Limits of Distributed Quantum Computing

GRAN SASSO SCIENCE INSTITUTE

Francesco d'Amore

Based on [STOC '24, STOC '25a, STOC '25b, SODA '26]

Joint works with Amirreza Akbari, Alkida Balliu, Sebastian Brandt, Filippo Casagrande, Xavier Coiteux-Roy, Massimo Equi, Rishikesh Gajjala, Barbara Keller, Fabian Kuhn, François Le Gall, Henrik Lievonen, Darya Melnyk, Augusto Modanese, Dennis Olivetti, Shreyas Pai, Marc-Olvier Renou, Václav Rozhon, Gustav Schmid, Jukka Suomela, Lucas Tendick, Isadora Veeren

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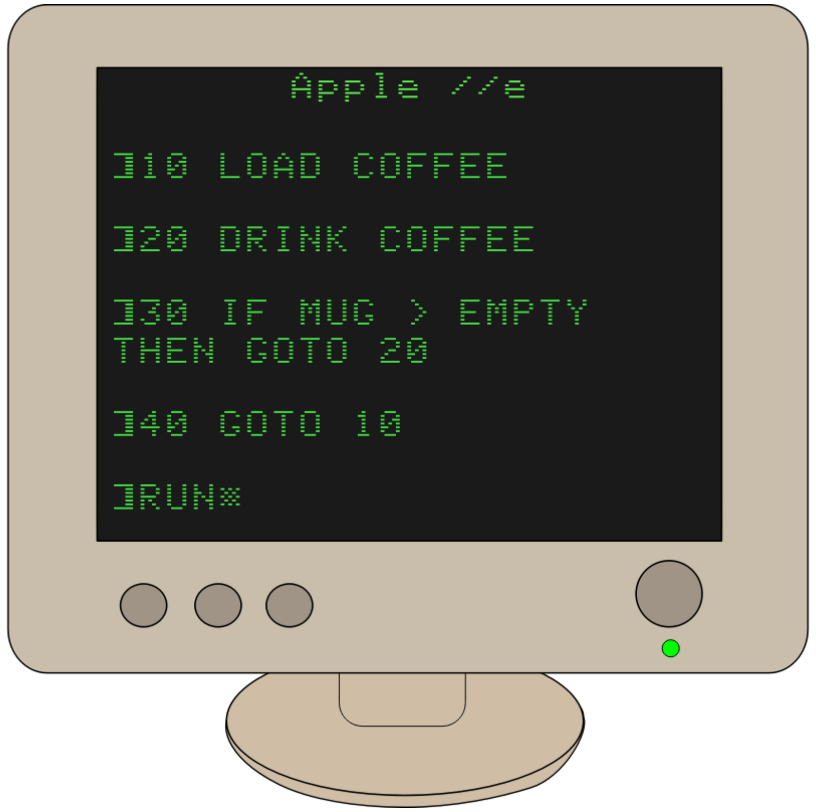
- Write a program $\mathcal{A}$ for a single computer (e.g., for $(\Delta + 1)$-coloring a graph)
  - use commands like *send a message through this communication port*, etc.



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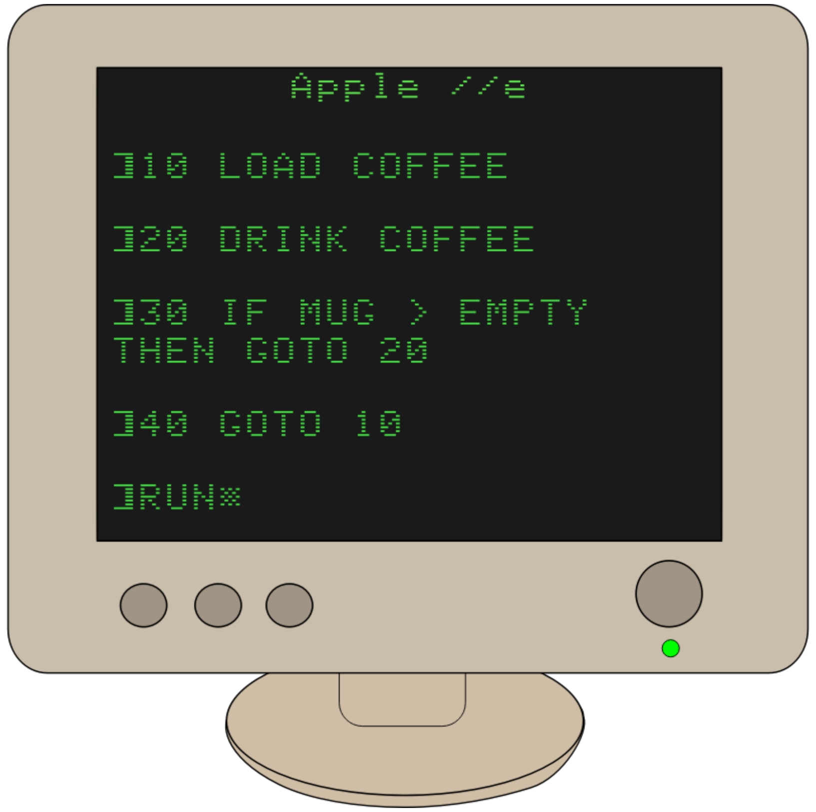

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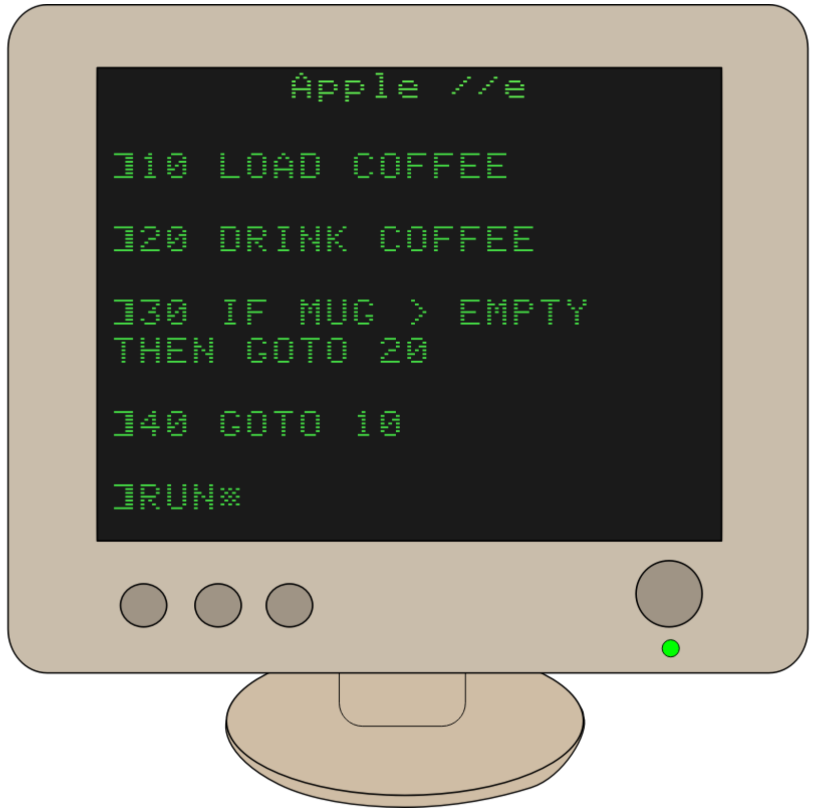

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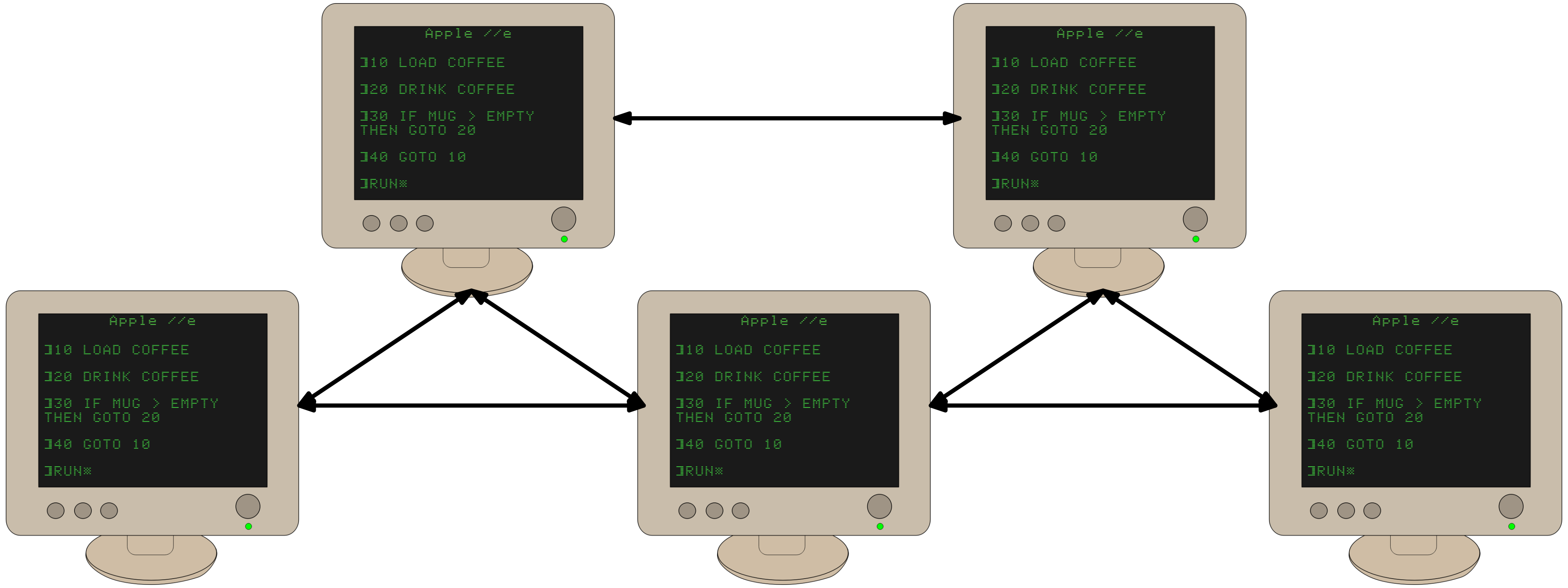

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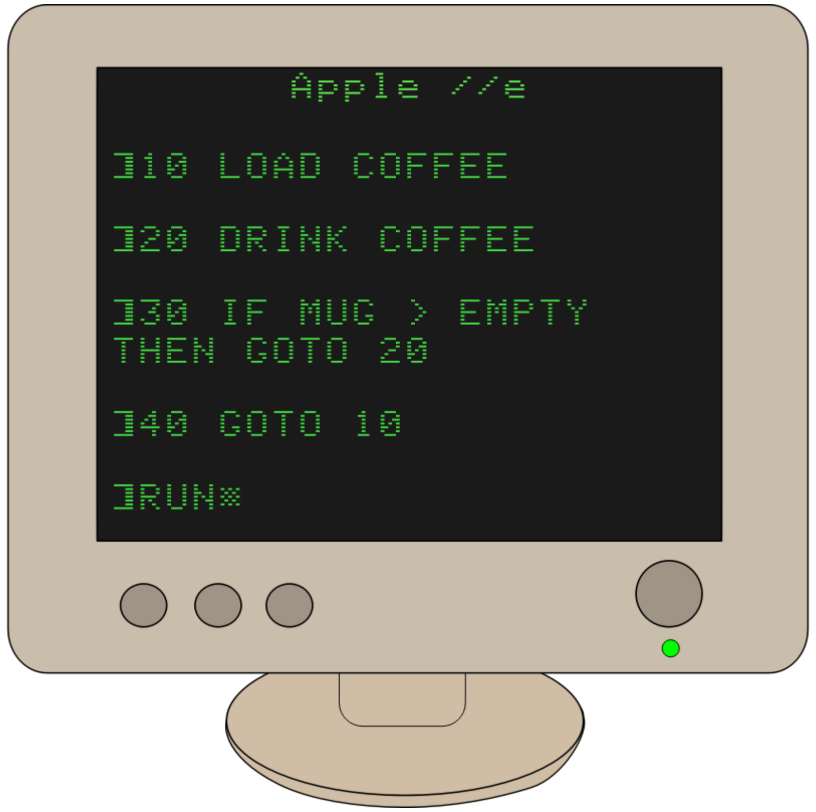
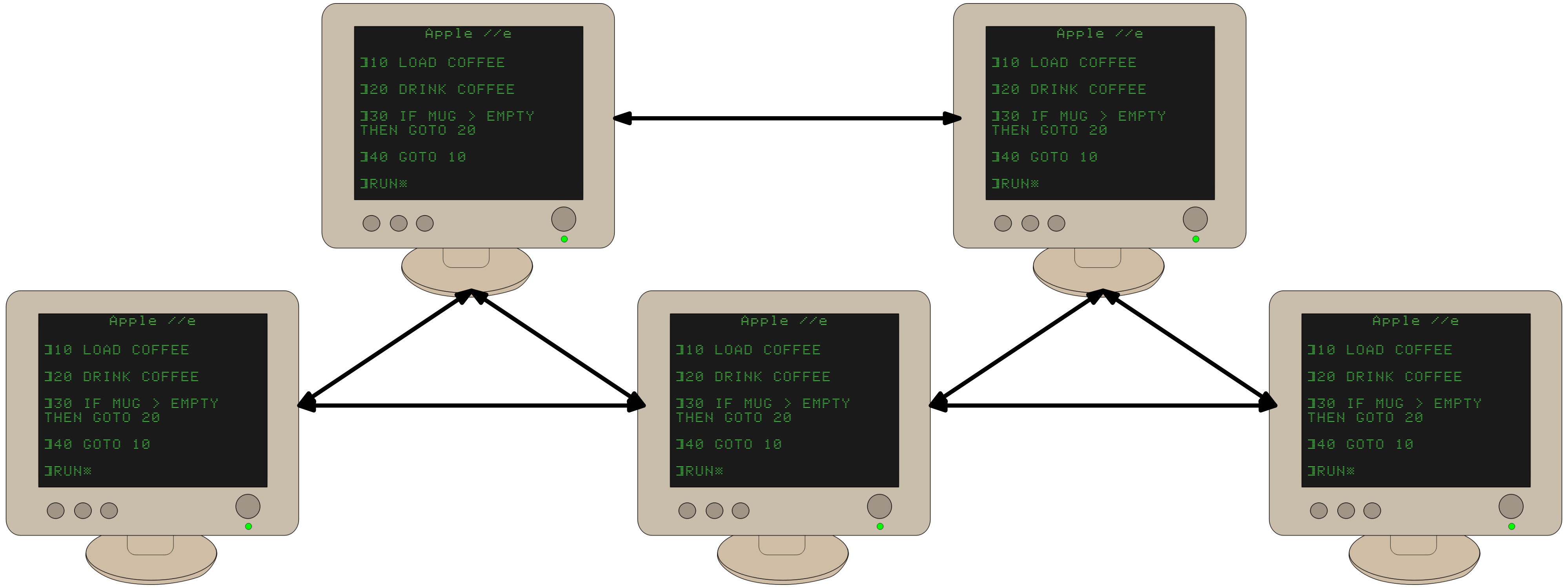
- **Switch everything on**, see what happens





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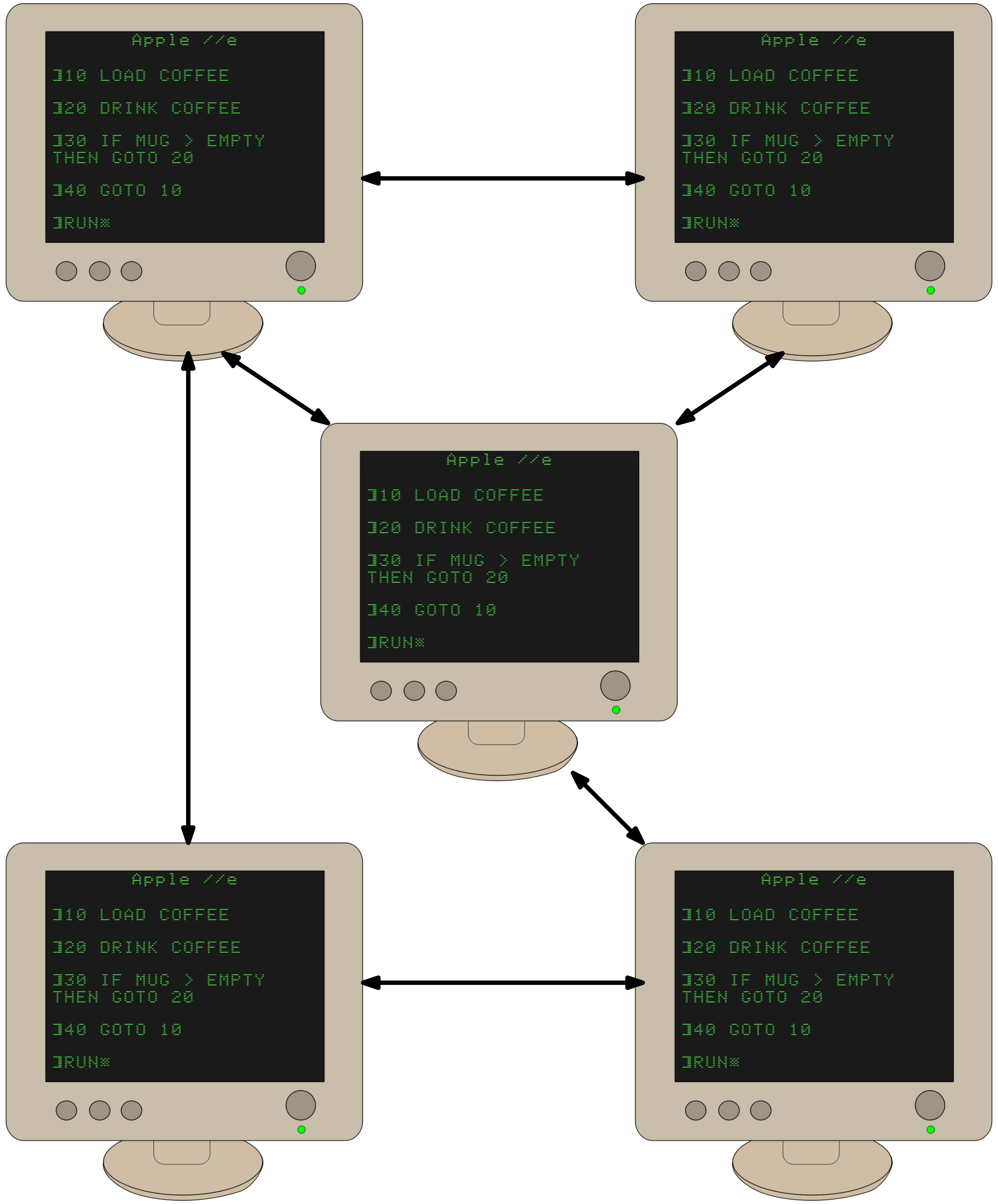

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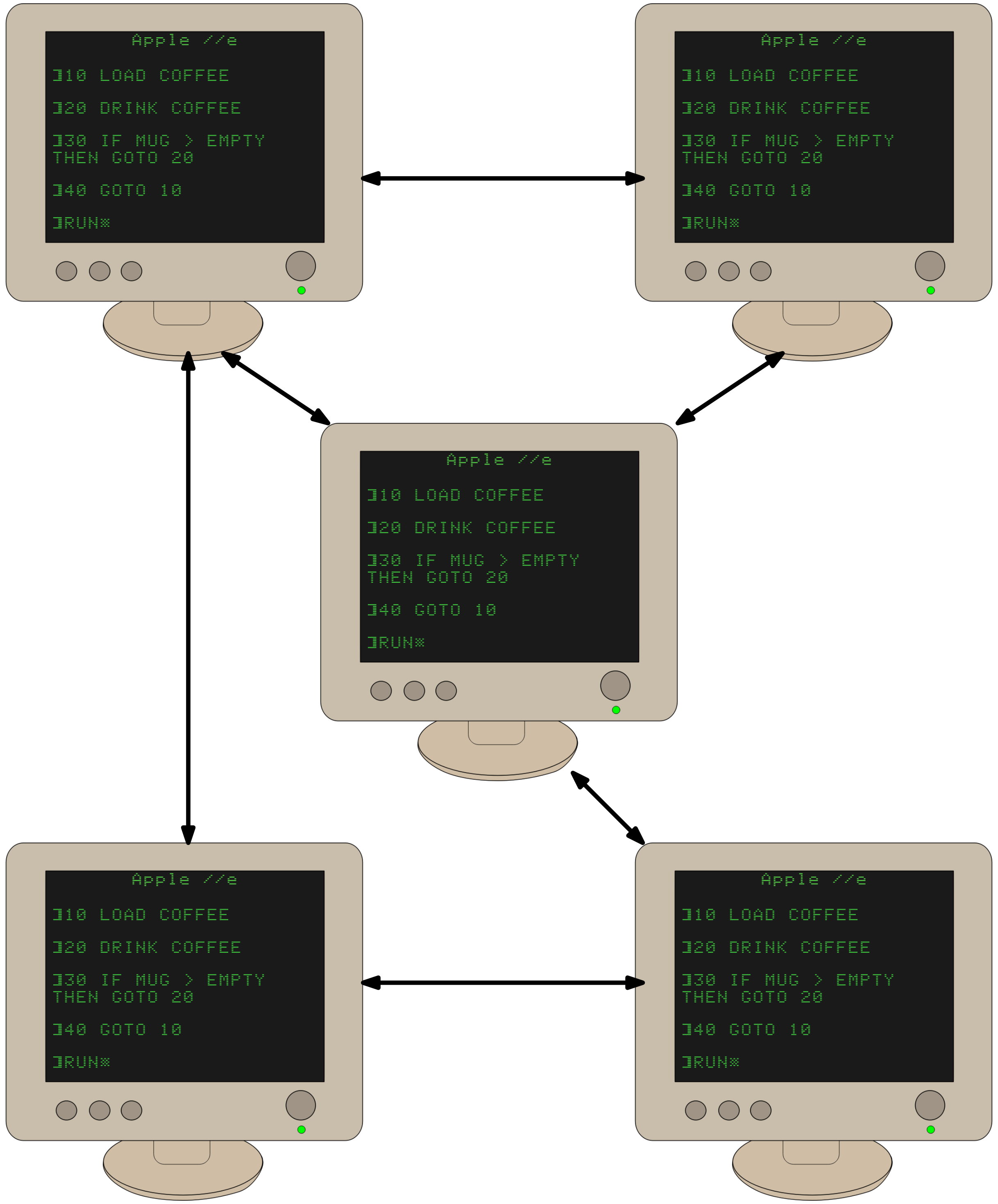

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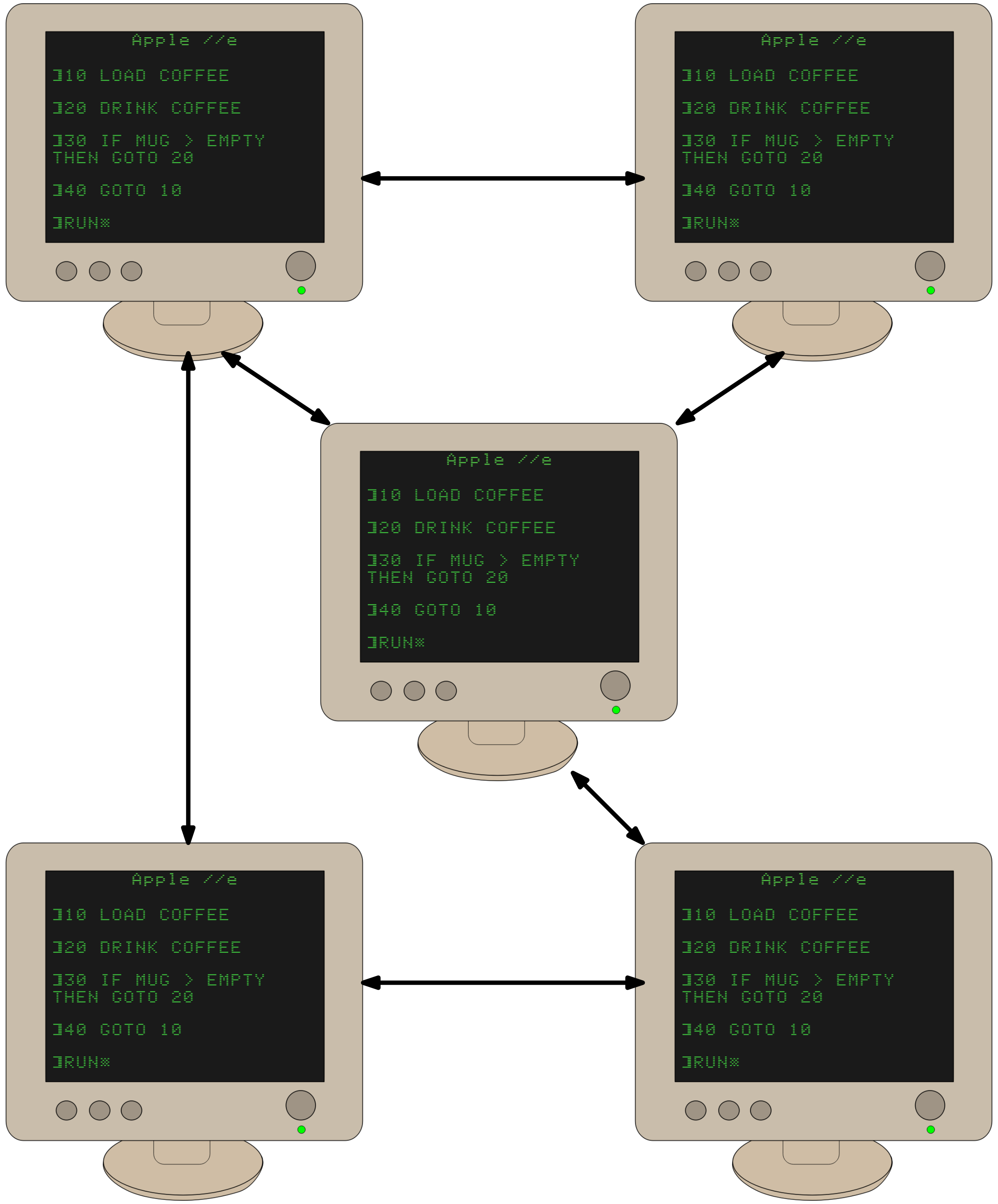

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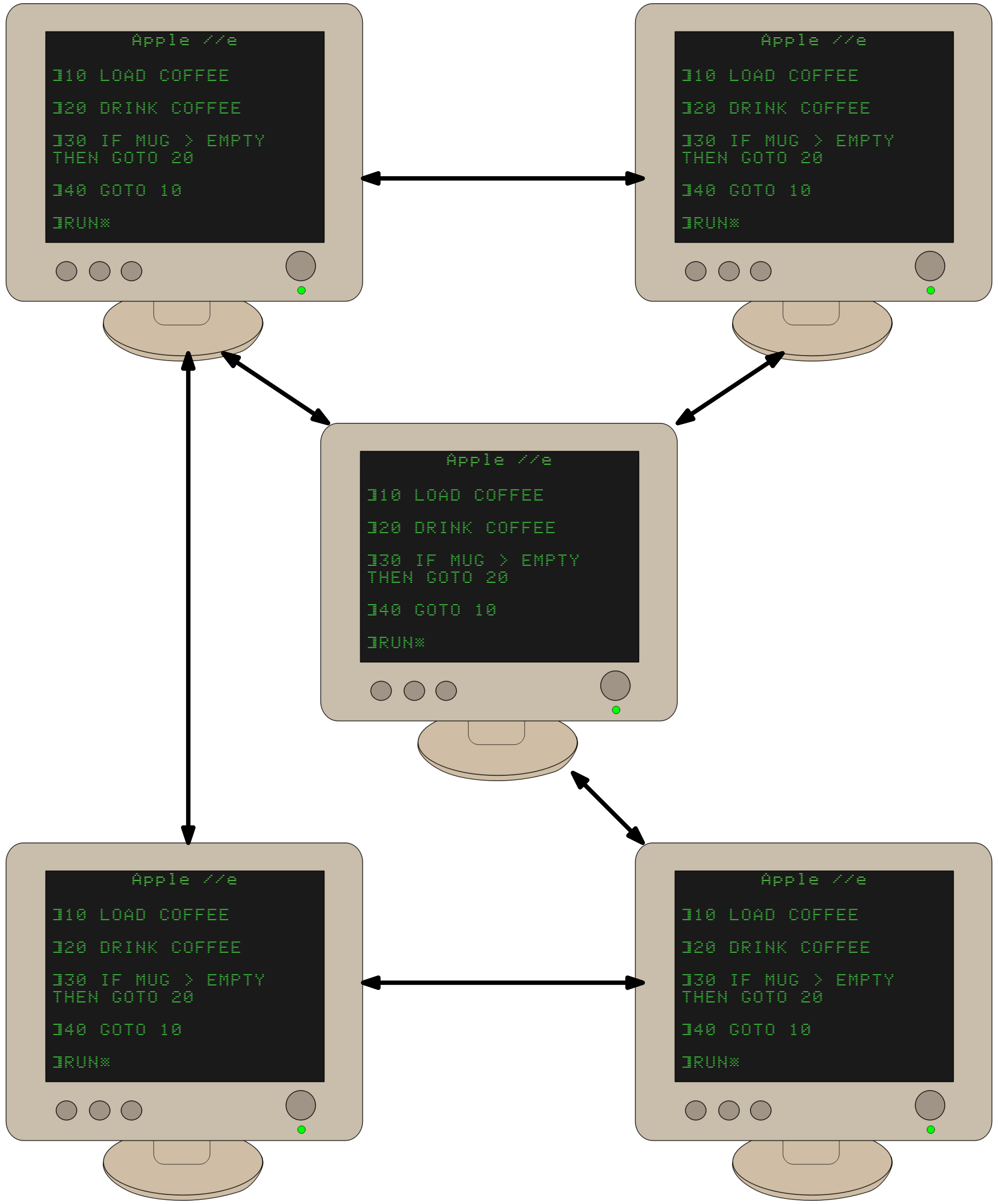

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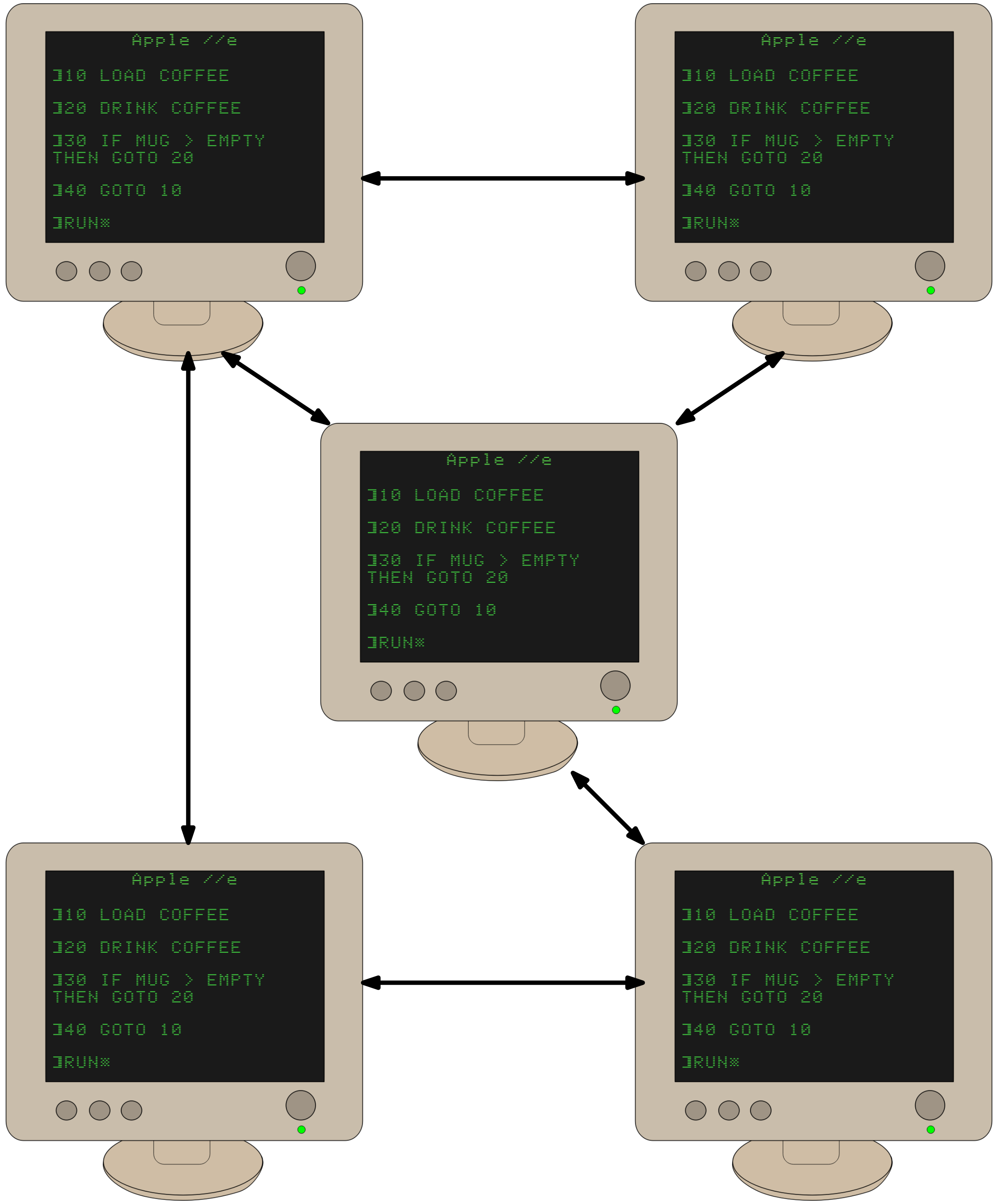
- **Challenge**: what to do in the middle of a network?



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- **Other possibility**: *randomness*. Each node has access to independent source of randomness

The LOCAL model

[Linial FOCS '87 & SICOMP '92]

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  - graph $G = (V, E)$ with $|V| = n$
  - E: communication links
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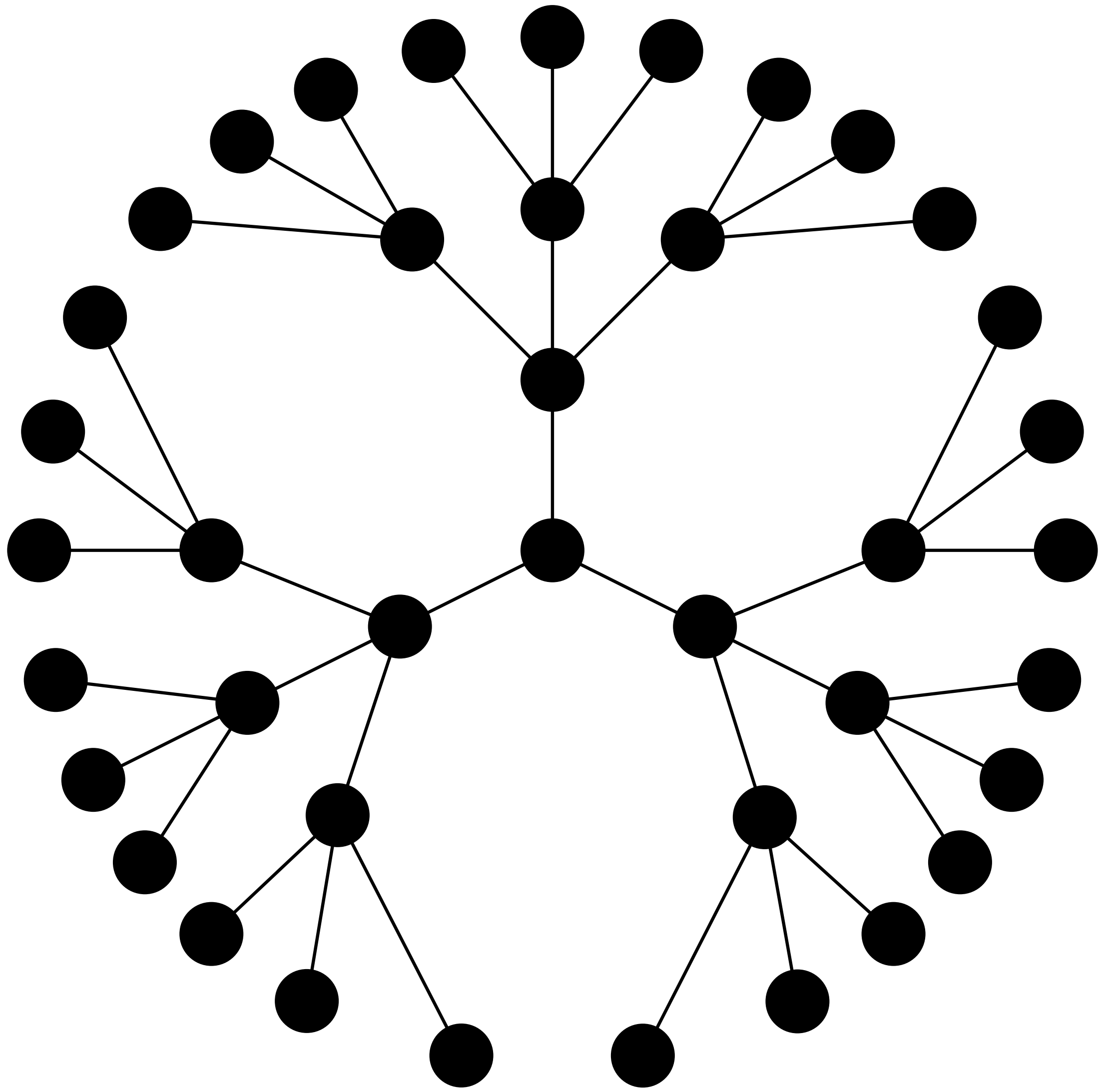

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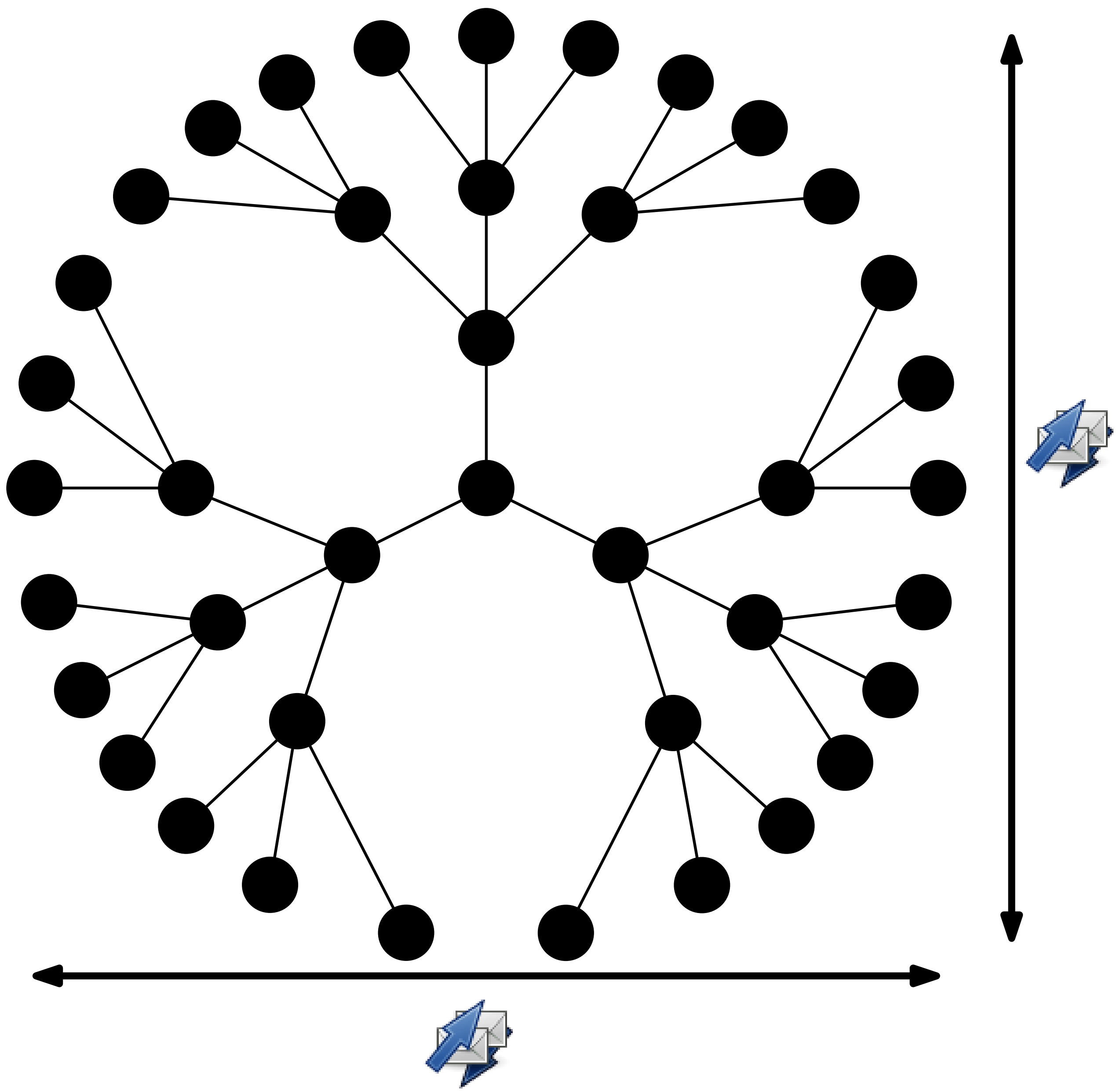

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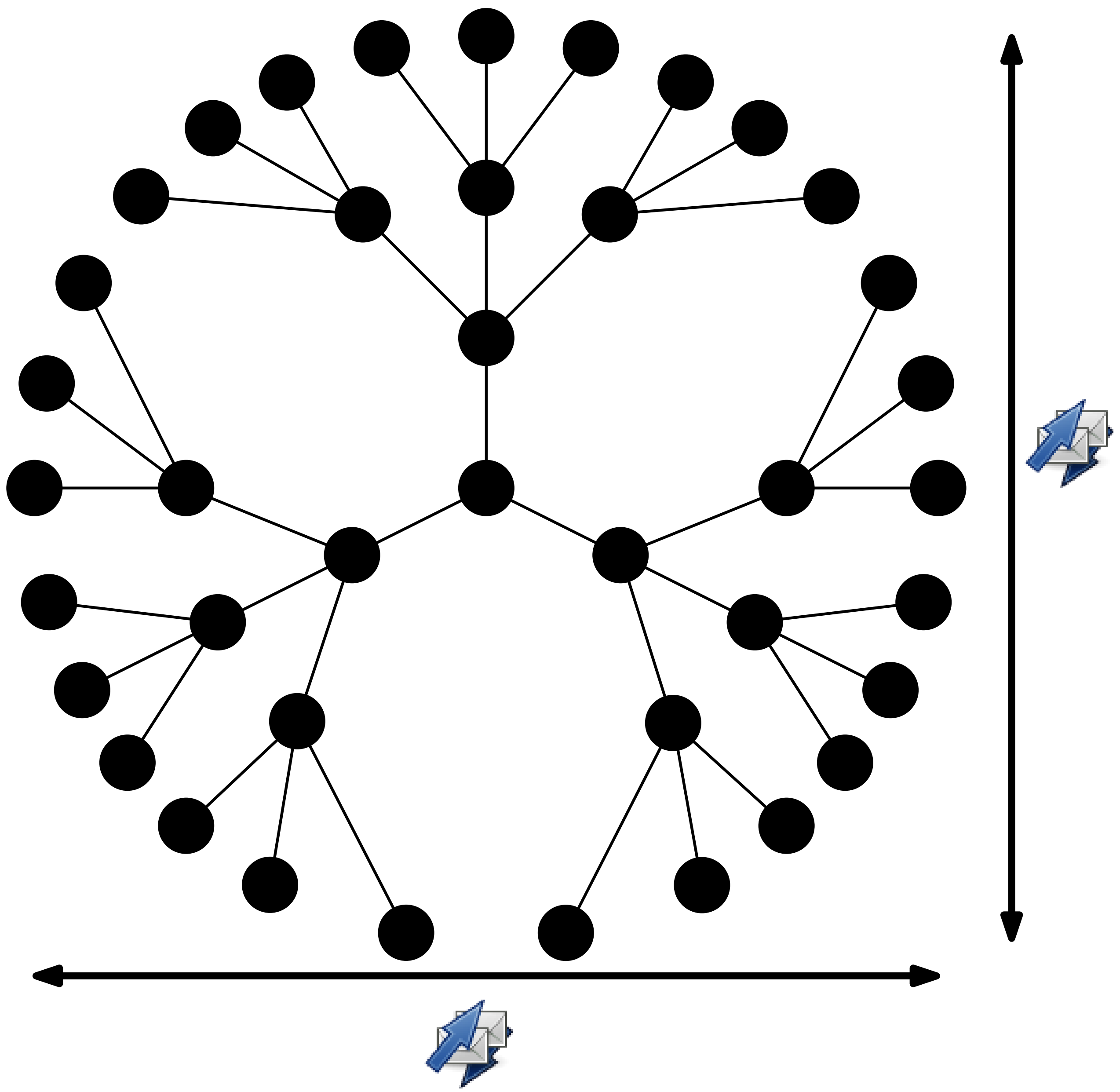

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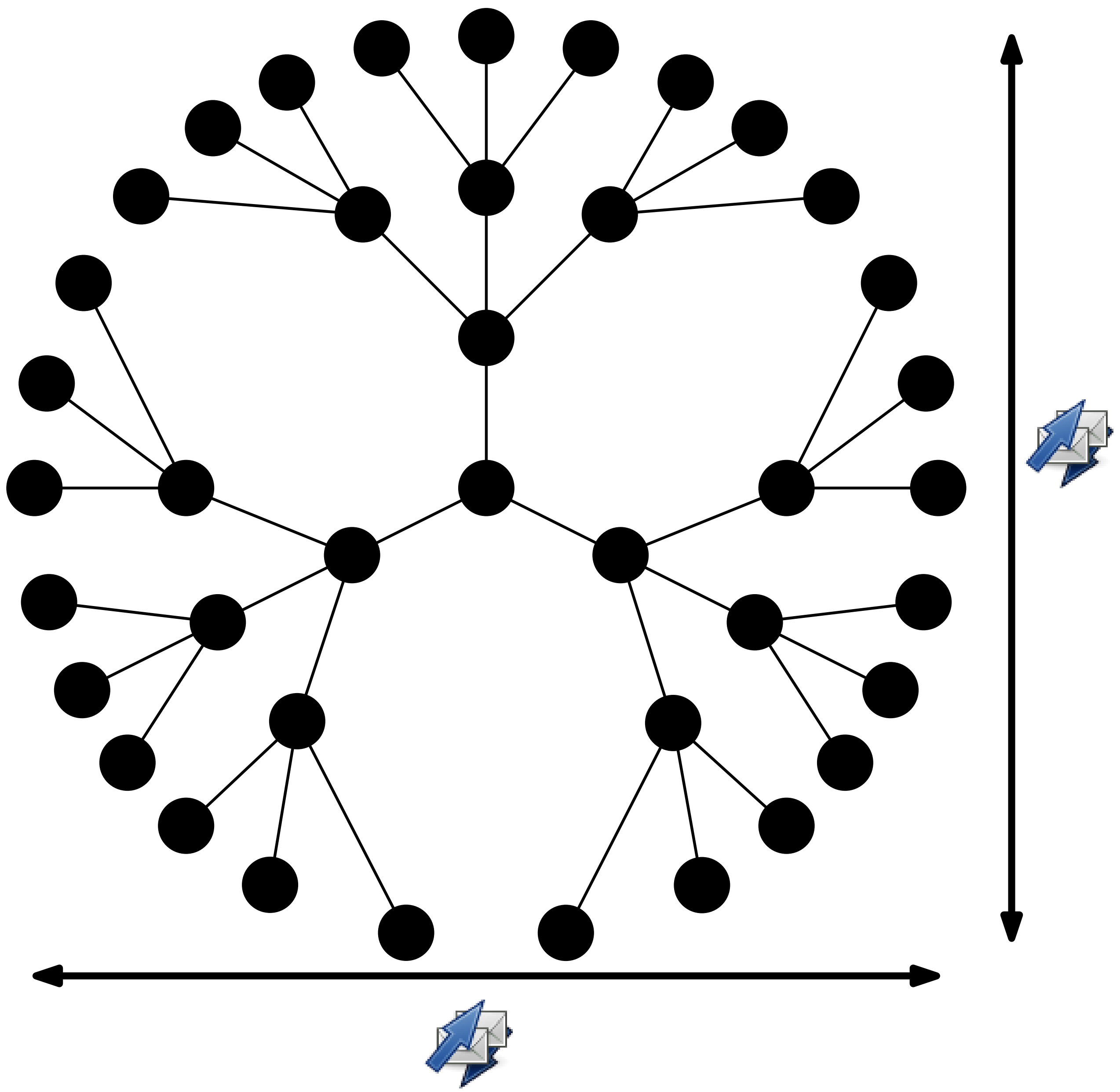

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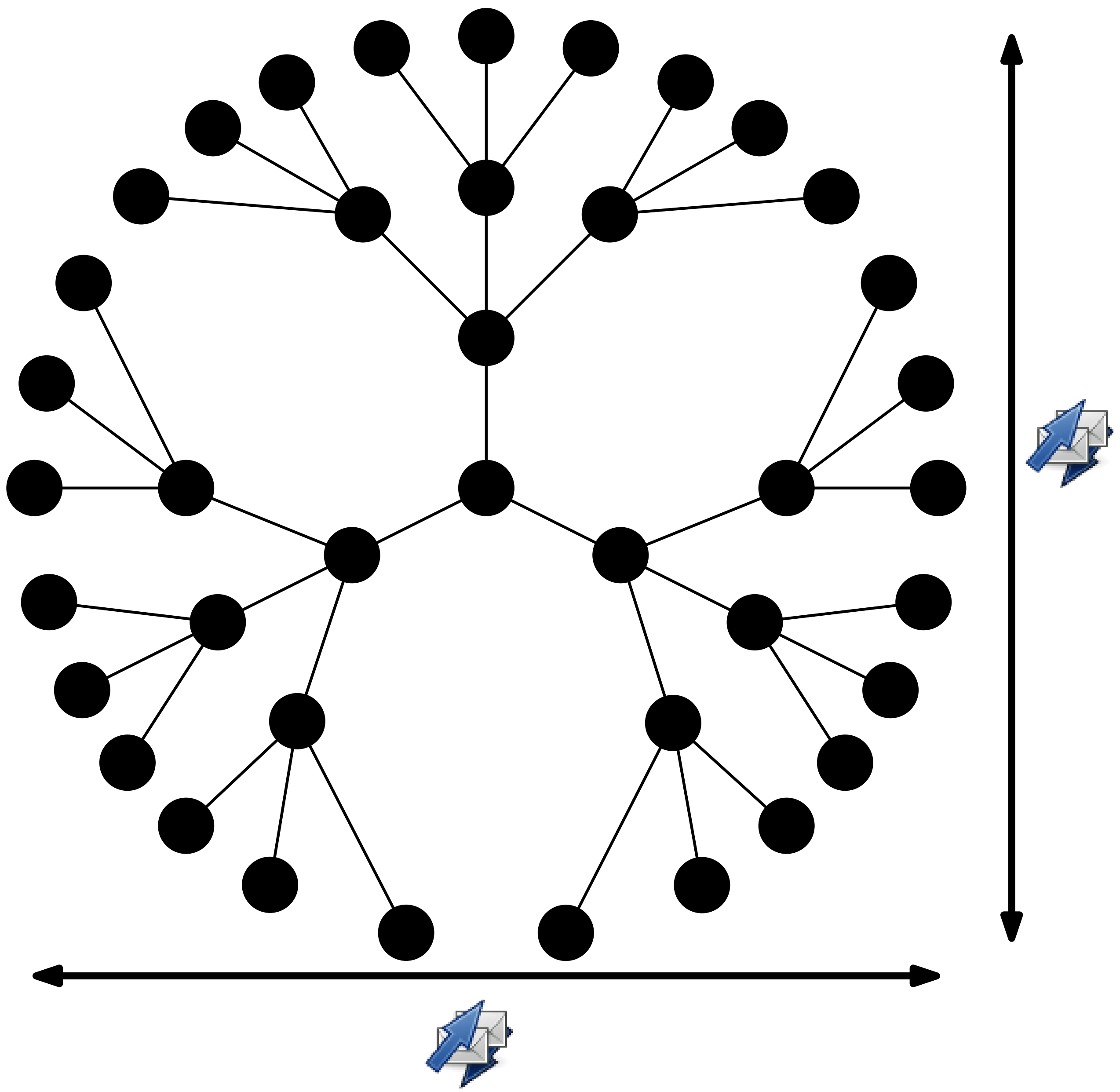

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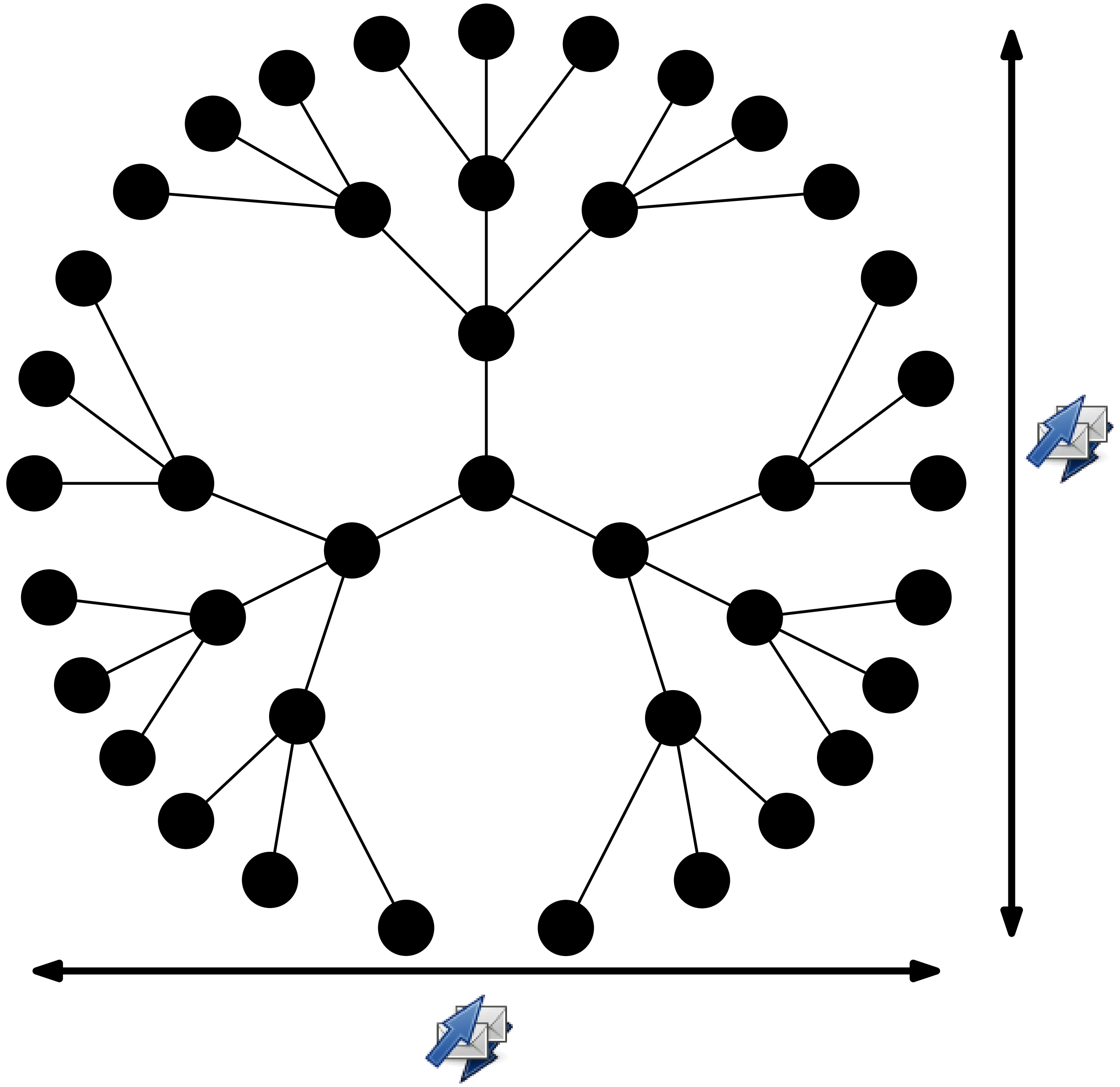

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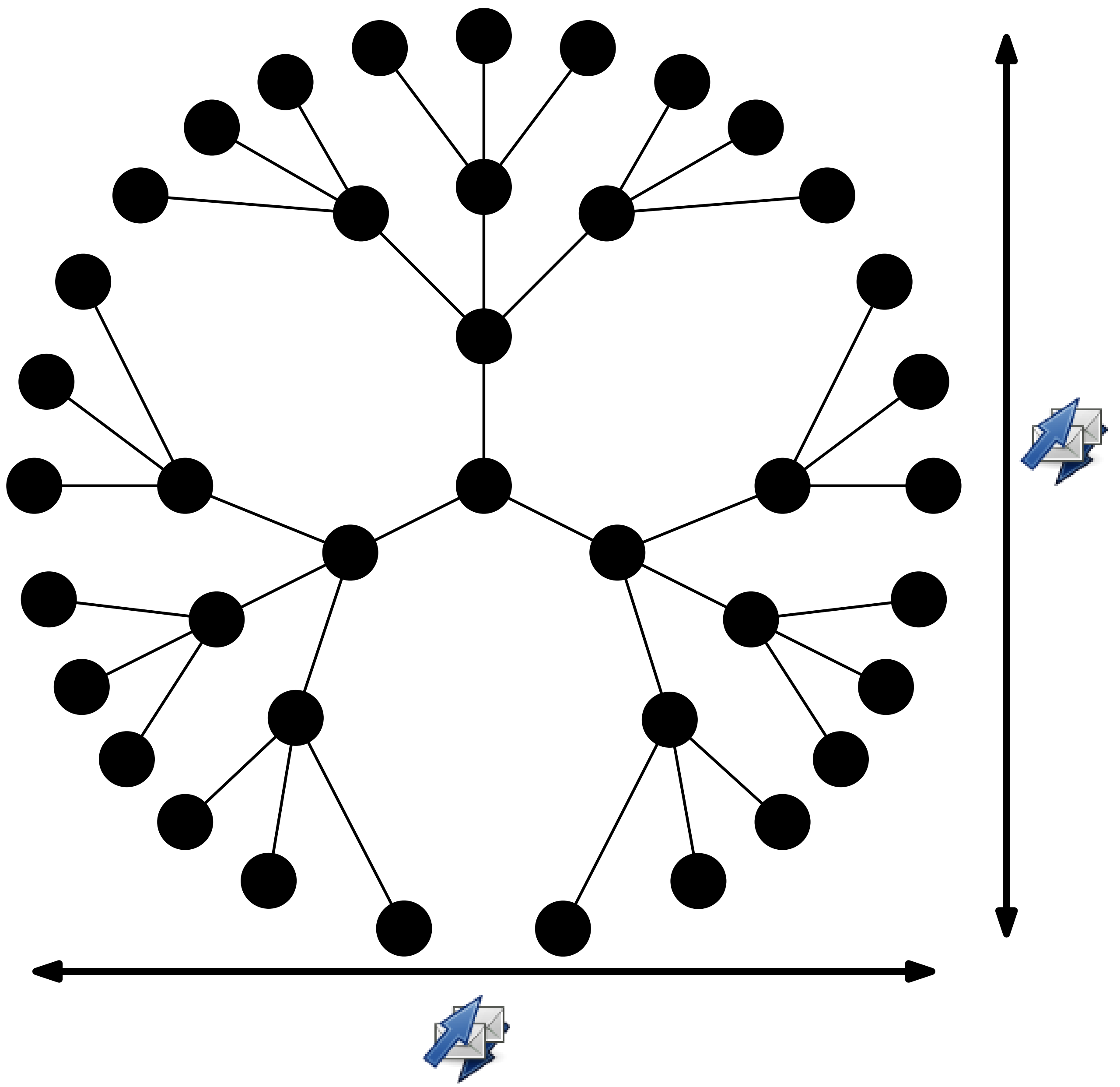
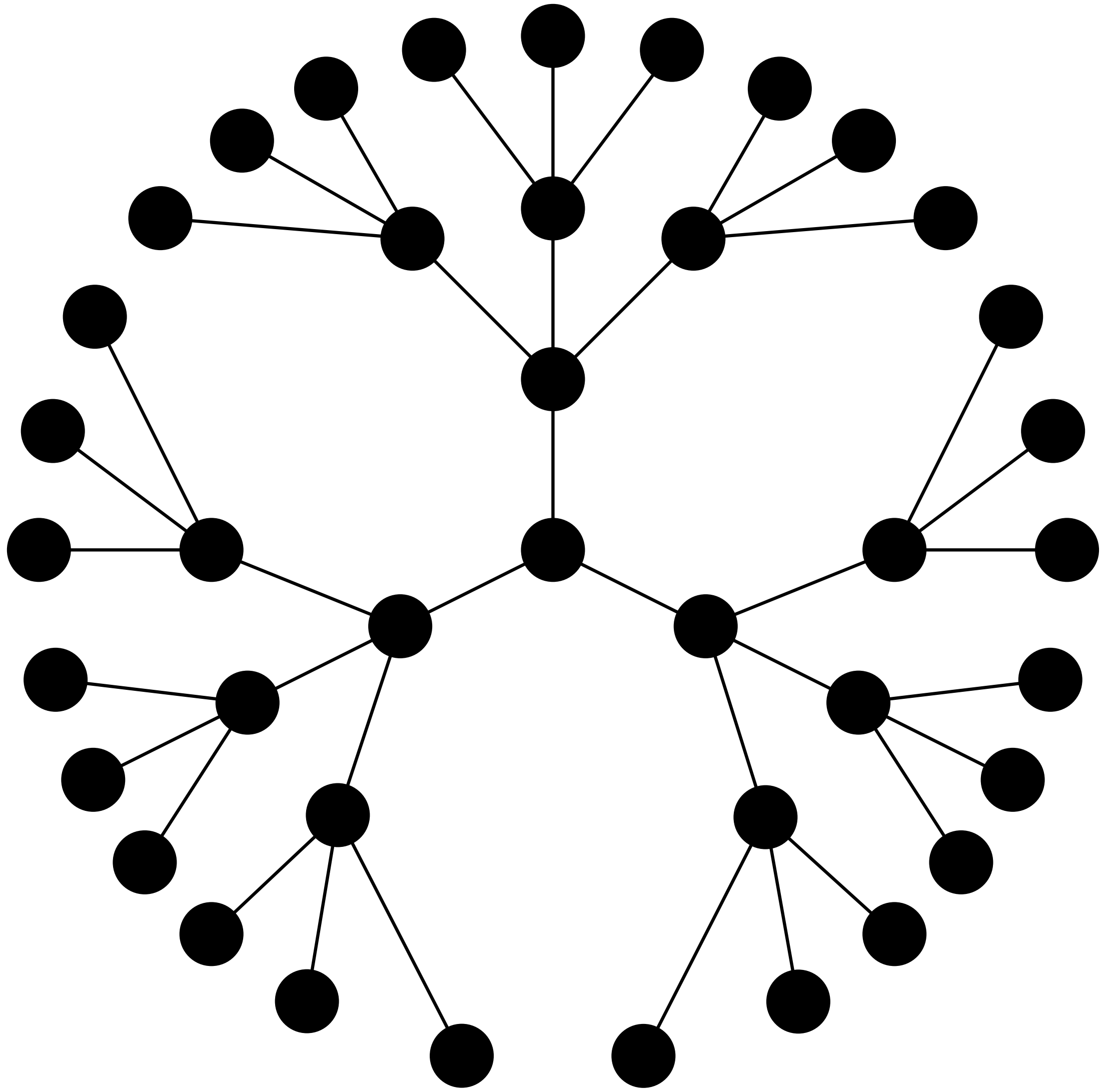


Local view

Complexity measure: number of communication rounds



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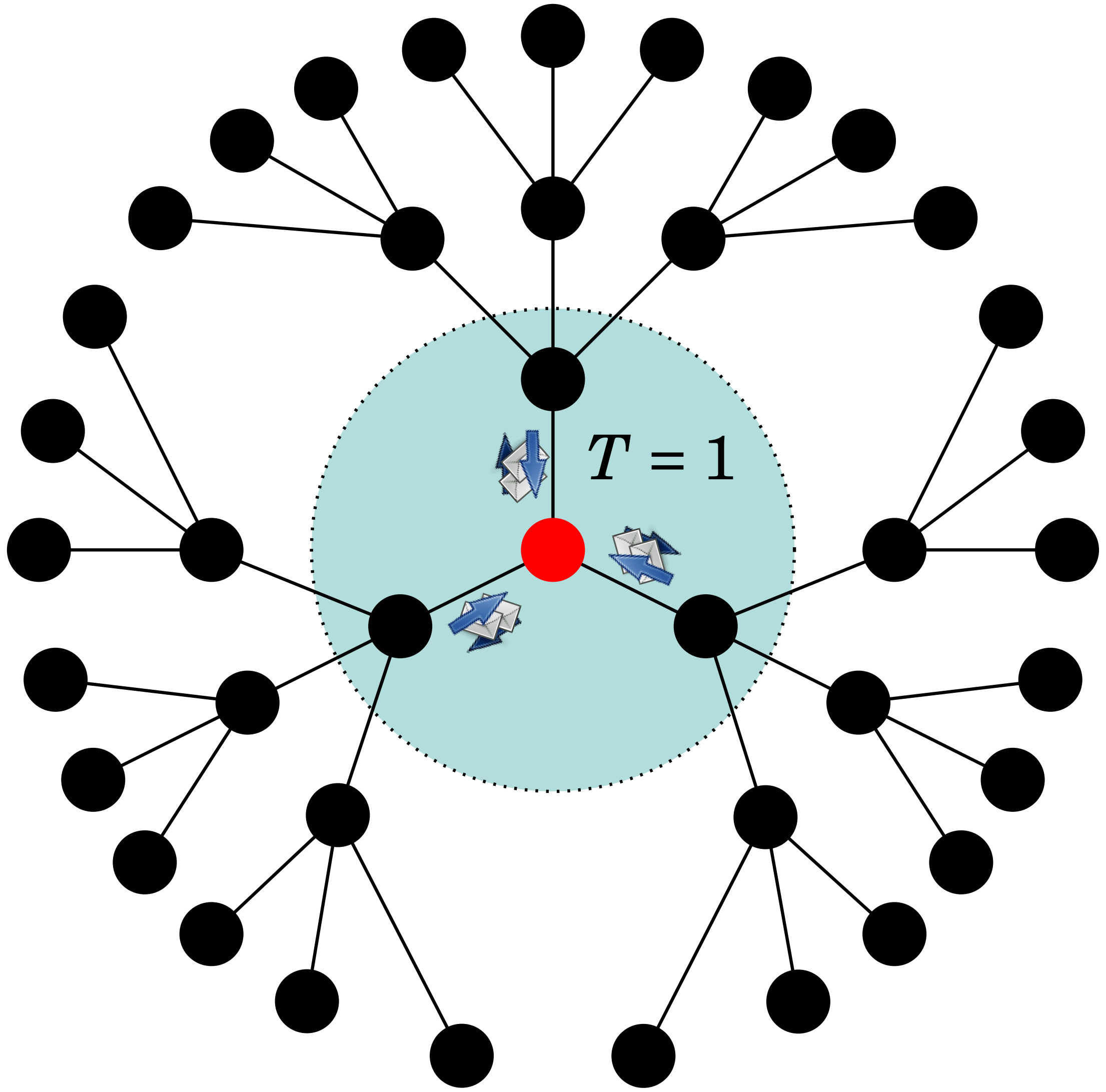

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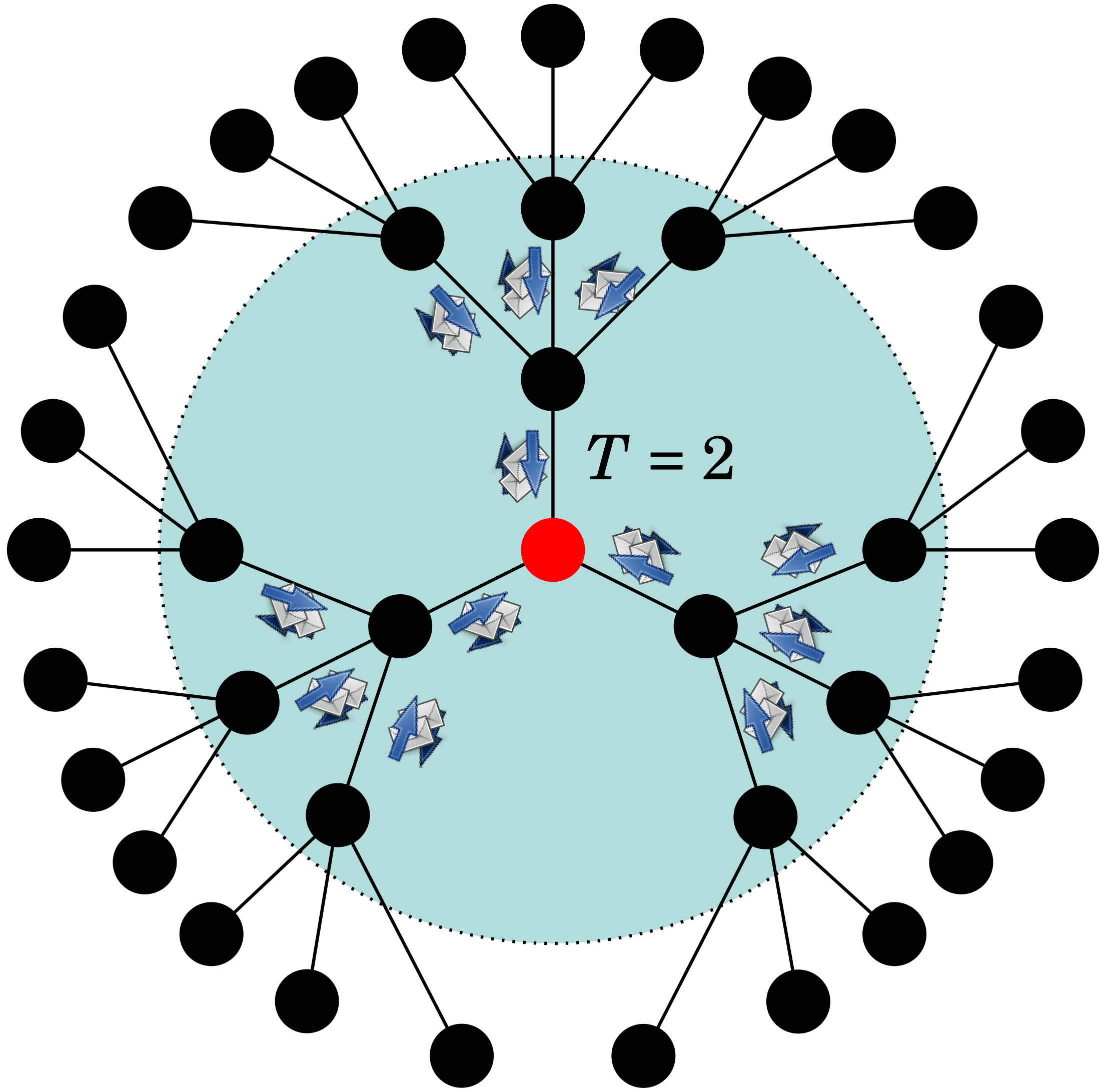

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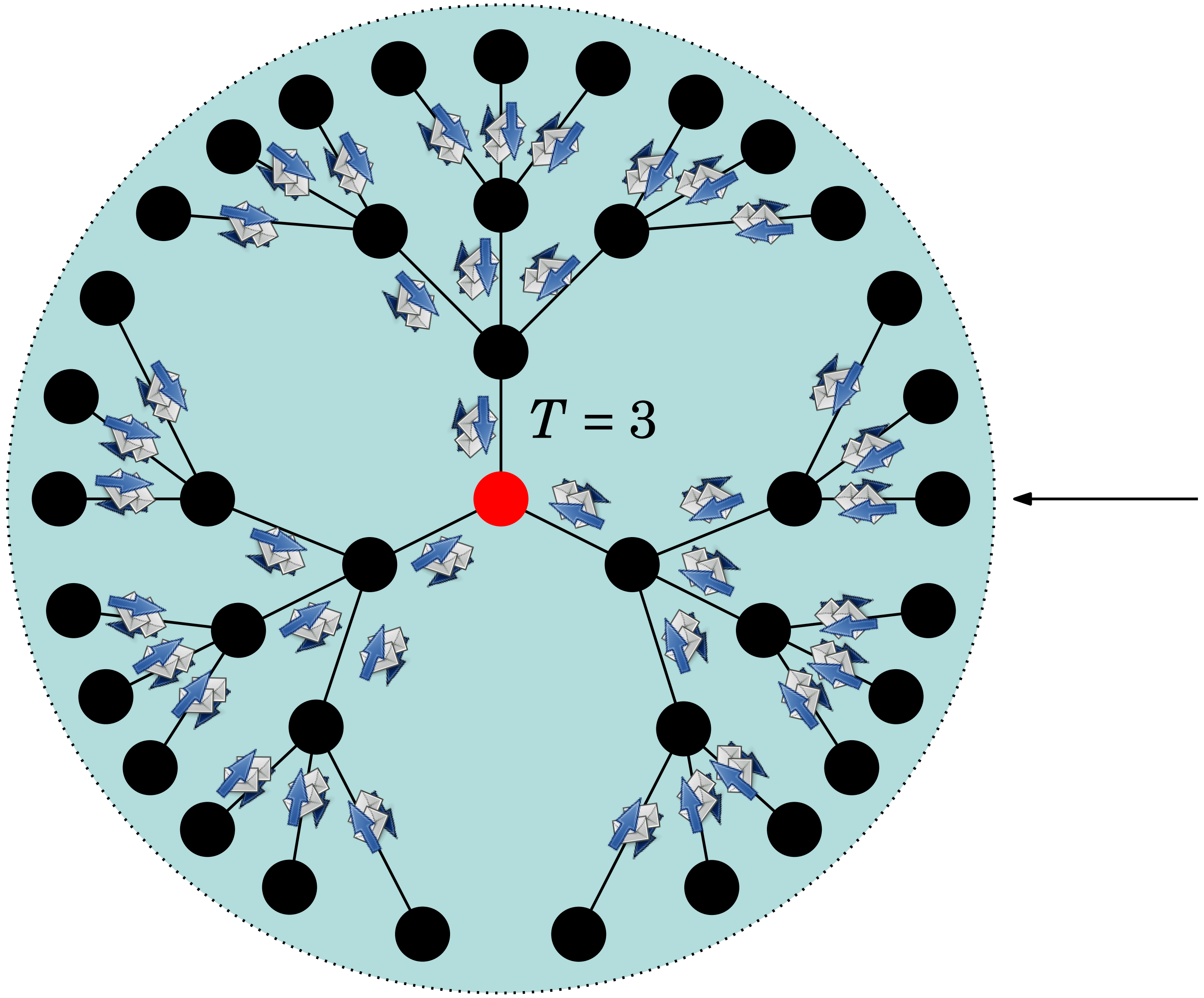

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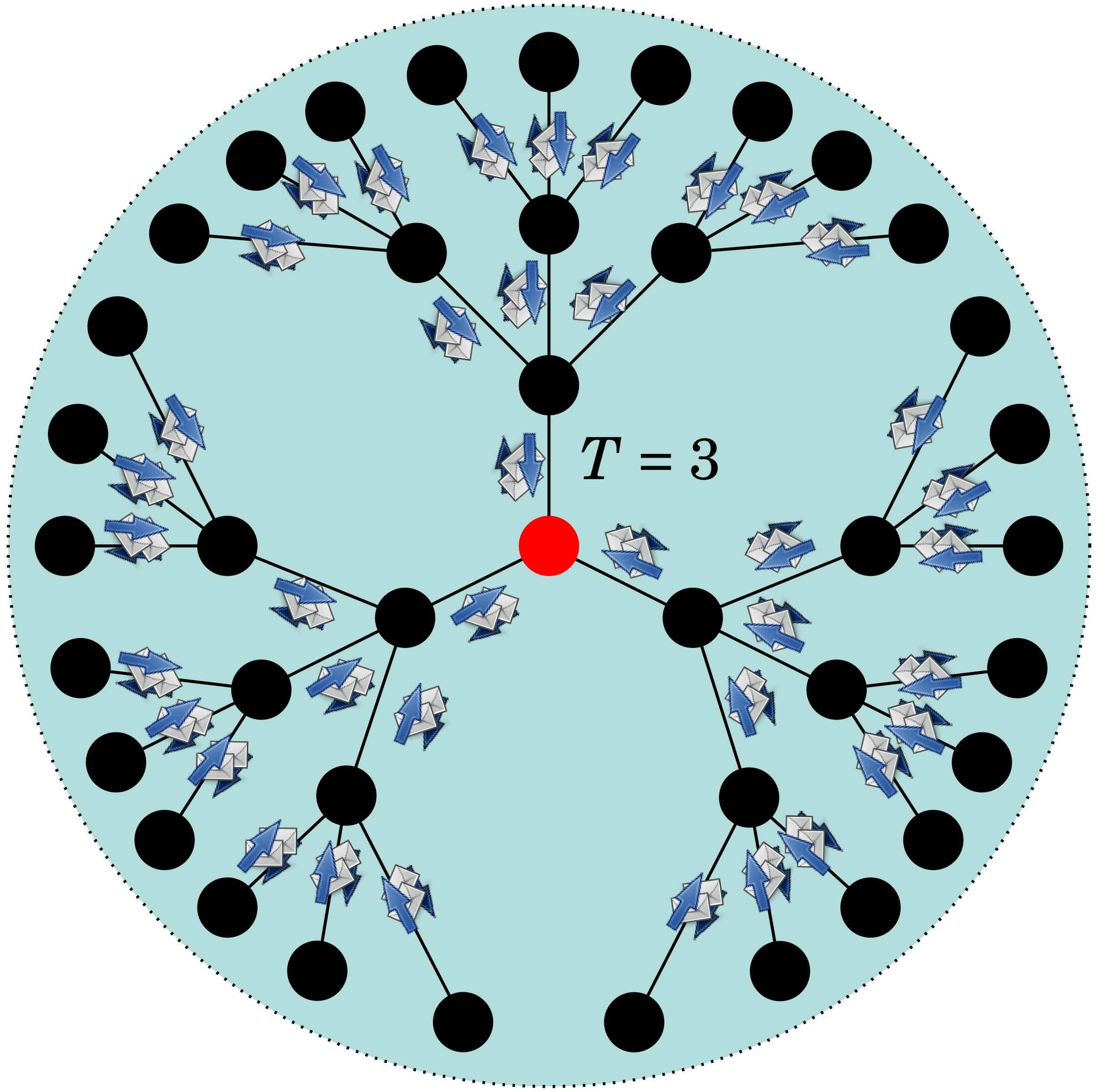

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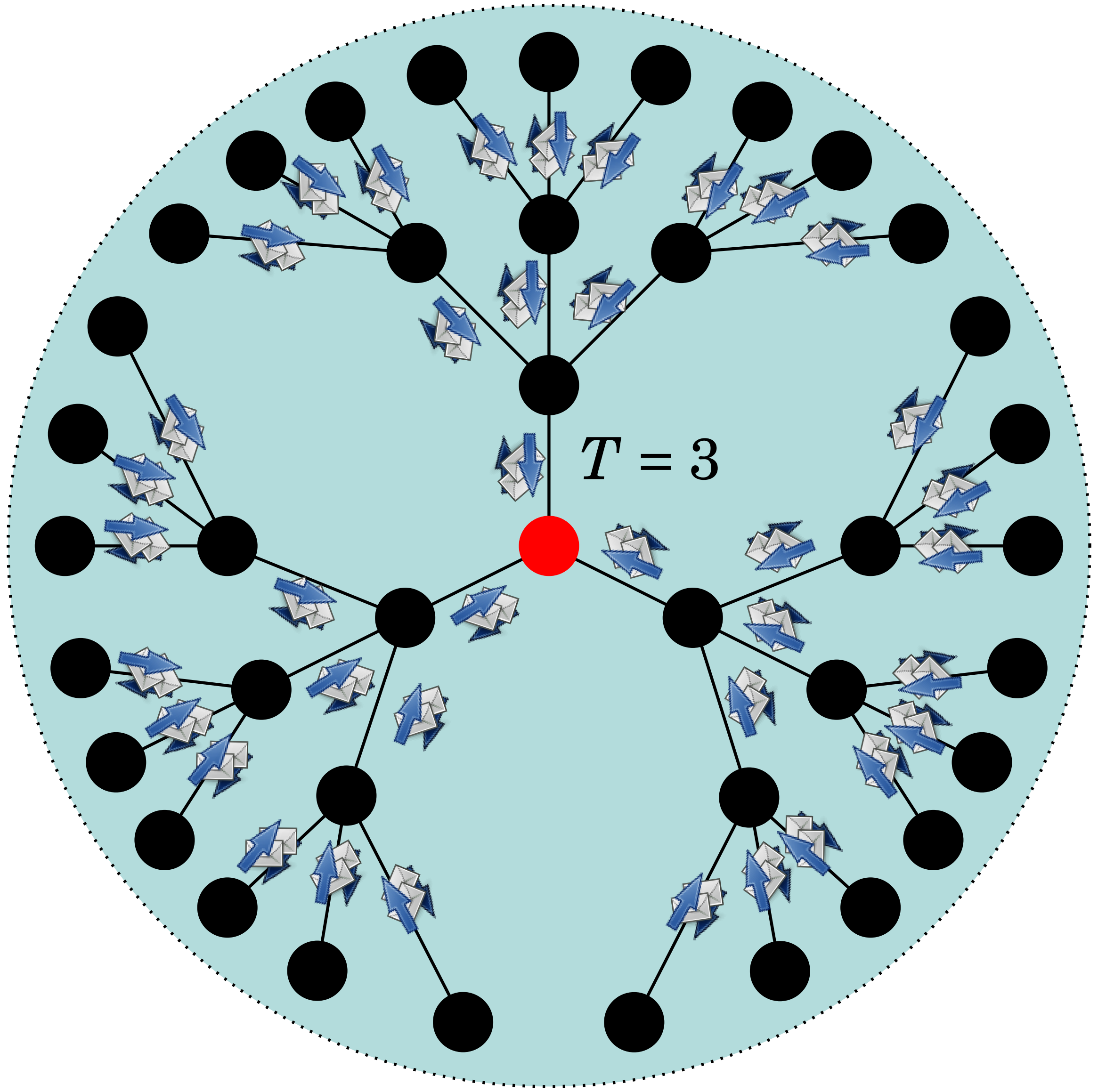

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- **Equivalence**: T-round algorithm ≈ funtion mapping radius-T neighborhoods to local outputs
- Locality $T = \mathrm{diam}(G) + 1$ is always sufficient to solve any problem: gathering algorithm

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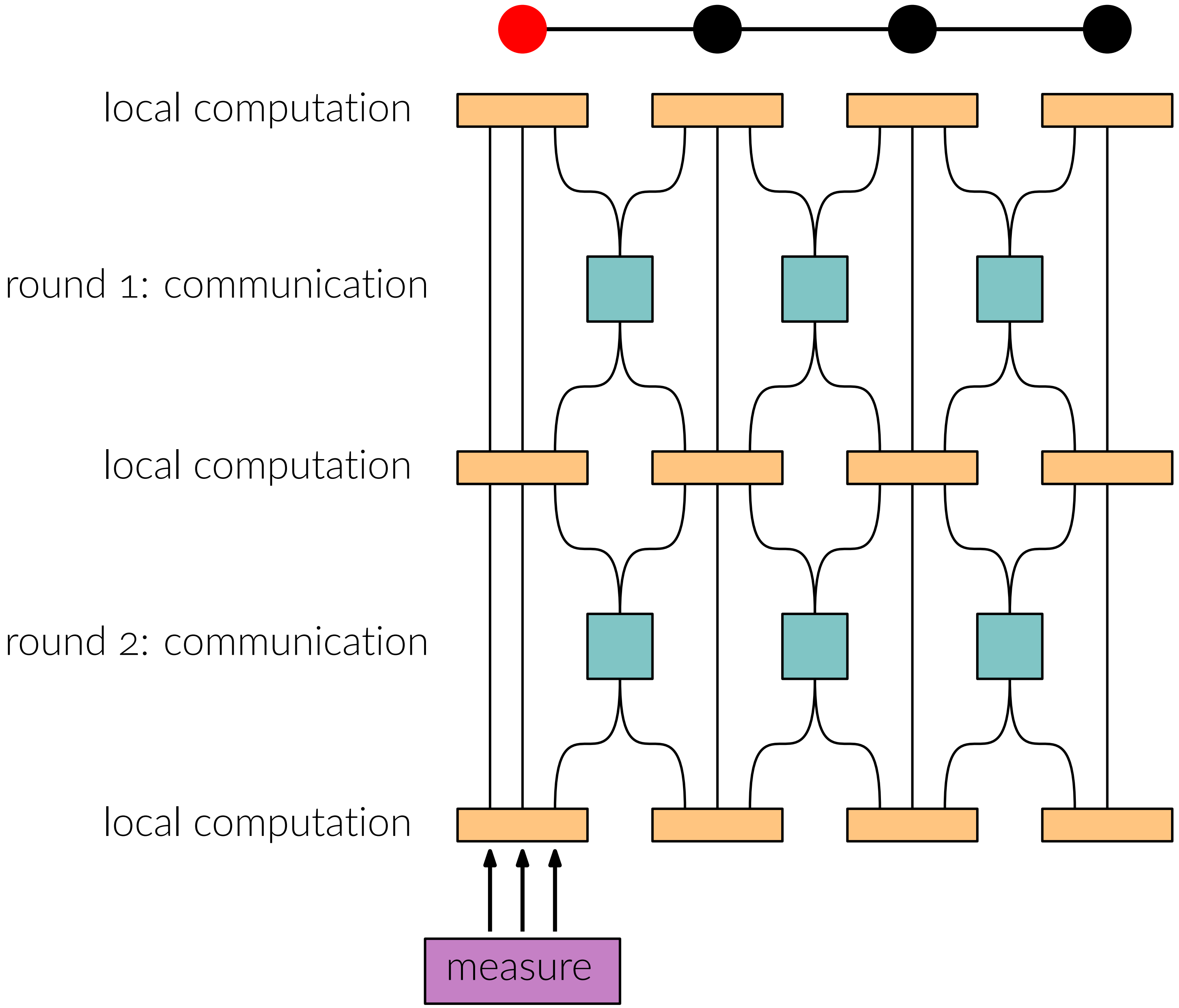
[Gavoille et al., DISC '09]

- **Distributed system** of n quantum processors/nodes
  - quantum computation
  - quantum communication (qubits)
  - output: measurement of qubits



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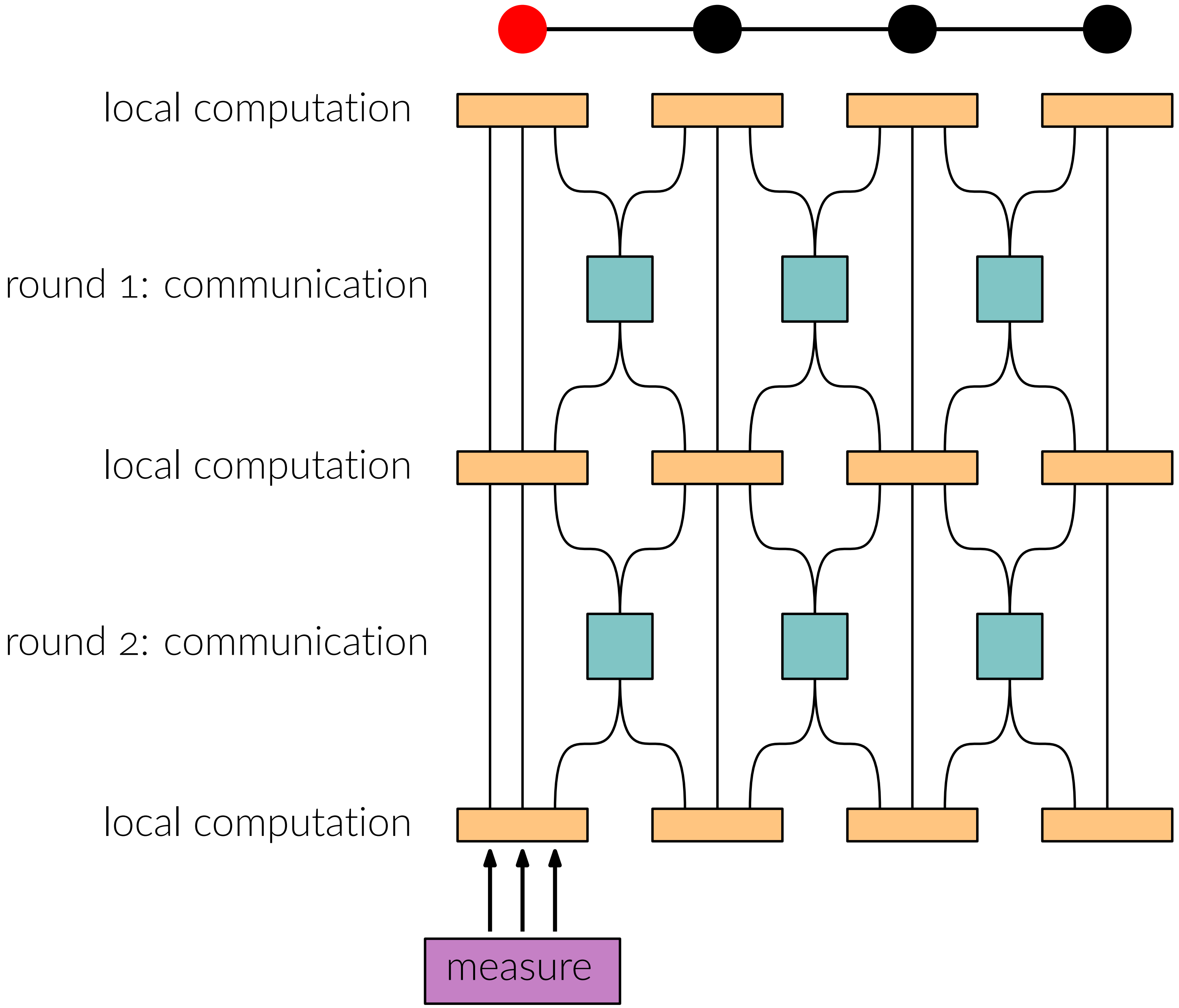

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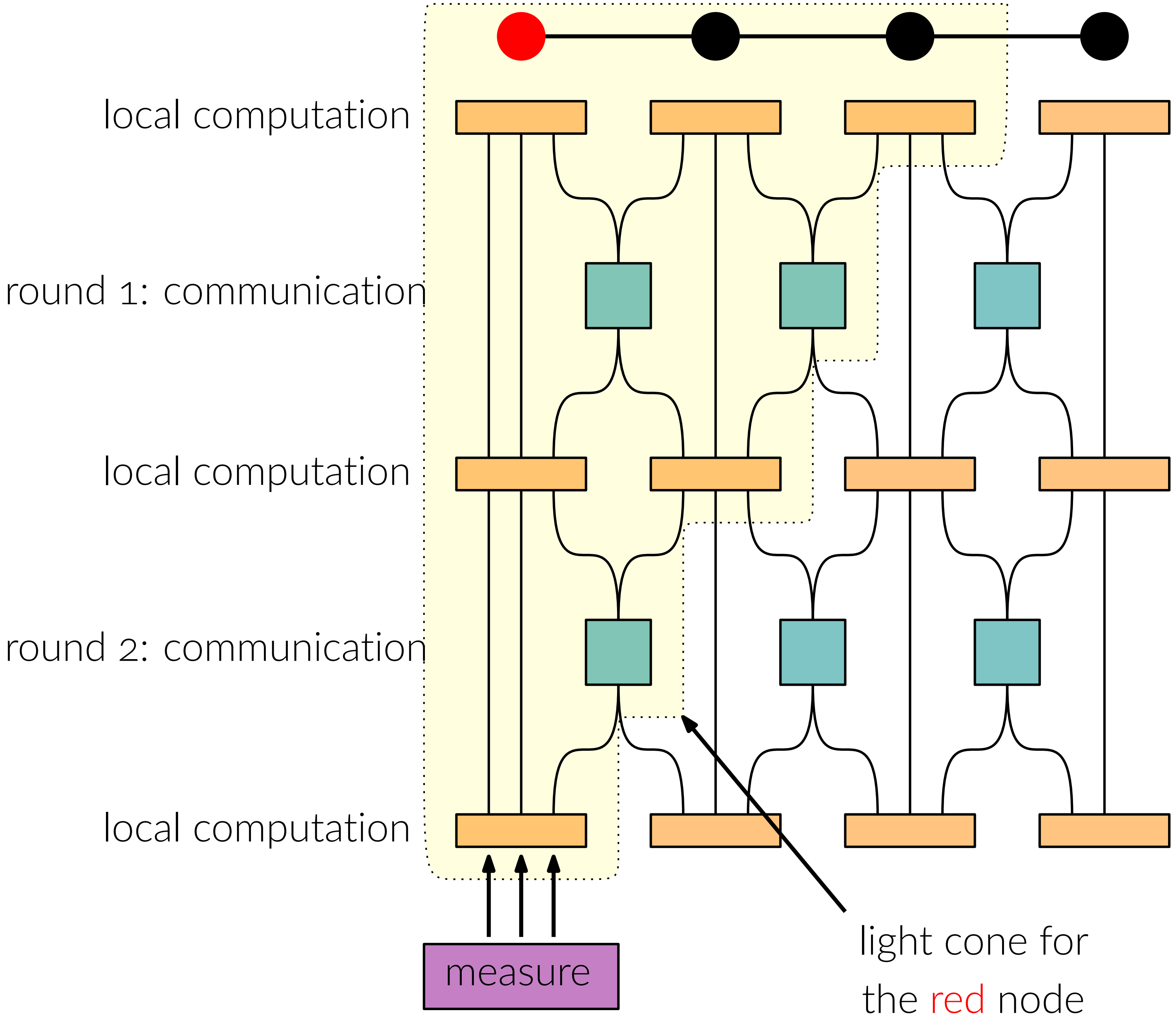

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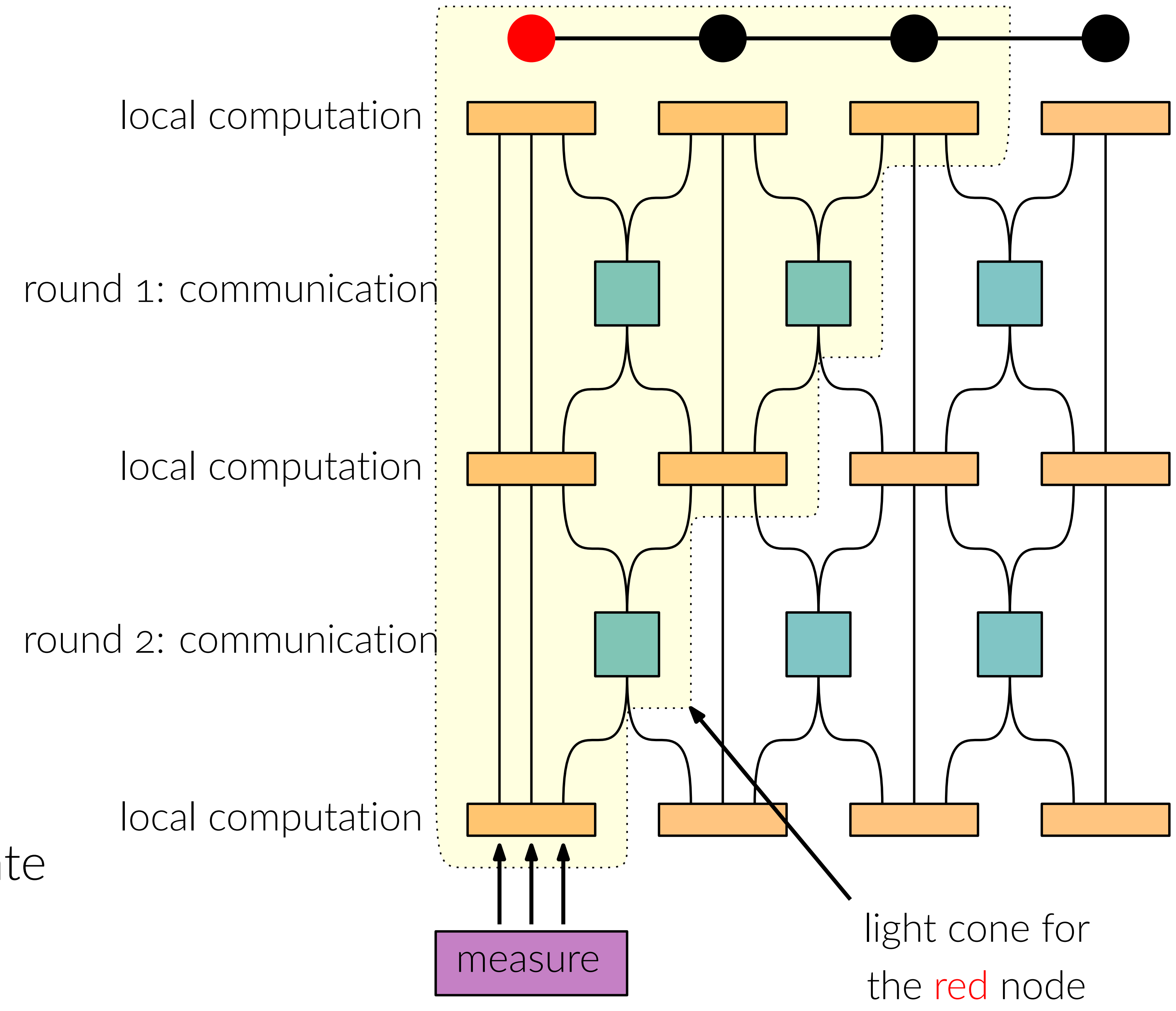
- Still, locality identifies how *far* nodes need to communicate



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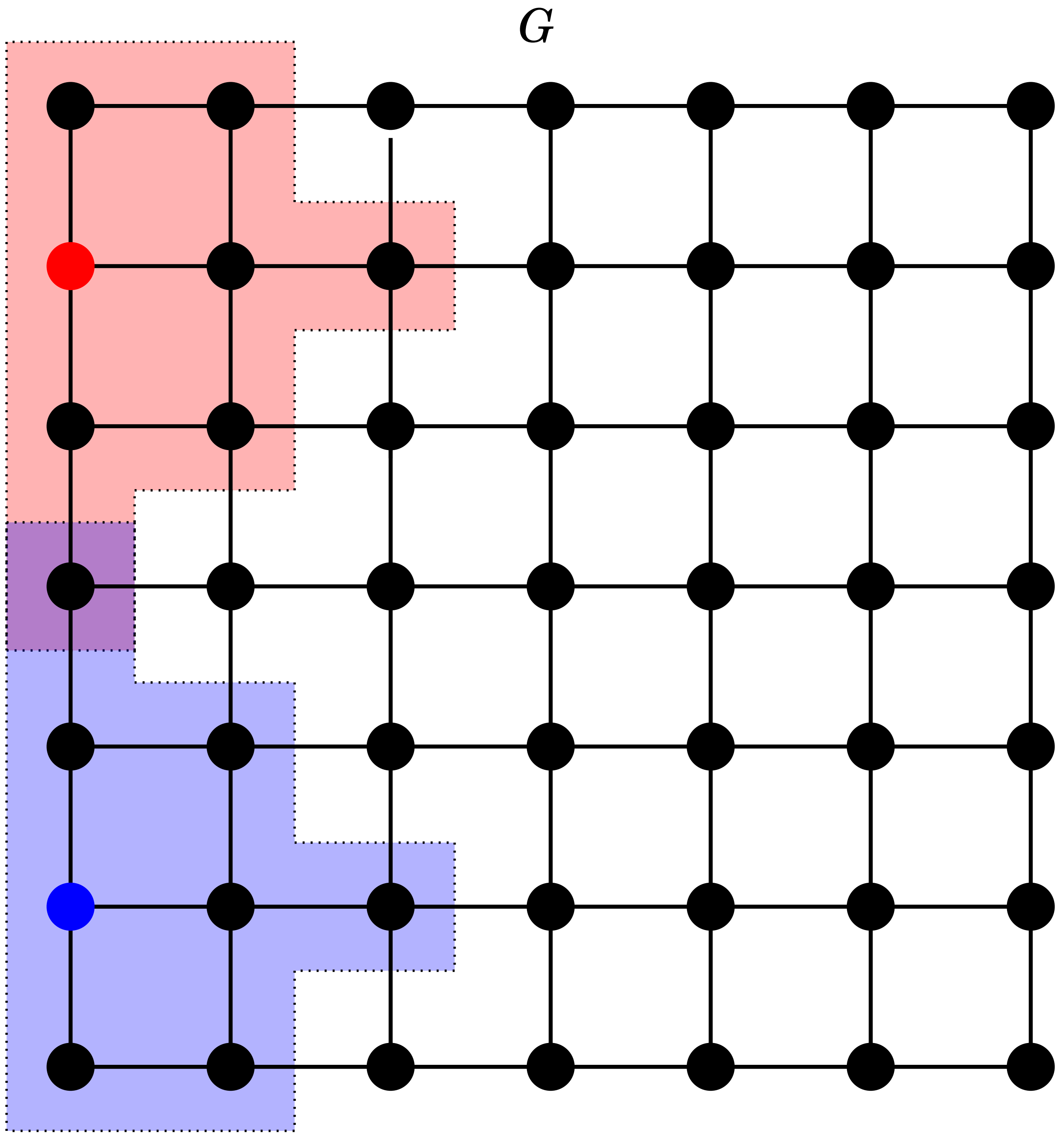

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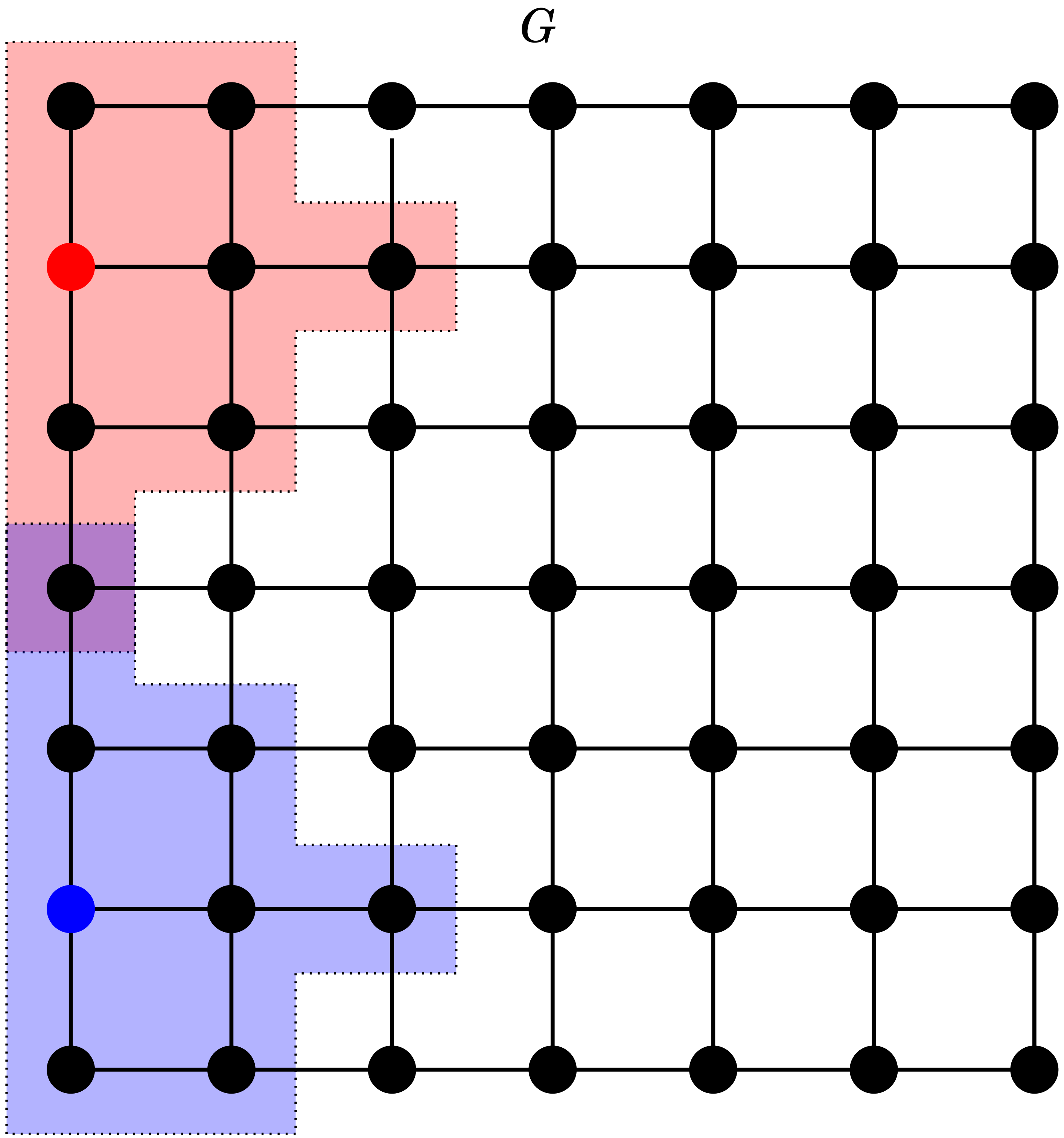

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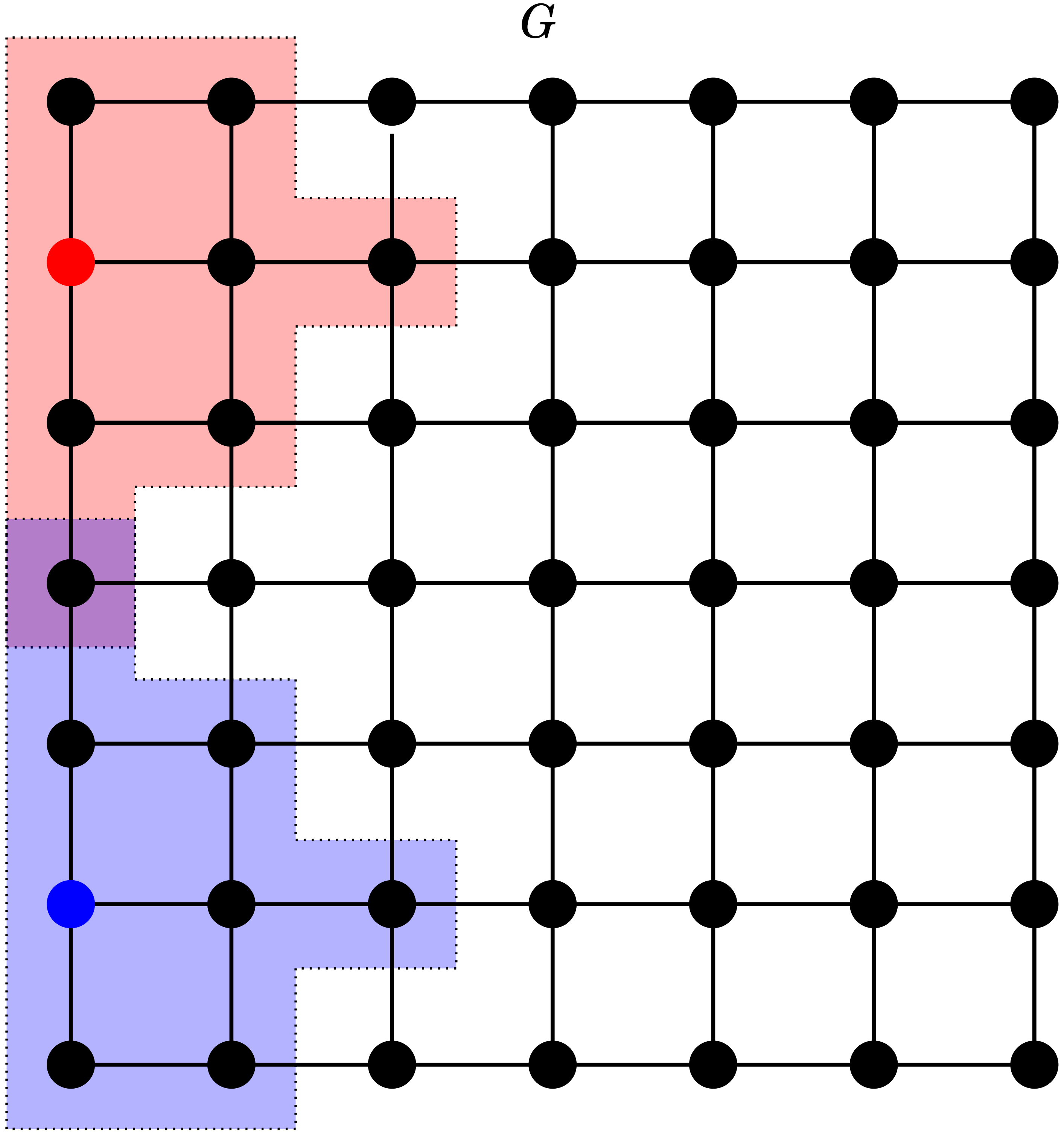

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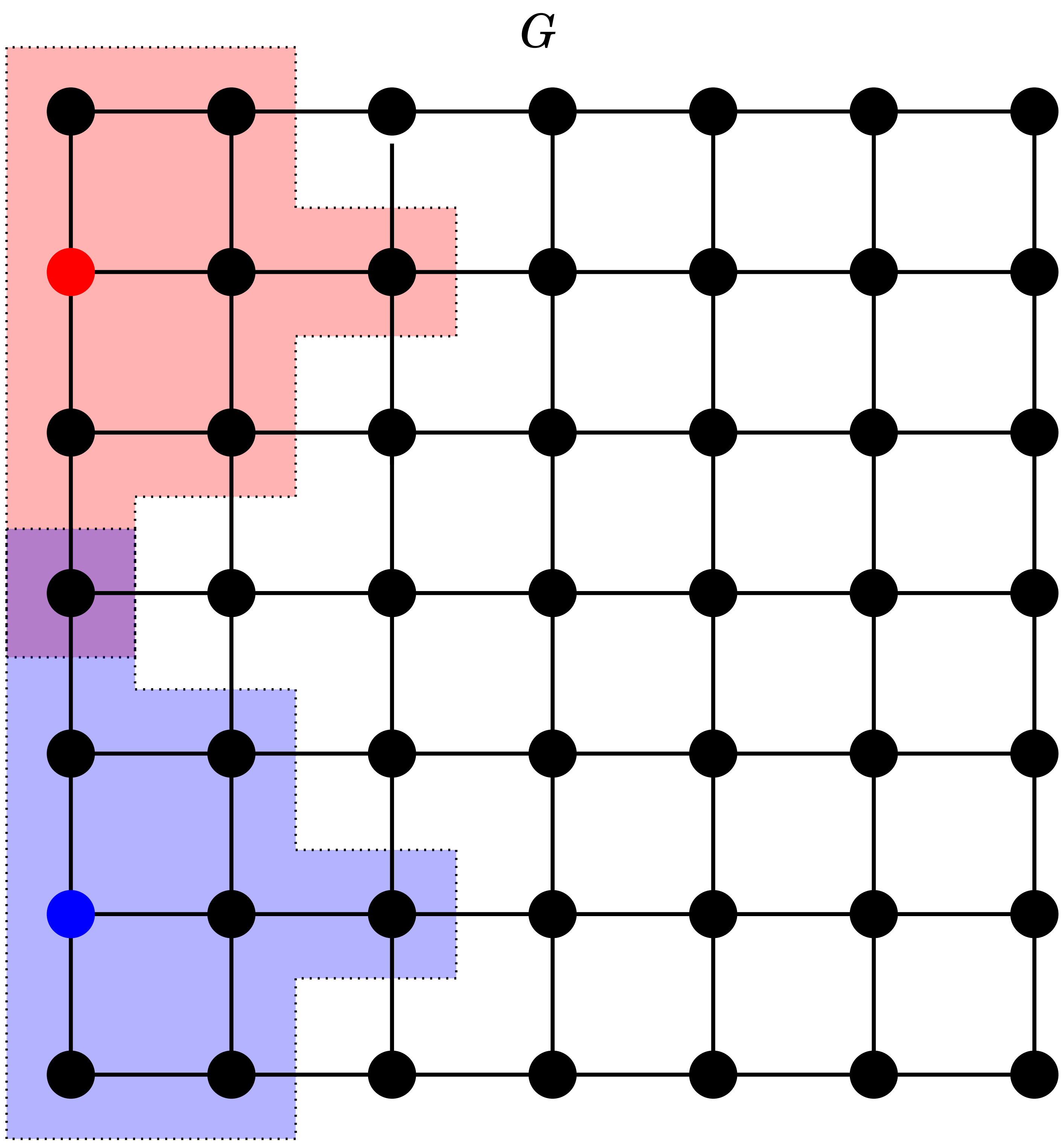

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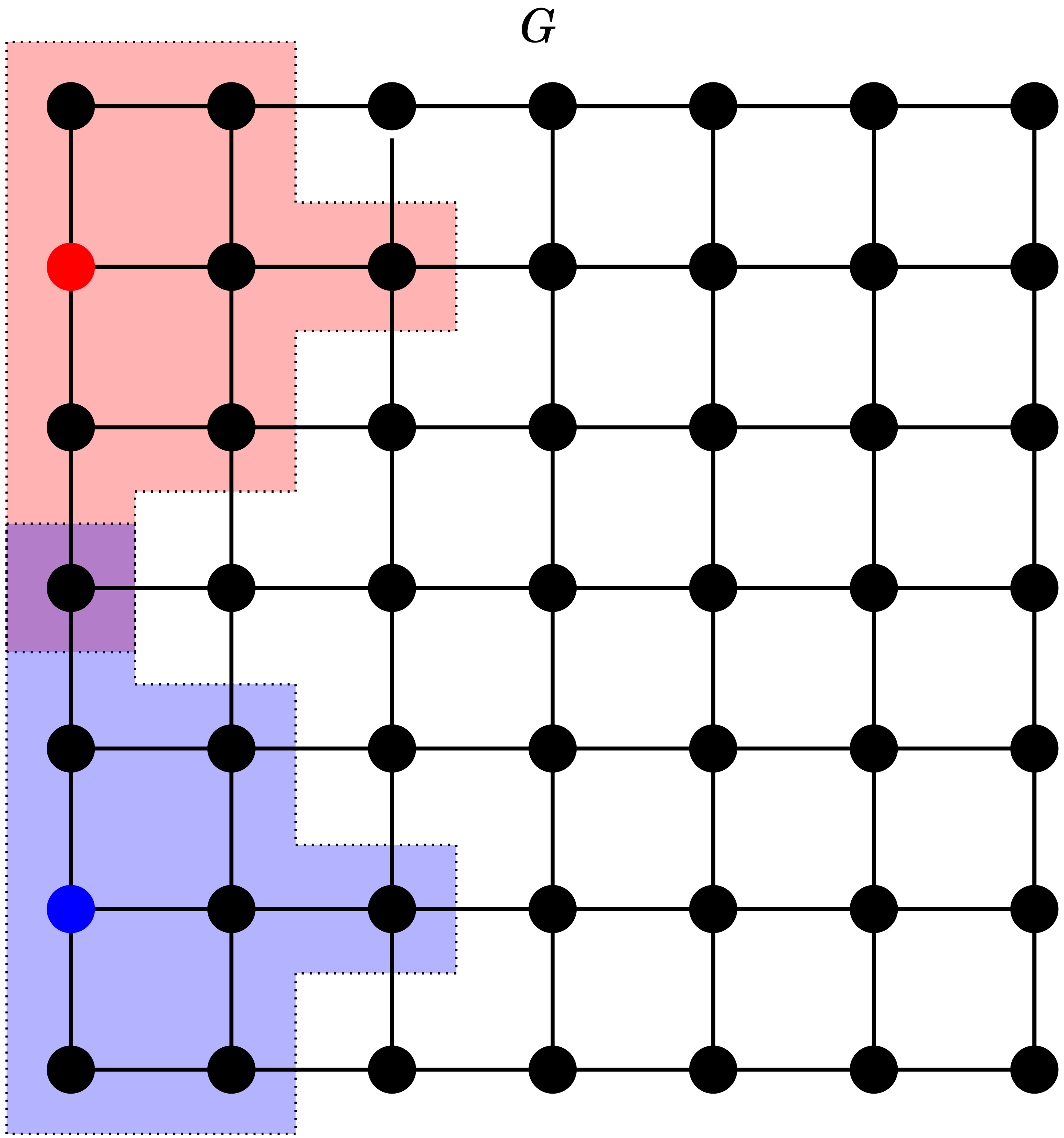

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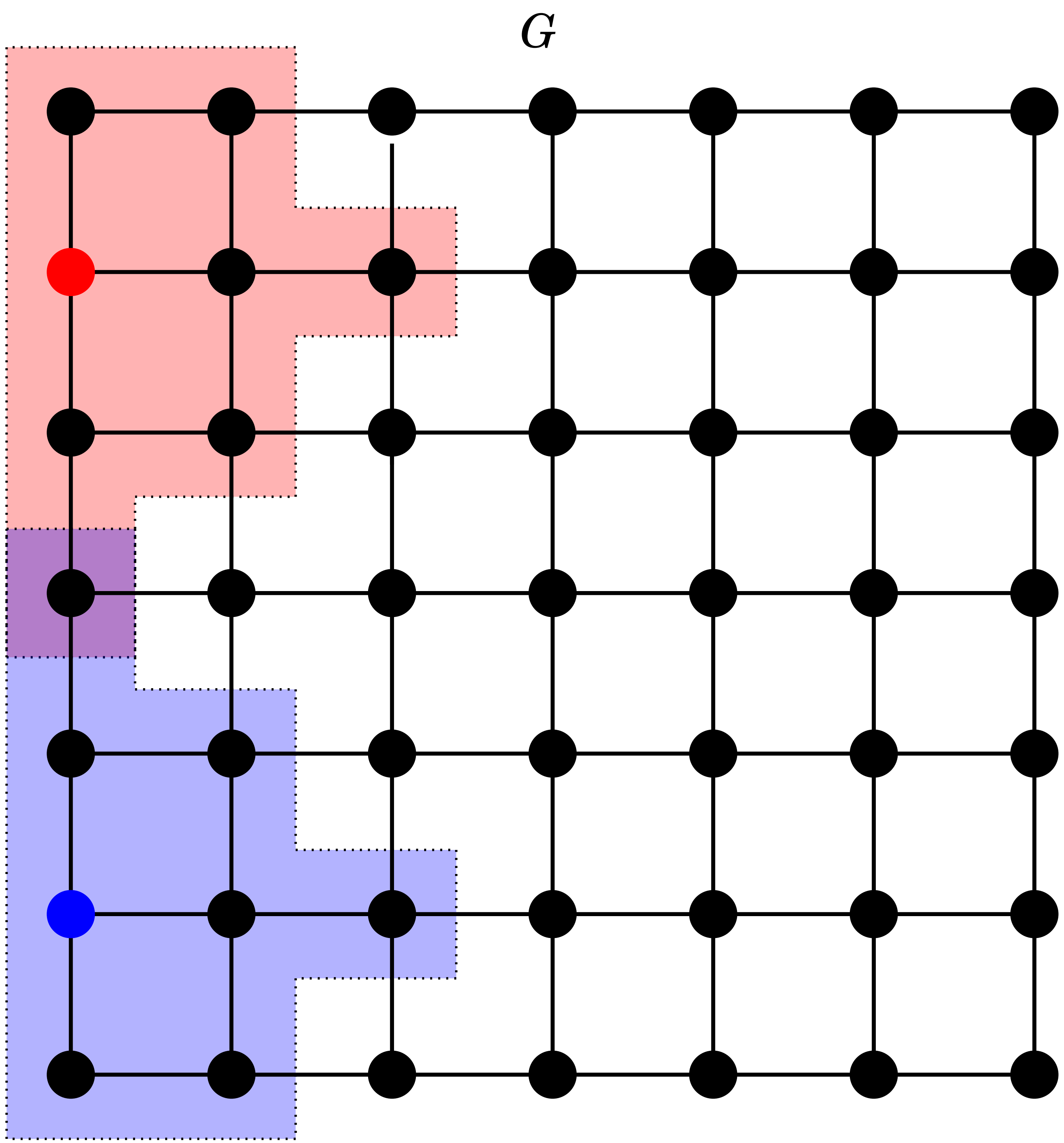
- **Question**: *is there any graph problem that admits quantum advantage?*



Locally checkable labeling (LCL) problems

[Naor and Stockmeyer, STOC '93 & SICOMP '95]

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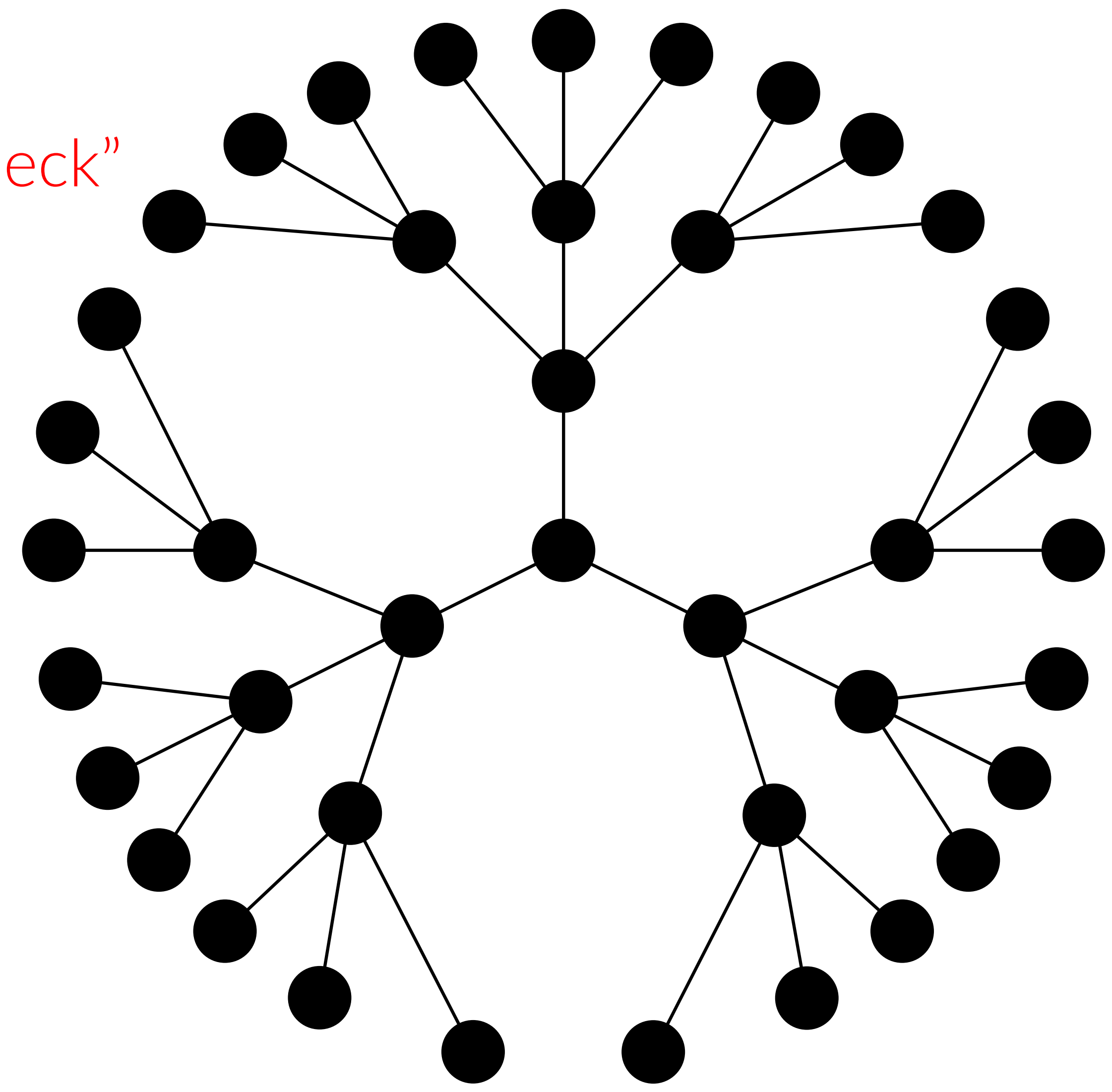
  - coloring, maximal independent set, maximal matching, etc.



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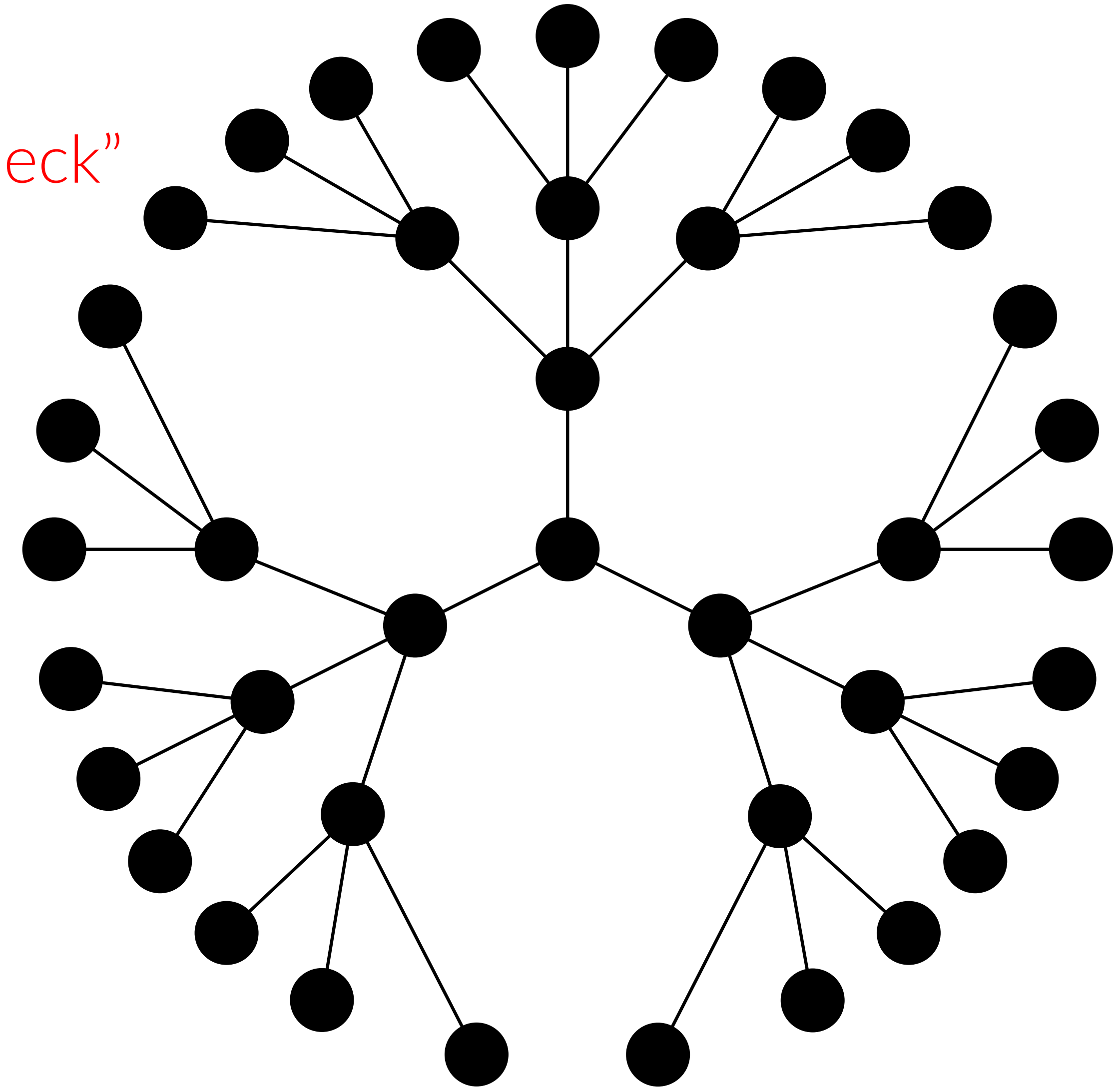

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  - radius $r = \Theta(1)$
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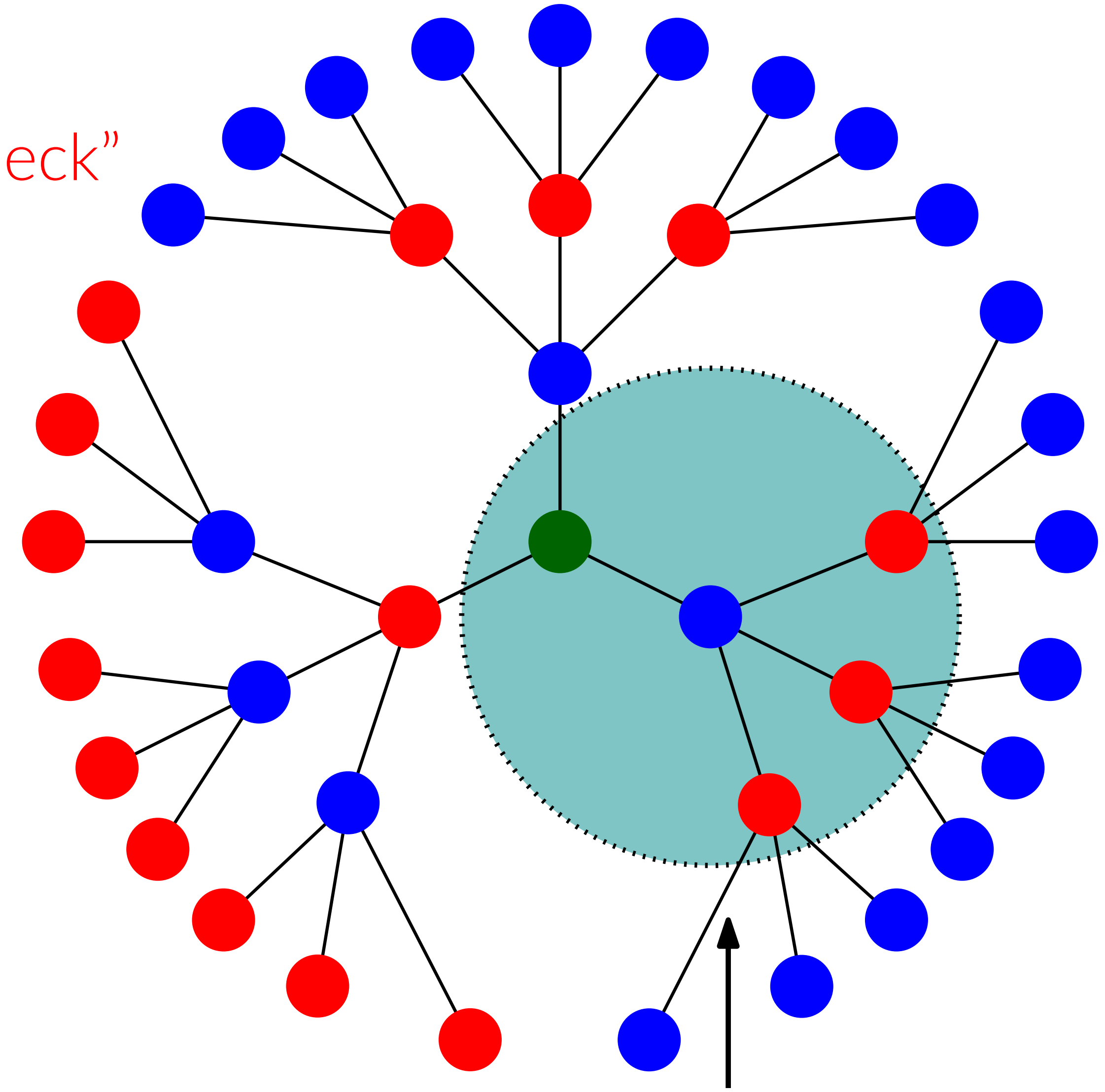

3-**coloring**: the blue node checks if its color is different from those of its neighbors

valid LCL

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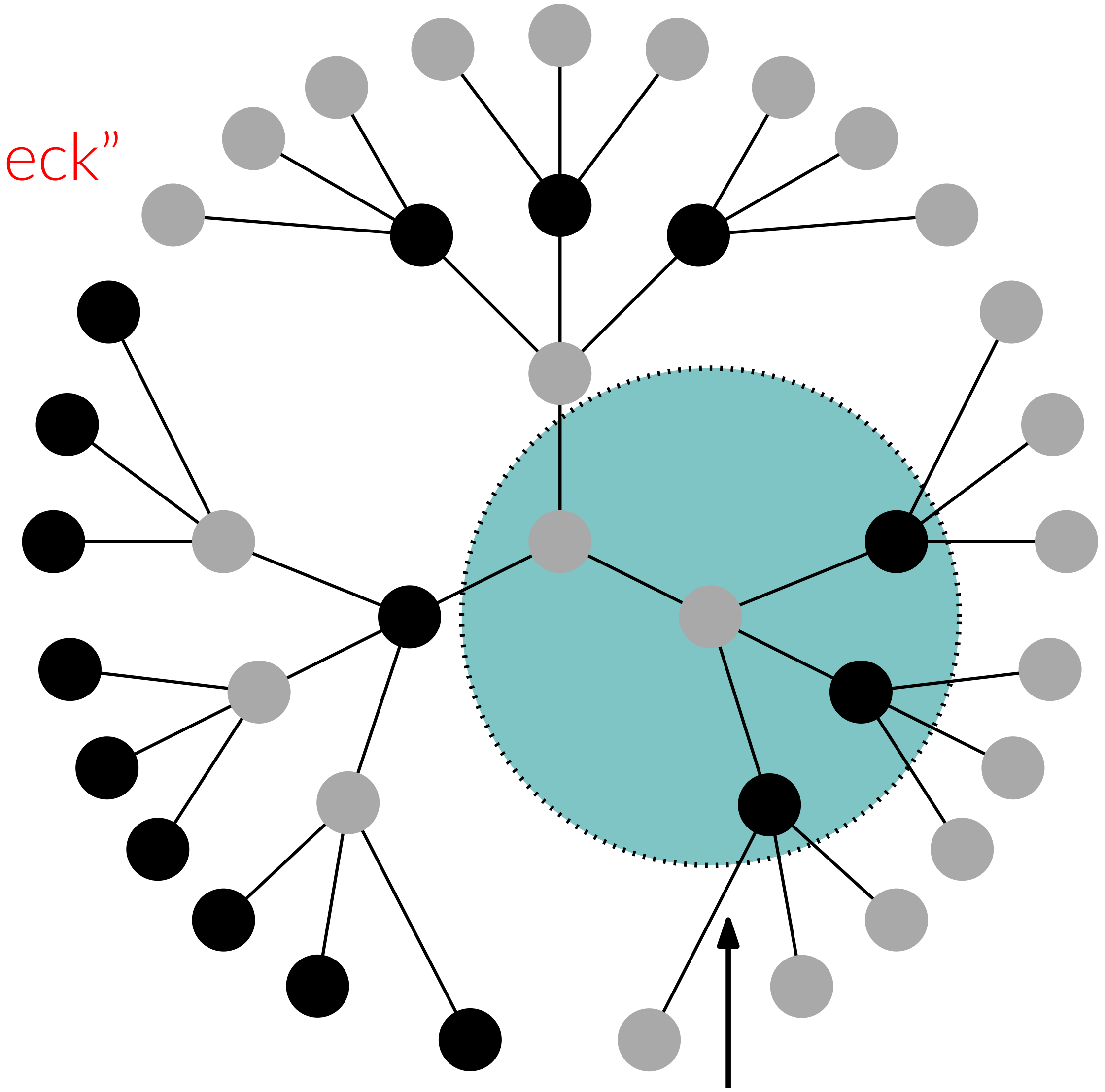

MIS: each node checks if it is in the IS or if it has a neighbor in the IS

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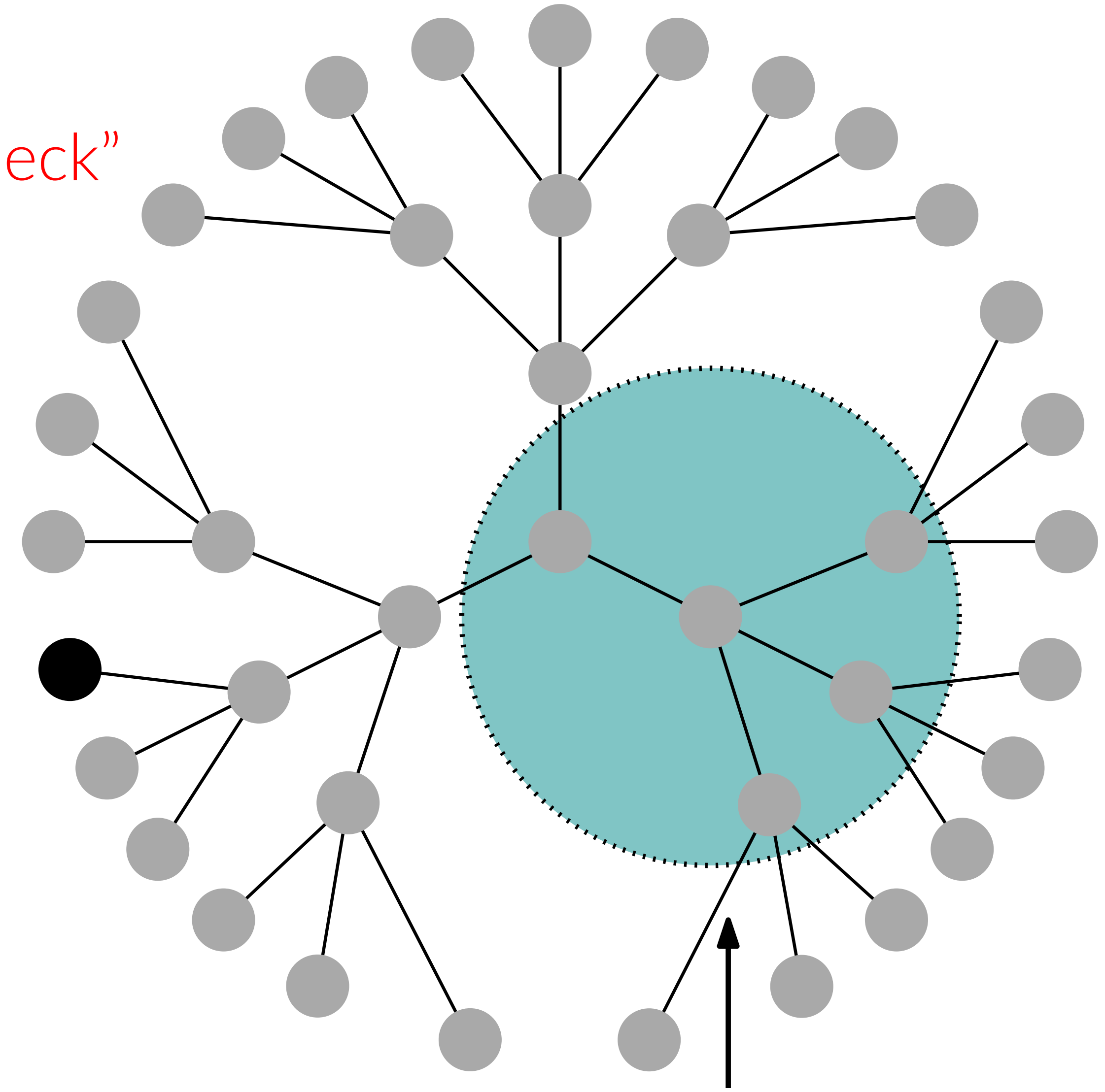

Leader election: the checking radius should be $r = \mathrm{diam}(G)$

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Locally checkable labeling (LCL) problems

[Naor and Stockmeyer, STOC '93 & SICOMP '95]

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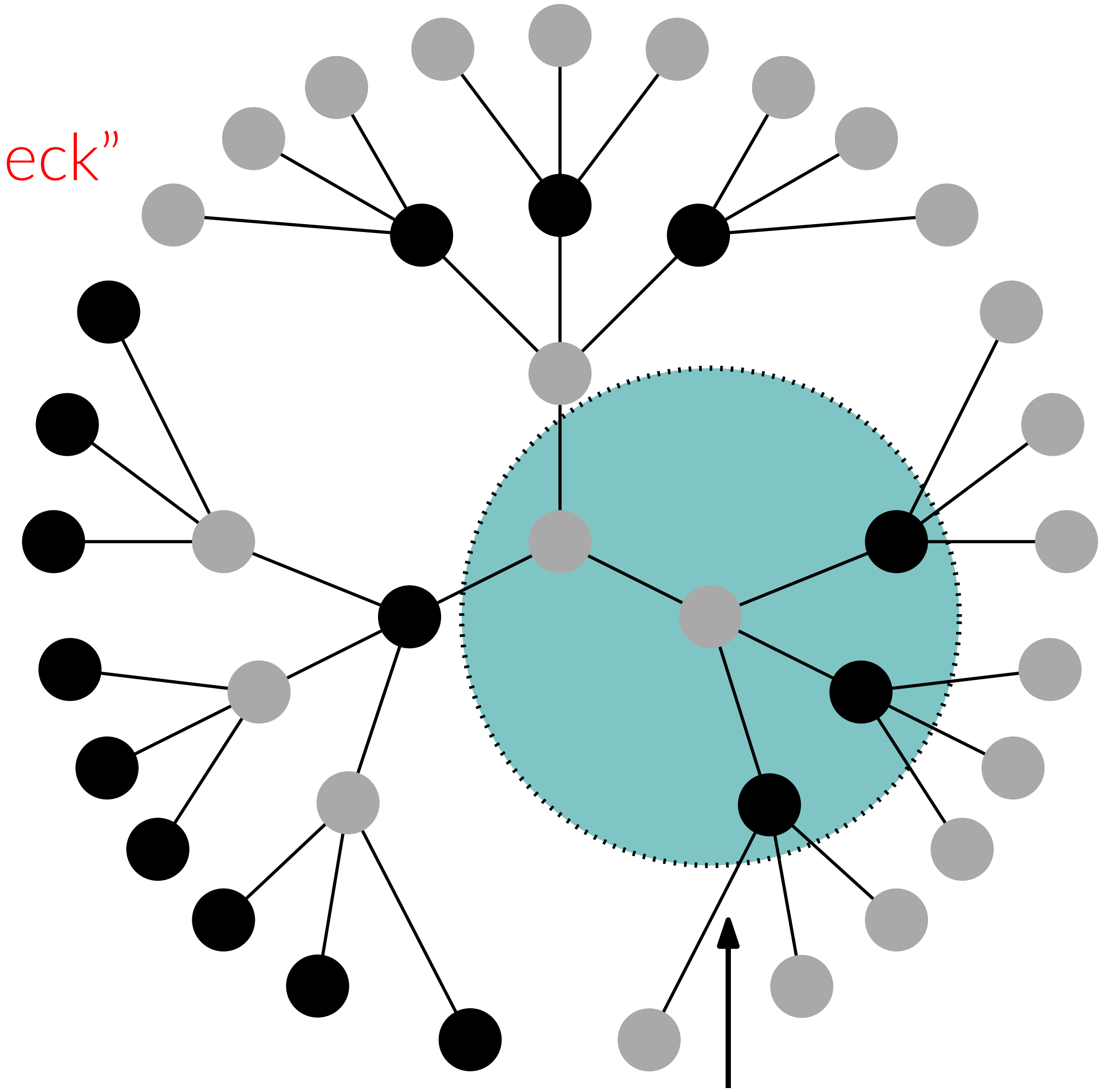

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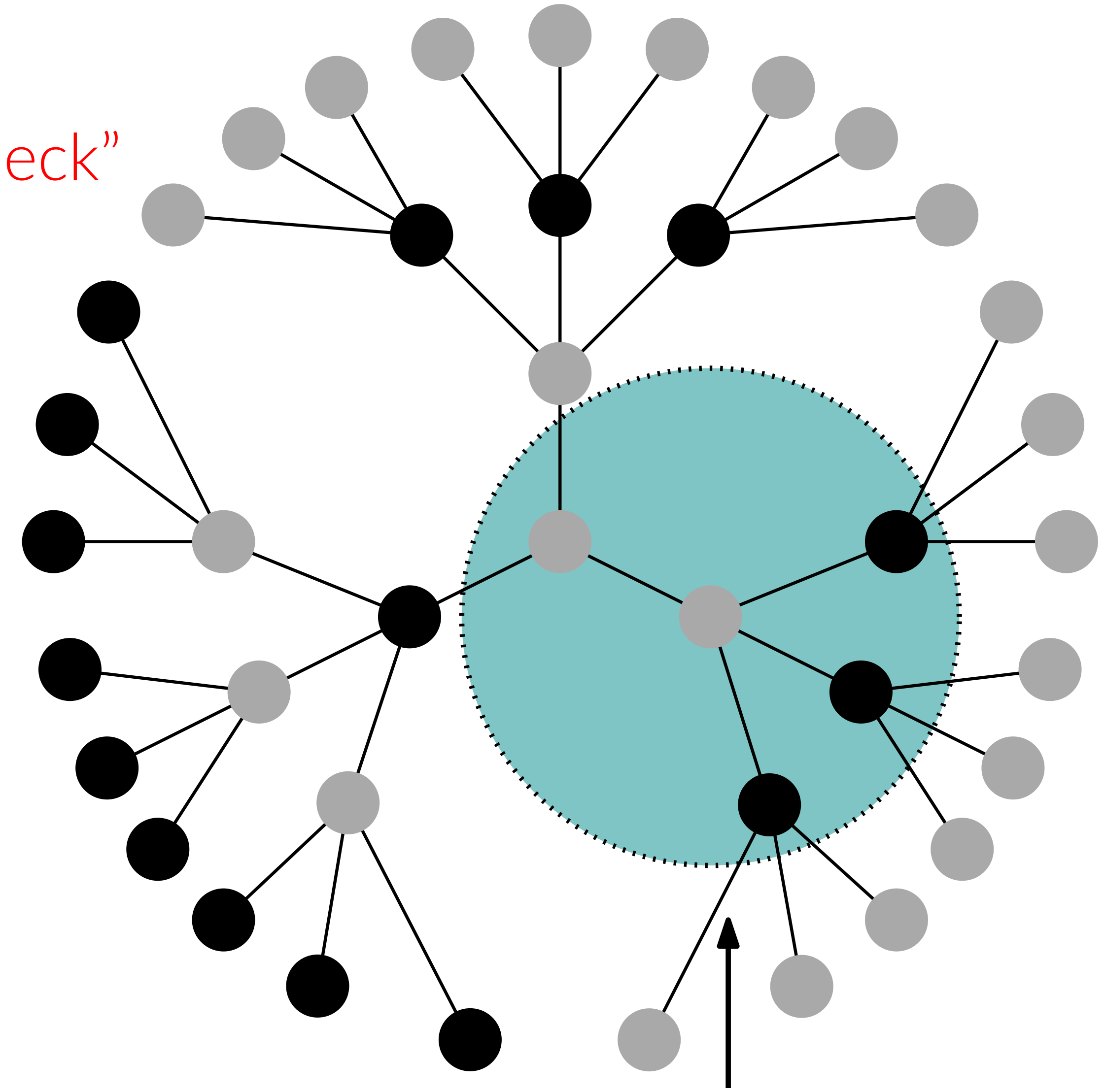

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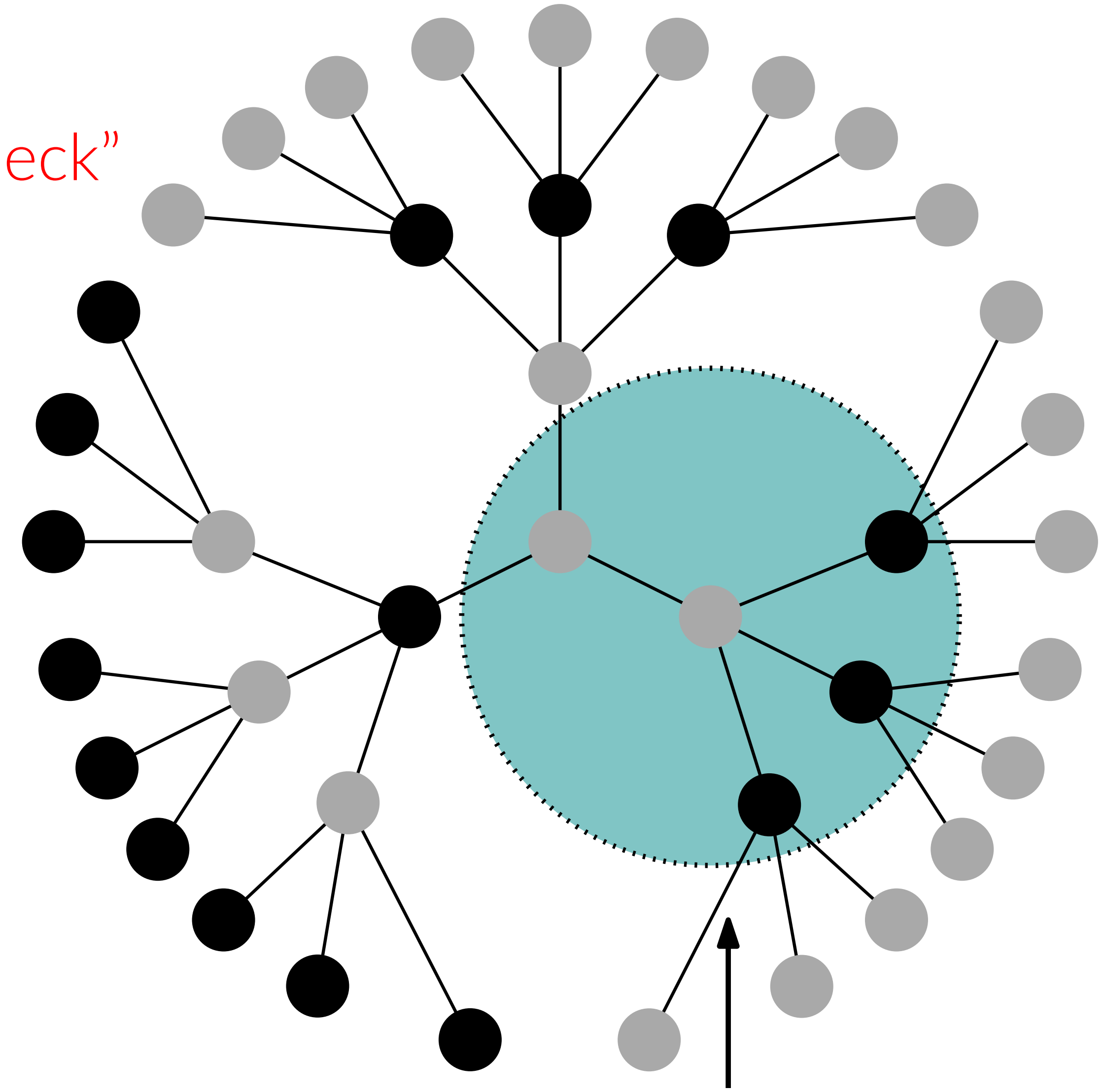

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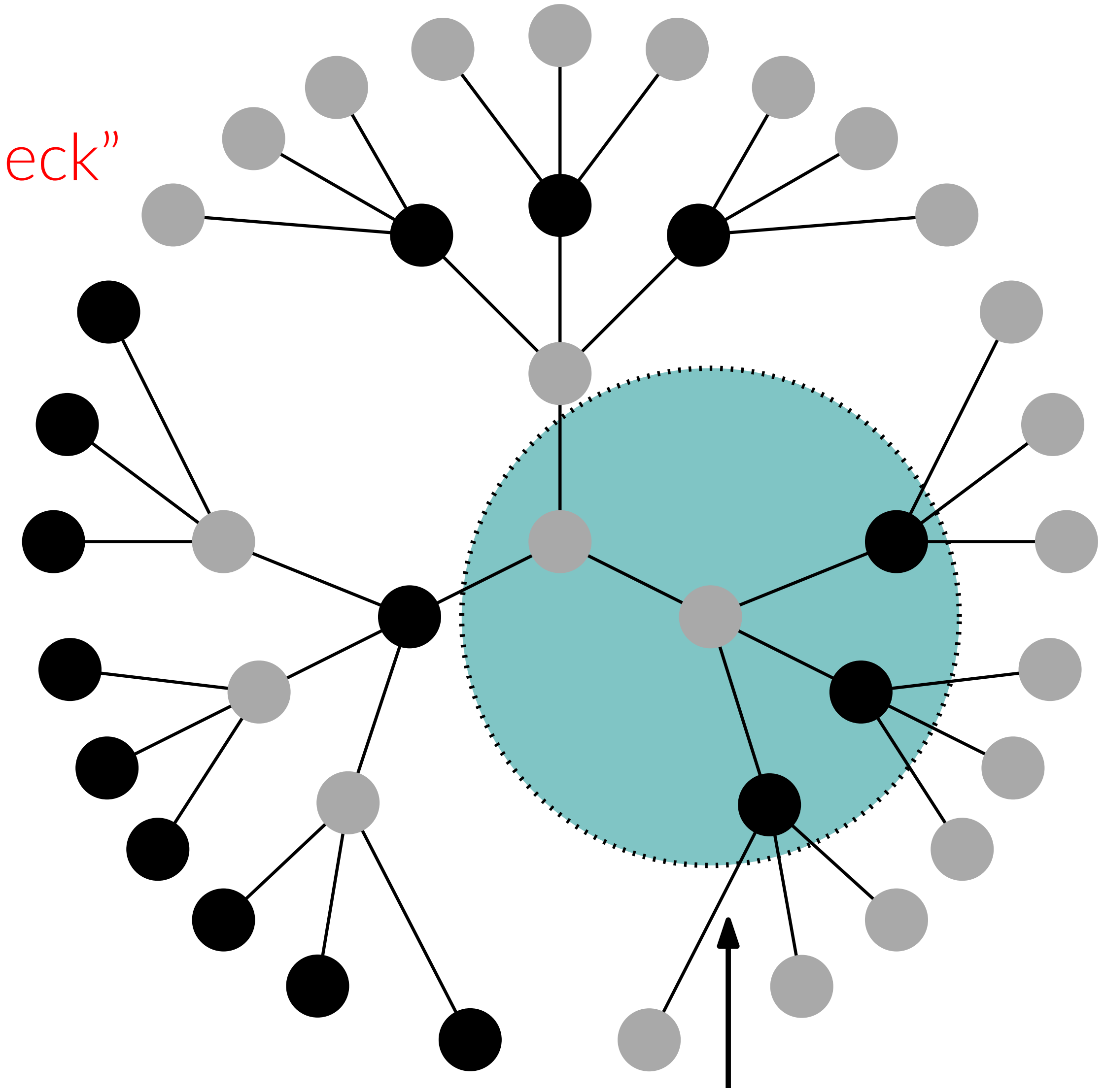

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Complexity landscape of LCL problems

- **Paths and cycles**

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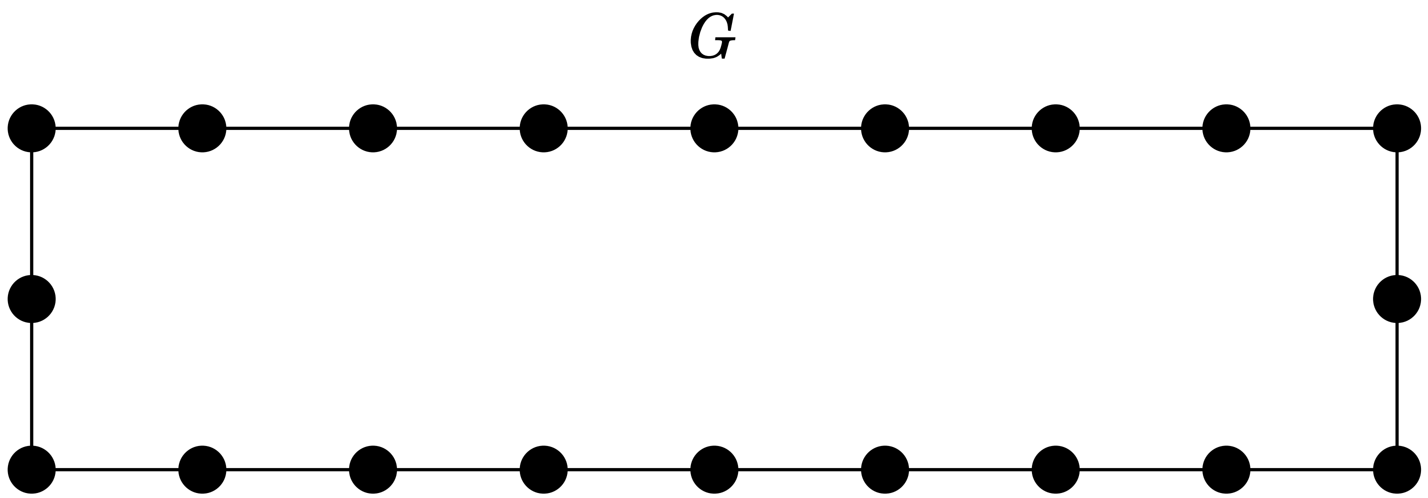

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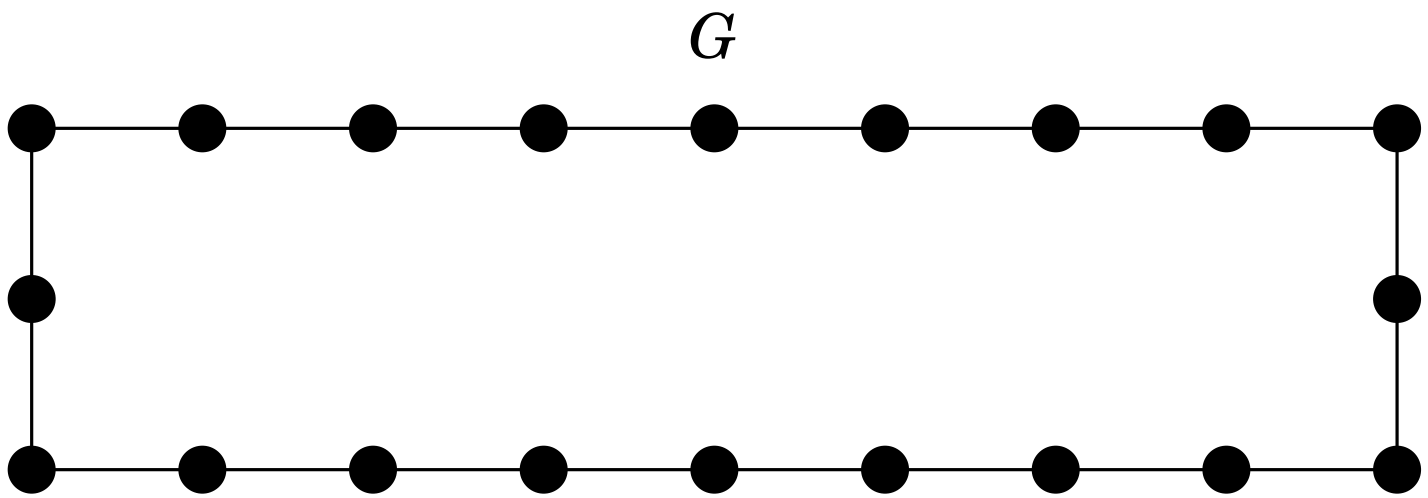

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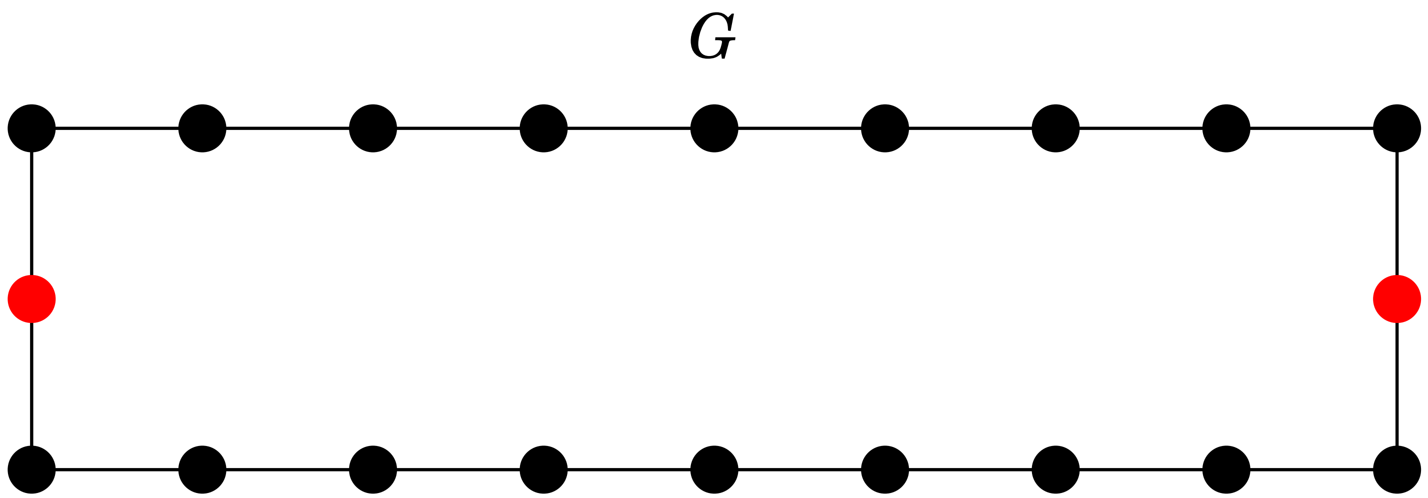

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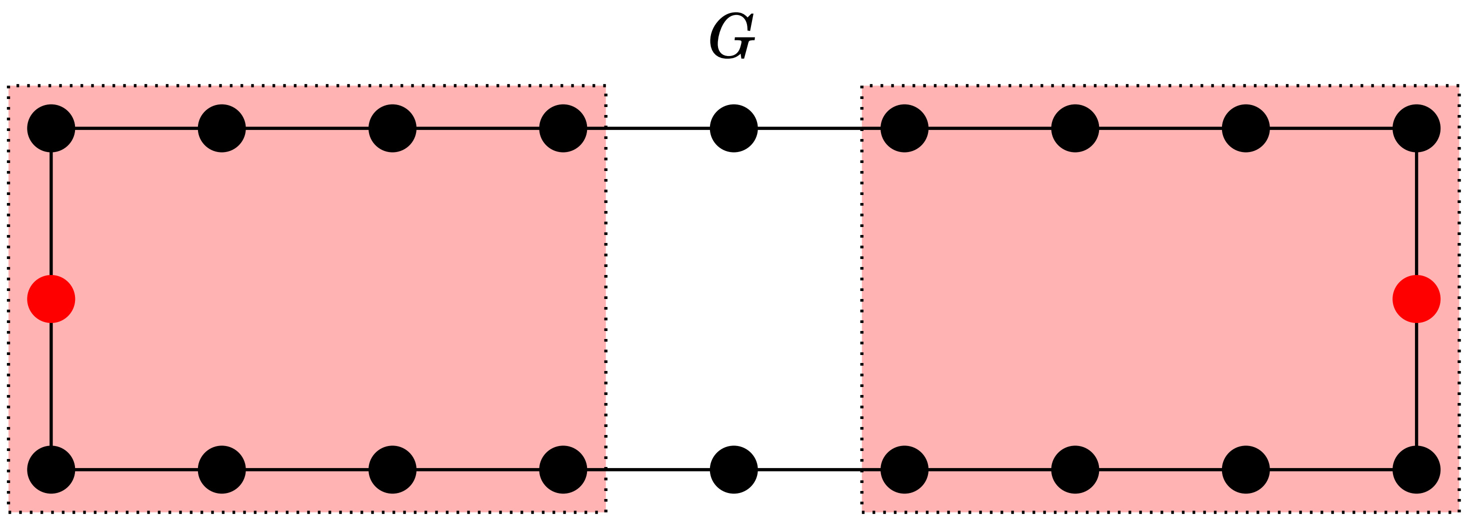

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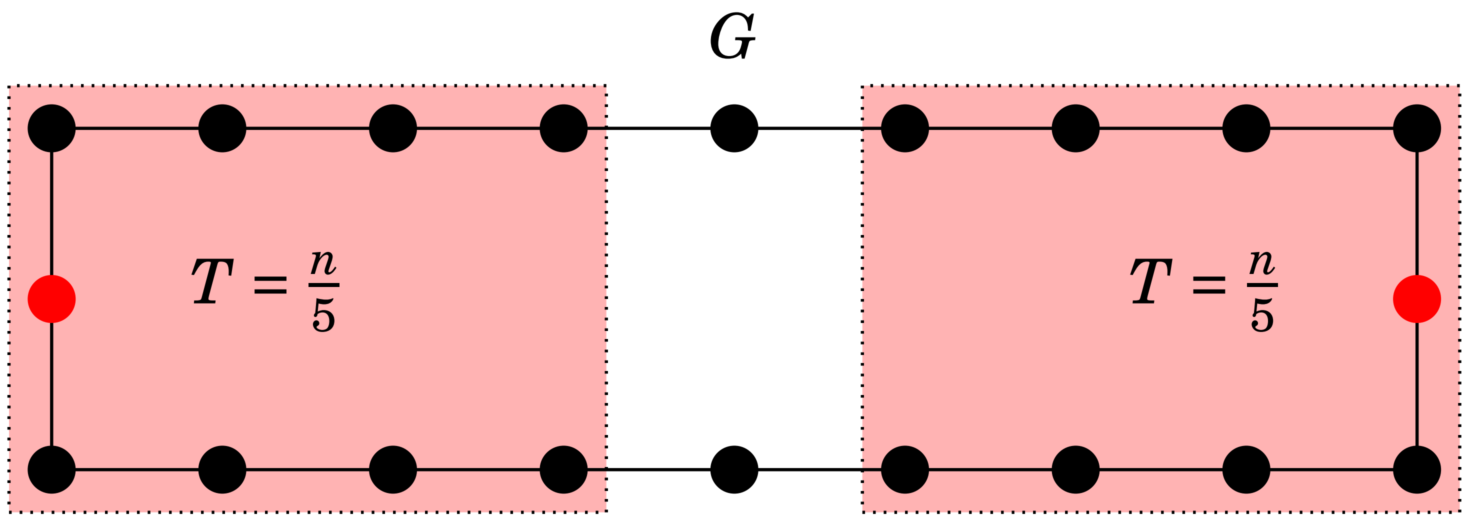

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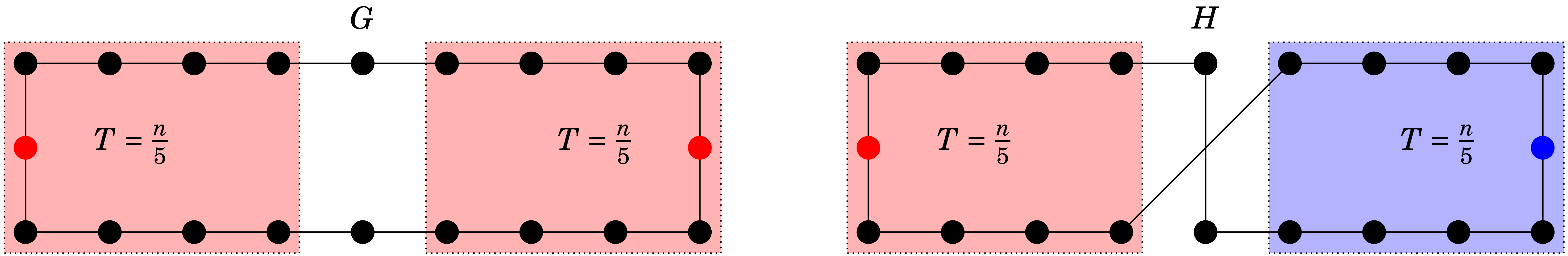

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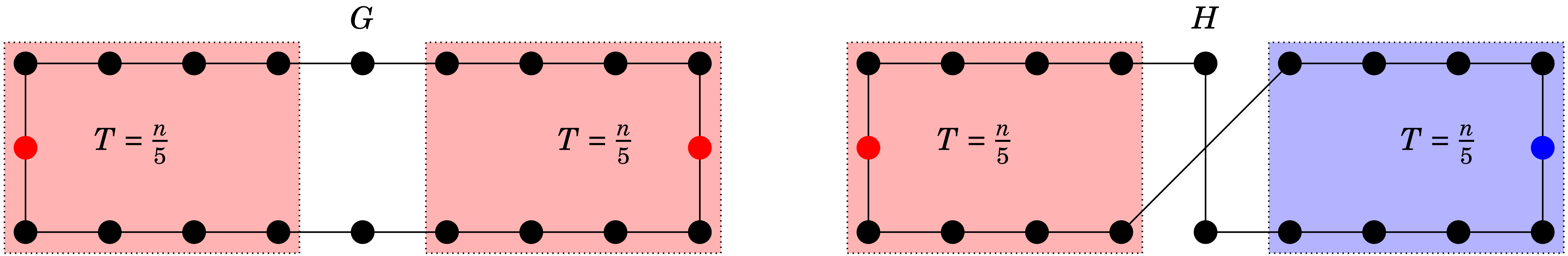

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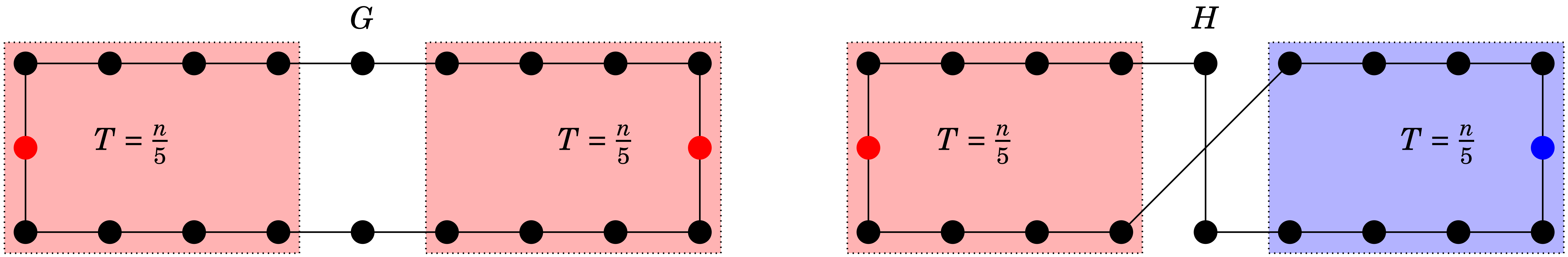

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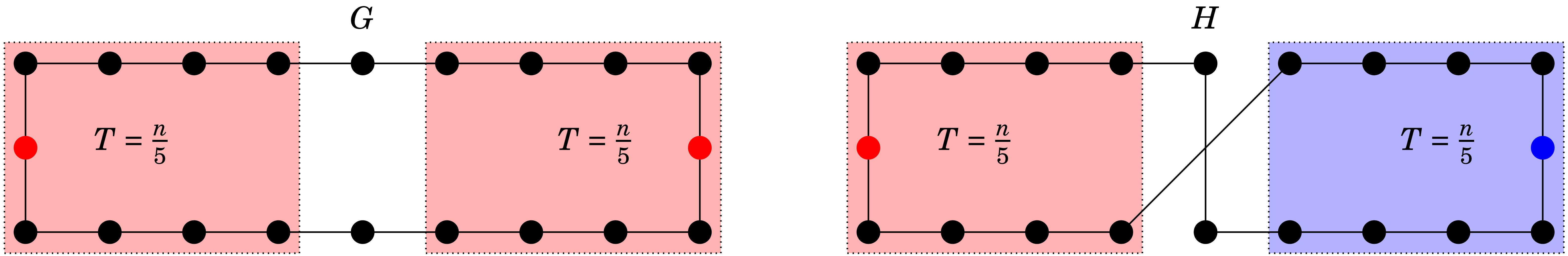

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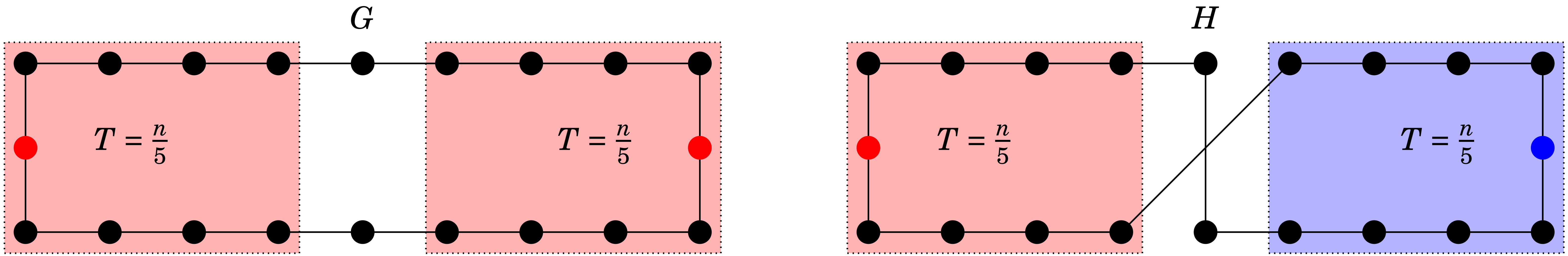

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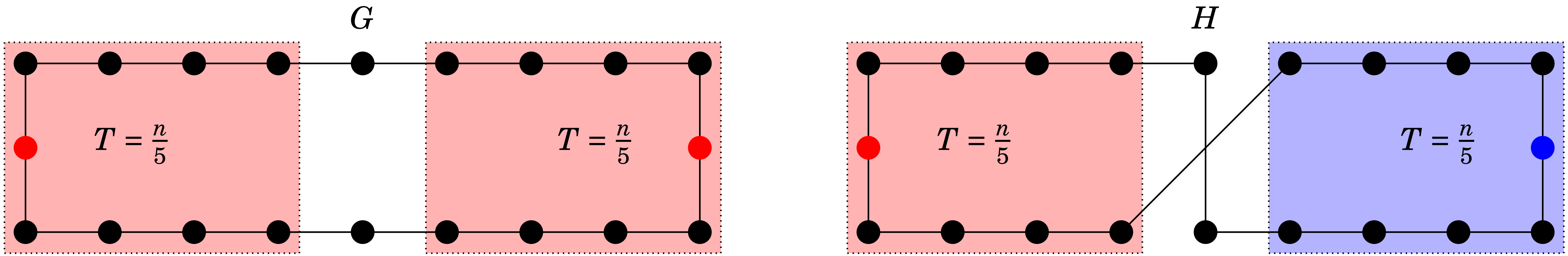

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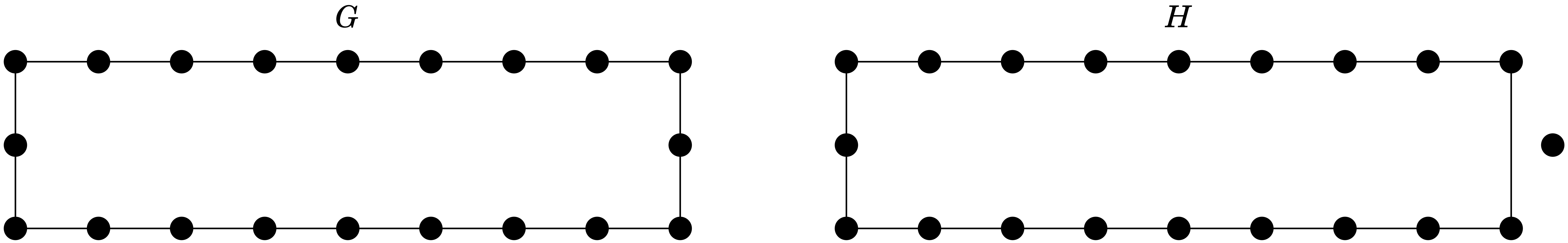

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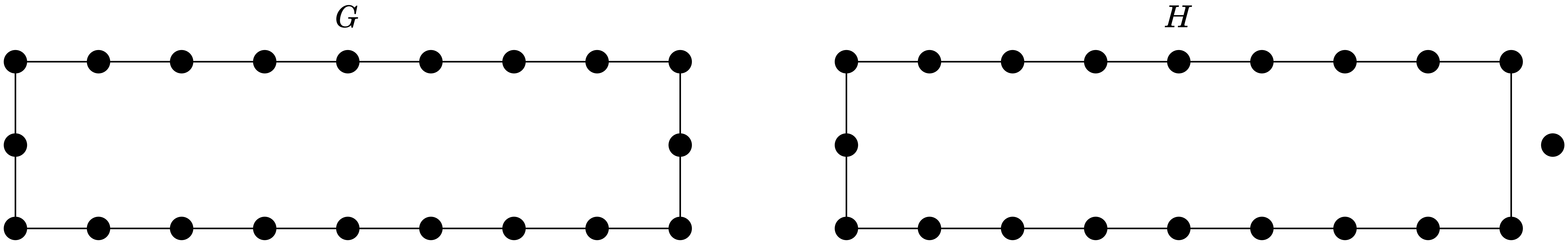

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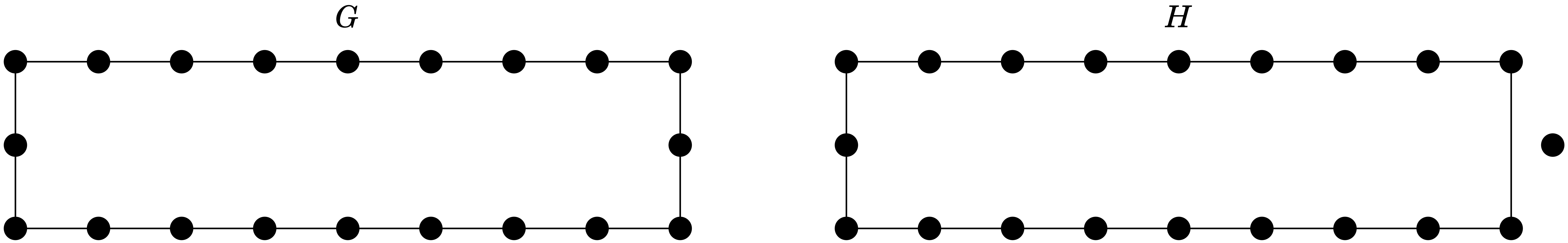

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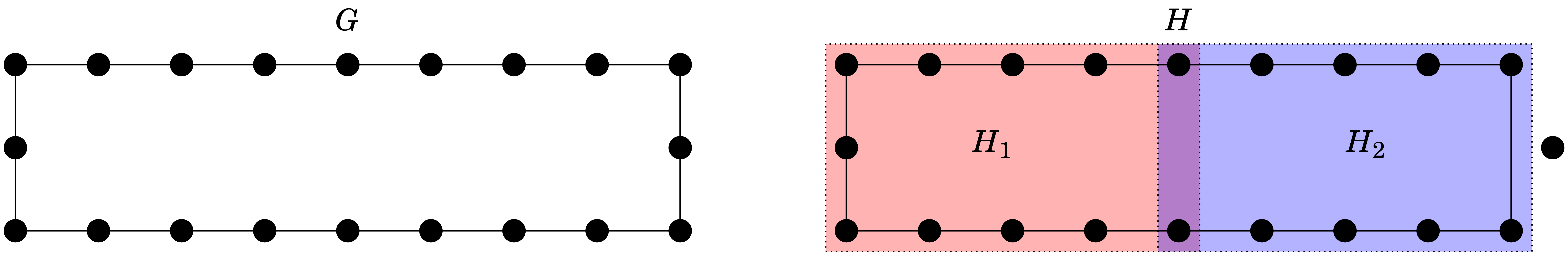

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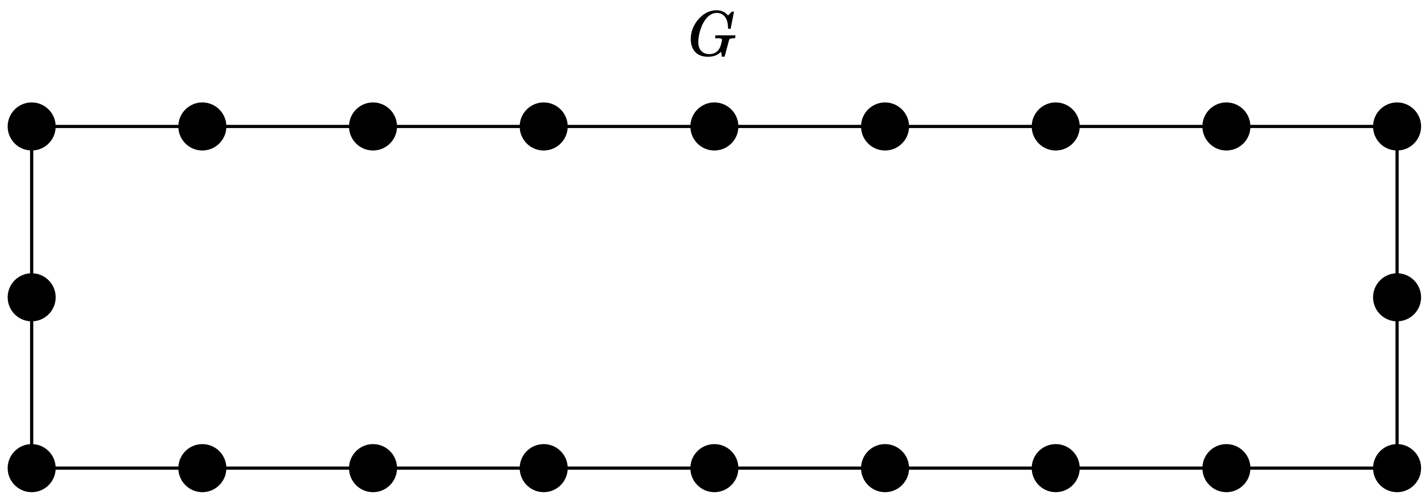

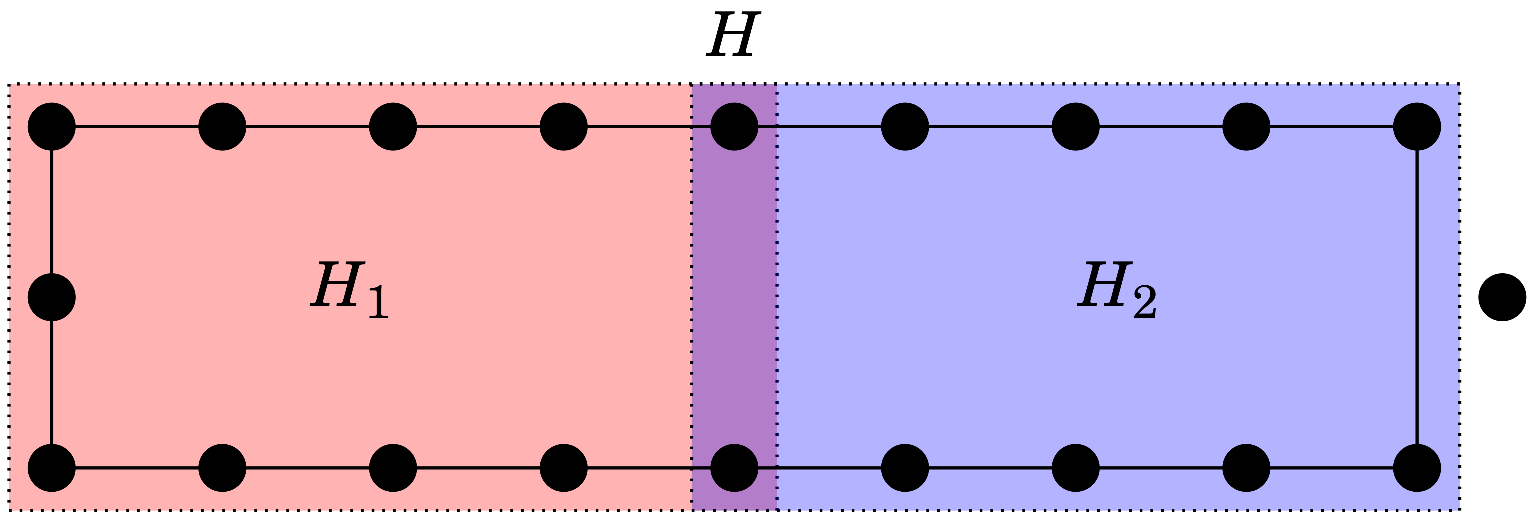

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Graph-existential indistinguishability (randomized)

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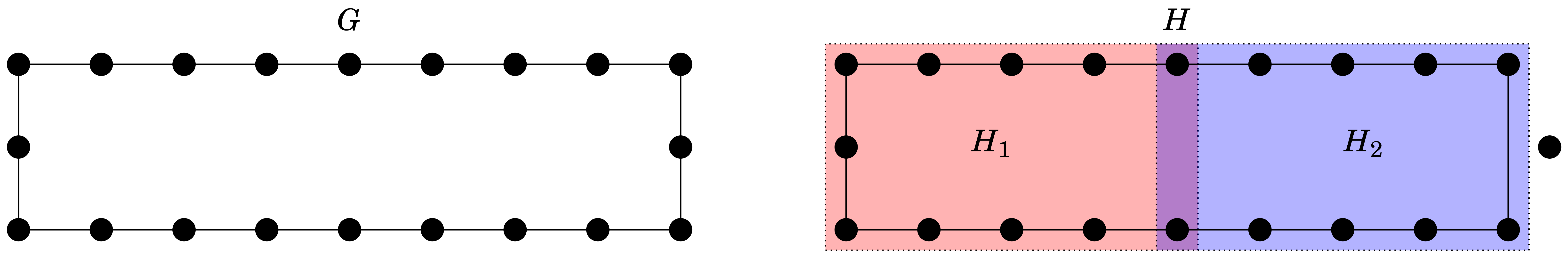

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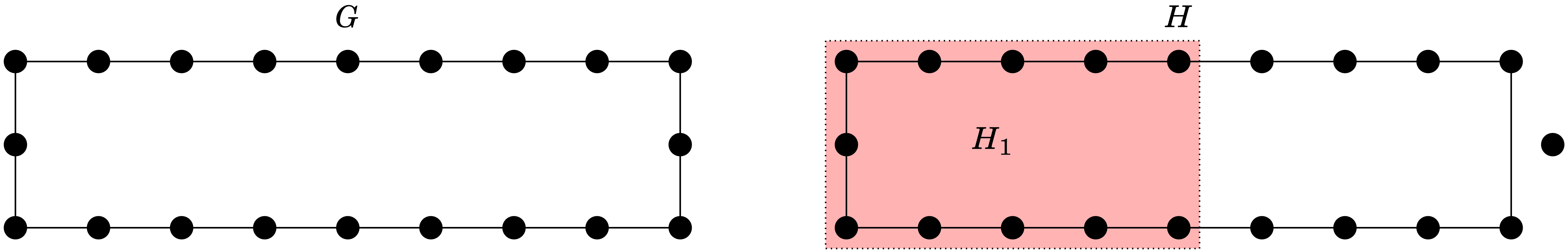

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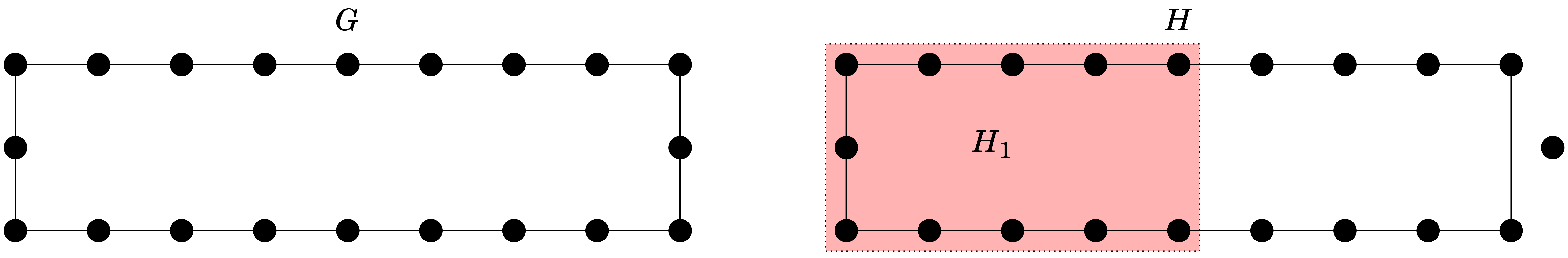

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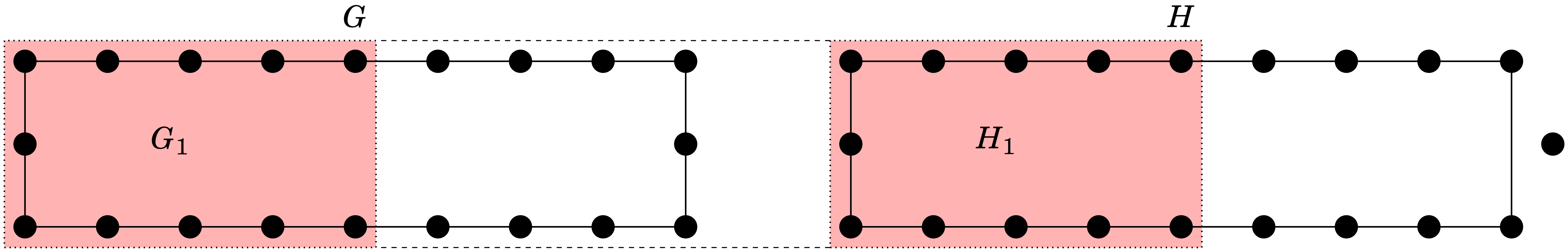

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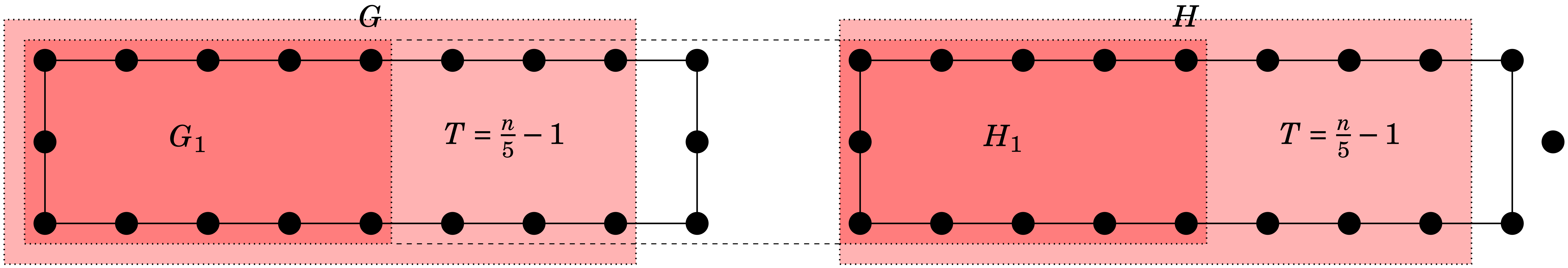

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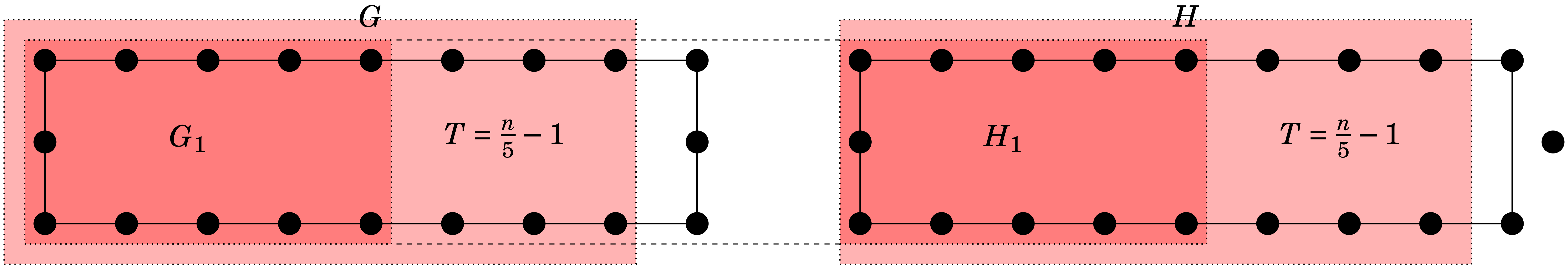

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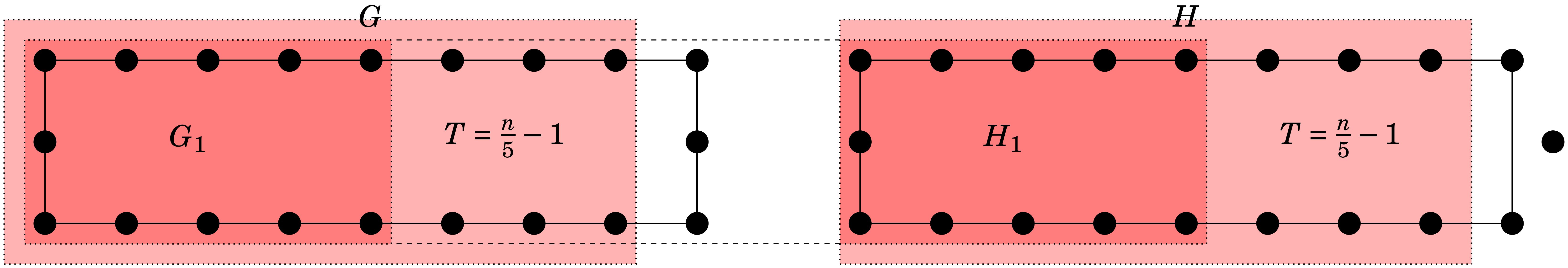

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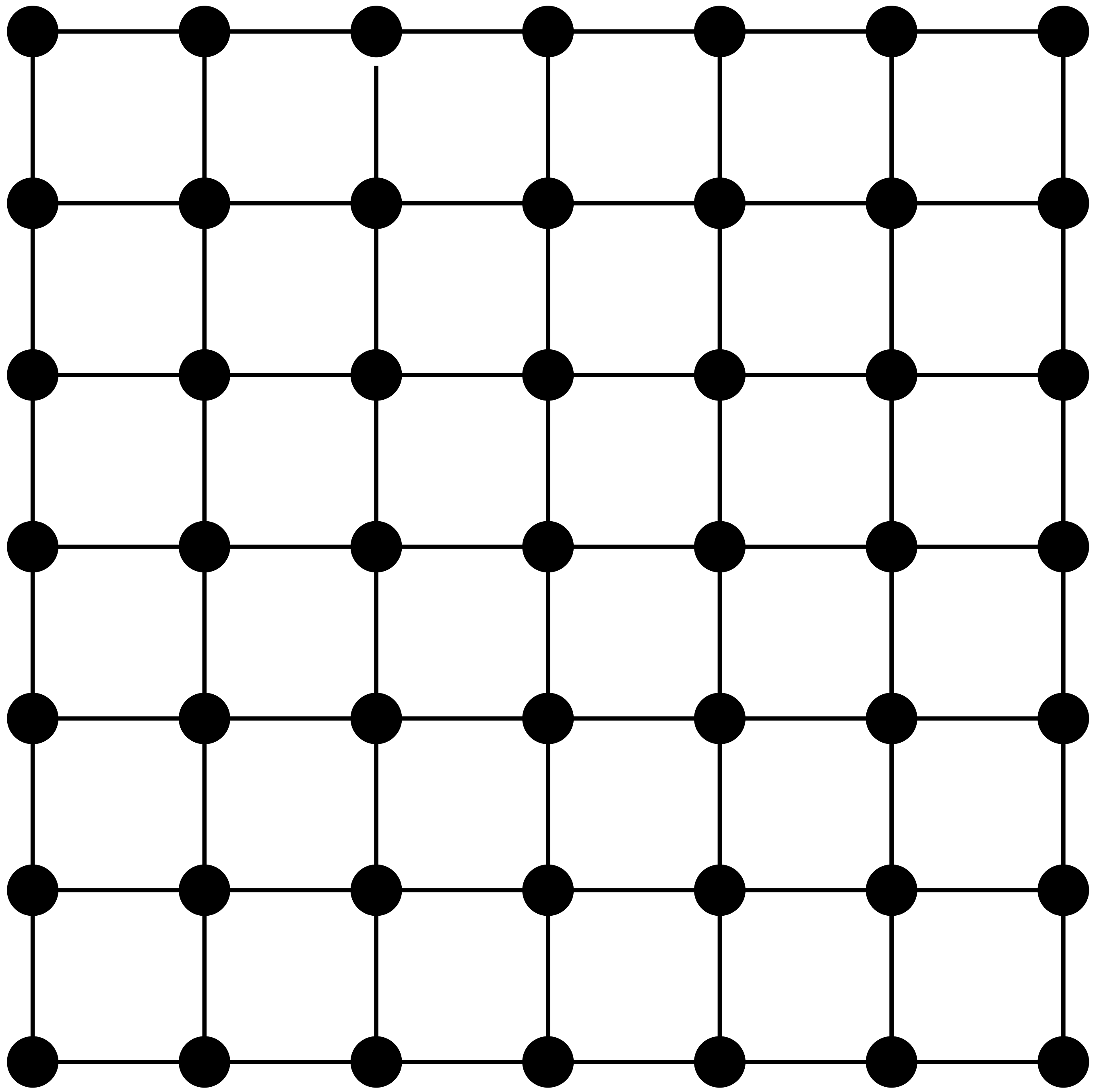
Table of content

Properties of distributed algorithms

- **Run** a $\mathbf{2}$-round algorithm $\boldsymbol{A}$ in $\boldsymbol{G}$



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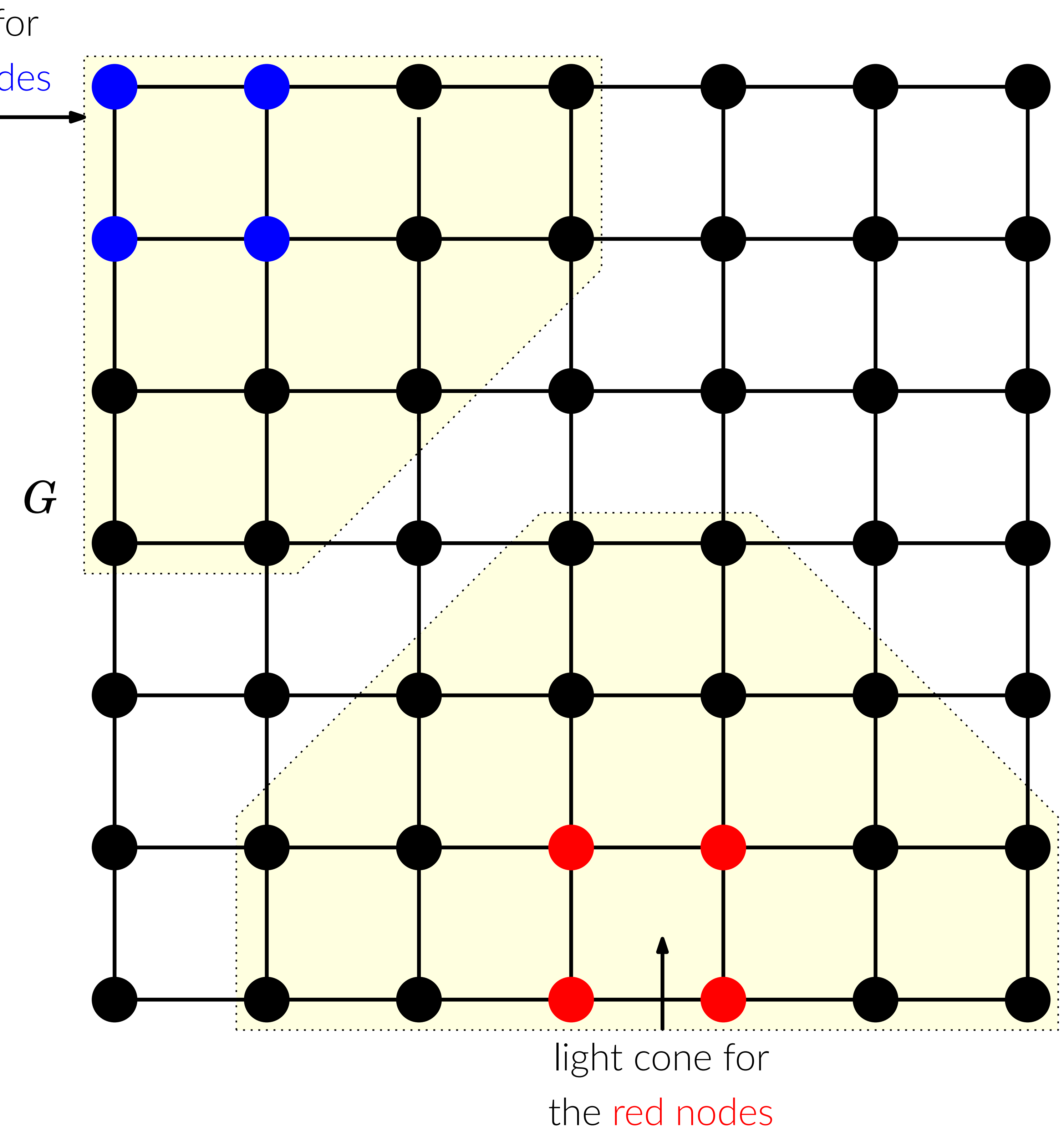

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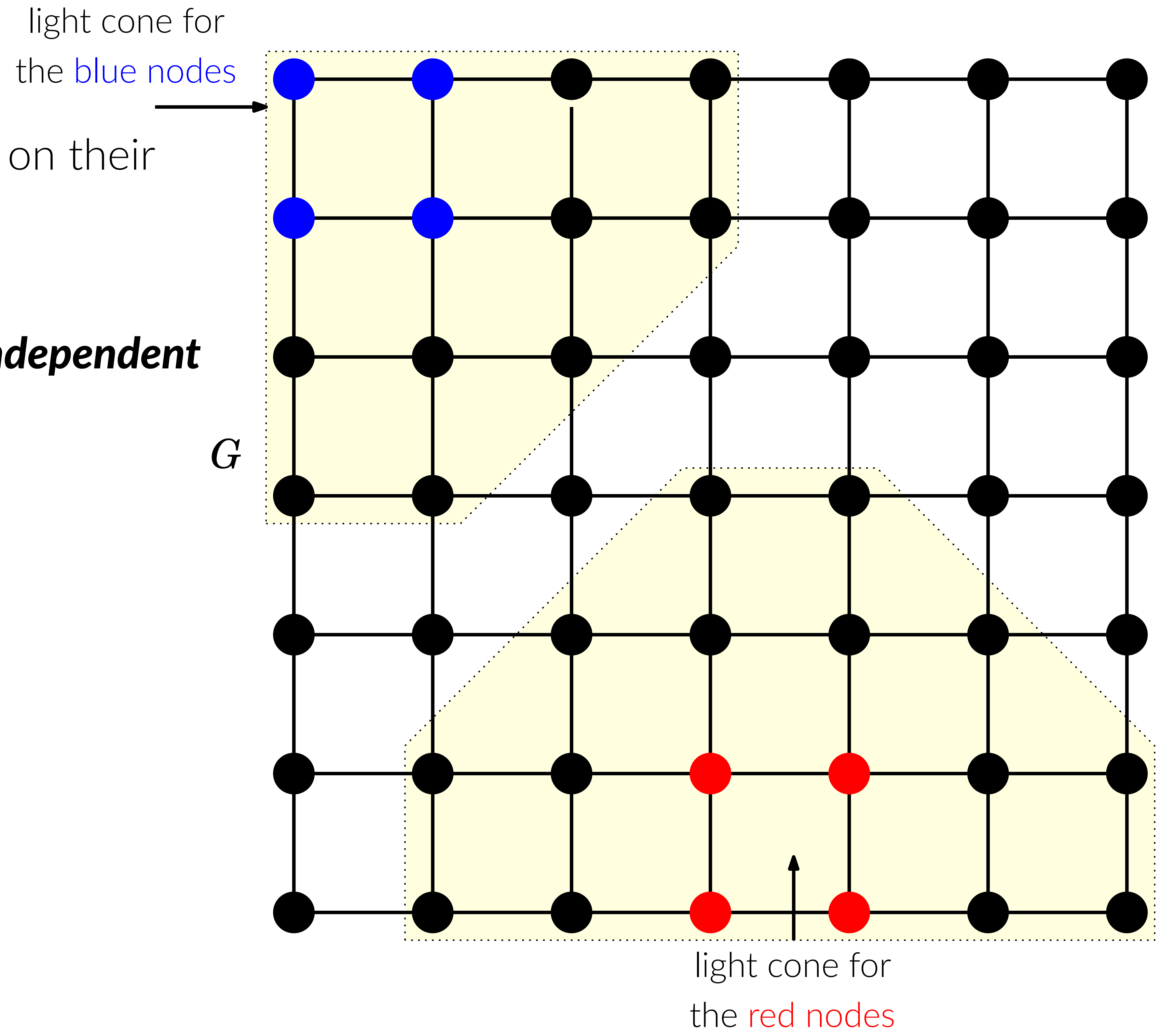

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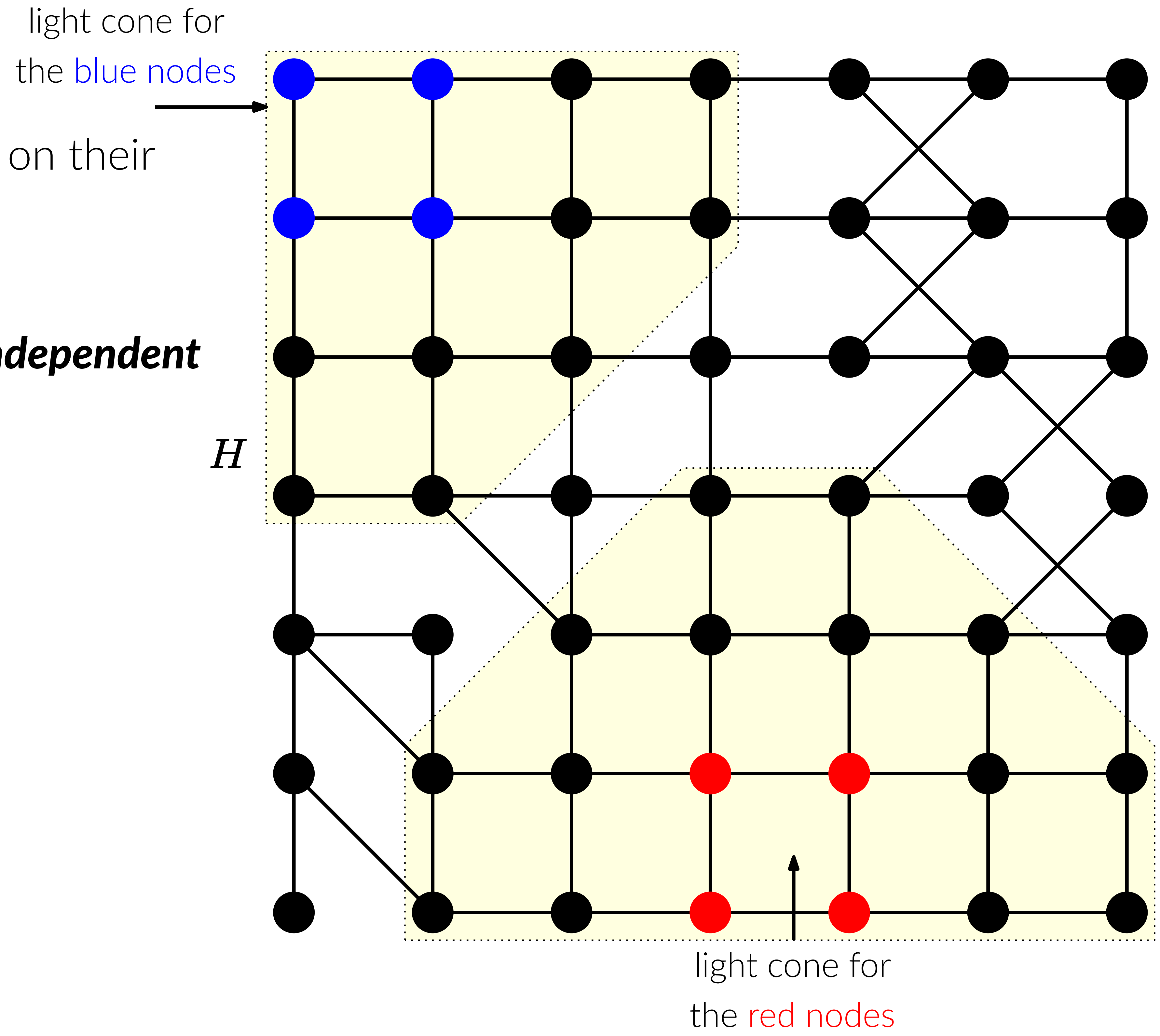

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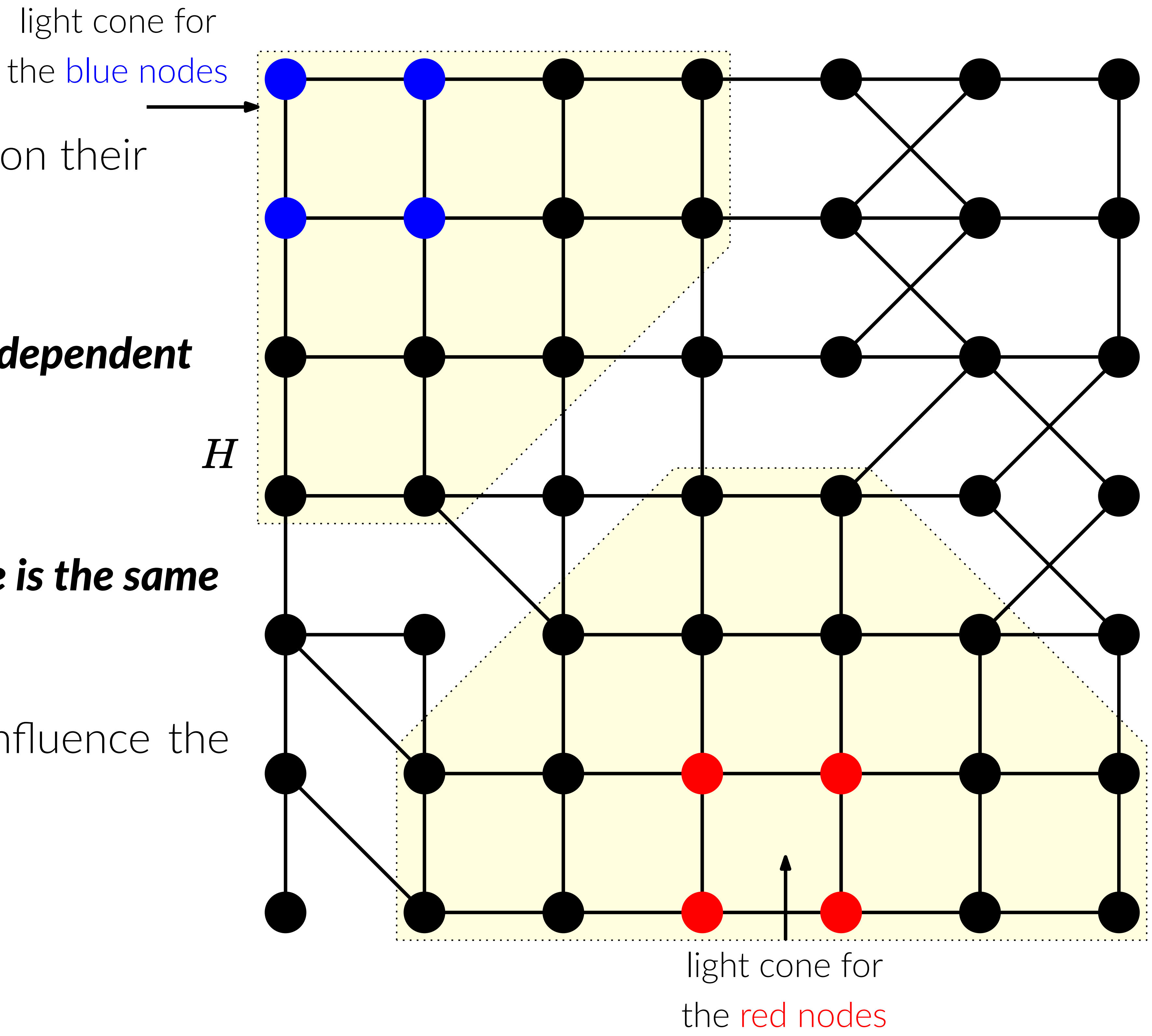
- **Output distributions** remains ***the same*** if ***light cone is the same***
  - non-signaling property
  - changes that are beyond 2-hops away do not influence the output distribution
  - also known as causality



Abstracting output distributions

- A T-round distributed algorithm yields an **output distribution** with the following **properties**:
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$$\boxed{\text{bounded-dependence model}} \longrightarrow \boxed{\text{non-signaling LOCAL}}$$

locality T =
independence at distance $2T + 1$ plus
non-signaling beyond distance T

locality T =
non-signaling beyond distance T

[Holroyd and Liggett, Forum of Mathematics, Pi '14]
[STOC '25a]

* finitely-dependent distributions if $T = O(1)$

[Gavoille, Kosowski, and Markiewicz, DISC '09]
[Arfaoui and Fraigniaud, PODC '12 & SIGACT News '14]

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The non-signaling model

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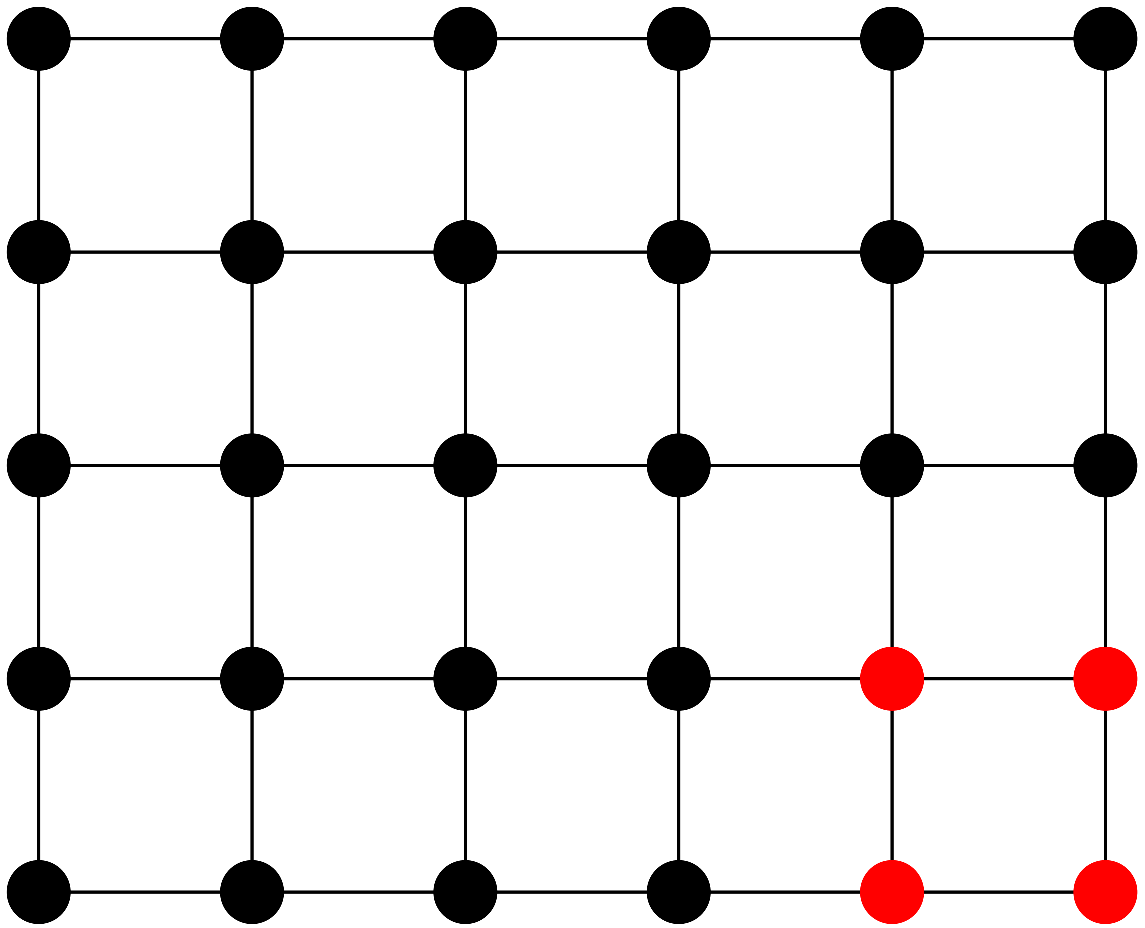
[STOC '24]



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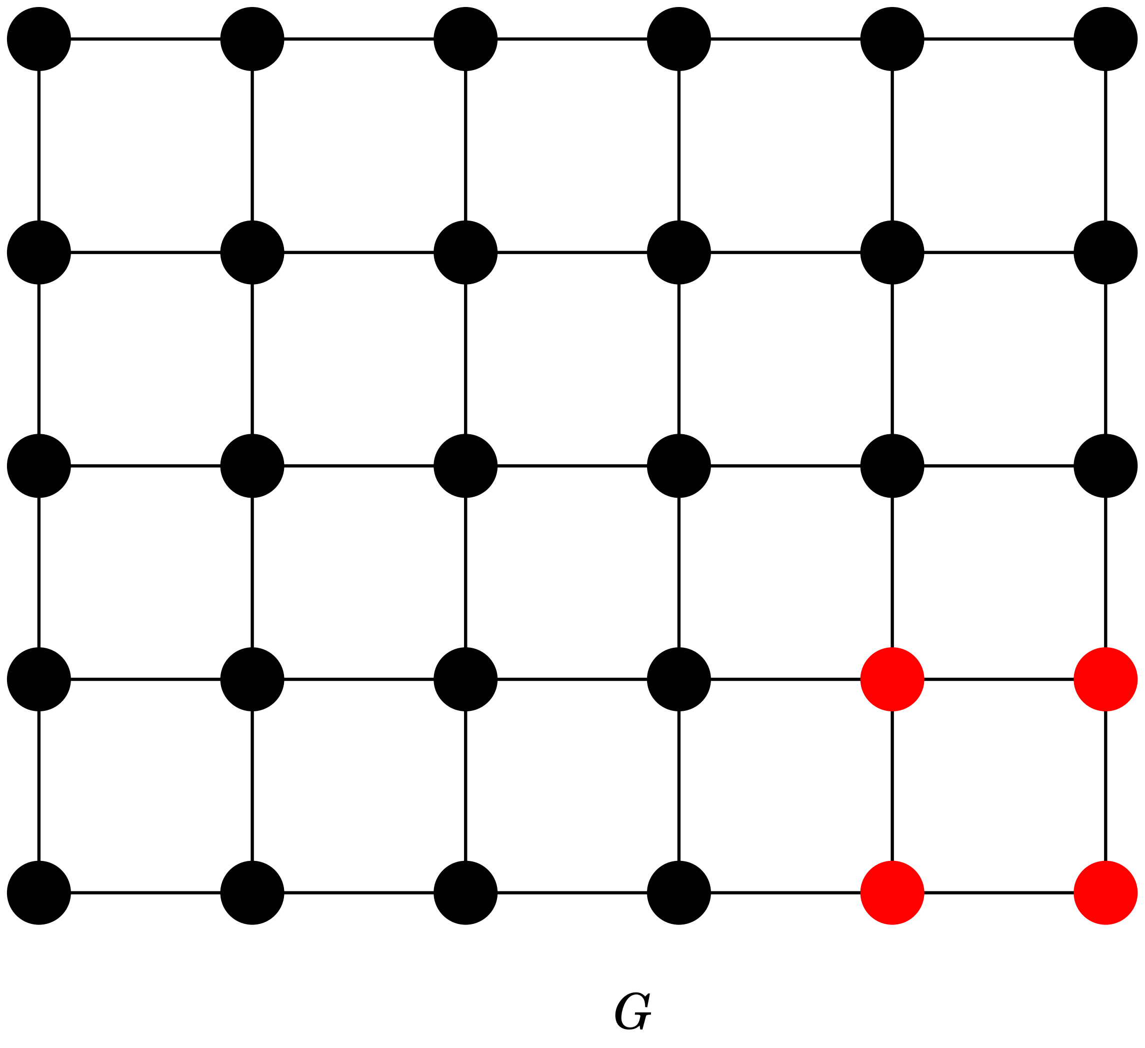

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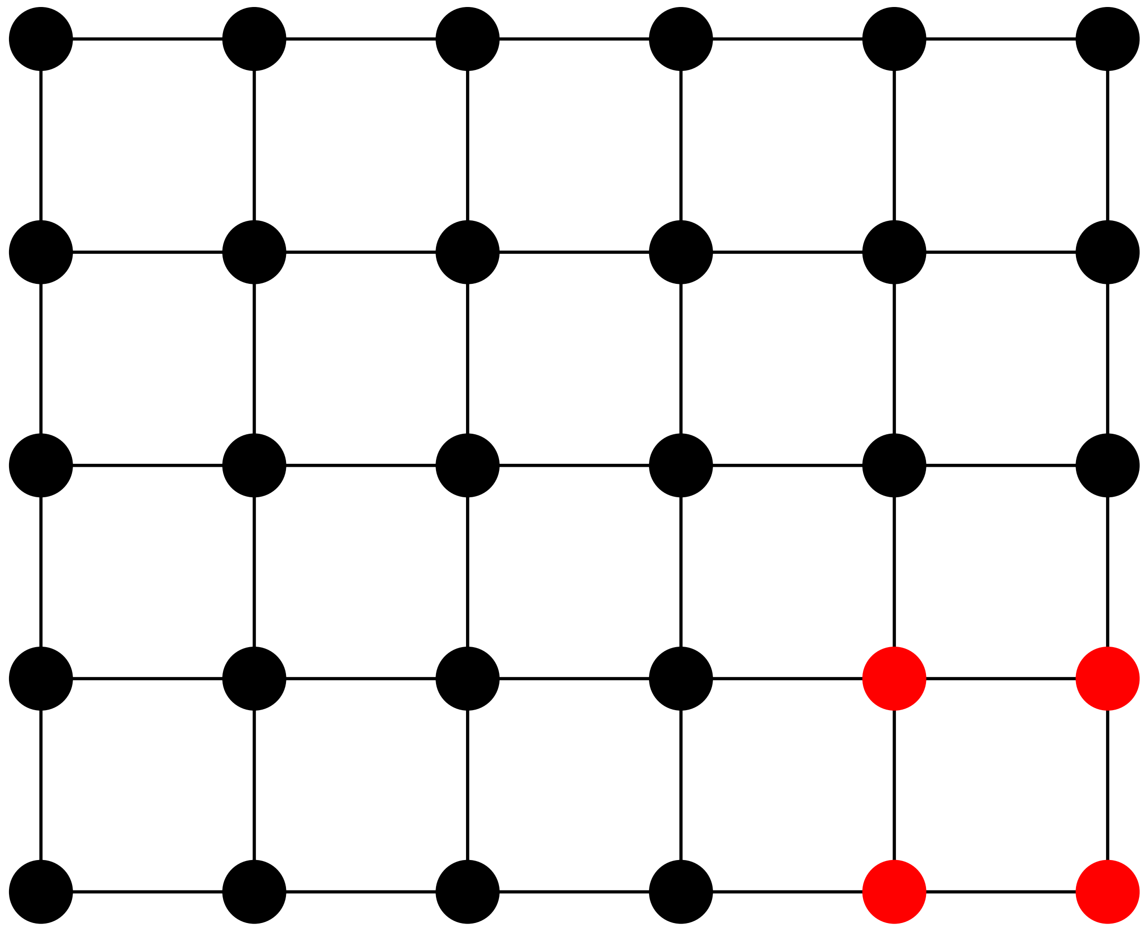

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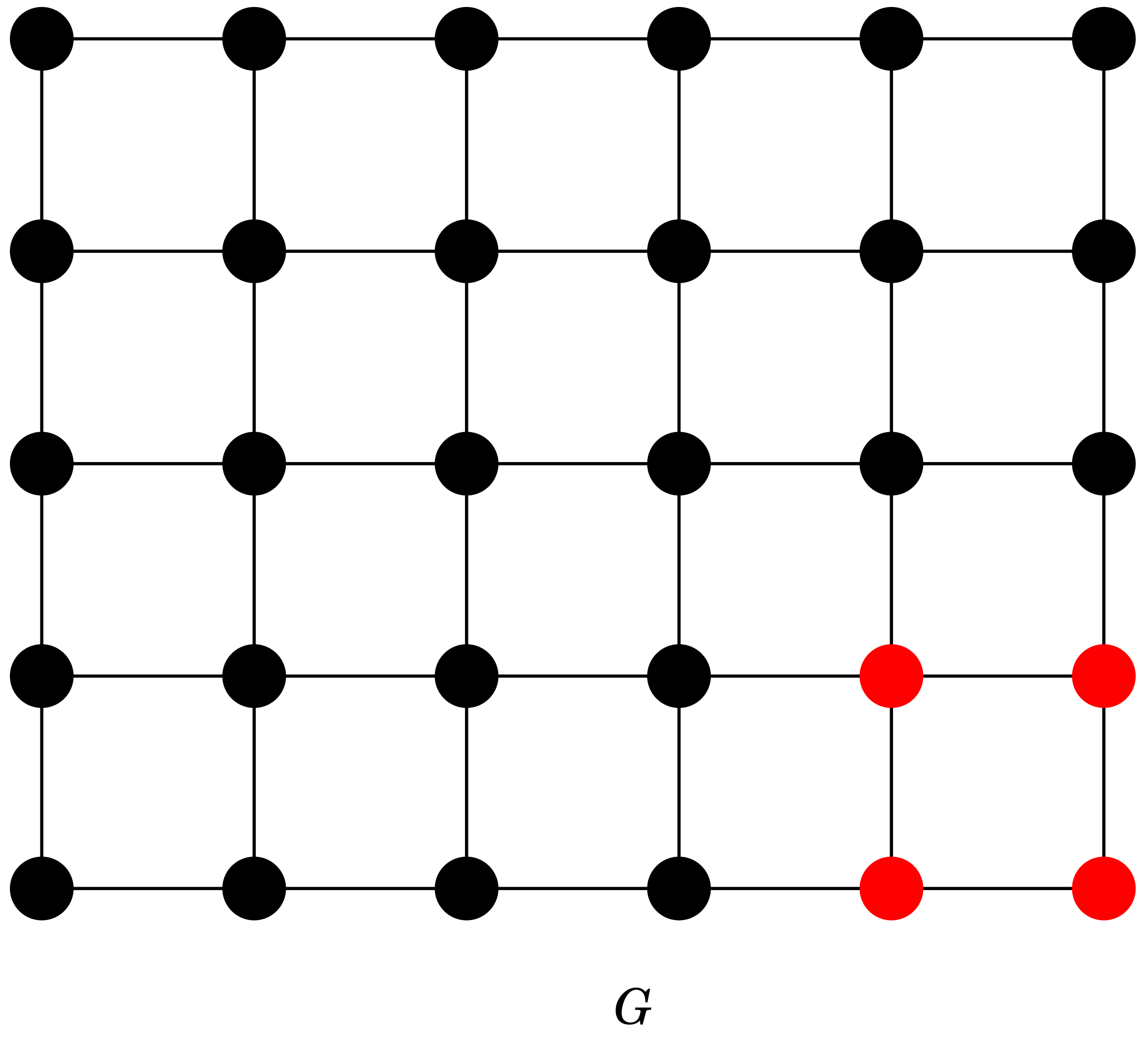

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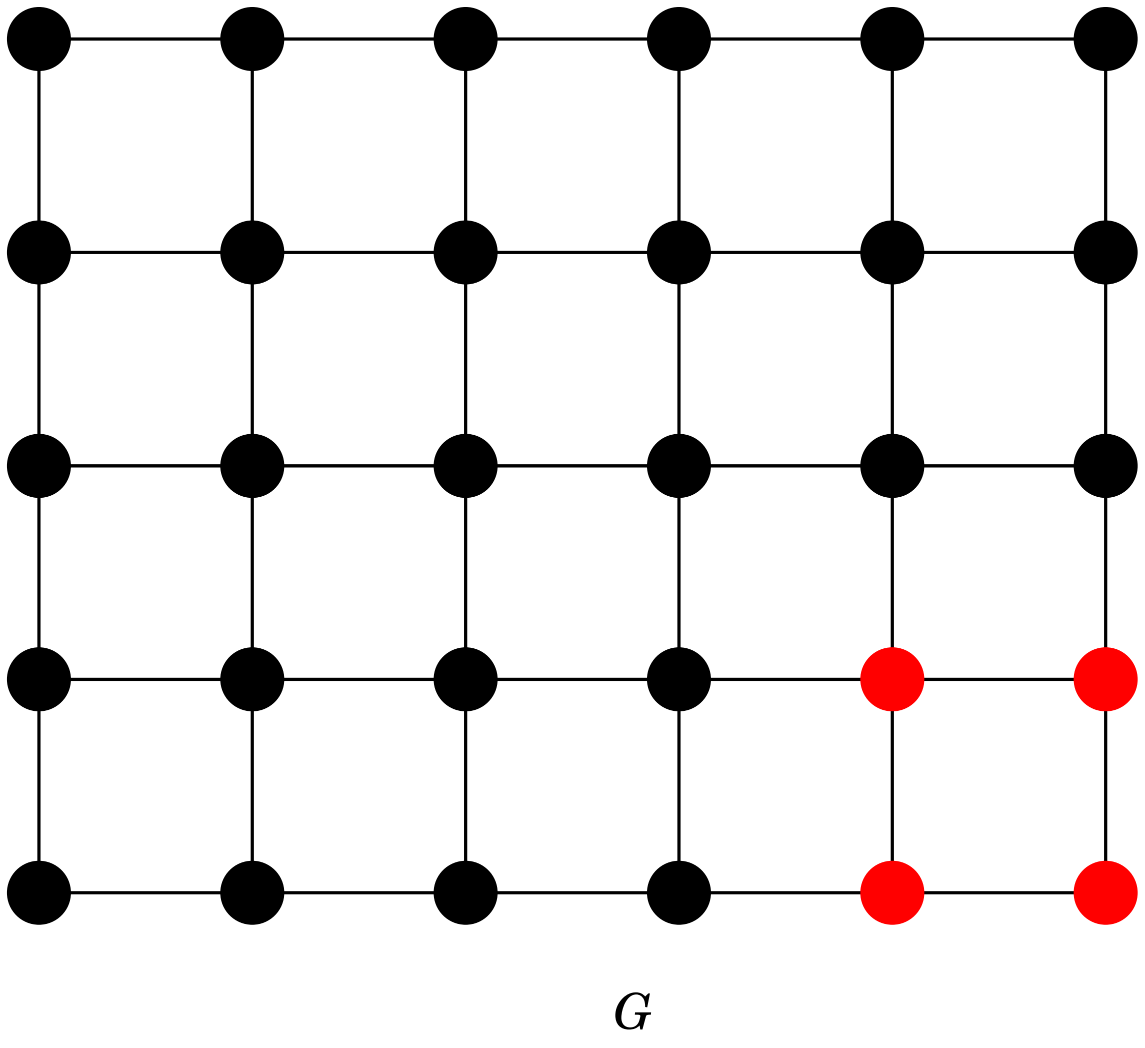

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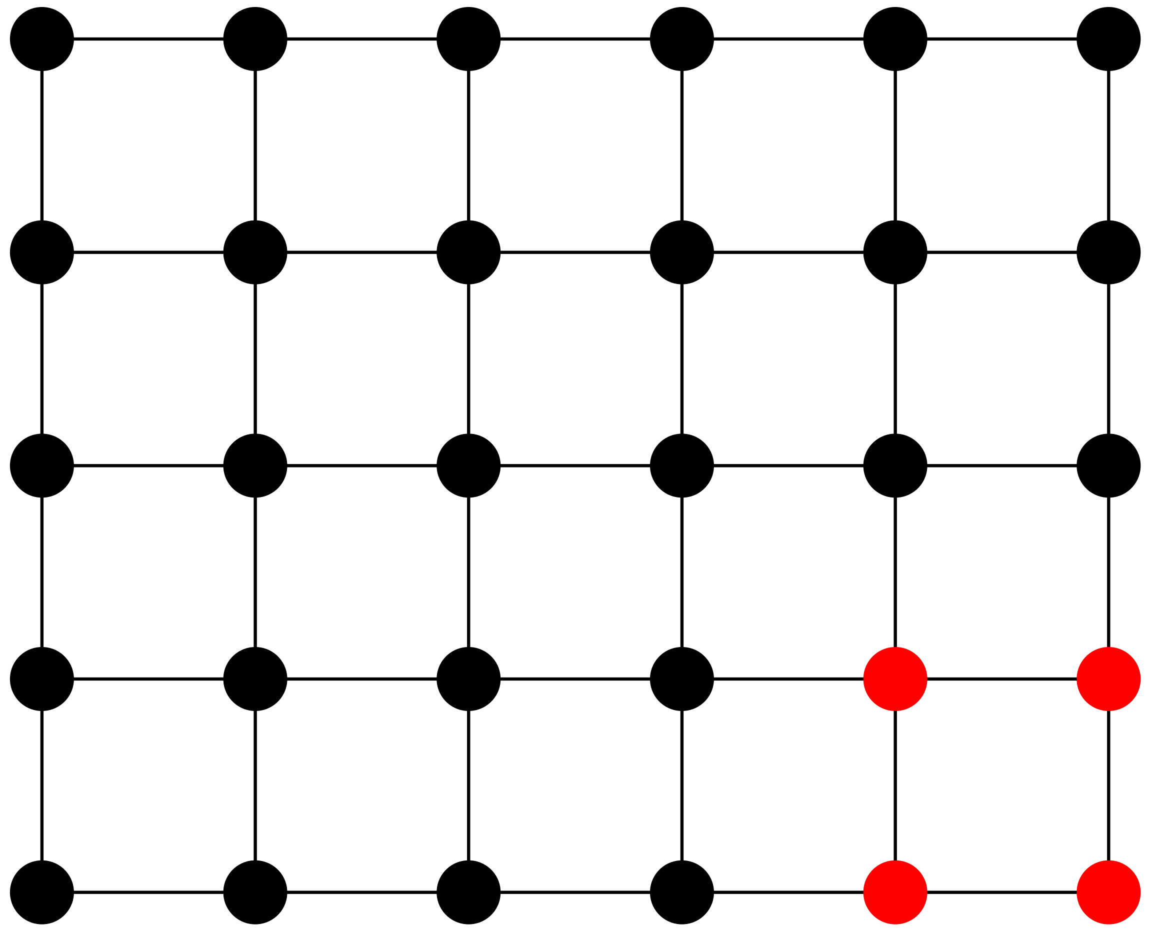

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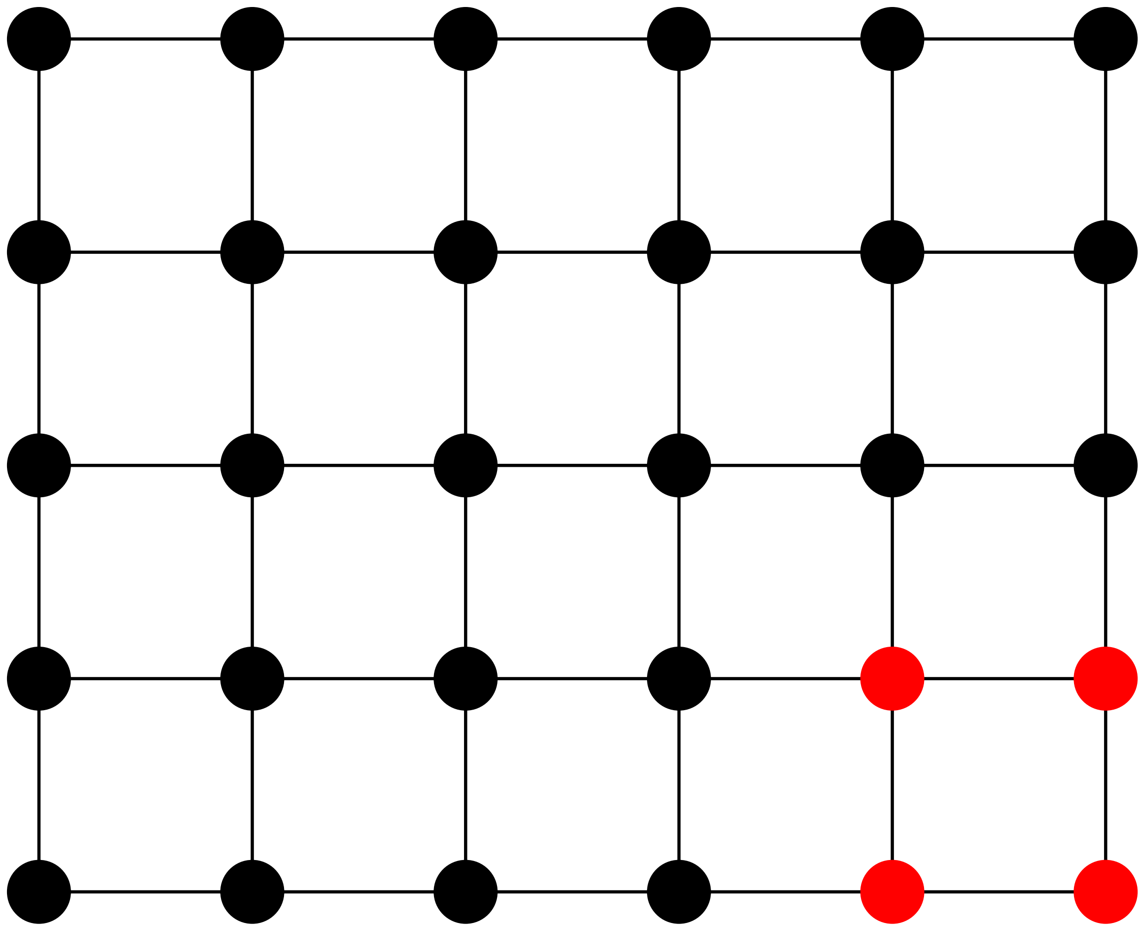

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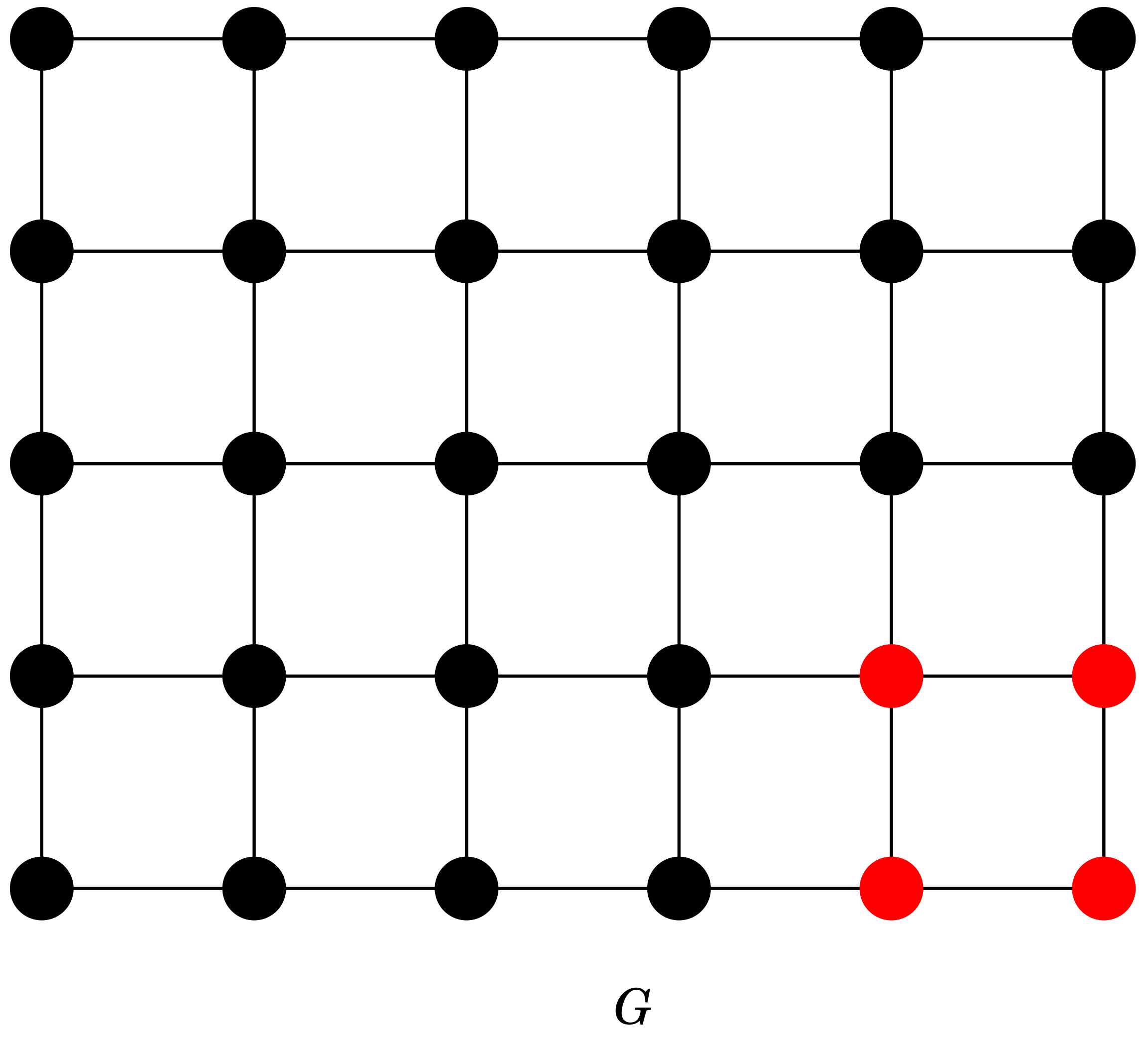

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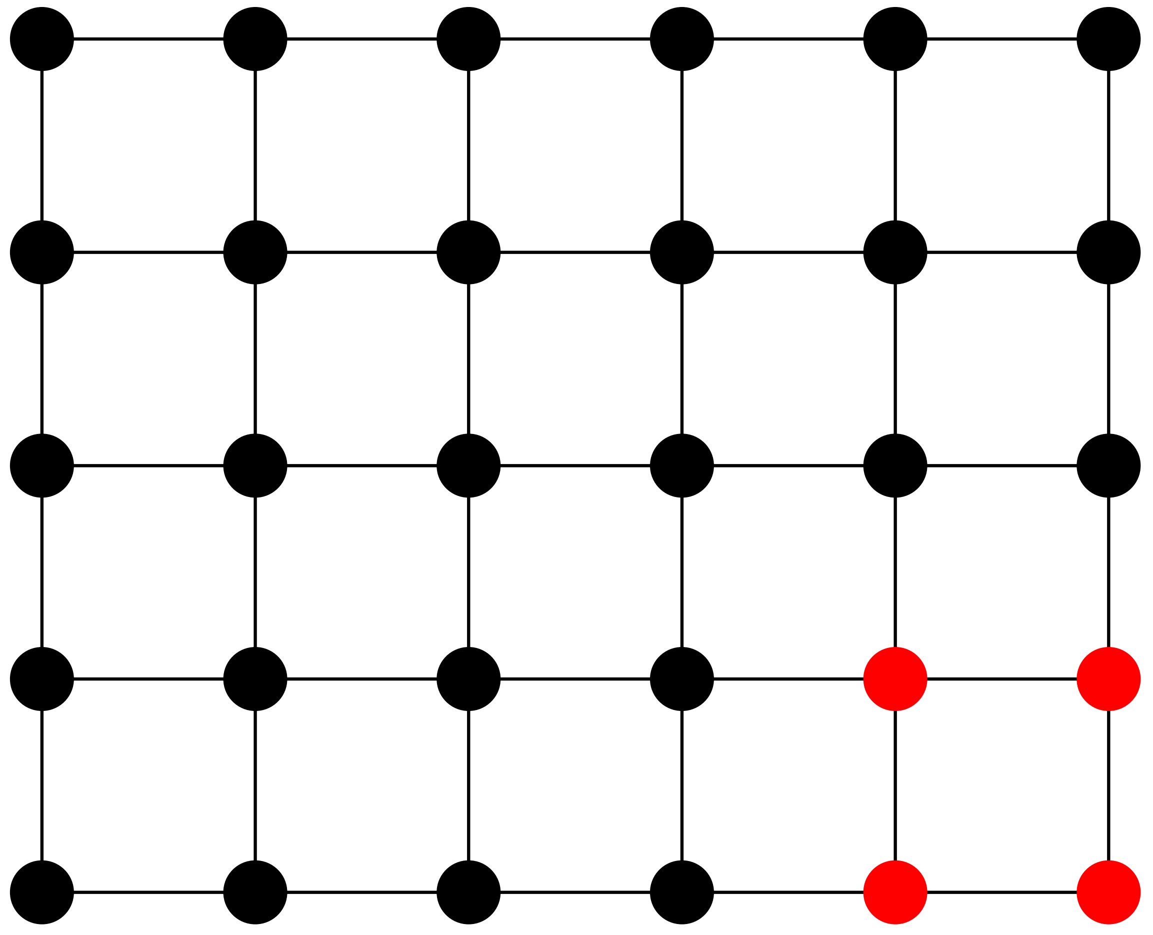

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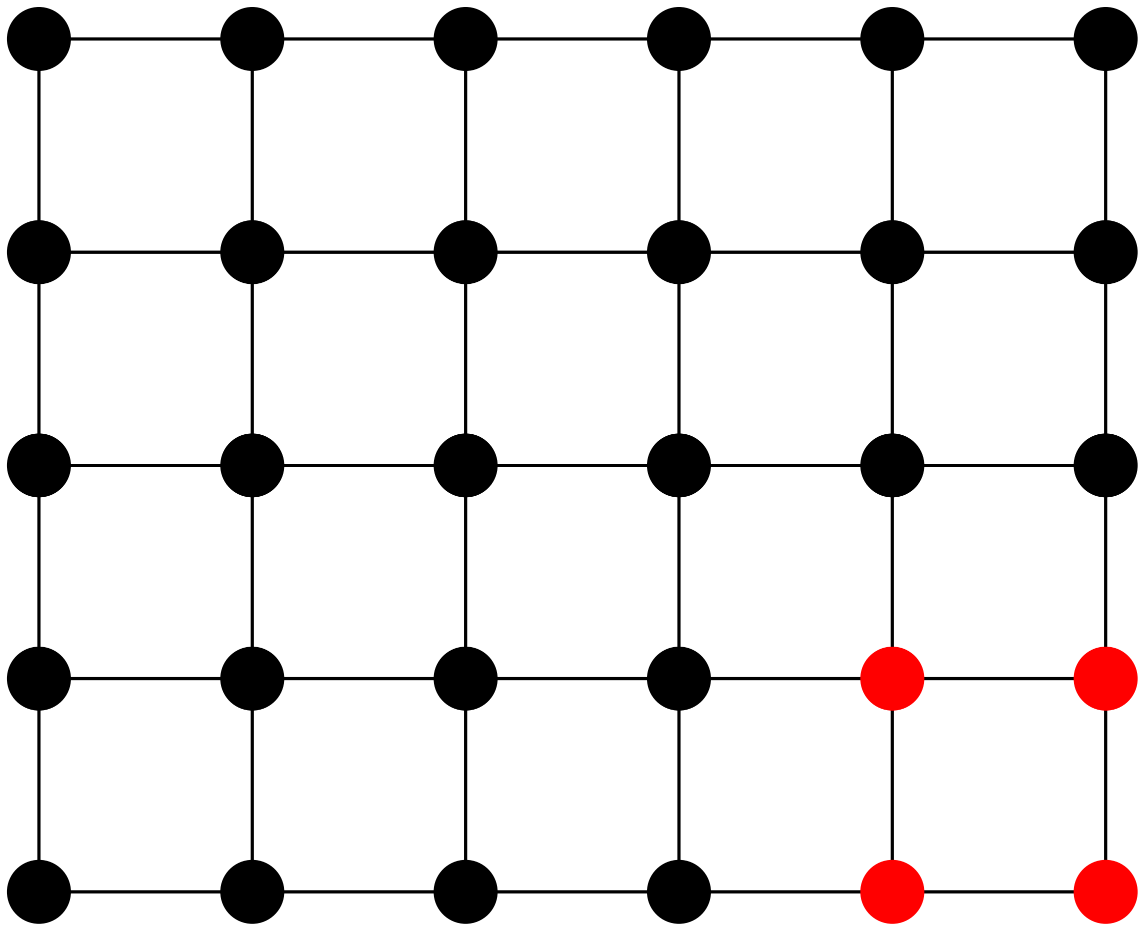

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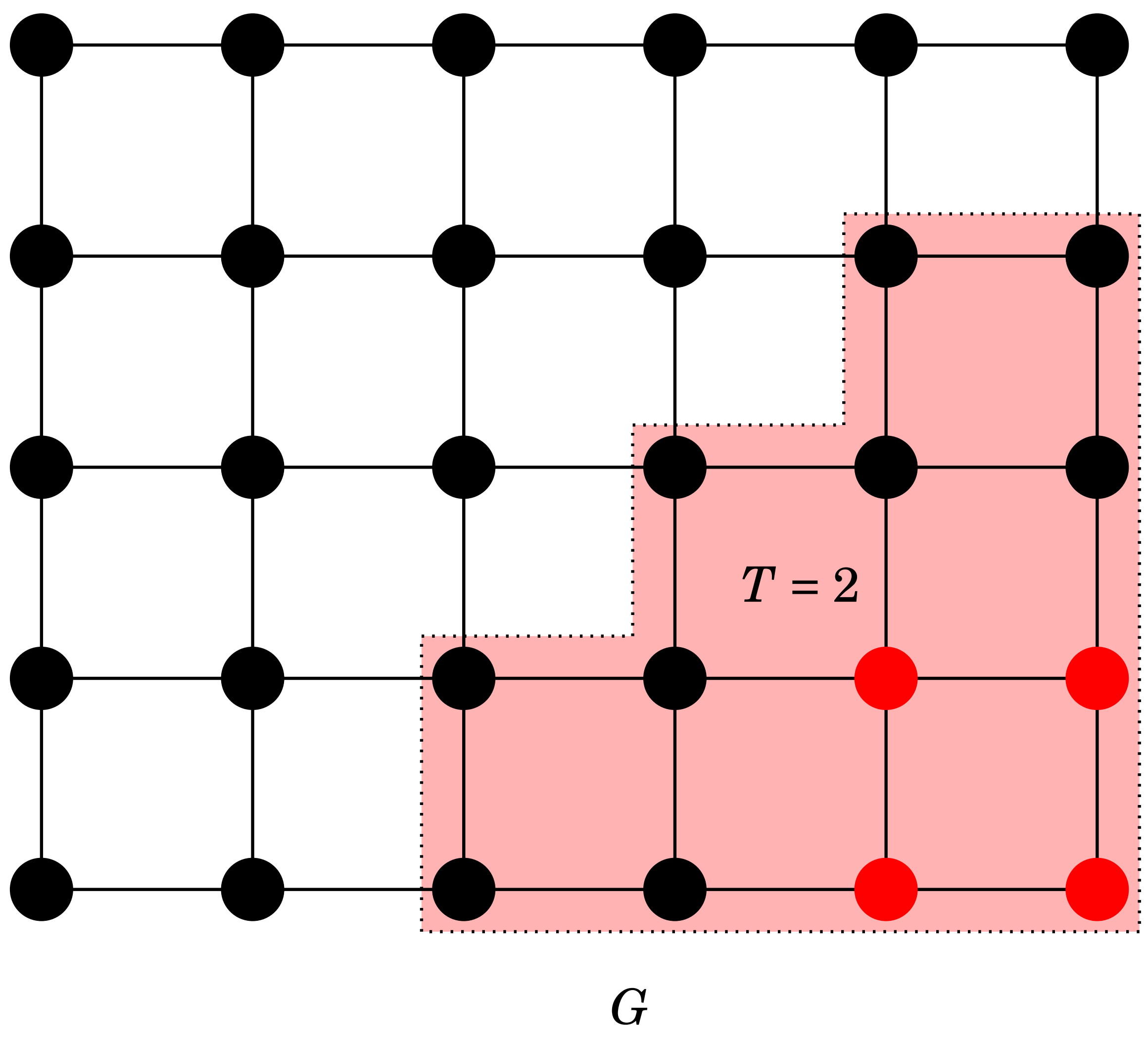

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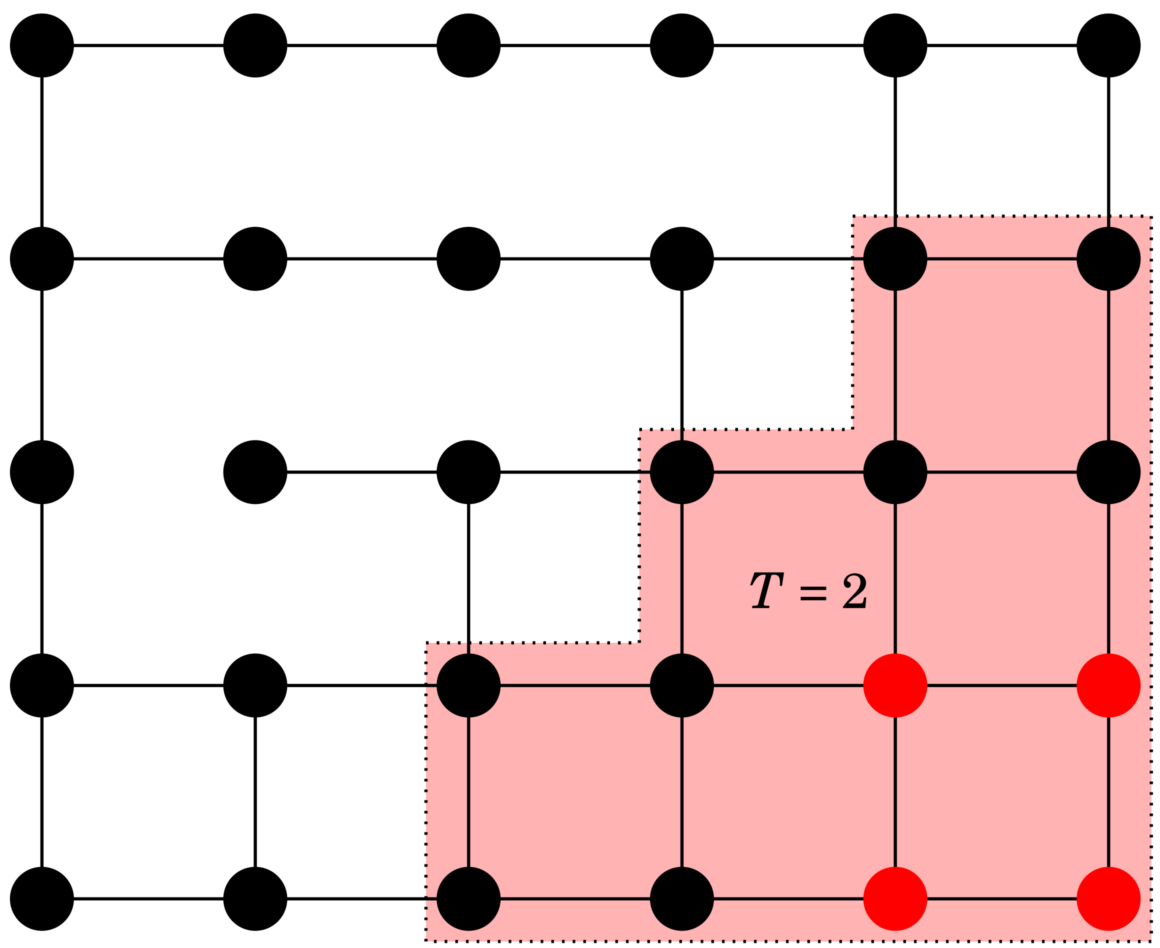

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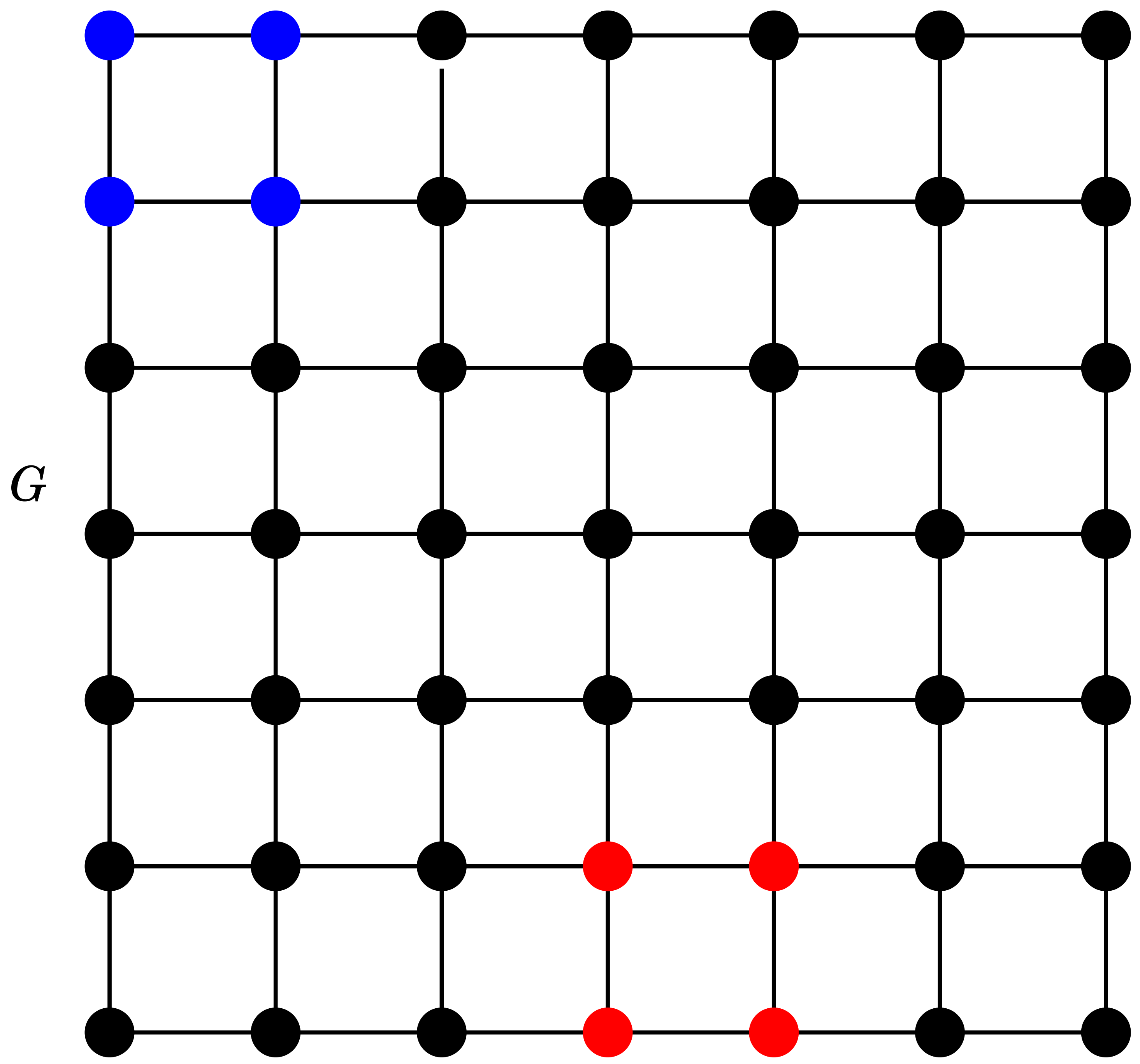

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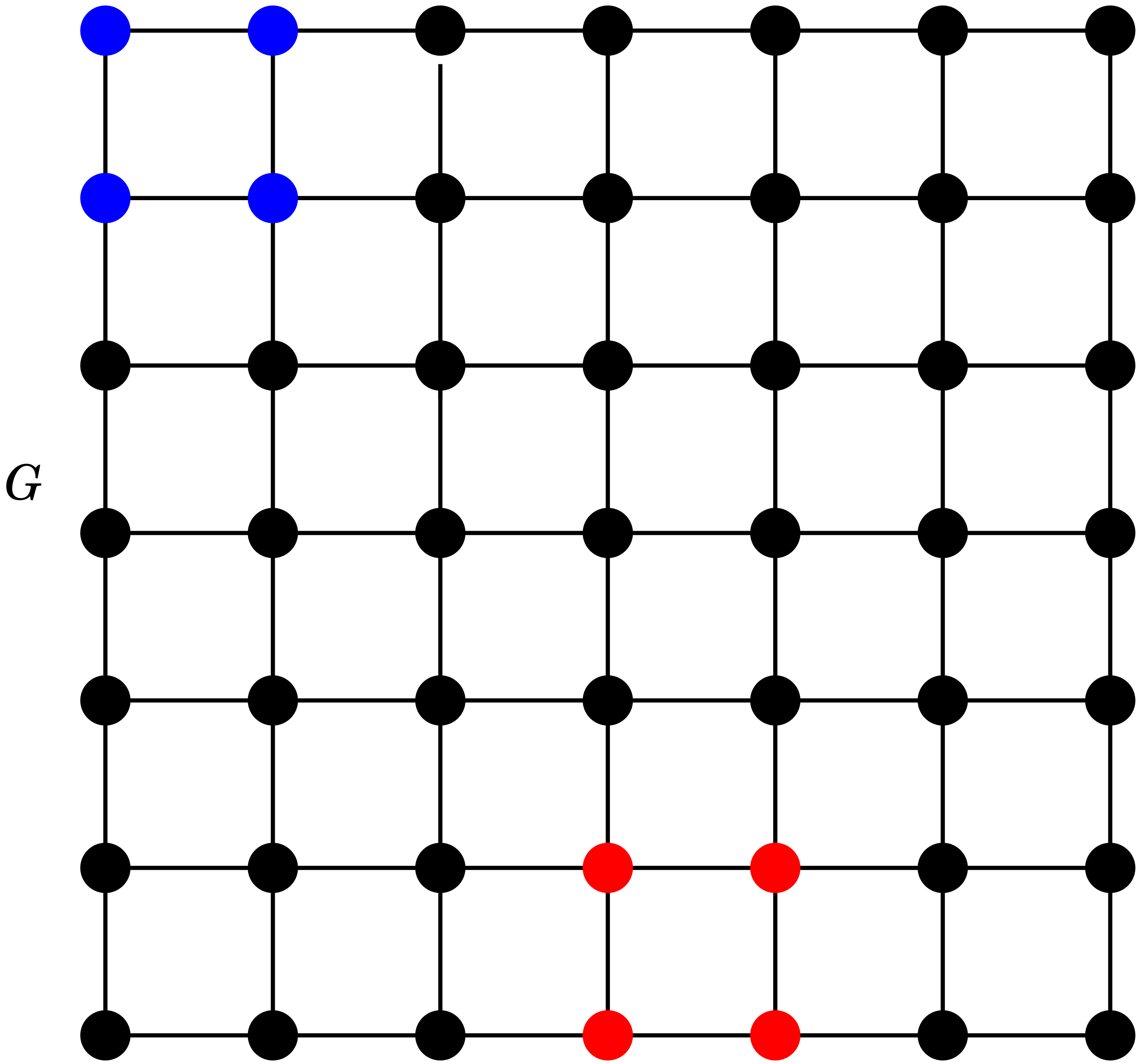

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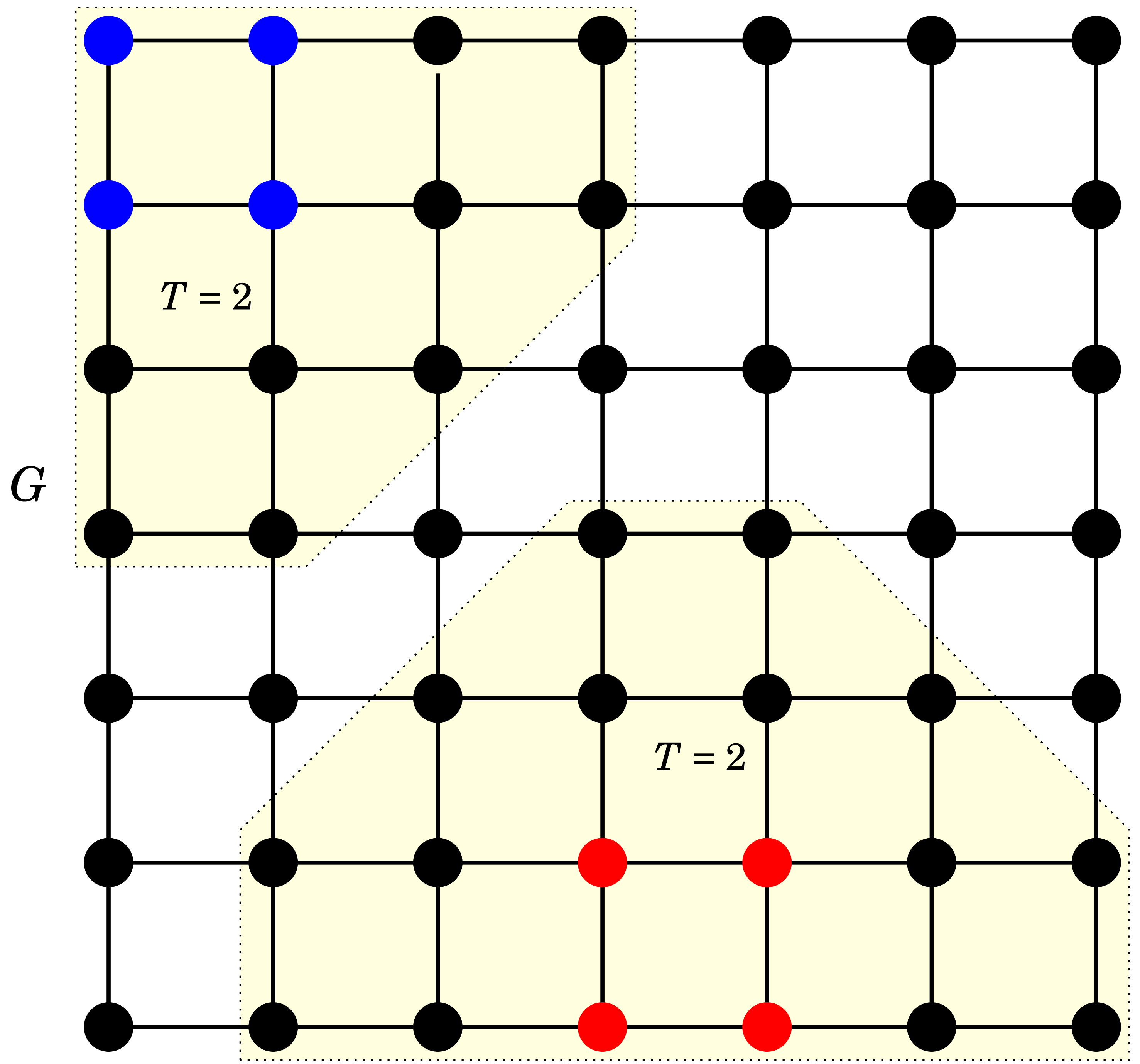

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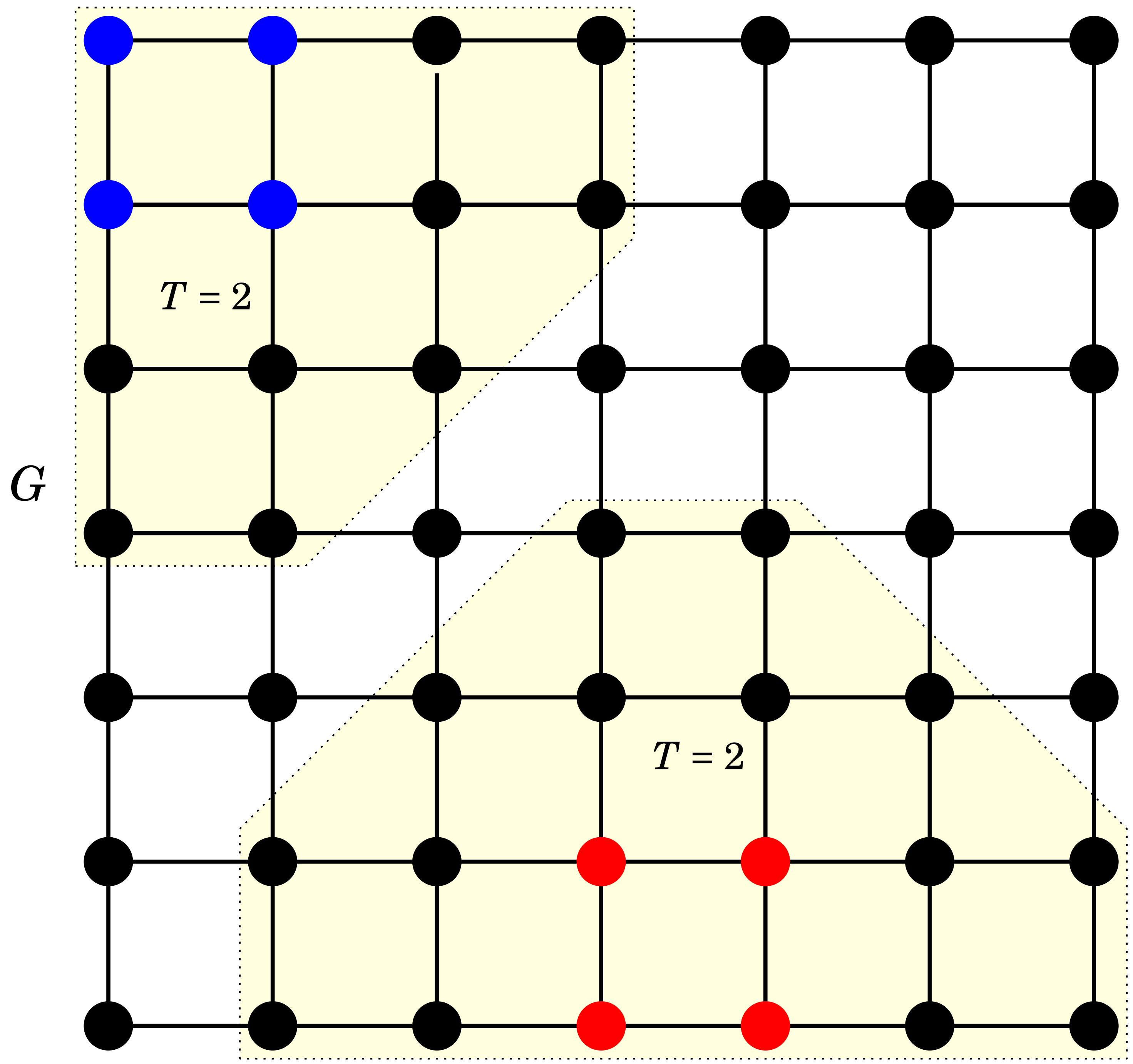

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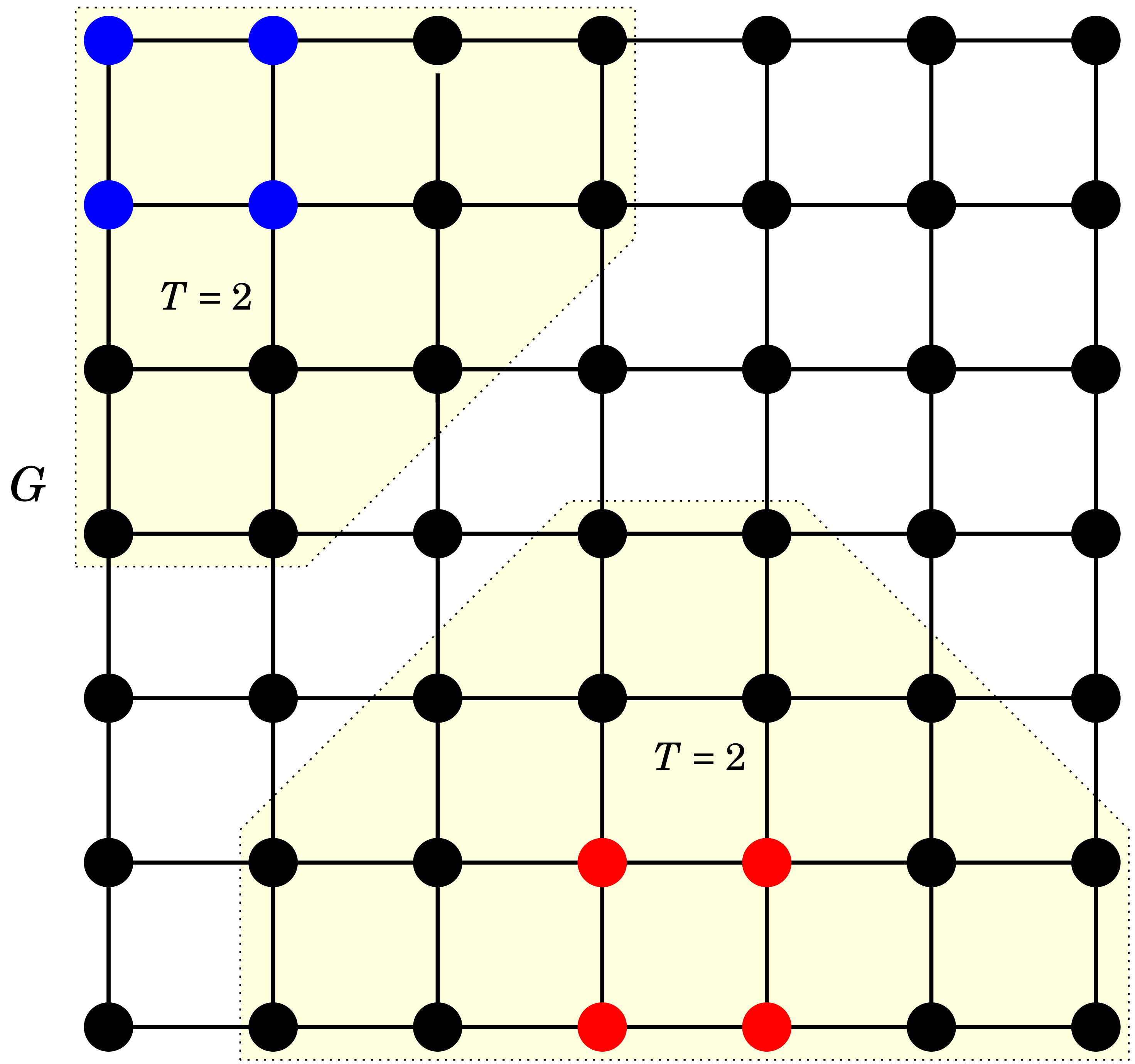
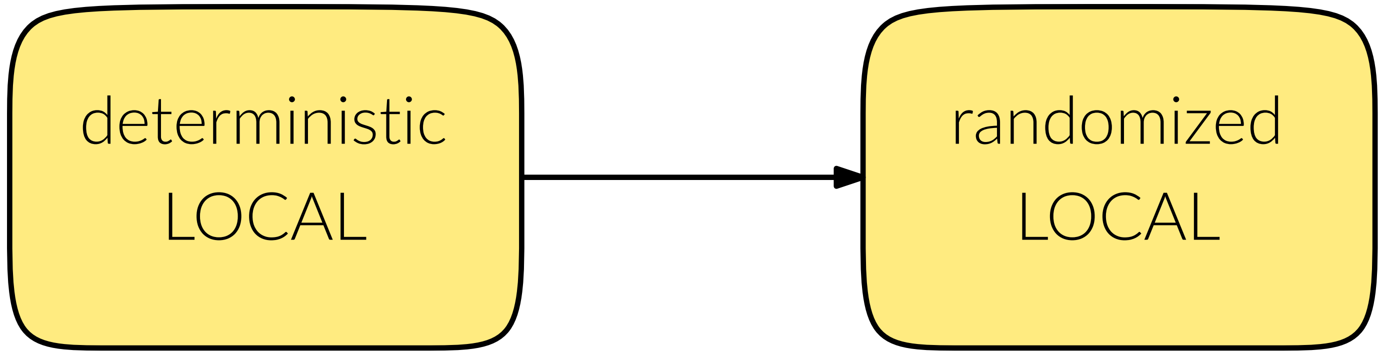


Relations among models

- $X \to Y$ means that locality T in X becomes locality $O(T)$ in Y



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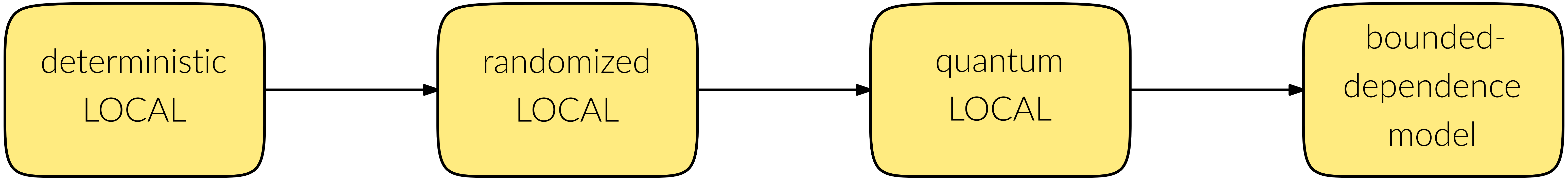

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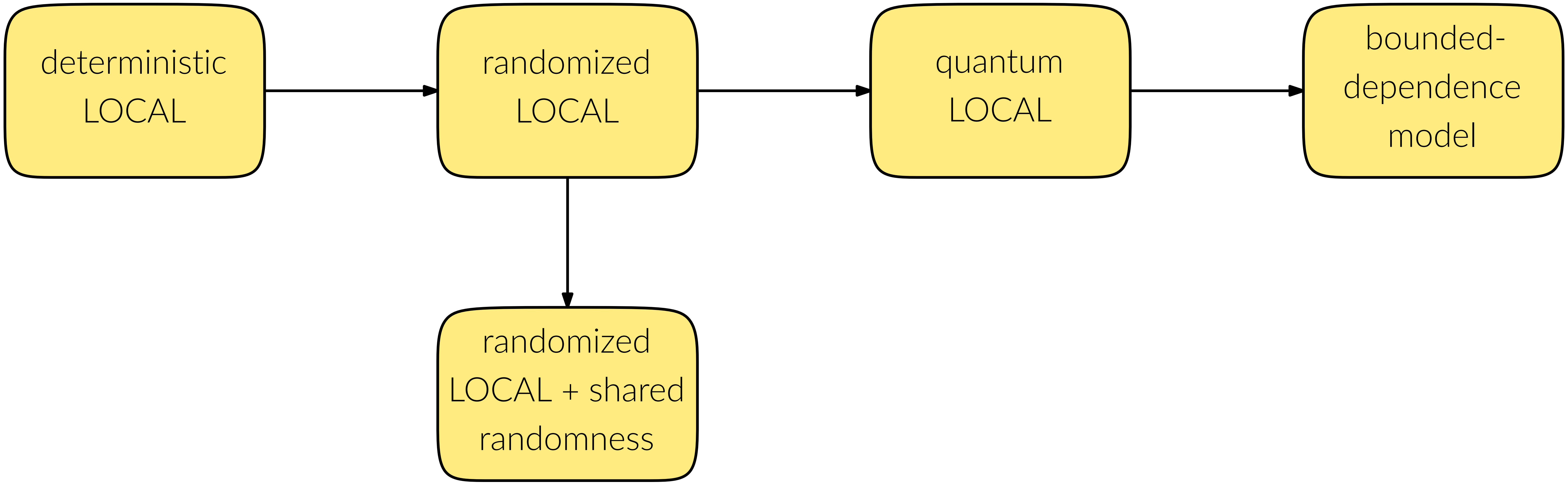

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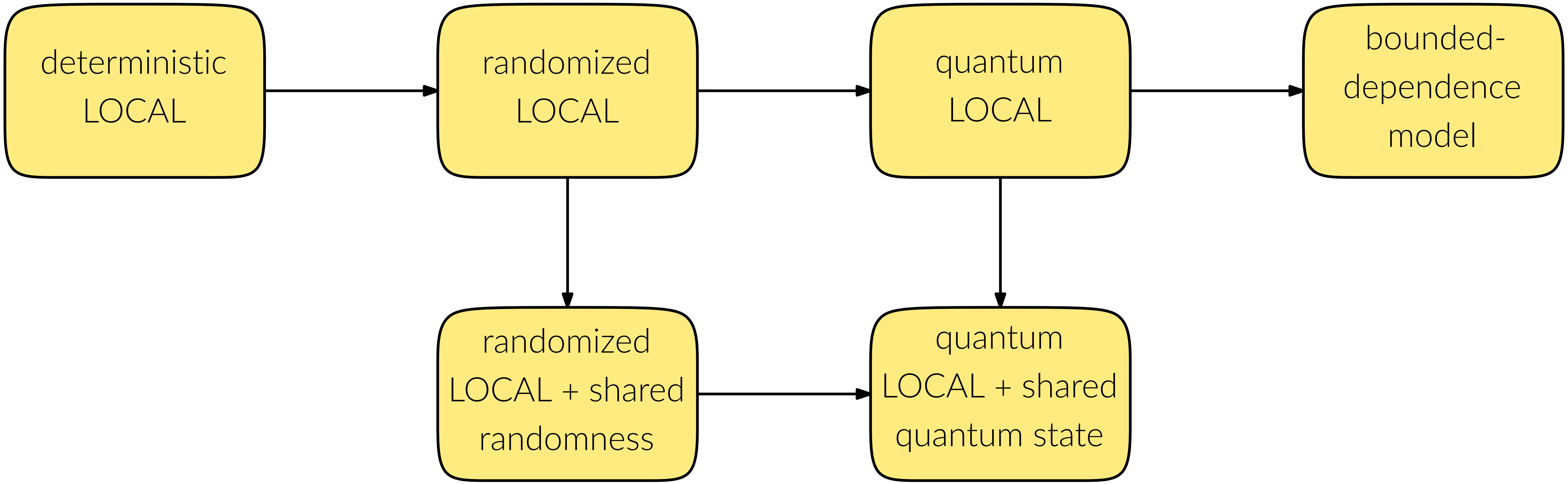

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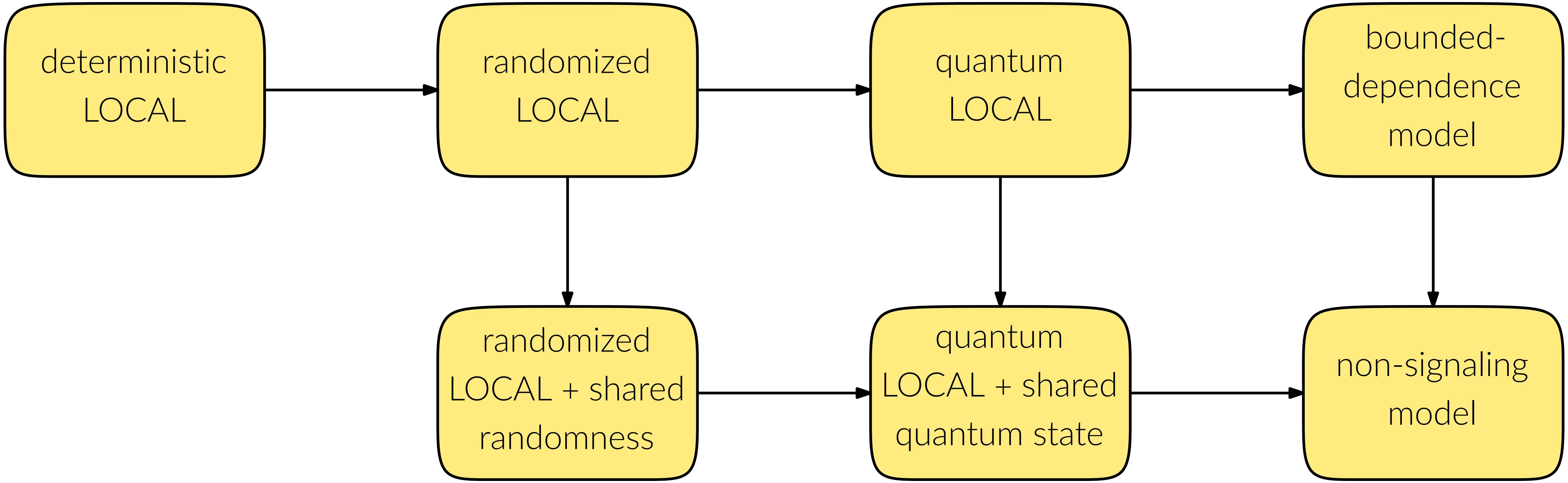

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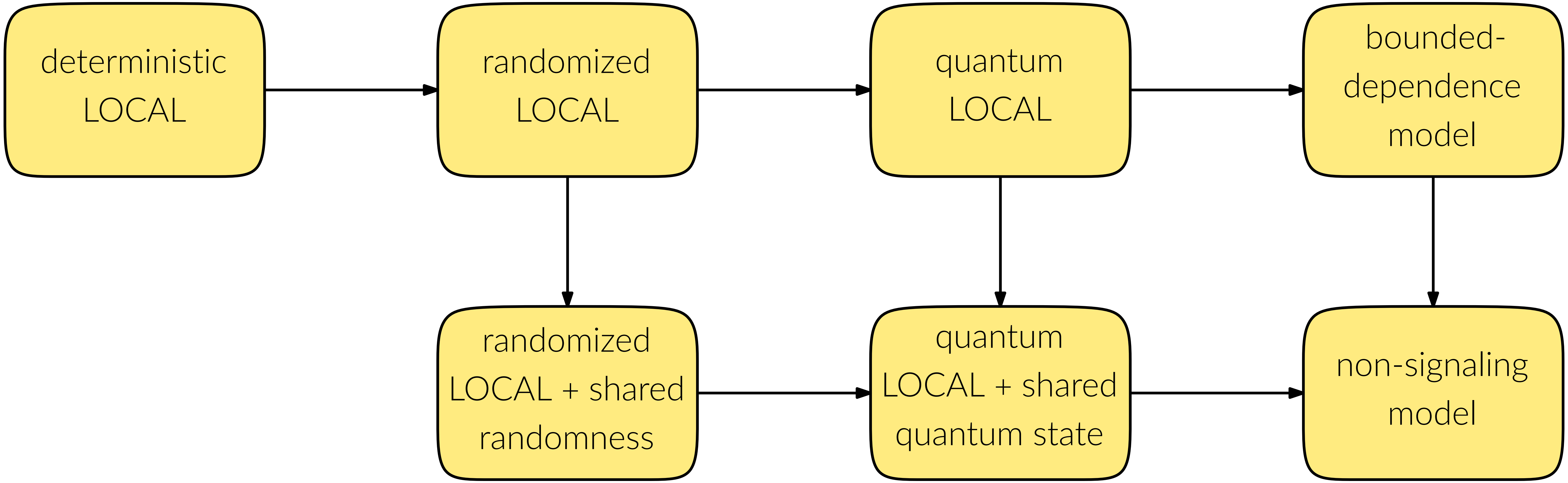

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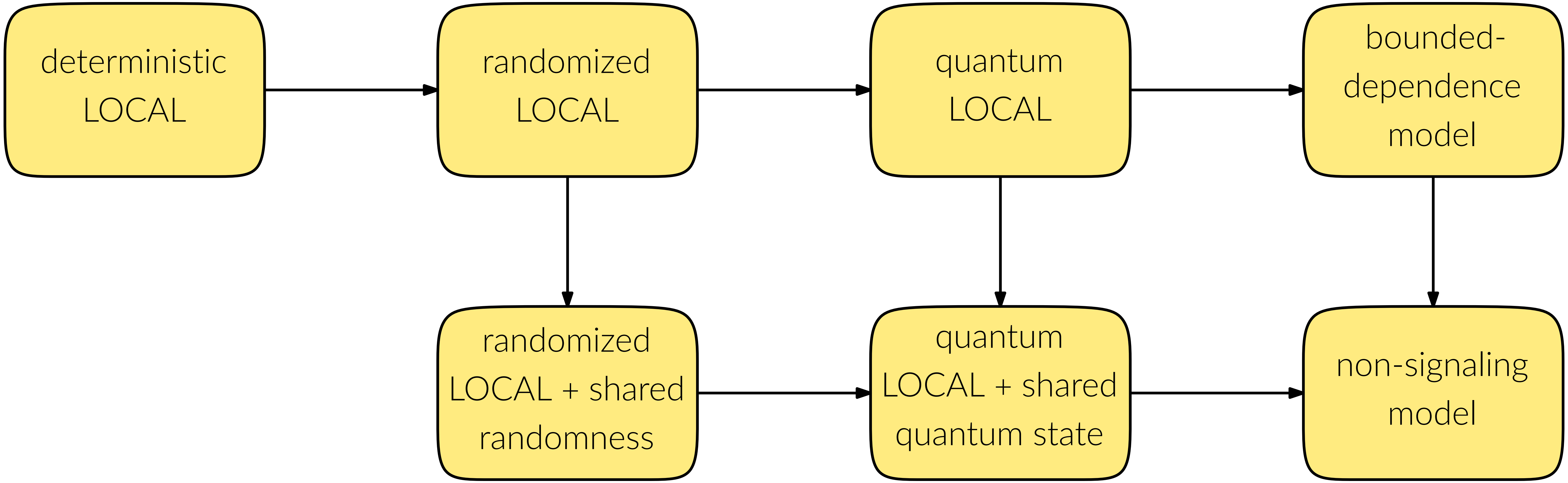
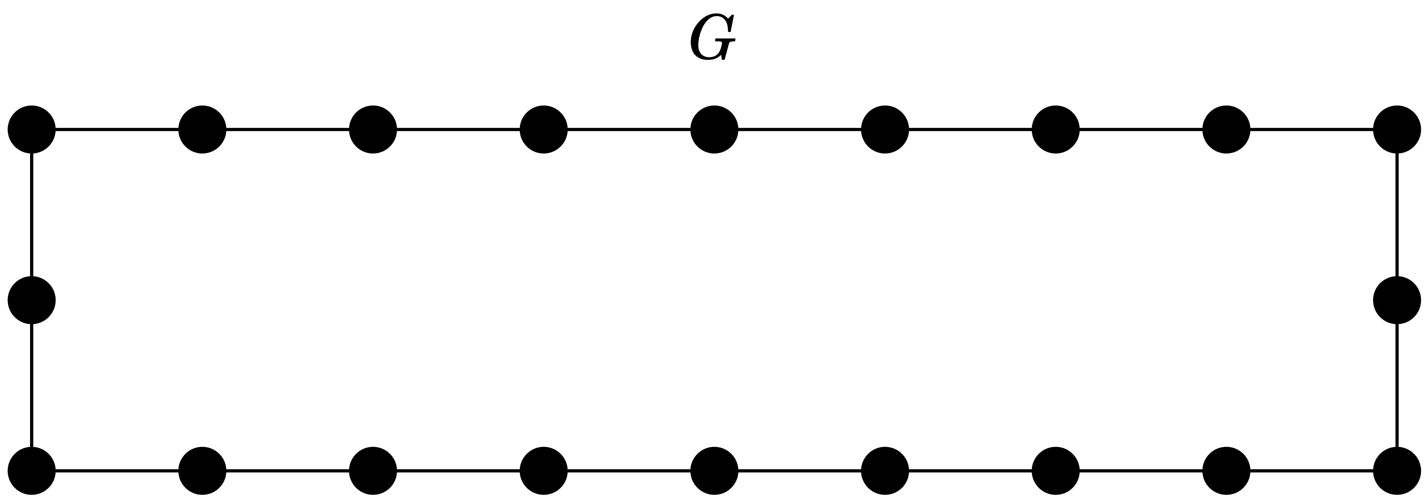


- Is it possible to **"sandwich" quantum-LOCAL** between weaker and stronger models?

  - yes!

Indistinguishability argument

- **Problem**: 2-coloring paths & even cycles in the *non-signaling* model



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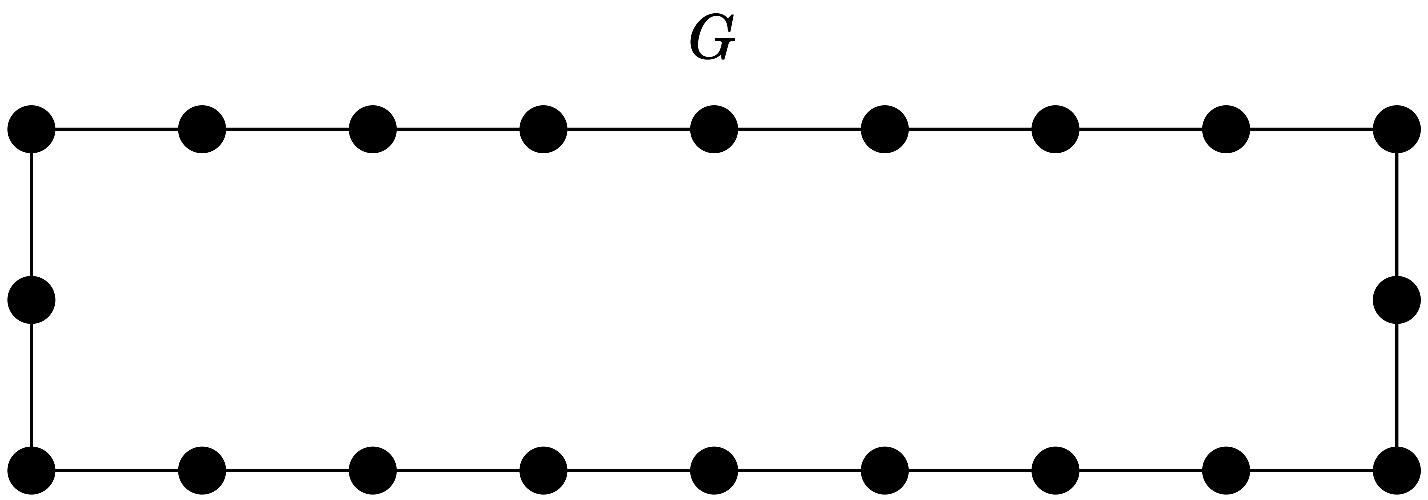

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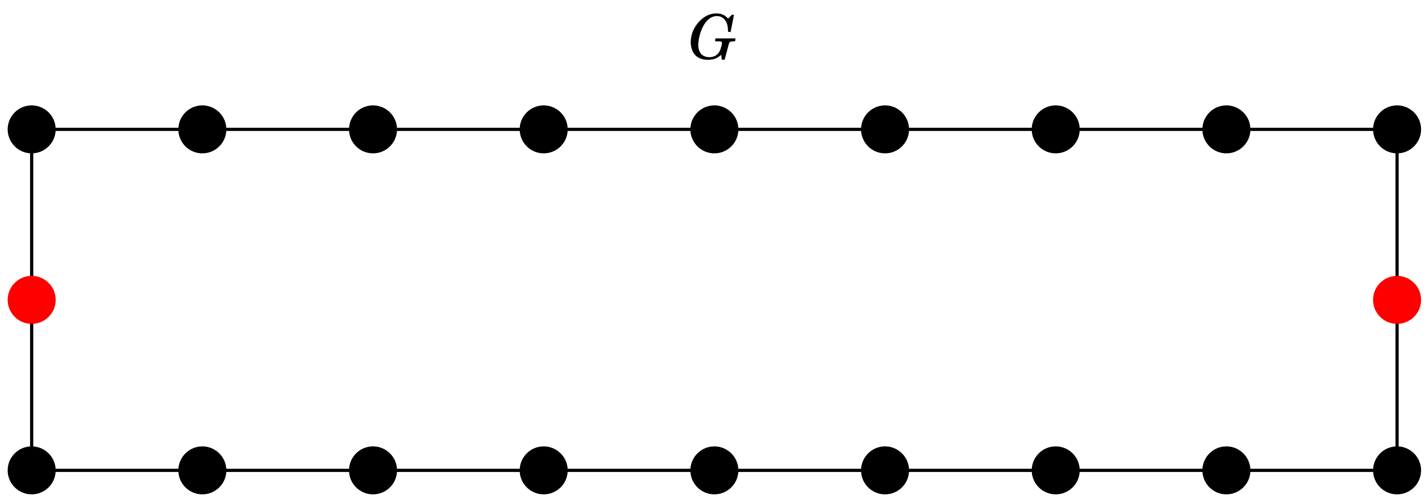

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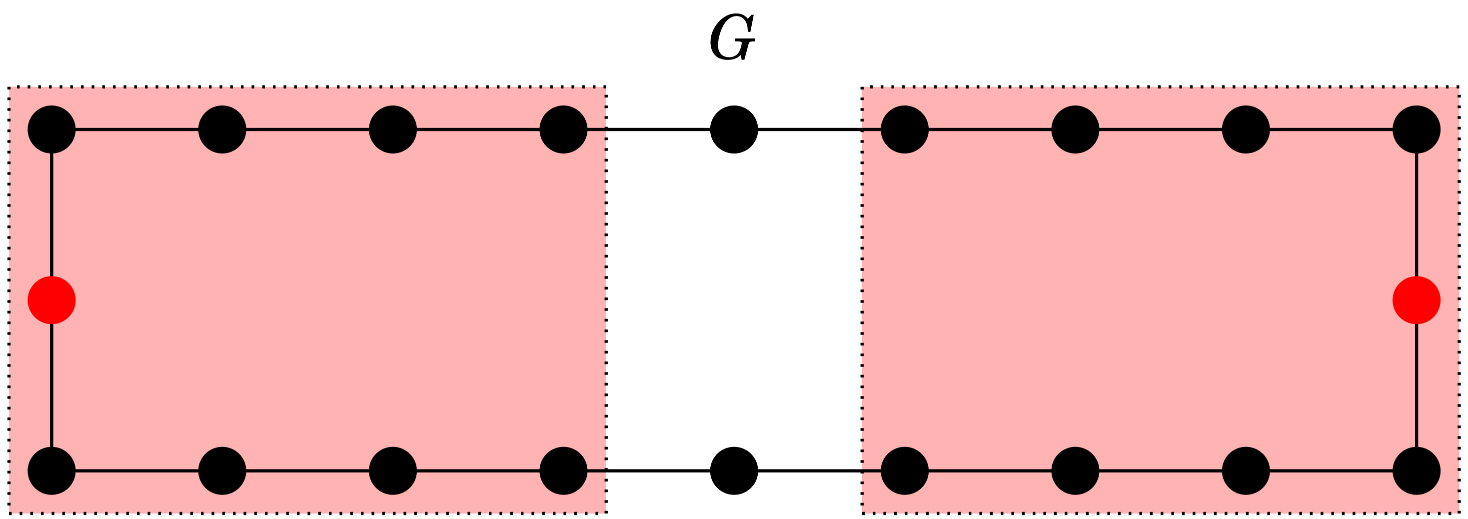

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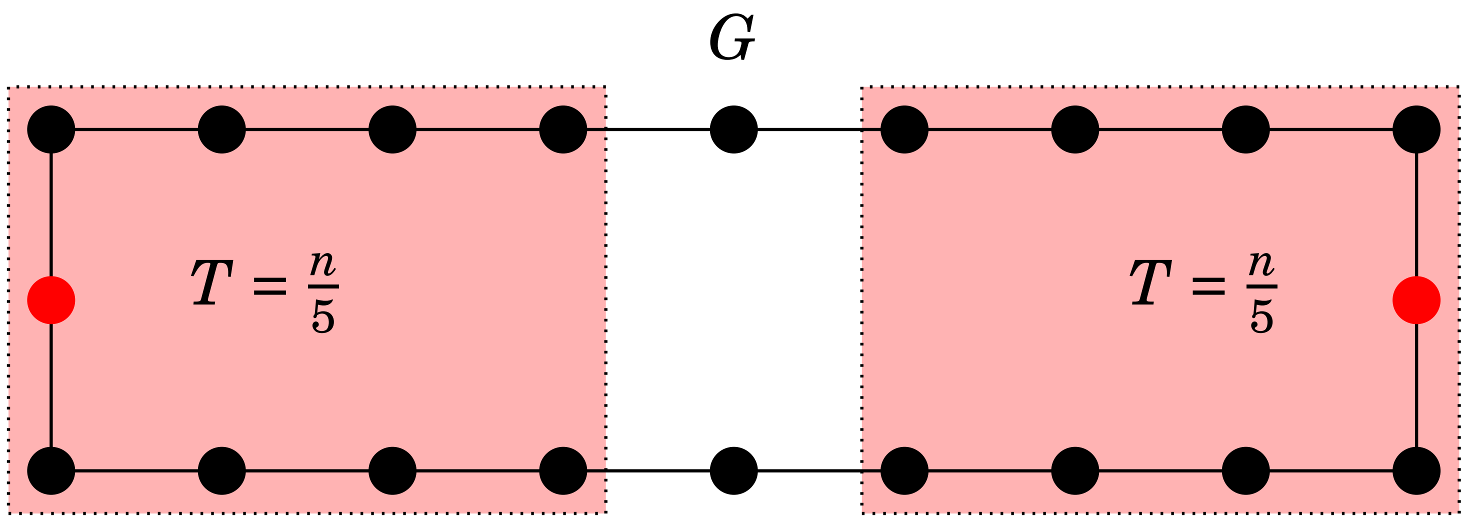

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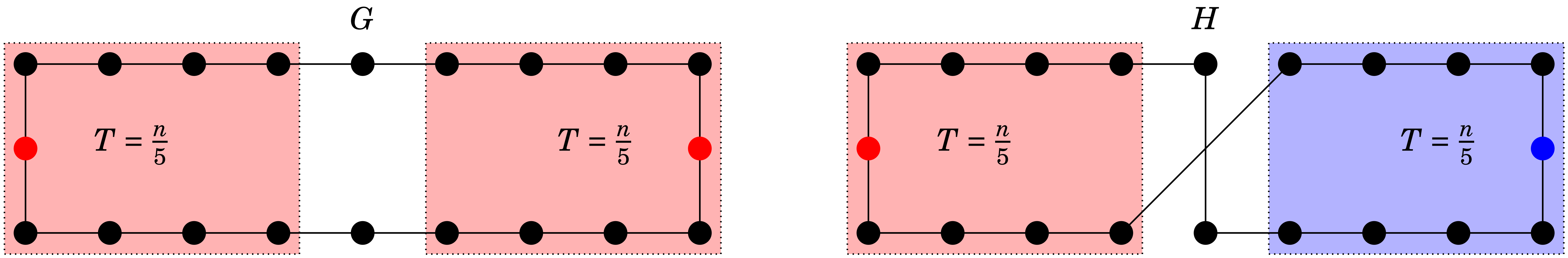

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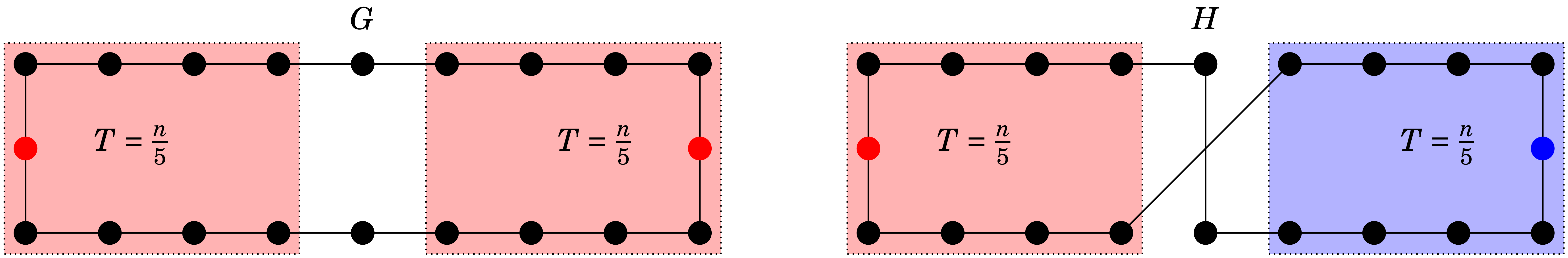

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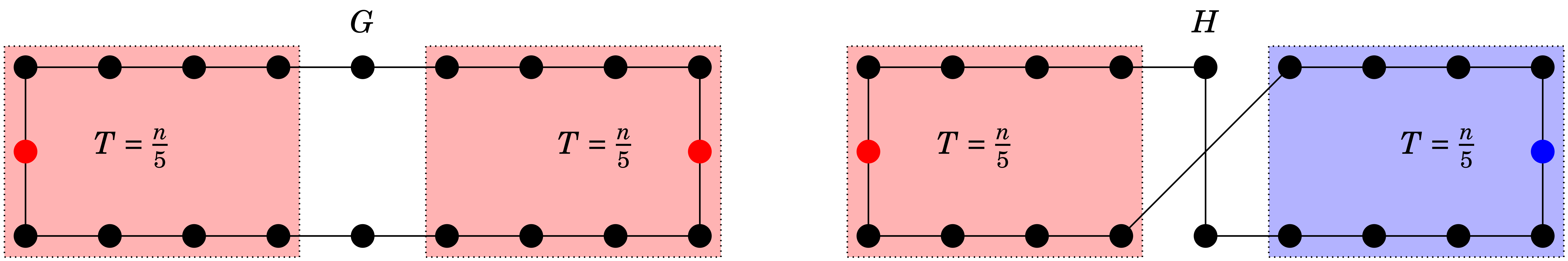

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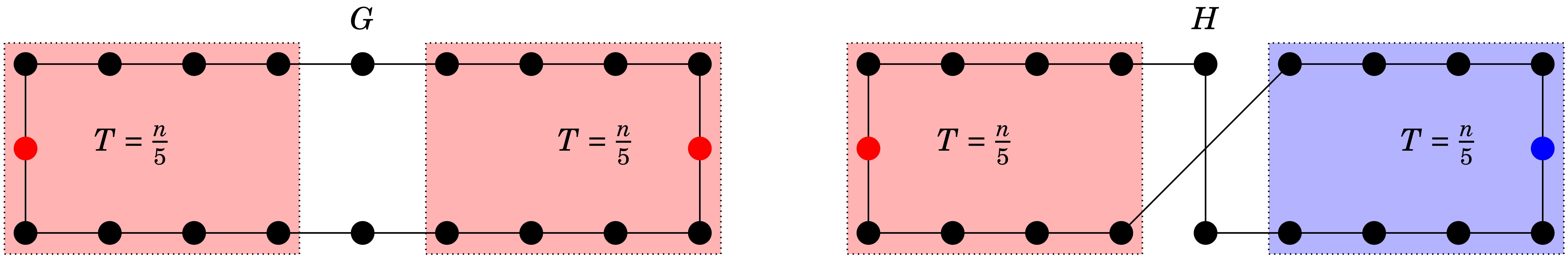

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- **Global problem**: complexity $\Theta(n)$

Graph-existential indistinguishability

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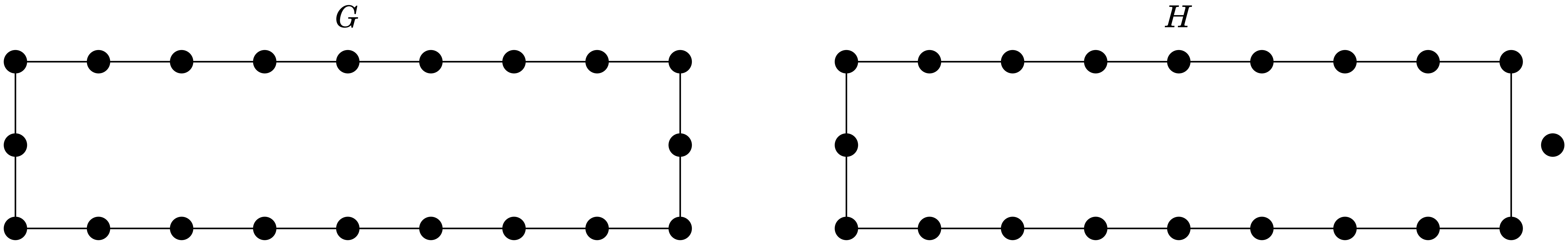

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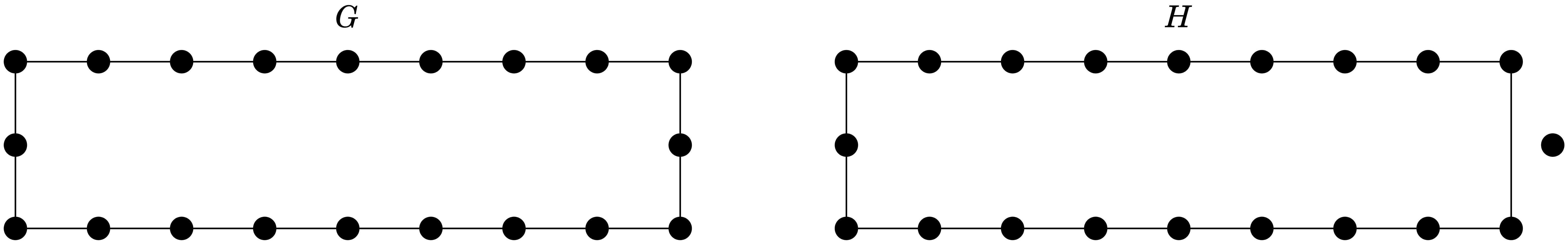

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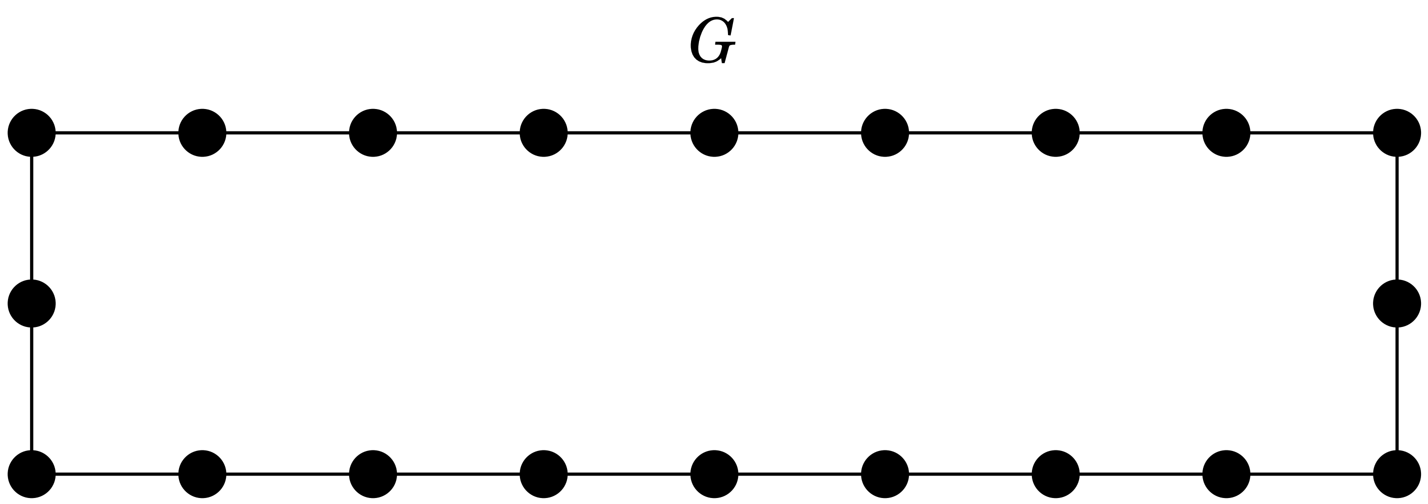

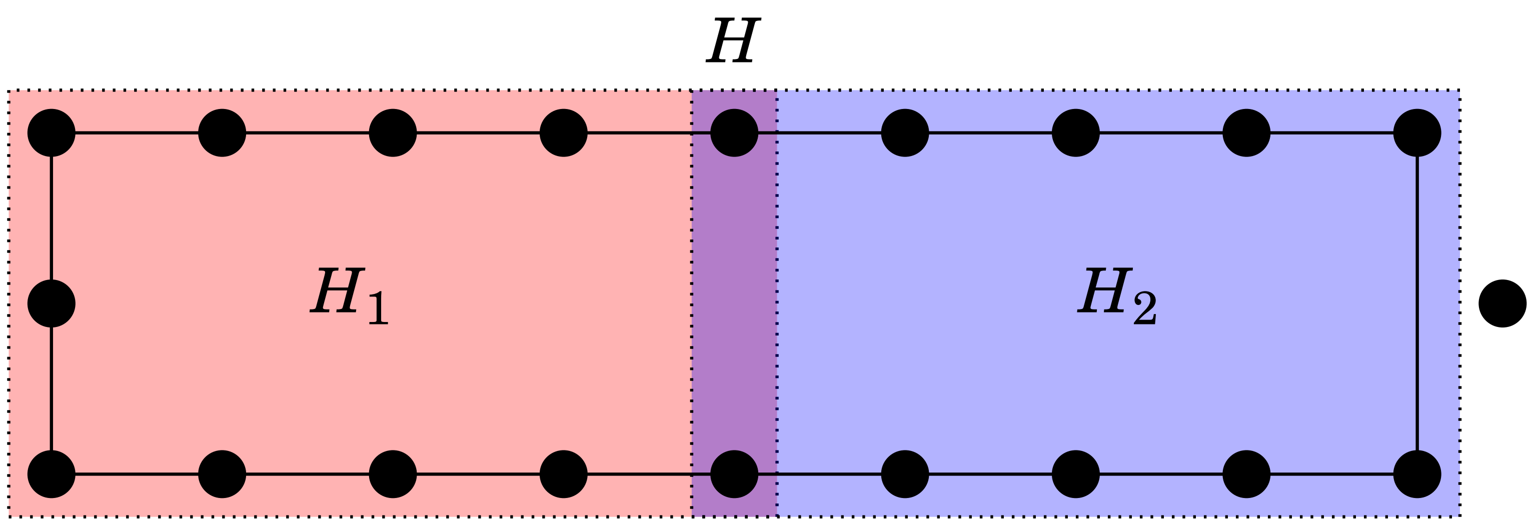

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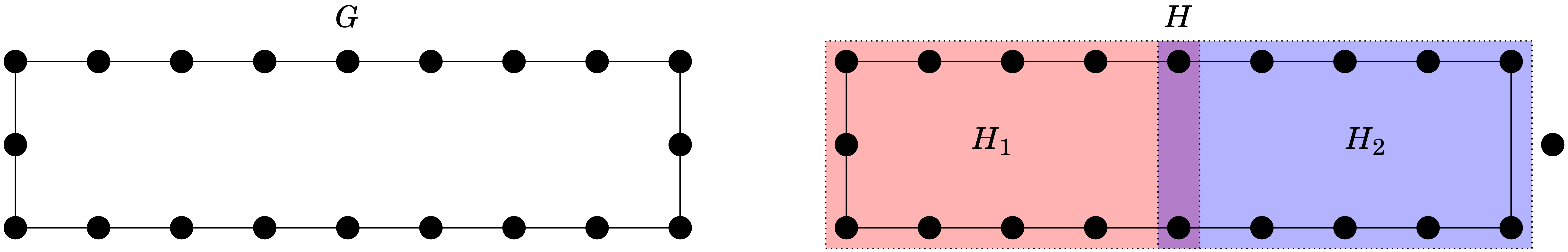

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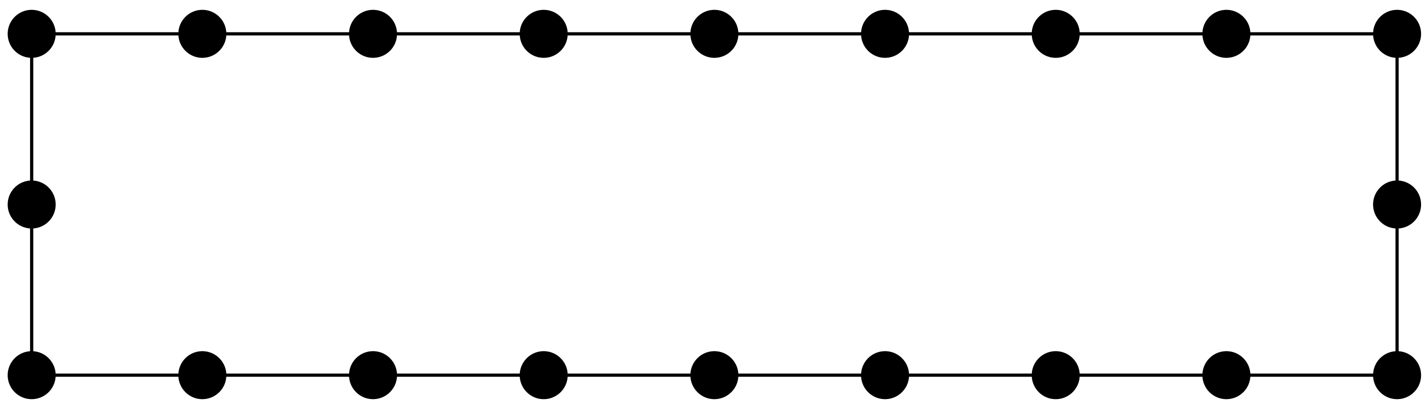

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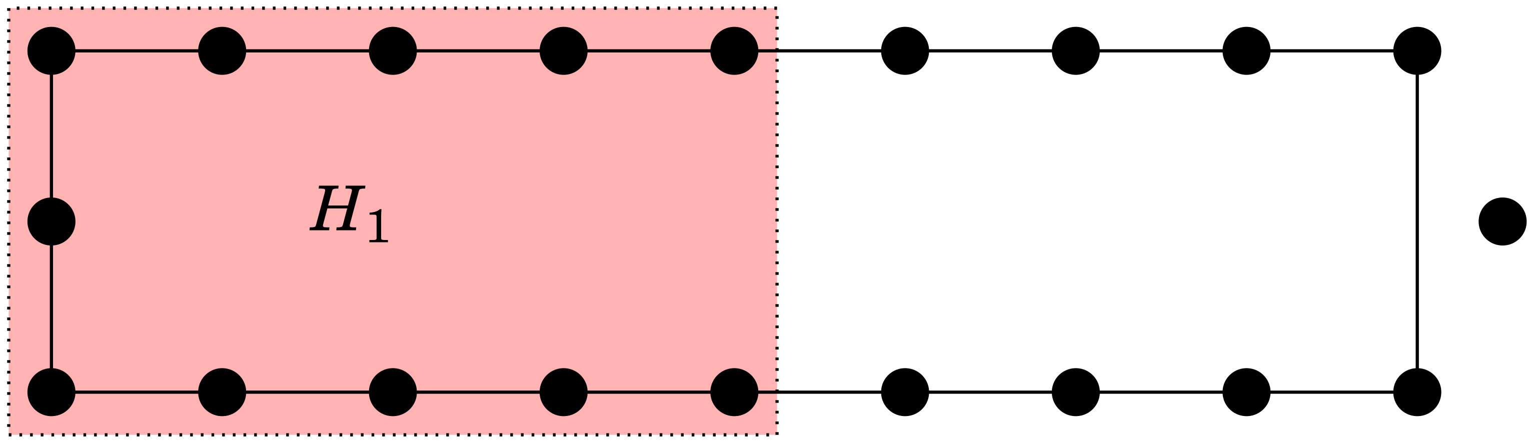

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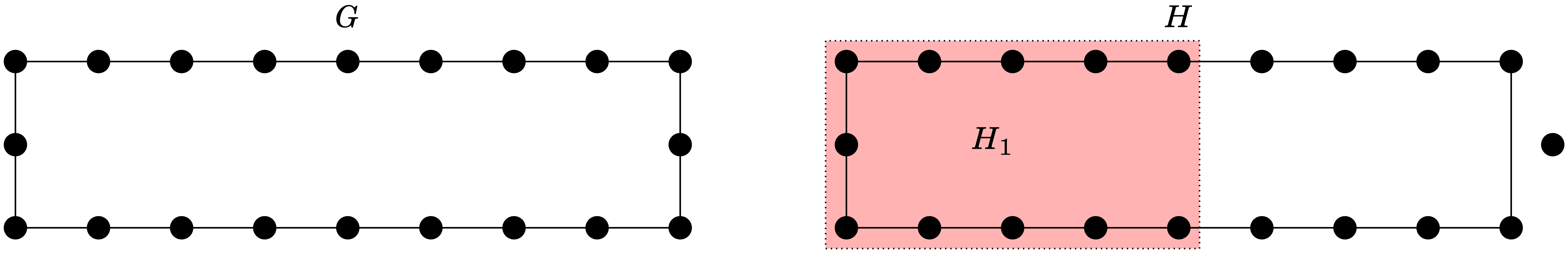

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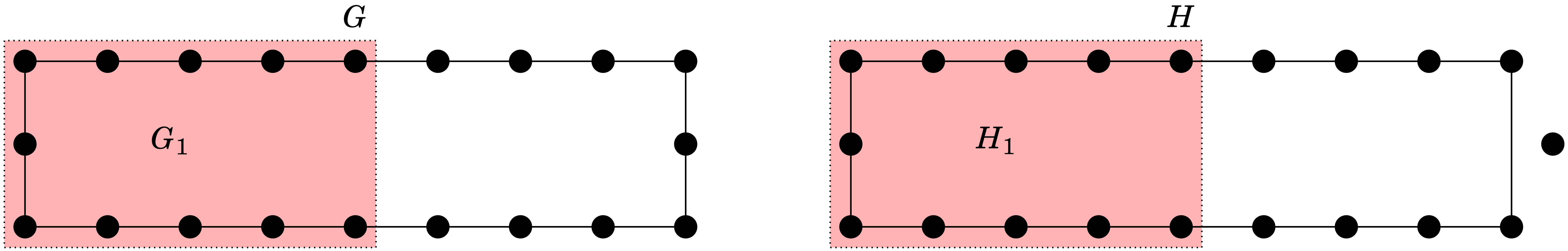

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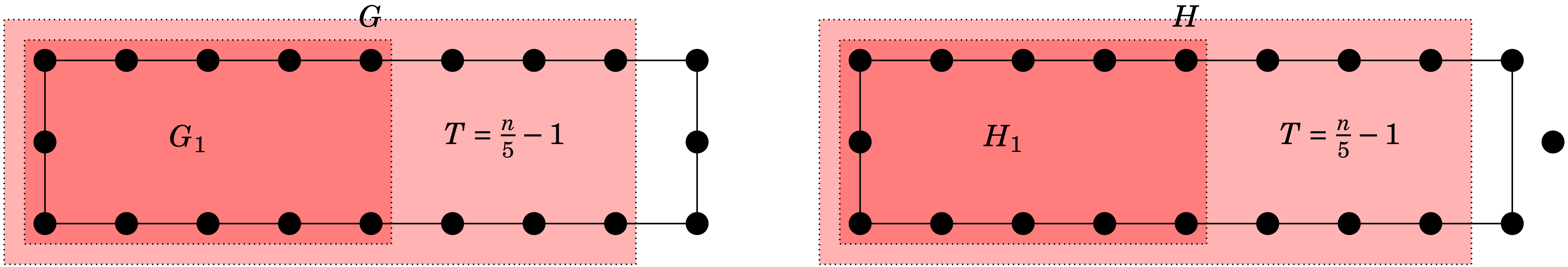

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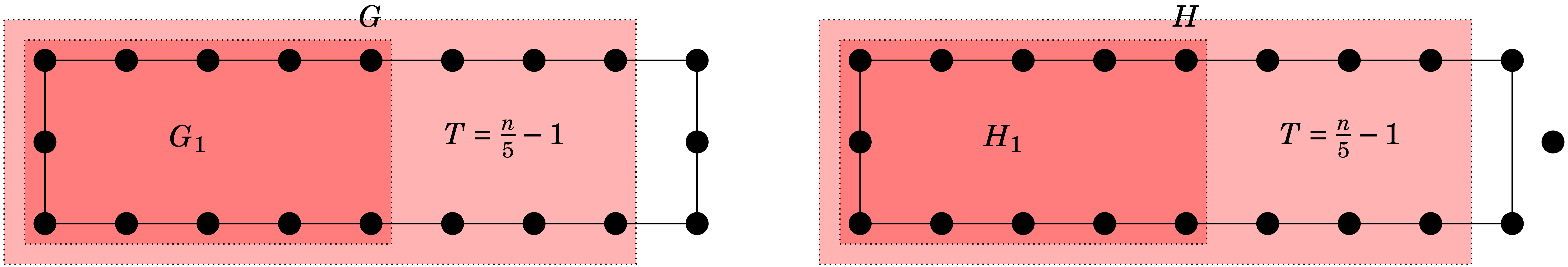

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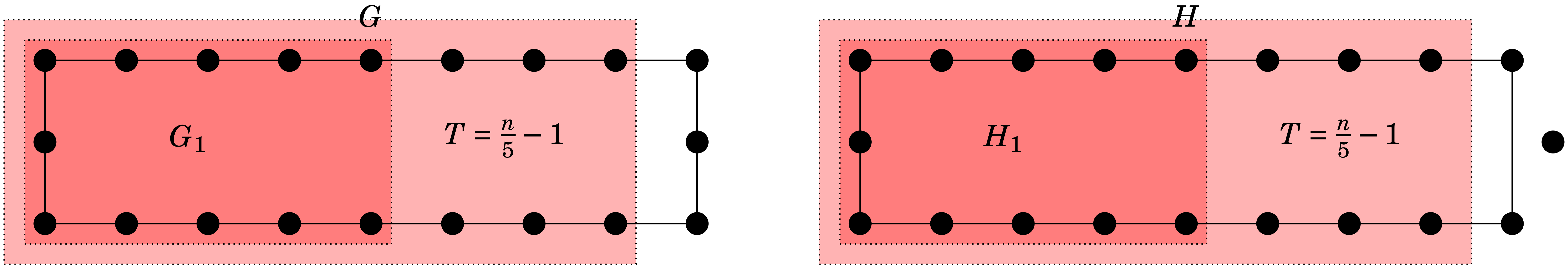

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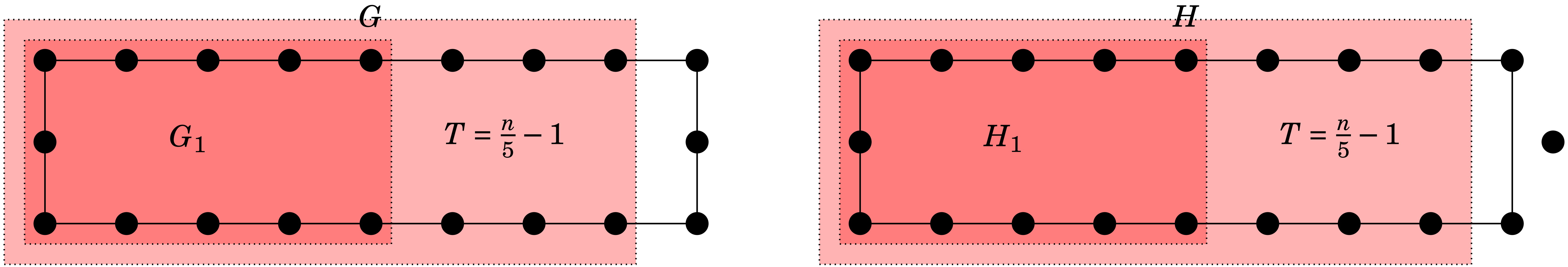

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Table of content

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Problem: c-coloring χ-chromatic graphs [STOC '24]

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Some results: non-signaling model

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What about other known lower bounds? E.g., 3-coloring cycles has complexity $\Theta(\log^\star n)$ [Linial, FOCS '87]

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- Is there any hope to **rule out quantum advantage for LCLs of complexity $\Theta(\log^\star n)$ in classical LOCAL**?
  * using stronger models

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- Can we similarly **rule out quantum advantage for 3-coloring cycles**? Classical complexity $T = \Theta(\log^\star n)$
  - no! *There is a finitely-dependent distribution that* 3*-colors paths and cycles* ($T = O(1)$)
  - [Holroyd and Liggett, Forum of Mathematics, Pi '14]
  - [Holroyd, Hutchcroft, and Levy, Electronic Communications in Probability '18]

- Is there **any quantum-LOCAL algorithm that 3-colors paths and cycles with locality** $T = o(\log^\star n)$?
  - major open question

- Is there any hope to **rule out quantum advantage for LCLs of complexity $\Theta(\log^\star n)$ in classical LOCAL**?
  * using stronger models
  - no!
  - For any LCL Π on bounded degree graphs, *there is a finitely-dependent distribution* ($T = O(1)$) *solving* Π
  - [STOC '25a]

Bounding general quantum advantage

- **Theorem** [SODA '26]: given any LCL Π

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Relations among models

- $X \to Y$ means that locality T in X becomes locality $O(T)$ in Y

T-dependent distribution $\implies O(\sqrt{Tn}\text{ poly}\log(n))$-time deterministic LOCAL algorithm

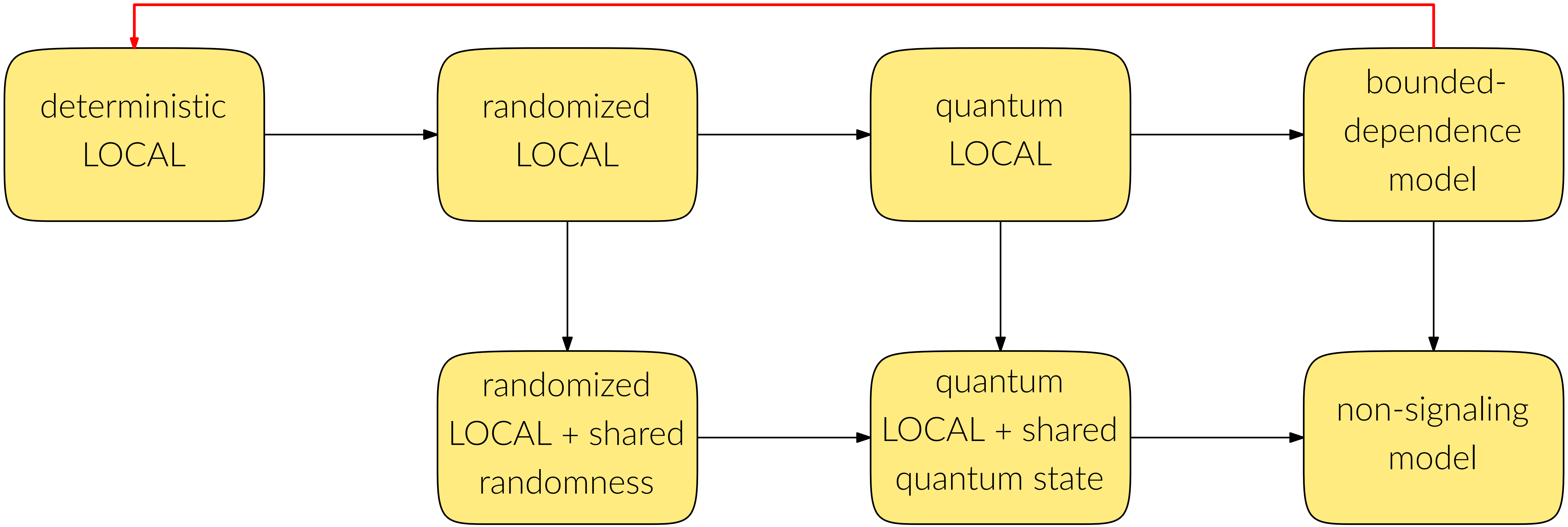

Table of content

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- **Quantum-LOCAL**: can we do something better? [STOC '25b, SODA '26]

Actual quantum advantage for LCLs

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THANKS! Questions?

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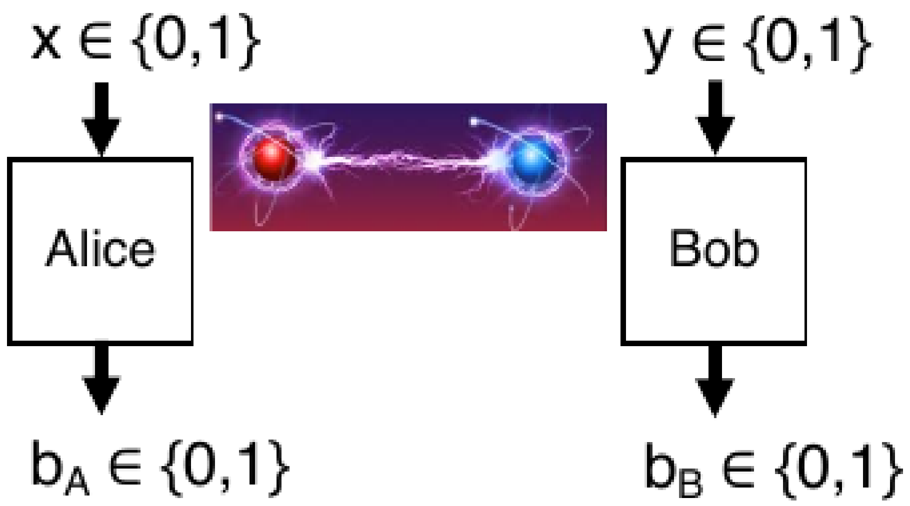
Non-signaling & quantum games

Alice

Bob

- Both Alice and Bob receive an input bit x and y in $\{0,1\}$
- They must output a bit each a and b according to some rule

CHSH Game [Clauser, Horne, Shimony, Holt 1969]



winning condition: $b_A \oplus b_B = xy$

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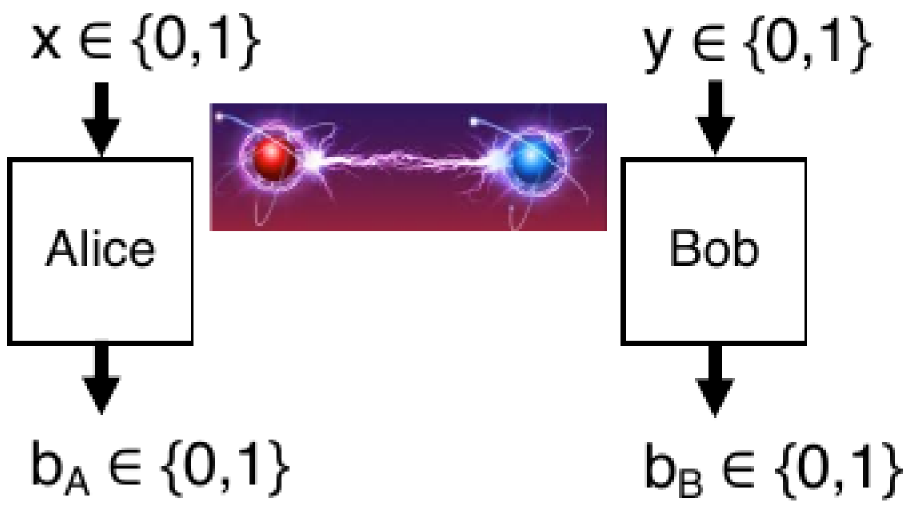

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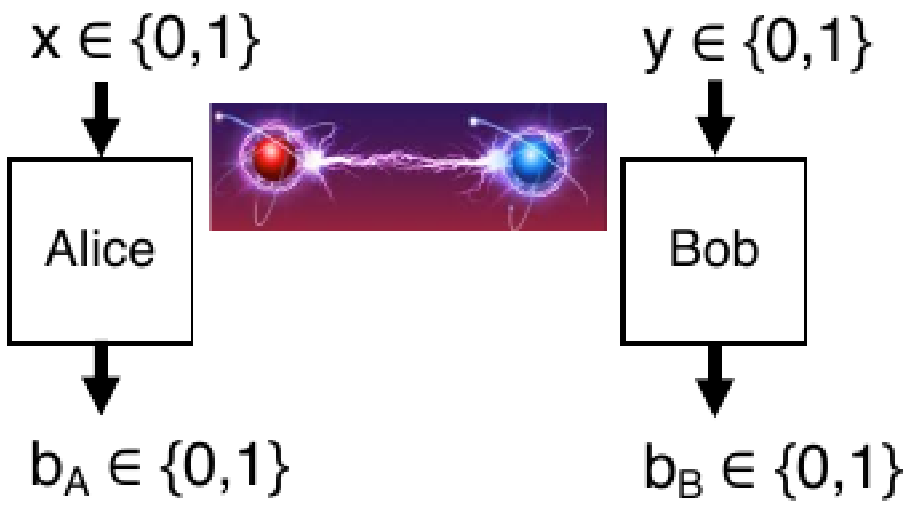

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Inputs	0, 0	0, 1	1, 0	1, 1
Outputs	0, 0	0, 0	0, 0	0, 1
	1, 1	1, 1	1, 1	1, 0

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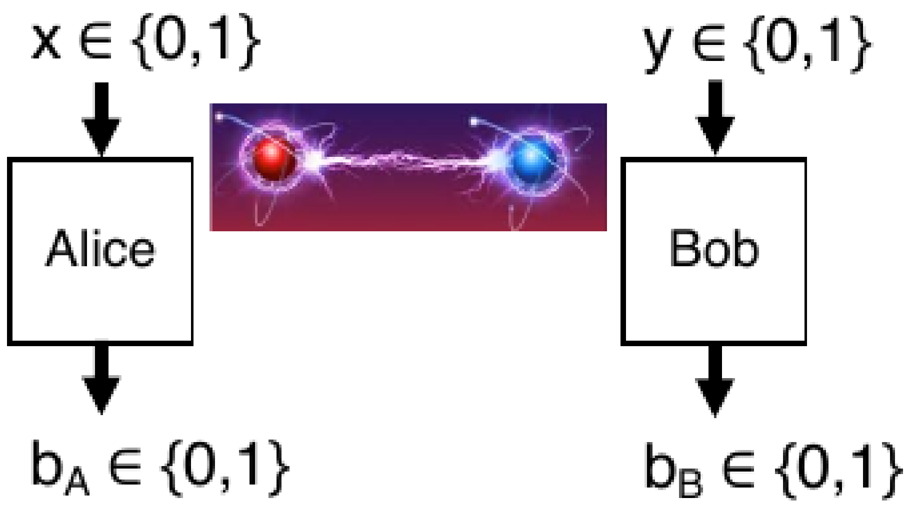

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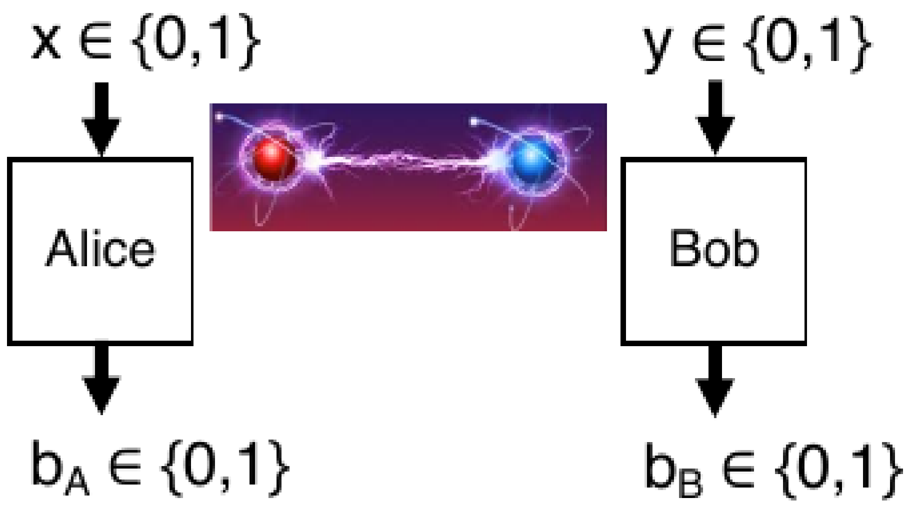

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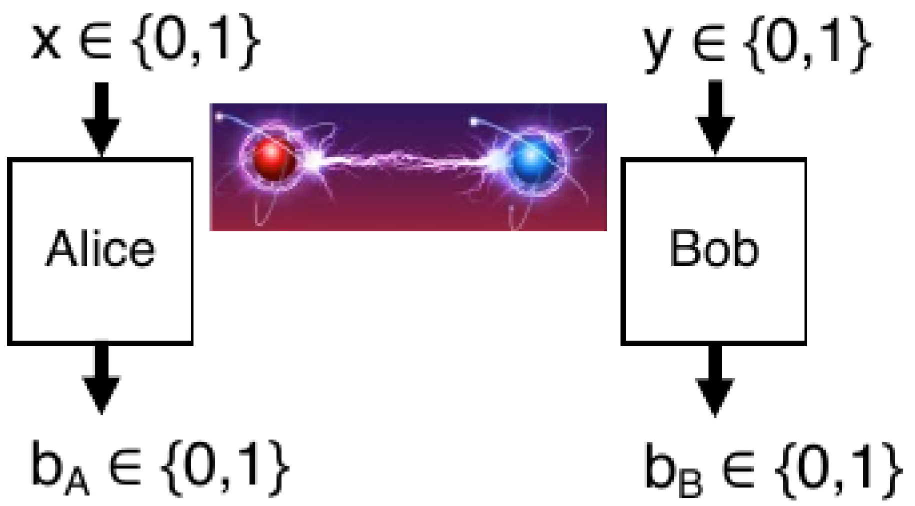

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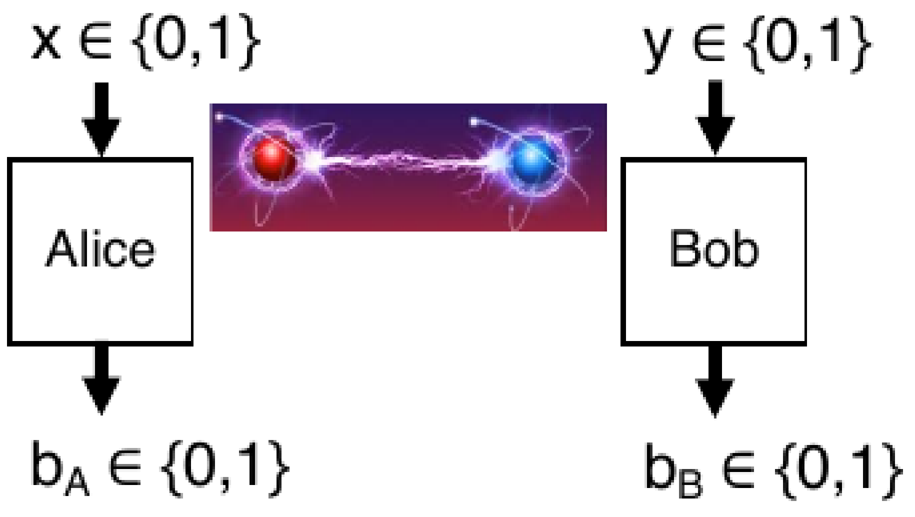

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  - *strategy*: sample u.a.r. among the correct solutions

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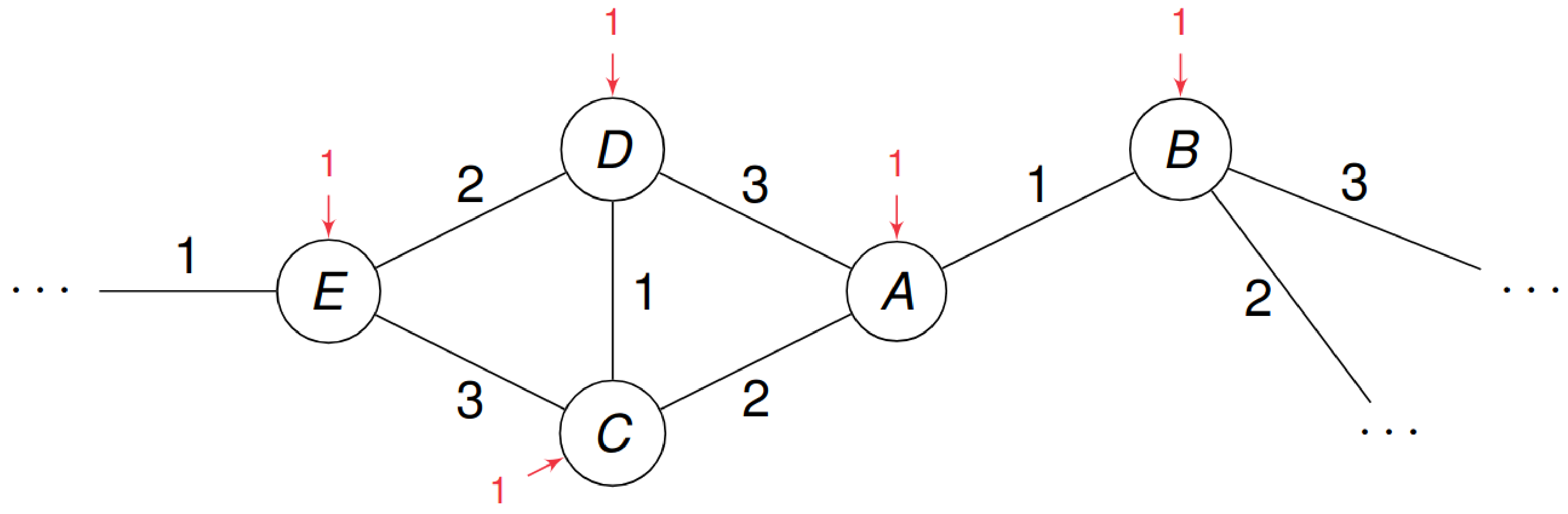

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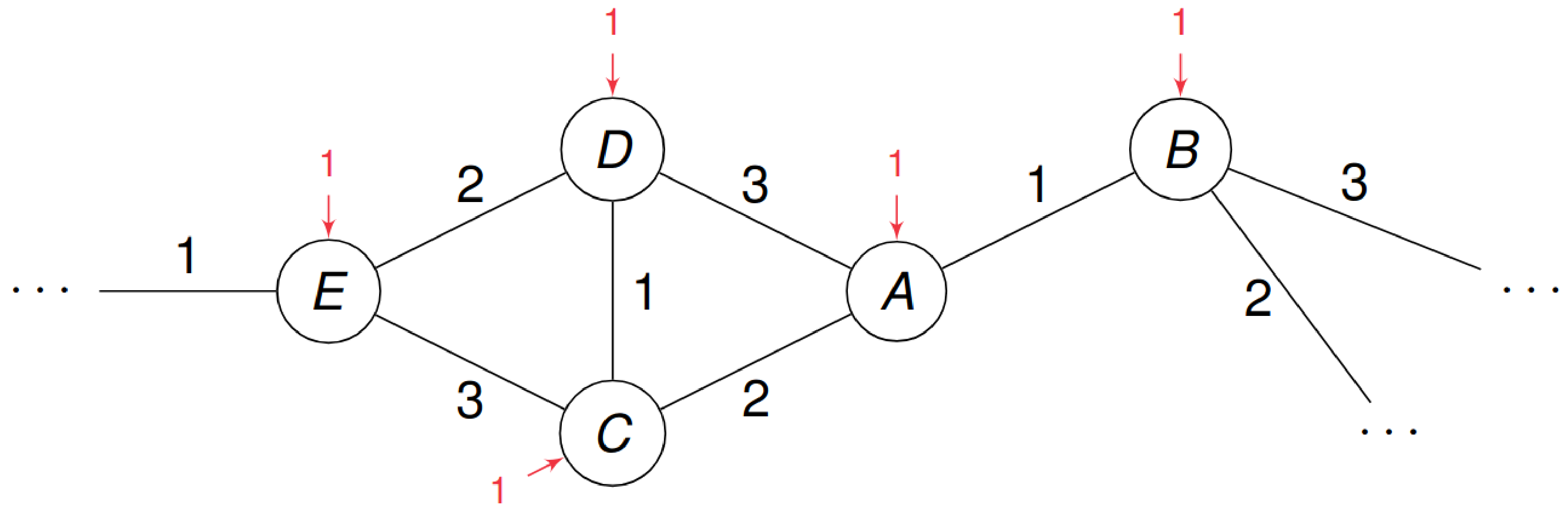

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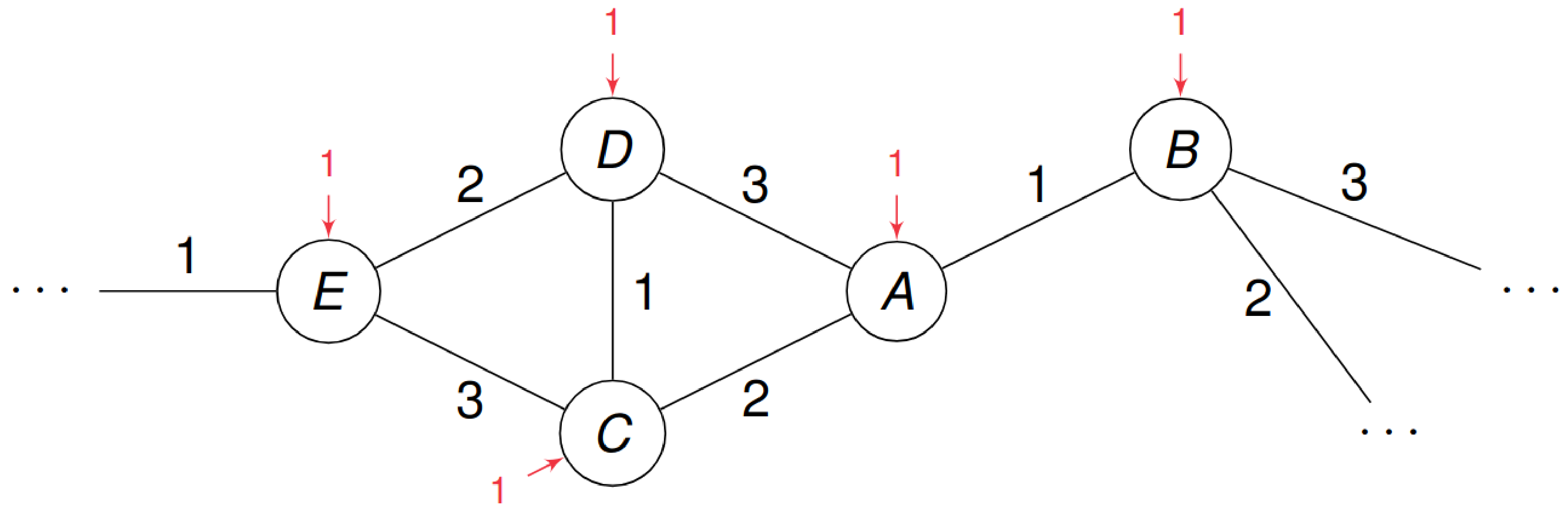

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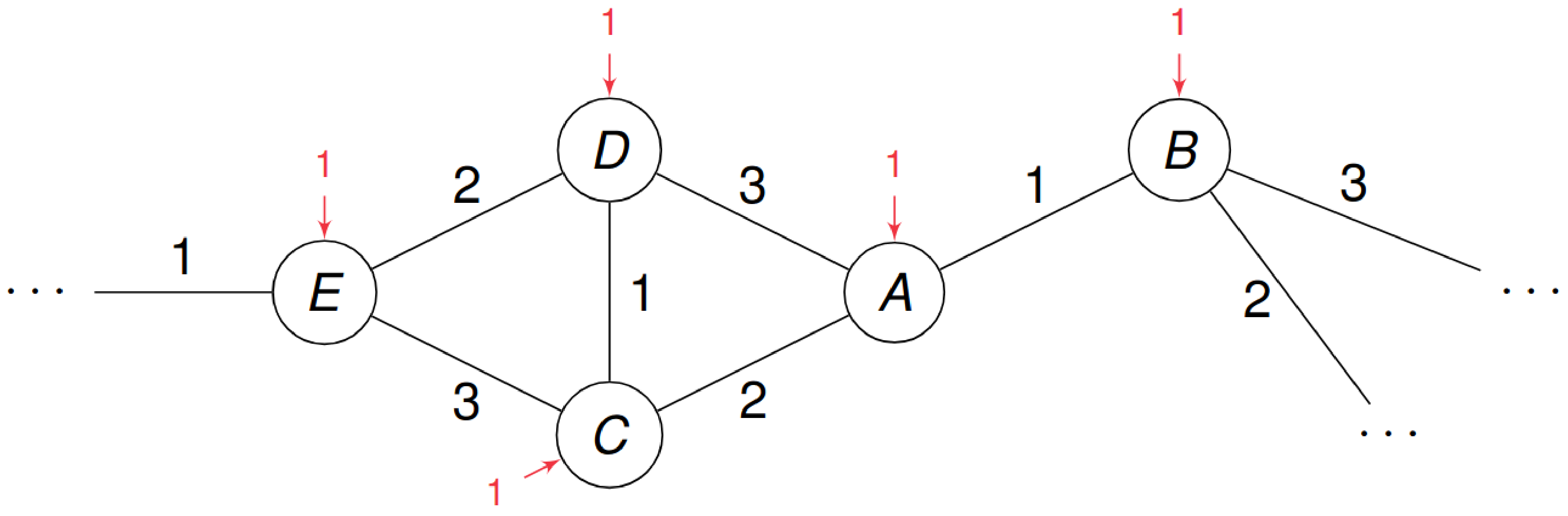

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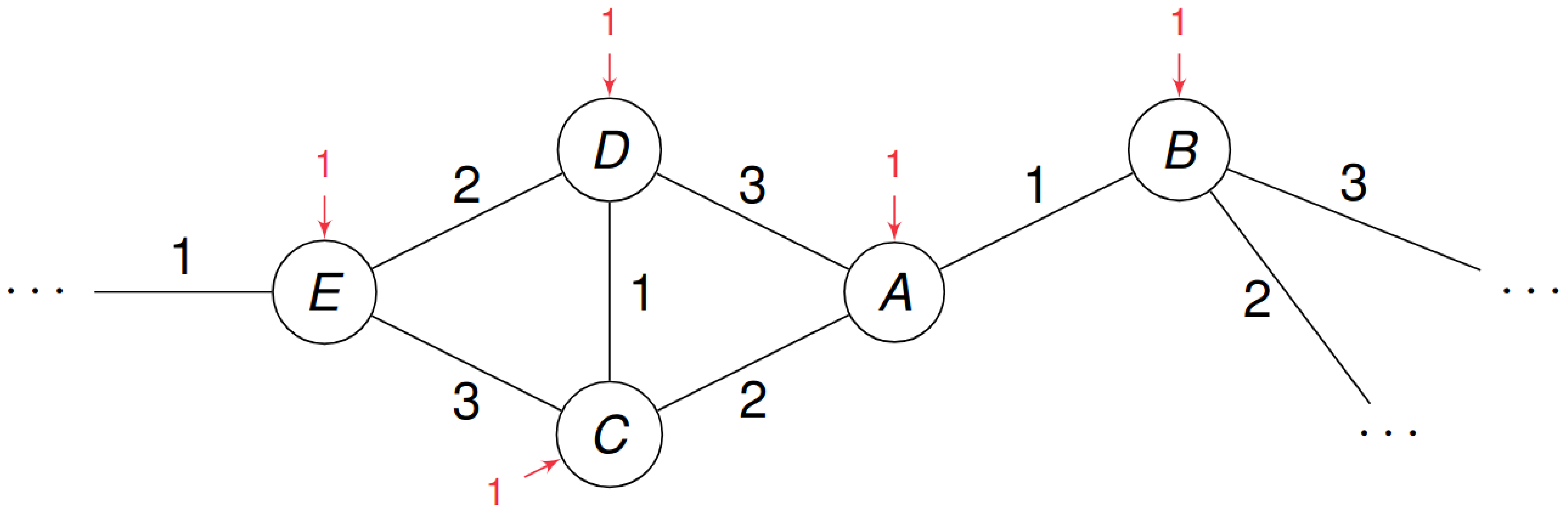

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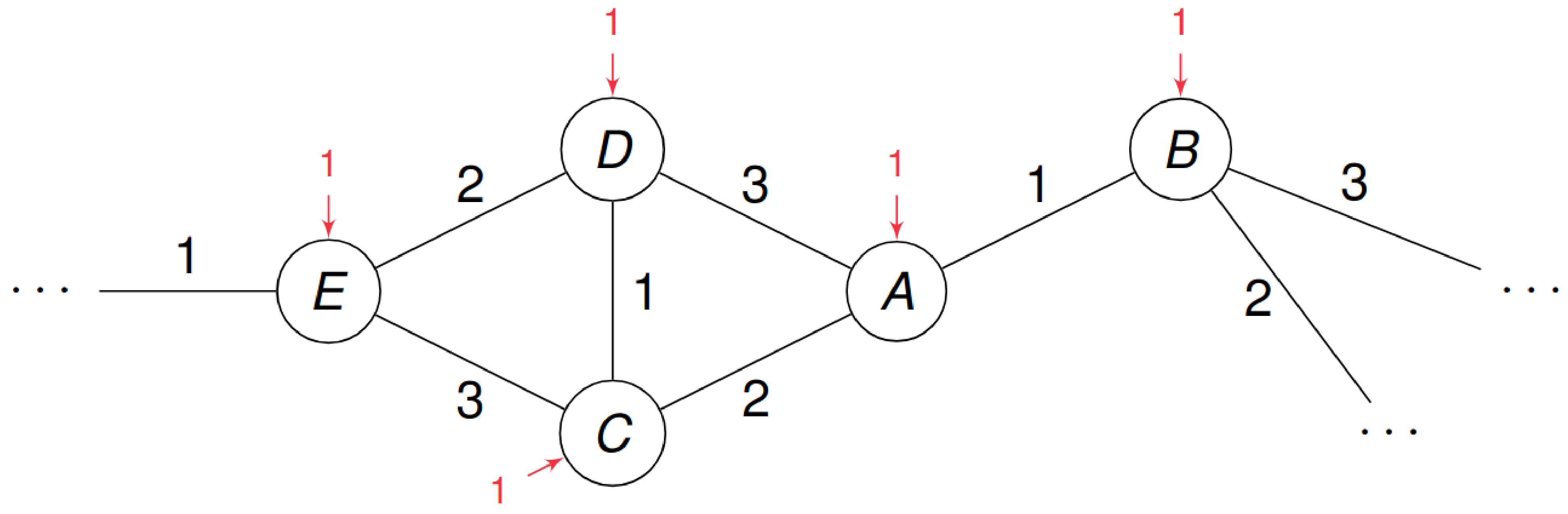

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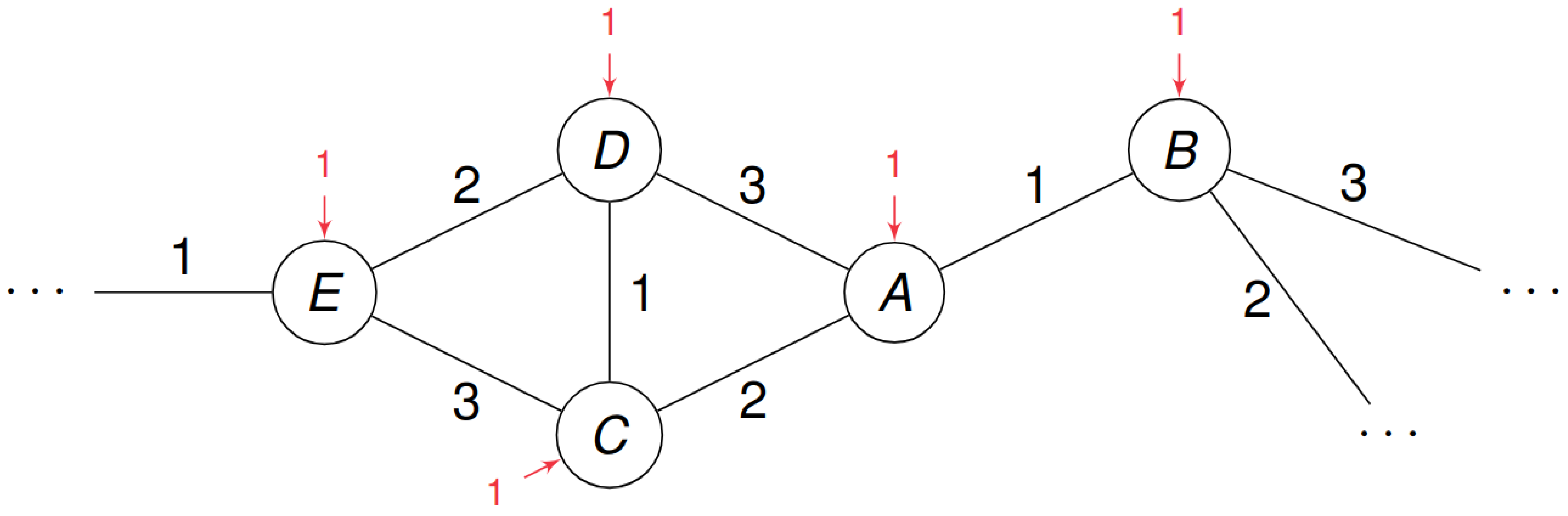

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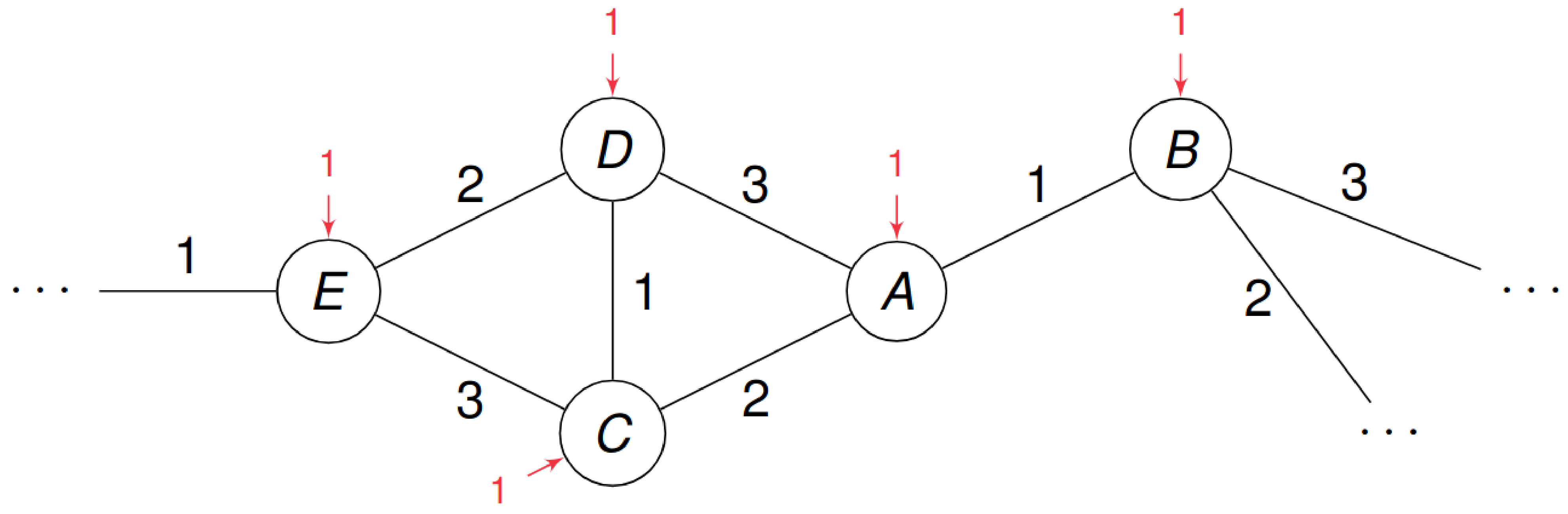

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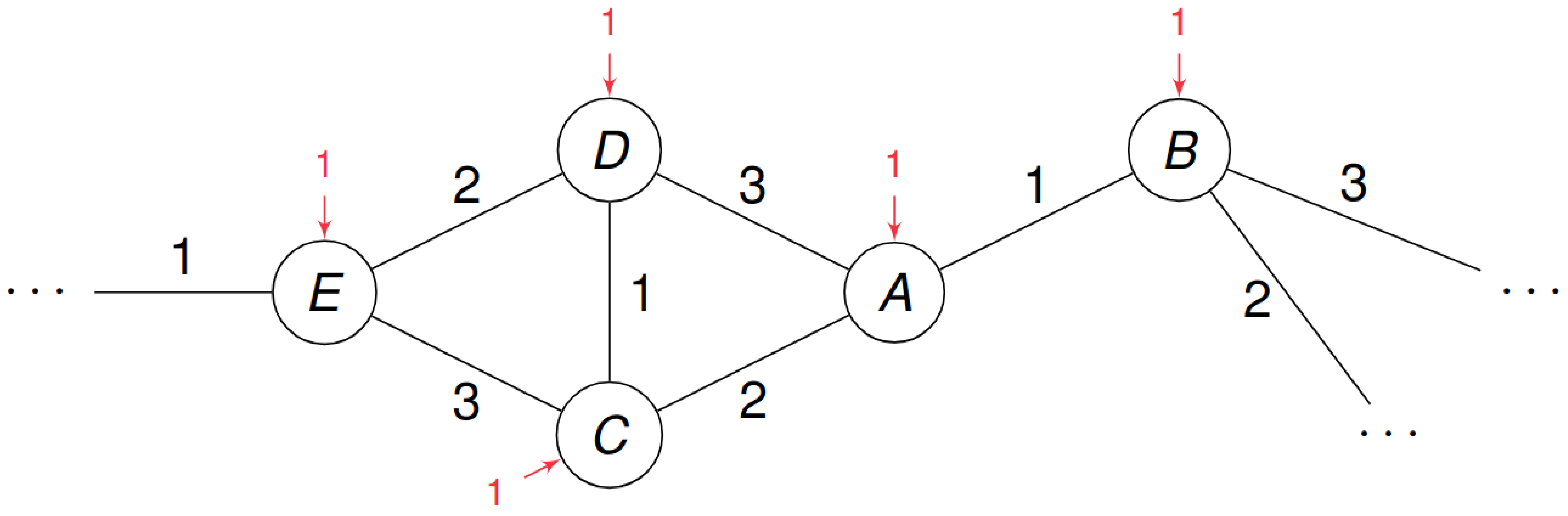

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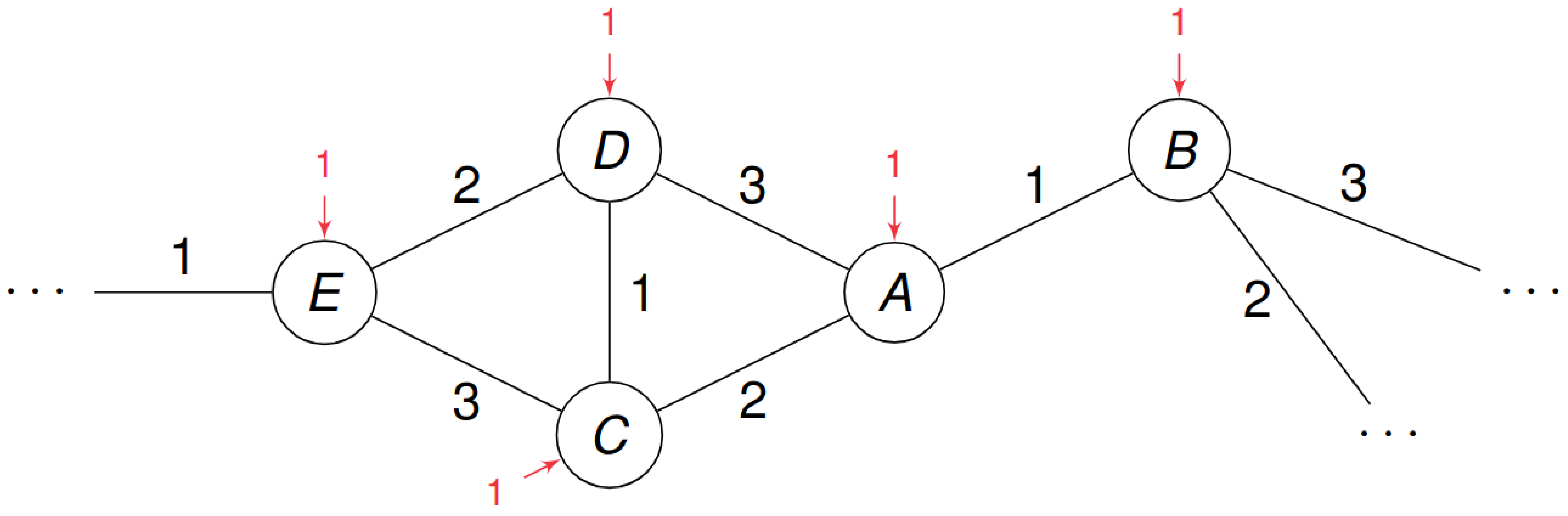

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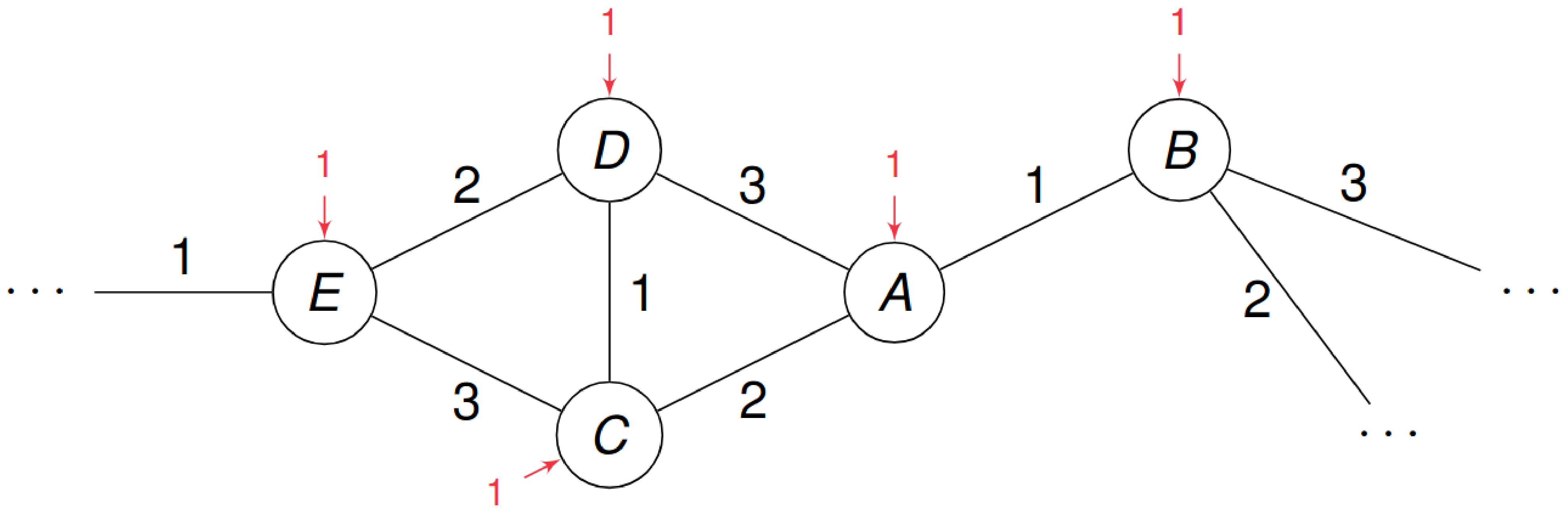

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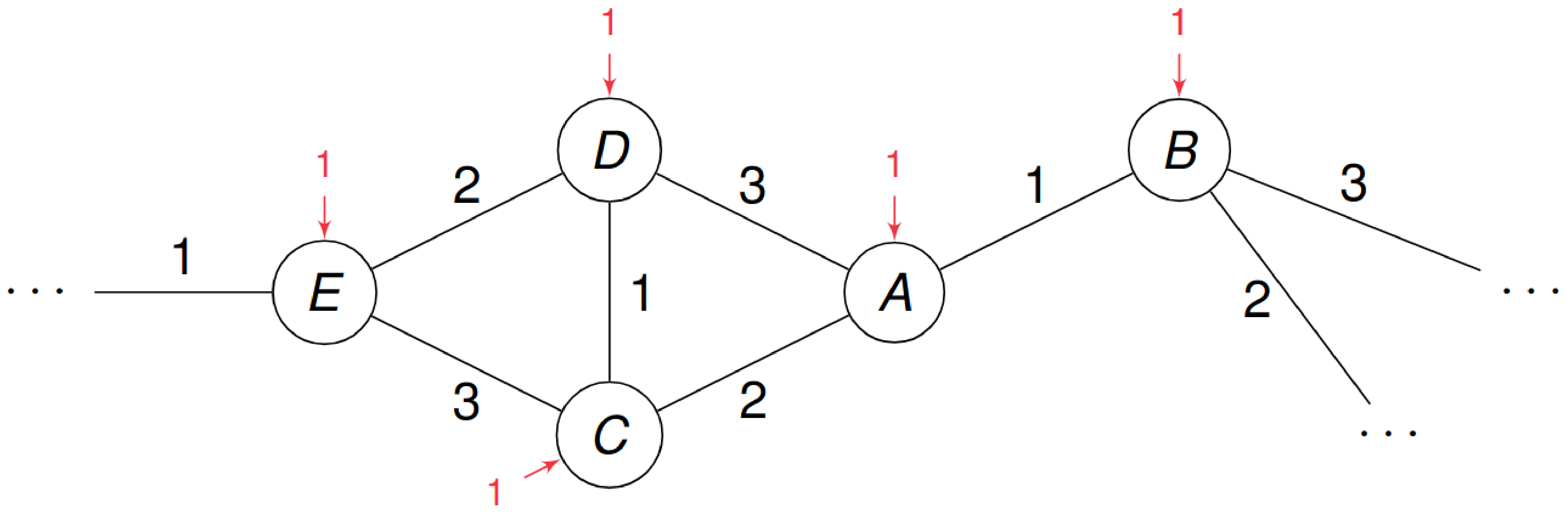

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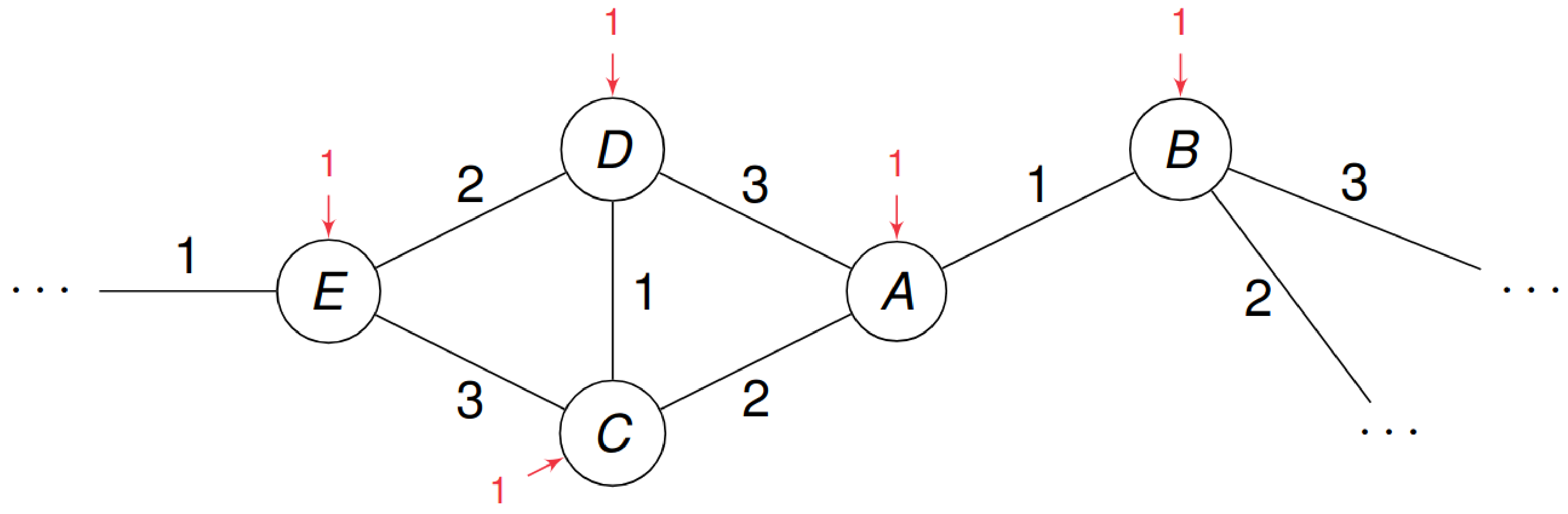

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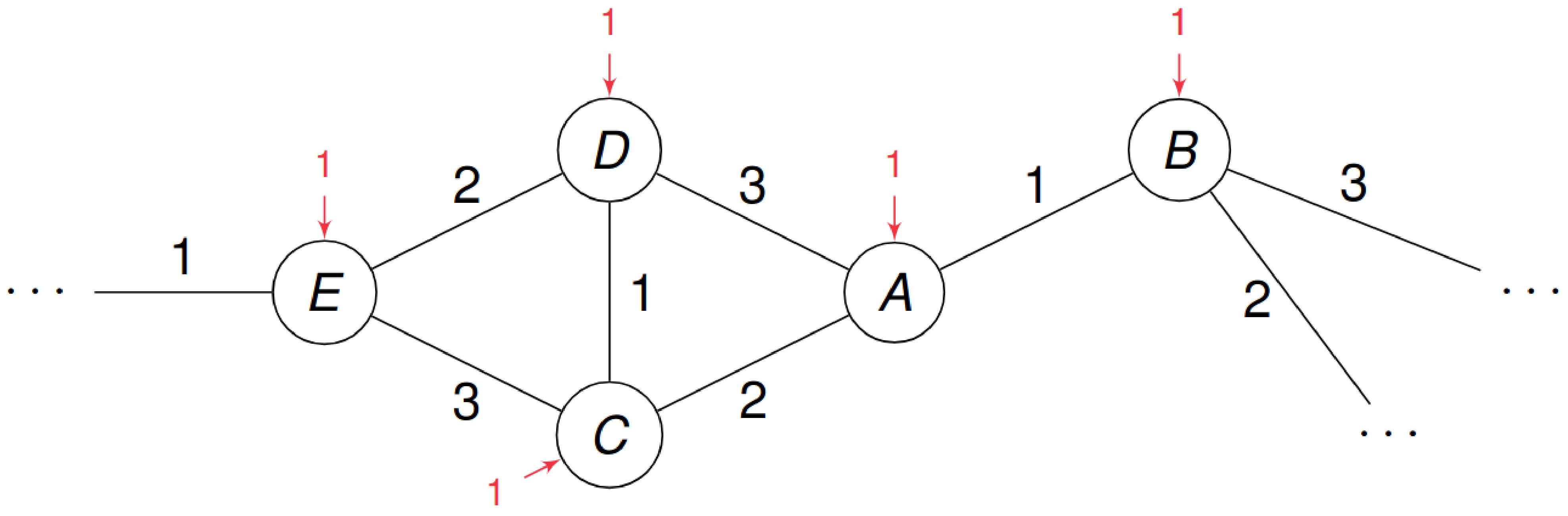

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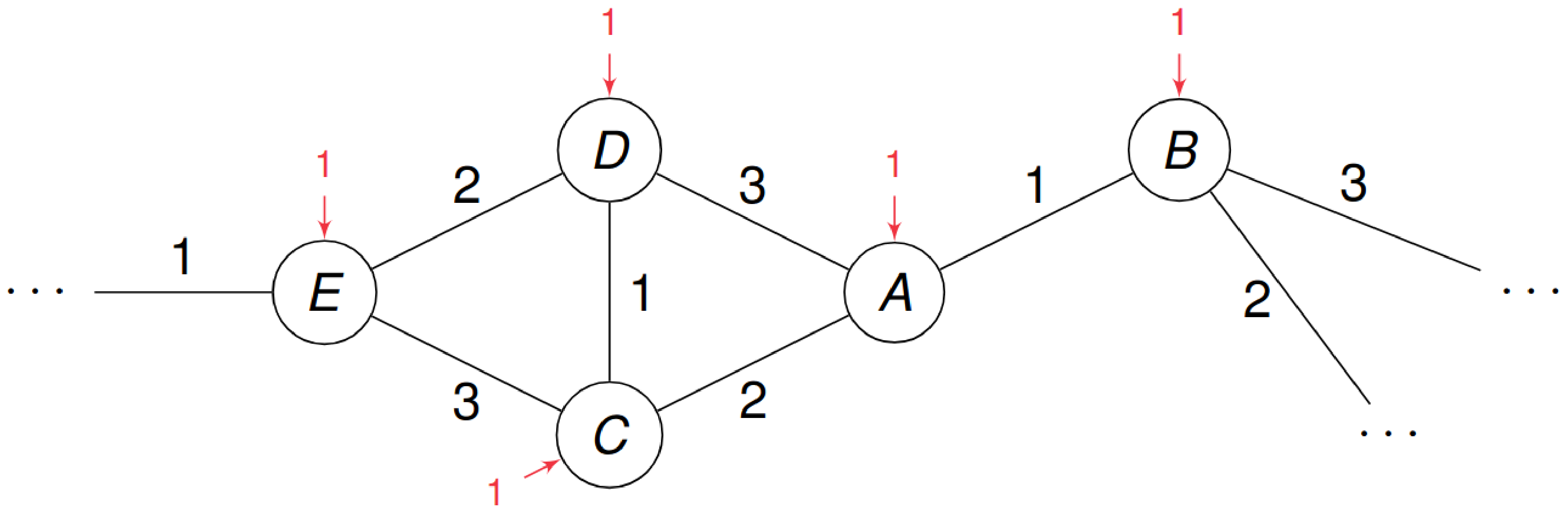

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  - we need a better game (win 100%)

GHZ game

- **Greenberger-Horne-Zeilinger game**

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Bob

Charlie

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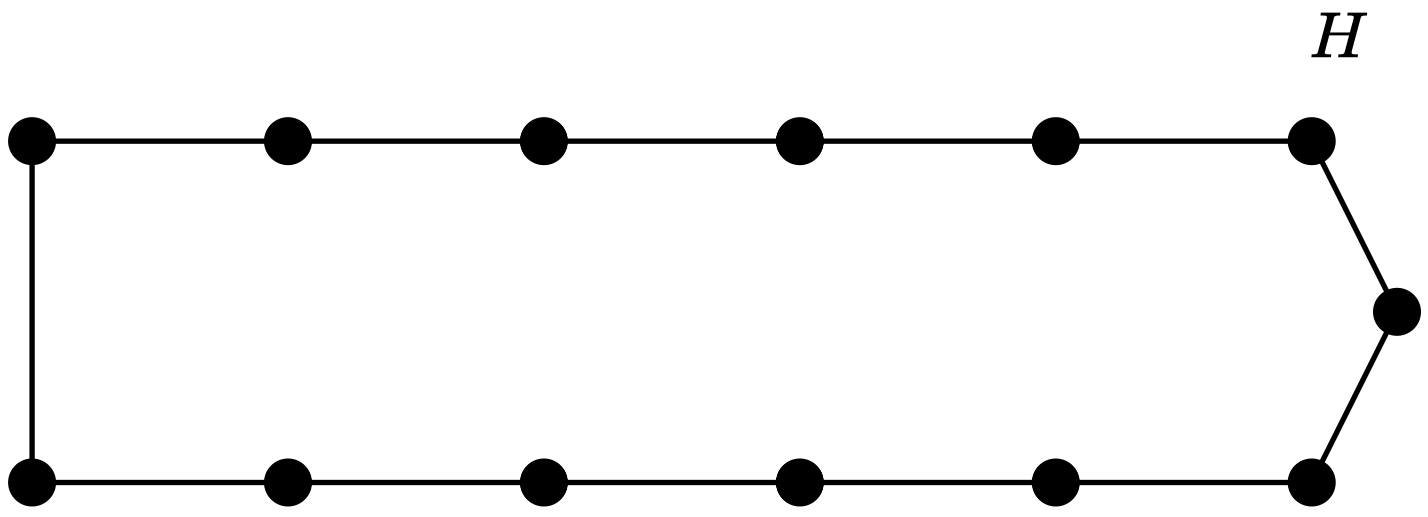
- Classical complexity $\Theta(\Delta)$, quantum complexity 1 round (just to share the quantum state)

Boosting failure probability: randomized-LOCAL

Problem: $\mathbf{2}$-coloring $\mathbf{2}$-chromatic graphs

- randomized LOCAL



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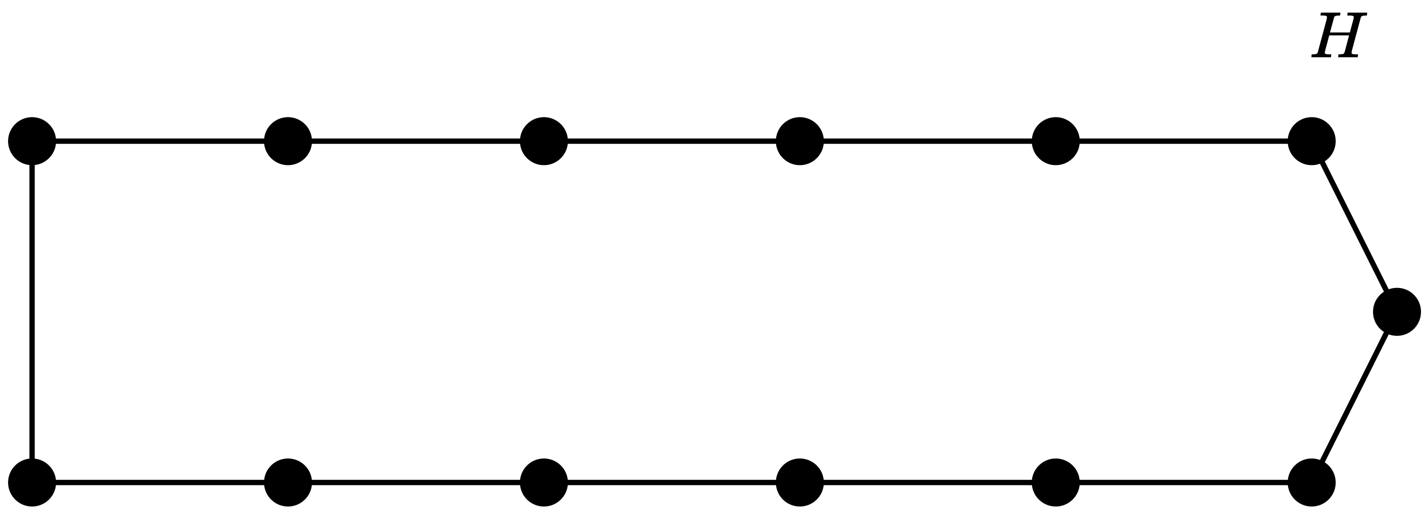

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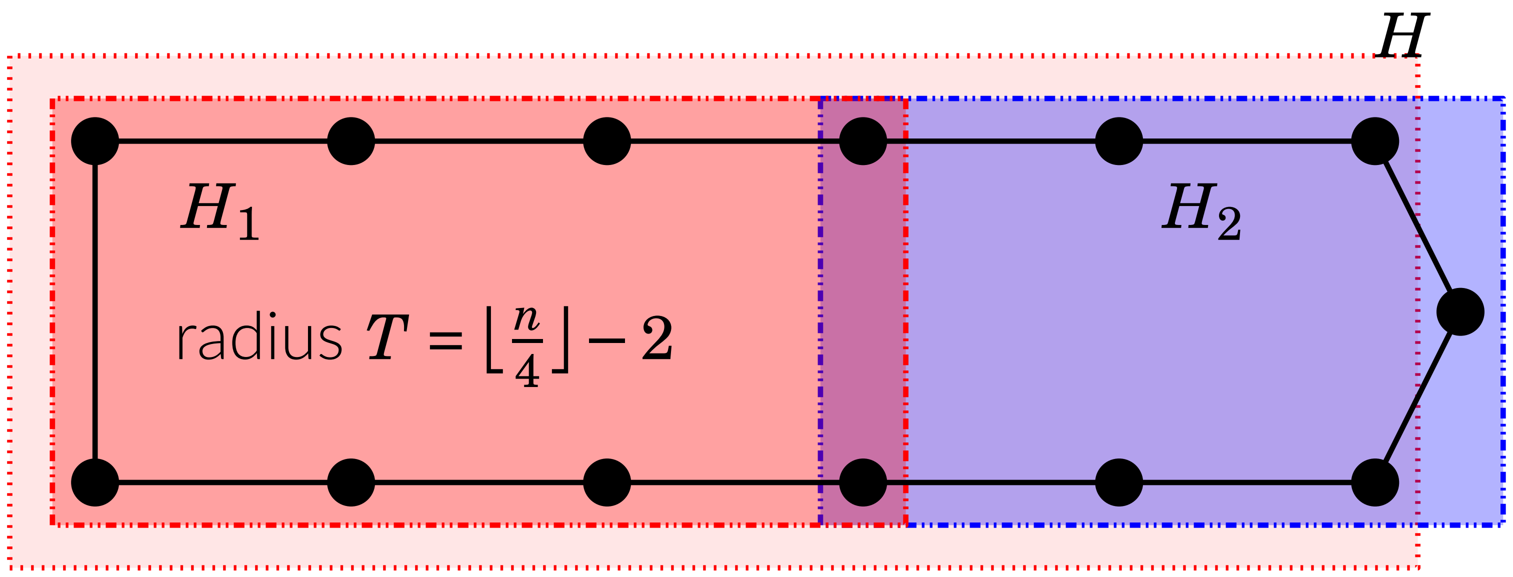

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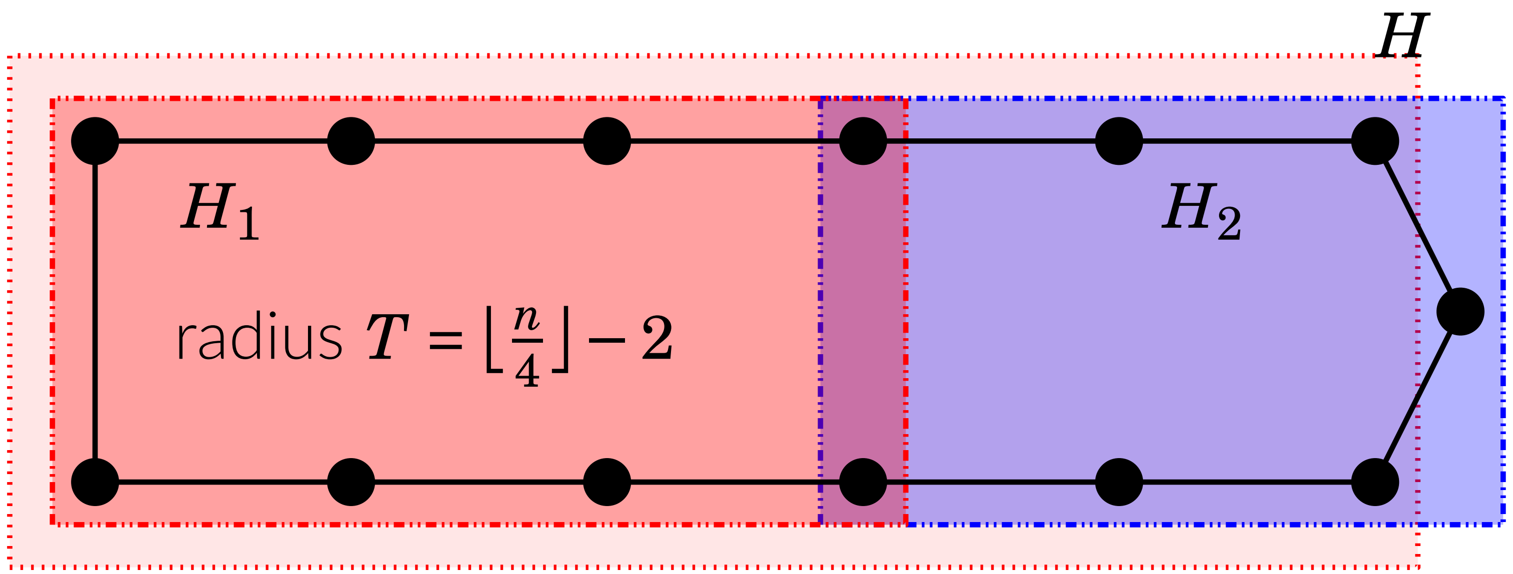

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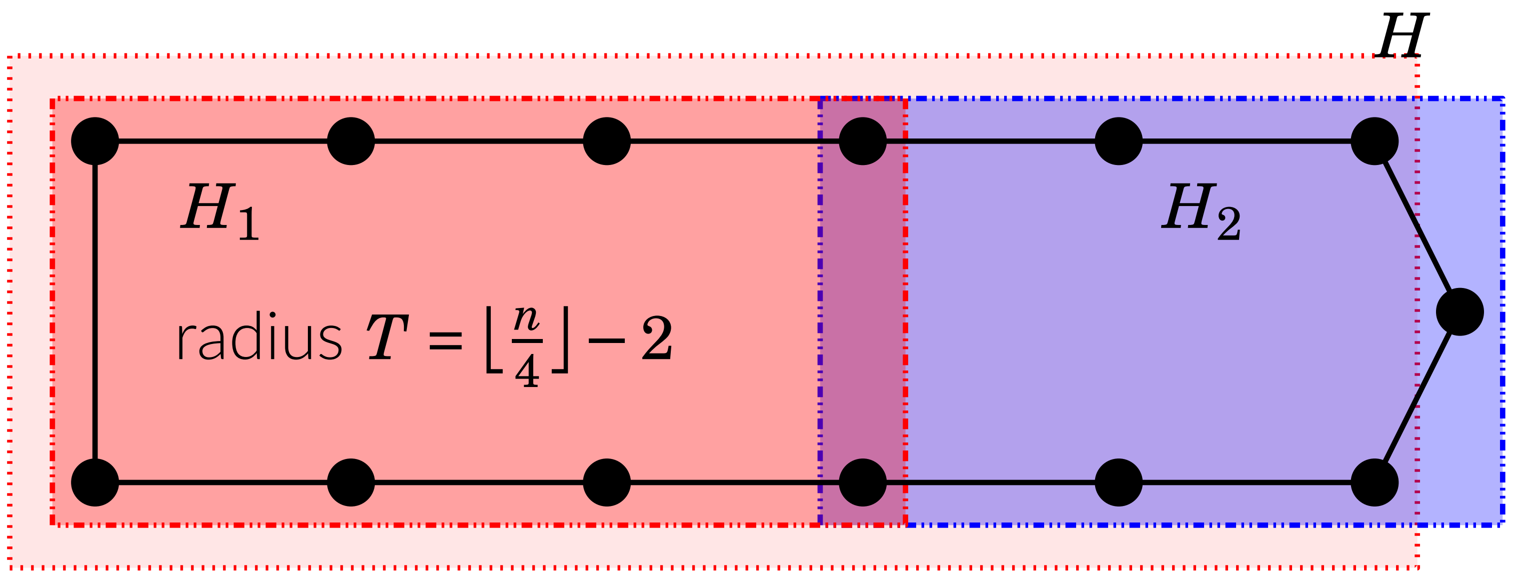

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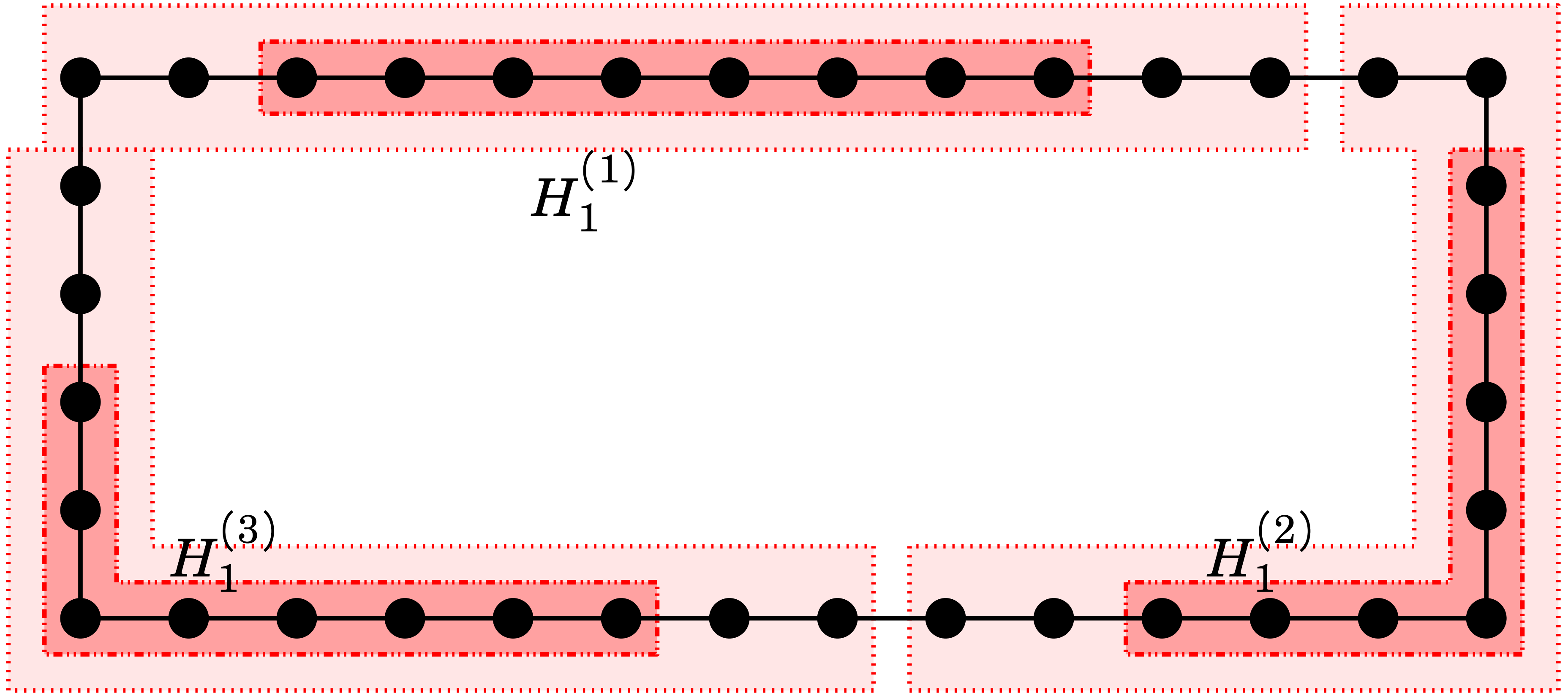

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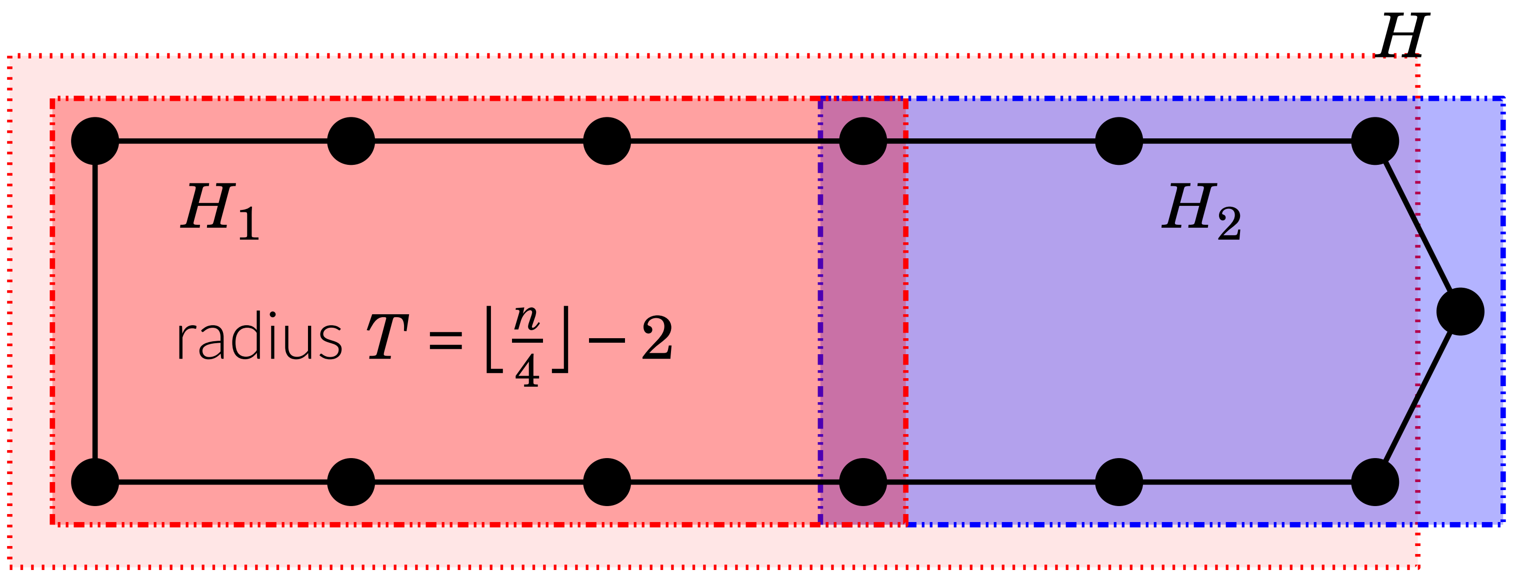

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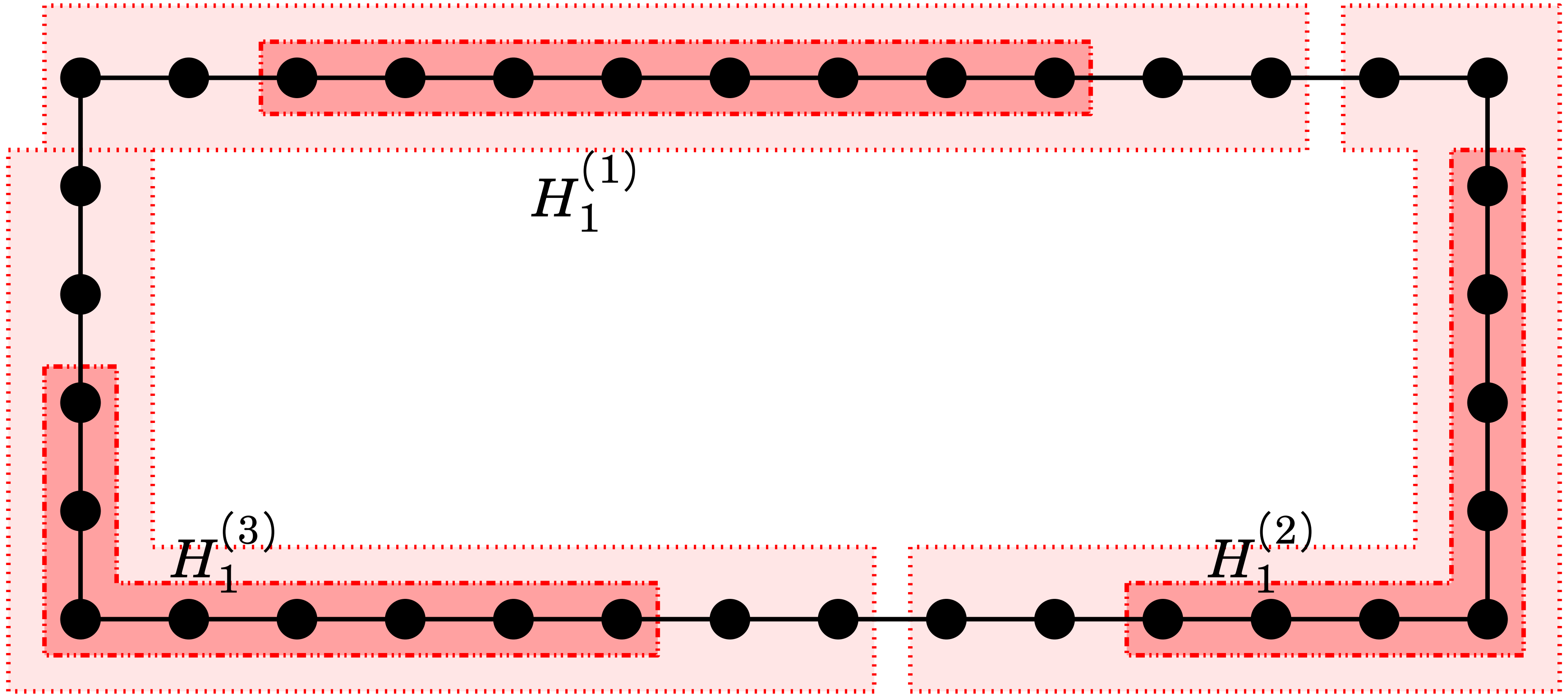

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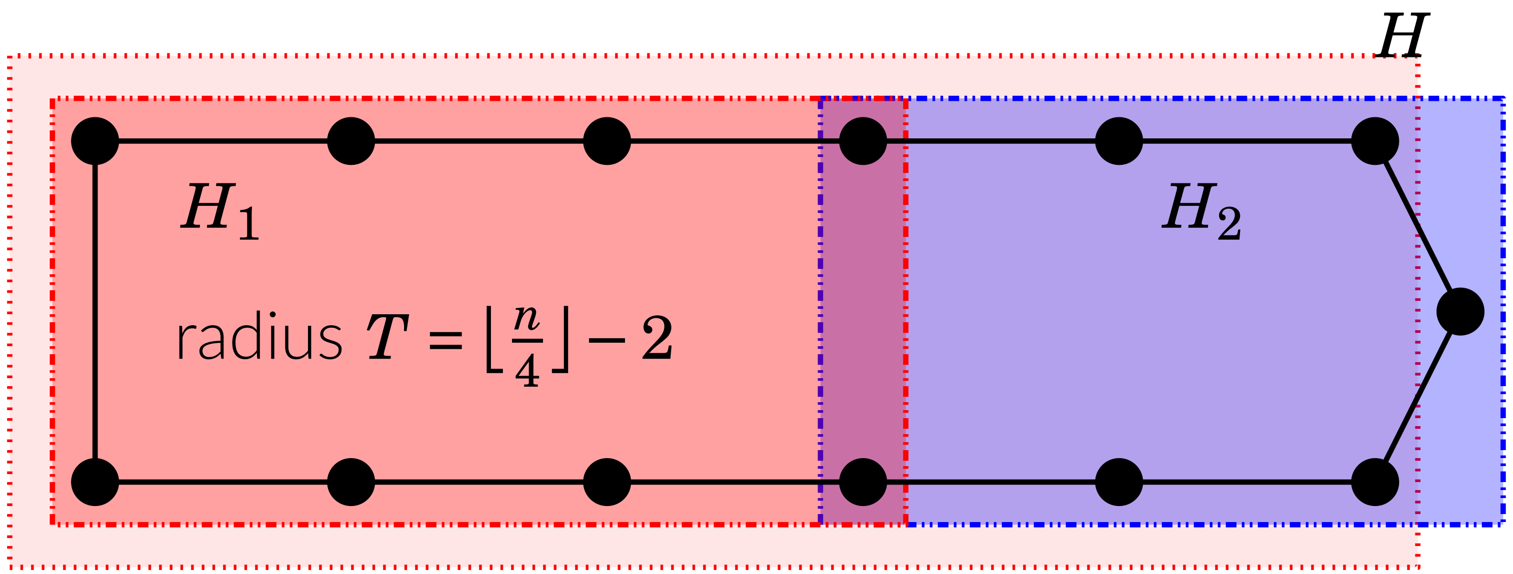

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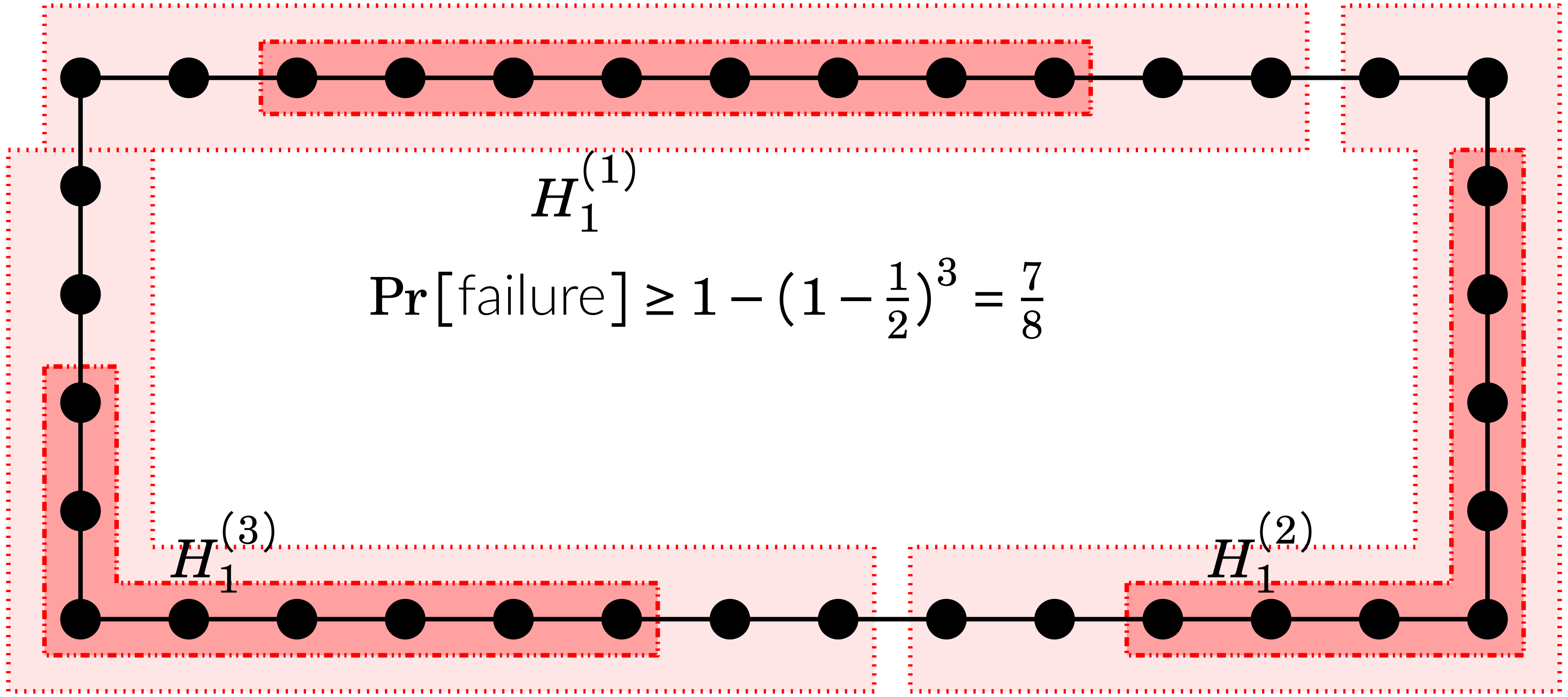

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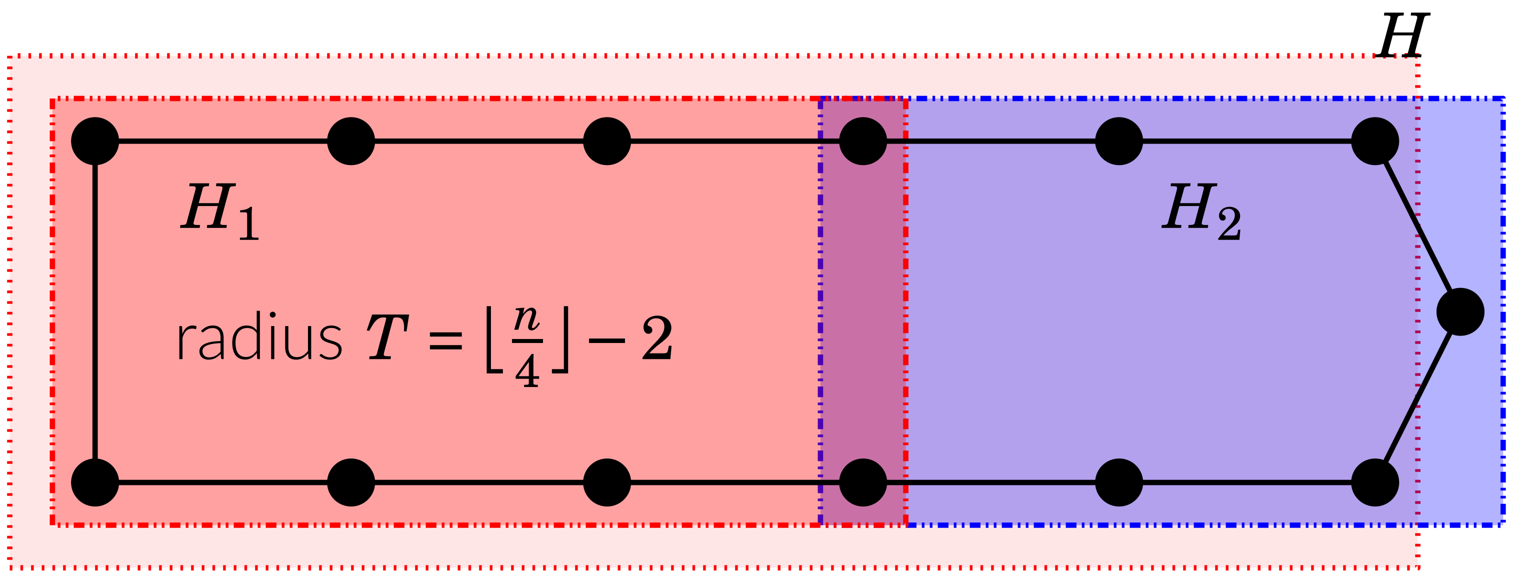

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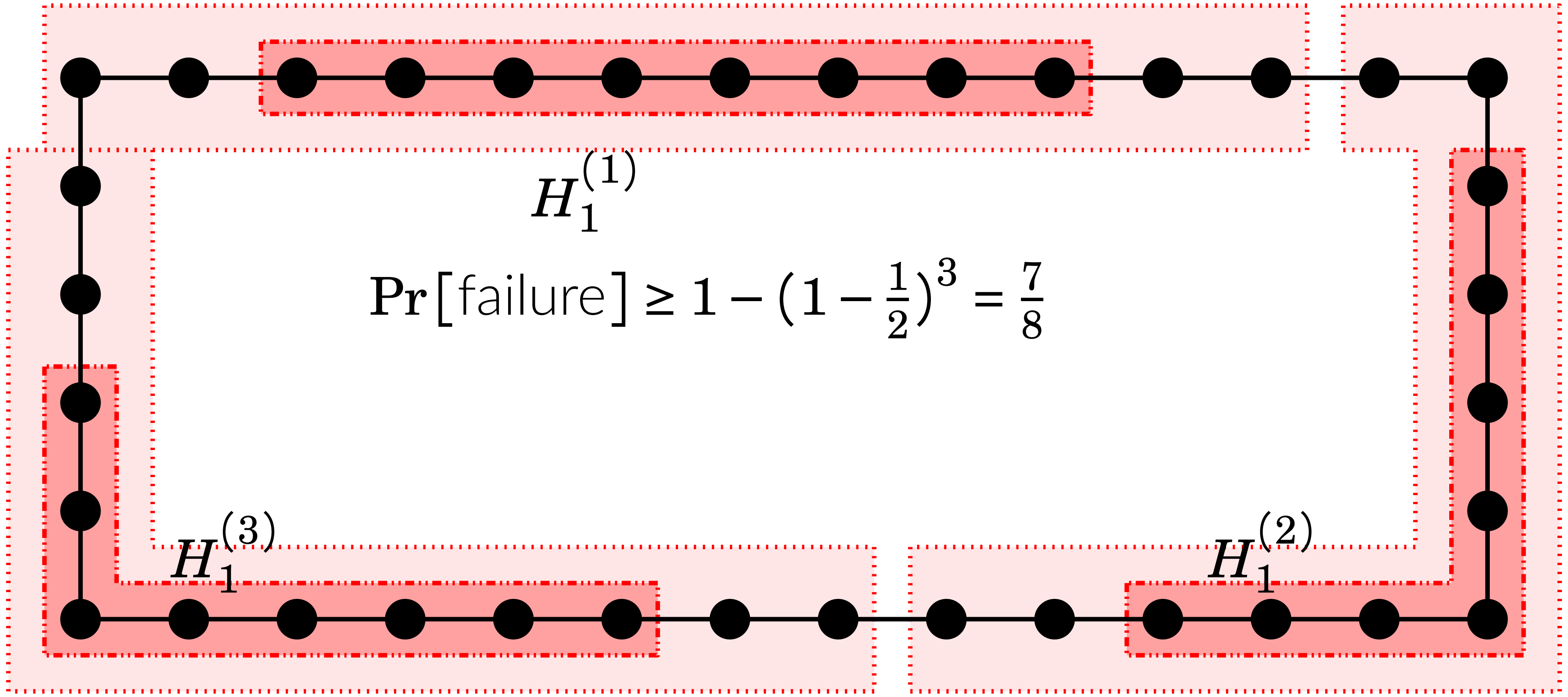

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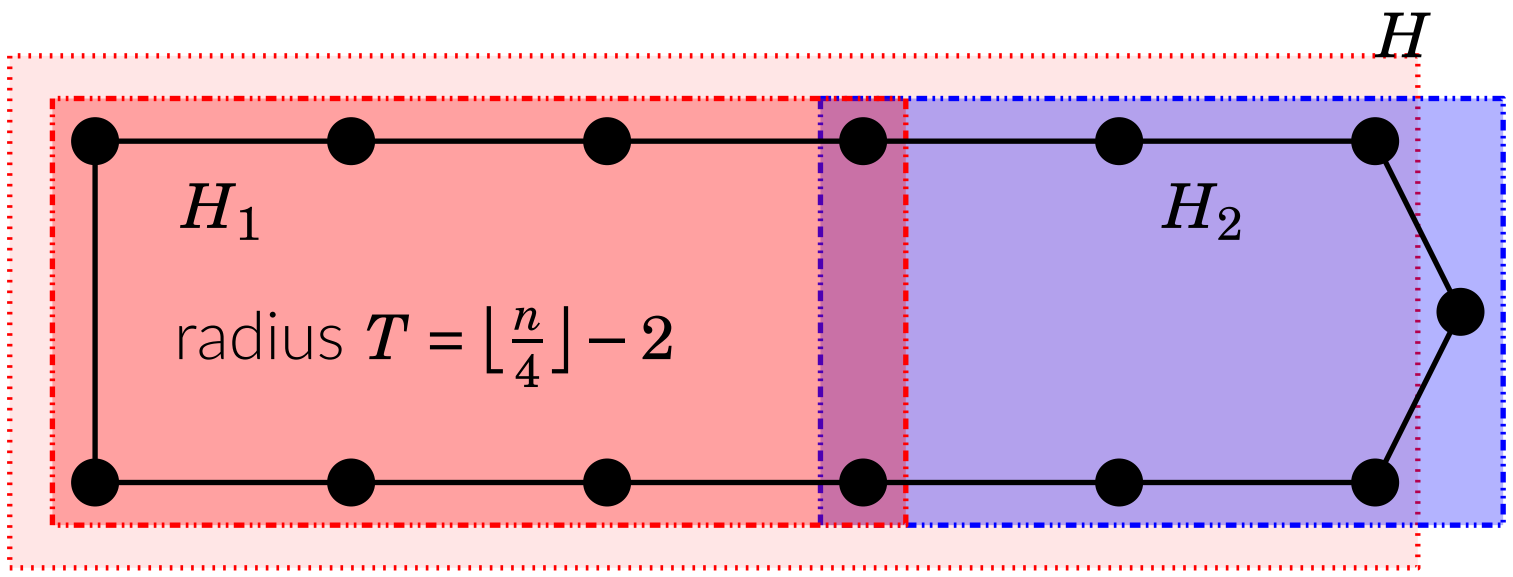

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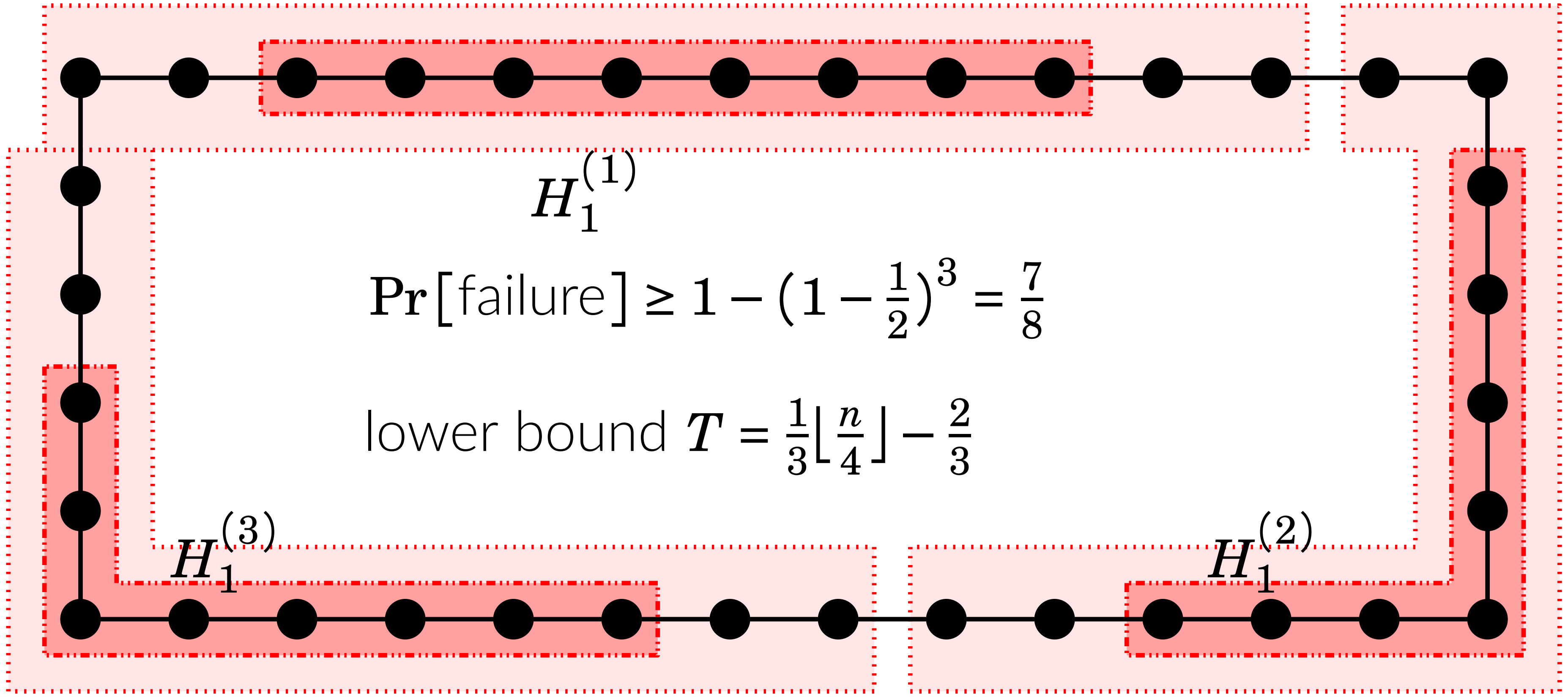

Problems in non-signaling: cloning and dependency

The **non-signaling** model:

- produces non-signaling outcomes

- outcome: function assigning to inputs (G, x) a distribution over labelings $\{(y_i, p_i)\}_{i \in I}$
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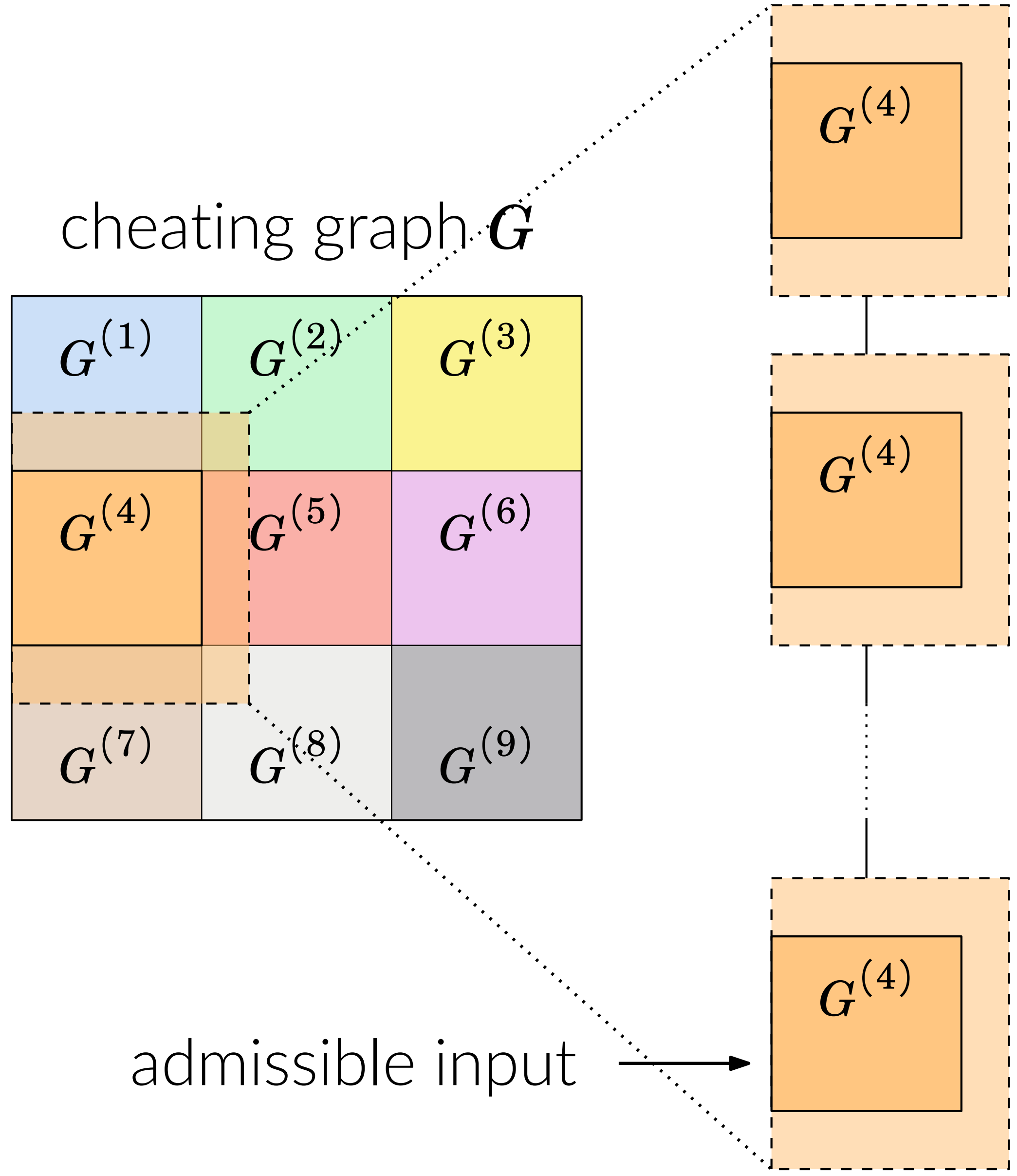

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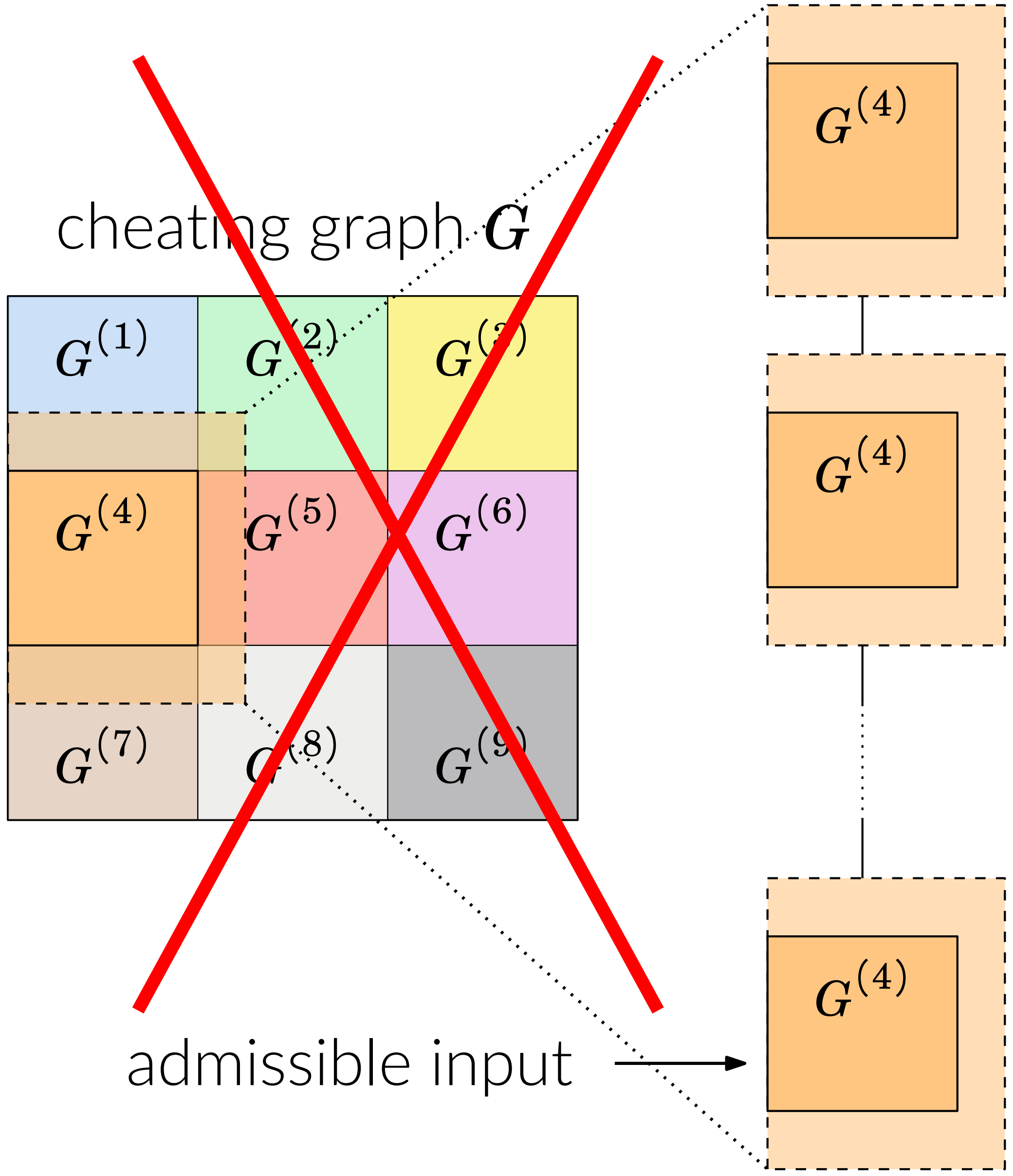

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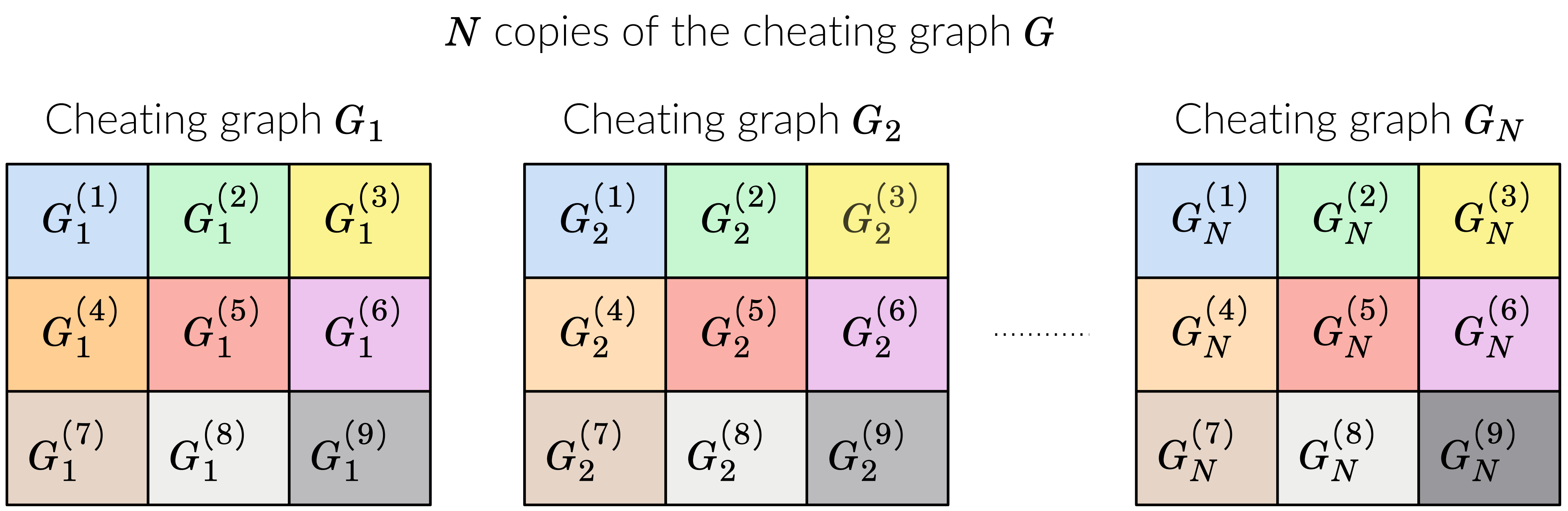

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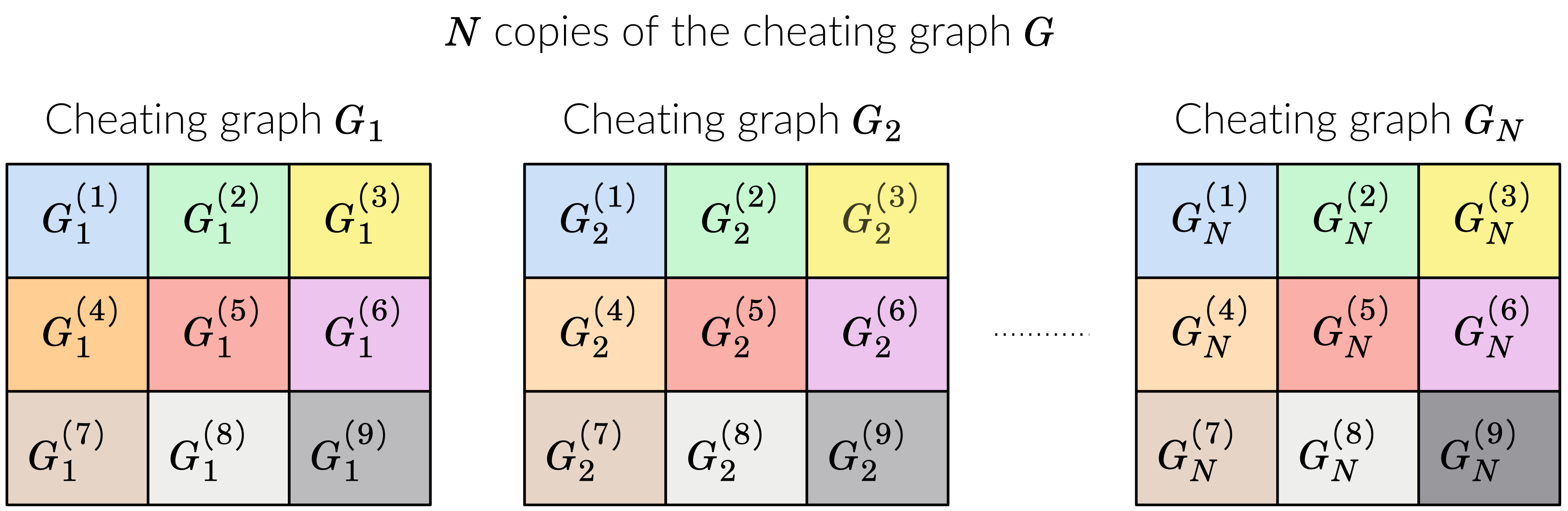

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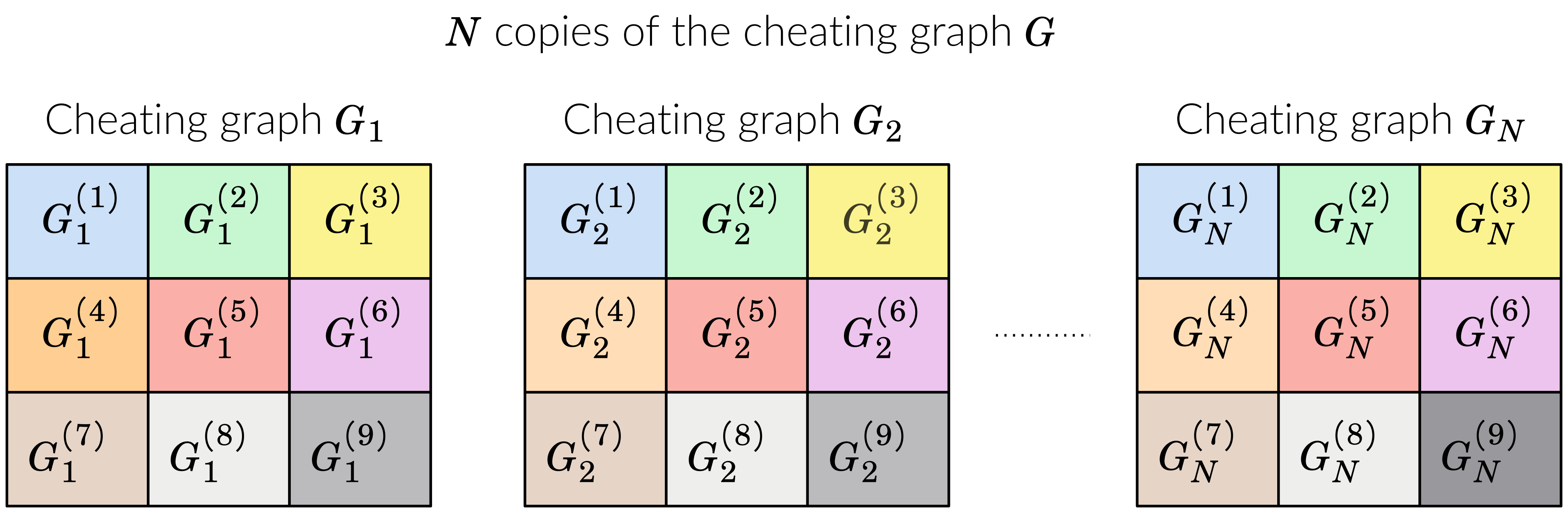

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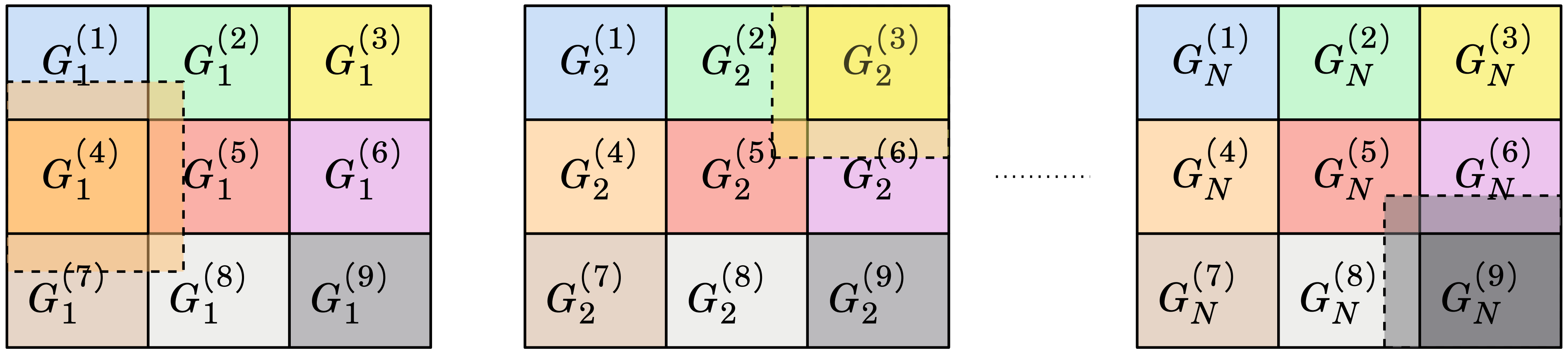
Cheating graph G_1 Cheating graph G_2 Cheating graph G_N



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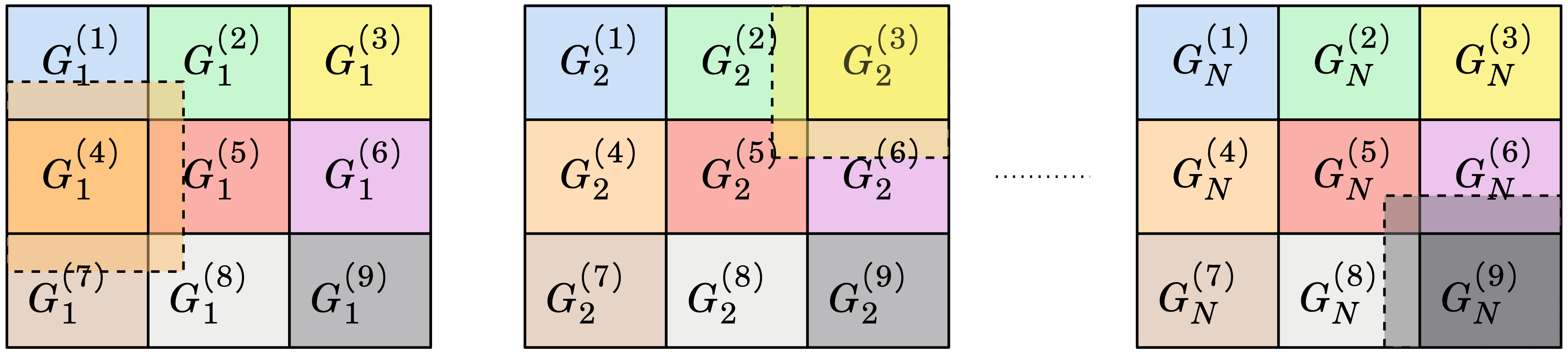

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