Dynamics for Multi-Agent System Coordination in Noisy and Stochastic Environments

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COATI team

Based on joint work with A. Clementi, G. Giakkoupis, E. Natale, and I. Ziccardi

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Natural algorithms

Algorithms designed by evolution over millions of years [Chazelle, SODA 2009]

- migrating geese
- flocking cranes
- fish baitball
- prey-predator systems
- synchronously flashing fireflies

"Georgia Aquarium Fish" by Mike Johnston

Natural algorithms

The computational lens help catching behavioral properties of biological systems

- bird flocking convergence time [Chazelle, SODA 2009]
- slime mold computing shortest paths [Bonifaci et al., Journal of Theoretical Biology 2012]

slime mold

flocking birds

"Flocking birds" by davepatten, CC BY-NC-SA 2.0.

Natural algorithms

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On the other hand, biological systems help designing new algorithms for well-known problems

- distributed maximal independent set from the fly's nervous system [Afek et al., SCIENCE 2011]

slime mold

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"Flocking birds" by davepatten, CC BY-NC-SA 2.0.

Distributed computing tasks

Often, systems of interacting agents performing collective tasks

Other than MIS and bird flocking:

- Information spreading: schooling fish [Rosenthal et al., PNAS 2015]
- Reaching agreement: molecules [Carrol, Nature Immunology 2004], bacteria [Bassler, Cell 2002], social insects [Franks et al., 2002] (e.g. bees [Reina et al., Physical Review E 2017])
- Collective search: ants and bees [Feinermant et Korman, Distributed Computing 2017]

foraging ants

"The Blueberry Hunters" by bob in swamp, CC BY 2.0.

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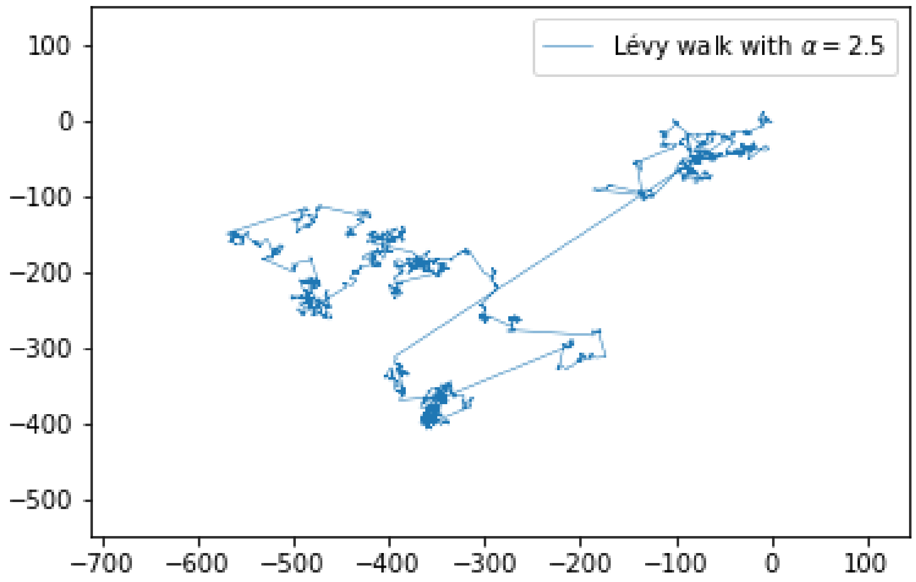
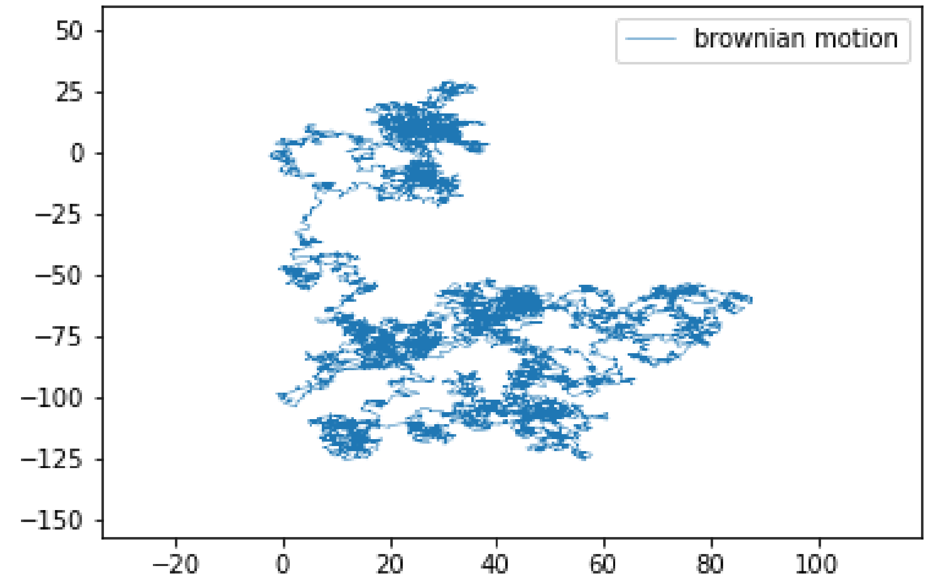
2. Lévy walks and the ANTS problem
3. Opinion dynamics for the consensus problem with uniform communication noise

The Lévy walk





Lévy walk (informal):
A Lévy walk is a random walk whose step-length density distribution is proportional to a power-law, namely, for each $d \in \mathbb{R}^+$, $f(d) \sim 1/d^\alpha$, *for some* $\alpha > 1$

Note: the speed of the walk is constant

Movement models and foraging theory

Lévy walks are used to model **movement patterns** [Reynolds, Biology Open 2018]

Examples:

- T cells within the brain
- swarming bacteria
- midge swarms
- termite broods
- schools of fish
- Australian desert ants
- a variety of molluscs

Rhytidoponera mayri workers. Credit: Associate Professor Heloise Gibb, La Trobe University

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Widely employed in the foraging theory

Lévy walk optimality

Foraging theory

- distribution of food locations in $\mathbb{R}^n$
- uninformed agent searching for food

[Viswanathan et al., Nature 1999]: Lévy walk with exponent $\alpha = 2$ is optimal in any dimension, with some assumptions

maximum expected food discovery rate

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maximum expected food discovery rate

Other search problems

- a target in the bidimensional thorus $\mathbf{T}$
- uninformed agent searching for it

[Guinard et Korman, Sciences Advances 2021]: (truncated) Lévy walk with exponent $\alpha = 2$ is optimal

as fast as possible

The Lévy flight foraging hypothesis

Formulation of an evolutionary hypothesis

The Lévy flight foraging hypothesis [Viswanathan et al., Physics of Life Reviews 2008]: since Lévy flights/walks optimize random searches, biological organisms must have therefore evolved to exploit Lévy flights/walks

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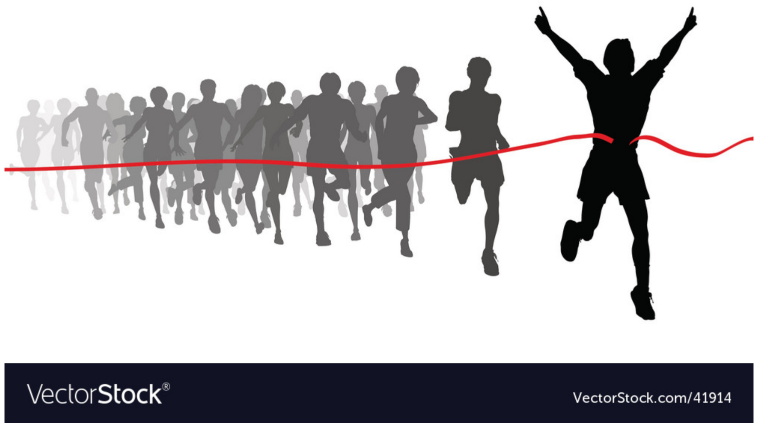
Seems there is a special exponent $\alpha = 2$

We test this hypothesis by focusing on a distributed search problem:

- the ANTS (Ants Nearby Treasure Search) problem



The ANTS problem

Introduced by [Feinerman et al., PODC 2012]:

Setting:

- k (mutually) independent agents start moving on $\mathbb{Z}^2$ from the origin
- time is synchronous and marked by a global clock
- one special node $\mathcal{P} \in \mathbb{Z}^2$, the *target*, placed by an adversary at unknown (Manhattan) distance ℓ from the origin

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Task: find the target as fast as possible

Lower bound: for any $k \geq 1$, and for any search algorithm $\mathcal{A}$, the hitting time to find $\mathcal{P}$ is $\Omega\left(\ell^2/k + \ell\right)$ both with constant probability and in expectation

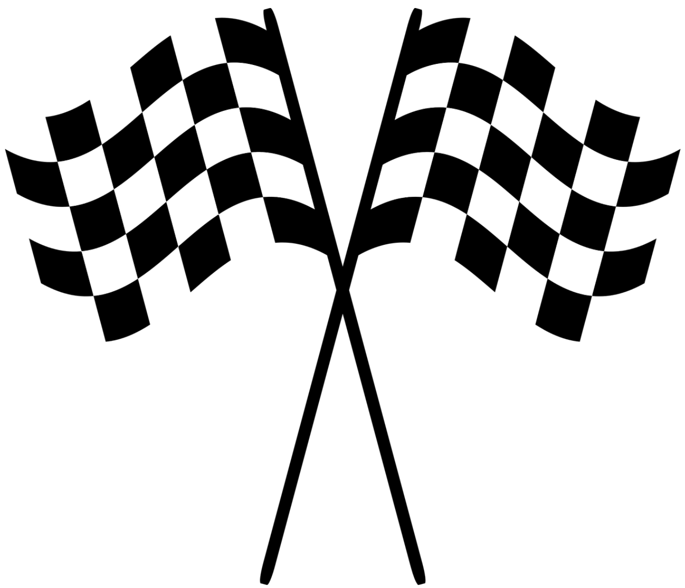

Image by OpenClipart-Vectors from Pixabay

Our contributions

Based on the work [Clementi et al., PODC 2021]

(i) we give the first definition of Lévy walk in the discrete setting in $\mathbb{Z}^2$, which is natural and time-homogeneus

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(ii) to the best of our knowledge, we give the first analysis of the hitting time distribution of k parallel walks

(iii) we show how the Lévy walks can be employed to give a natural, almost-optimal solution to the ANTS problem (no advice, no communication)

Our contributions

(i) DEFINITION OF DISCRETE LÉVY WALK

(ii) ANALYSIS OF THE PARALLEL HITTING TIME

(iii) ALGORITHM FOR THE ANTS PROBLEM

Defining the discrete Lévy walk

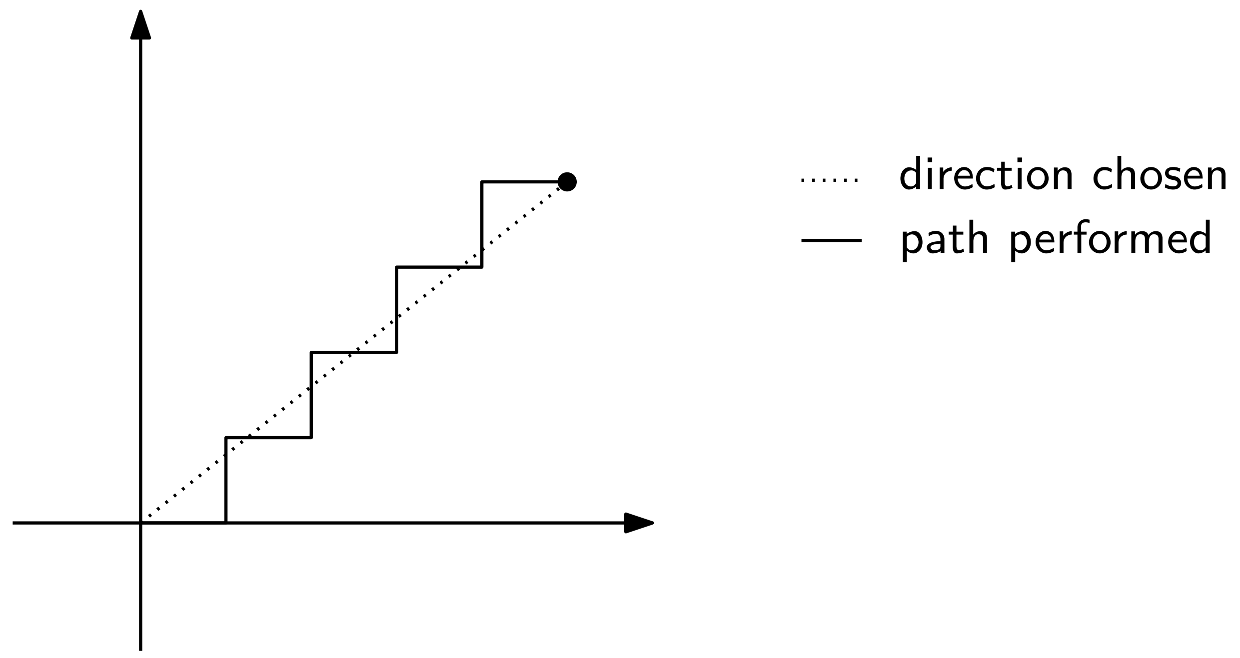
Two choices to make:

- define the jump-length distribution
- define a notion of approximating a line-segment

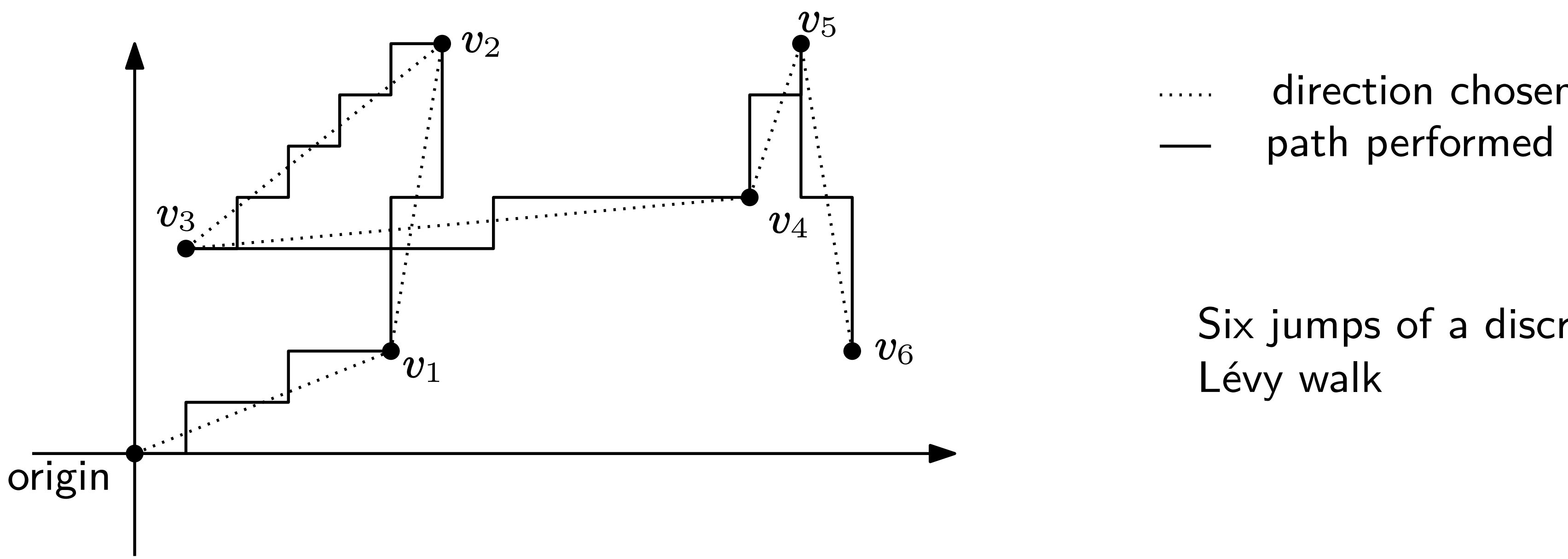
Jump length distribution

- $d = 0$ with probability $1/2$
- $d \geq 1$ with probability c_α / d^α

Approximation of a line-segment



Discrete Lévy walk



Six jumps of a discrete Lévy walk

Let $\alpha > 1$ be a real value

Lévy walk: the agent

a) chooses a distance $d \in \mathbb{N}$ as follows: $d = 0$ w.p. $1/2$, and $d \geq 1$ w.p. c_α / d^α
b) chooses a destination u.a.r. among those at distance d
c) walks along an approximating path for d steps, one edge at a time, crossing d nodes
d) repeats the procedure

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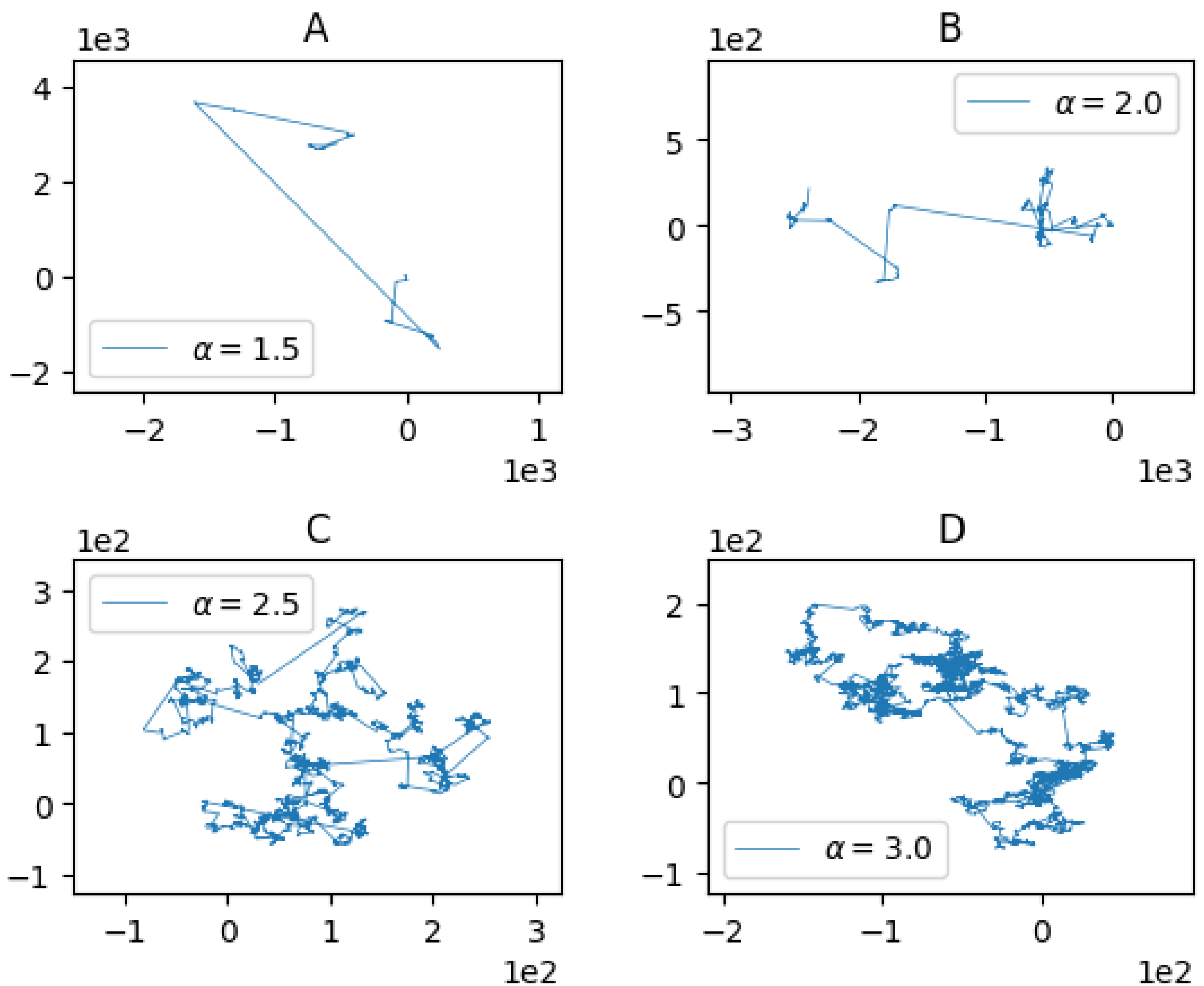
α-behavior of Lévy walks

- $1 < \alpha \leq 2$ ballistic diffusion (fig.s A and B)
- $2 < \alpha < 3$ super diffusion (fig. C)
- $3 \leq \alpha$ normal diffusion (fig. D)



Intuitive explanation

Expected jump-length

- $1 < \alpha \leq 2$: $\int_1^\infty x^{-\alpha+1} \mathsf{d}x = \infty$
- $2 < \alpha$: $\int_1^\infty x^{-\alpha+1} \mathsf{d}x = \Theta(1)$

Jump-length second moment

- $1 < \alpha \leq 3$: $\int_1^\infty x^{-\alpha+2} \mathsf{d}x = \infty$
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The secret lies in the range $2 < \alpha < 3$...

Three ranges for k and ℓ

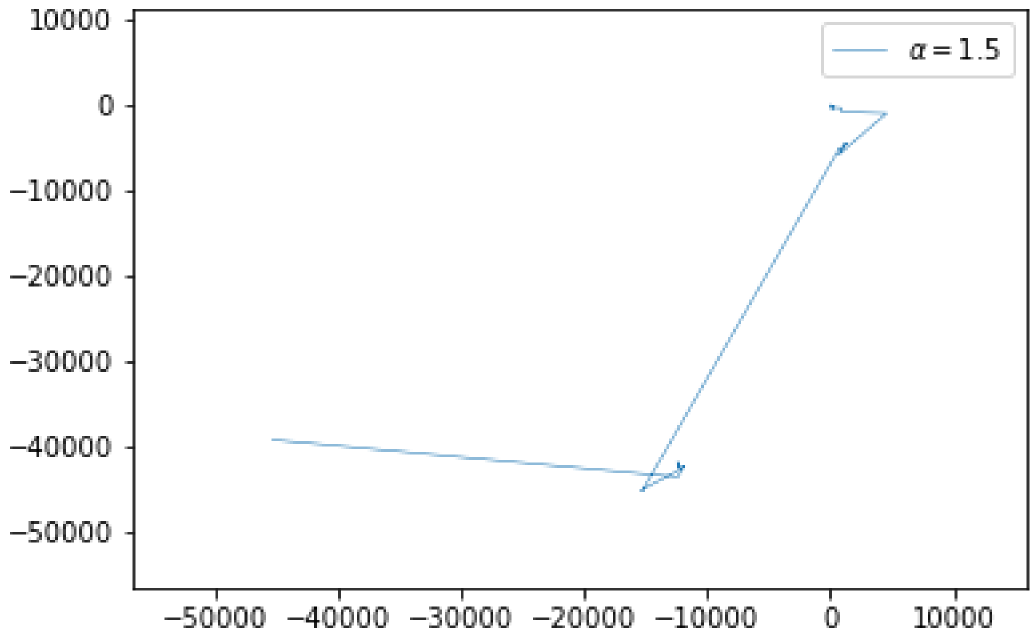
Recall: ℓ target distance, k number of agents

Three different possible settings:

1. close target: $\ell \leq k/\text{polylog}\,(k)$
2. far target: $k/\text{polylog}\,(k) \leq \ell \leq \exp\left(k^{\Theta(1)}\right)$
3. very far target: $\exp\left(k^{\Theta(1)}\right) \leq \ell$

Close target: $\ell \leq k/\text{polylog}\,(k)$

Best strategy $=$ ballistic walks: any α in $(1, 2]$



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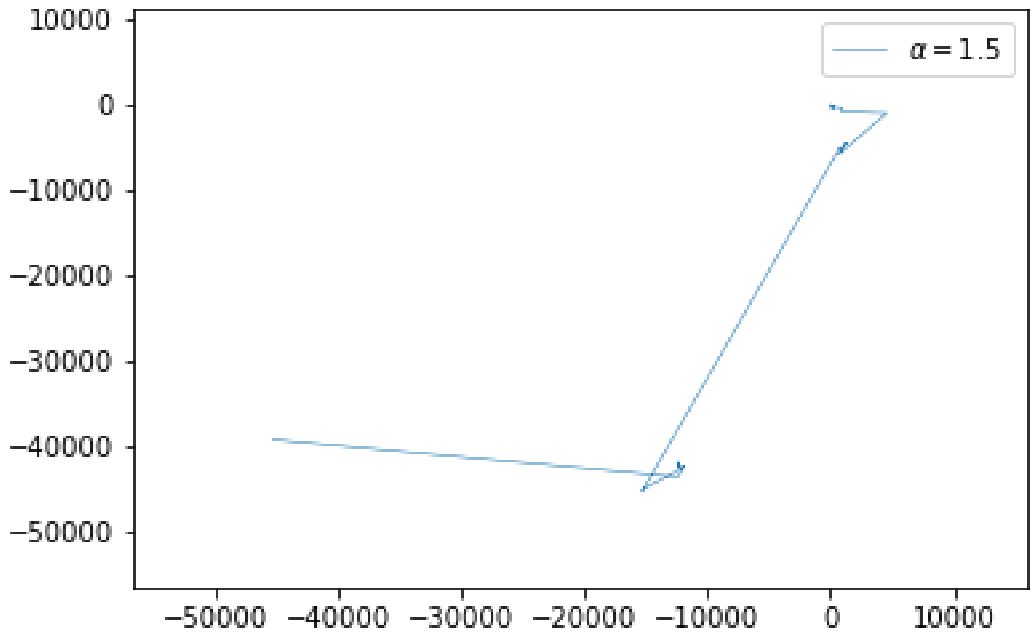

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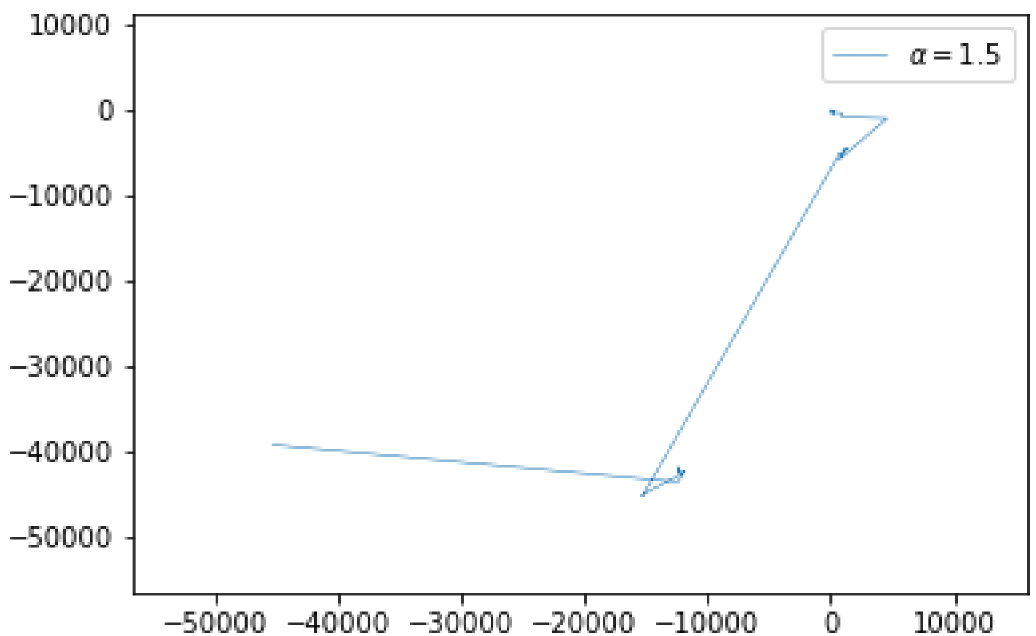
Recall: an event E depending on a parameter ℓ holds with high probability in ℓ if $\mathbb{P}\,(E) \geq 1 - \ell^{-\Theta(1)}$



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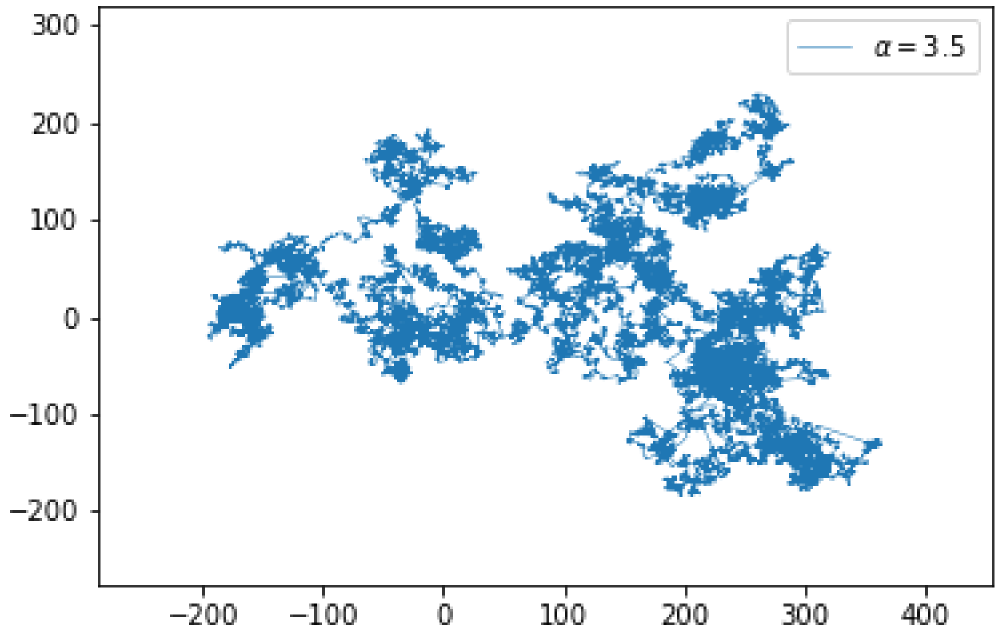
Best strategy $=$ diffusive walks: any α in $[3, +\infty)$ (brownian-like behavior)

With probability 1, the walks will eventually find the target



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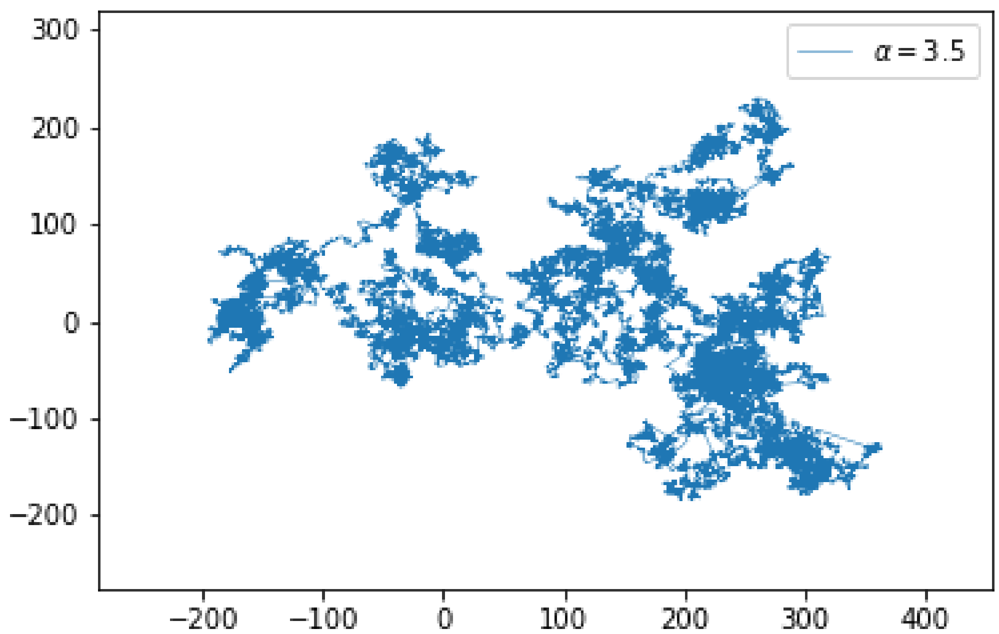

If $\alpha = 3 - \epsilon$, with high probability the target is not found

Far target: $k/\text{polylog}\,(k) \le \ell \le \exp\left(k^{\Theta(1)}\right)$

Best strategy: ... it depends!

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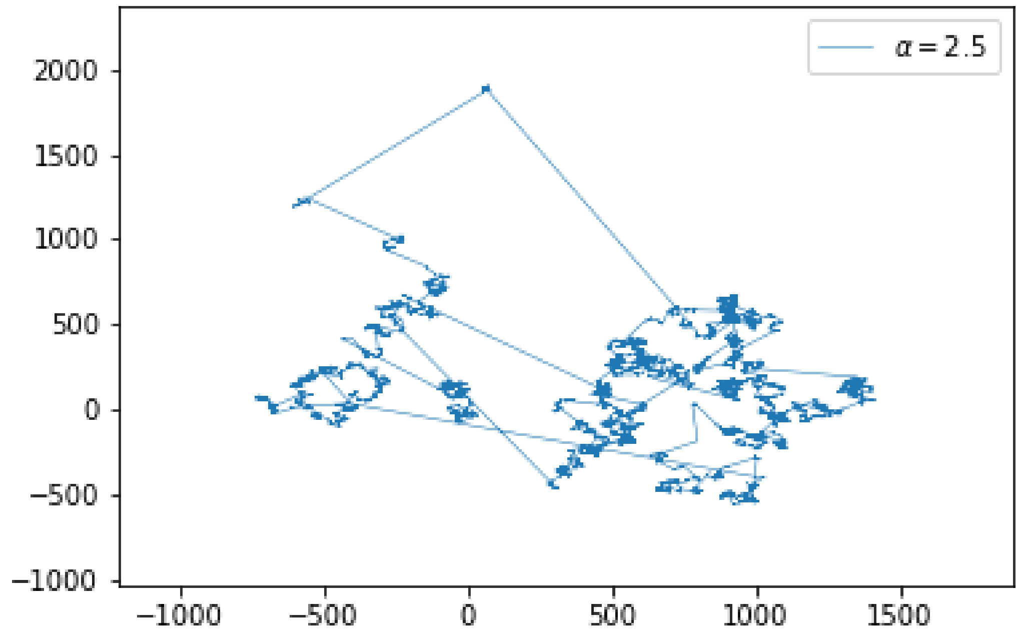
Fix $\alpha^\star = 3 - \log k / \log \ell$: super-diffusive range



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The followings hold w.h.p. in ℓ

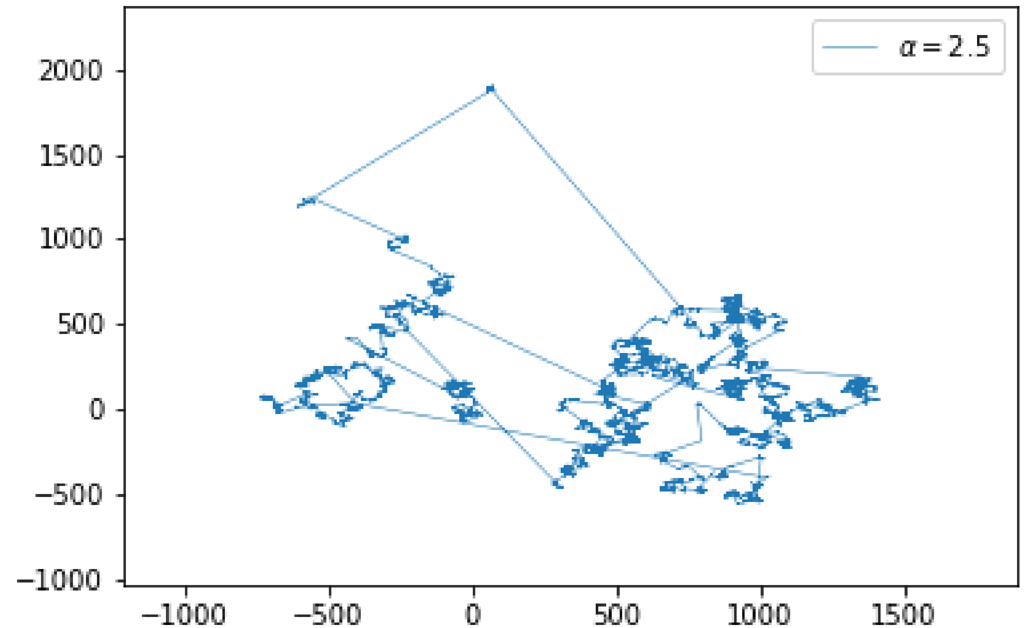
- if $\alpha = \alpha^\star + \mathcal{O}(\log\log\ell/\log\ell)$, the hitting time is

$$\mathcal{O}\left(\left(\ell^2/k + \ell\right)\text{polylog}(\ell)\right)$$

- if $\alpha = \alpha^\star + \epsilon$, the hitting time is

$$\Omega\left(\left(\ell^2/k + \ell\right)\ell^c\right),$$

  for some constant $c > 0$
- if $\alpha = \alpha^\star - \epsilon$ the hitting time is *infinite*



But... No advice, no communication!

How can we find $\alpha^\star$?

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We don't have to!

Algorithm: *each agent* u *samples u.a.r. a real number* $\alpha_u \in (2, 3)$*. Then, it performs a discrete Lévy walk with exponent* α_u

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If $\ell \leq \exp\left(k^{\Theta(1)}\right)$, the hitting time is $\mathcal{O}\left(\left(\ell^2/k + \ell\right) \text{polylog}\left(\ell\right)\right)$ w.h.p.

The idea behind the algorithm

Fix some $\epsilon = \mathcal{O}\left(\log\log\ell/\log\ell\right)$

We use: $\ell < \exp\left(k^{\Theta(1)}\right)$ ($\iff$ $k \geq$ polylog (ℓ)) $+$ Chernoff bound

$\implies$ at least $\Theta\left(\epsilon k\right)$ agents sample an exponent in the range $\left(\alpha^\star - \epsilon, \alpha^\star + \epsilon\right)$ w.h.p.

$\Theta\left(\epsilon k\right)$ agents are sufficient to ensure high probability to find the target fast enough

Recap

In this work, we

- provide a definition of a discrete version of the Lévy walk
- analyze the hitting time of k parallel Lévy walks
- show that for any choices of k and ℓ from a wide range, Lévy walks are an almost-optimal search strategy for the ANTS problem

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    - very natural and time-homogeneus random process
    - does not improve the optimal solution

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- show that for any choices of k and ℓ from a wide range, Lévy walks are an almost-optimal search strategy for the ANTS problem
    - very natural and time-homogeneus random process
    - does not improve the optimal solution
- mathematically corroborate the Lévy flight foraging hypothesis
- argue the non (universal) optimality of exponent $\alpha = 2$

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The consensus problem

Input: system of n agents supporting opinions, with a communication network

Task: designing a protocol which **brings the system** in finite time to a configuration such that

1. all agents support the same opinion (AGREEMENT)
2. the final opinion is among the initial ones (VALIDITY)
3. the agreement keeps on unless external events occur (STABILITY)

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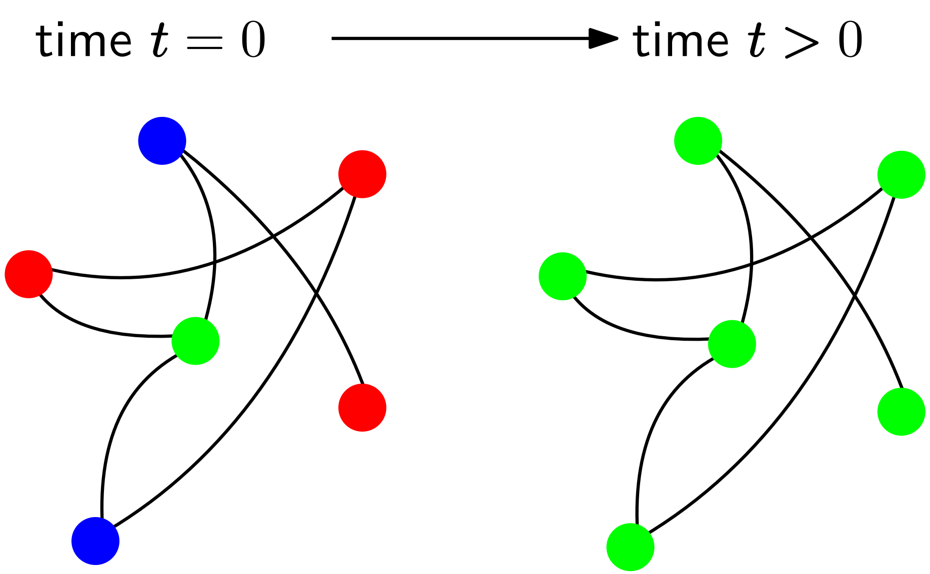
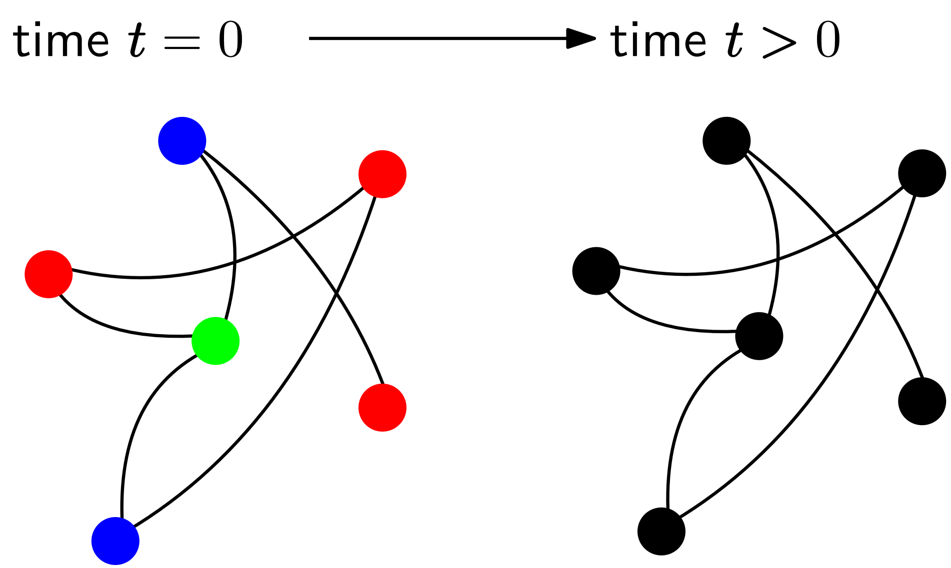


valid consensus



non valid consensus

The majority consensus problem

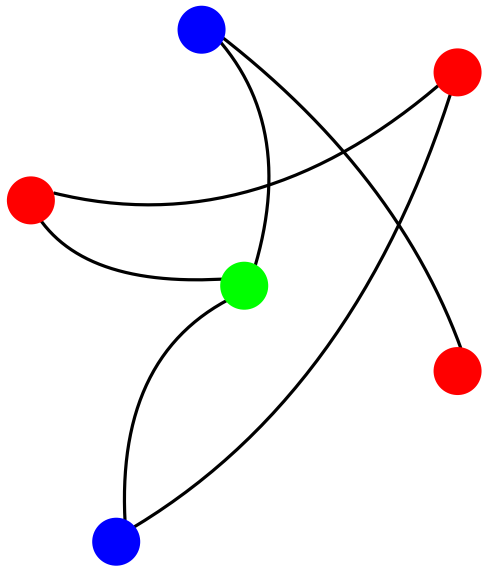
1. AGREEMENT
2. ~~VALIDITY~~
3. STABILITY

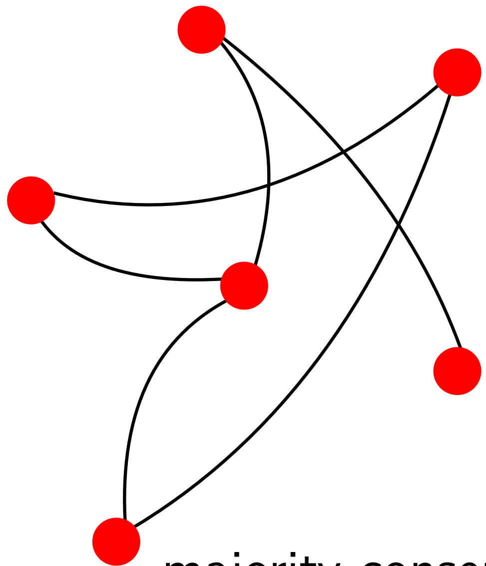
$\longrightarrow$

1. AGREEMENT
2. **MAJORITY**
3. STABILITY

2. MAJORITY property: the final opinion is the **initial majority** one

time $t = 0$ $\longrightarrow$ time $t > 0$





majority consensus

Opinion dynamics for the consensus problem

Opinion dynamics: class of simple, lightweight parallel protocols for the consensus problems

Many have been investigated, including:

- Voter Model [Hassin and Peleg, Inf. Comput. 2001]
- Averaging dynamics [Becchetti et al., SODA 2017]

linear dynamics

- 3-Majority [Becchetti et al., SODA 2016]
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non-linear dynamics

Majority update-rules and the undecided state dynamics have biological inspirations [Reina et al., Physical Review 2017] [Condon et al., Nat. Computing 2020] [Chaouiya et al., PLOS ONE 2013]

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Often, settings with adversarial Byzantine failures are investigated

Not realistic in biological scenarios; rather, uniform noise [Feinerman et al., PODC 2014]

Uniform communication noise

Inspired by [Feinerman et al., PODC 2014], [Freignaud and Natale, PODC 2016]

Σ set of k opinions, $p \in [0, 1/2]$ constant

When u looks at v's opinion x

a) with probability $1 - p$, u sees x
b) with probability p, u sees y where y is chosen u.a.r. in Σ

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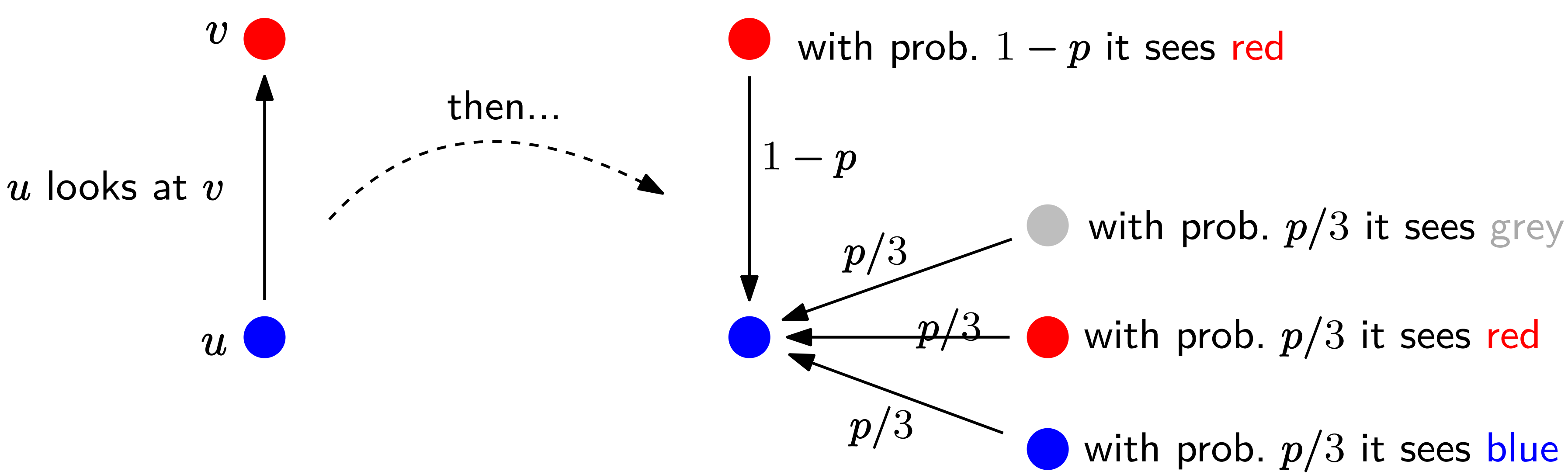
Example: $k = 3$



The dynamics

3-Majority dynamics: each node u

1. samples 3 neighbors u.a.r.
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$u \backslash v$	opinion i	opinion j	undecided
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Overview of results in noiseless settings [Becchetti et al., SIGACT News 2020]

Our contribution

Based on [d'Amore et al., SIROCCO 2020], [d'Amore et Ziccardi, SIROCCO 2022]

We study the **Undecided-State** dynamics and the **3-Majority** dynamics with $k = 2$ opinions in the presence of uniform noise in the compelete graph

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For the **3-Majority** dynamics: phase-transition

$p < 1/3$:
- a value $\bar{s} = \Theta(n)$ exists such that the bias of the system reaches the interval $I_\varepsilon = [(1-\varepsilon)\bar{s}, (1+\varepsilon)\bar{s}]$ in time $\mathcal{O}(\log n)$ w.h.p., and keeps in I_ε for time poly(n) w.h.p. $\longrightarrow$ **almost-consensus**
- if the initial bias is $\Omega(\sqrt{n \log n})$, we have almost-majority consensus

$p > 1/3$:
- in time $\mathcal{O}(\log n)$ the bias becomes bounded by $\mathcal{O}(\sqrt{n \log n})$ and keeps bounded for time poly(n) wh.p. $\longrightarrow$ **victory of noise**
- there is constant probability to switch majority within time $\mathcal{O}(\log n)$

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For the **3-Majority** dynamics: phase-transition

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- a value $\bar{s} = \Theta(n)$ exists such that the bias of the system reaches the interval $I_\varepsilon = [(1-\varepsilon)\bar{s}, (1+\varepsilon)\bar{s}]$ in time $\mathcal{O}(\log n)$ w.h.p., and keeps in I_ε for time poly(n) w.h.p. $\longrightarrow$ **almost-consensus**
- if the initial bias is $\Omega(\sqrt{n \log n})$, we have almost-majority consensus

$p > 1/3$:
- in time $\mathcal{O}(\log n)$ the bias becomes bounded by $\mathcal{O}(\sqrt{n \log n})$ and keeps bounded for time poly(n) wh.p. $\longrightarrow$ **victory of noise**
- there is constant probability to switch majority within time $\mathcal{O}(\log n)$

For the **Undecided-State**: similar behavior

- phase-transition at $p = 1/2$
- less characterized, more complex

Techniques

For **consesus**:

- symmetry breaking: the bias has enough standard deviation to break symmetry (drift analysis results)
- applying concentration inequalities to construct a process M_t such that $M_{t+1} \leq (1-\delta)M_t$ w.h.p. as long as the bias is outside $I_\varepsilon = [(1-\varepsilon)\bar{s}, (1+\varepsilon)\bar{s}]$ but at least $\Omega(\sqrt{n \log n})$
- chain rule + union bound

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For **victory of noise**:

- constructing a super-martingale N_t such that $\mathbb{E}\left[N_{t+1} | \mathcal{F}_t\right] \leq (1-\delta)N_t$ involving the bias
- concentration arguments for super-martingales [Lehre and Witt, ISAAC 2014] implying that the bias reaches 0

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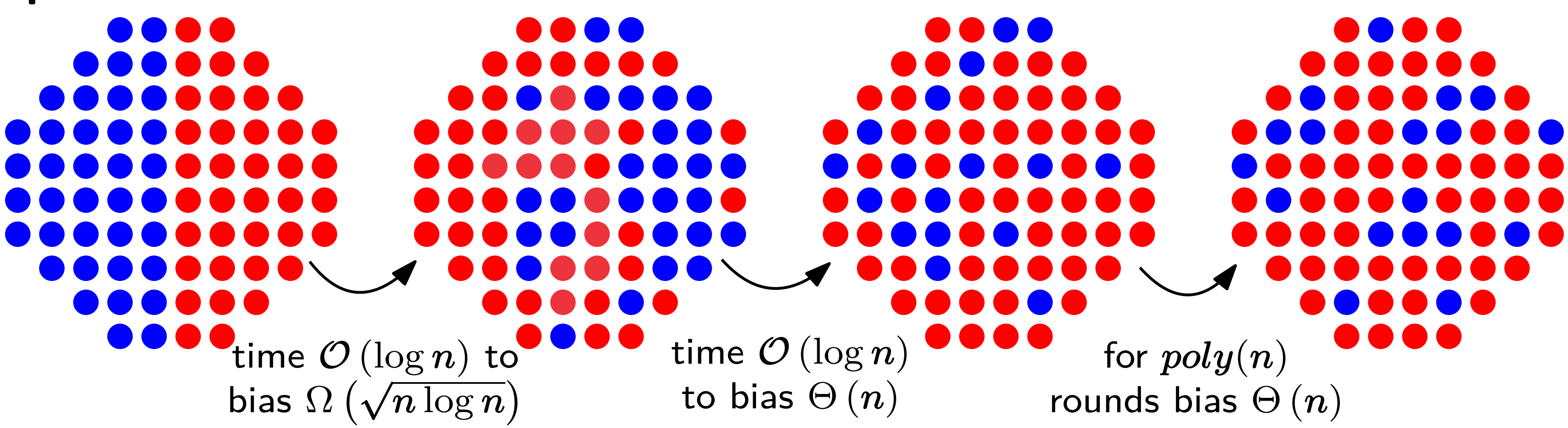
Example: consensus



Discussion

- **Undecided-State** dynamics and **3-Majority** dynamics are not implemented by biological systems
  - despite the bio-inspiration, highly abstract model
  - aiming to capture fundamental phenomena that (**very loosely**) relates to many biological systems

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- **Undecided-State** dynamics more resilient to noise than **3-Majority** dynamics
  - phase-transitions: $p = 1/2$ vs $p = 1/3$
    - $p = 1/2$ (USD) means half of the communications are non-noisy on average
    - $p = 1/3$ (3-Maj) means 2 out of 3 pulled opinions are non-noisy on average

- Are noise-thresholds independent of k?

- What about sparser topologies, e.g. expanders?

The End

THANK
YOU