Planning with the Assembly Calculus

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COATI

Inria
INVENTEURS DU MONDE NUMÉRIQUE

Joint work with

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Columbia University

Emanuele Natale
COATI, CNRS

Christos Papadimitriou
Columbia University

COATI'S SEMINAR
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Brain & Mind

How does the brain beget the mind?

How do the intelligence, reasoning, languange (etc.) emerge from neurons and synapses?

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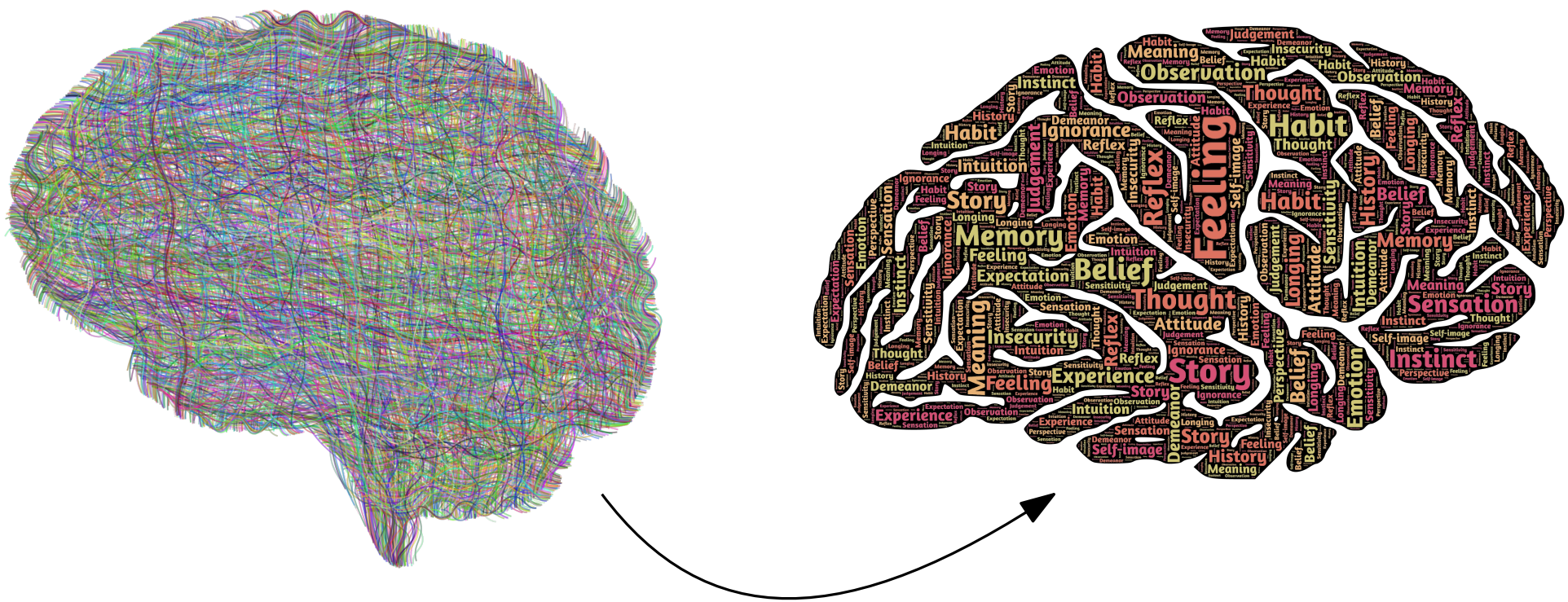

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Difficulty: huge gap of scale and methodology between Experimental Neuroscience and Cognitive Science

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Nobel laureate Richard Axel: *We do not have a* **logic** *for the transformation of neural activity to thought and action. I consider discerning [this* **logic***] as the most important future direction in Neuroscience* [Axel, Neuron 2018]

The Assembly Calculus

Recently proposed formal computational system [Papadimitriou et al., PNAS 2020]

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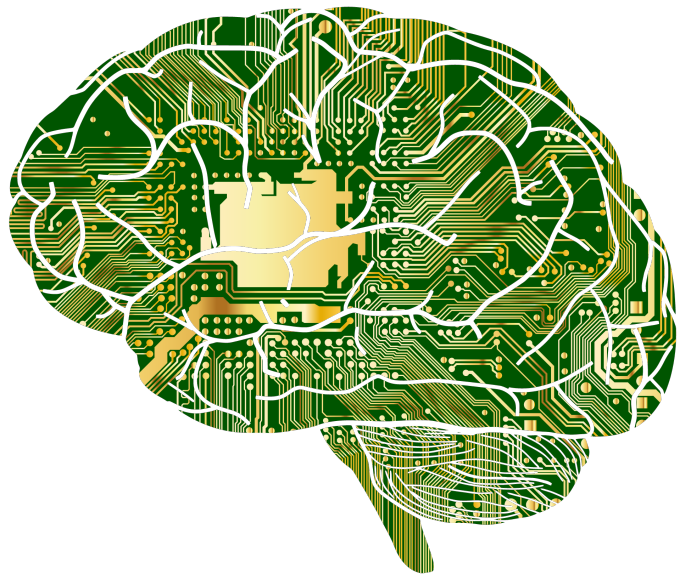

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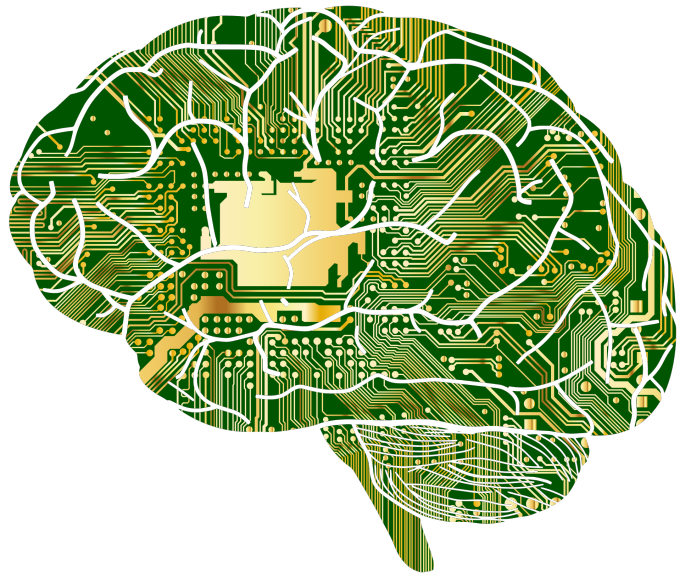
Components:

- *assemblies*
- *operations*



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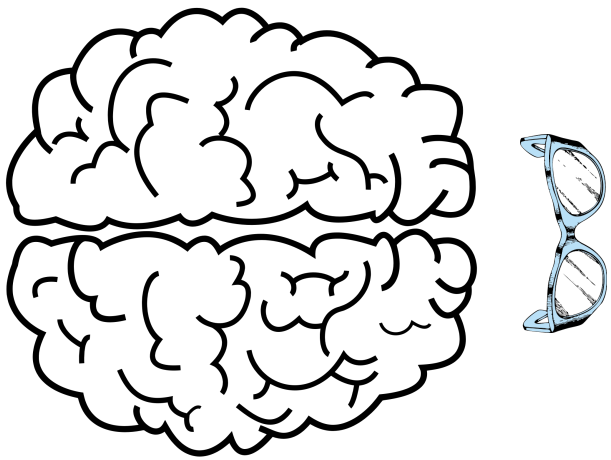

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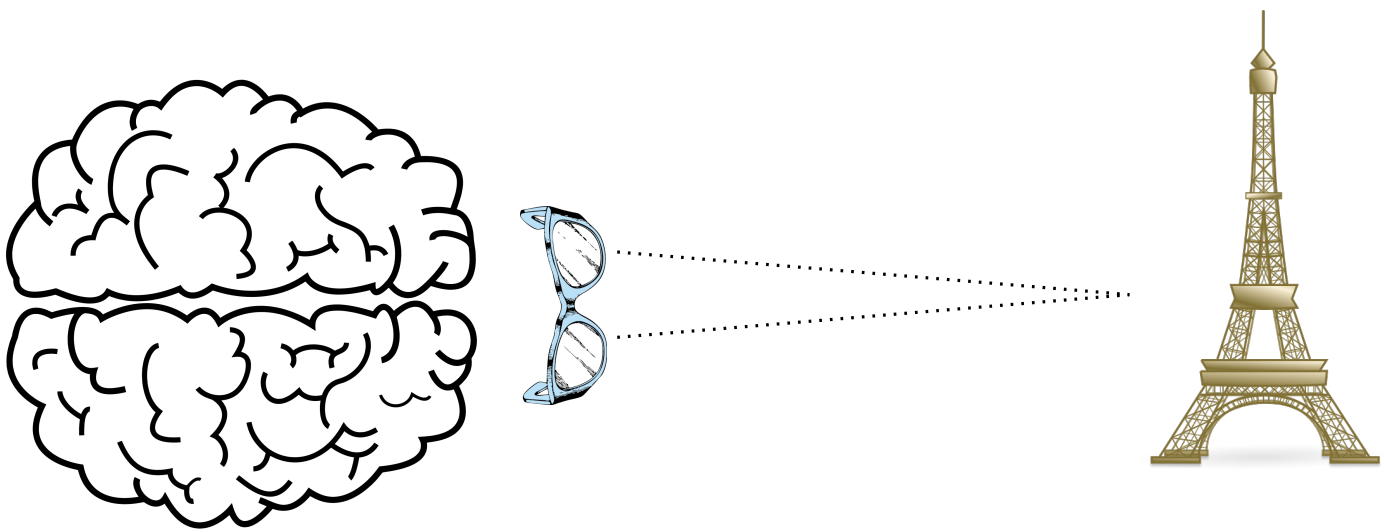

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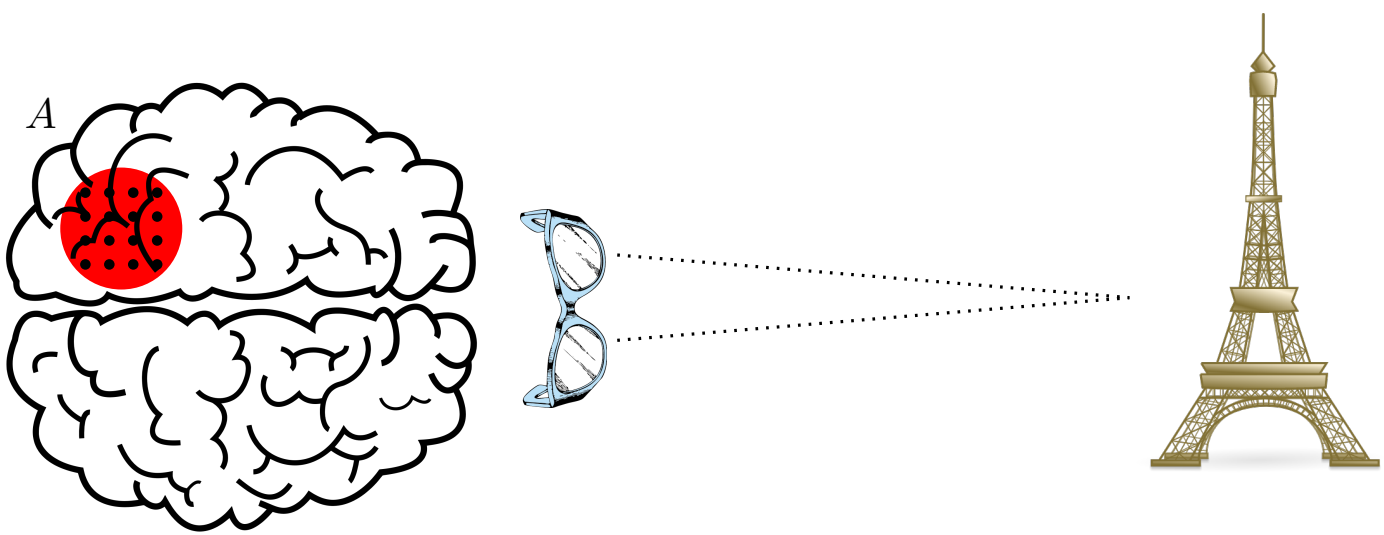

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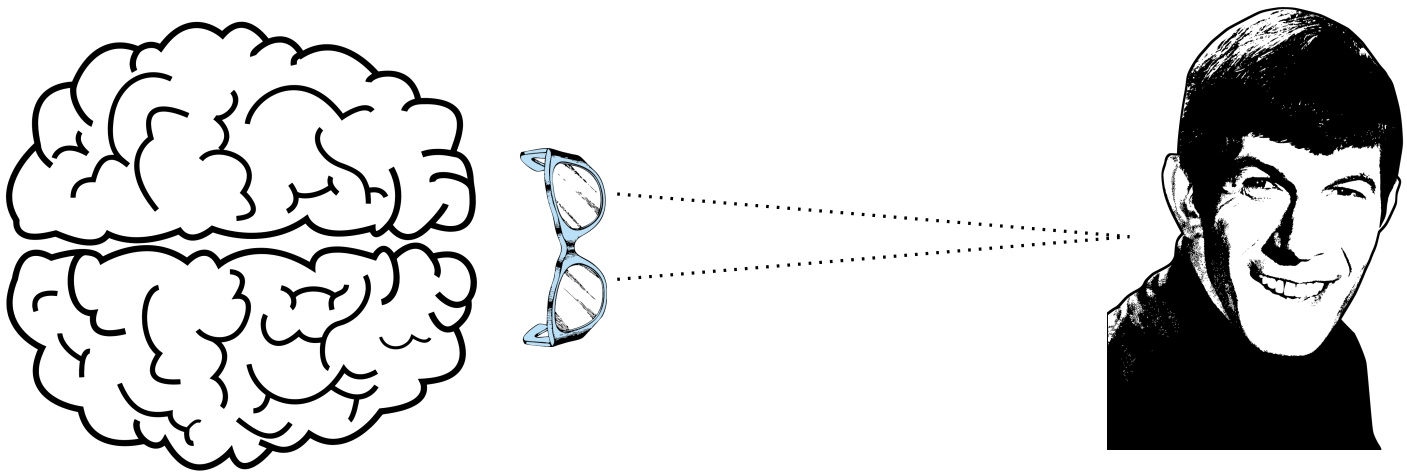

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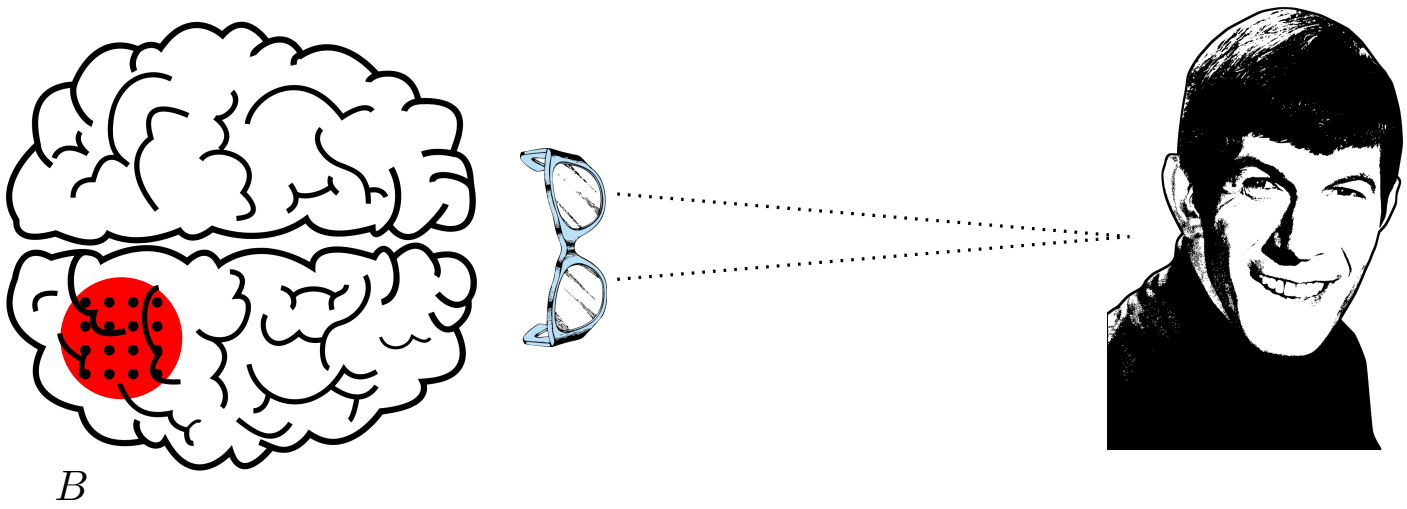
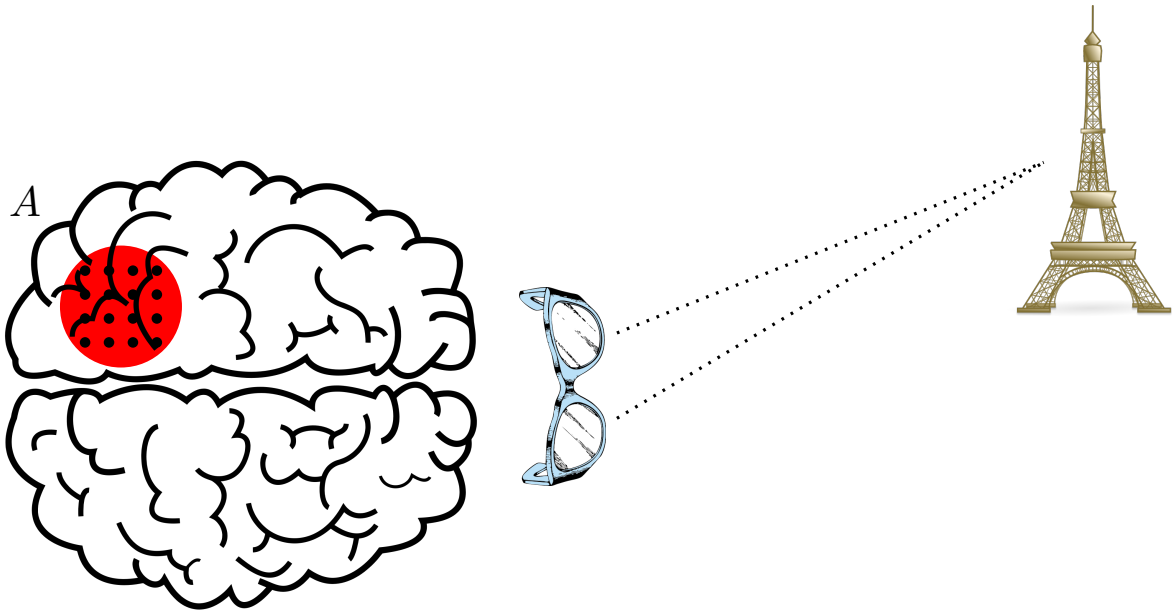


Operations with Assemblies

Experiment by [Ison et al., Neuron 2015]



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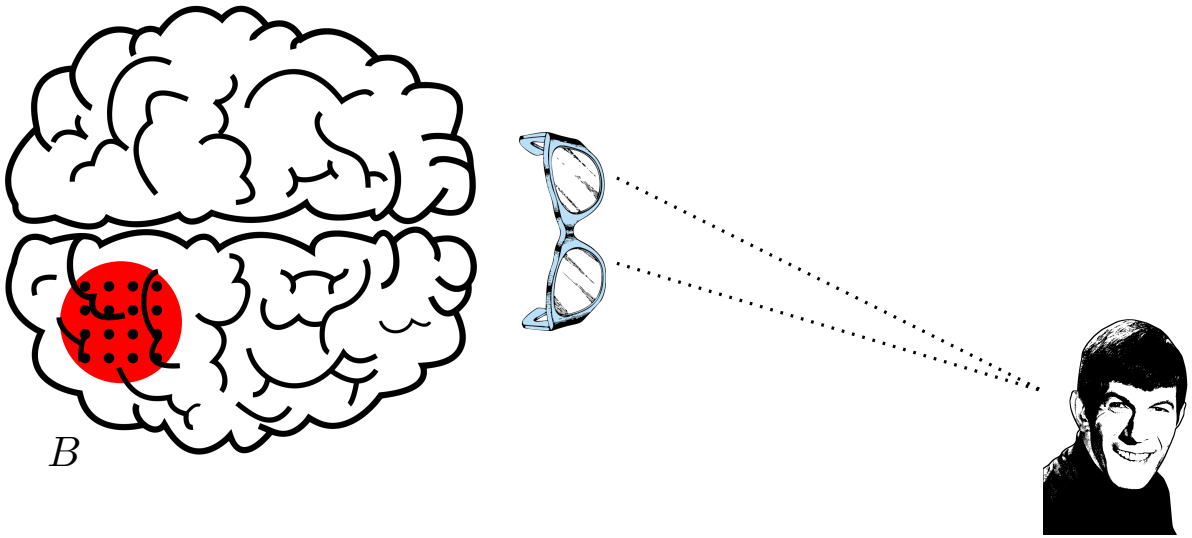

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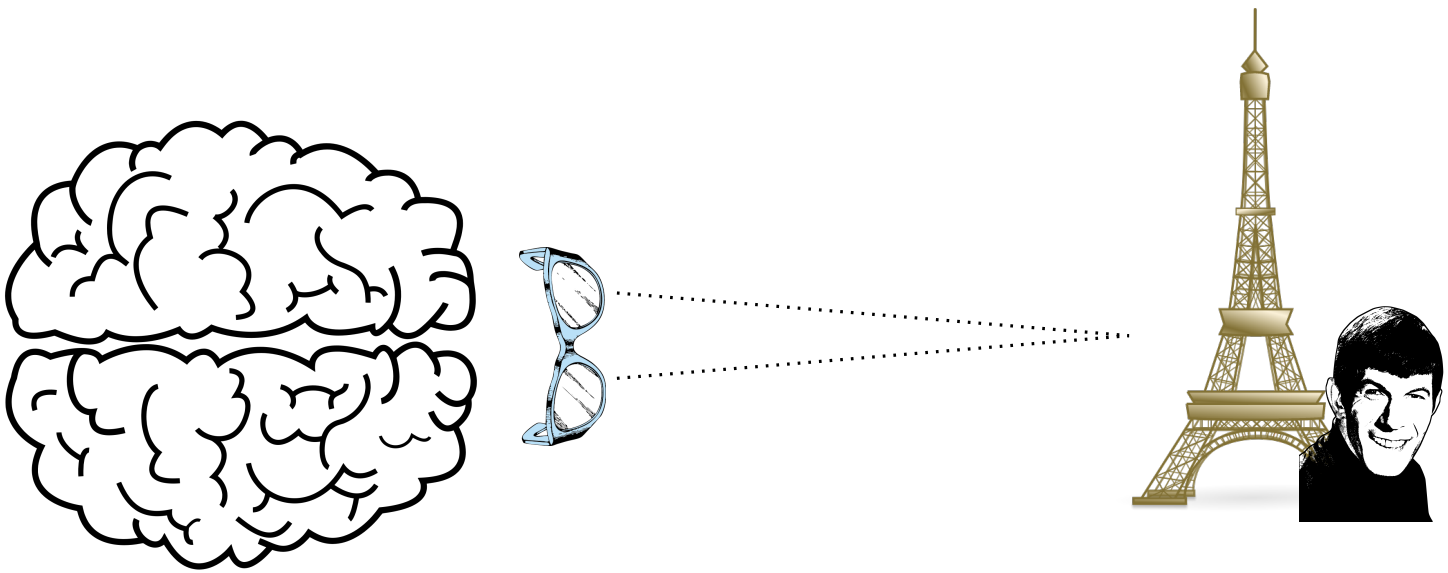

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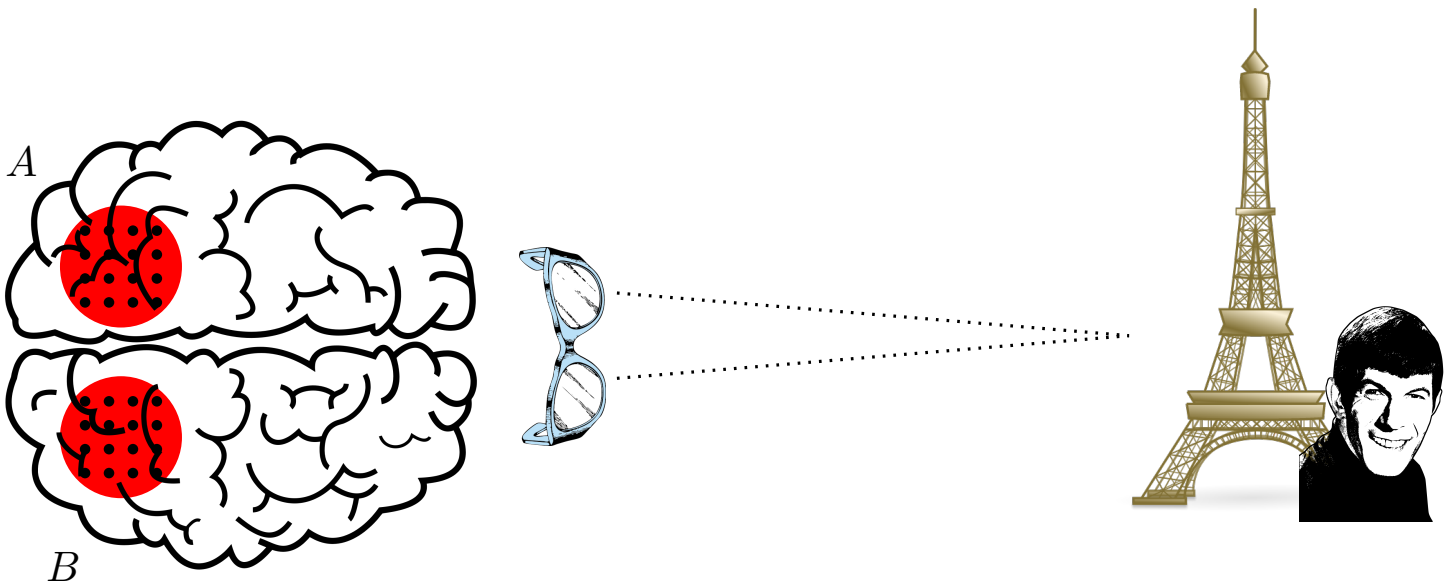

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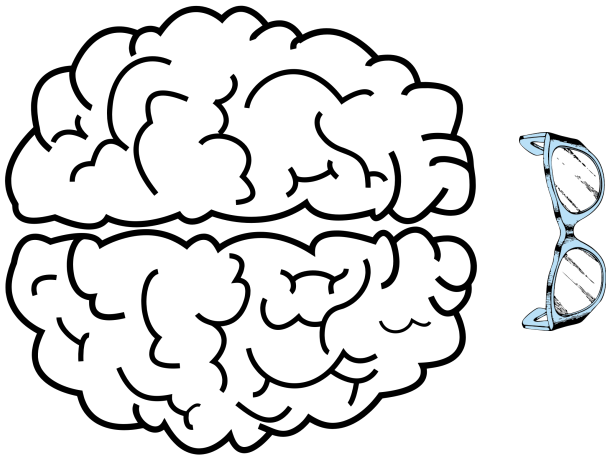

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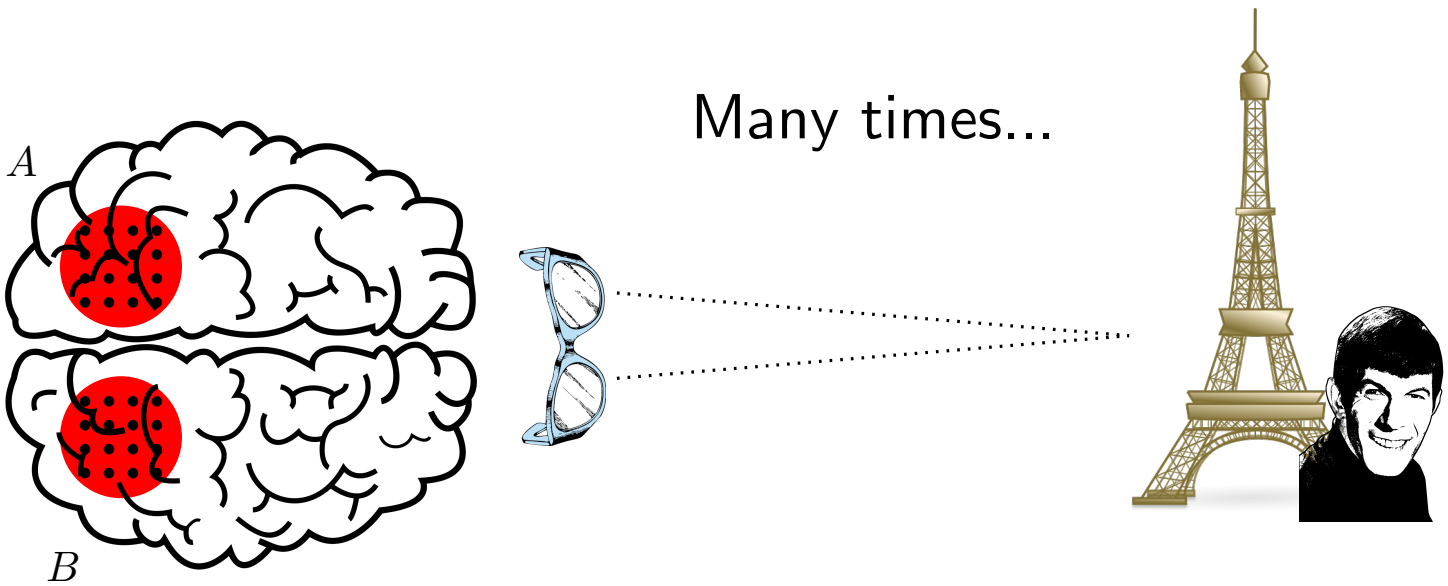

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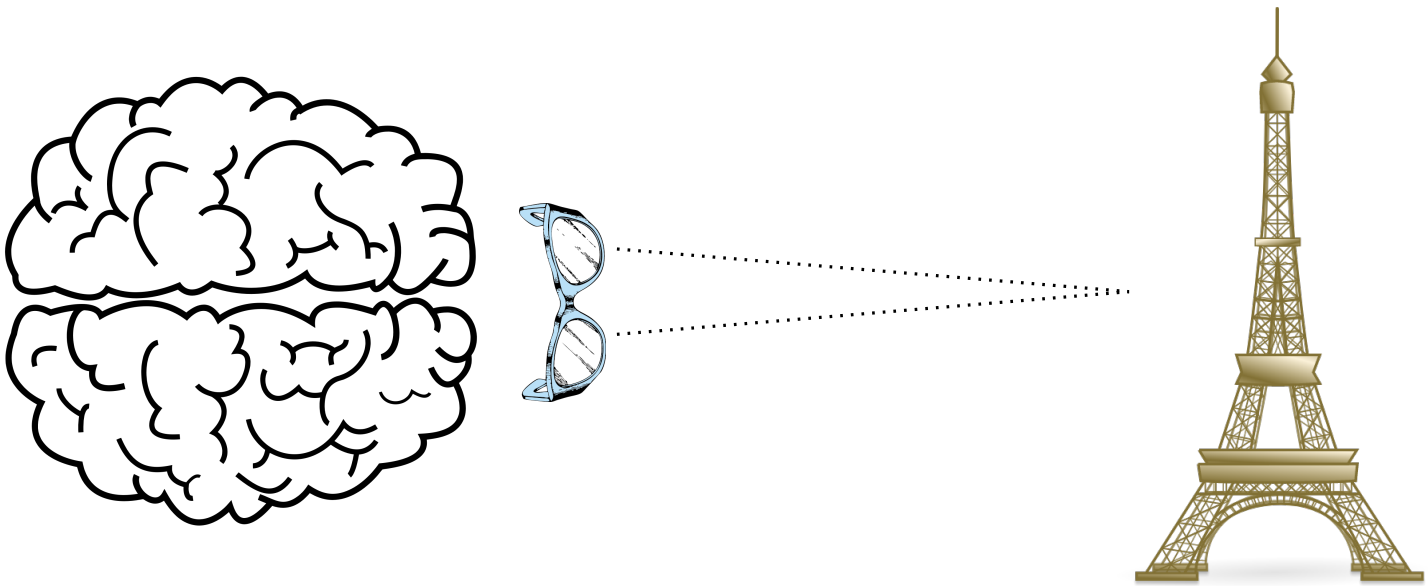

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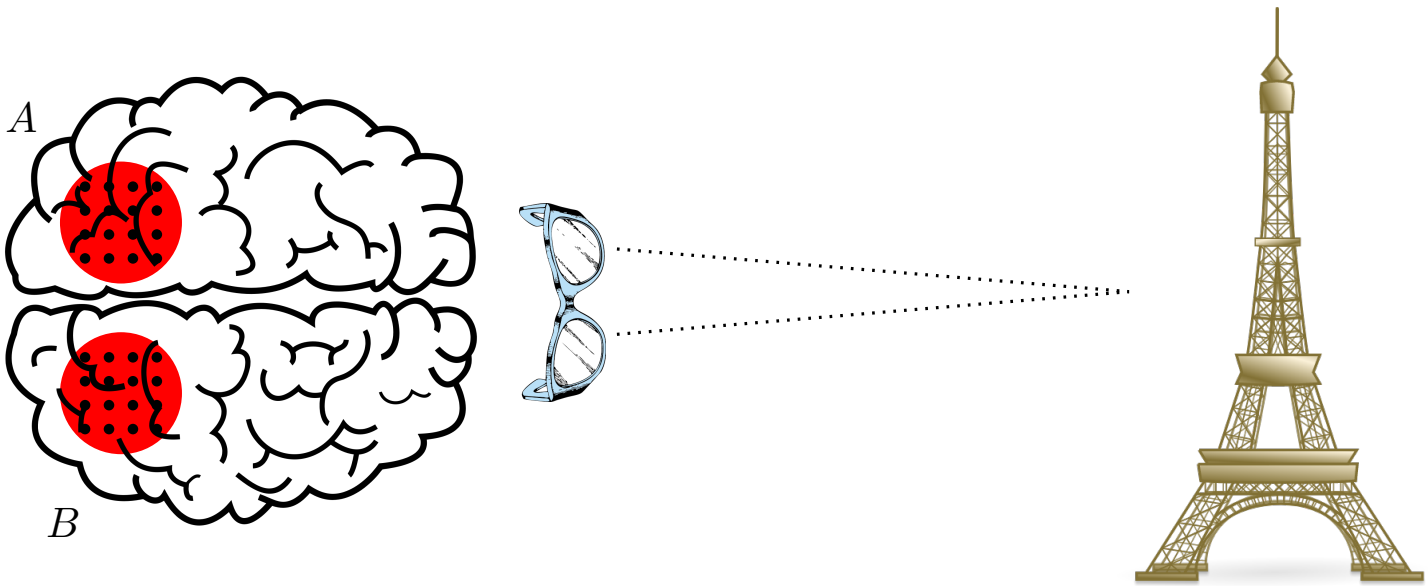

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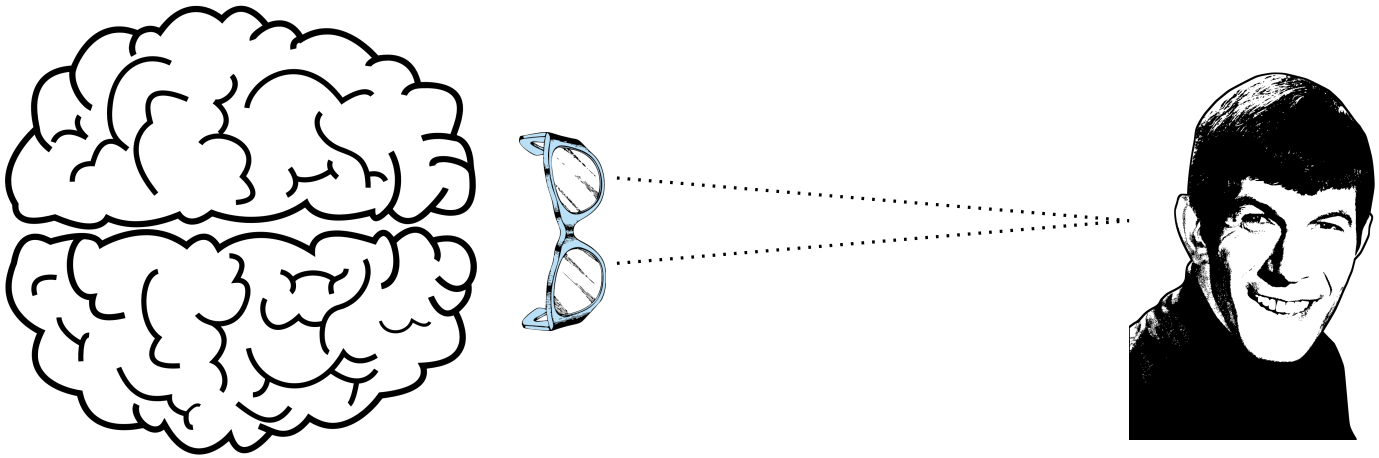

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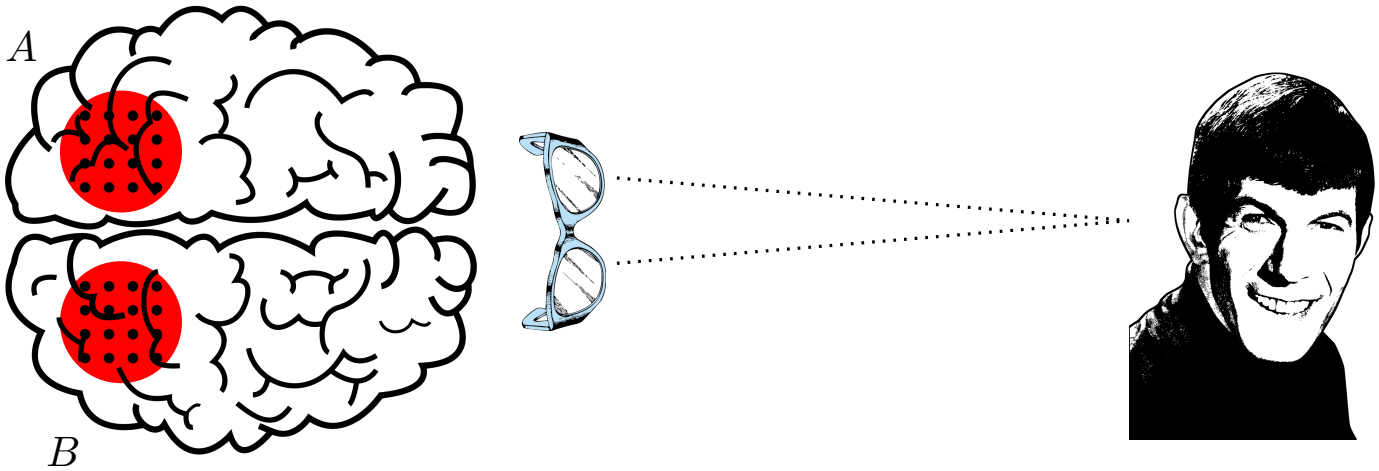

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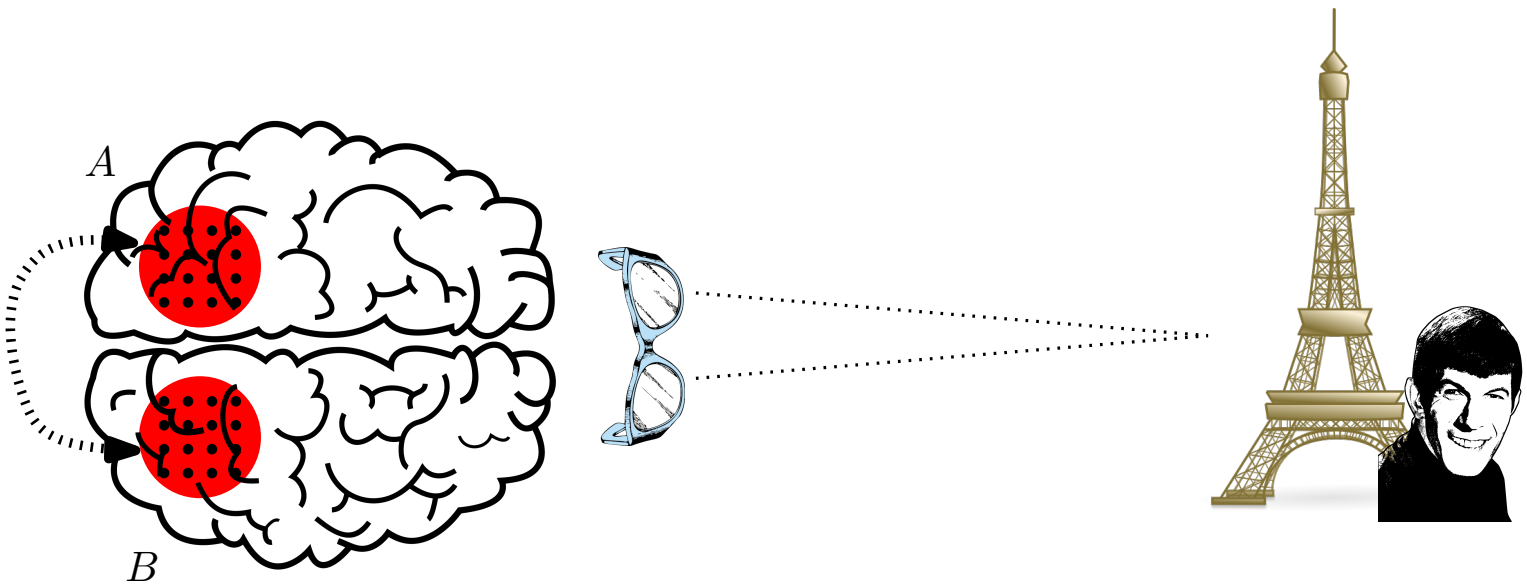

Association of assemblies (and others...)

The Model

- **Brain area**: $A = G_{n_A, p_A}$ Erdös Rényi graph

- *Recall* $G_{n,p} = (V, E)$:
  - $|V| = n$
  - $\forall x, y \in V$, $(x, y) \in E$ with probability p

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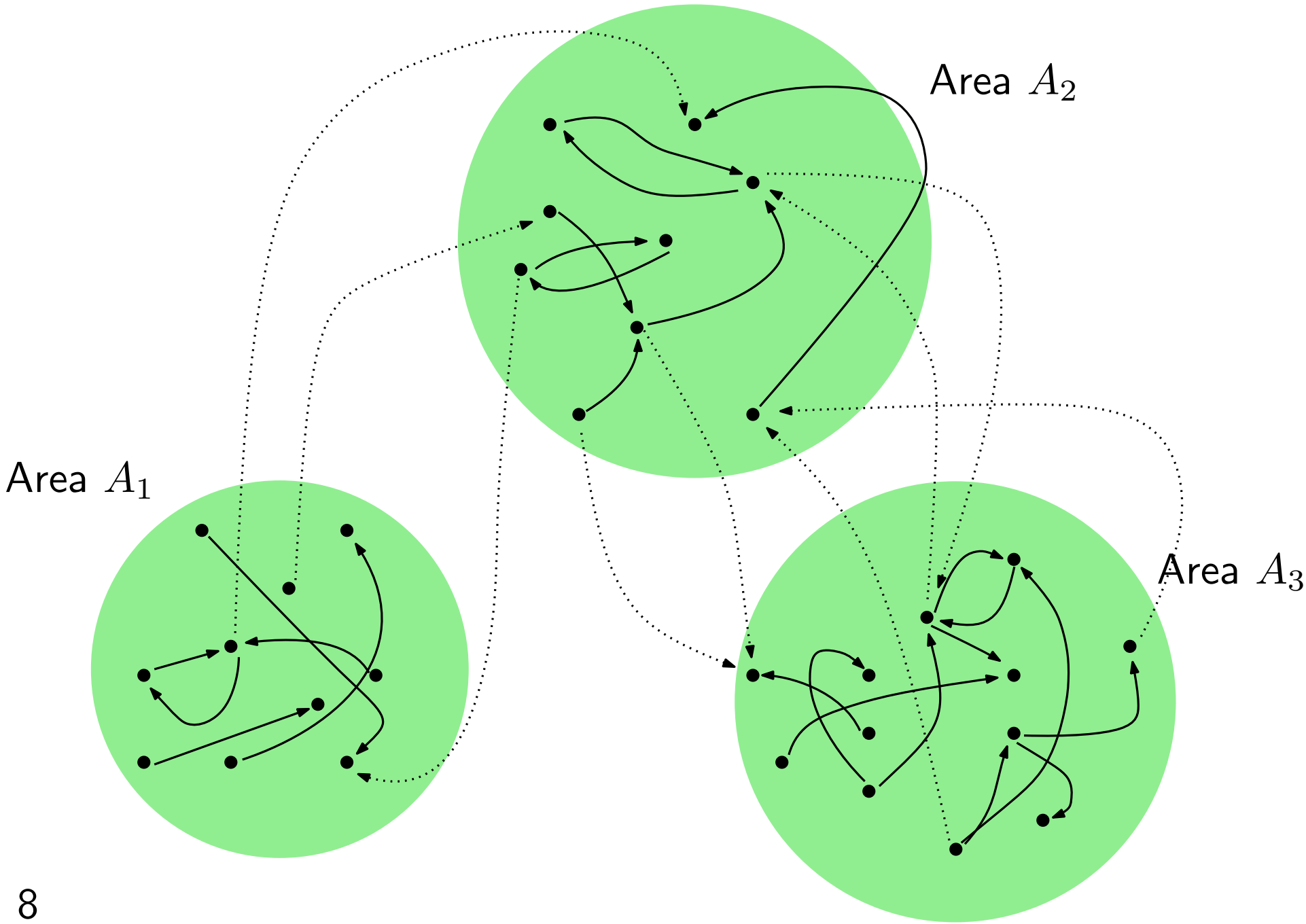
- set of brain areas $\mathcal{S} = \{A_1, A_2, \ldots, A_m\}$
- $\mathcal{C} \subseteq \mathcal{S} \times \mathcal{S}$ set of ordered pairs of brain regions

- **The Brain**: $\mathcal{B} = (V_{\mathcal{B}}, E_{\mathcal{B}})$ with
  - $V_{\mathcal{B}} = V_{A_1} \cup \cdots \cup V_{A_m}$
  - $E_{\mathcal{B}} = E \cup E_{A_1} \cup \ldots E_{A_m}$
  - E: $\forall (A, B) \in \mathcal{C}$, $\forall x \in A, y \in B$, $(x, y) \in E$ with probability $p_{A,B}$

Example of a Brain



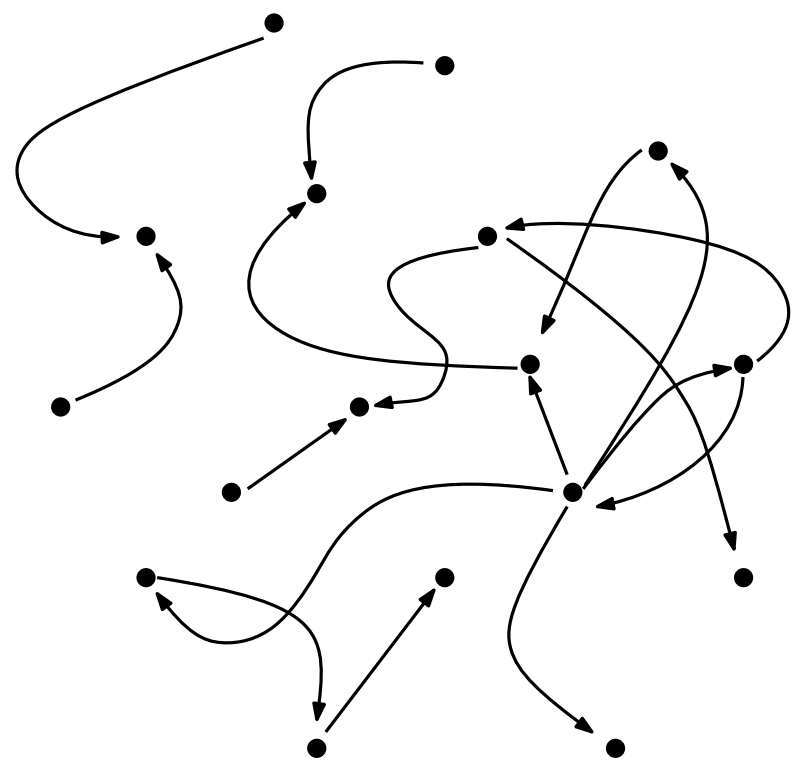
The Dynamics

Neuron = vertex **Fiber** = edge

All fiber weights initialized to be 1

Input for a neuron:

- external input (number)
- sum of incoming fiber weights whose sources have fired



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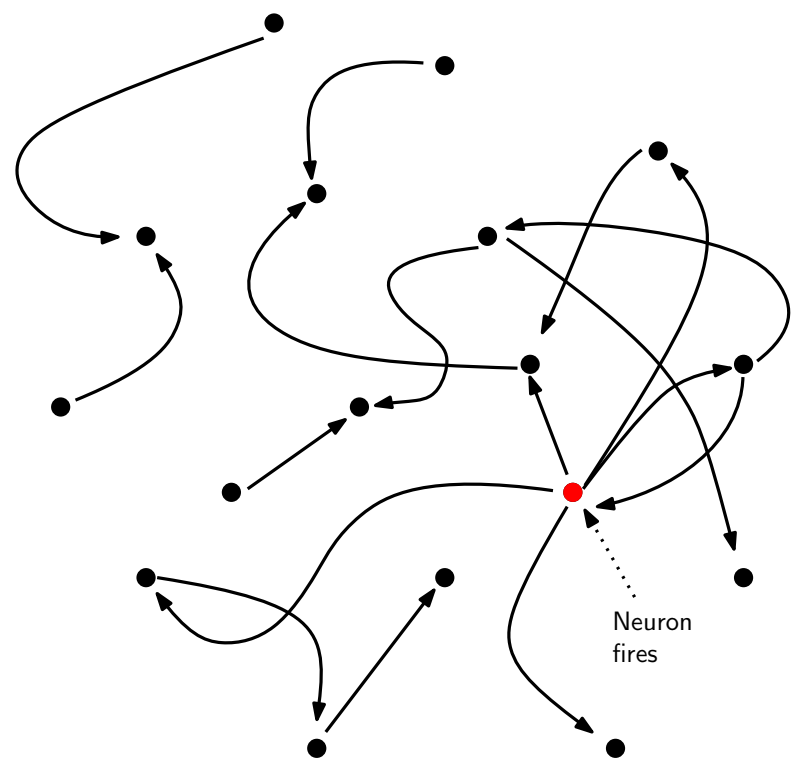

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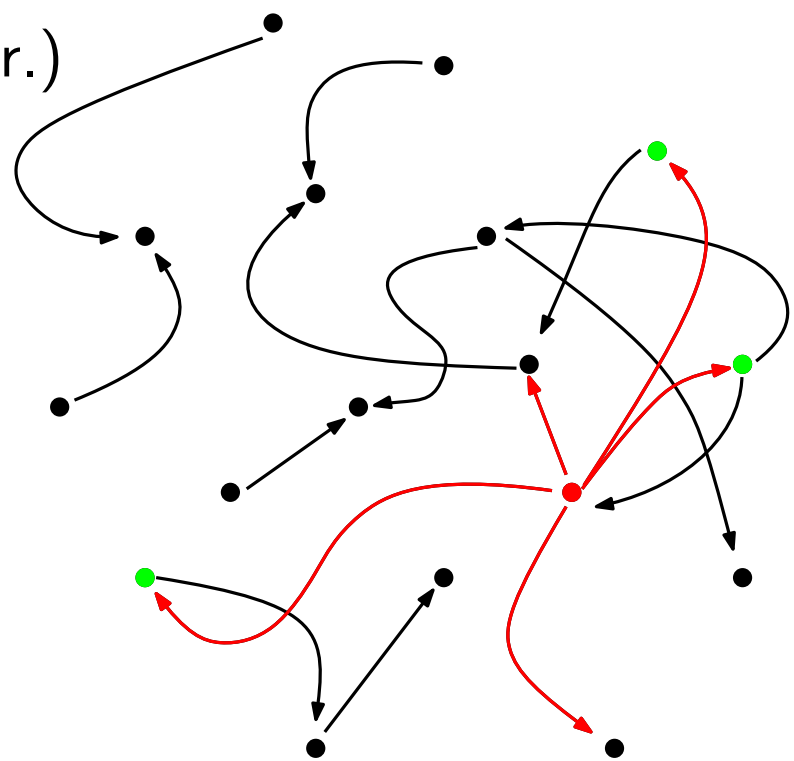
Neuron fires if it is among the k neurons with the highest input (**CAP**)

Here (toy): $k = 3$ (ties broken u.a.r.)



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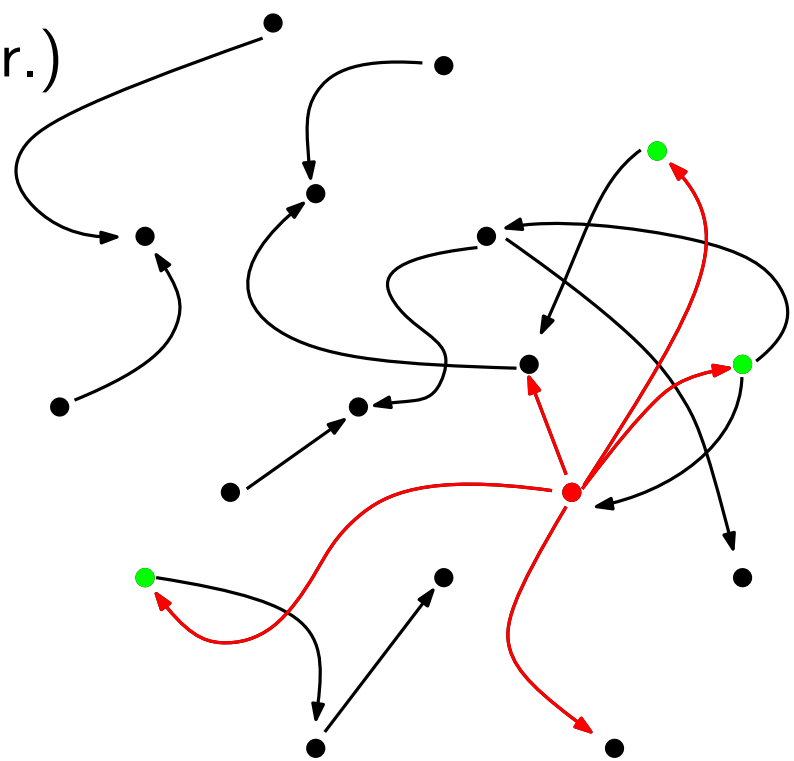
Upon fiber endpoint activation:

weight = old_weight $\cdot (1 + \beta)$

Hebbian plasticity



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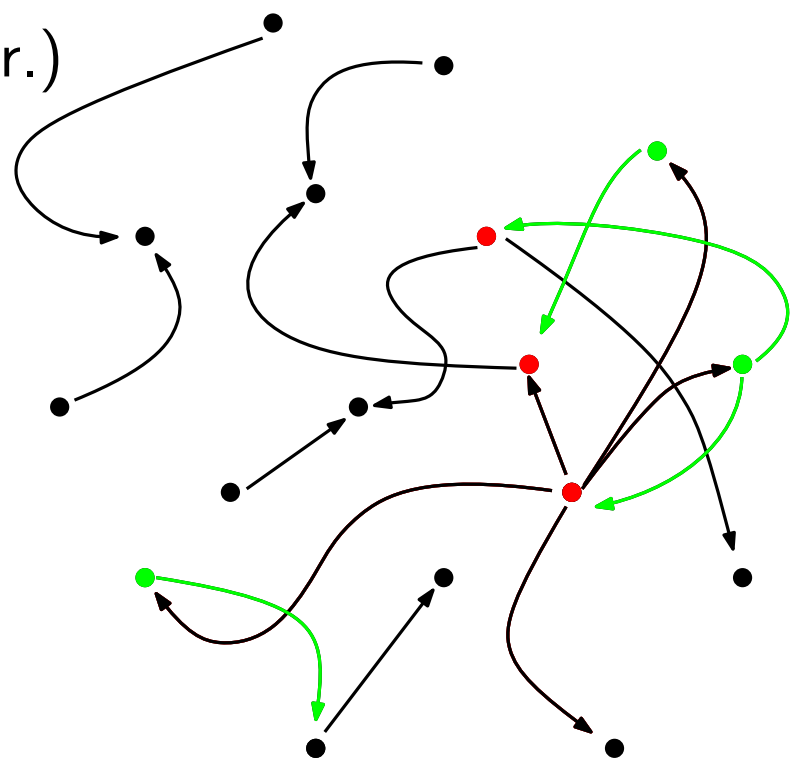

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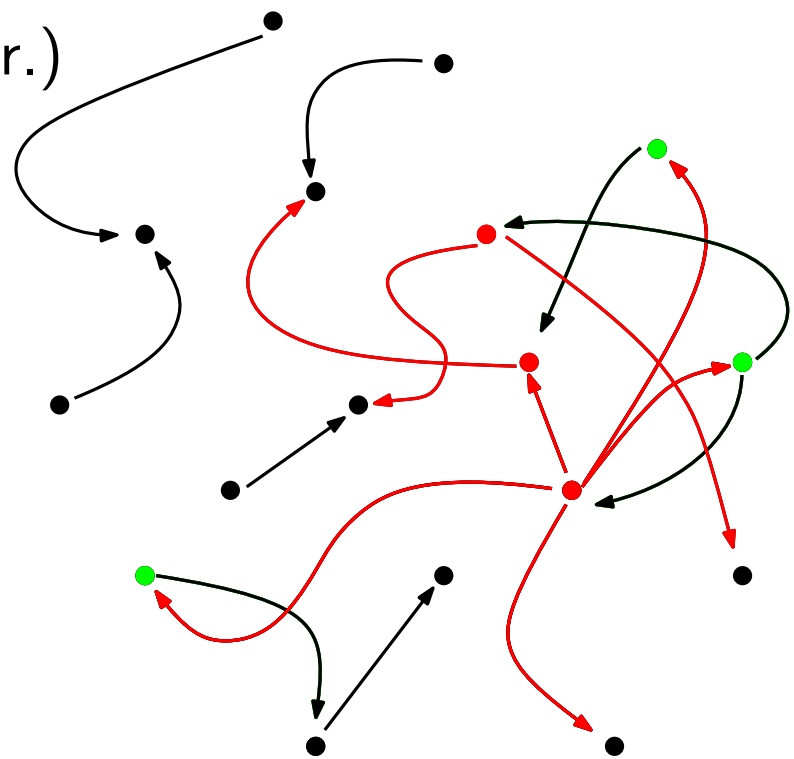

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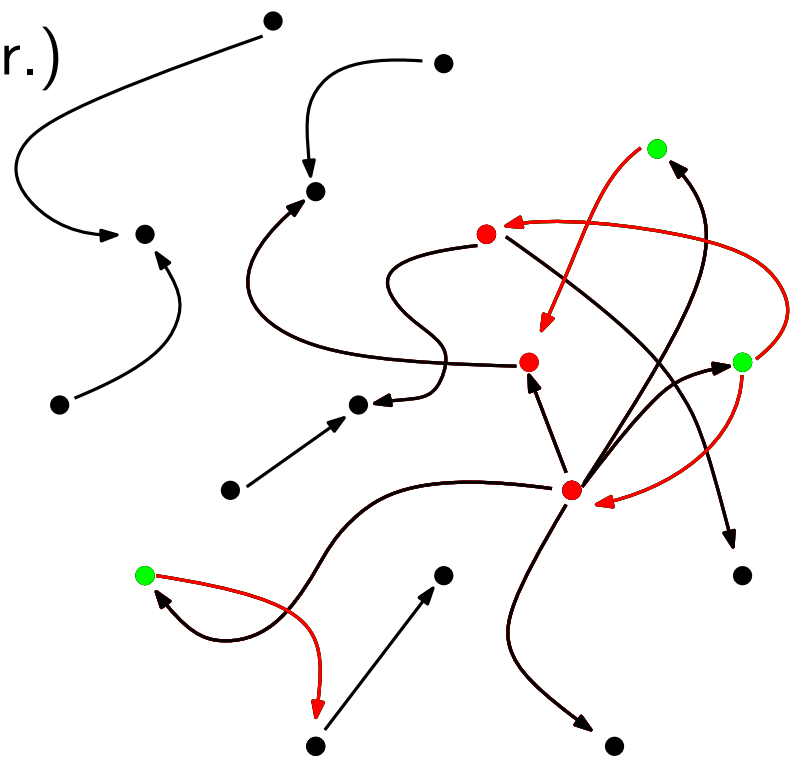

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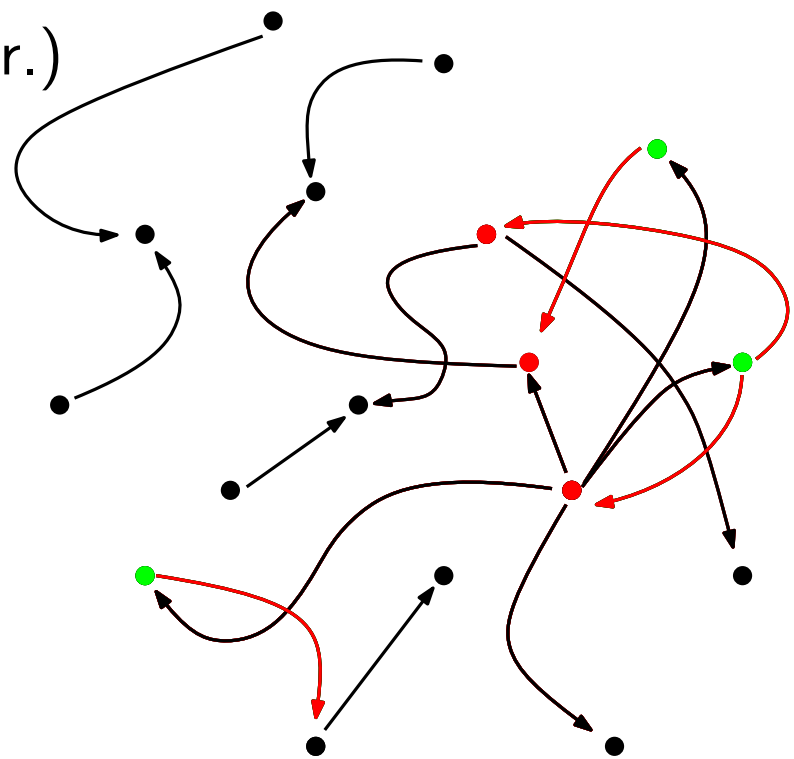

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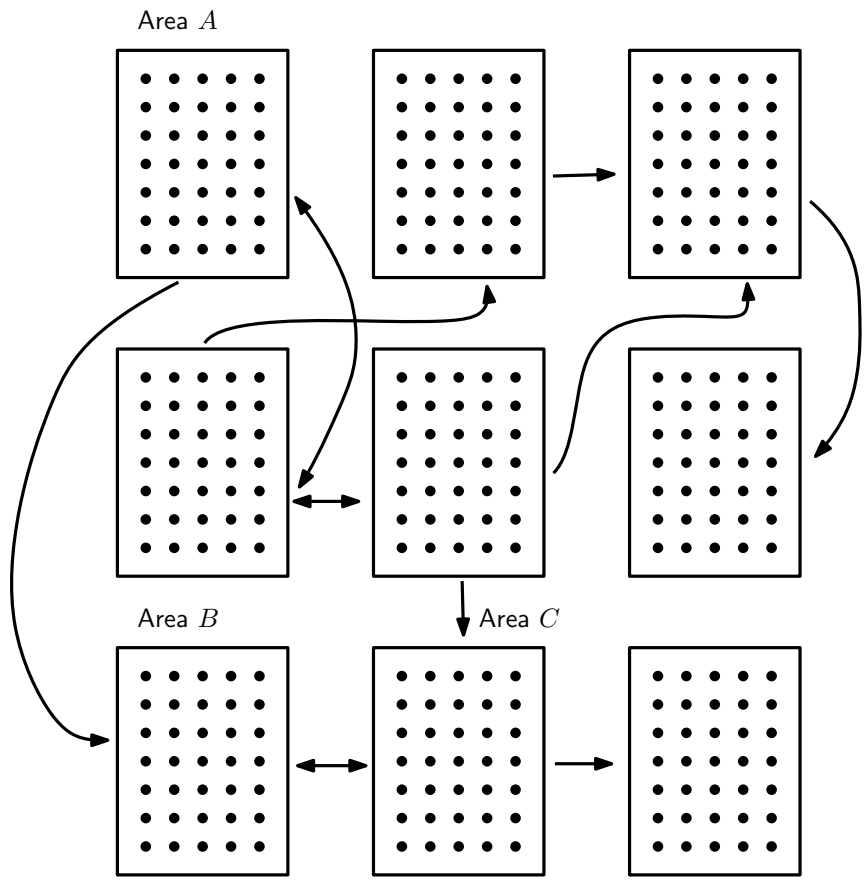
Assembly: *stable* set of k neurons

Operations

The brain and its areas

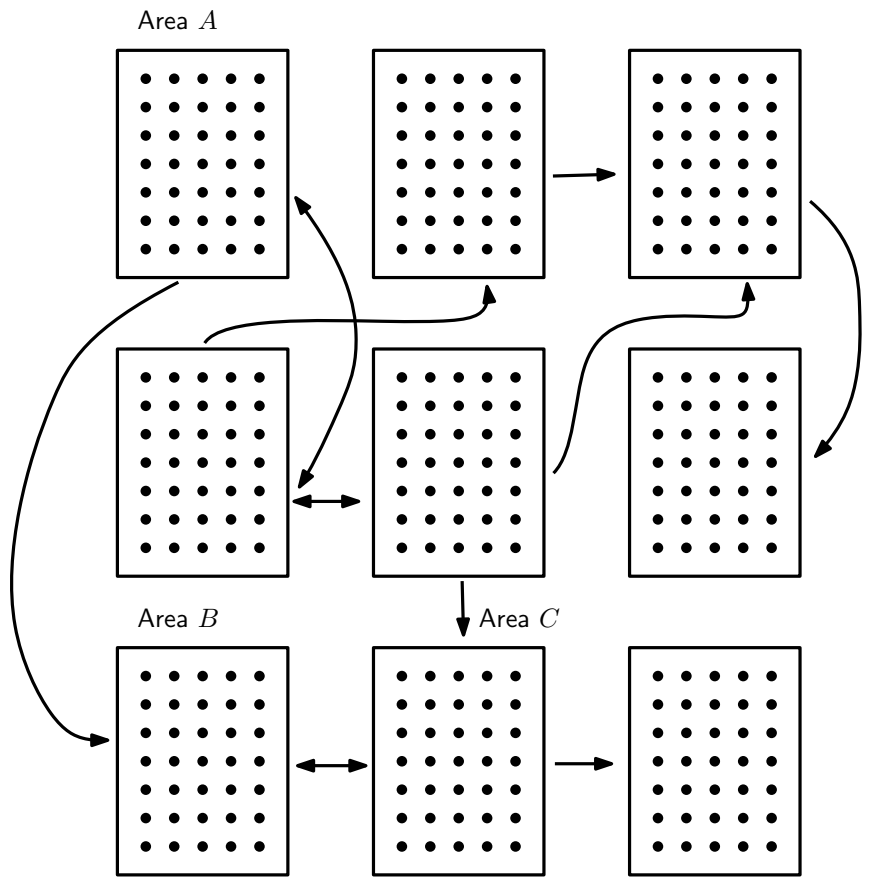


Operations

- **Inhibition**: an area can be inhibited, that is, its neurons cannot fire

Inhibition is accomplished through populations of inhibitory neurons, whose firing prevents other neurons from firing [Mitropolsky et al., TACL 2021]

The brain and its areas



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Disinhibition inhibits a population of inhibitory neurons, which currently inhibit the area [Mitropolsky et al., TACL 2021]

We assume we can do the same with fibers

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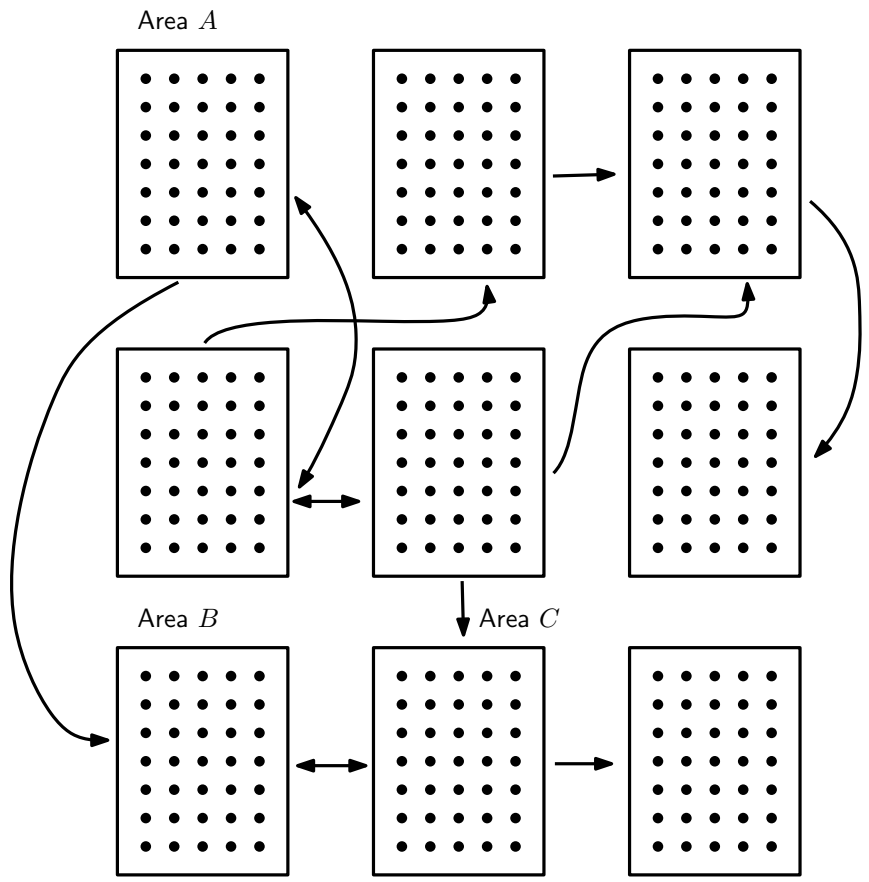

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For areas A, B, C:

- `inhibitArea` (A)

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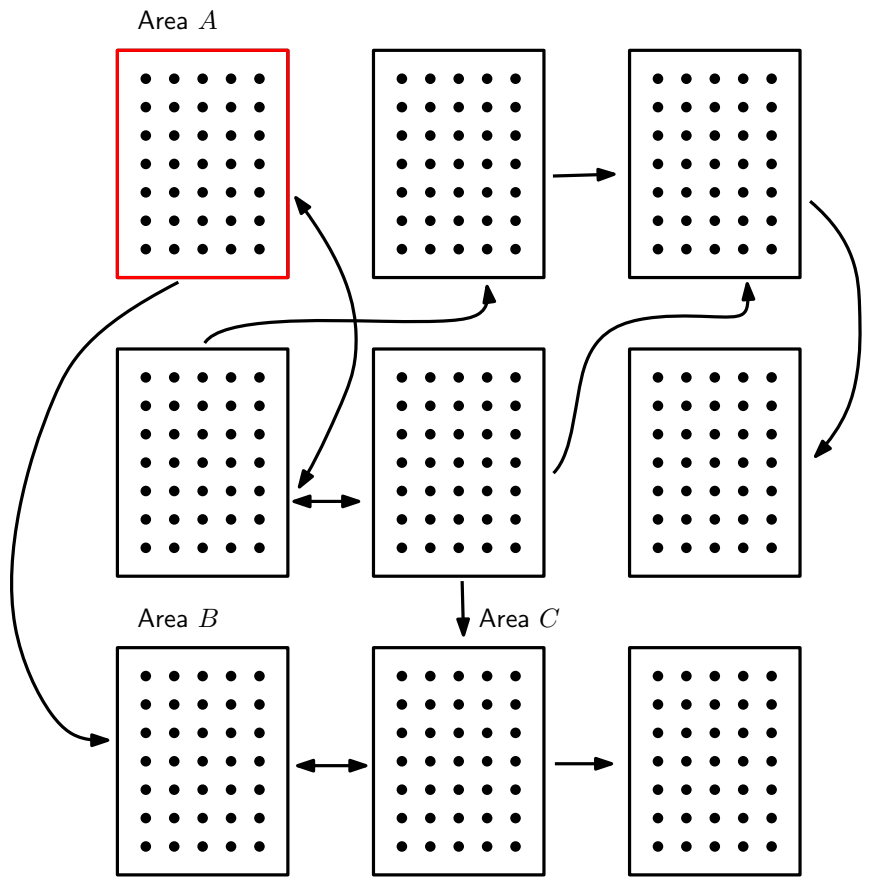

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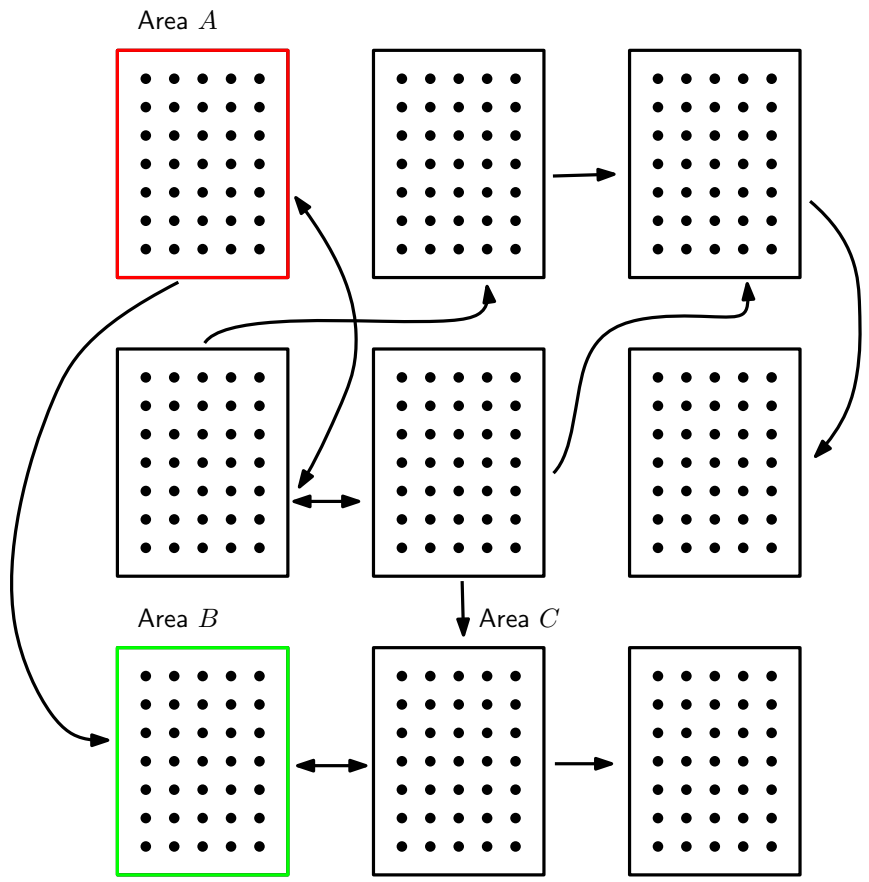

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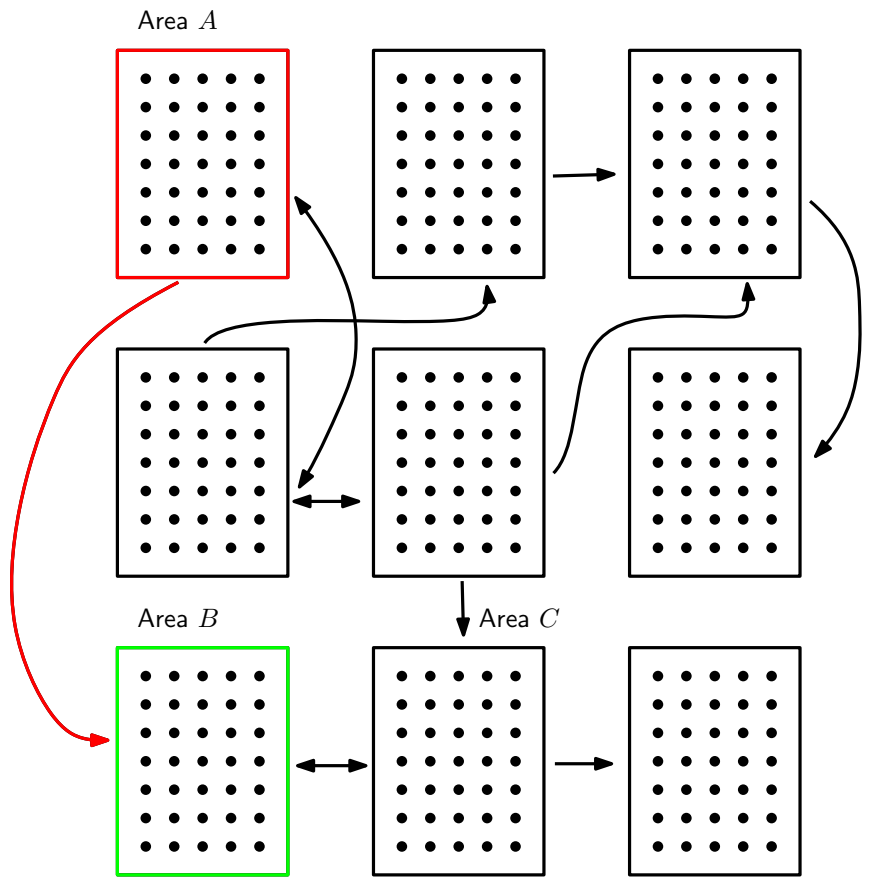

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For areas A, B, C:

- `inhibitArea` (A)
- `inhibitFiber` (A, B)
- `disinhibitArea` (B)

Operations

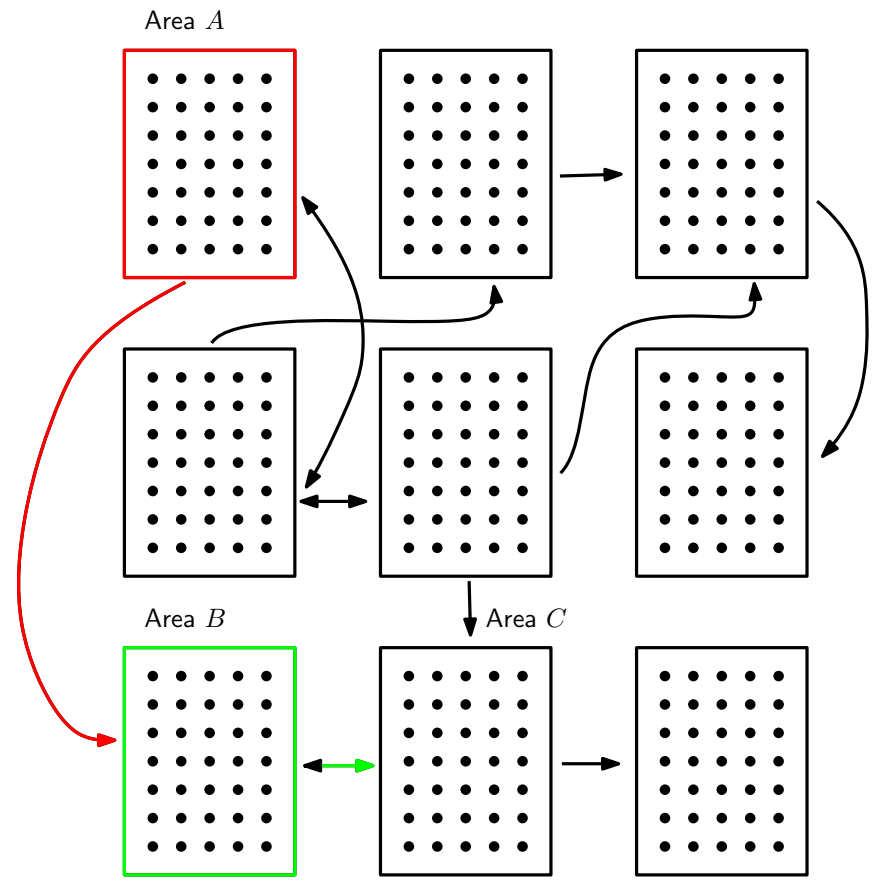
- **Inhibition**: an area can be inhibited, that is, its neurons cannot fire

Inhibition is accomplished through populations of inhibitory neurons, whose firing prevents other neurons from firing [Mitropolsky et al., TACL 2021]

- **Disinhibition**: an inhibited area can be disinhibited, that is, its neurons can now fire

Disinhibition inhibits a population of inhibitory neurons, which currently inhibit the area [Mitropolsky et al., TACL 2021]

The brain and its areas



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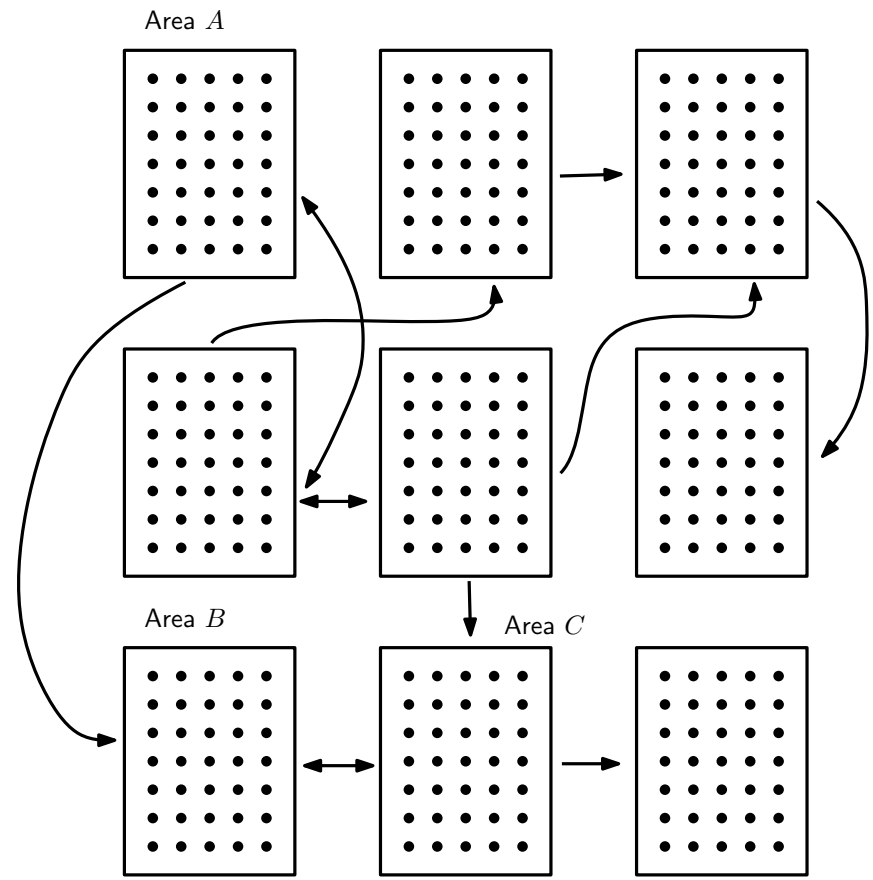

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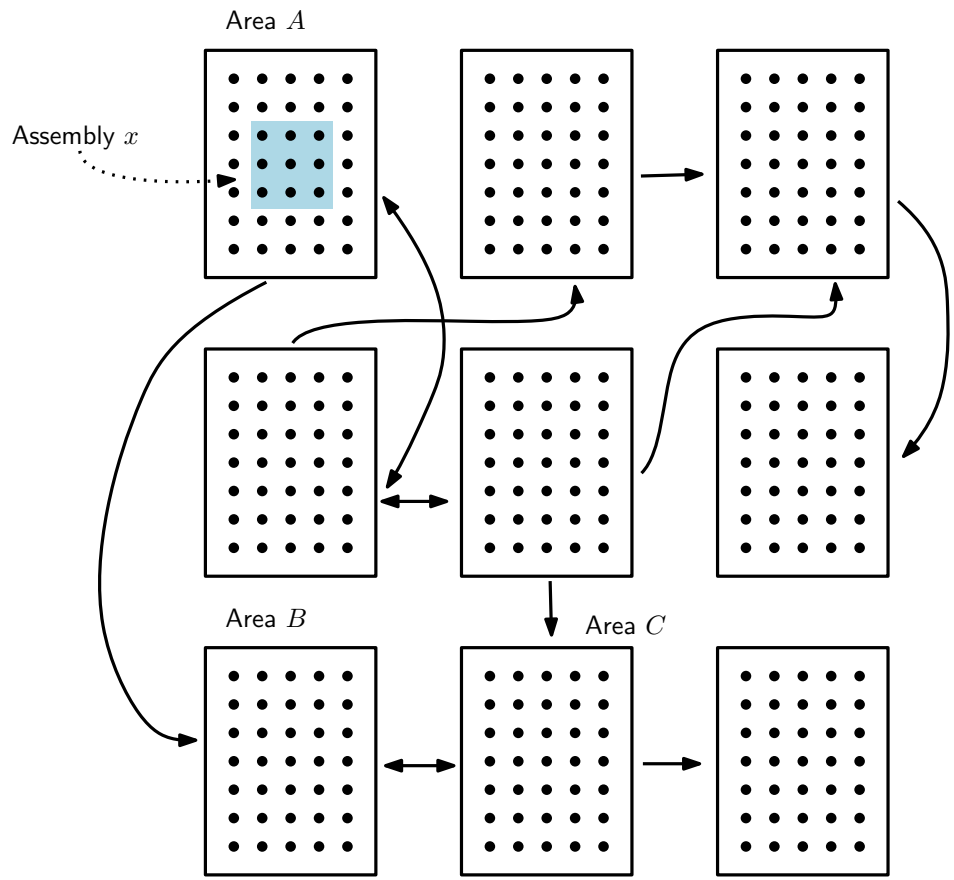

Operations

- **Projection**: the last active assembly x in area A starts firing repeatedly into area B until an assembly y is formed

This process eventually converges under a wide range of parameters w.h.p. [Papadimitriou et Vempala, ITCS 2019]

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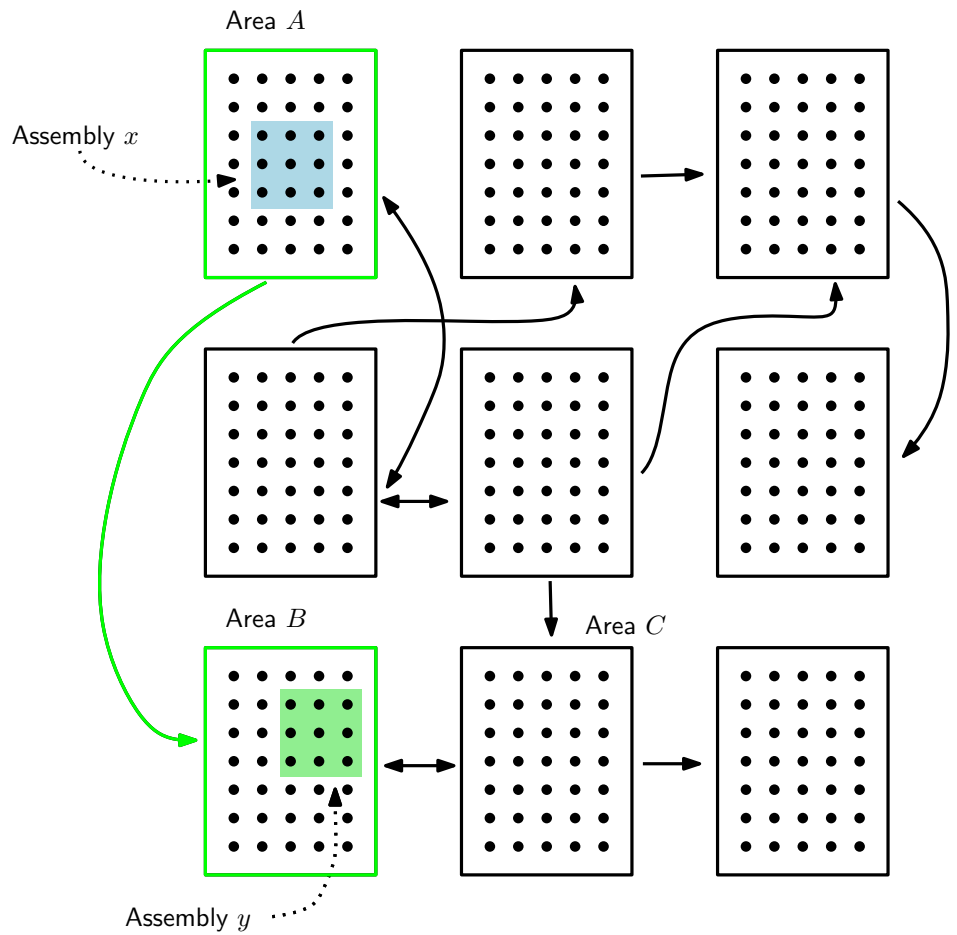

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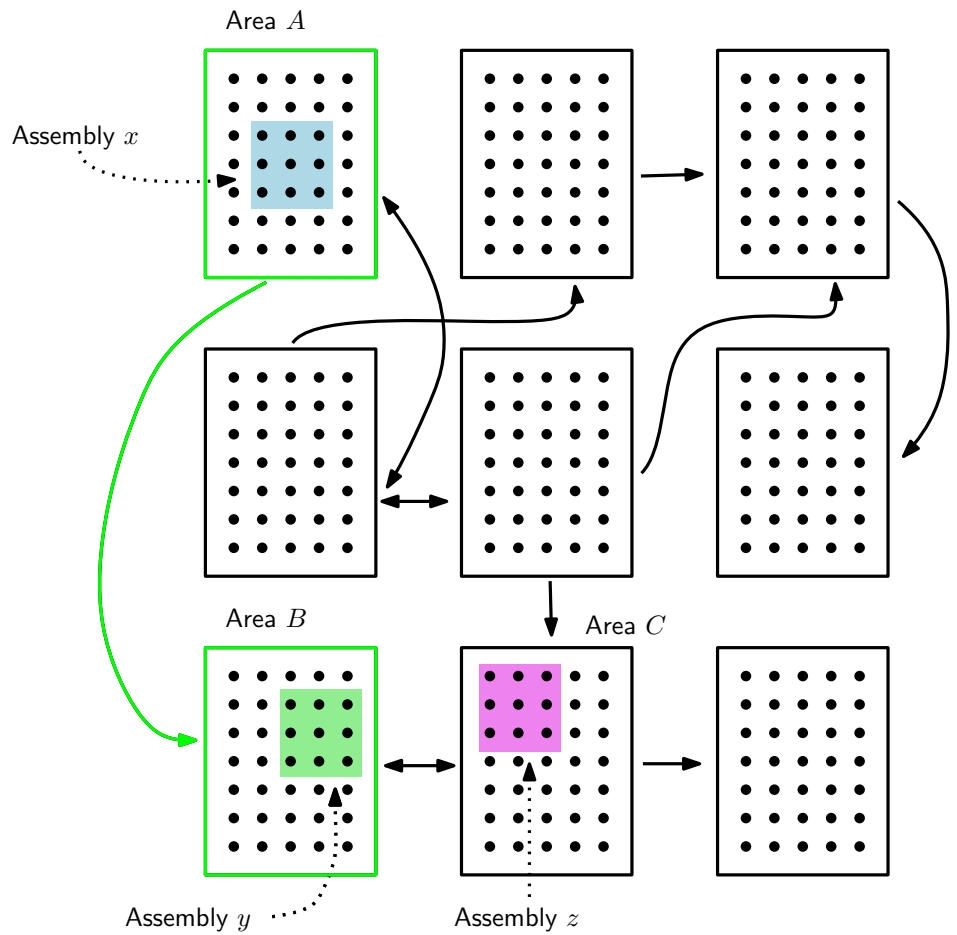

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Linked: the firing of one results into the firing of the other

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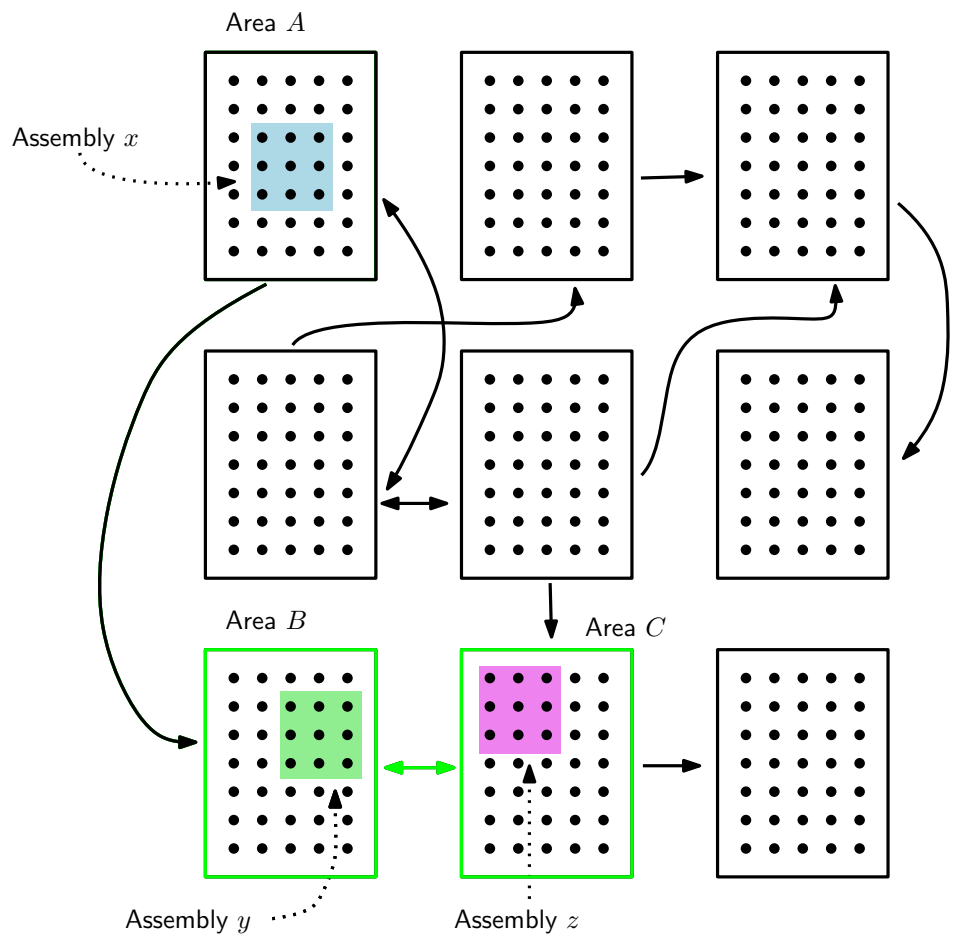

Remember?

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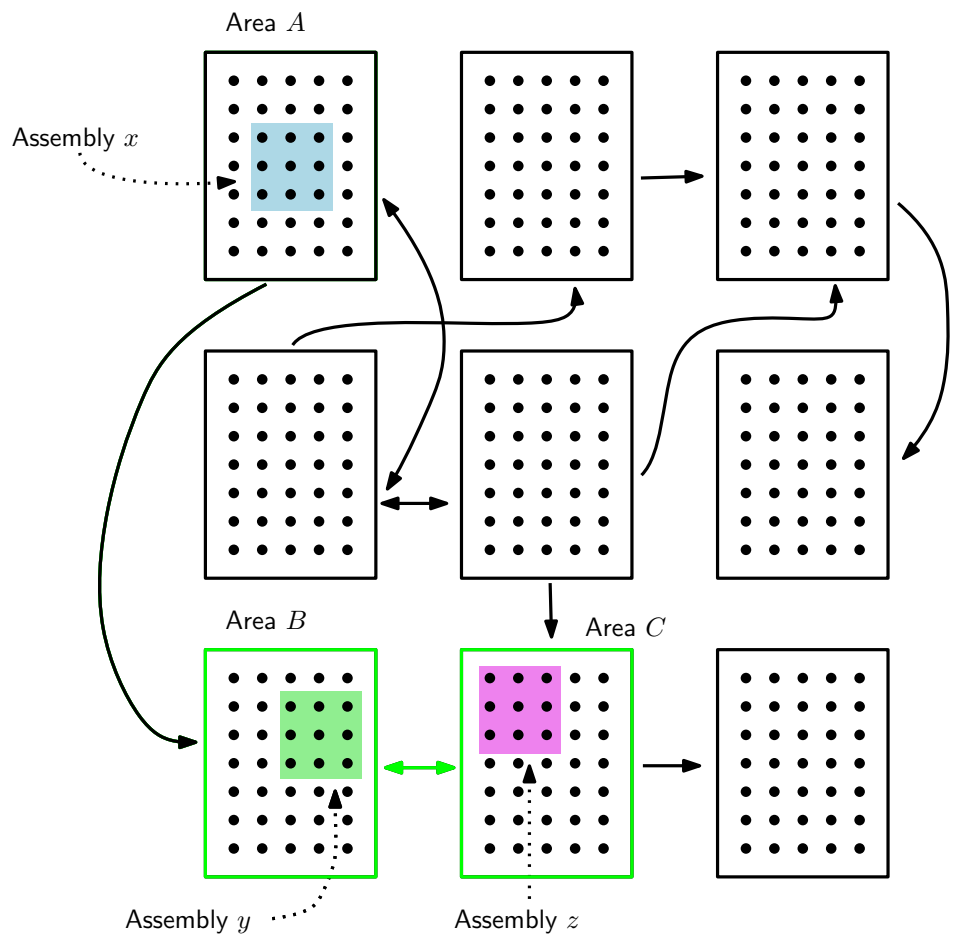


For areas A, B, C:

- `project` (A, B)
- `associate` (B, C)

Remember?

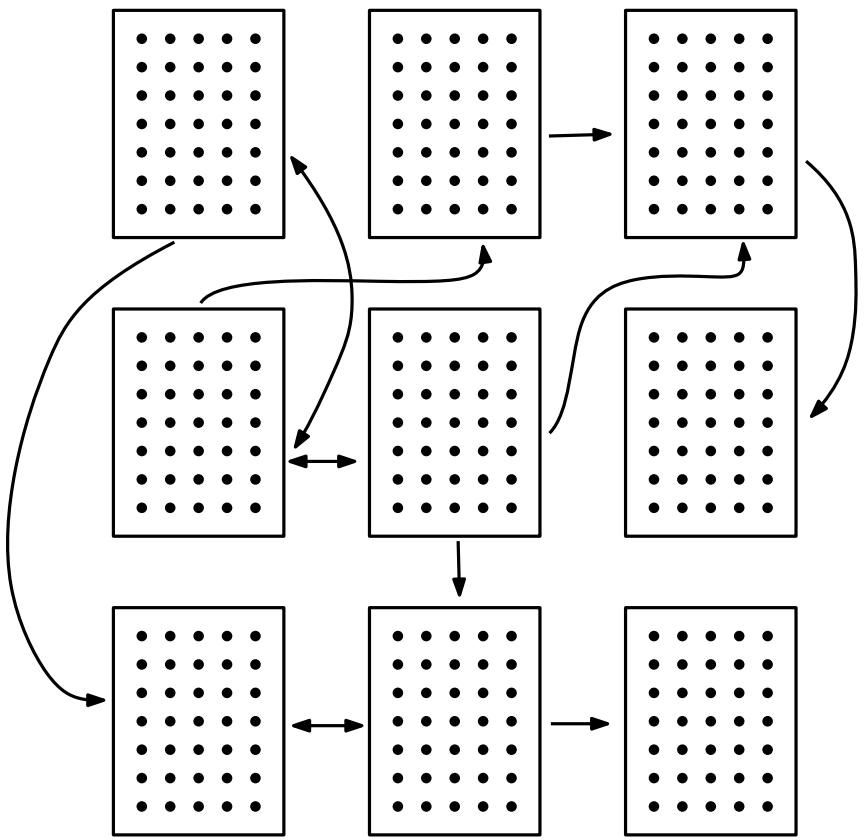
Operations

- **Strong projection**: enhancement of the projection operation

Dynamics on the subgraph induced by all disinhibited areas and fibers:

- all active assemblies start projecting in adjacent areas, forming new assemblies, and so on, until stability

The brain and its areas



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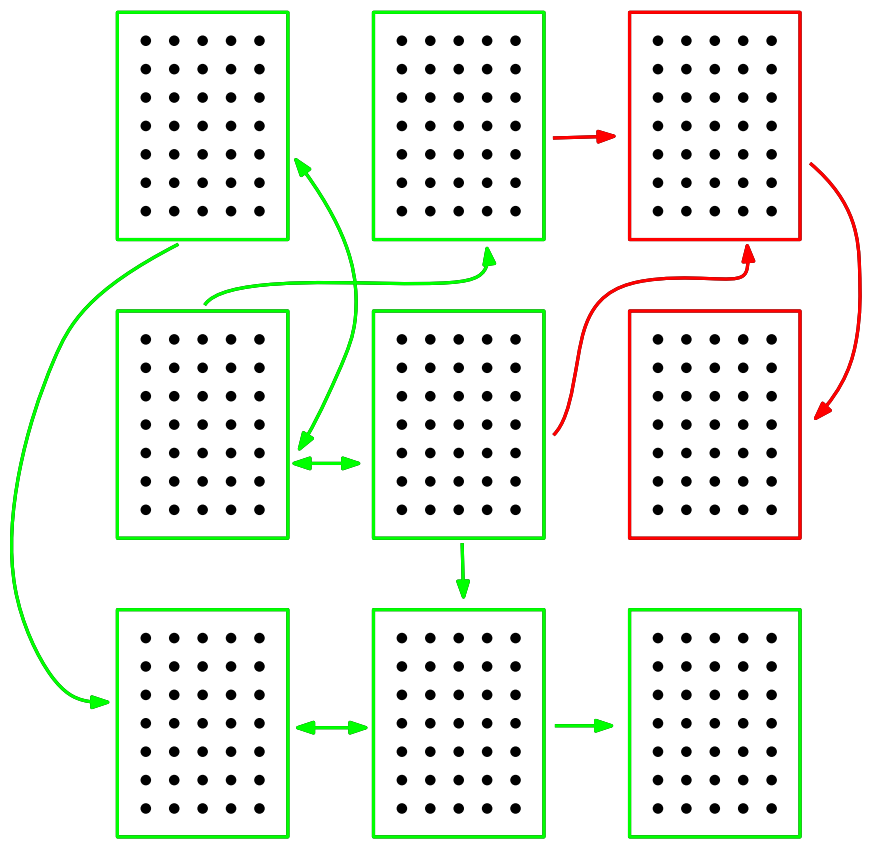

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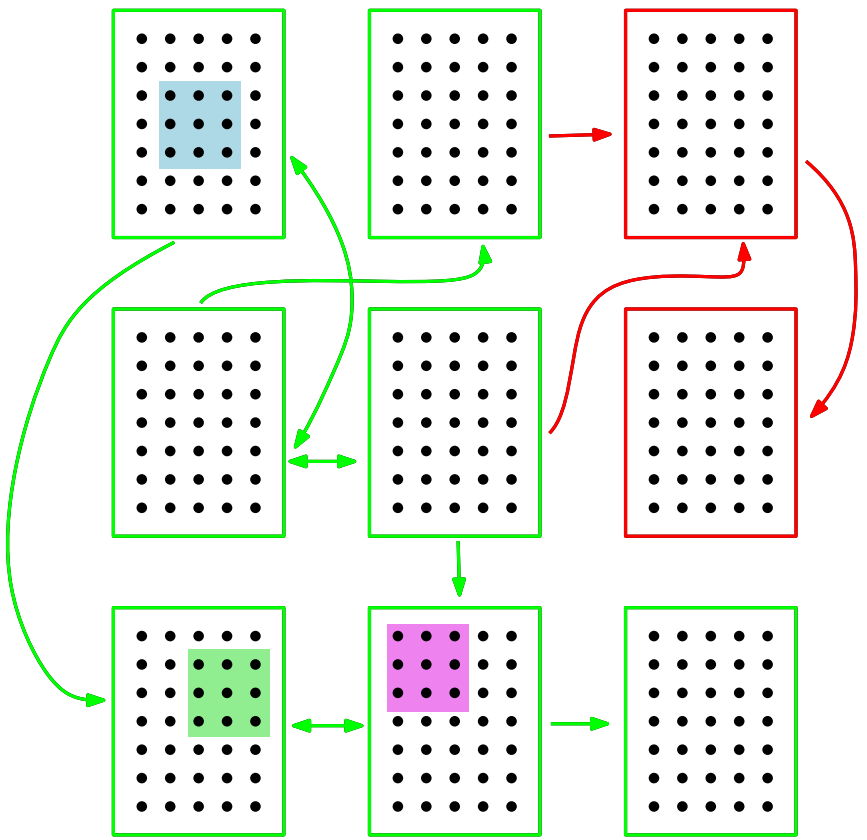

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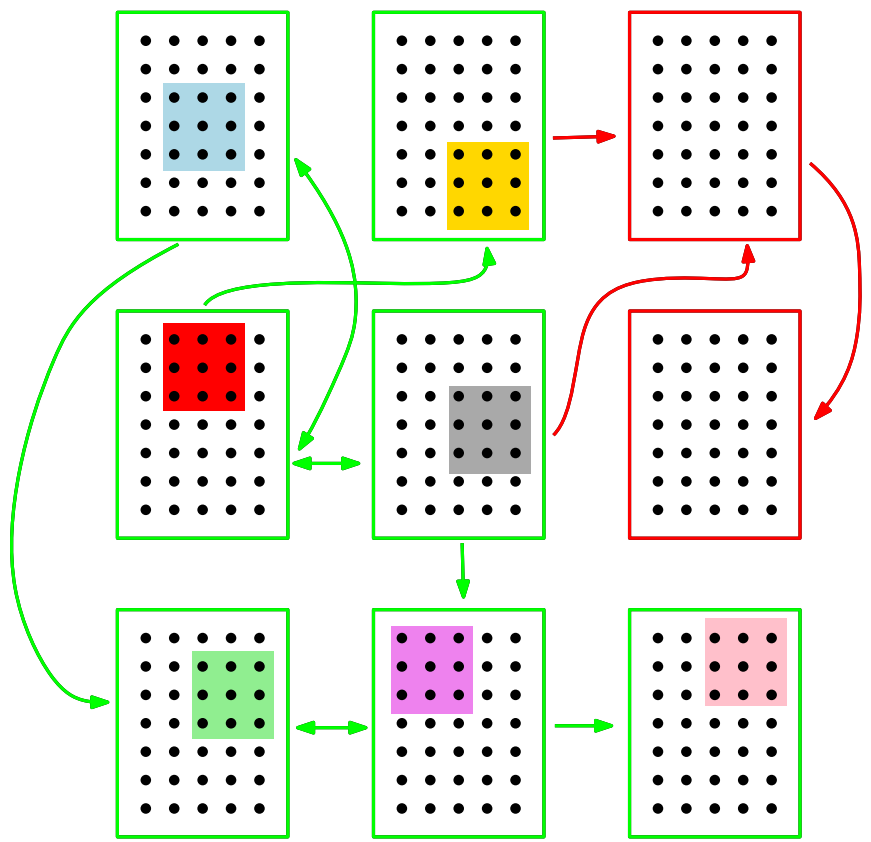

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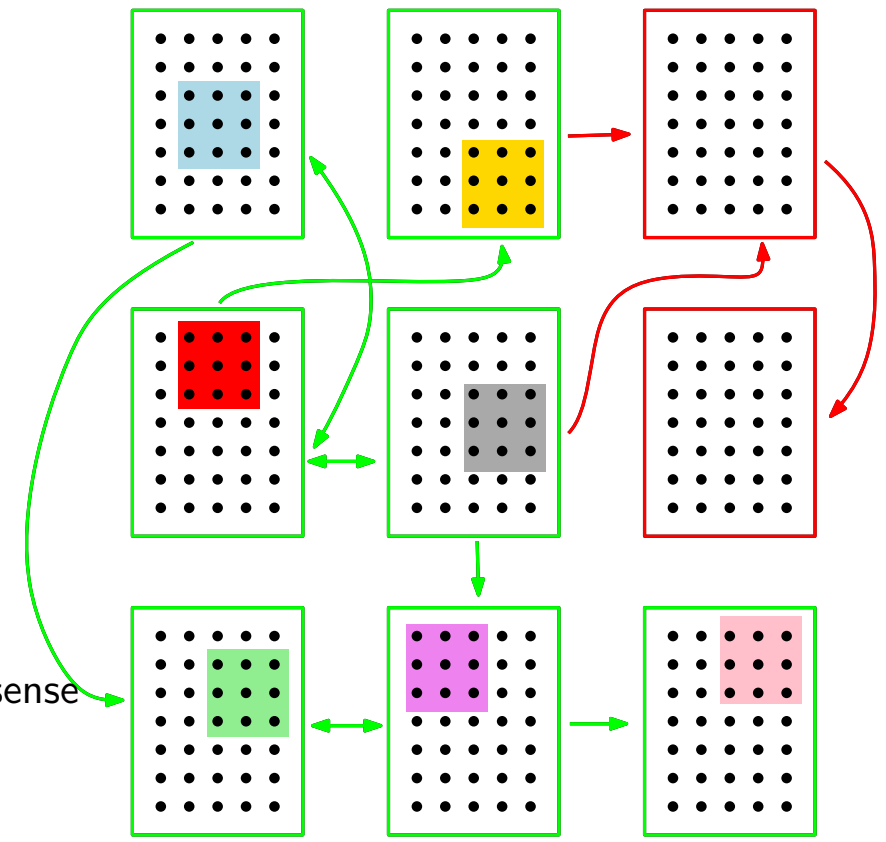
The brain and its areas



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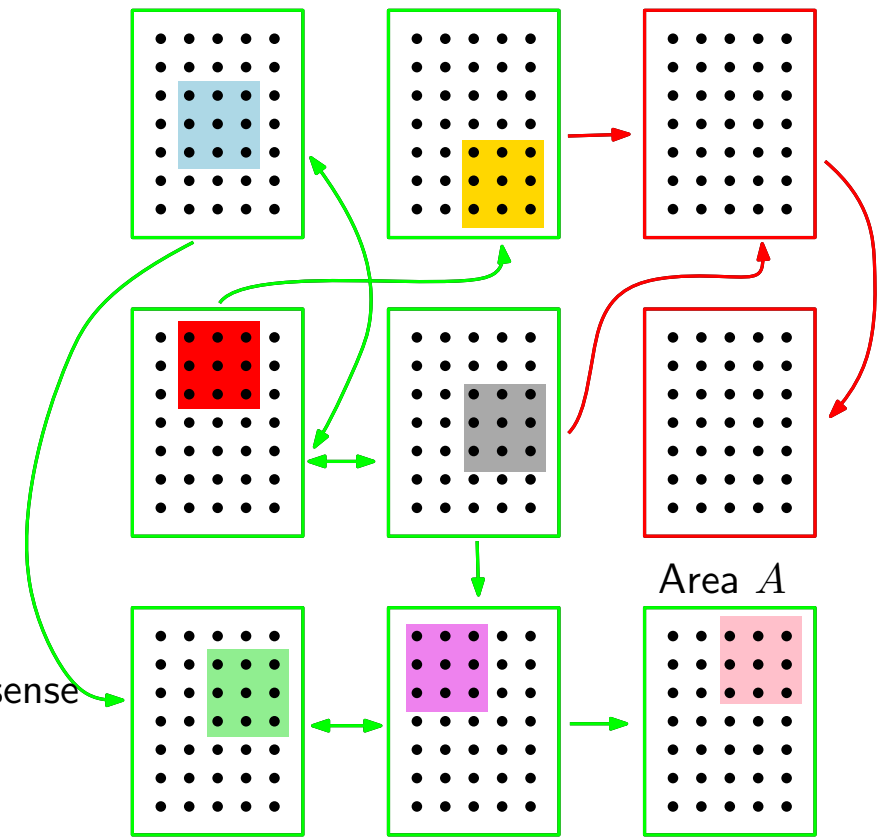

We write `strongProject()` and `read`(A) for a brain area A

Related Works

- [Legenstein et al., ITCS 2018] shows analytically how assemblies emerge from stimula

- [Papadimitriou et Vempala, ITCS 2019] analyzes the convergence of processes defined by operations with assemblies

- [Papadimitriou et al., PNAS 2020] introduces the formal model of the assembly calculus and proves it's Turing complete
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Our Contribution

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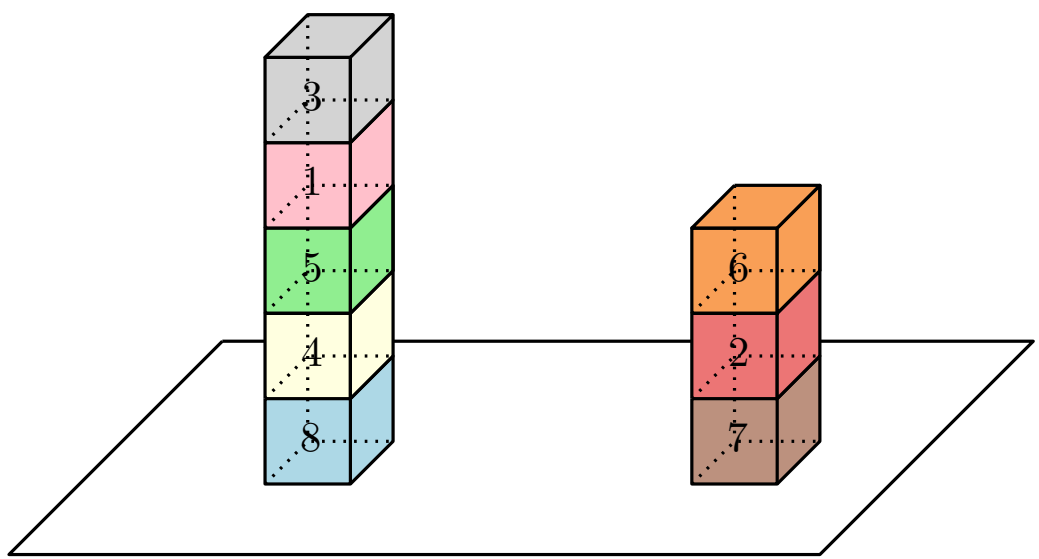

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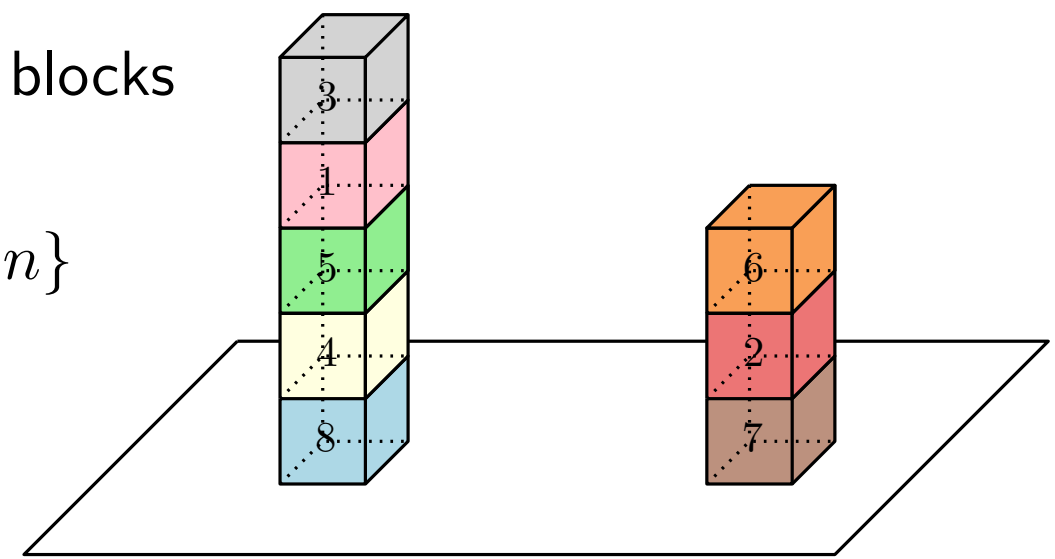
Blocks world: n unique blocks labelled in $\{1, \ldots, n\}$, organized as follows

- set of stacks $\{S_i\}_i$
- each stack is a sequence of blocks $S_i = (b_1^{(i)}, \ldots, b_{k_i}^{(i)})$
- $\cup_i \{b_1^{(i)}, \ldots, b_{k_i}^{(i)}\} = \{1, \ldots, n\}$



The Planning Task

Possible moves:

- move a block from the top of a stack to the table
- move a block from the top of a stack to the top of another stack
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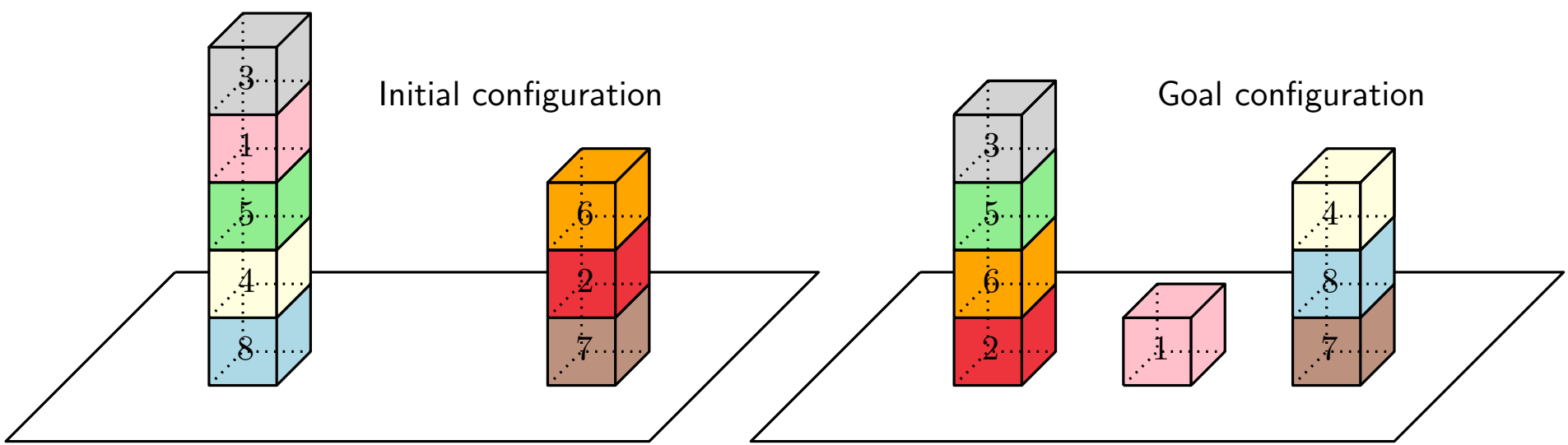

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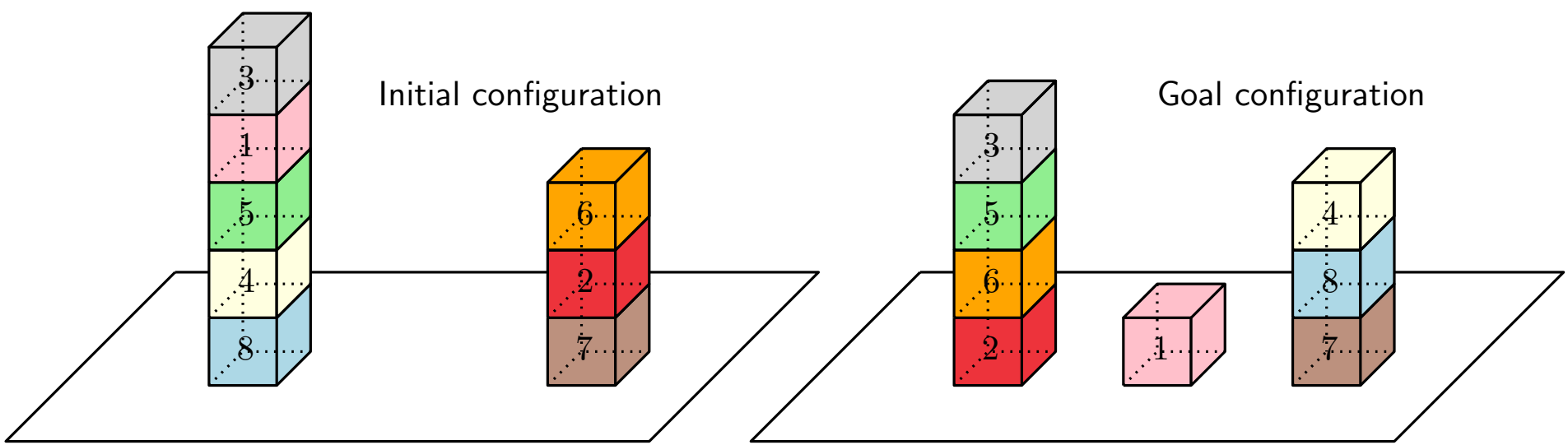
Output:

- the sequence of moves



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Brute-force strategy: *move all blocks to the table and place them correctly, one by one*

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We implement both of them...

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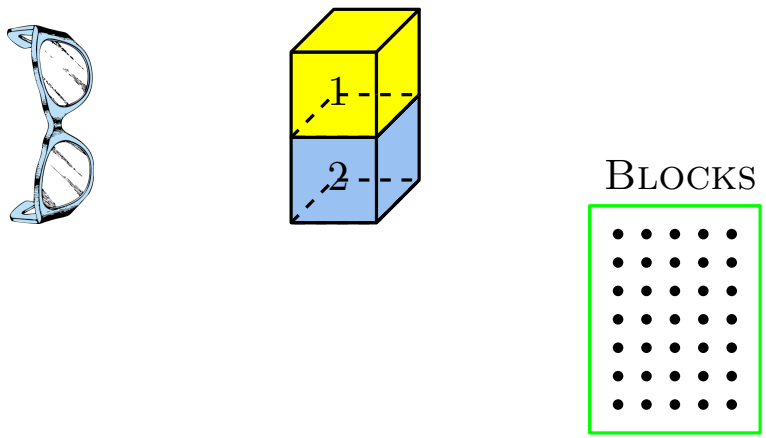

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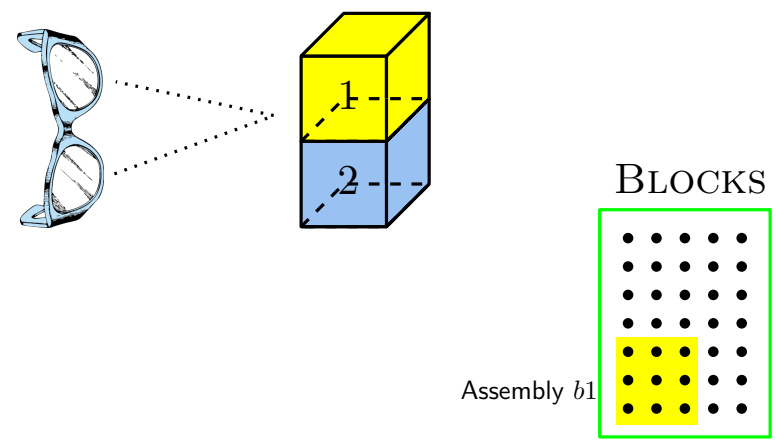

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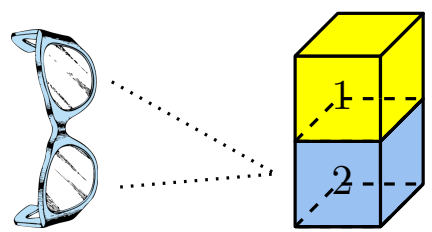

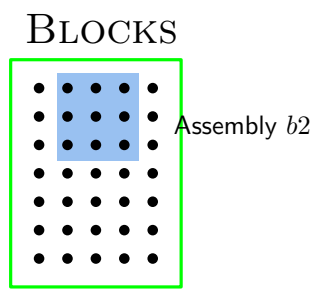

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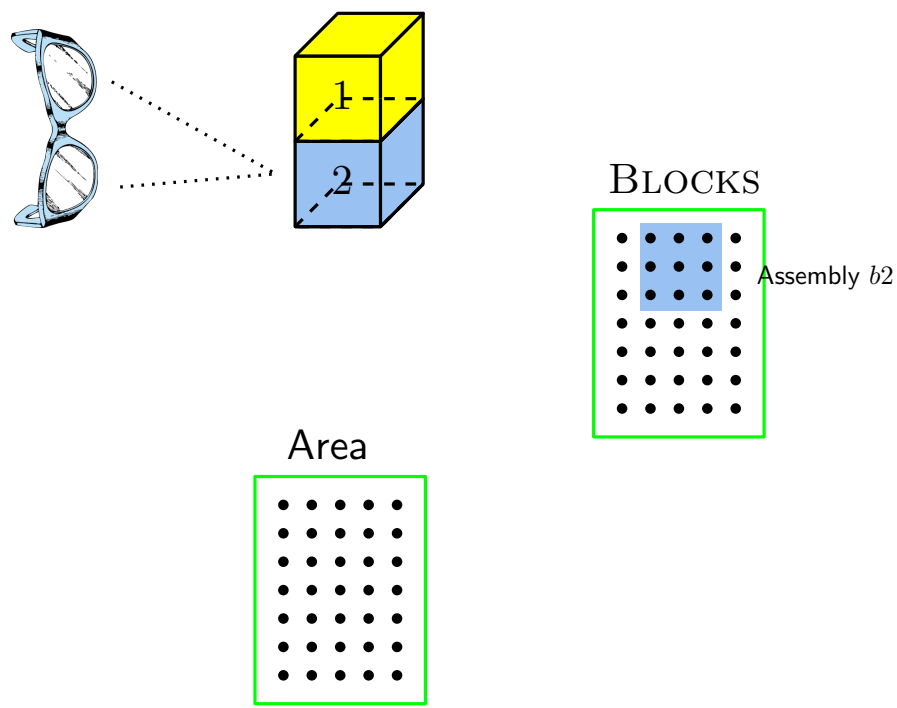

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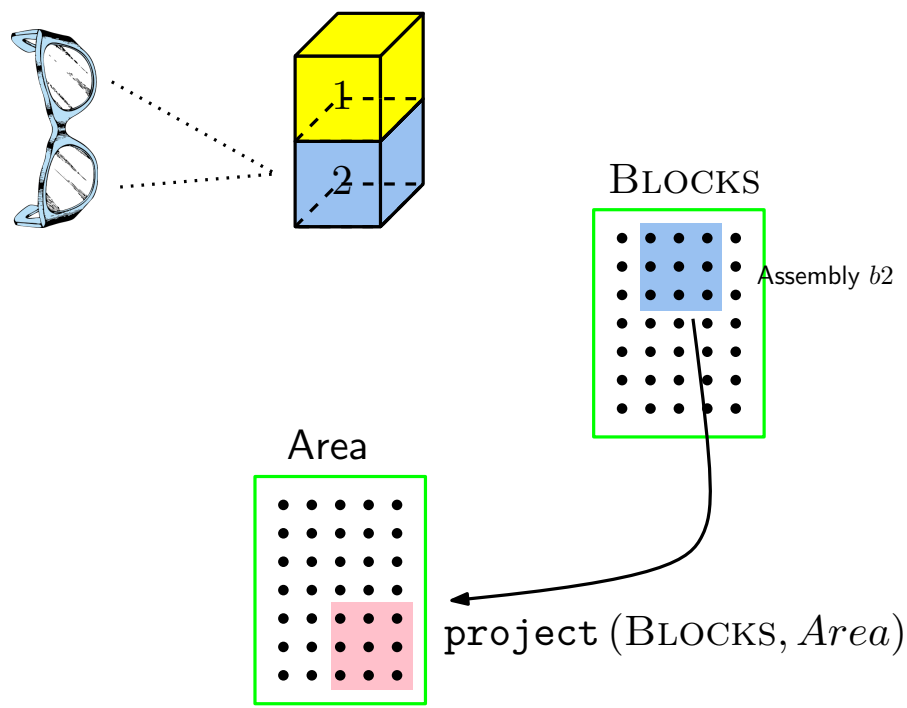

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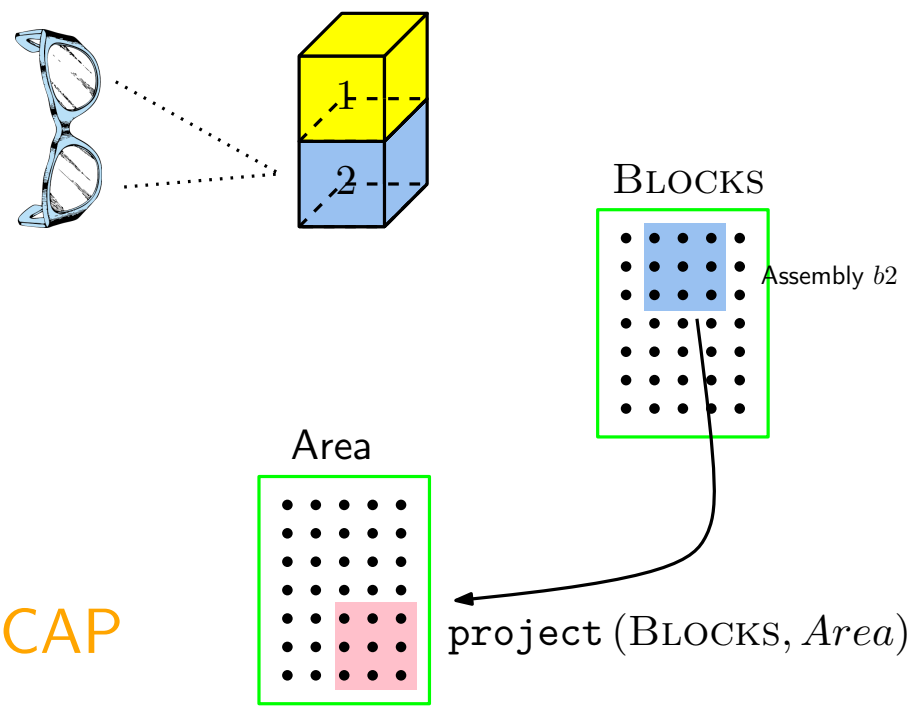
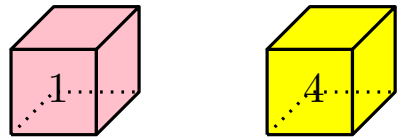
Each area has n neurons, the same Erdös-Renyi prob. p, the same k as the CAP

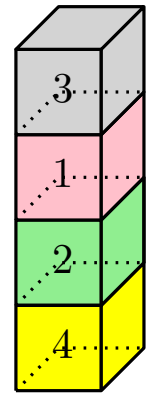


A Toy Example

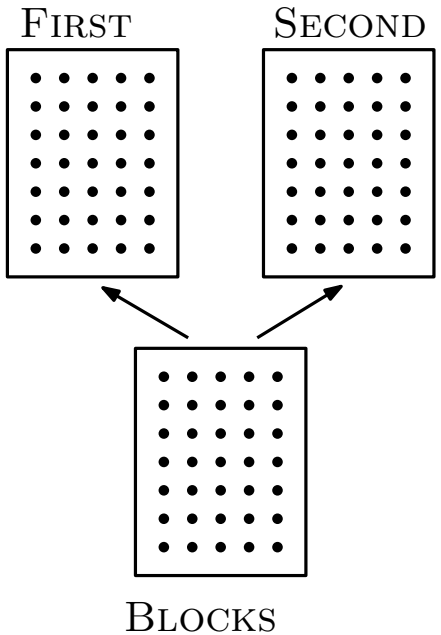
Given a stack of blocks and two of its blocks a, b*, is* a **above** b*?*

E.g., is block 1 above block 4?





Assumptions: 3 brain areas, Blocks, First, Second



IsAbove Algorithm

Algorithm 1: IsAbove (S, x, y)

input: a stack S of blocks $b_1, b_2, \ldots, b_s$;
two blocks x, y with $x, y \in S$.

```
1  disinhibitArea({Blocks, First});
2  disinhibitFiber({(Blocks, First)});
3  fire(x);
4  strongProject();
5  inhibitArea({First});
6  disinhibitArea({Second});
7  disinhibitFiber({(Blocks, Second)});
8  fire(y);
9  strongProject();
10 foreach i with 2 ≤ i ≤ s do
11     inhibitArea({Second});
12     disinhibitArea({First});
13     fire(b_i);
14     if read(First) then return true ;
15     inhibitArea({First});
16     disinhibitArea({Second});
17     fire(b_i);
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19 end
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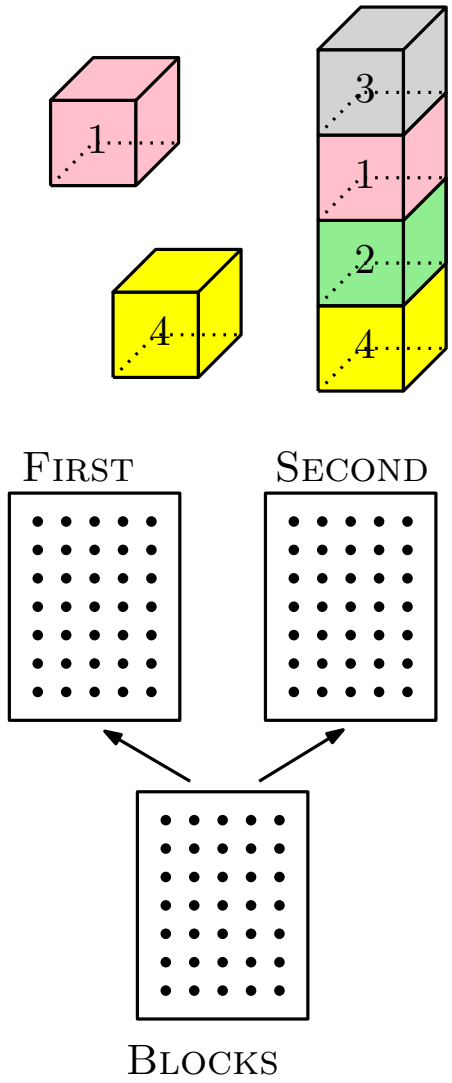

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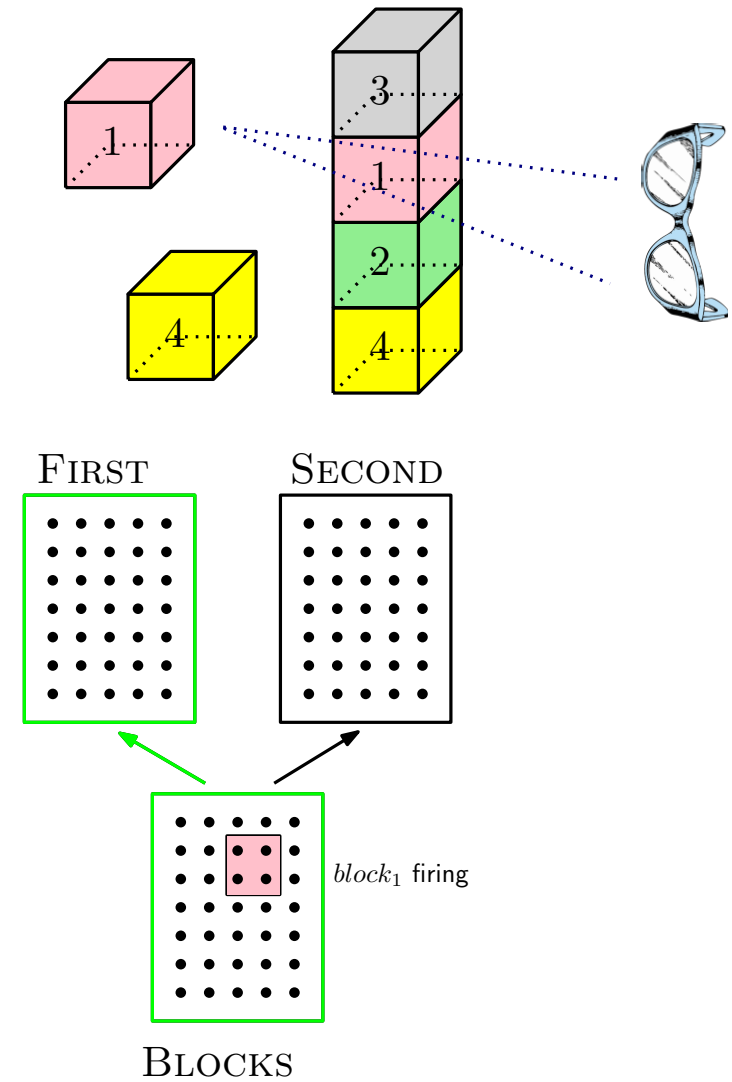

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4  strongProject();
5  inhibitArea({First});
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7  disinhibitFiber({(Blocks, Second)});
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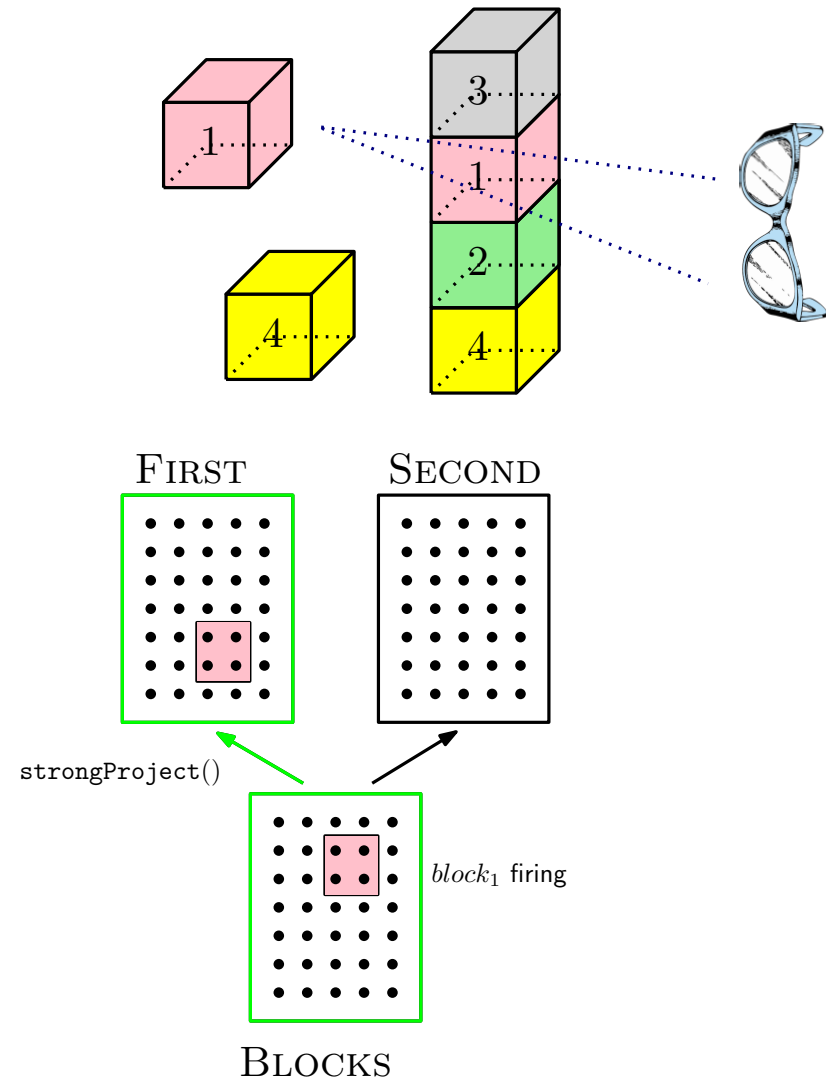

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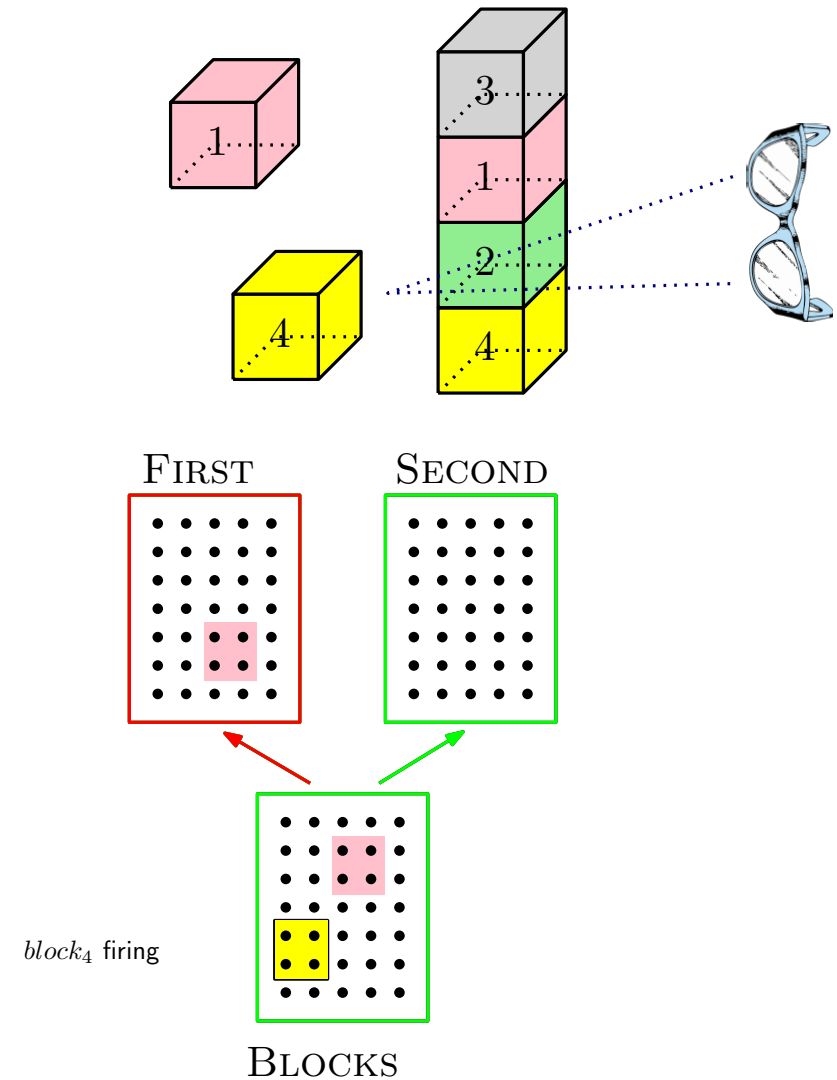

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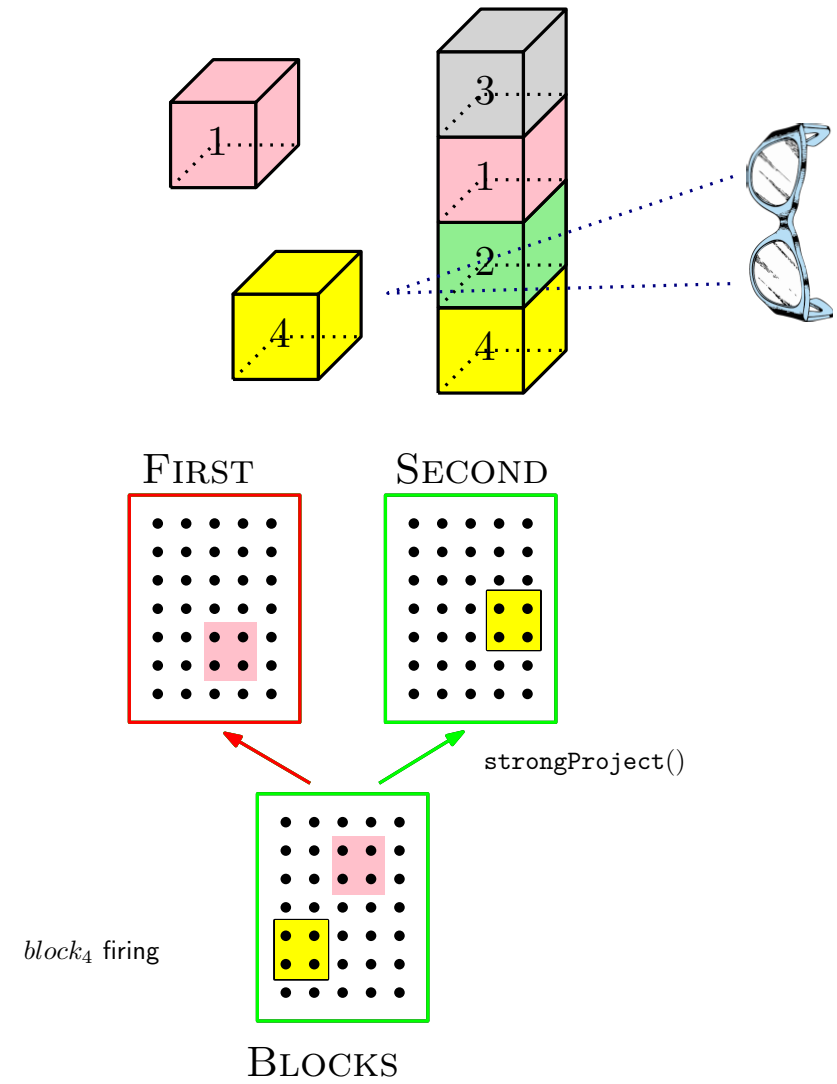

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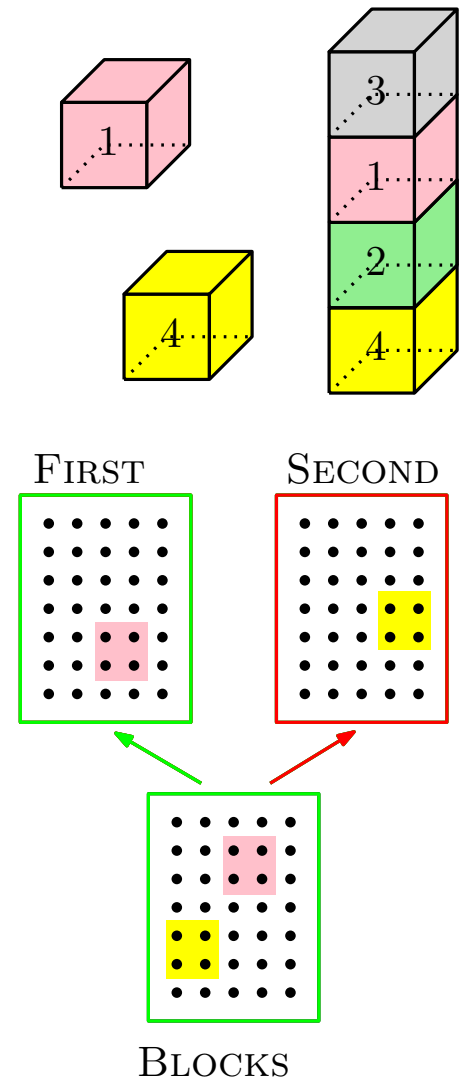

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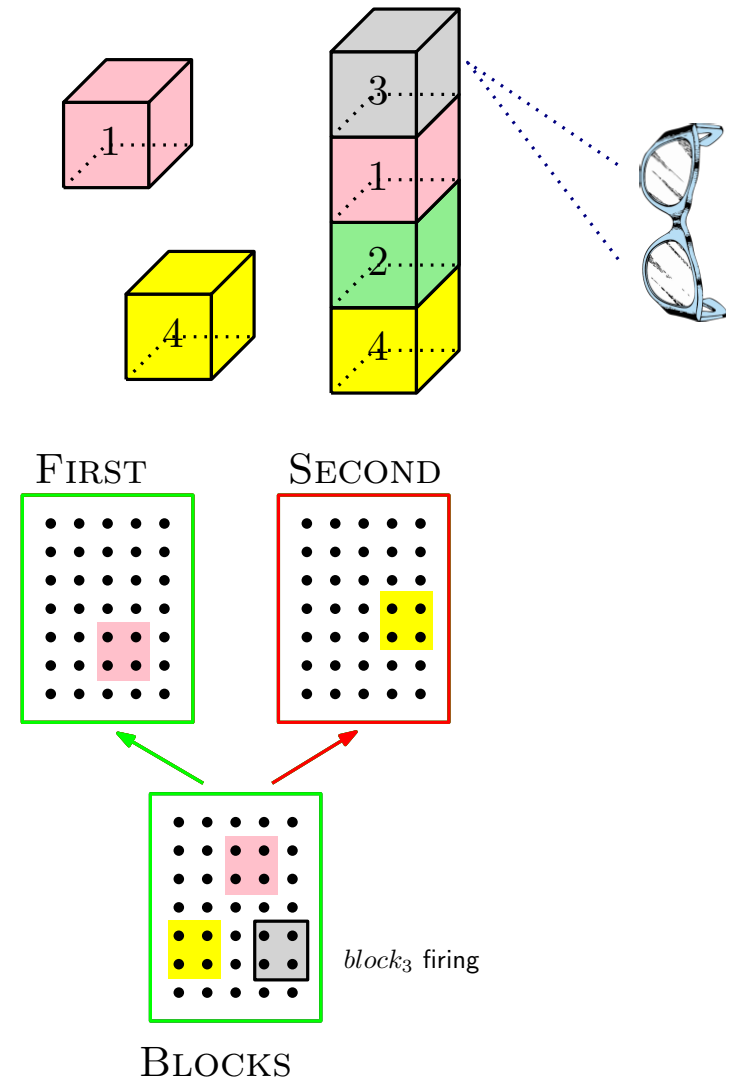

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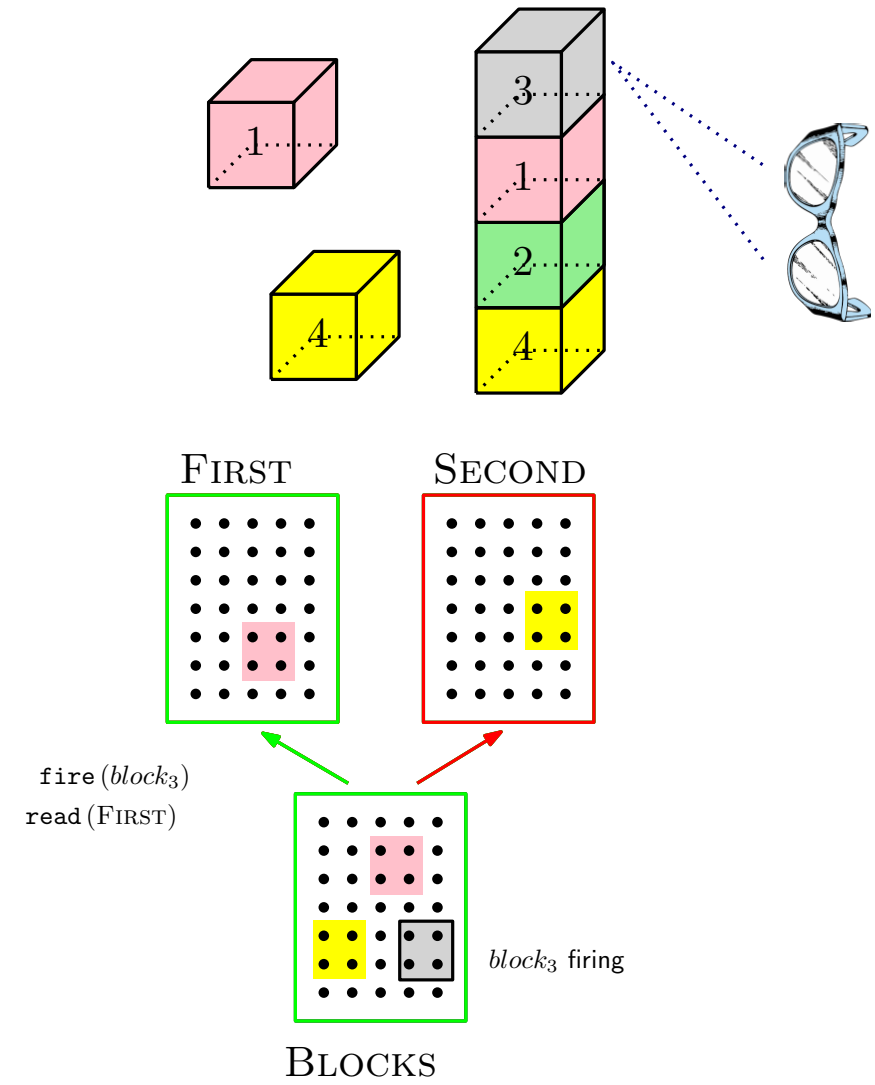

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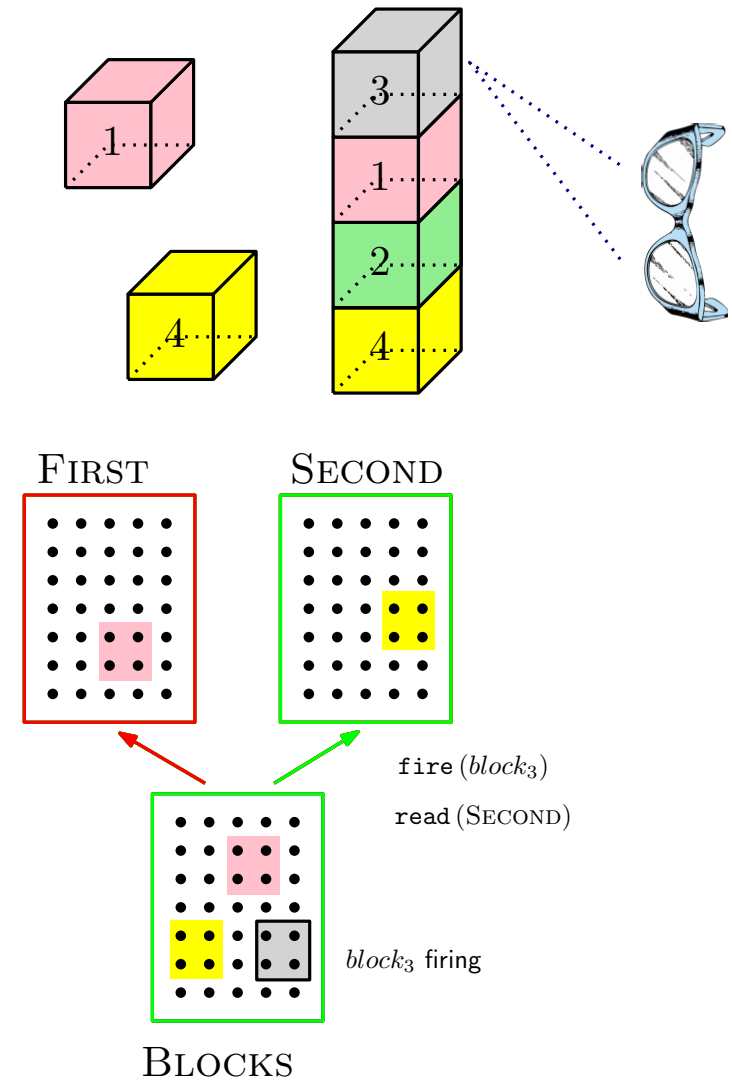

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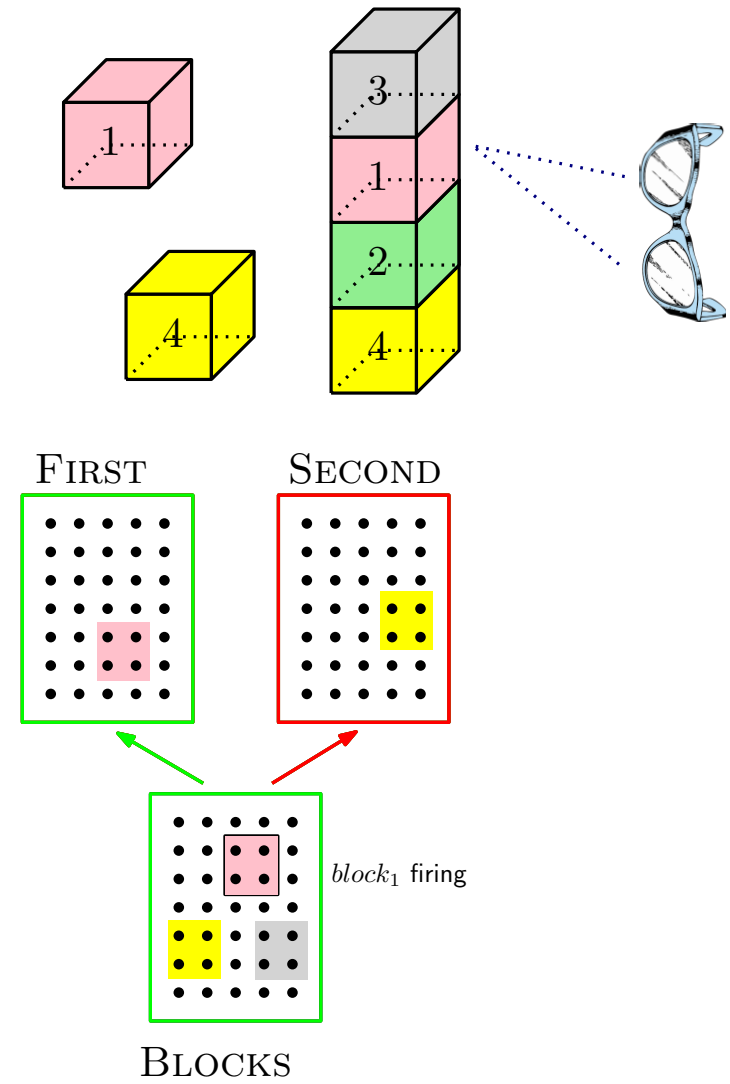

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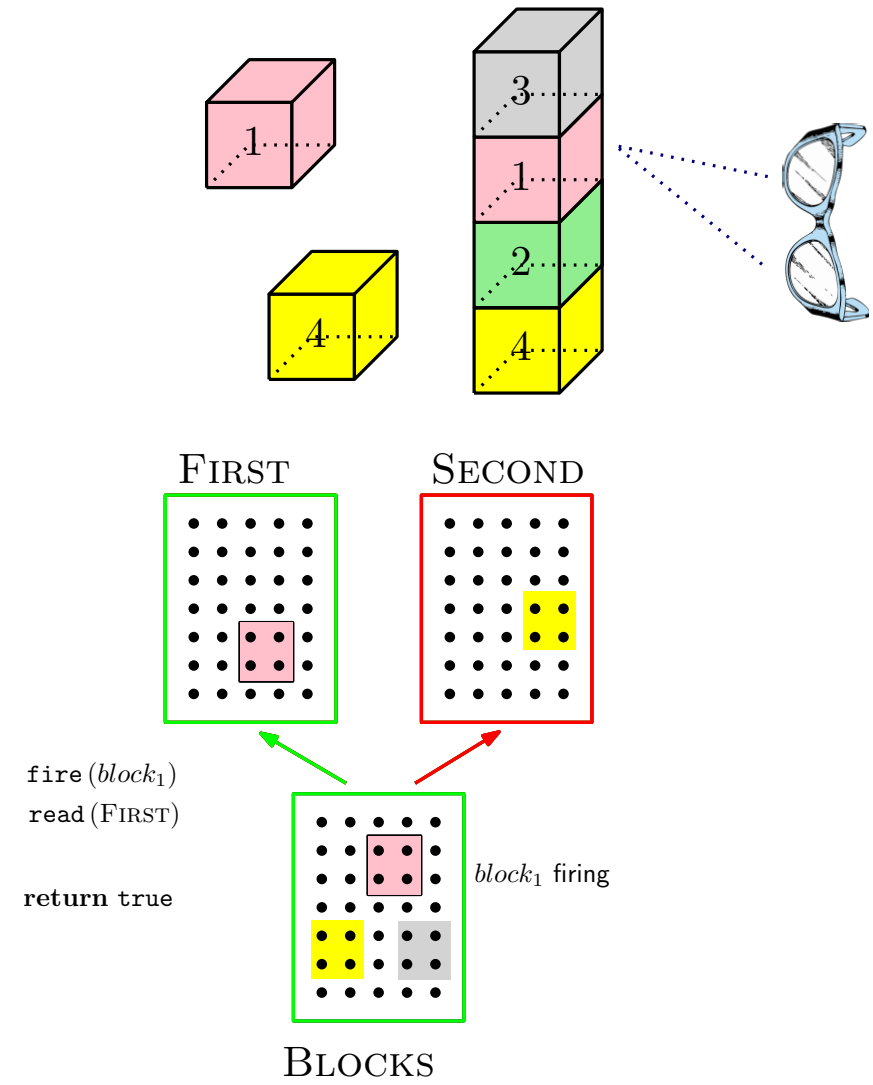

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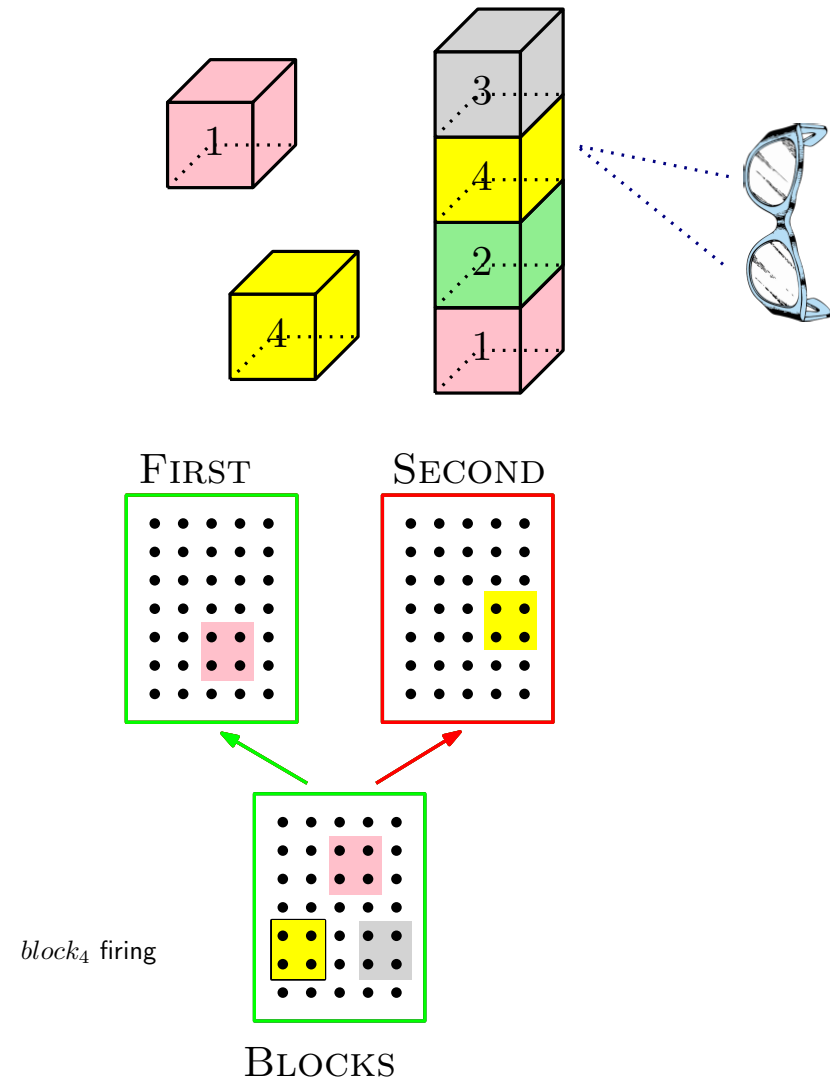

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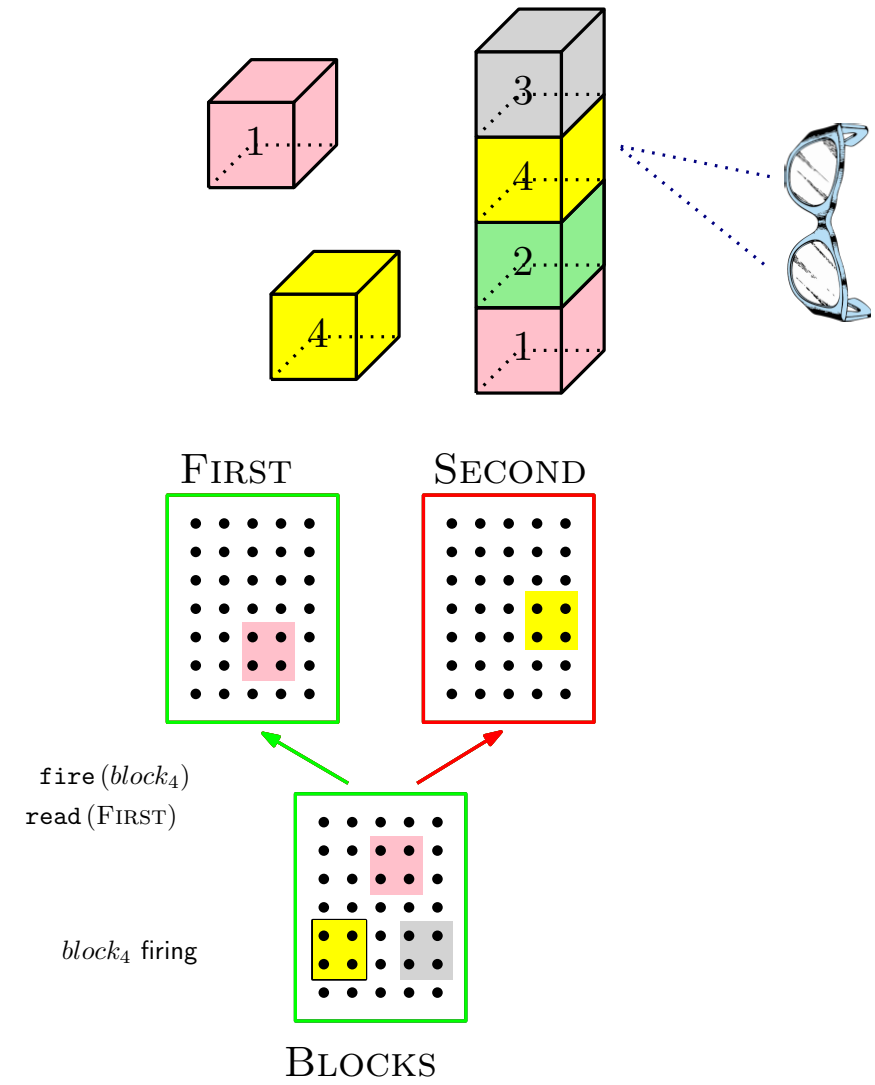

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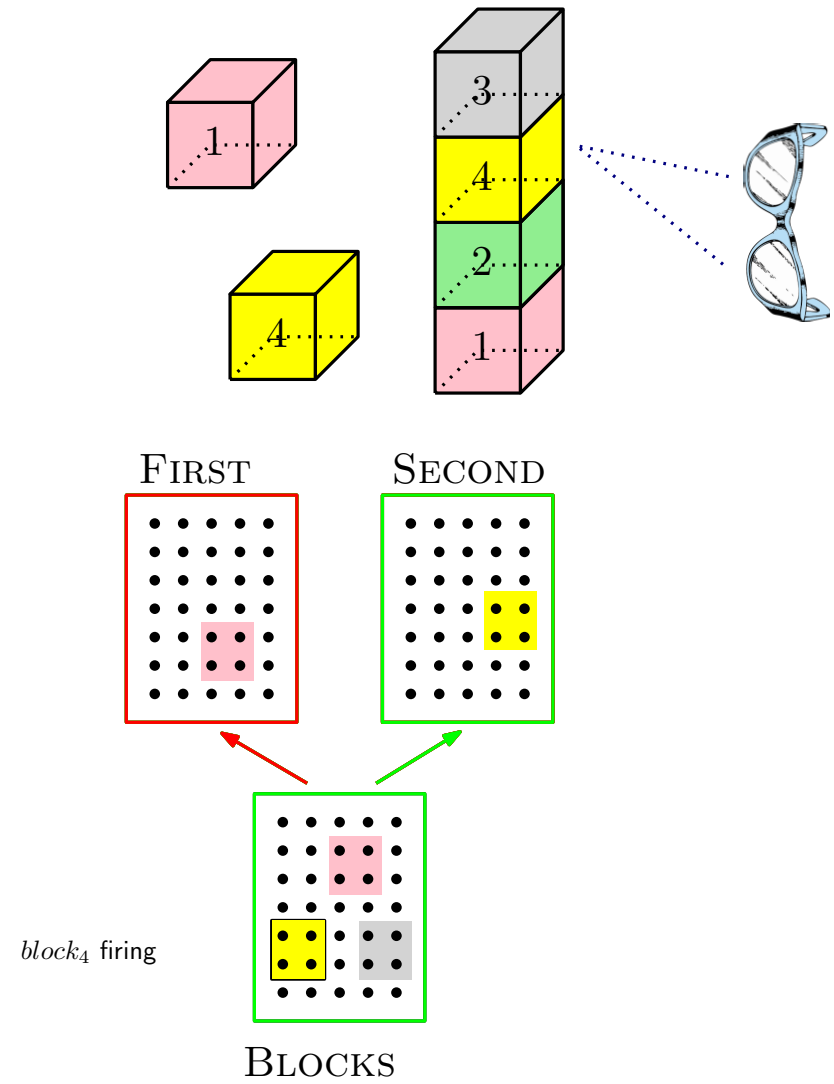

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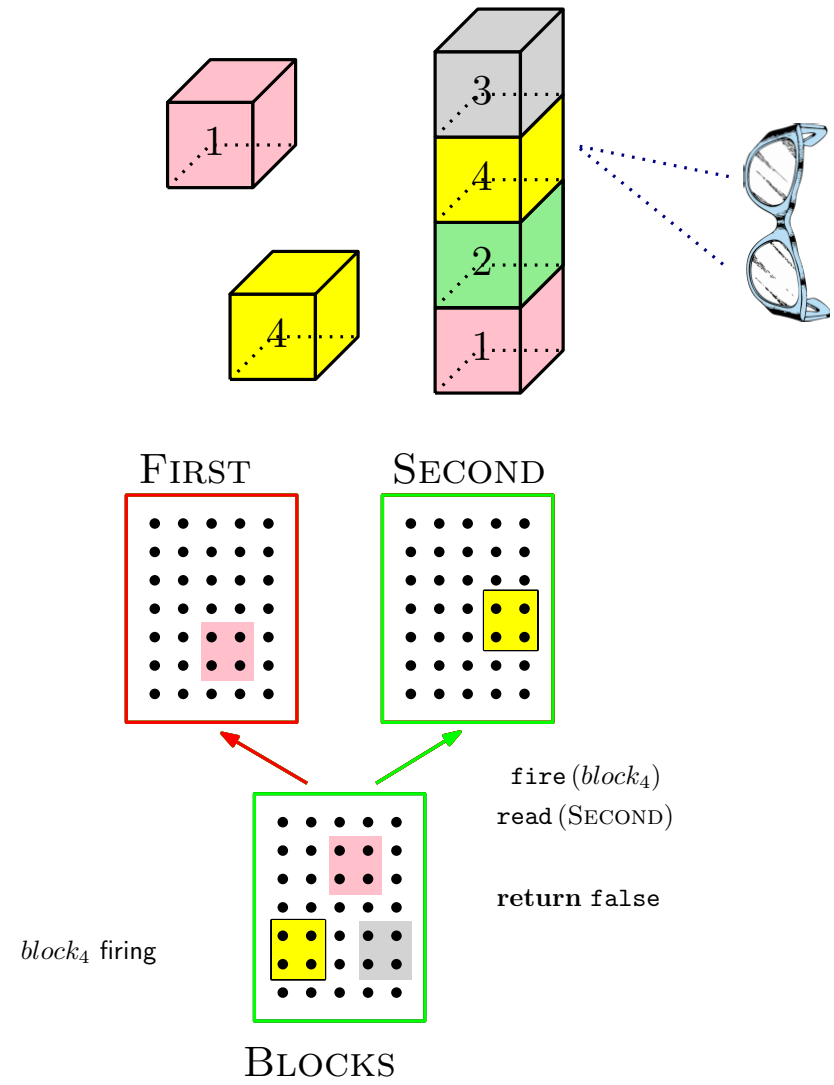

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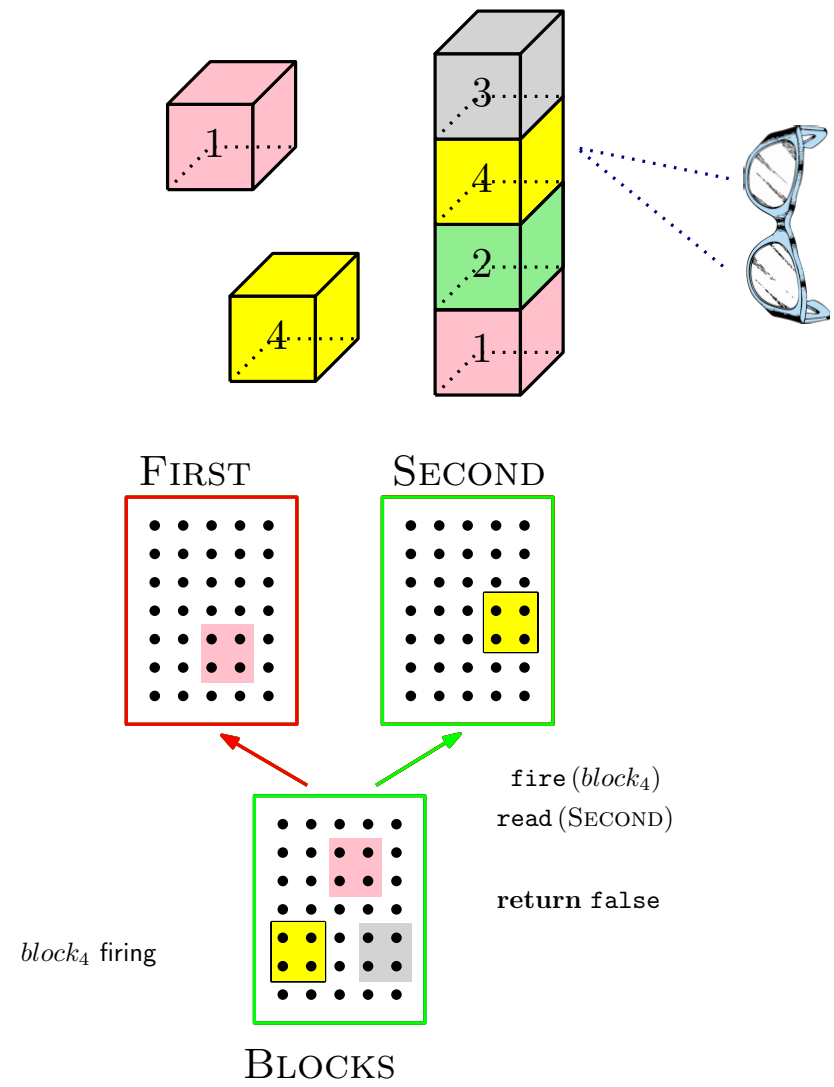
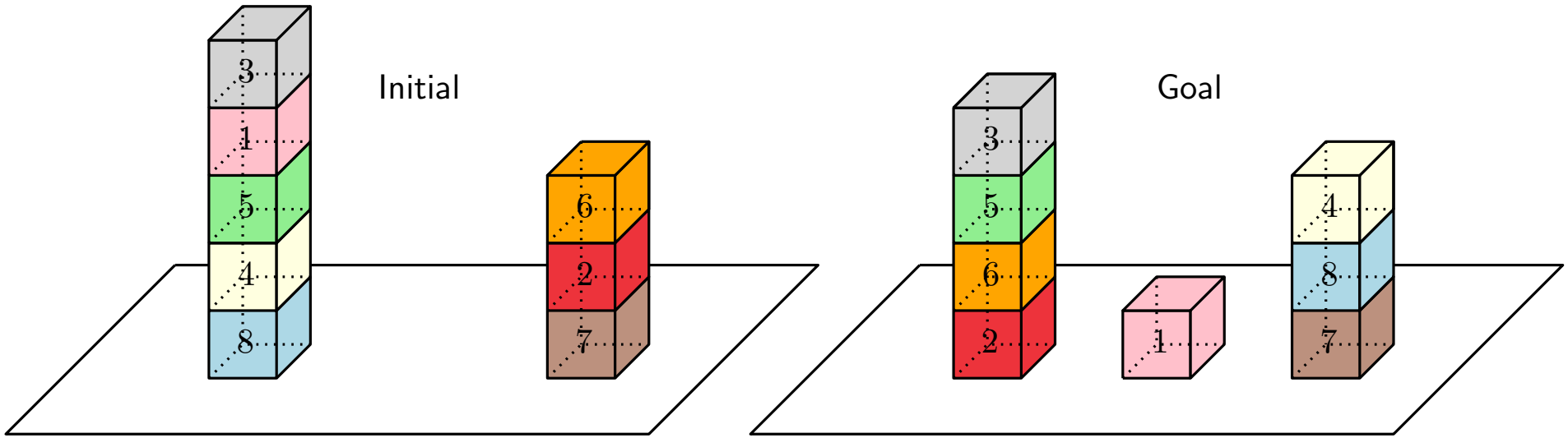


How does the brain understand a positive (negative) answer?

The answer corresponds to an assembly activating in a particular brain area

Back to the Planning Problem



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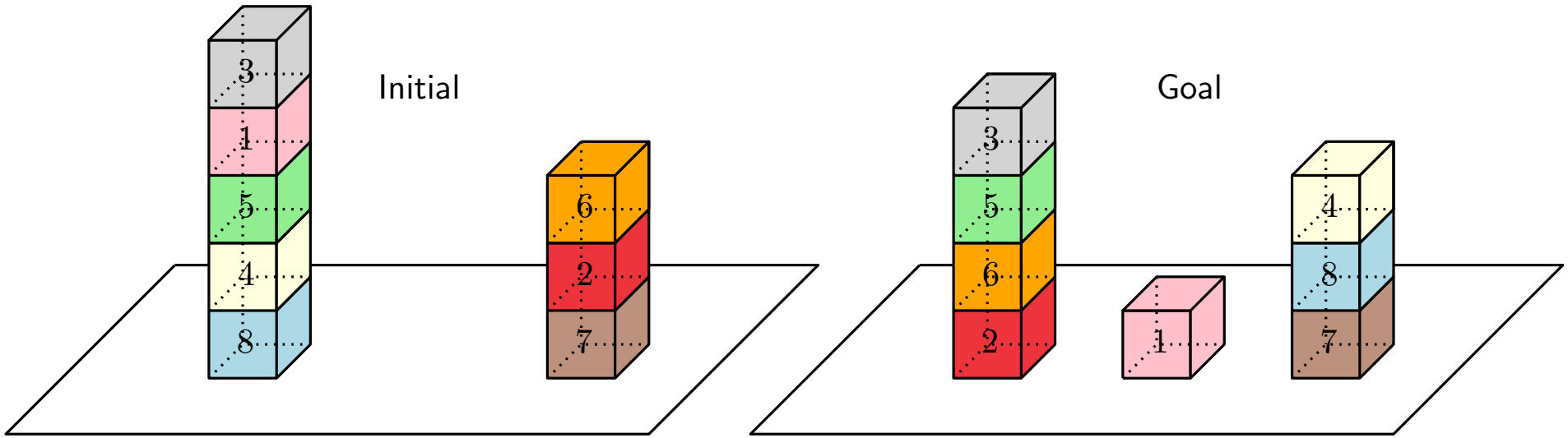

- **First**: way to represent the initial and the goal stacks in the brain

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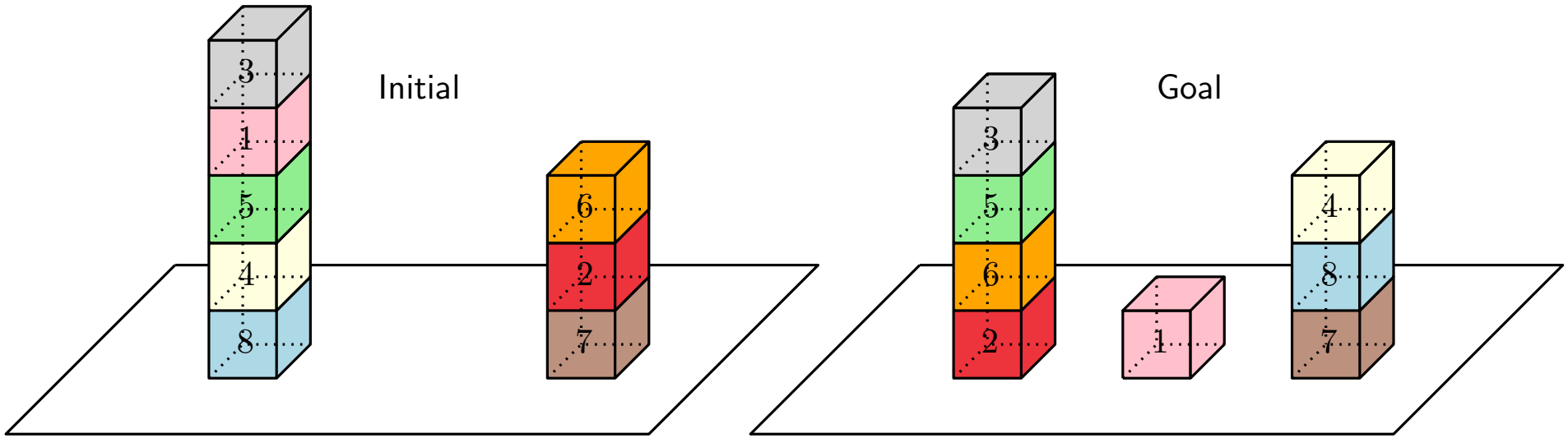

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A Parser

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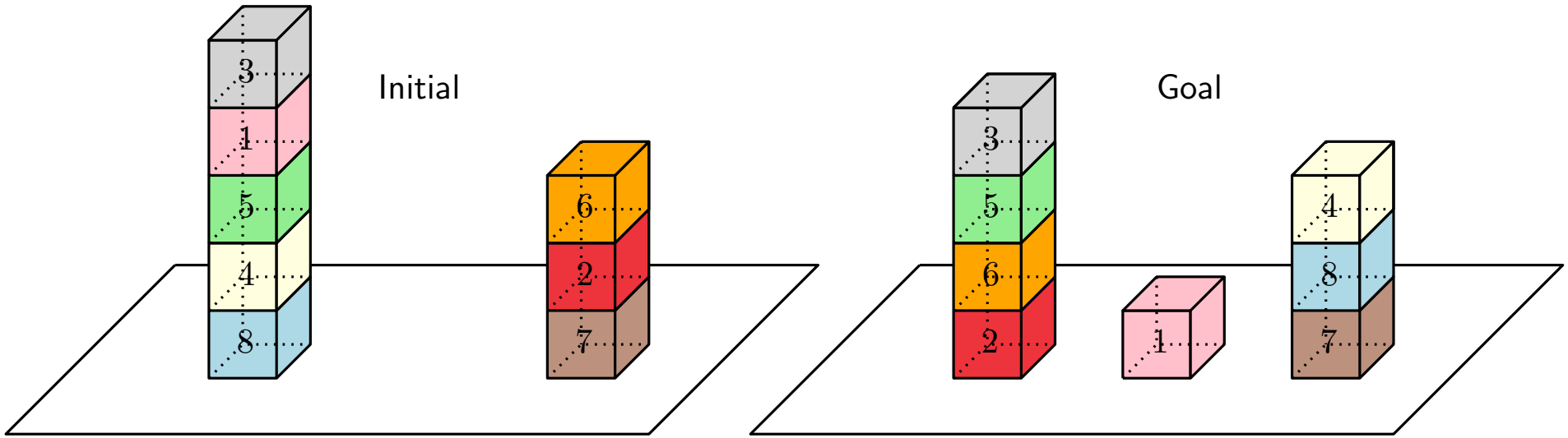


- **First**: way to represent the initial and the goal stacks in the brain

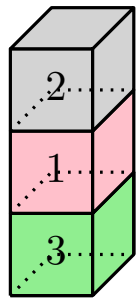
A Parser

- **Second**: way to implement actions and represent them

The Parser

Assumptions:

- single stack
- brain as undirected graph
- 5 brain areas, BLOCKS, NODE_0, NODE_1, NODE_2, HEAD



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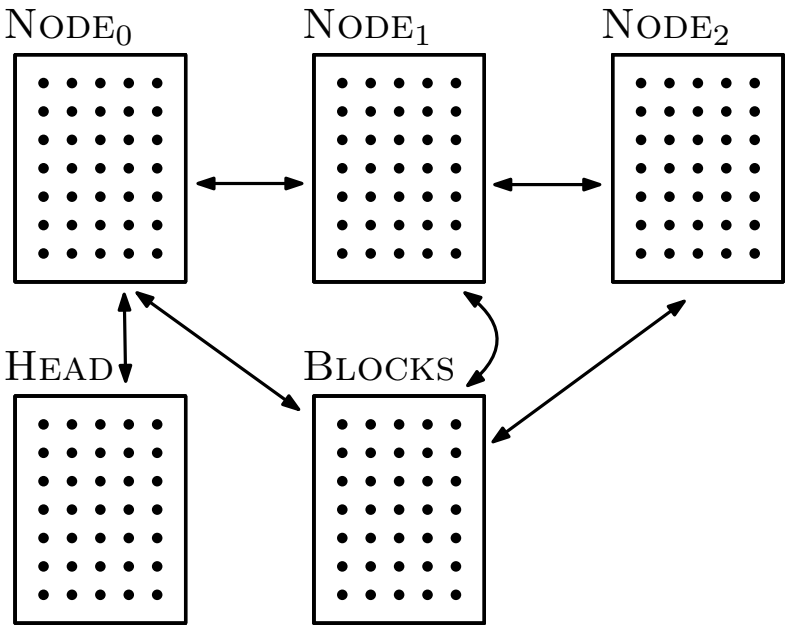

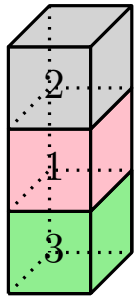

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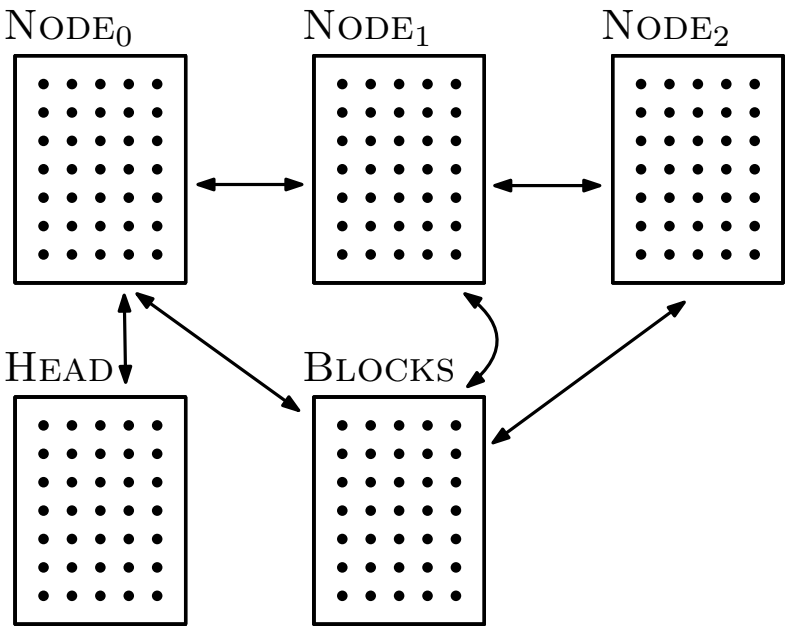

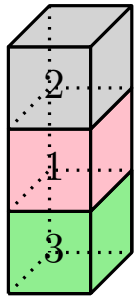

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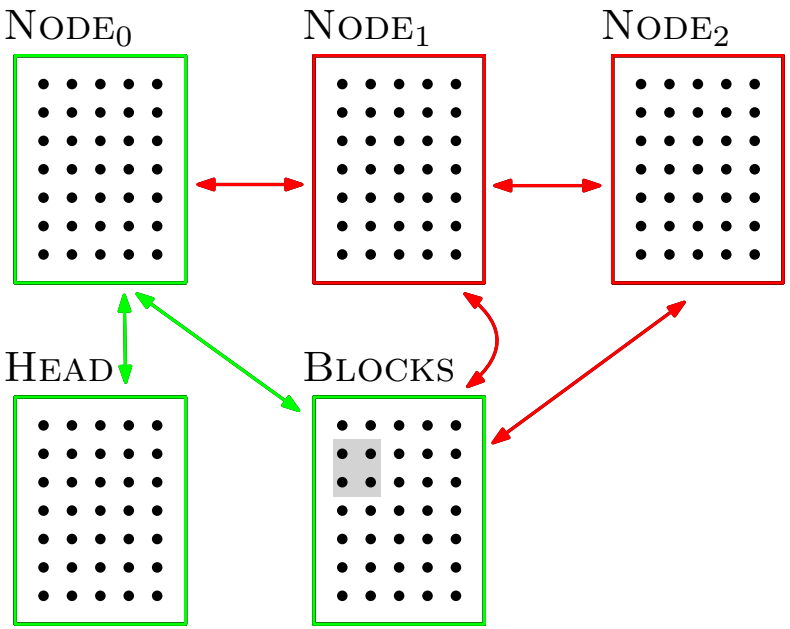

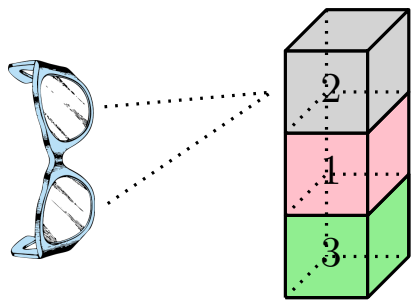

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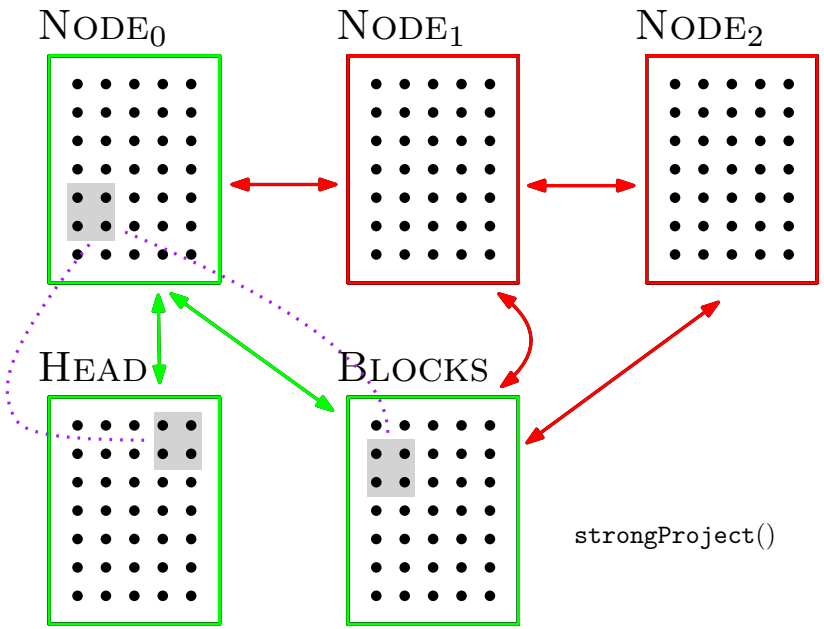

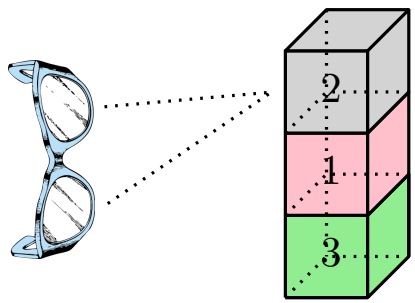

The Parser

Assumptions:

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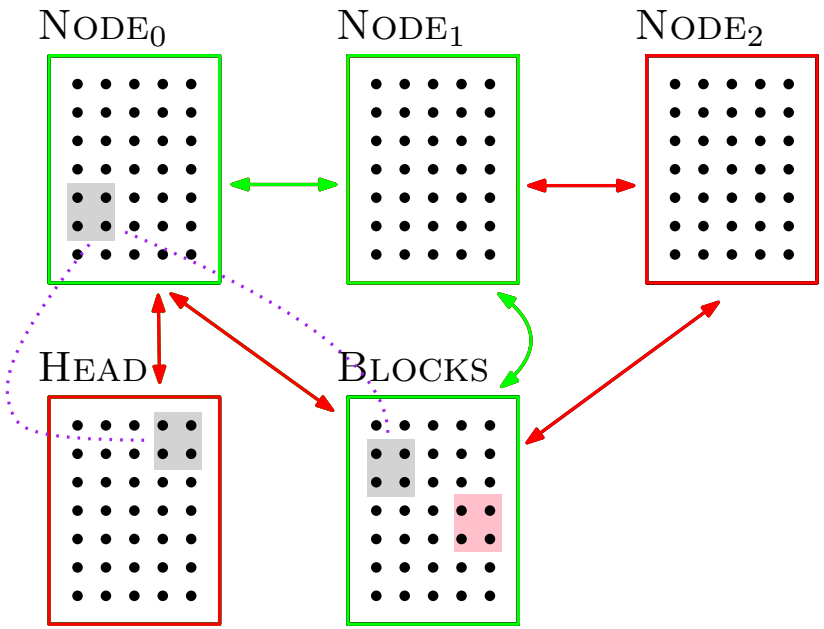

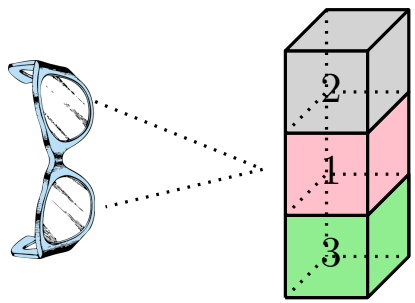

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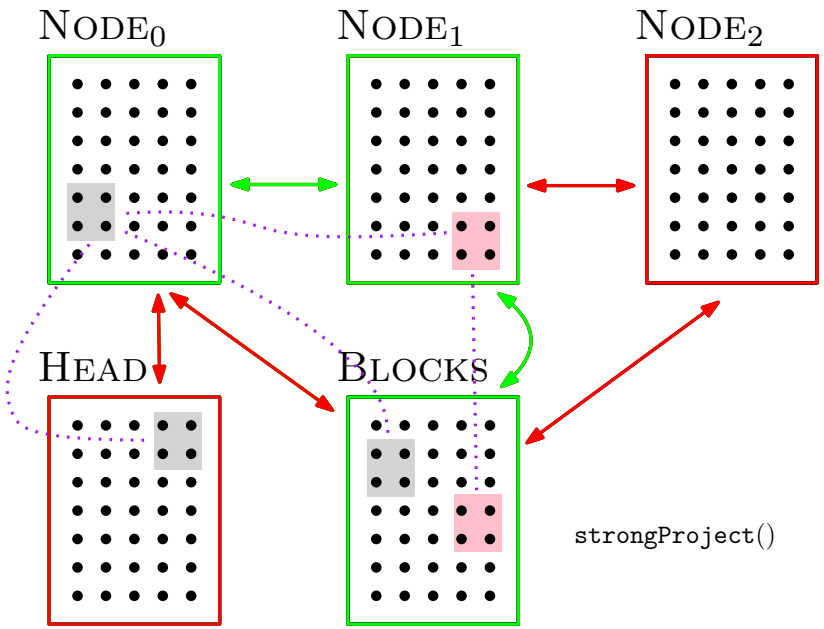

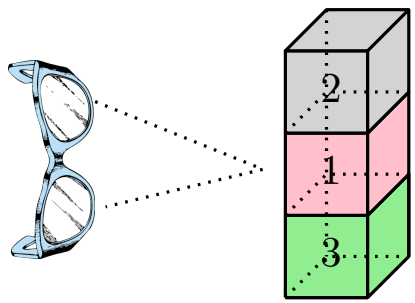

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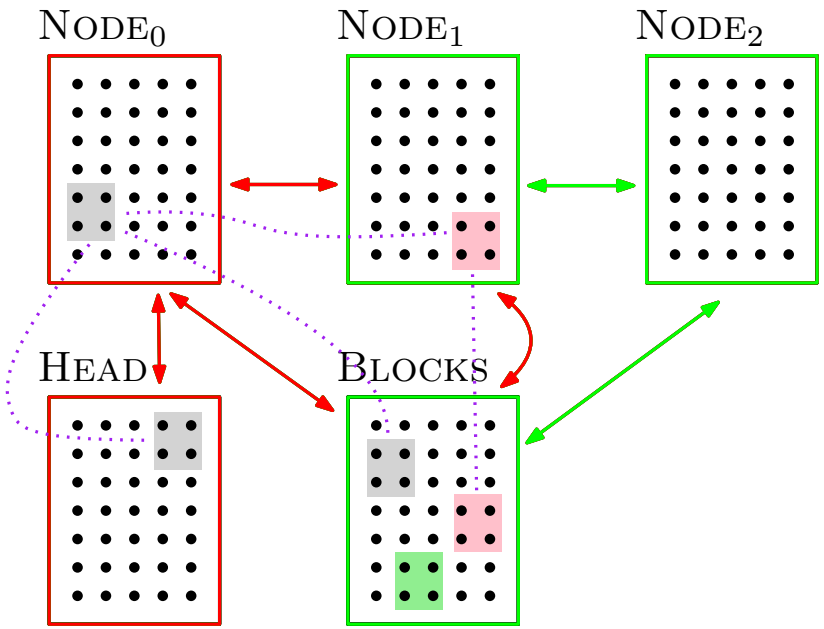

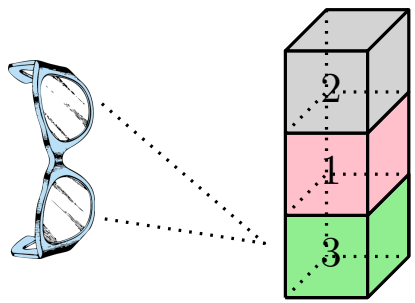

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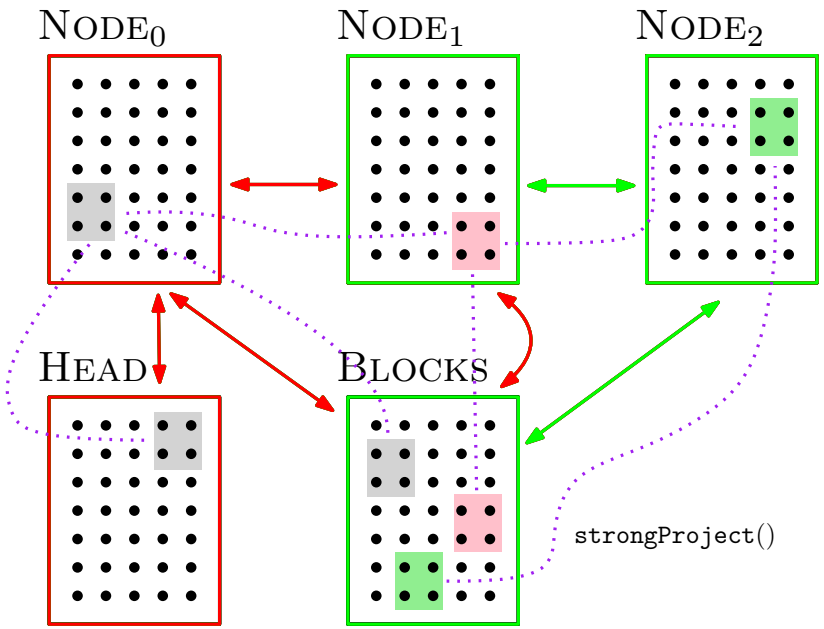

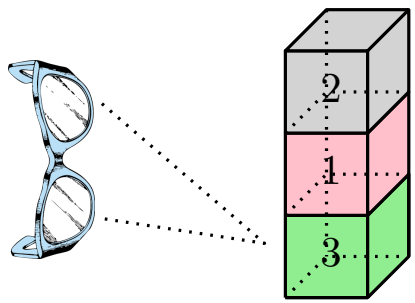

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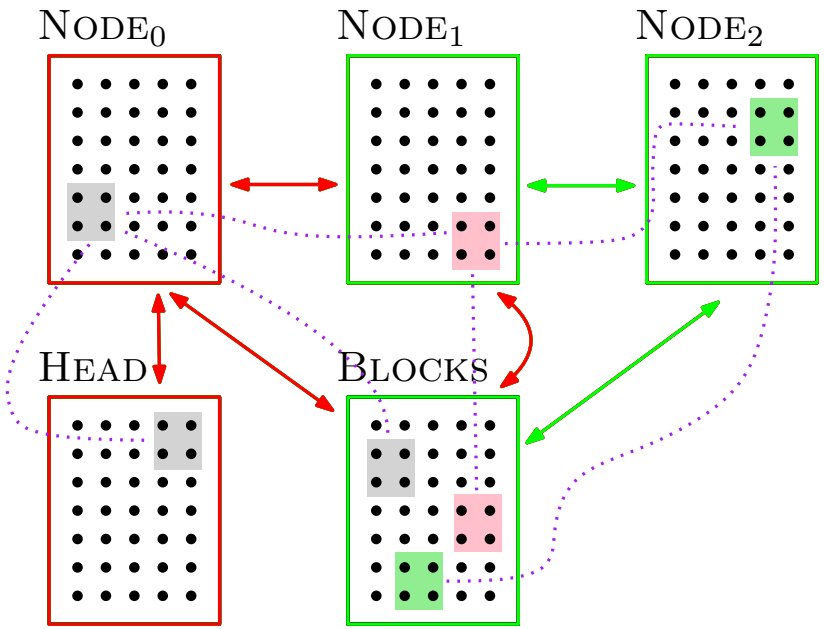

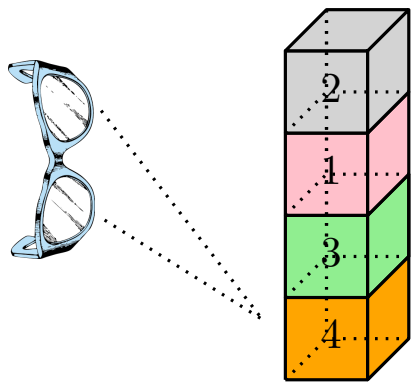

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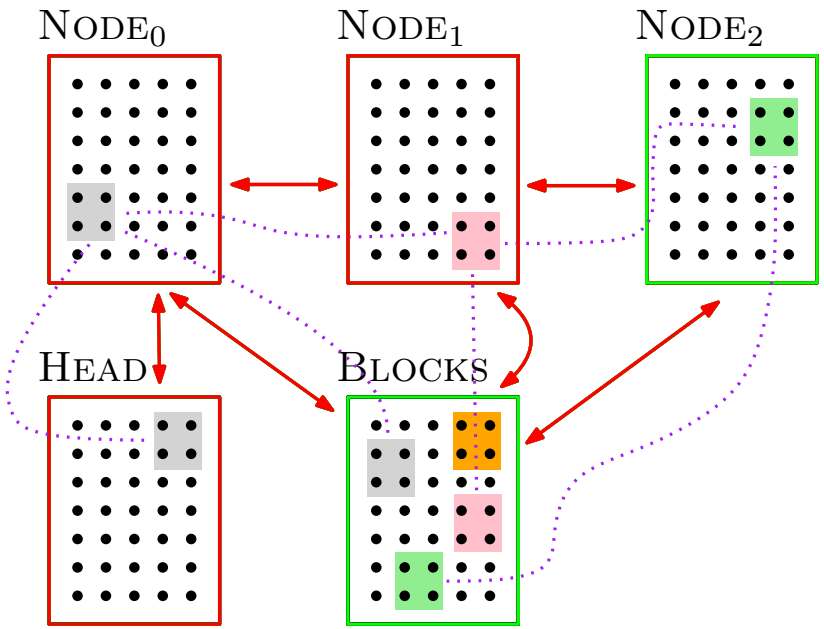

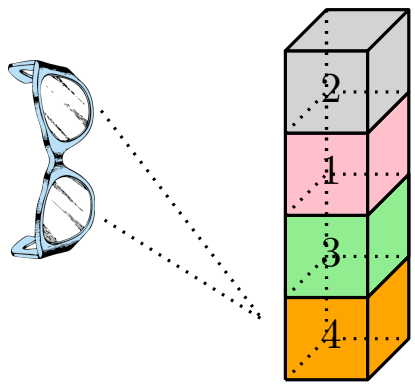

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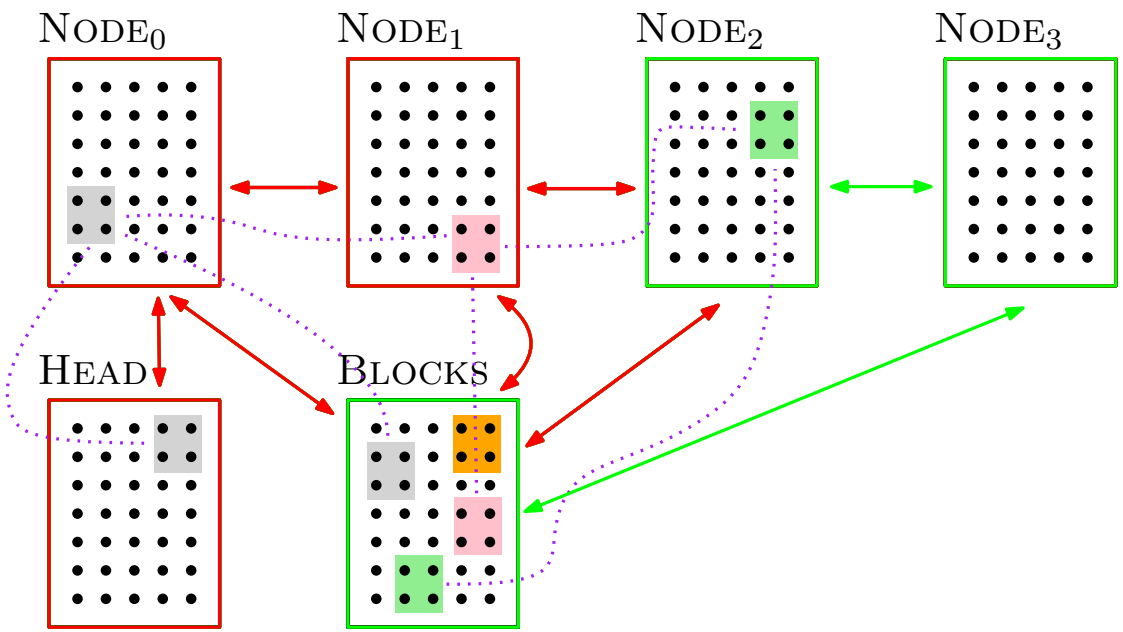

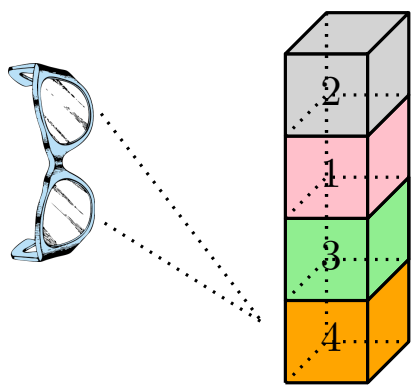

As many areas as the number of blocks... :(

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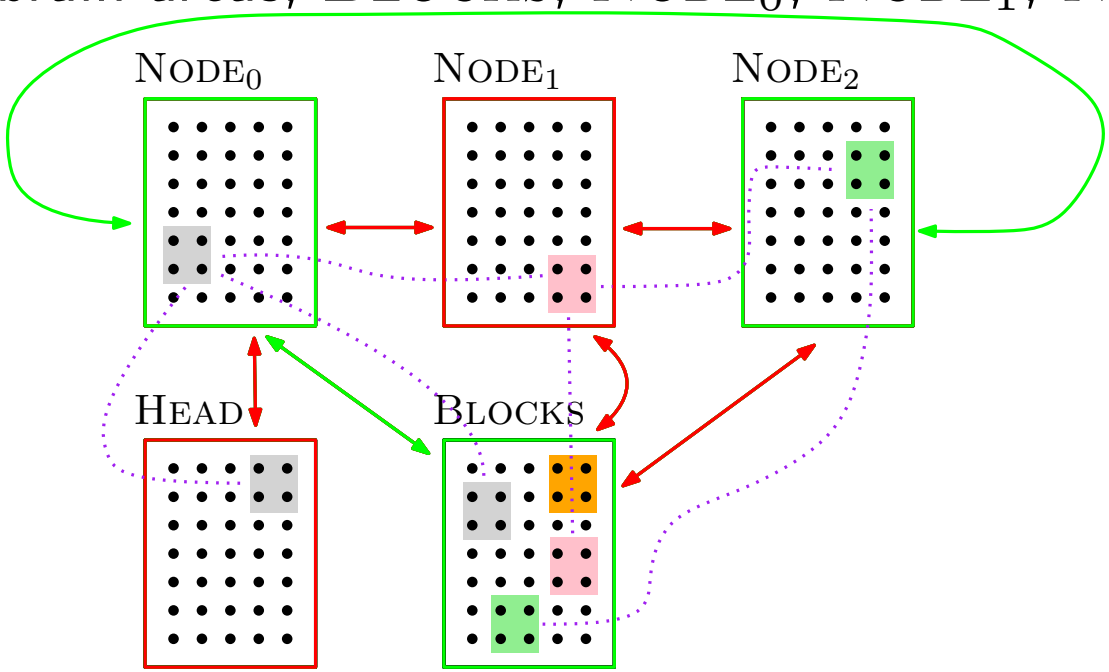

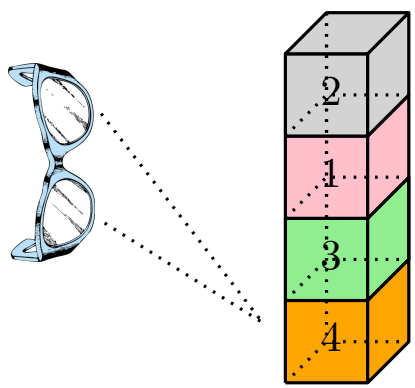

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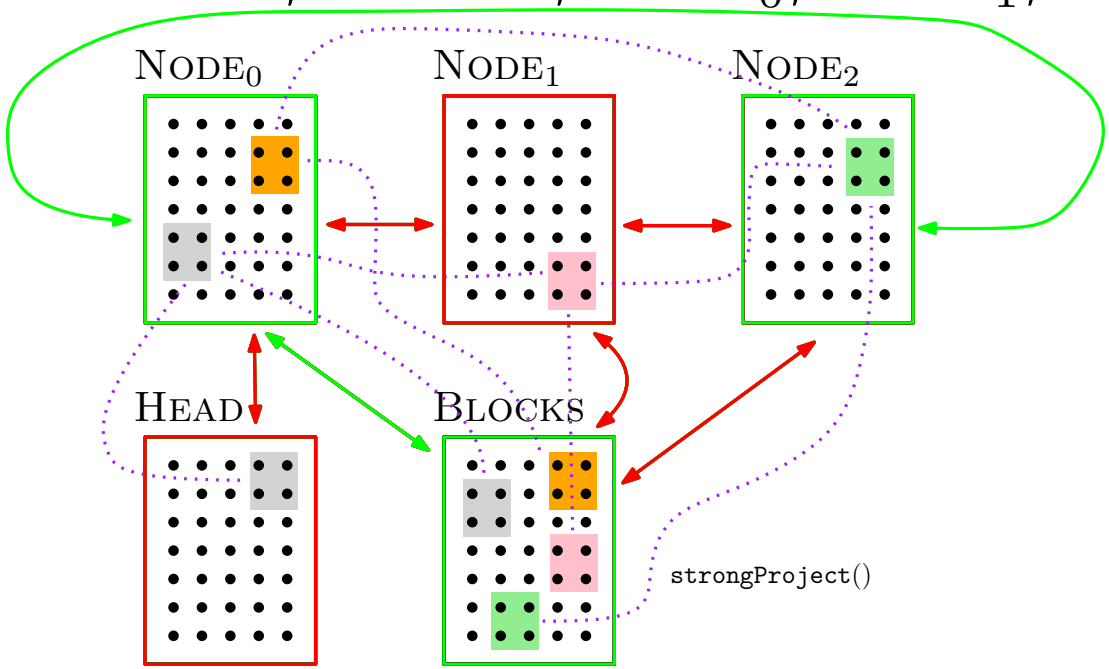

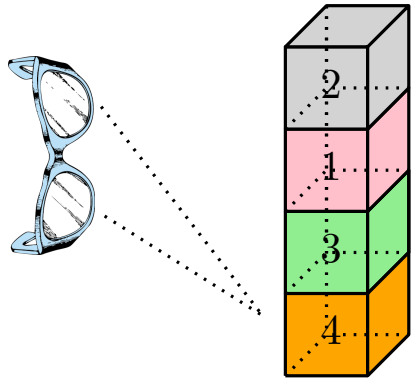

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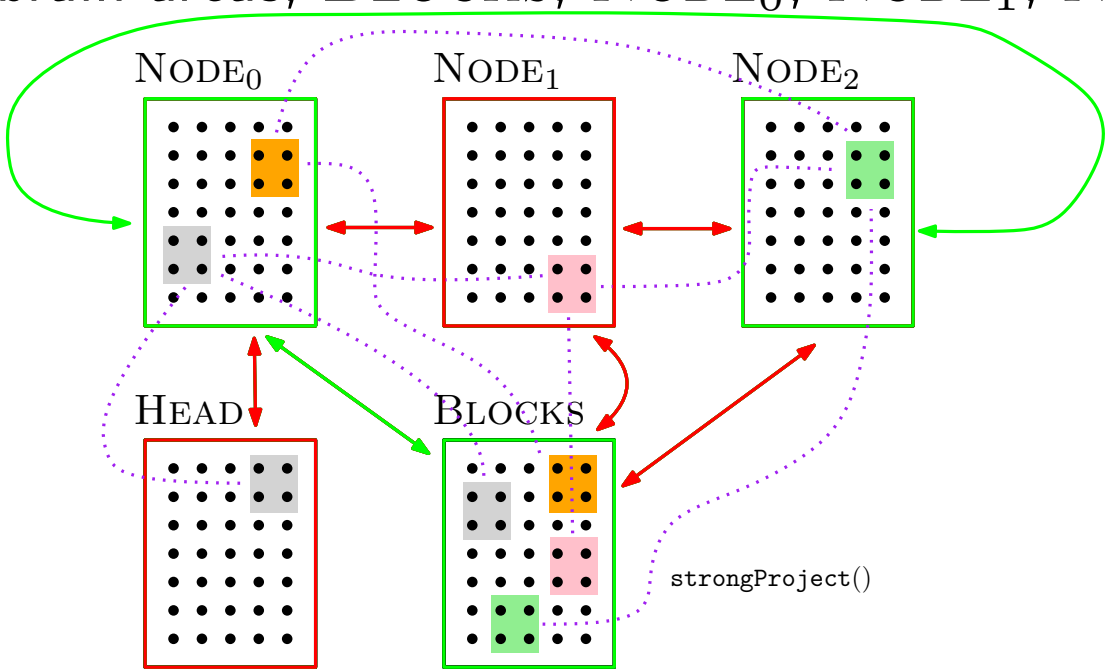
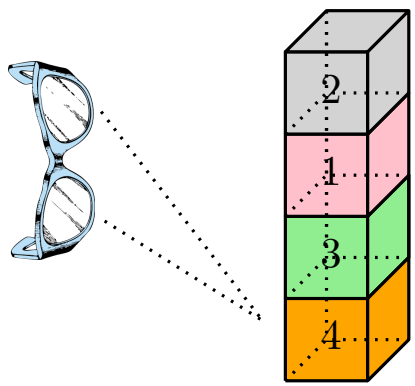




Chaining: novelty. Number of blocks represented depends only on n (number of neurons per area) and k (the CAP)

Parser Code

Algorithm 2: PARSER (S)

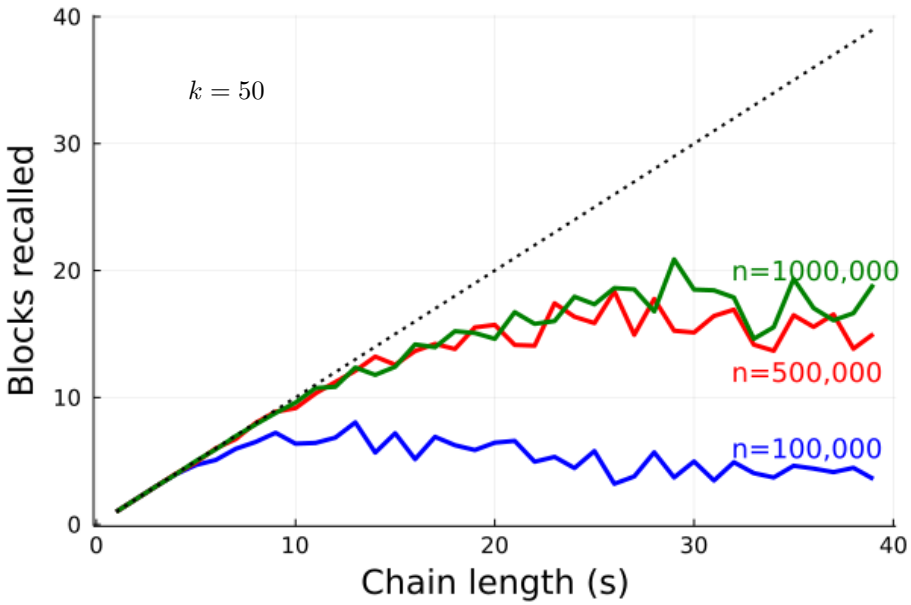
input: a stack S of blocks $b_1, b_2, \ldots, b_s$.

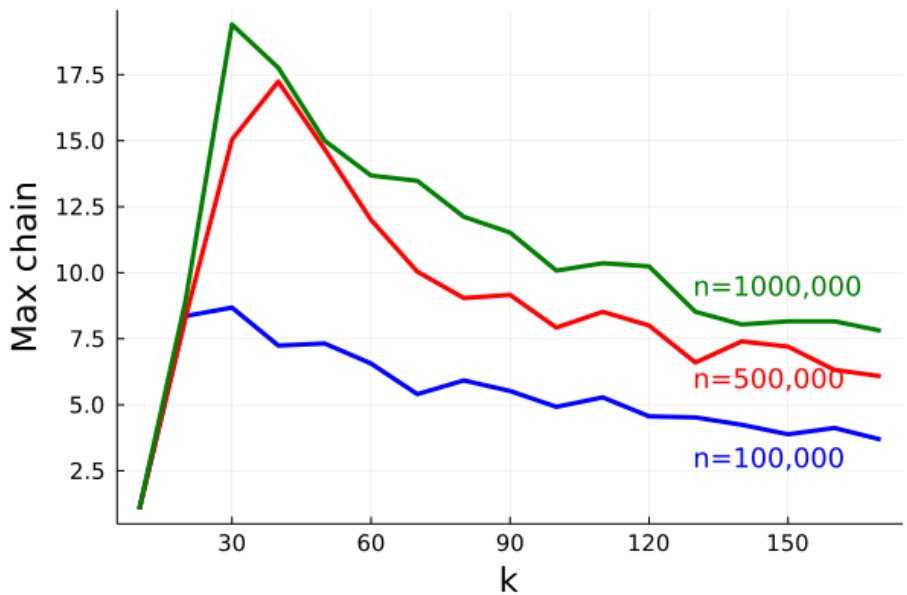
```
1  disinhibitArea({BLOCKS, HEAD, NODE_0});
2  disinhibitFiber({(HEAD, NODE_0), (NODE_0, BLOCKS)});
3  fire(b_1);
4  strongProject();
5  inhibitArea({HEAD});
6  inhibitFiber({(HEAD, NODE_0), (NODE_0, BLOCKS)});
7  foreach i with 2 ≤ i ≤ s do
8      p = (i − 2) mod 3; c = (i − 1) mod 3;
9      disinhibitArea({NODE_c});
10     disinhibitFiber({(NODE_p, NODE_c), (NODE_c, BLOCKS)});
11     fire(b_i);
12     strongProject();
13     inhibitArea({NODE_p});
14     inhibitFiber({(NODE_p, NODE_c), (NODE_c, BLOCKS)});
15 end
16 inhibitArea({BLOCKS, NODE_(s−1) mod 3});
```

Chaining Experiments

Due to overlap of assemblies, chaining may fail

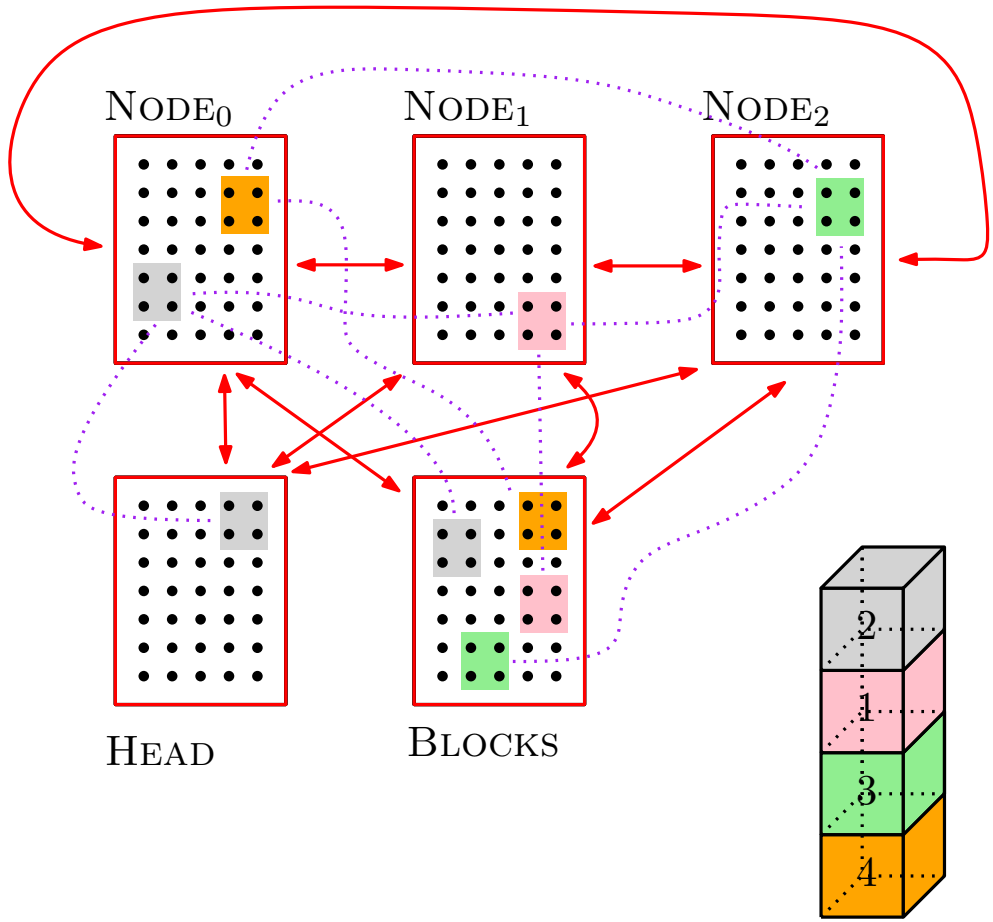
Experiments conducted averaging over 50 trials





Actions

- *Removing the top block*



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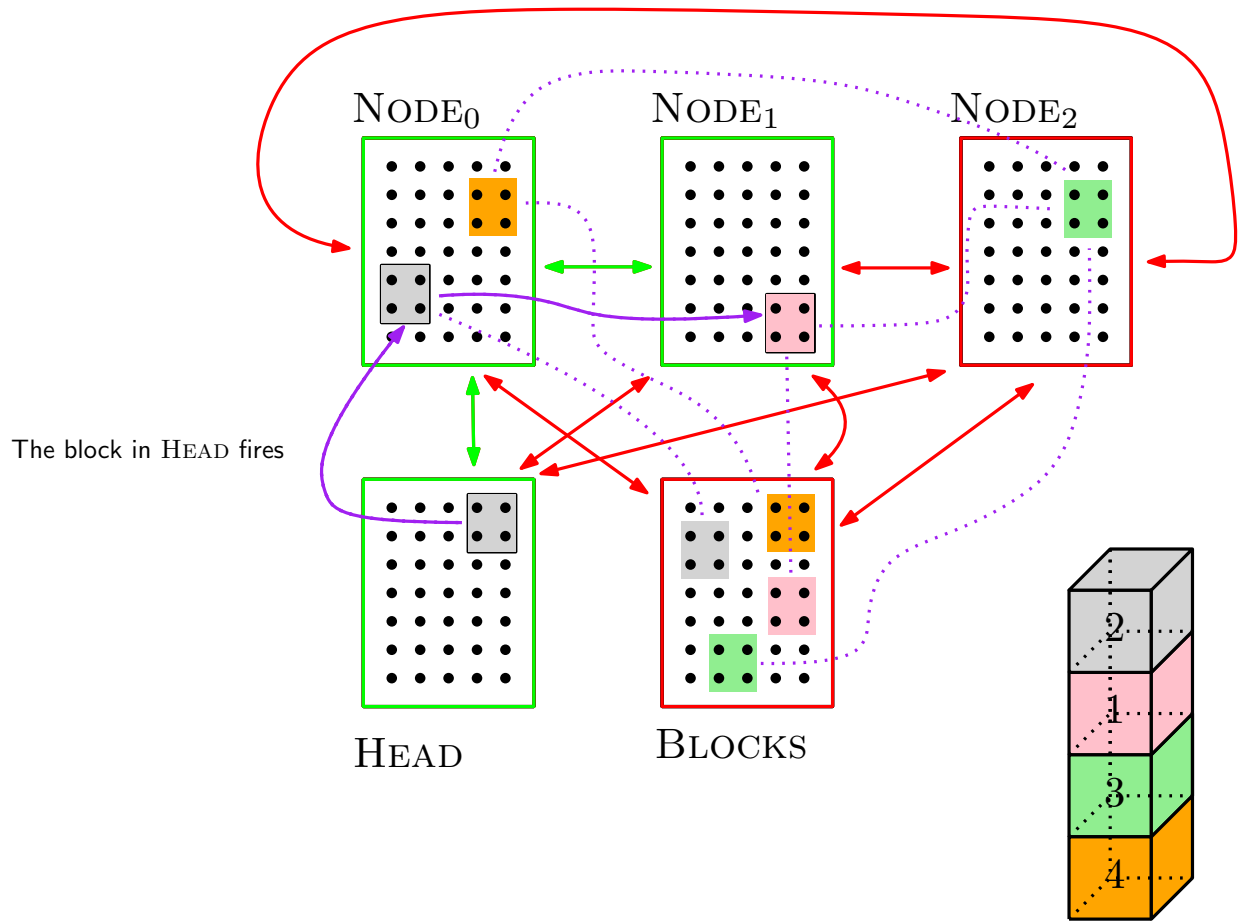


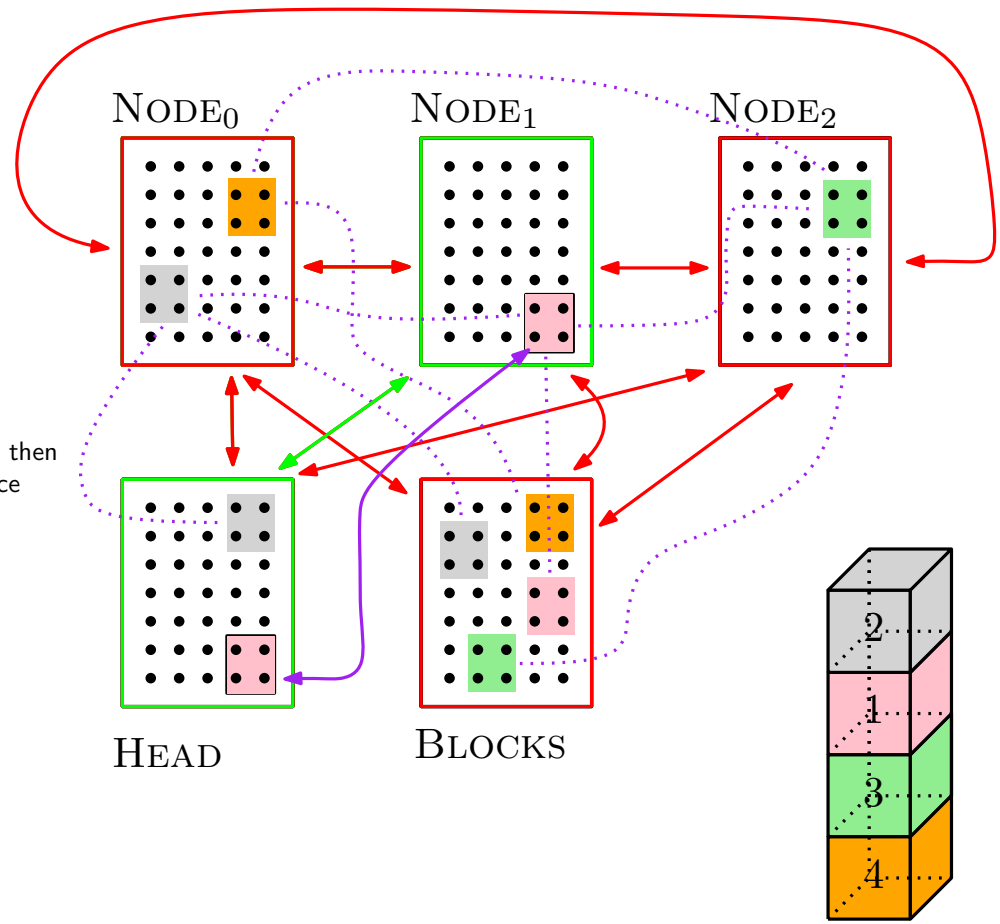
Actions

- *Removing the top block*

The last active assembly in HEAD will point to the new top

The model doesn't forget old connections, it simply doesn't use them again

Now, `project` $(\text{NODE}_1, \text{HEAD})$ and then `project` $(\text{HEAD}, \text{NODE}_1)$ to reinforce



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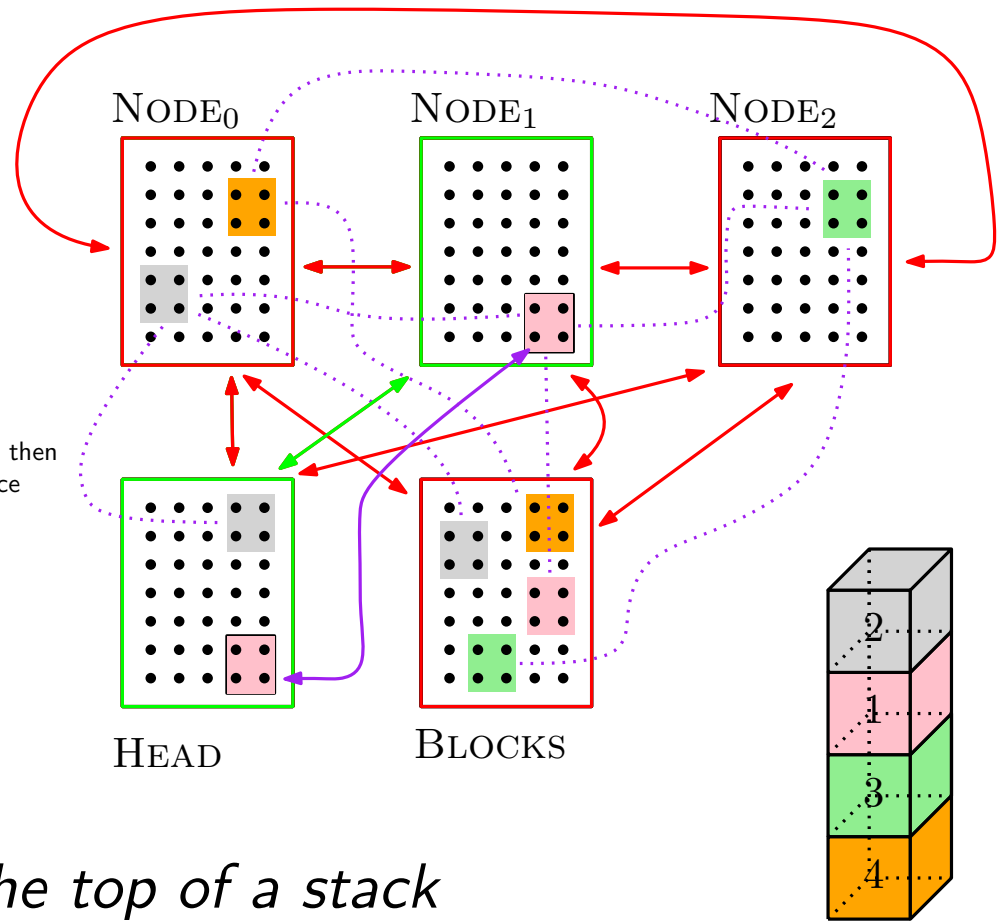
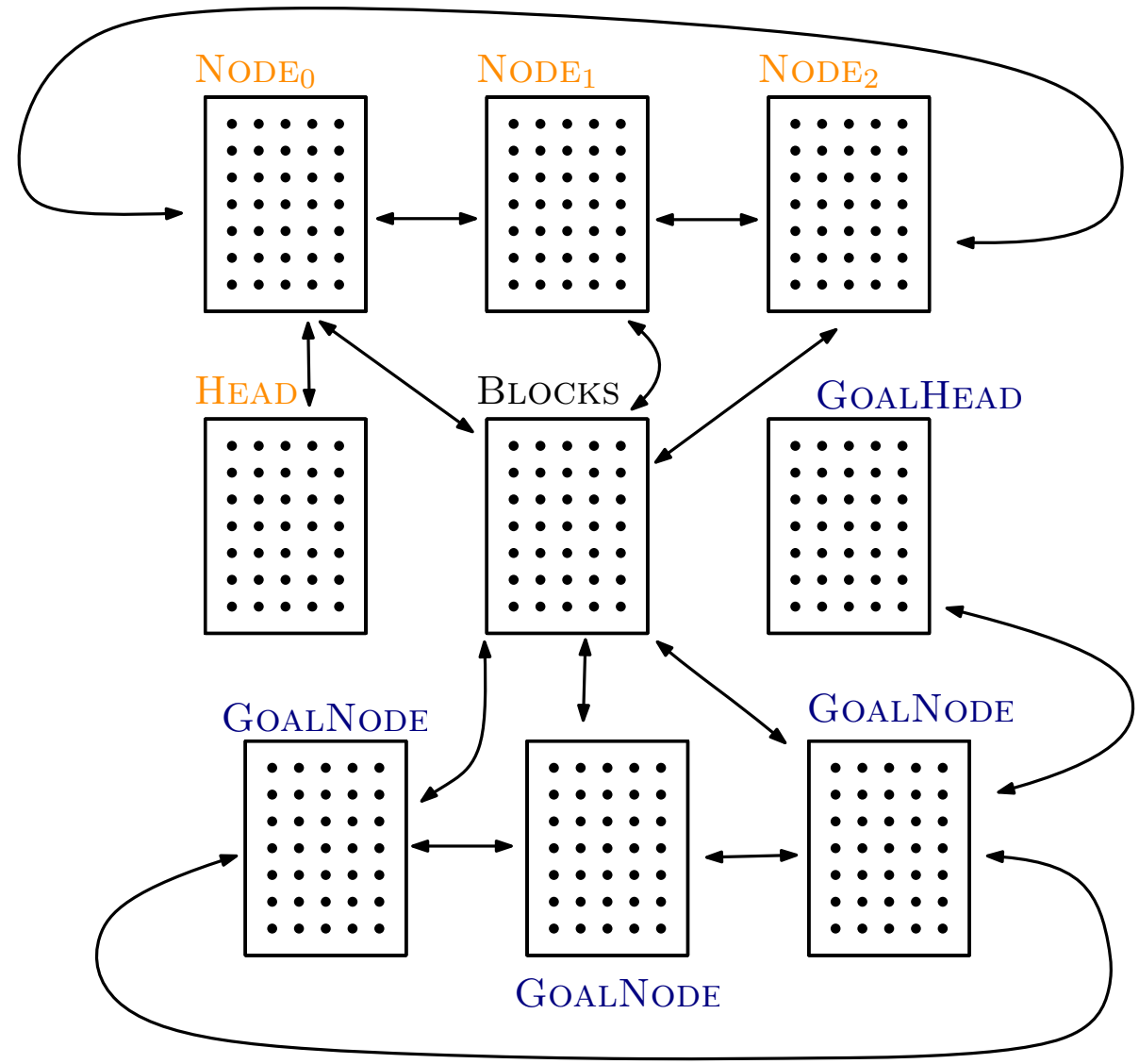


- *Put a block from the table to the top of a stack*

Comparisons

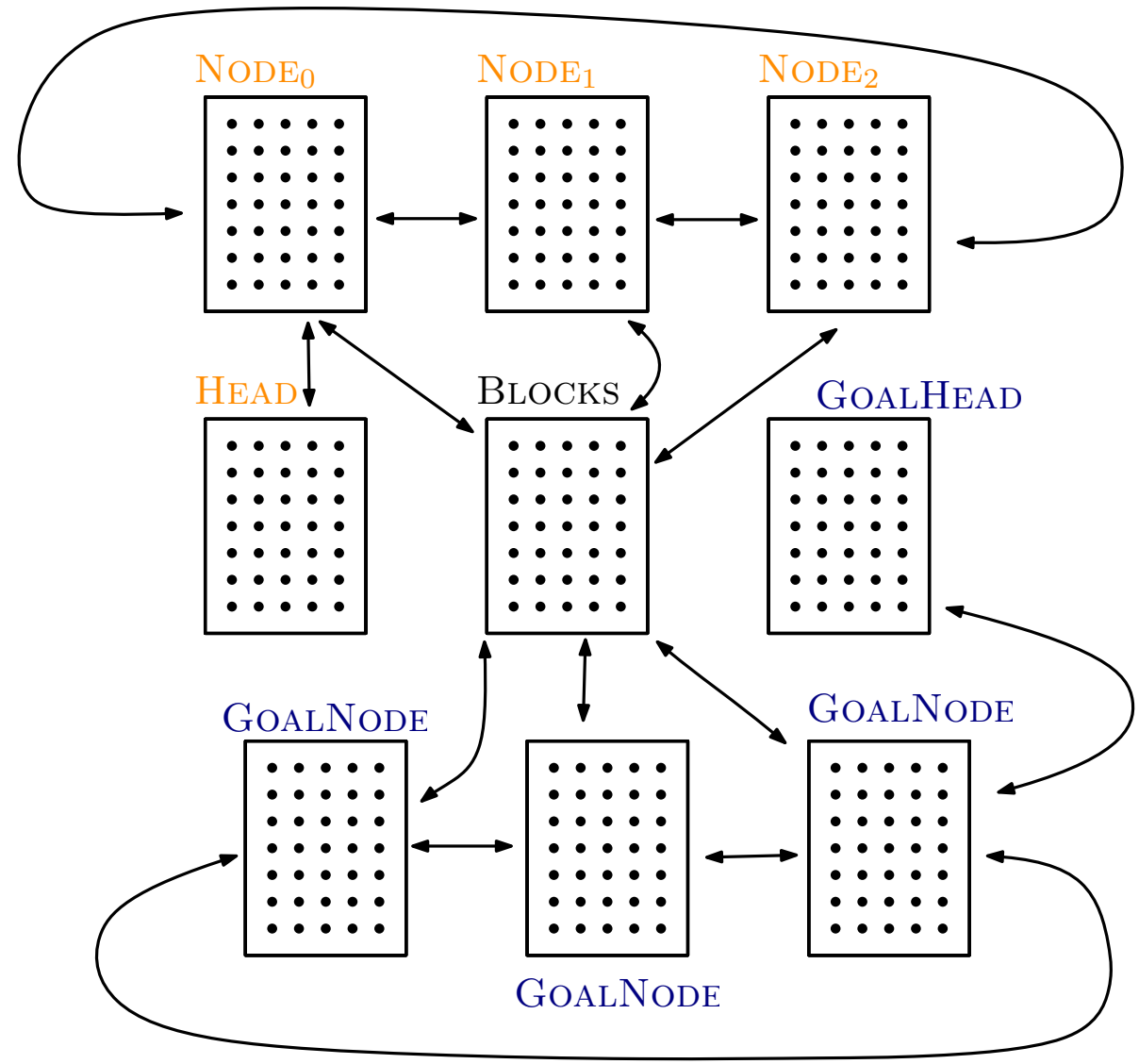
We have to save both the initial configuration and the goal configuration



Comparisons

We have to save both the initial configuration and the goal configuration

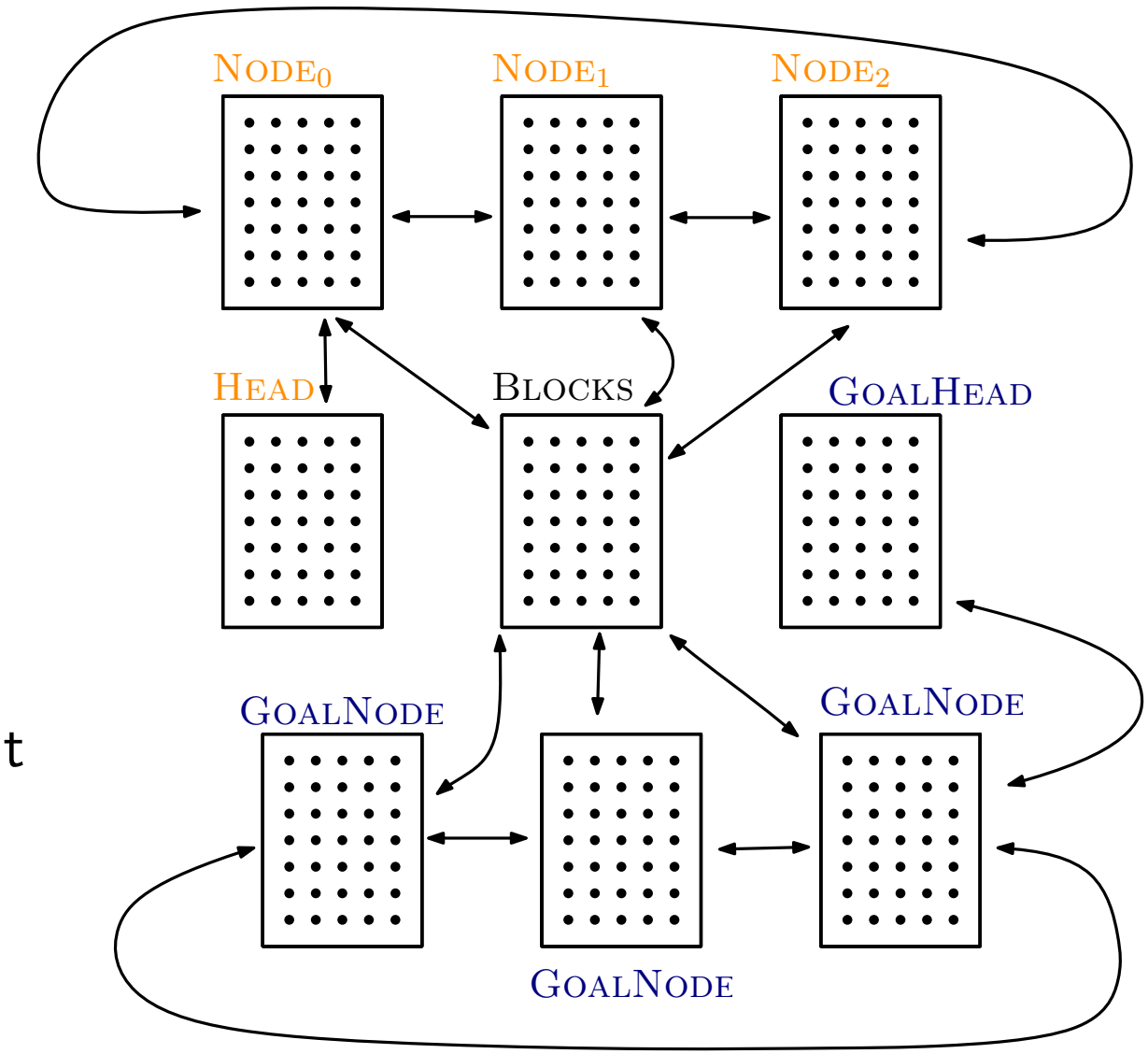
- *Intersect*: scrolls down the two stacks until reaching the last block, then starts comparing each pair of blocks by scrolling up



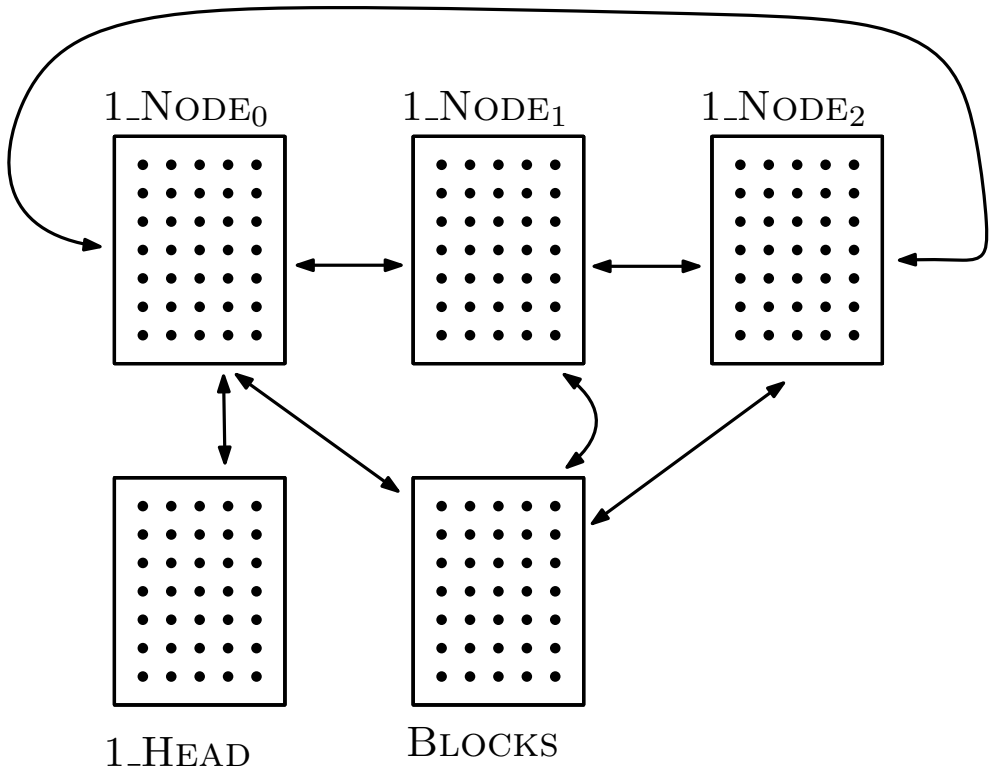
Comparisons

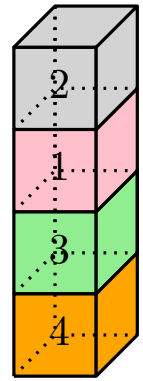
We have to save both the initial configuration and the goal configuration

- *Intersect*: scrolls down the two stacks until reaching the last block, then starts comparing each pair of blocks by scrolling up
- By combining these operations we implement the planning strategies



Multiple Stacks Case





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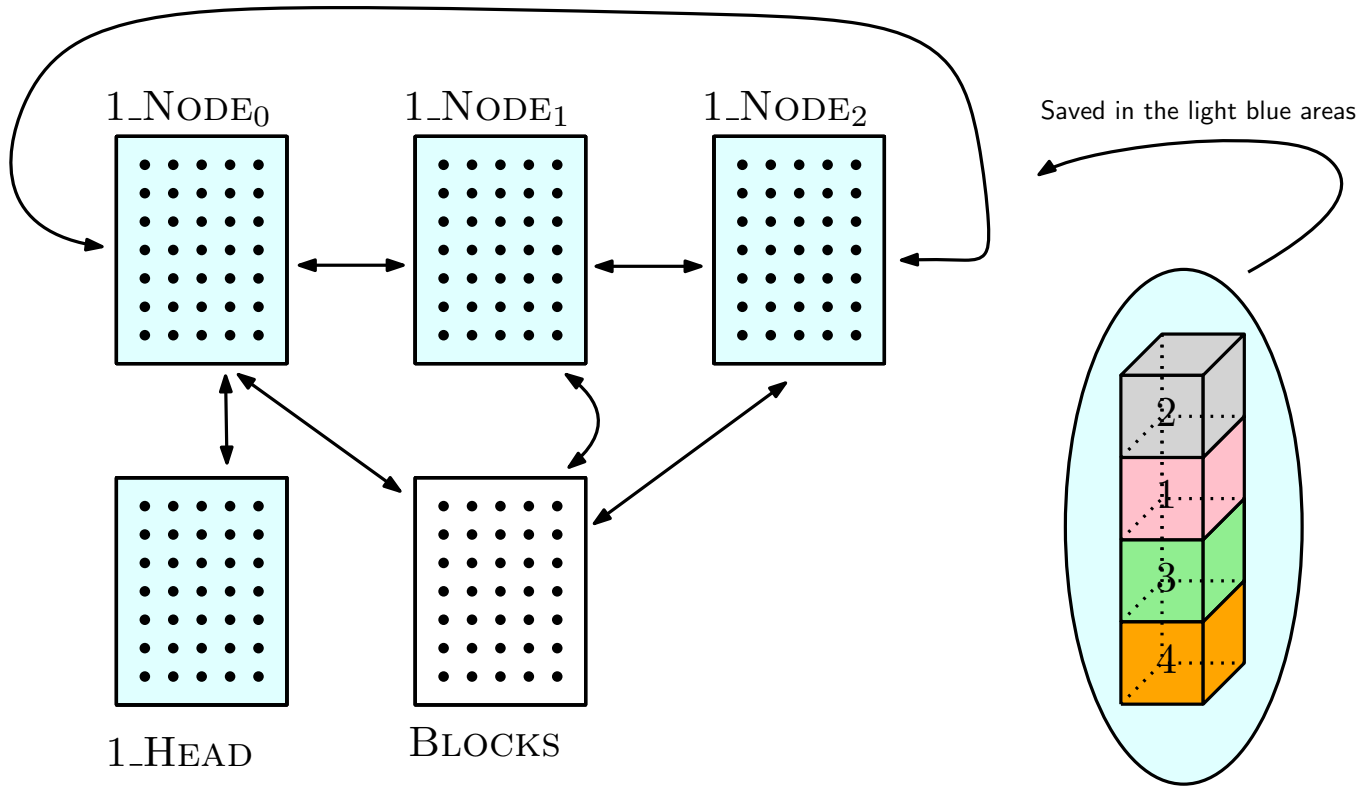

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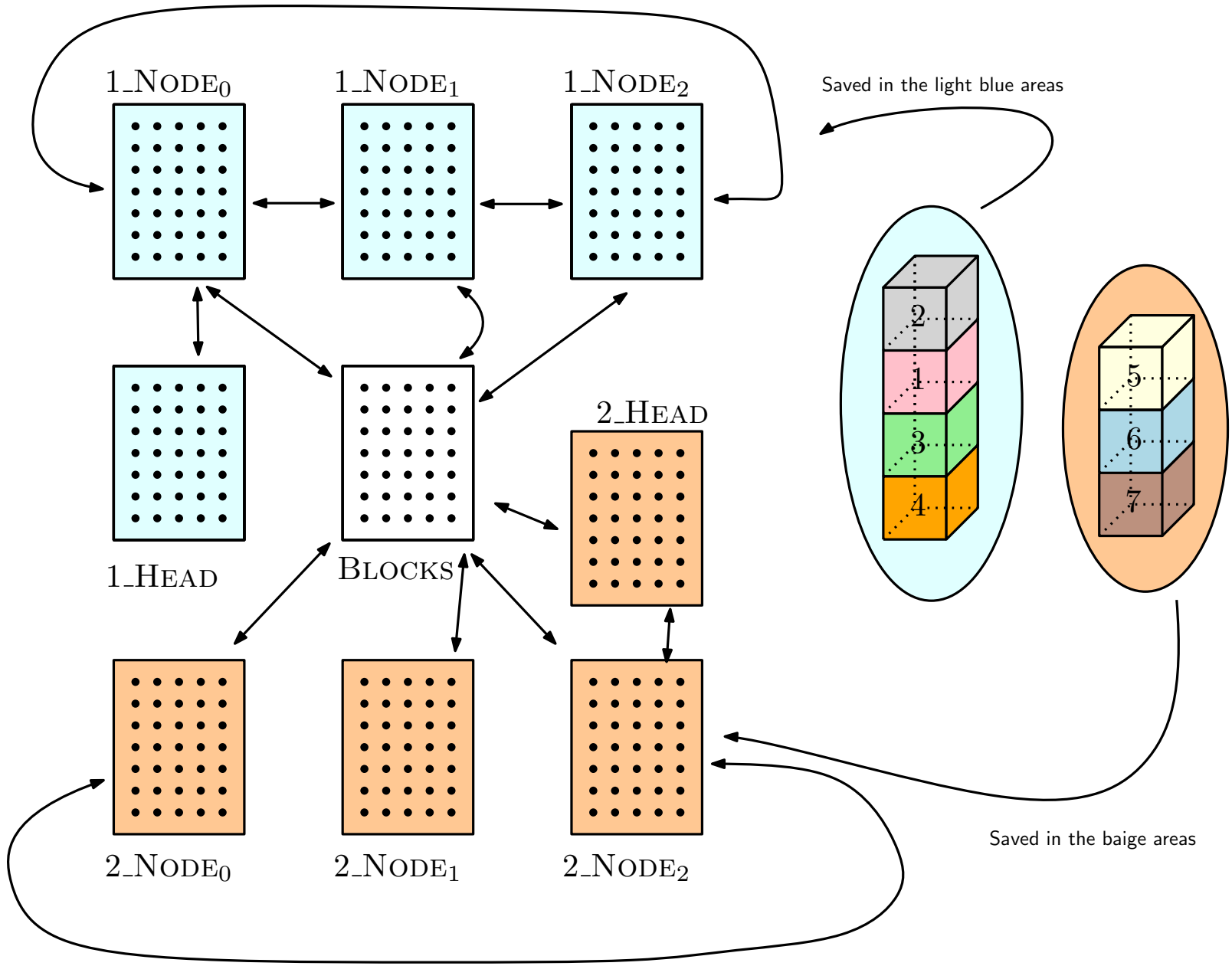
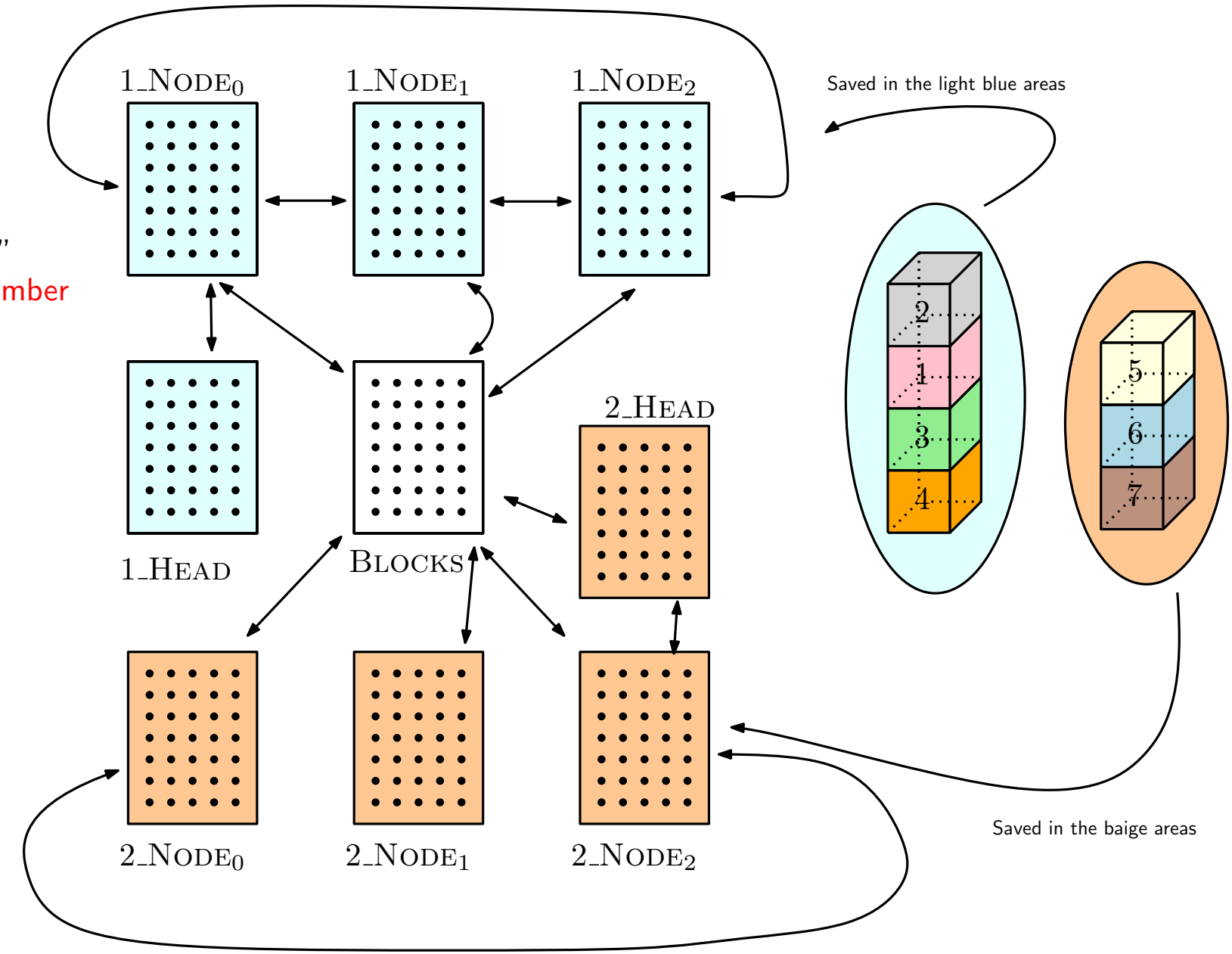


Multiple Stacks Case

As many "saving" regions as the number of stacks: **to be improved** (like chaining)



Discussion

- Demonstrated experimentally that reasonably large and complex programs in the assembly calculus can execute correctly and reliably
- Shown the realization of a *list-like data structure* which makes use of a *constant* number of brain regions
- Shown how simple manipulations of the data structure (removing or putting a block) can be realized by making use of a constant number of brain regions
- **Bottleneck**: the parsing. Its reliability depends on the ratio between the number of neurons and the size of the assemblies in each region
  - Must be the object of further investigation

Future Directions

After syntactic analysis in language [Mitropolsky et al., TACL 2021] and blocks world planning, what comes **next** as a compelling stylized cognitive function, which could be implemented in the AC?

- *Reasoning*
- *Planning and problem solving* in less specialized domains
- *Deductive tasks* in the context of logical and constraint-based formalisms

The End

Questions?