\ell = \omega(\log n)$

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No Memory		
Memory		

Overview

Parallel	$\ell = O(\log n)$	$\ell = \omega(\log n)$
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Memory		

Asynchronous	$\ell = O(\log n)$	$\ell = \omega(\log n)$
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Memory		

“Follow the Trend”

Parallel

Memory

Amos Korman & R.V.

Early Adapting to Trends: Self-Stabilizing Information Spread using Passive Communication.

PODC, 2022

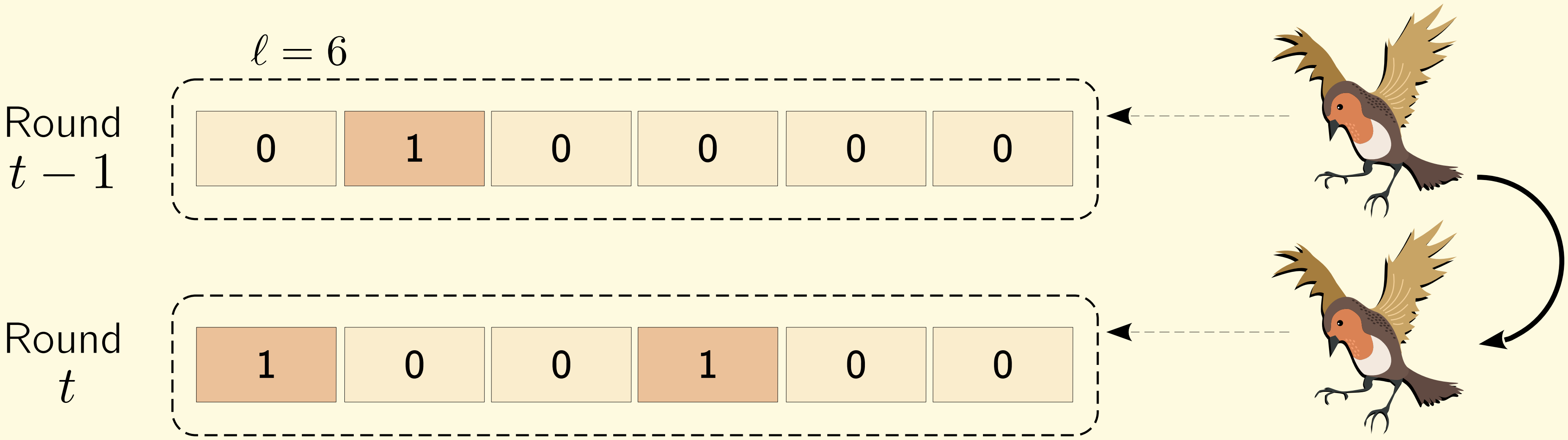
Follow the Trend

$\ell = 6$

Round
$t-1$

0	1	0	0	0	0

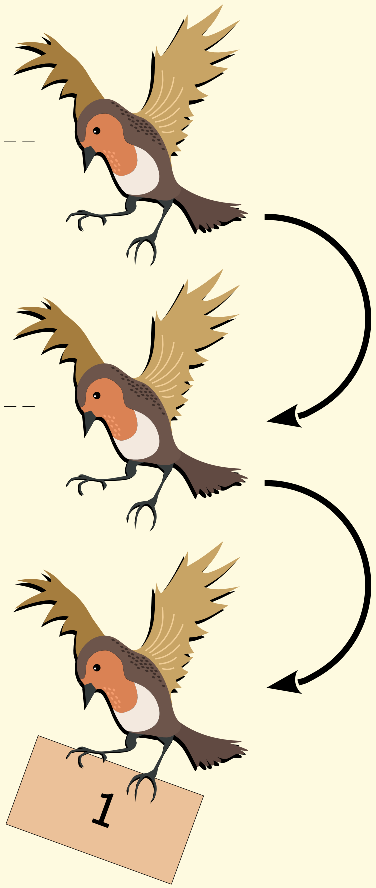
Follow the Trend



Follow the Trend

$\ell = 6$

Round						
$t-1$	0	1	0	0	0	0
t	1	0	0	1	0	0
$t+1$						



Follow the Trend

$\ell = 6$

Round $t-1$

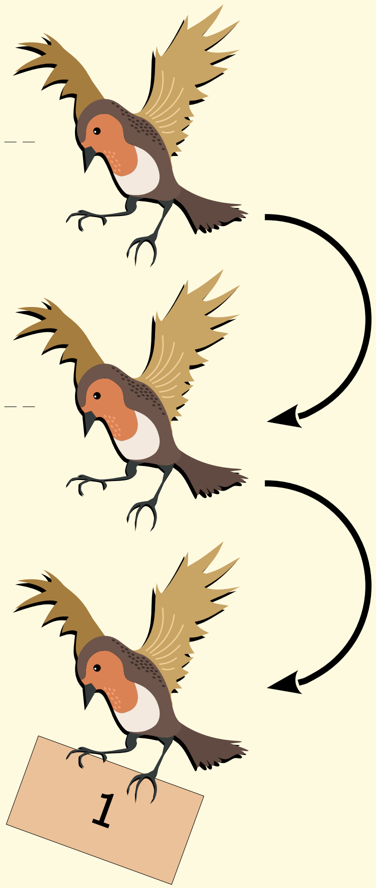
0	1	0	0	0	0

Round t

1	0	0	1	0	0

Round $t+1$

(In case of tie, keep the same opinion)



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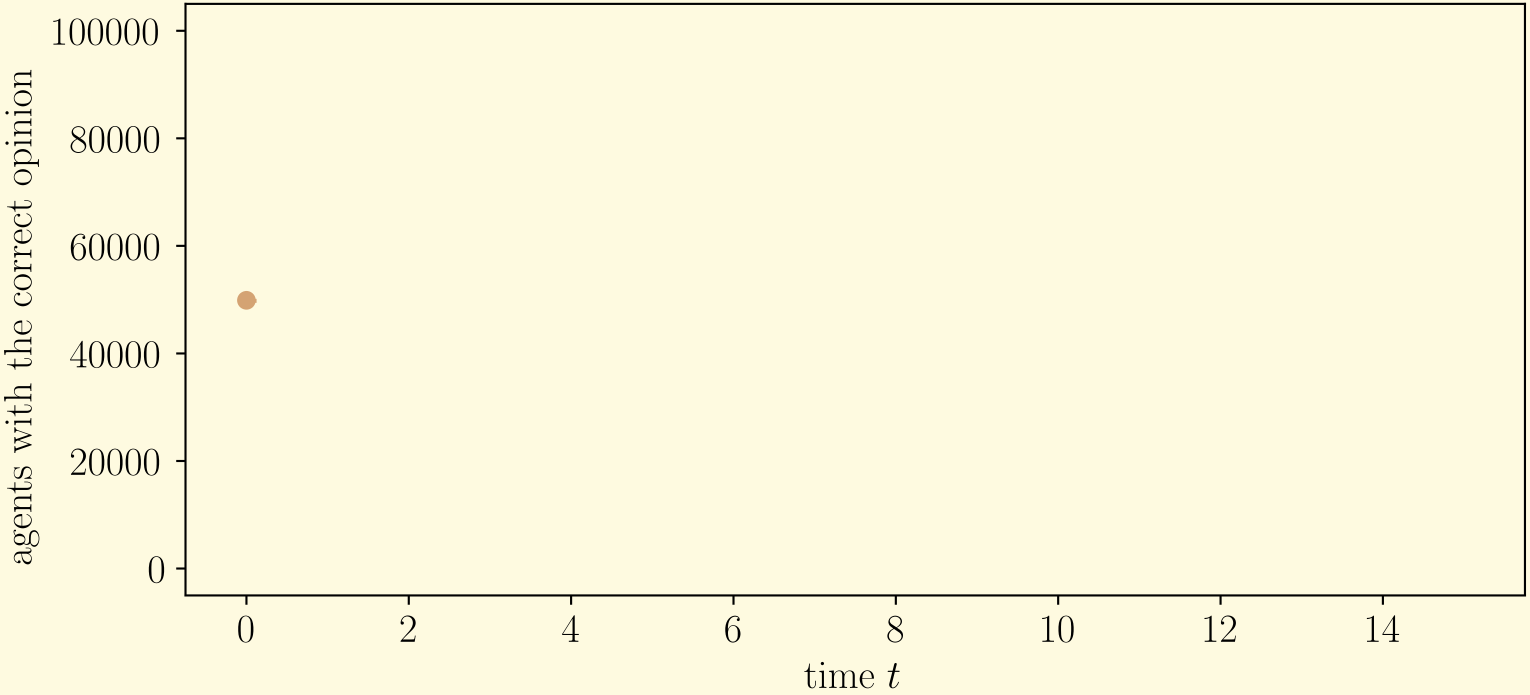

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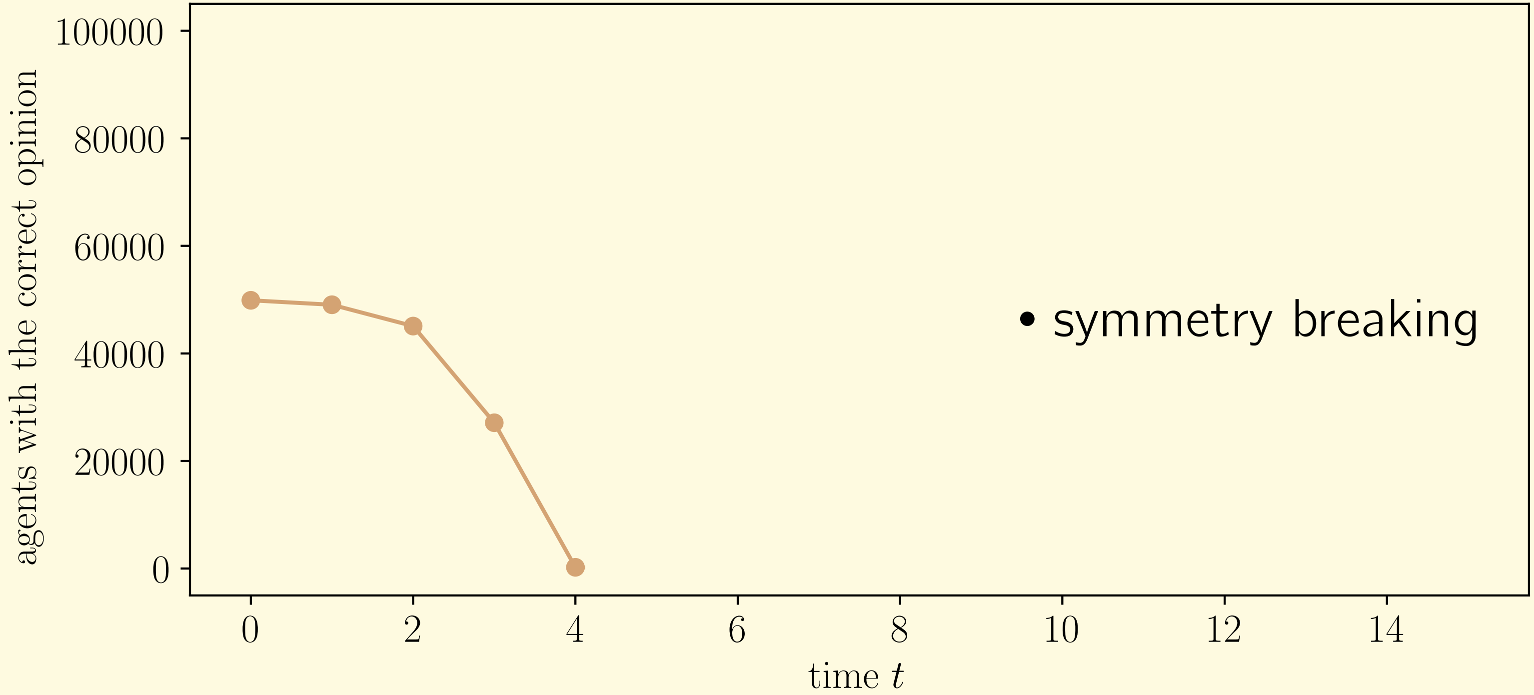
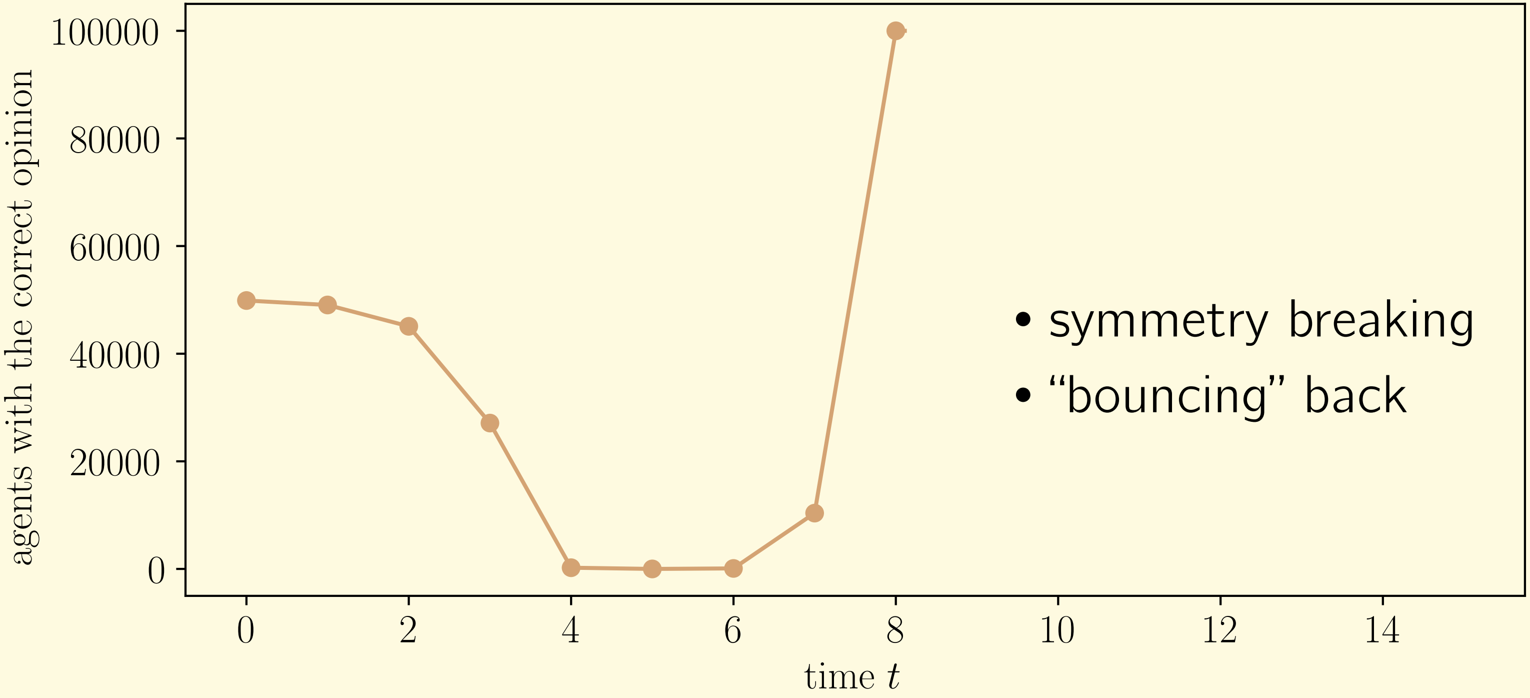


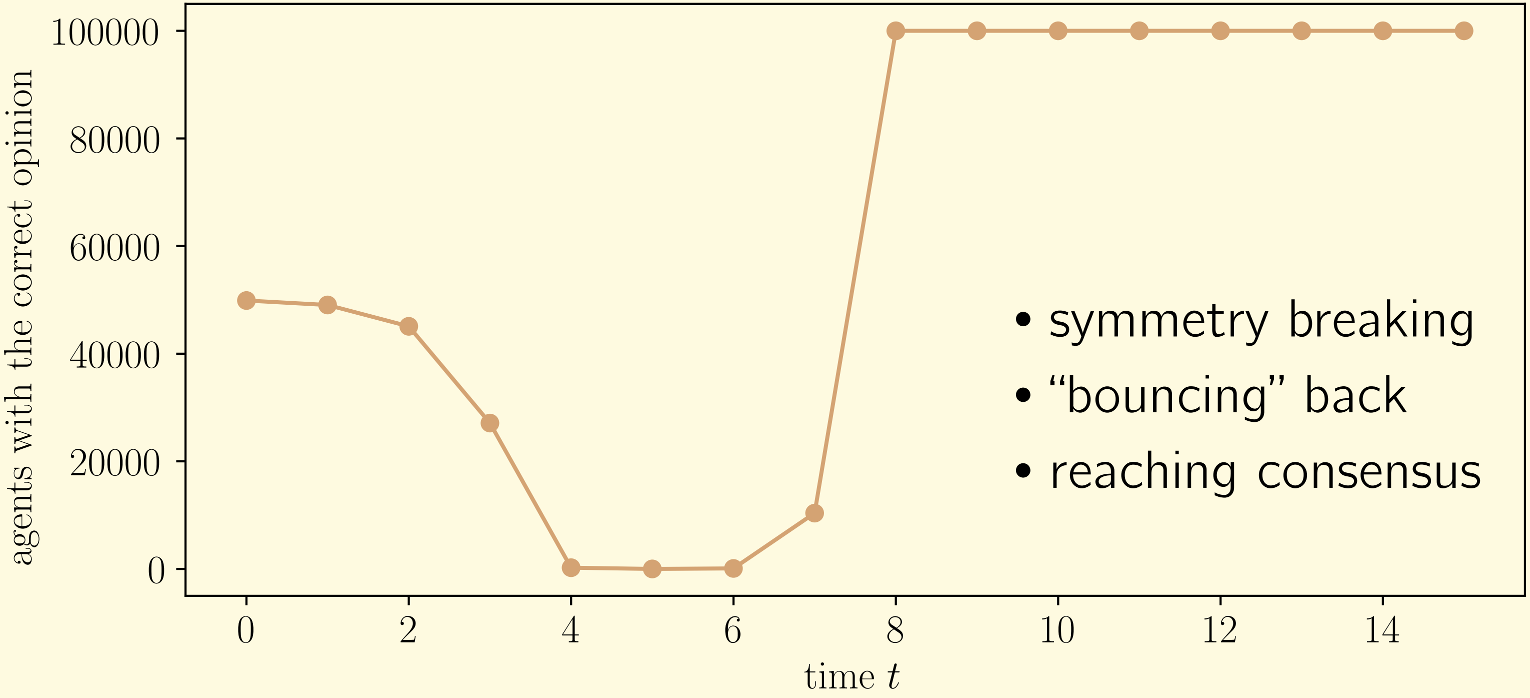
- symmetry breaking

Follow the Trend



- symmetry breaking
- “bouncing” back

Follow the Trend



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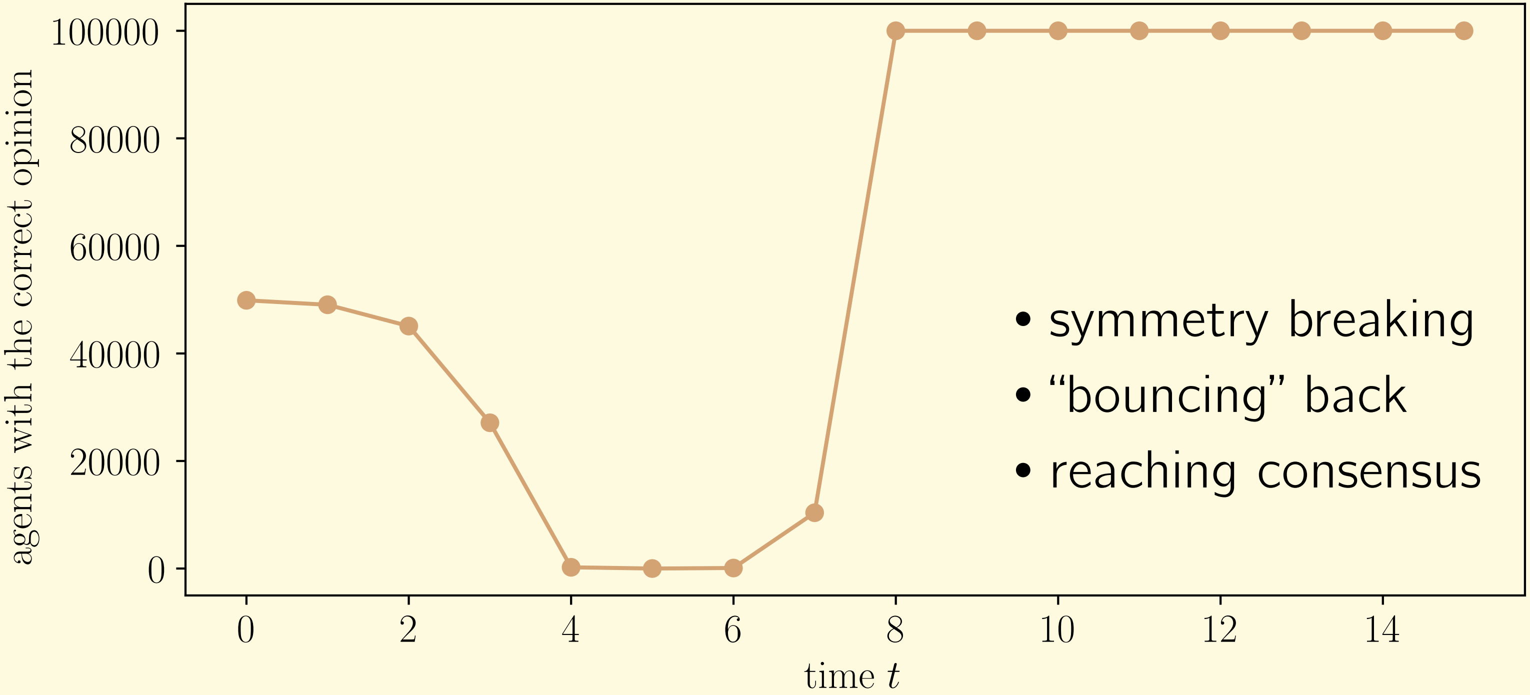

Theorem: Follow the Trend converges in $O(\text{polylog n})$ rounds w.h.p. when the sample size is at least logarithmic in n.

Overview

Parallel	$\ell = O(\log n)$	$\ell = \omega(\log n)$
No Memory		
Memory	Follow the Trend $O(\text{polylog n})$	

Asynchronous	$\ell = O(\log n)$	$\ell = \omega(\log n)$
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Memory	Follow the Trend $O(\text{polylog n})$ [Empirical]	

A Lower-bound in the memoryless, asynchronous setting

Asynchronous

No Memory

Luca Becchetti, Andrea Clementi, Amos Korman, Francesco Pasquale, Luca Trevisan and R.V.
On the Role of Memory in Robust Opinion Dynamics.
IJCAI, 2023

Lower bound in the memoryless, asynchronous setting

- **No Memory:** The process is a Markov chain $(X_t)_t$ on $\Sigma = \{0, \ldots, n\}$

(number of agents with opinion 1)

$$X_t = \quad 0 \qquad 1 \qquad 2 \qquad 3 \qquad \cdots \qquad n-1 \qquad n$$

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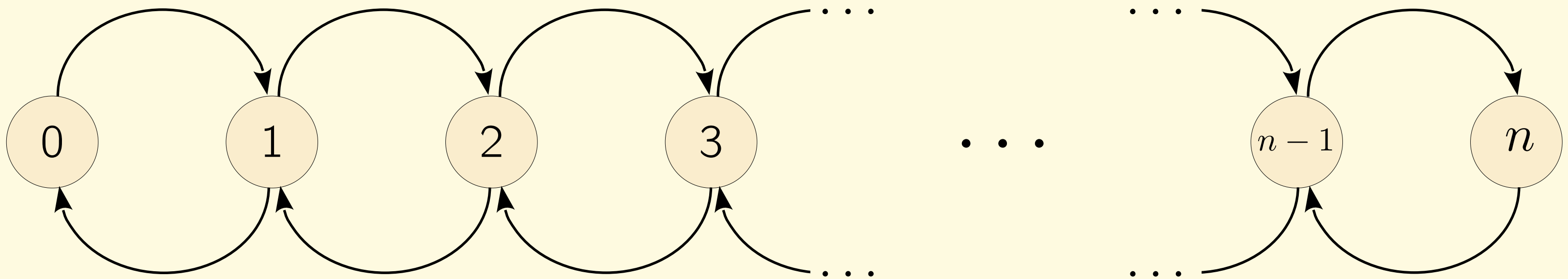

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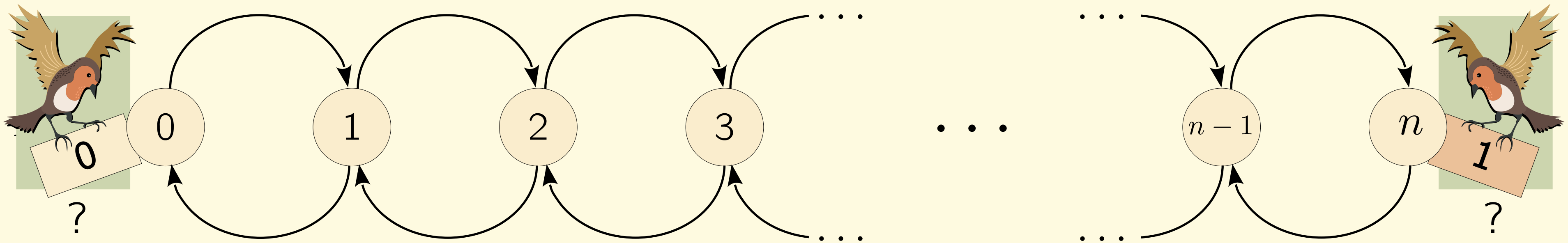

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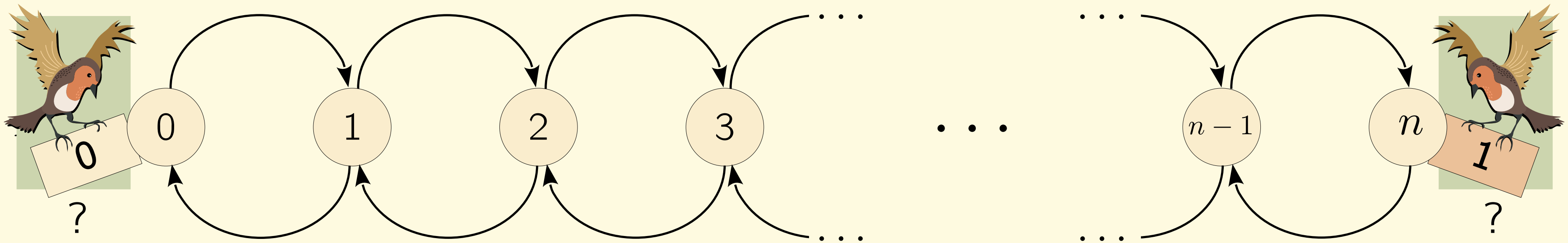

Observation: To solve *self-stabilizing* information spread, a protocol must go “fast” in both directions

Lower bound in the memoryless, asynchronous setting

Theorem: All protocols are slower than the unbiased random walk on Σ.

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Theorem: All protocols are slower than the unbiased random walk on Σ.

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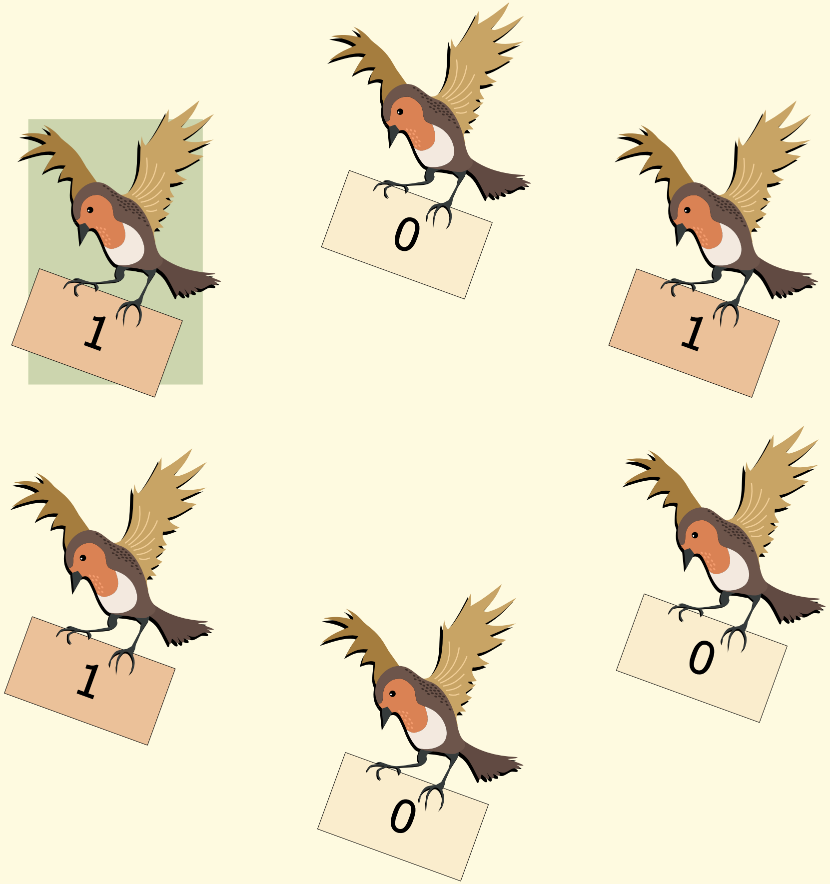
- Holds independently of the sample size ℓ

Lower bound is almost tight

"Voter" dynamics
Sample $\ell = 1$ agent u.a.r.
and copy its opinion



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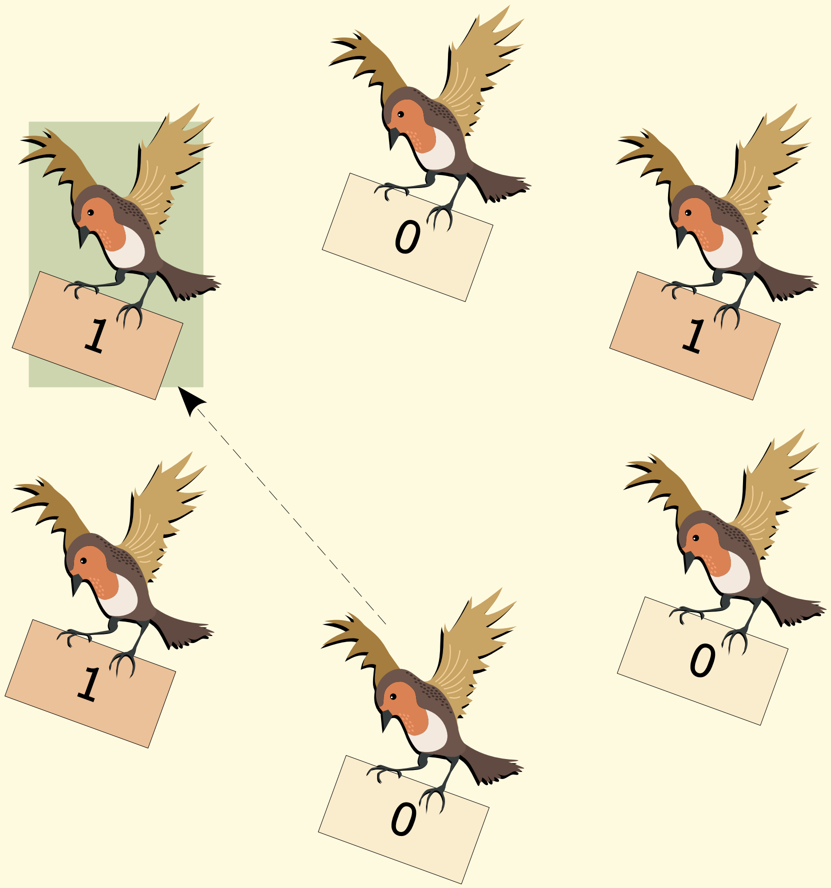

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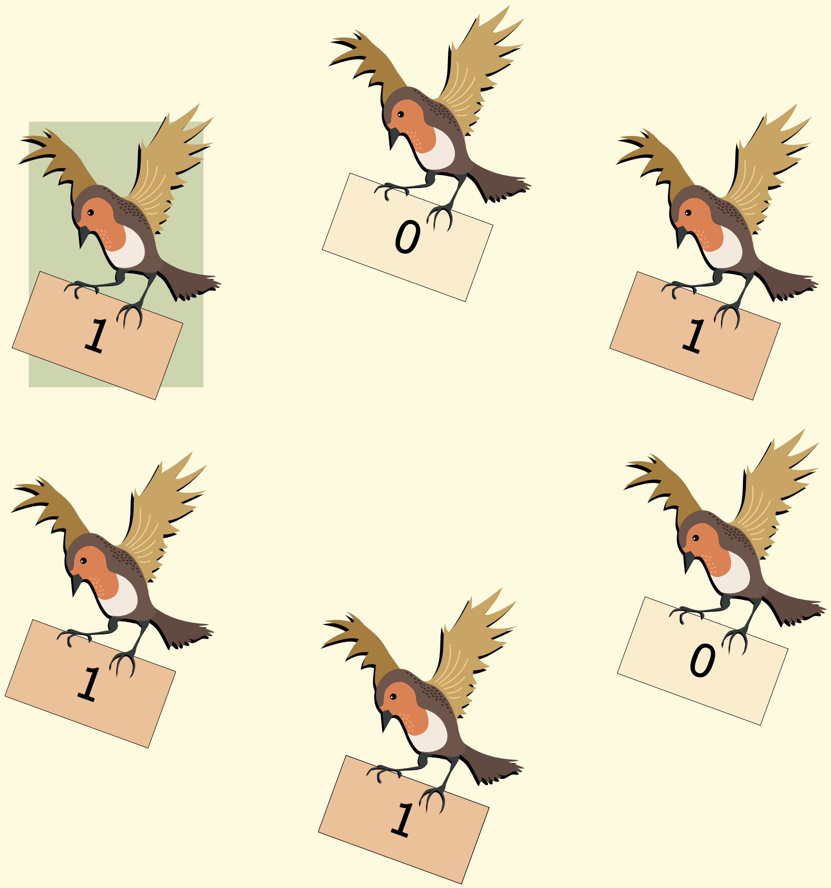

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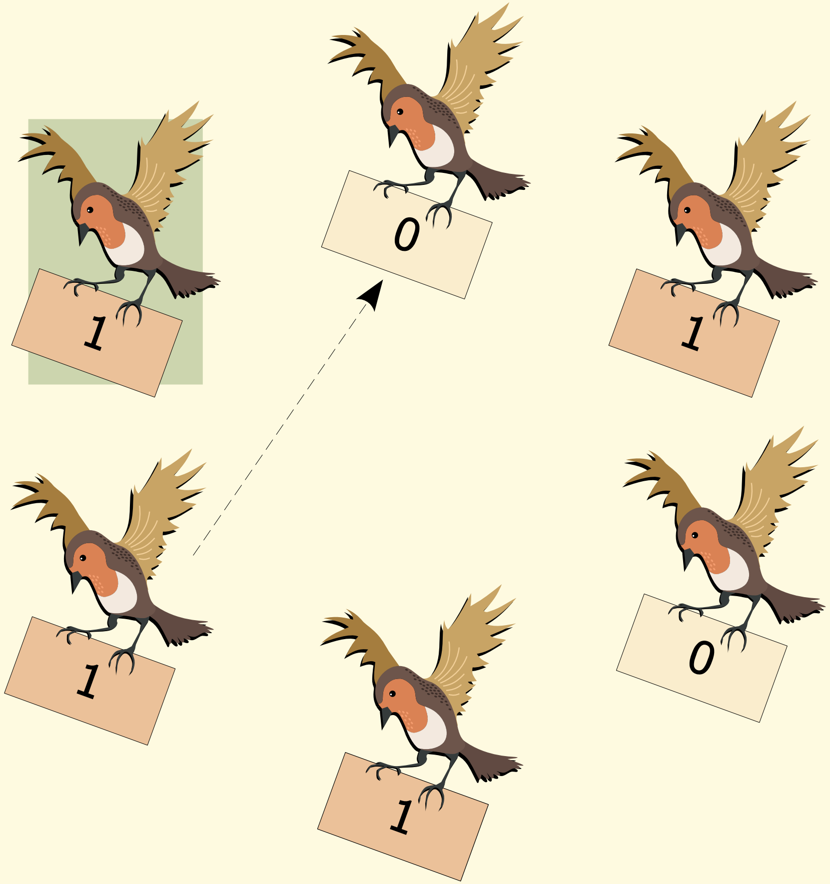

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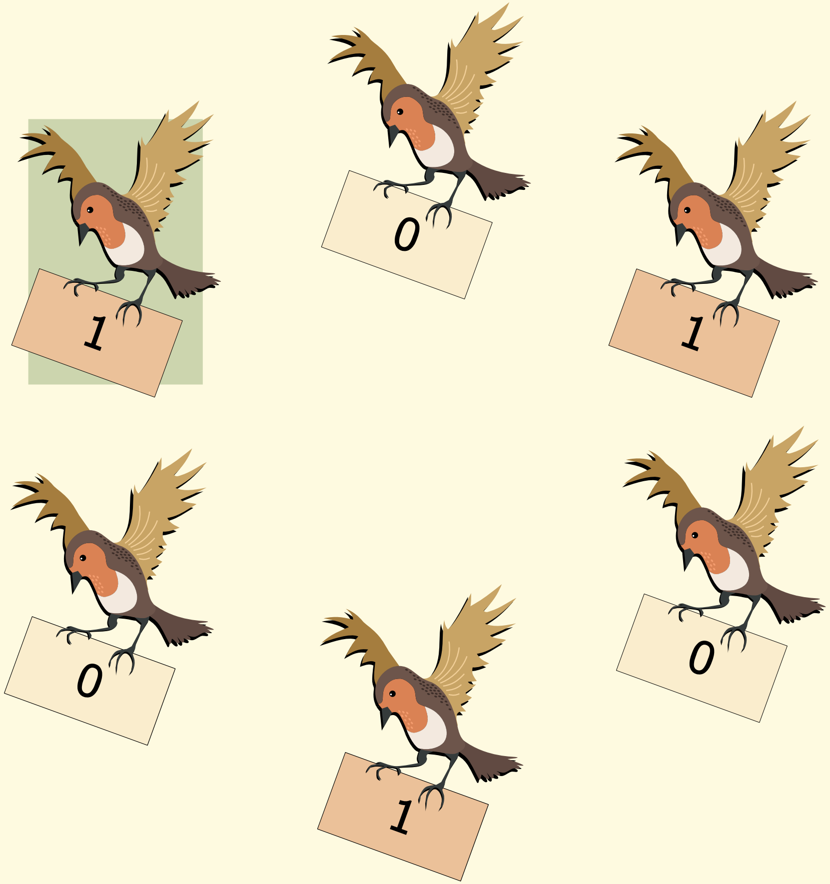

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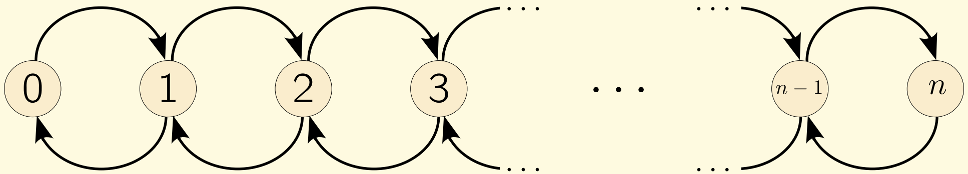
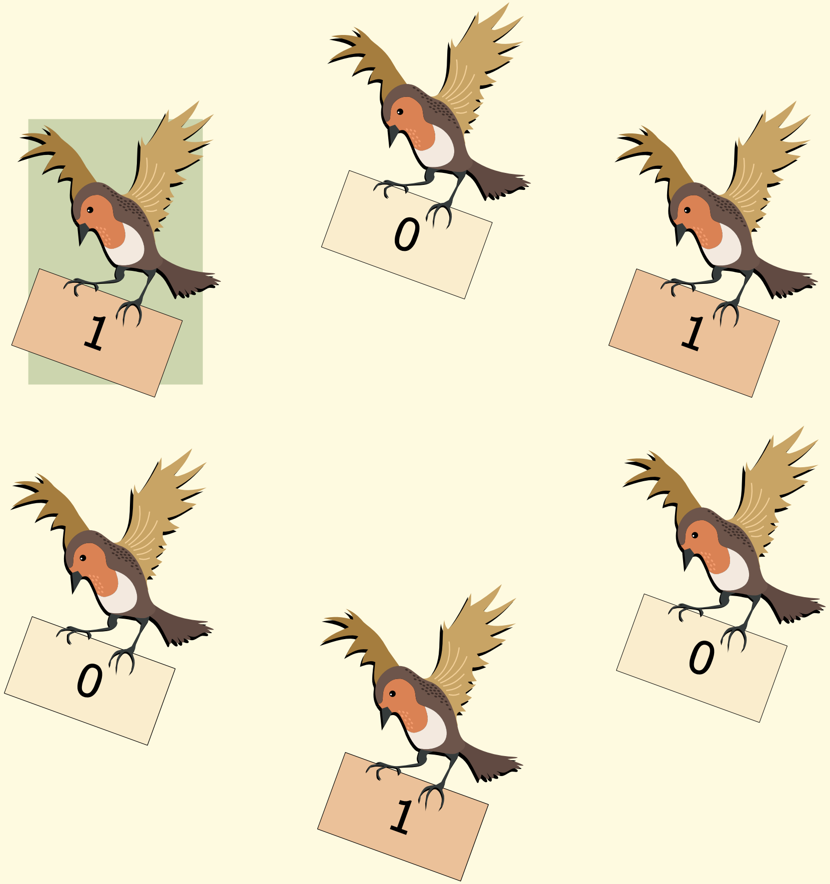


Unbiased protocol
(same probability to add *or* remove
an agent with opinion 1)



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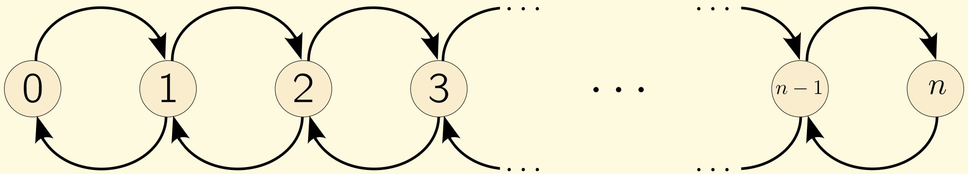

Unbiased protocol
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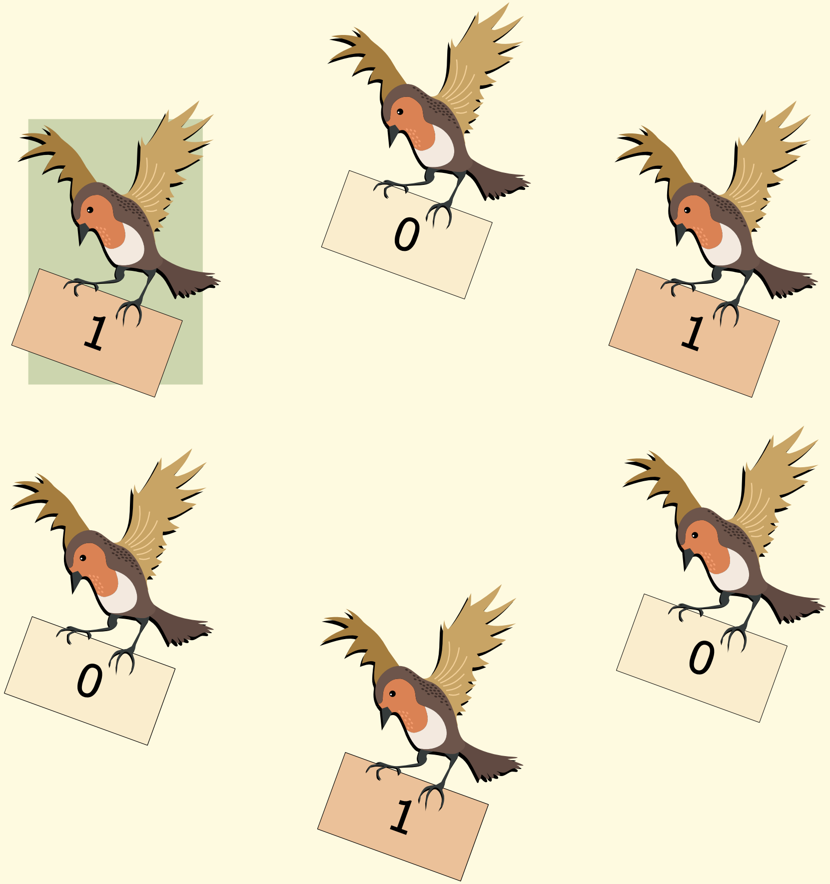
→ Running time $O(n \log n)$ activations per agent



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</table>

Memory matters

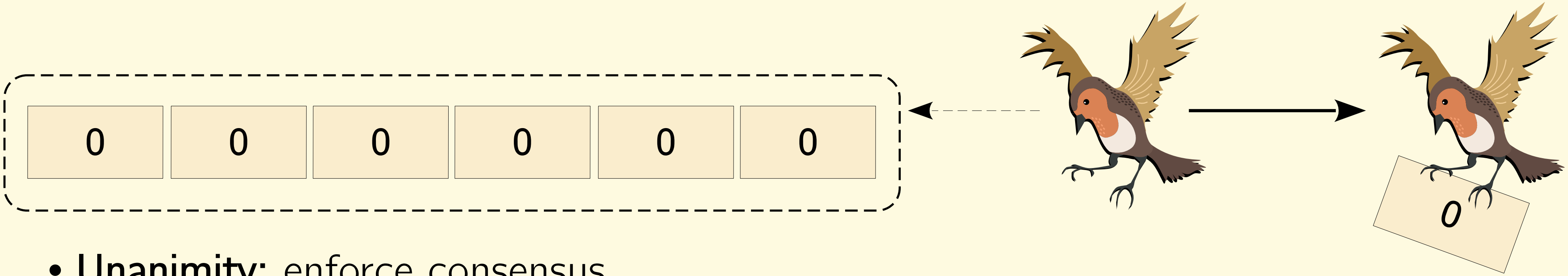
“Minority”

Parallel

No Memory

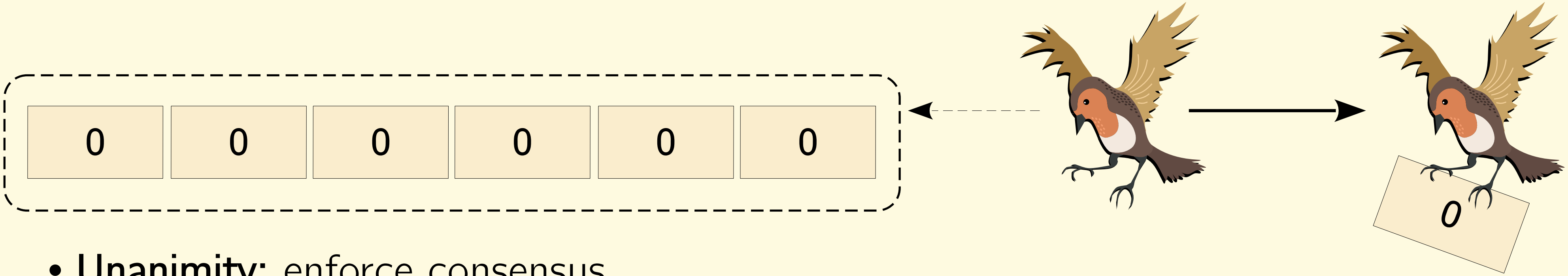
Work in progress with Luca Becchetti, Andrea Clementi, Amos Korman, Francesco Pasquale, Luca Trevisan, R.V. and Isabella Ziccardi

Minority

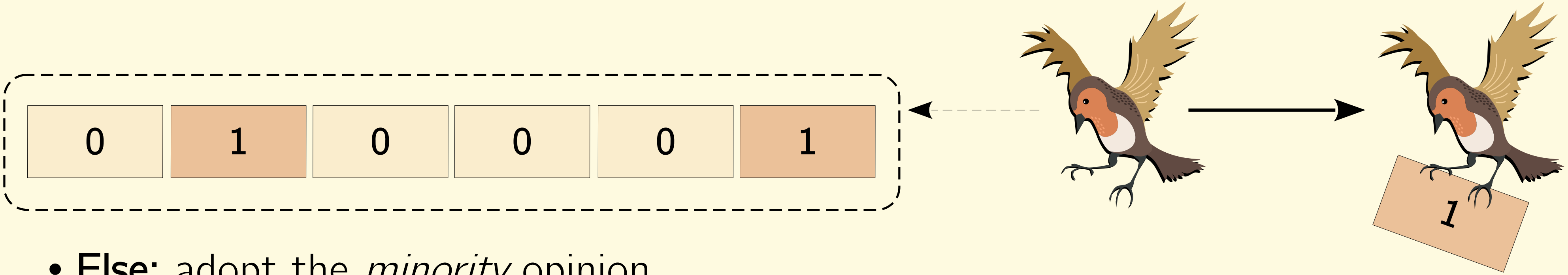


- **Unanimity:** enforce consensus

Minority

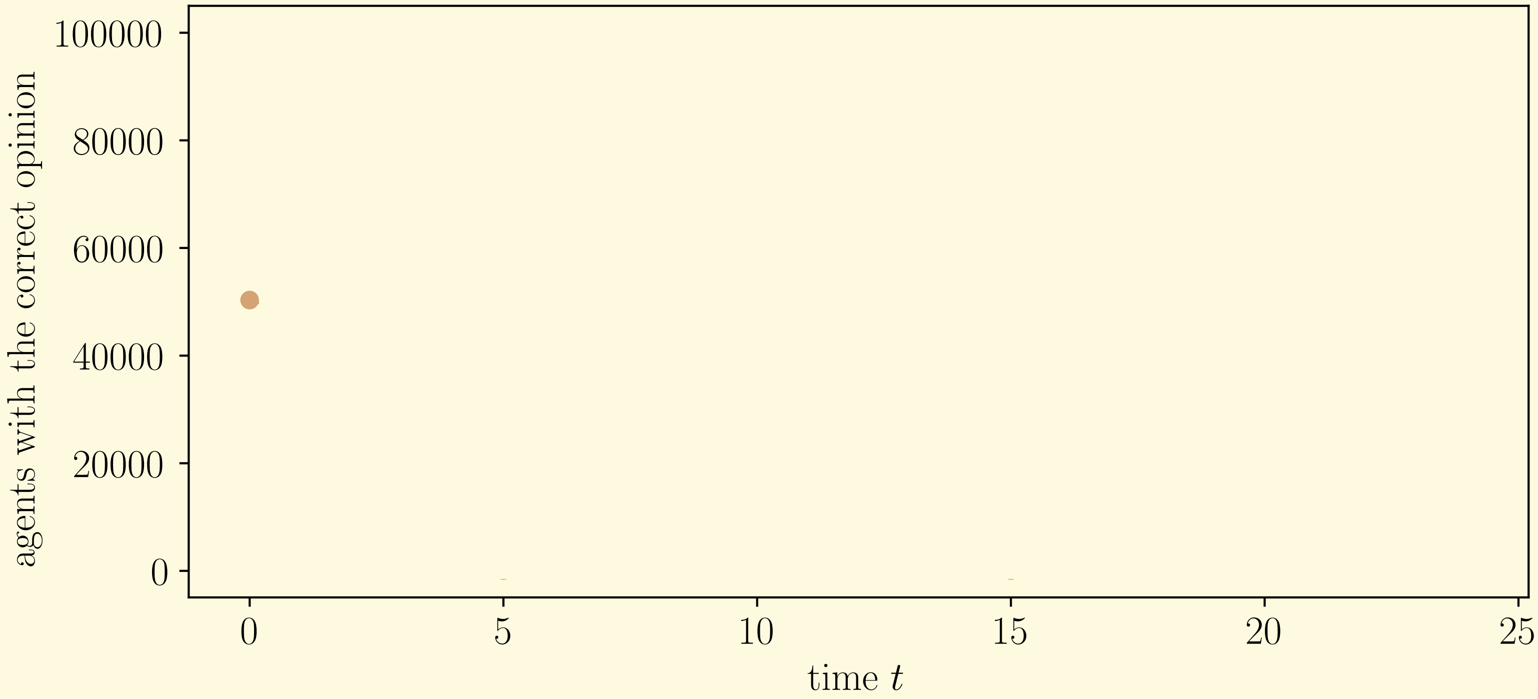


- **Unanimity:** enforce consensus

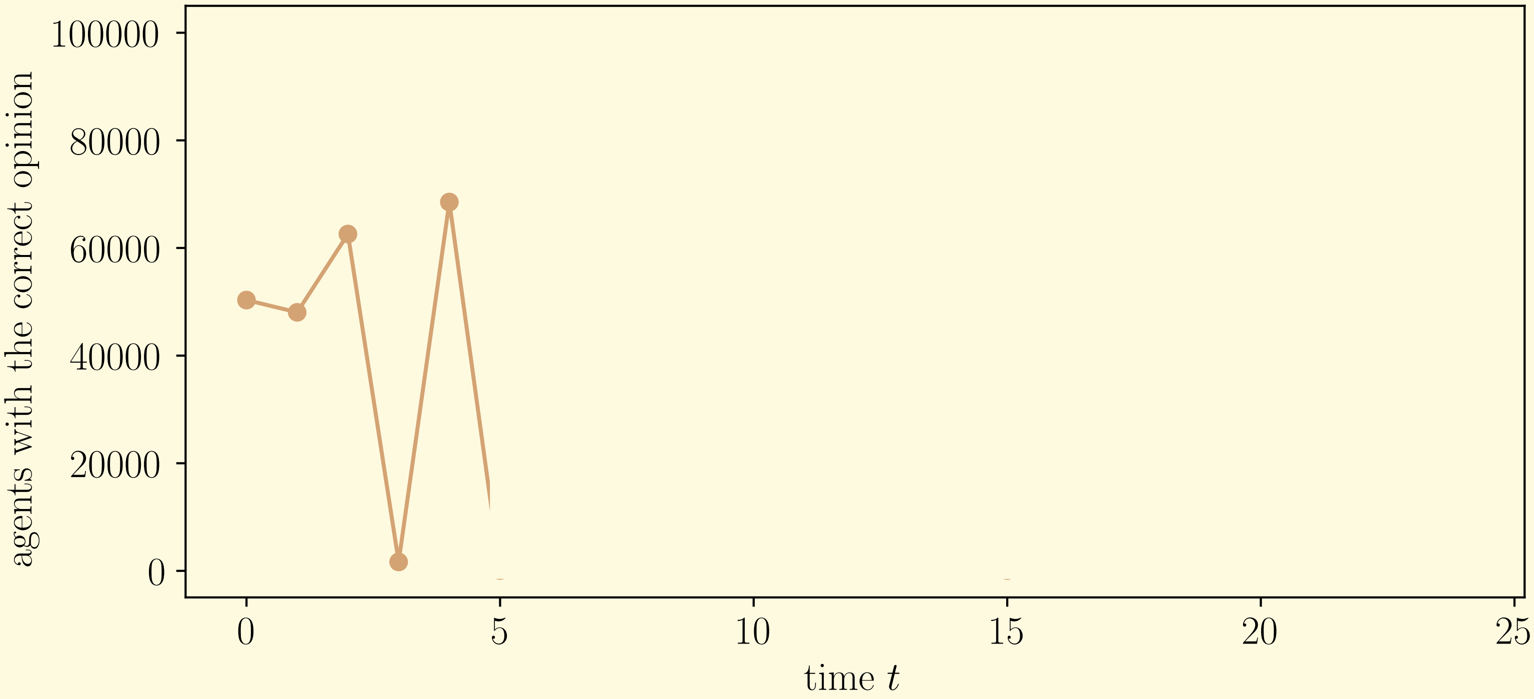


- **Else:** adopt the *minority* opinion

Minority

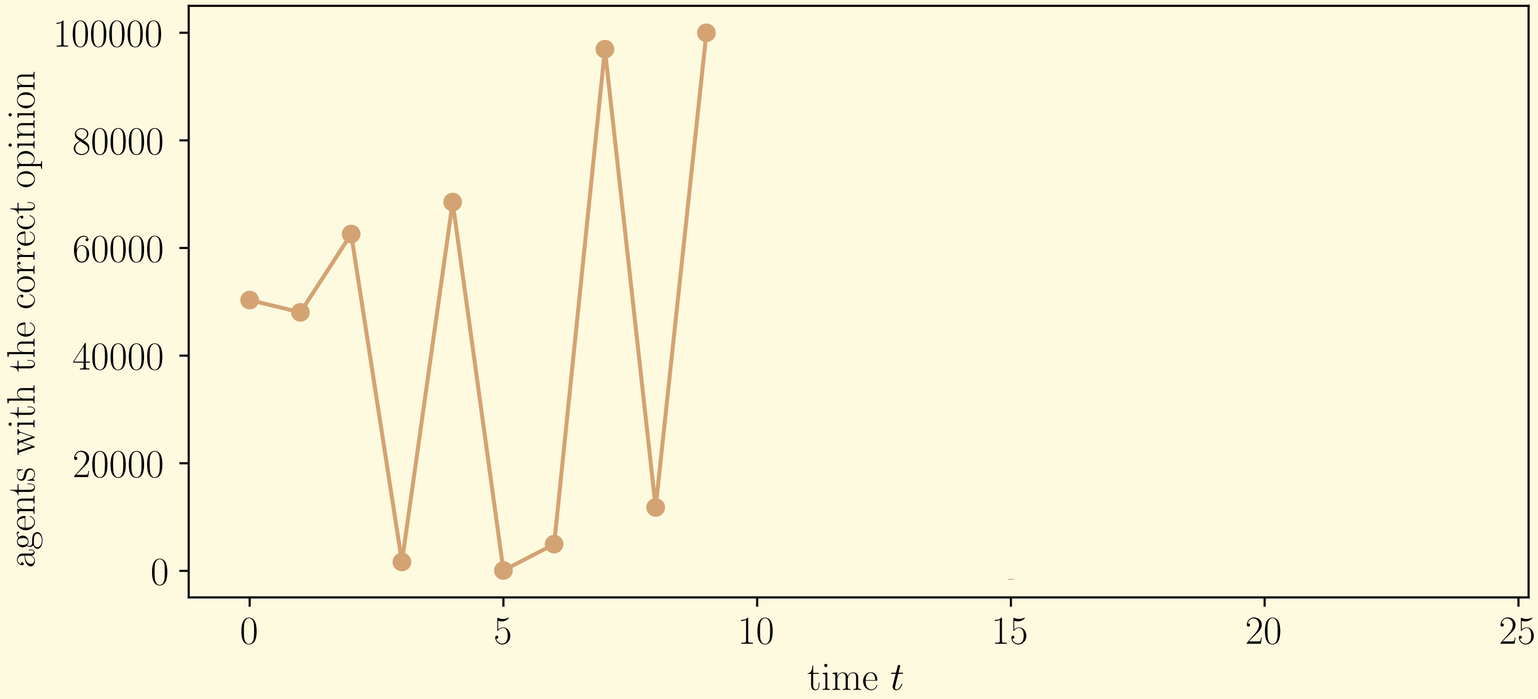

agents with the correct opinion
100000
80000
60000
40000
20000
0
0
5
10
15
20
25
time t

Minority



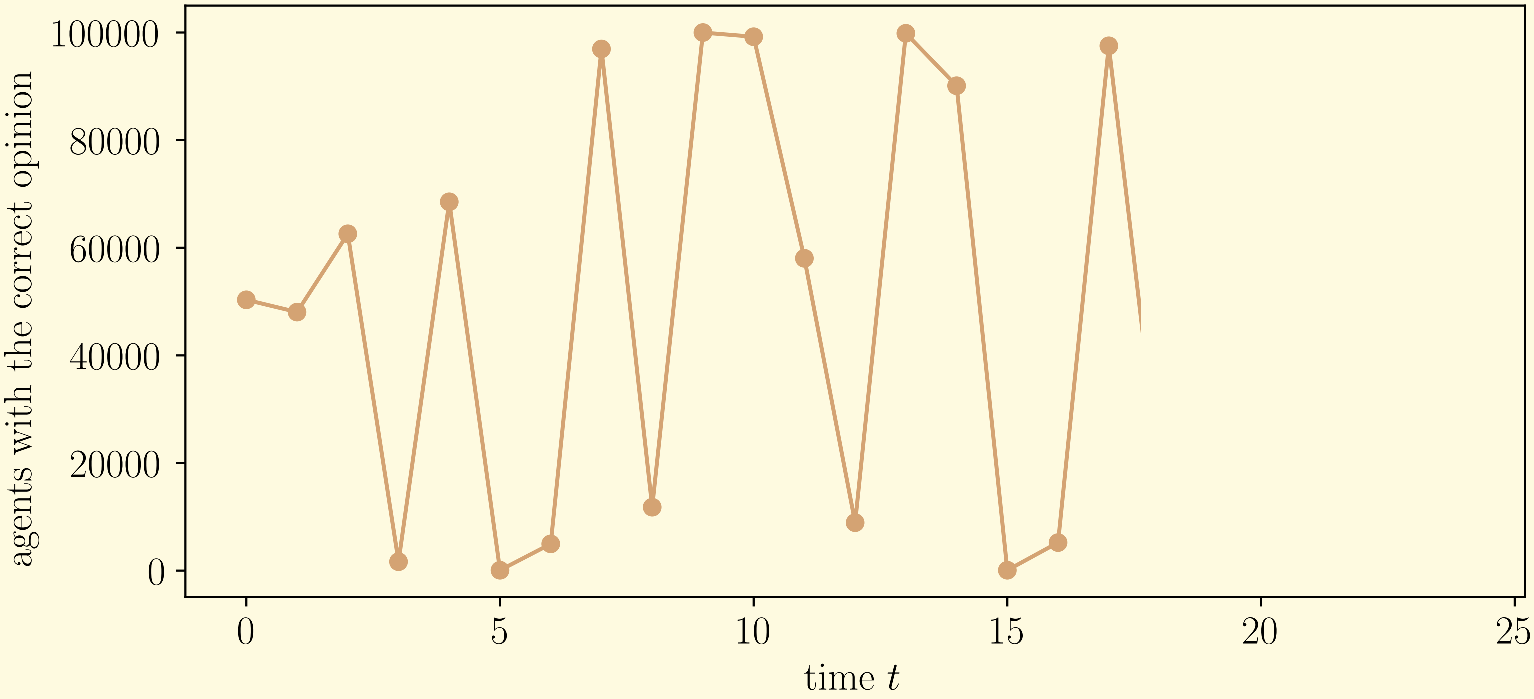
100000
80000
60000
40000
20000
0
agents with the correct opinion
0
5
10
15
20
25
time t

Minority



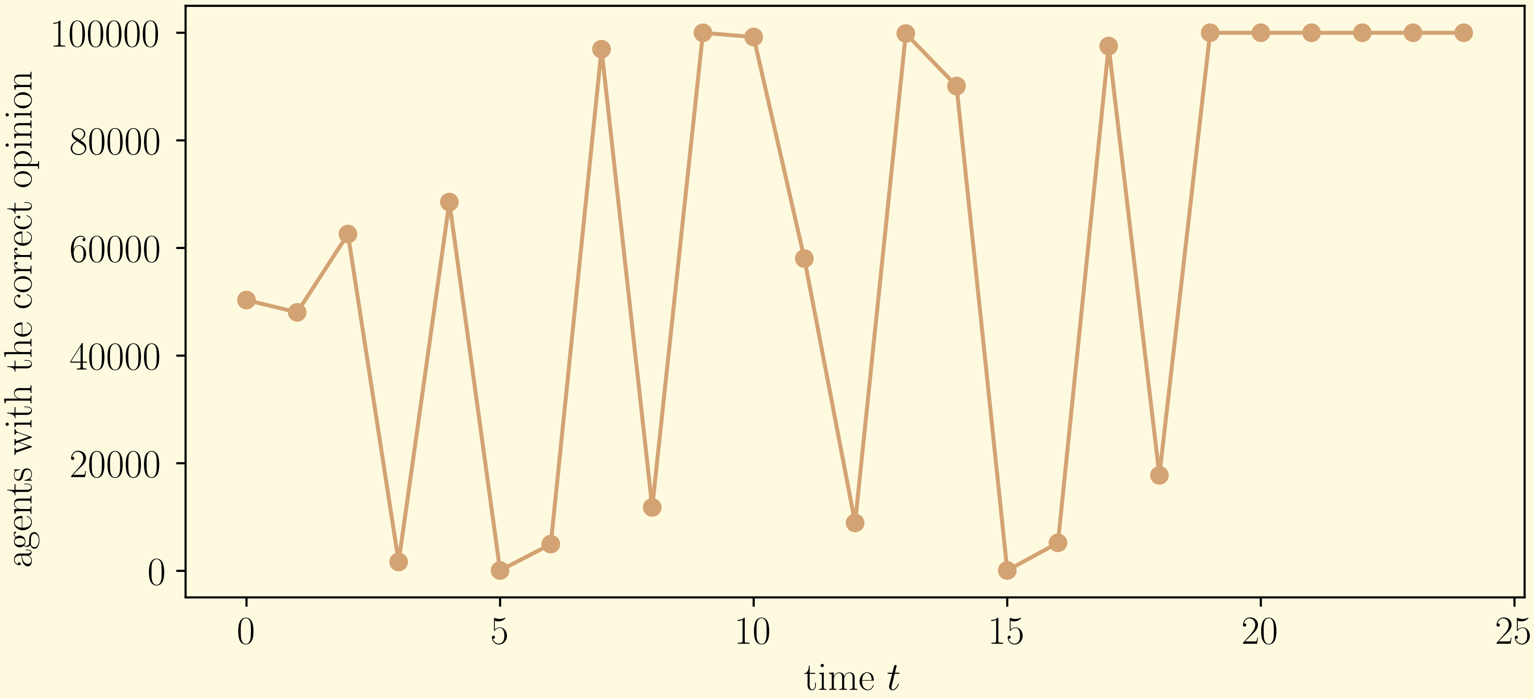
agents with the correct opinion
100000
80000
60000
40000
20000
0
0
5
10
15
20
25
time t

Minority



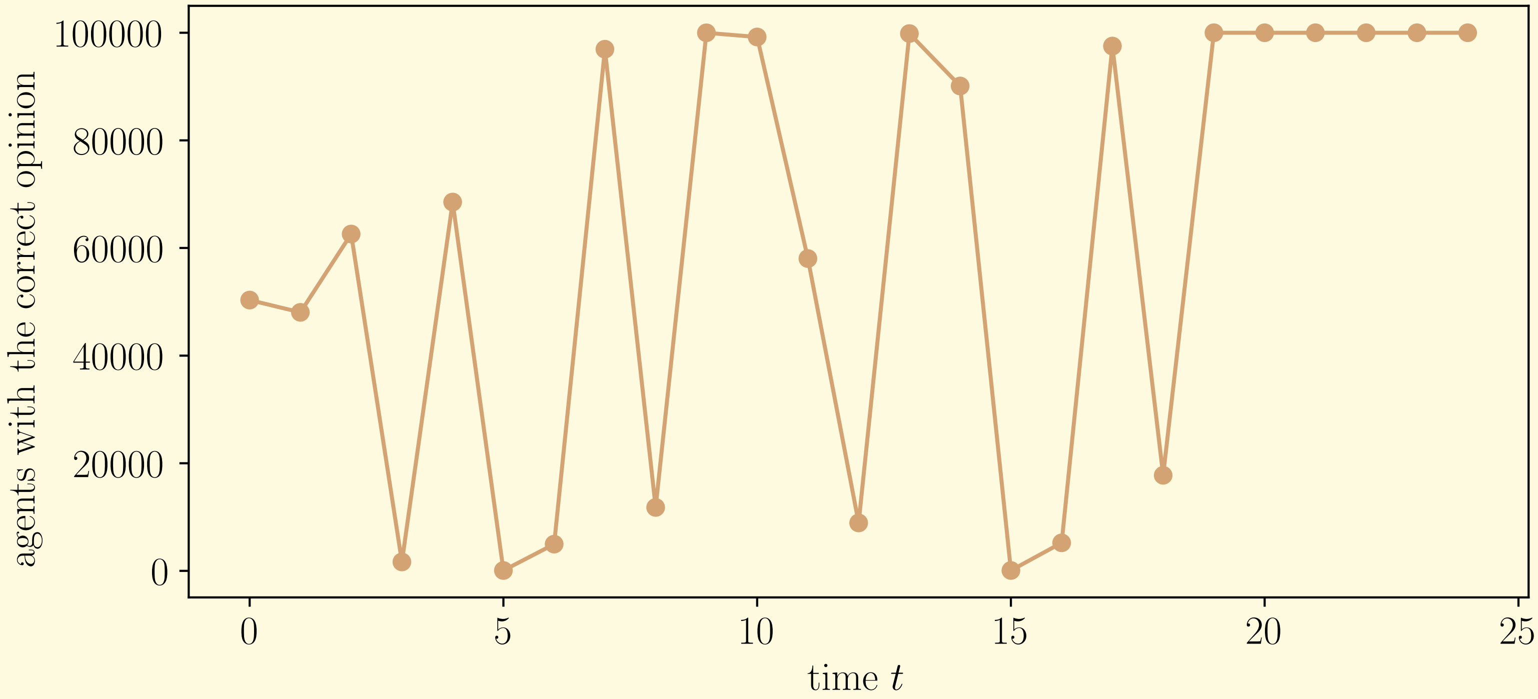
agents with the correct opinion
100000
80000
60000
40000
20000
0
0
5
10
15
20
25
time t

Minority



agents with the correct opinion
100000
80000
60000
40000
20000
0
0
5
10
15
20
25
time t

Minority



Theorem: Minority converges in $O(\text{polylog n})$ rounds w.h.p. when the sample size is at least $\sqrt{n}\log n$.

Overview

Parallel	$\ell = O(\log n)$	$\ell = \omega(\log n)$
No Memory		Minority $O(\text{polylog n})$
Memory	Follow the Trend $O(\text{polylog n})$	

Asynchronous	$\ell = O(\log n)$	$\ell = \omega(\log n)$
No Memory	$\Omega(n)$	
Memory	Follow the Trend $O(\text{polylog n})$	

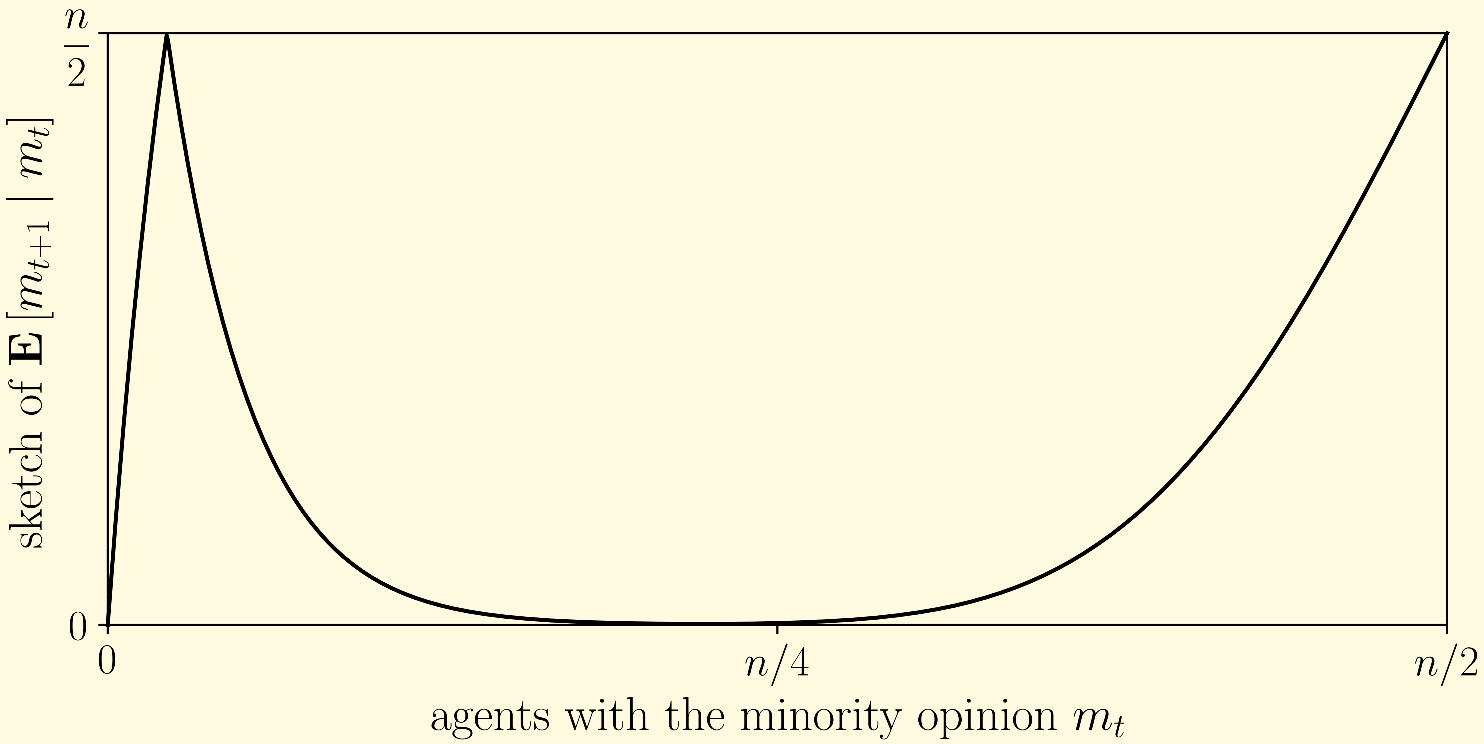
Overview

<table>
<tr><th>Parallel</th><th>$\ell = O(\log n)$</th><th>$\ell = \omega(\log n)$</th></tr>
<tr><td>No Memory</td><td></td><td>Minority
$O(\text{polylog n})$</td></tr>
<tr><td>Memory</td><td colspan="2">Follow the Trend
$O(\text{polylog n})$</td></tr>
</table>

Parallelism matters

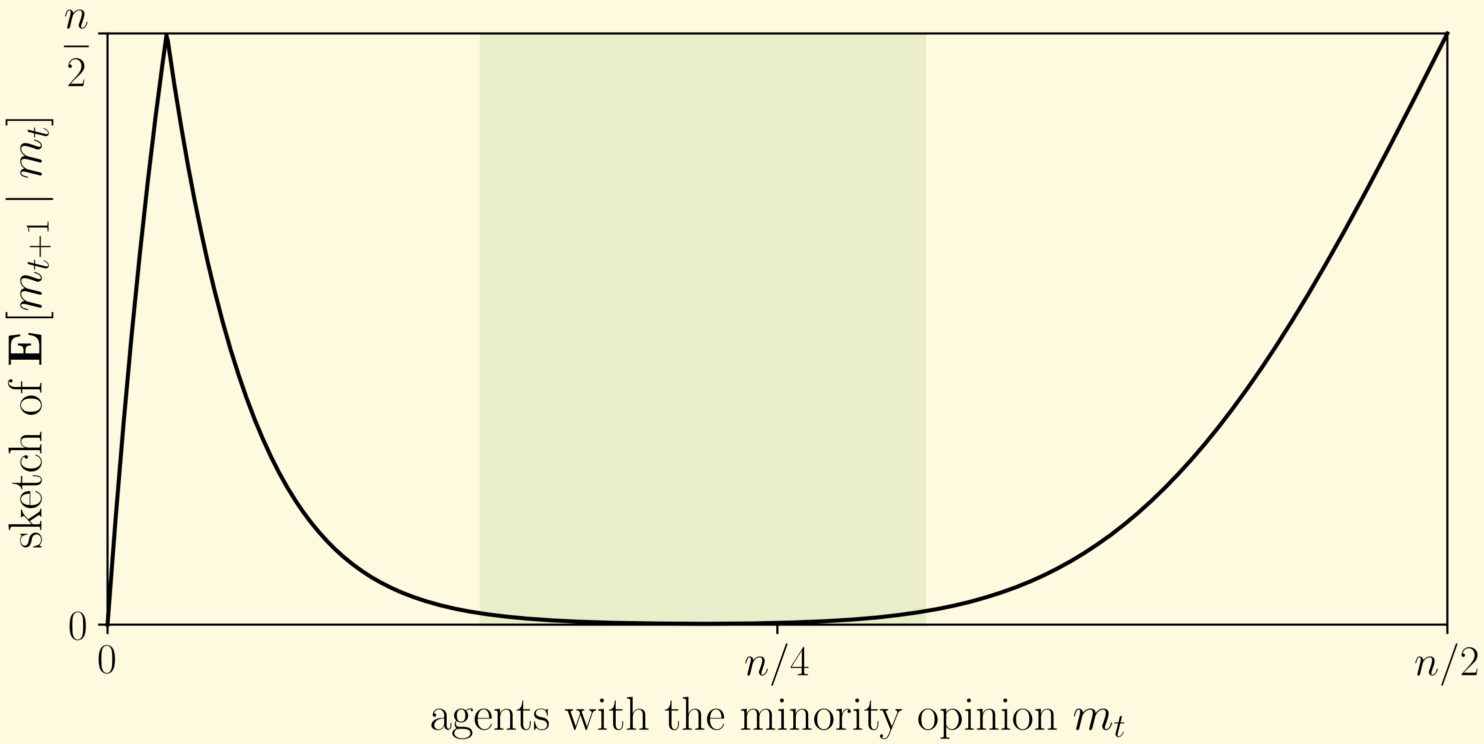
<table>
<tr><th>Asynchronous</th><th>$\ell = O(\log n)$</th><th>$\ell = \omega(\log n)$</th></tr>
<tr><td>No Memory</td><td colspan="2">$\Omega(n)$</td></tr>
<tr><td>Memory</td><td colspan="2">Follow the Trend
$O(\text{polylog n})$</td></tr>
</table>

Minority

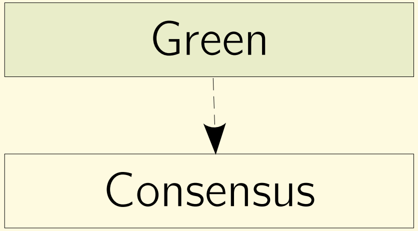


$\frac{n}{2}$
0
sketch of $\mathbf{E}[m_{t+1} \mid m_t]$
0
$n/4$
$n/2$
agents with the minority opinion m_t

Minority

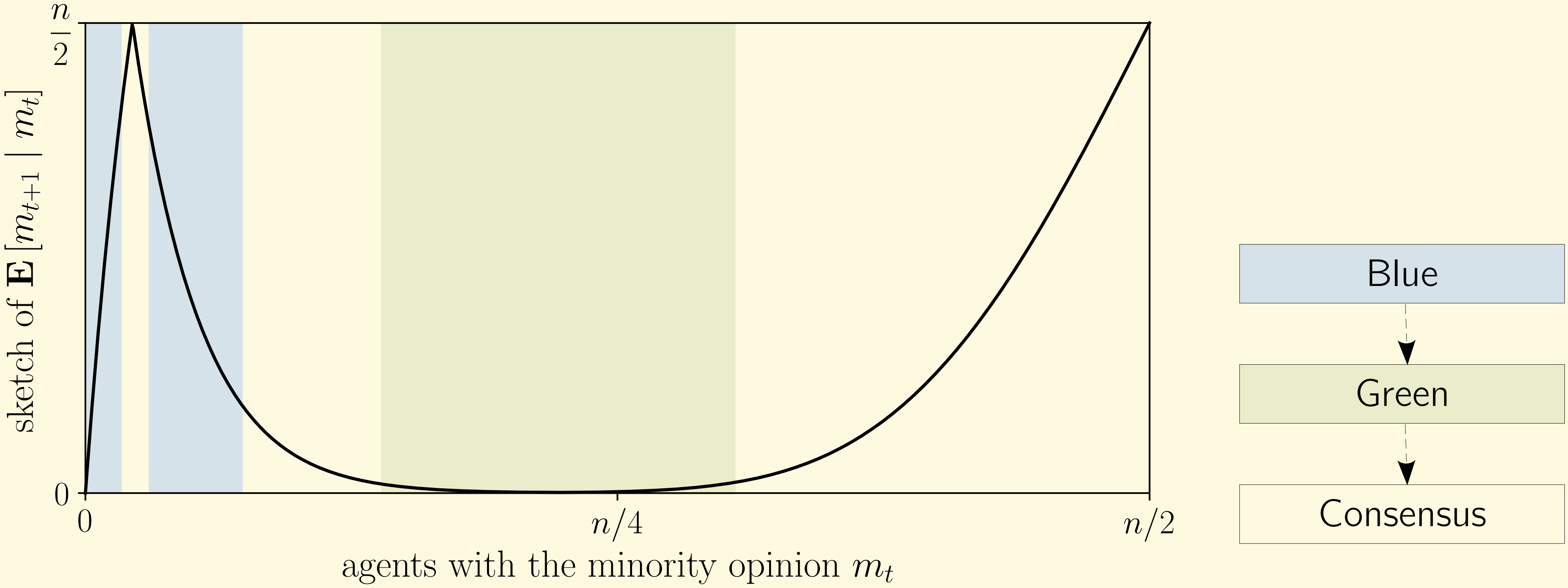


$\frac{n}{2}$
0
0
$n/4$
$n/2$
sketch of $\mathbf{E}[m_{t+1} \mid m_t]$
agents with the minority opinion m_t



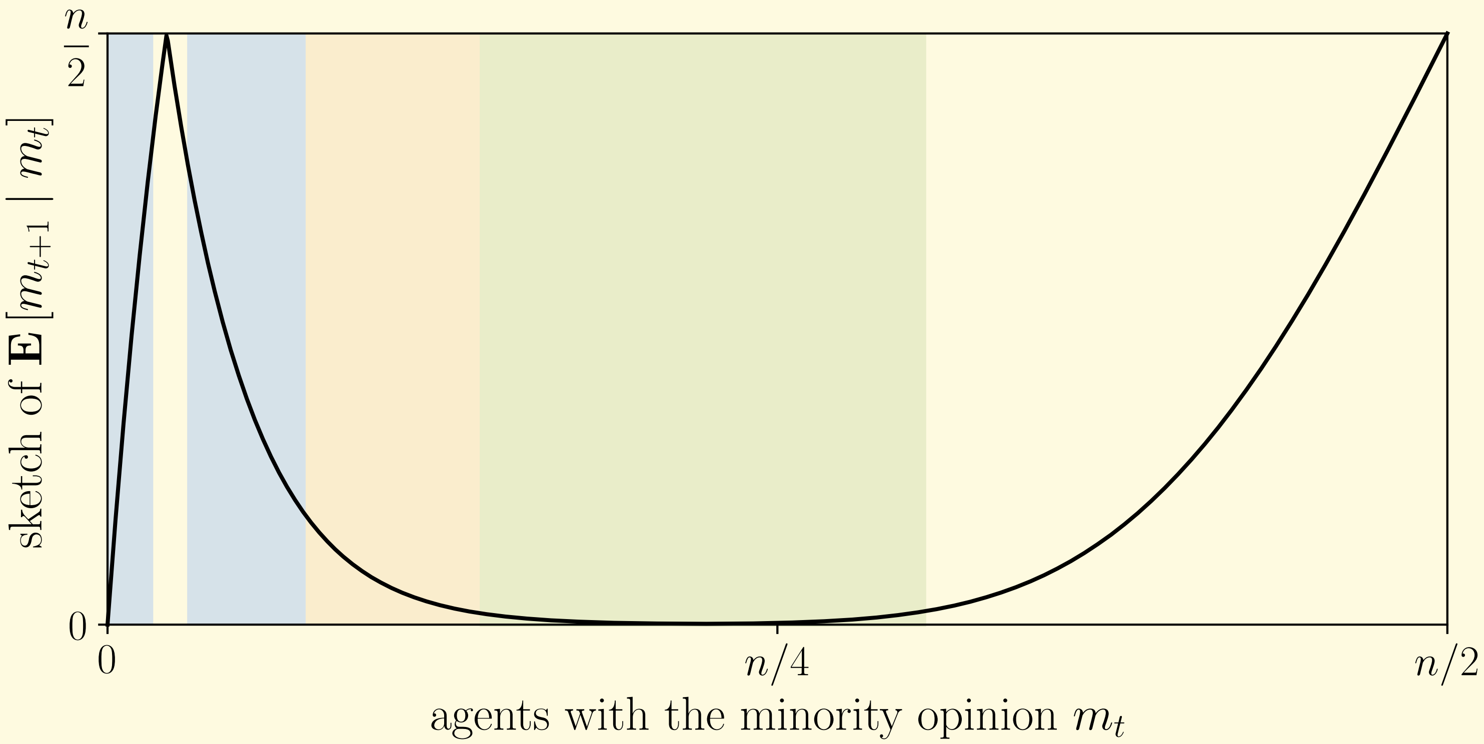
Green
Consensus

Minority

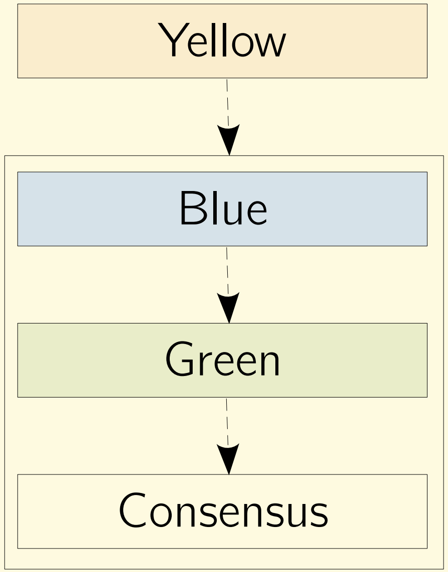


$\frac{n}{2}$
0
sketch of $\mathbf{E}[m_{t+1} \mid m_t]$
0
$n/4$
$n/2$
agents with the minority opinion m_t
Blue
Green
Consensus

Minority

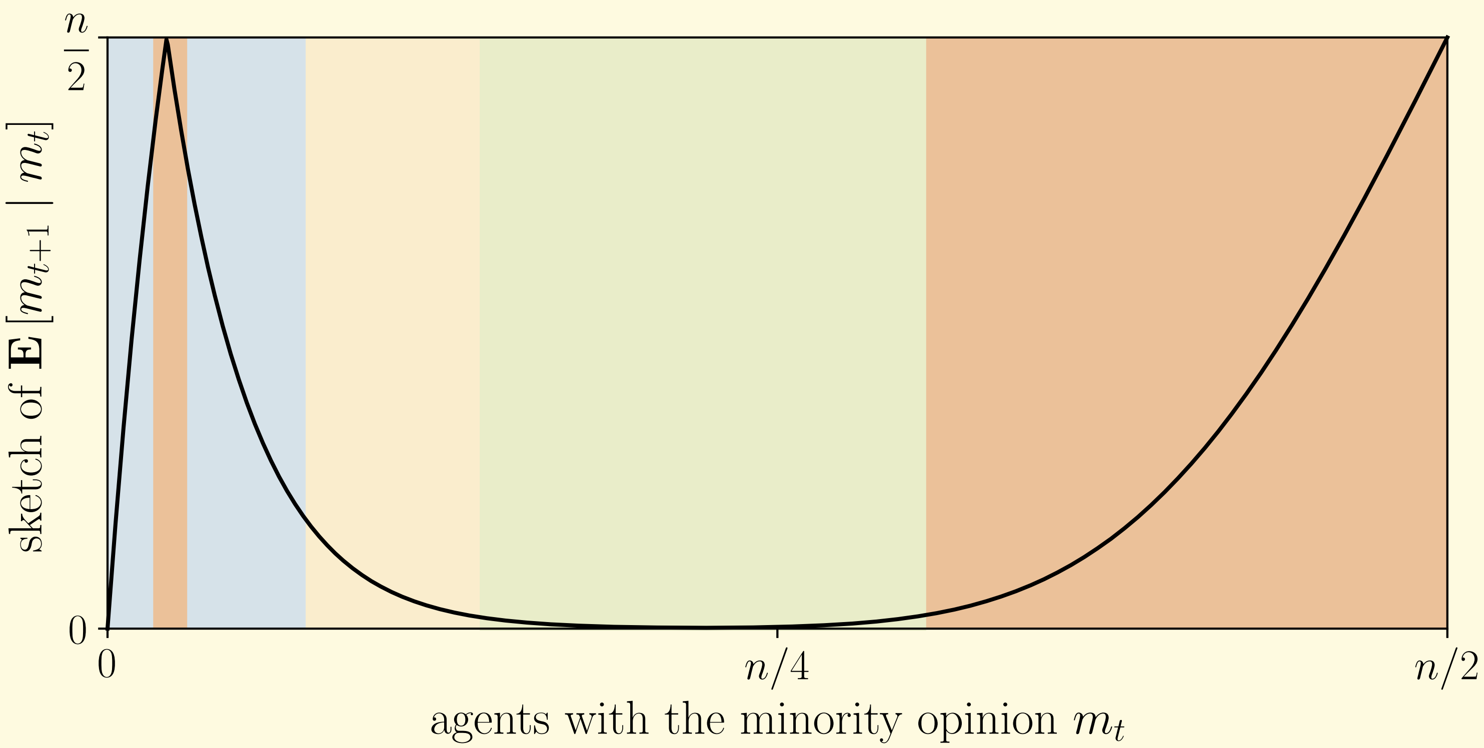


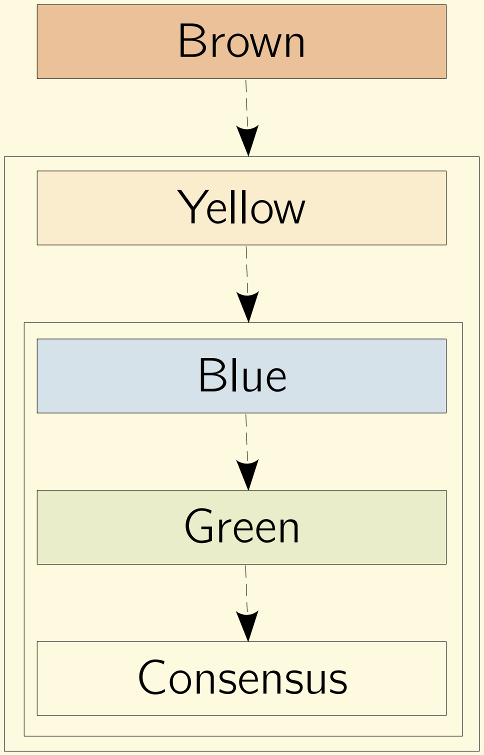
$\frac{n}{2}$
sketch of $\mathbf{E}[m_{t+1} \mid m_t]$
0
0
$n/4$
$n/2$
agents with the minority opinion m_t

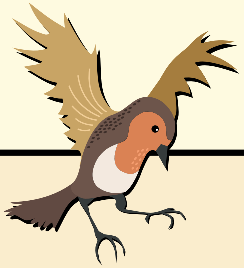


Yellow
Blue
Green
Consensus

Minority


$\frac{n}{2}$
0
sketch of $\mathbf{E}[m_{t+1} \mid m_t]$
0
$n/4$
$n/2$
agents with the minority opinion m_t


Brown
Yellow
Blue
Green
Consensus



Thank you!