Phase Transition of a Non-Linear Opinion Dynamics with Noisy Interactions

Francesco d'Amore

Joint work with:

- Andrea Clementi: *Università di Roma "Tor Vergata"*
- Emanuele Natale: *Inria, Cnrs, I3S, Université Côte d'Azur*

SIROCCO 2020

The Consensus Problem

- $\Sigma = \{1, 2, \ldots, k\}$ set of opinions/colors

MAS $\left\{ \right.$

- system of n agents/nodes initially colored with colors in Σ
- communication network for interactions
- agents may change color upon message exchange

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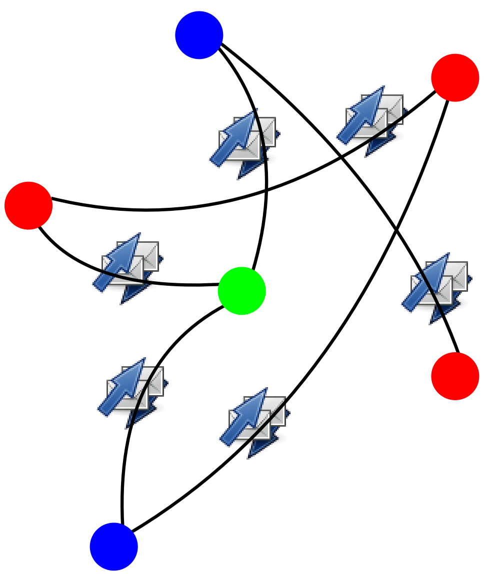

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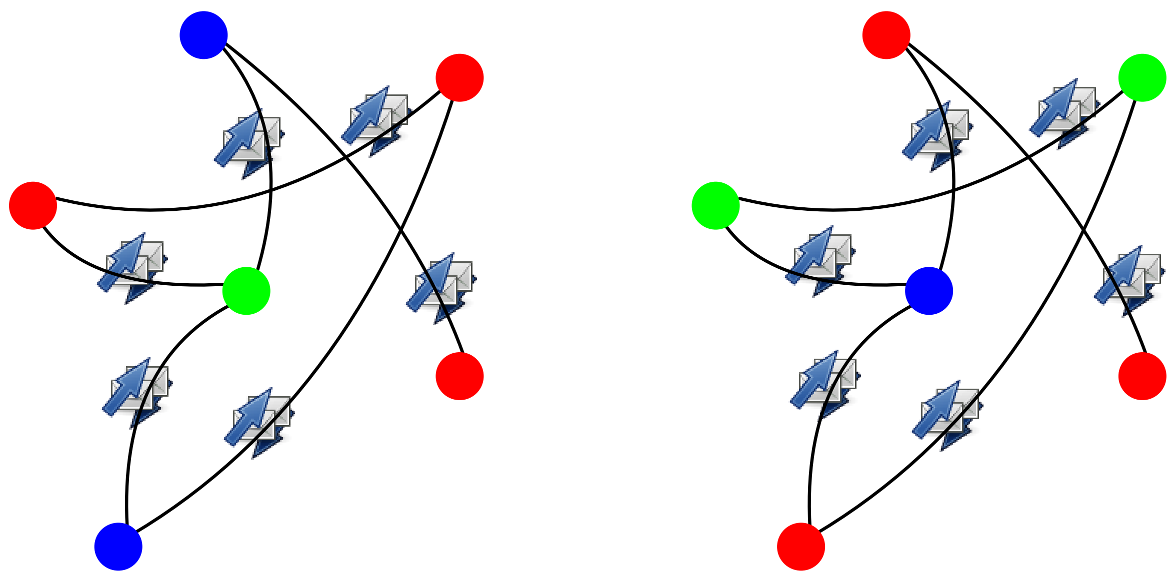

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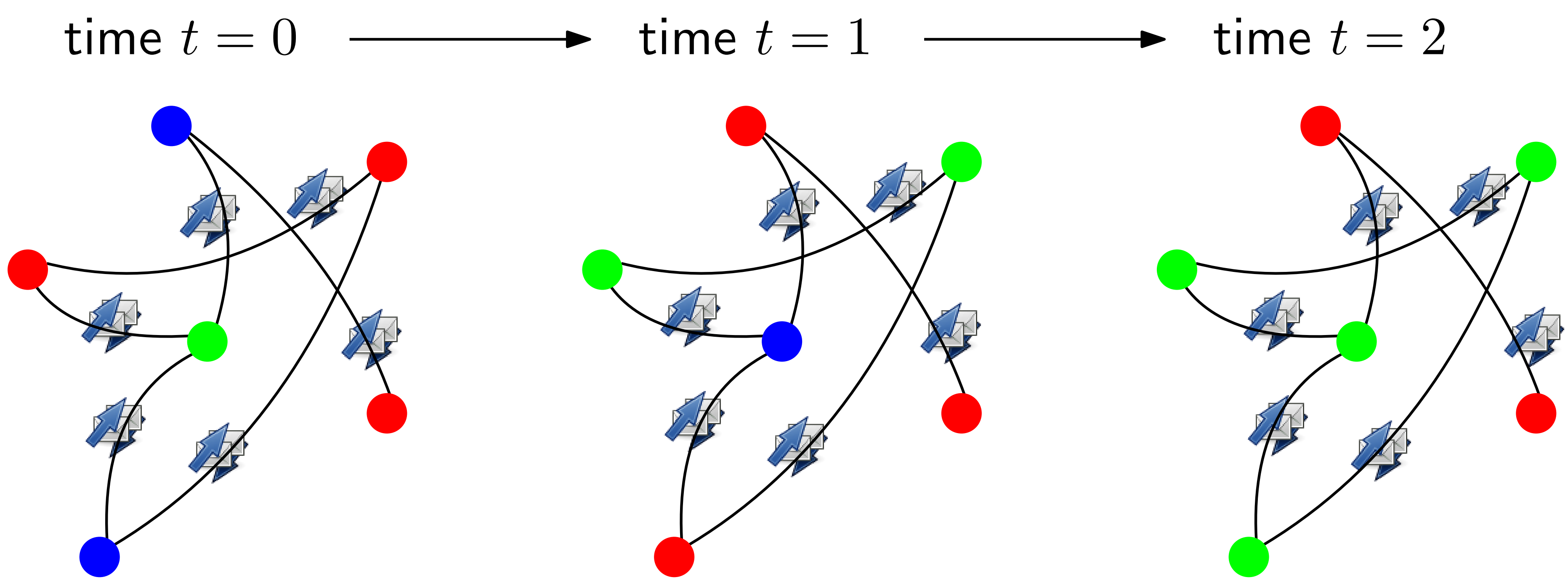

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Task: designing a protocol which brings the system in finite time to a configuration such that

1. all agents support the same color (*AGREEMENT*)
2. the final color is among the initial ones (*VALIDITY*)
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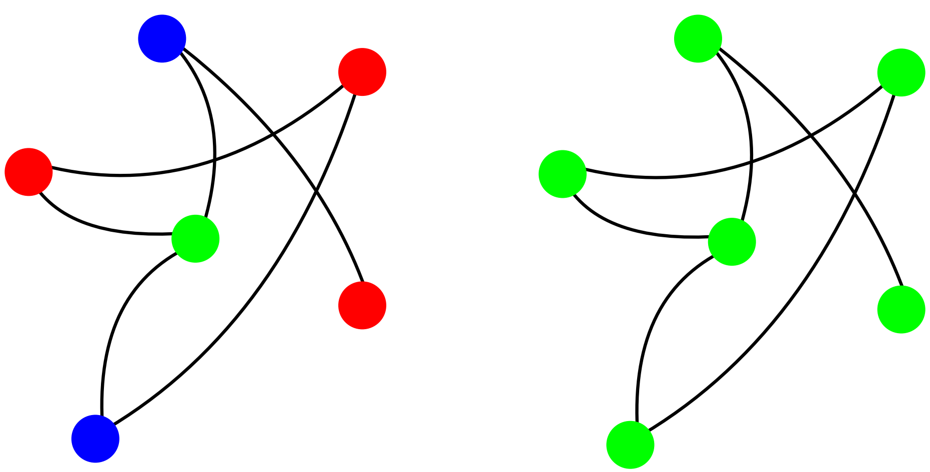

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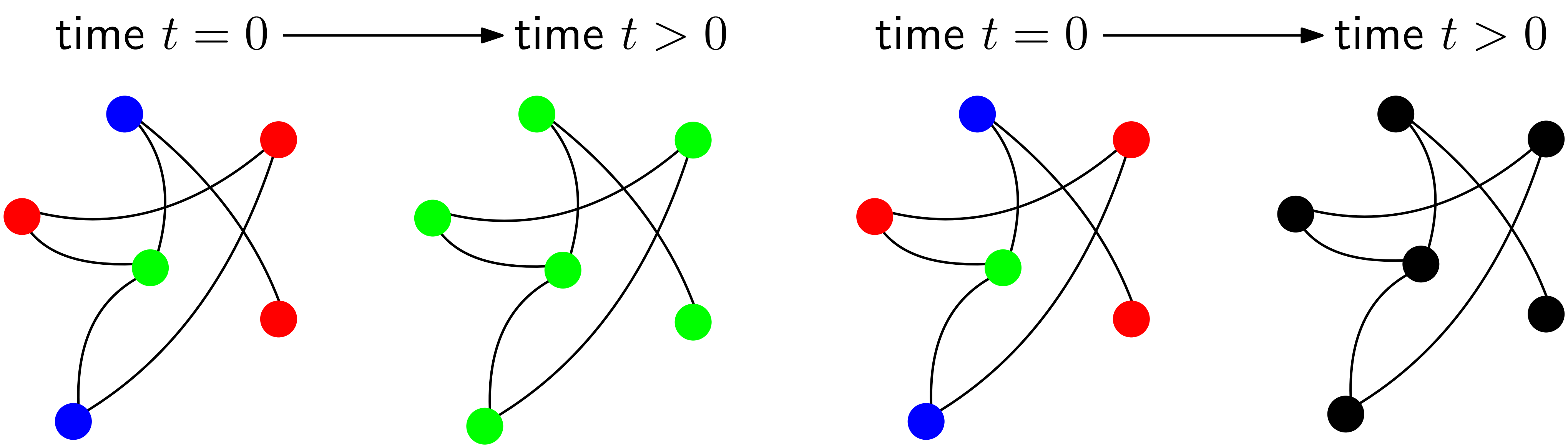

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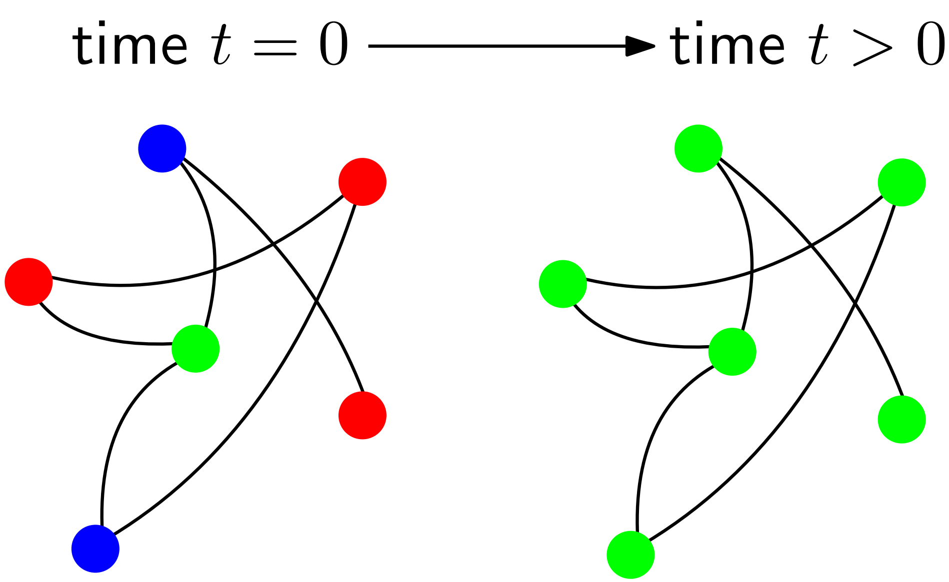

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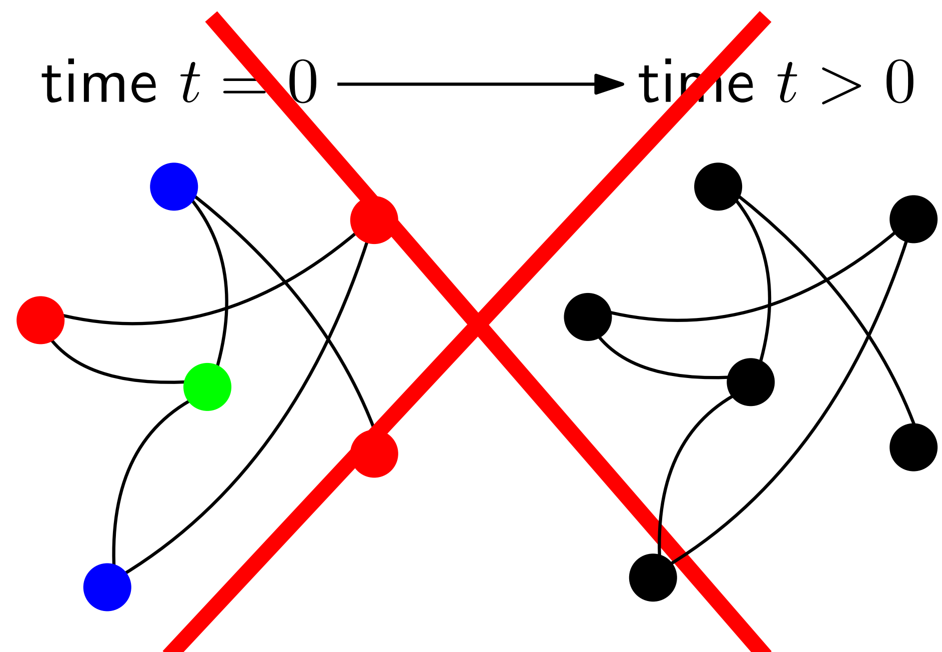

non-valid consensus

The Majority Consensus Problem

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2. VALIDITY
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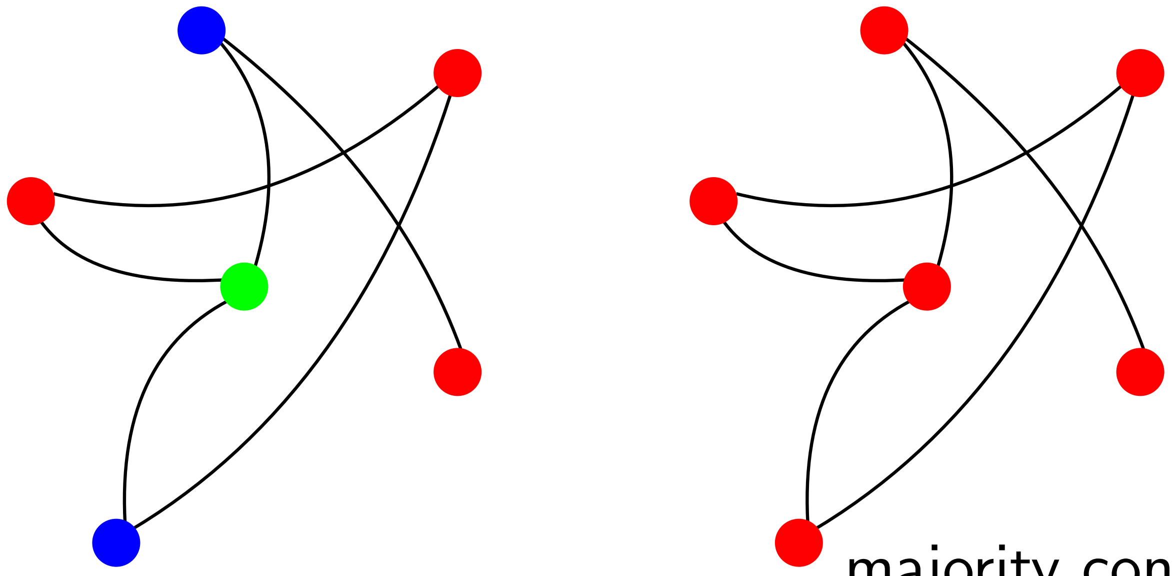

majority consensus

The (Majority) Consensus Problem

Lot of interest in many application domains:

- social networks [Mossel and Tamuz '17]
- biological systems [Feinerman et al. '17]
- sensor networks [Angluin et al. '08]
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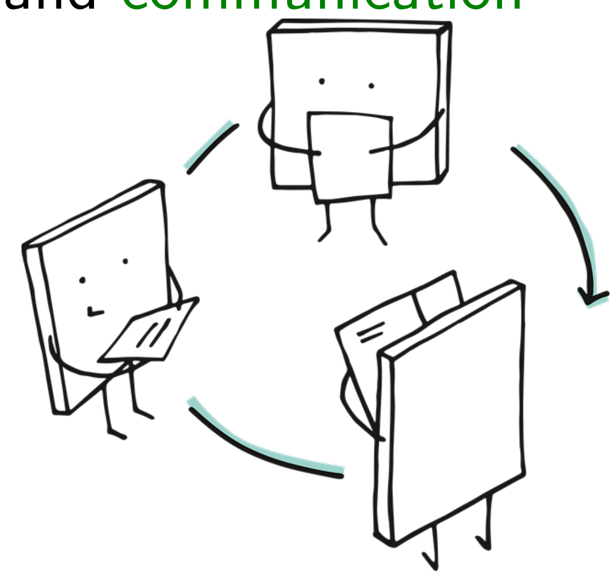
Investigation of opinion dynamics in chaotic systems

- mathematical models of how (decentralized) MAS reach consensus
- simple and lightweight: subject to memory and communication constraints
- efficient and resilient
- (majority) consensus is required w.h.p.

fail with prob. $< 1/n^{\Theta(1)}$



Some Literature

Largely studied opinion dynamycs:

- Voter Model [Hassin and Peleg '01]
- 3-Majority [Becchetti et al. '16]
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- linear dynamics
- slow convergence (polynomial even in K_n)
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3-Majority and 2-Choices:

- non-linear dynamics
- fast convergence (polylogarithmic even in sparse graphs with good expansion)
- guarantee majority consensus w.h.p.
- at least 2 bits of per-round communication complexity for each node

The Undecided-State Dynamics

- randomized, non-linear opinion dynamics for the (Majority) Consensus Problem [Angluin et al. '08], [Perron et al. '09]
- biologically inspired [Reina et al. '17], [Condon et al. '19]

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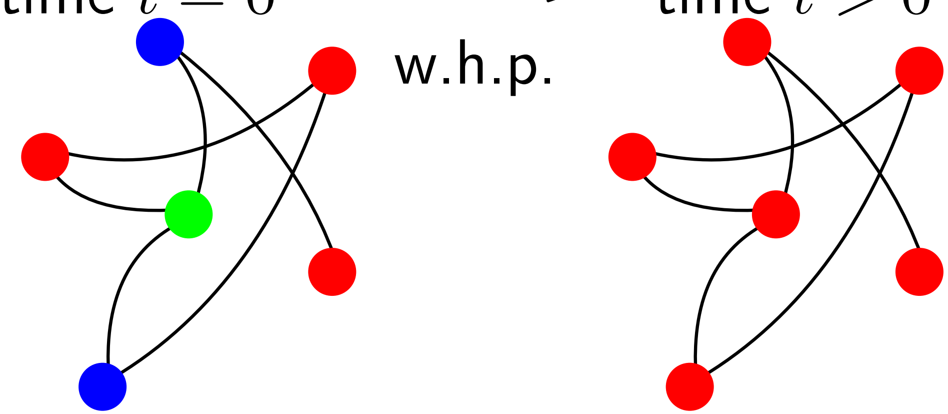
Definition (*w.h.p.*): an event E depending on a parameter $n \in \mathbb{N}$ holds *with high probability* w.r.t. n if

$$\mathbb{P}(E) \geq 1 - \frac{1}{n^{\Theta(1)}}$$

time $t = 0$ $\longrightarrow$ time $t > 0$

w.h.p.



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1. chooses a neighbor v u.a.r.
2. pulls v's color
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$u \backslash v$	color i	color j	undecided
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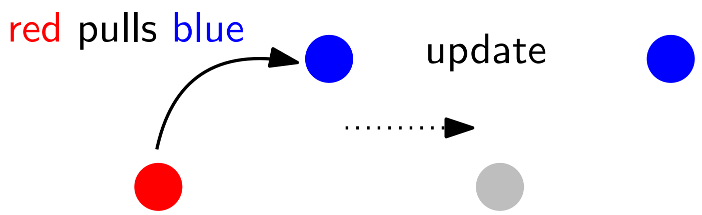

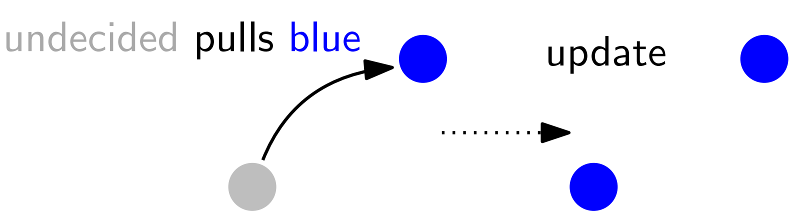

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Complete graph with six nodes, two colors

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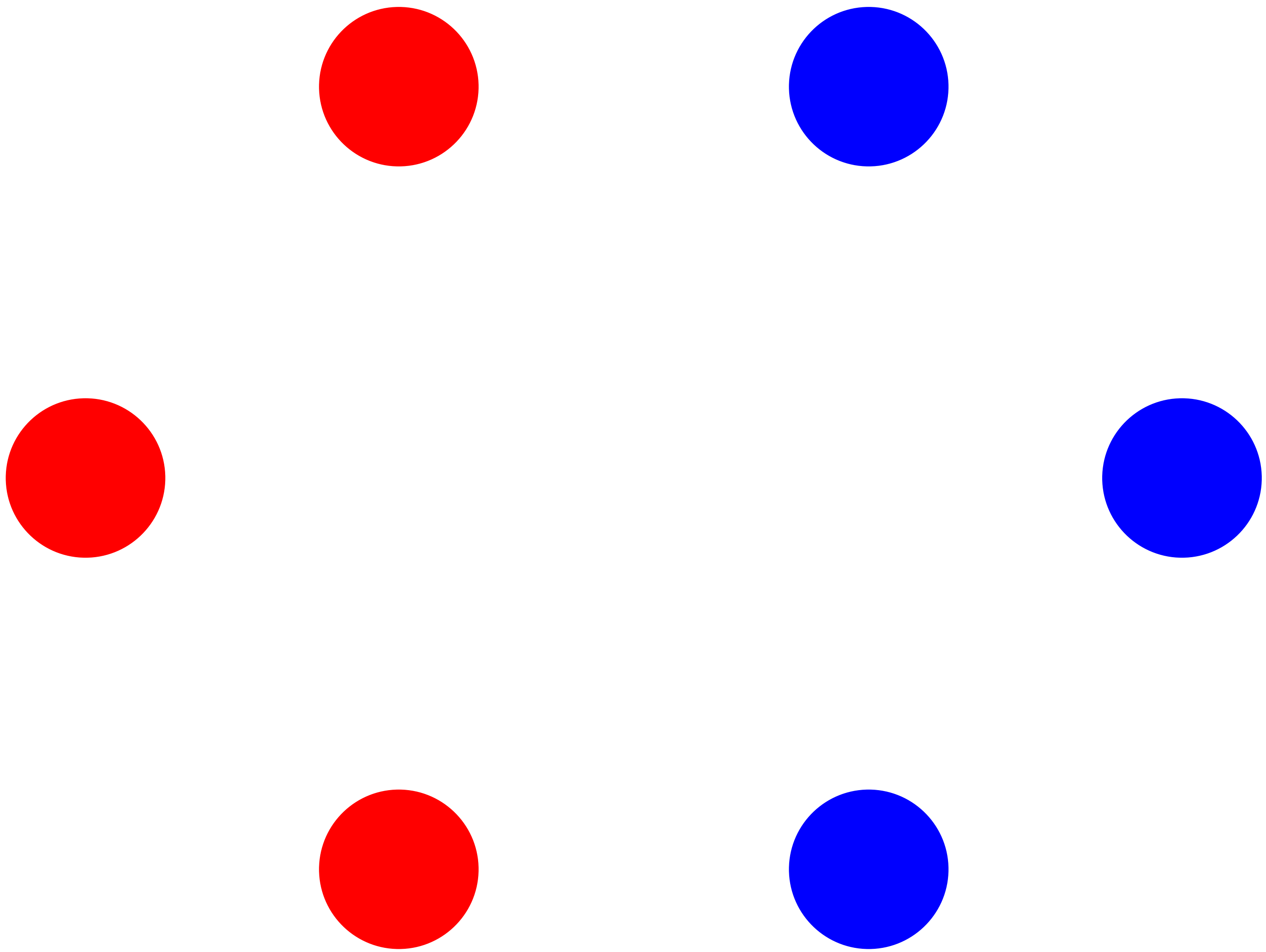

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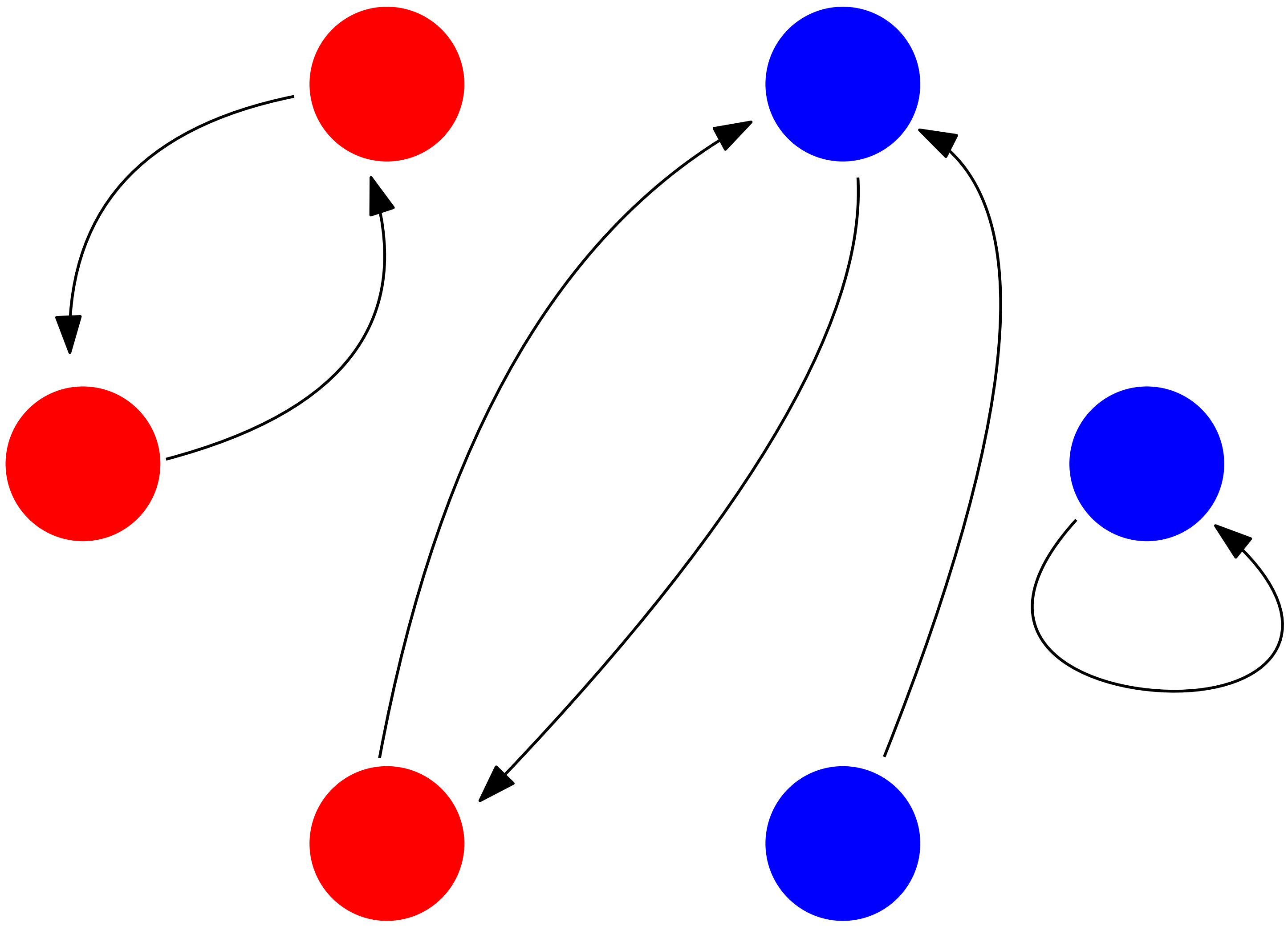

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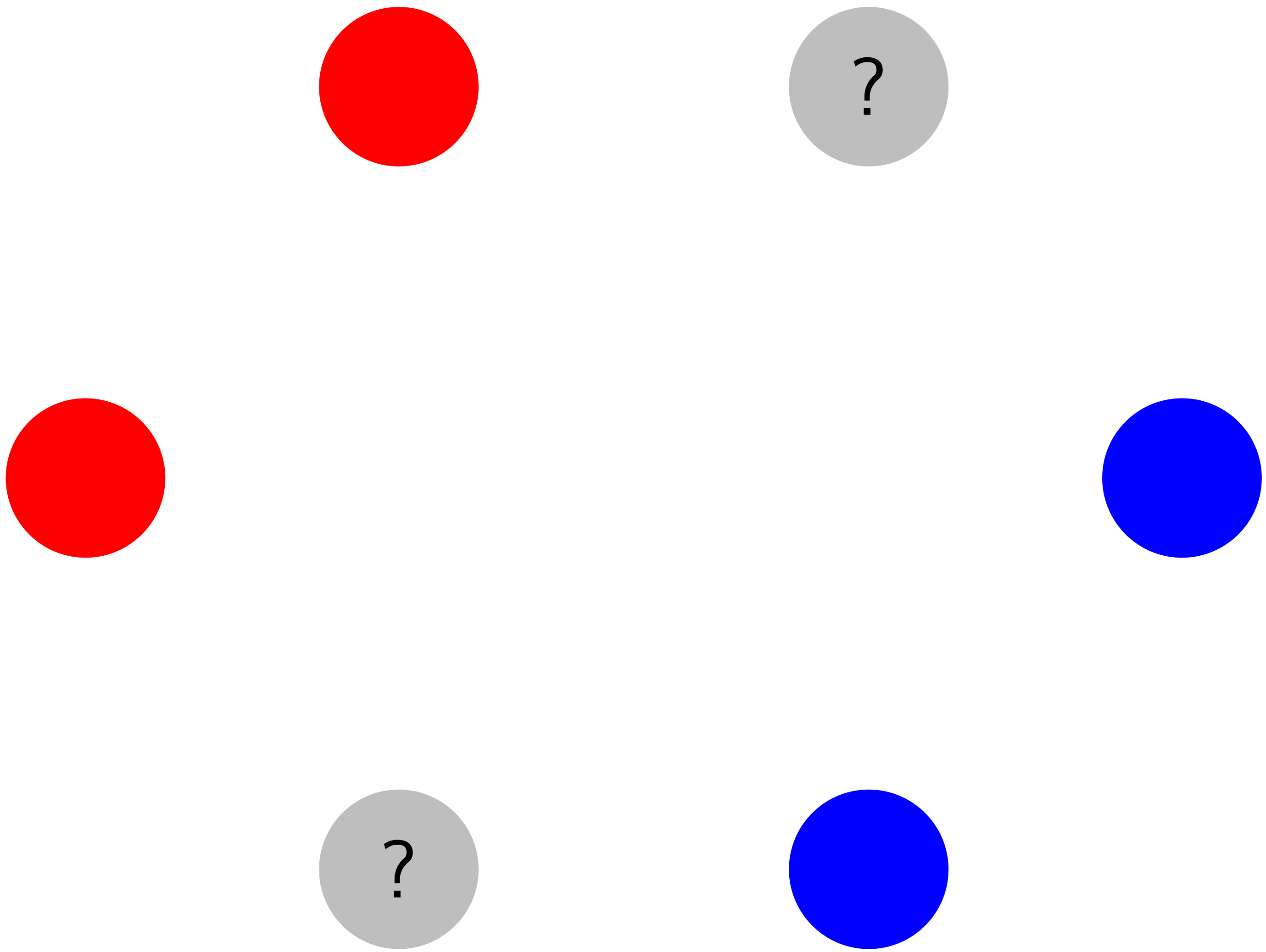

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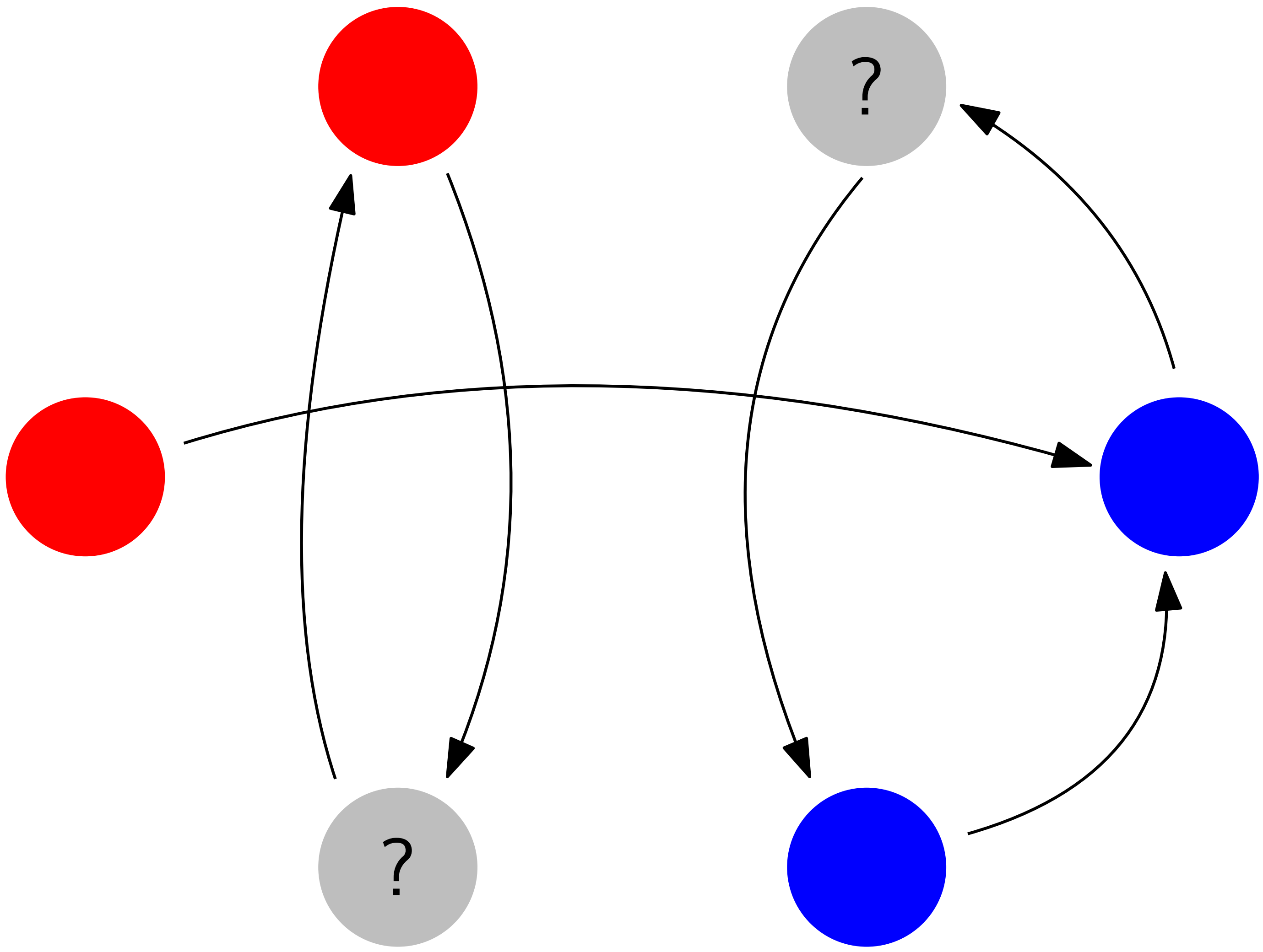

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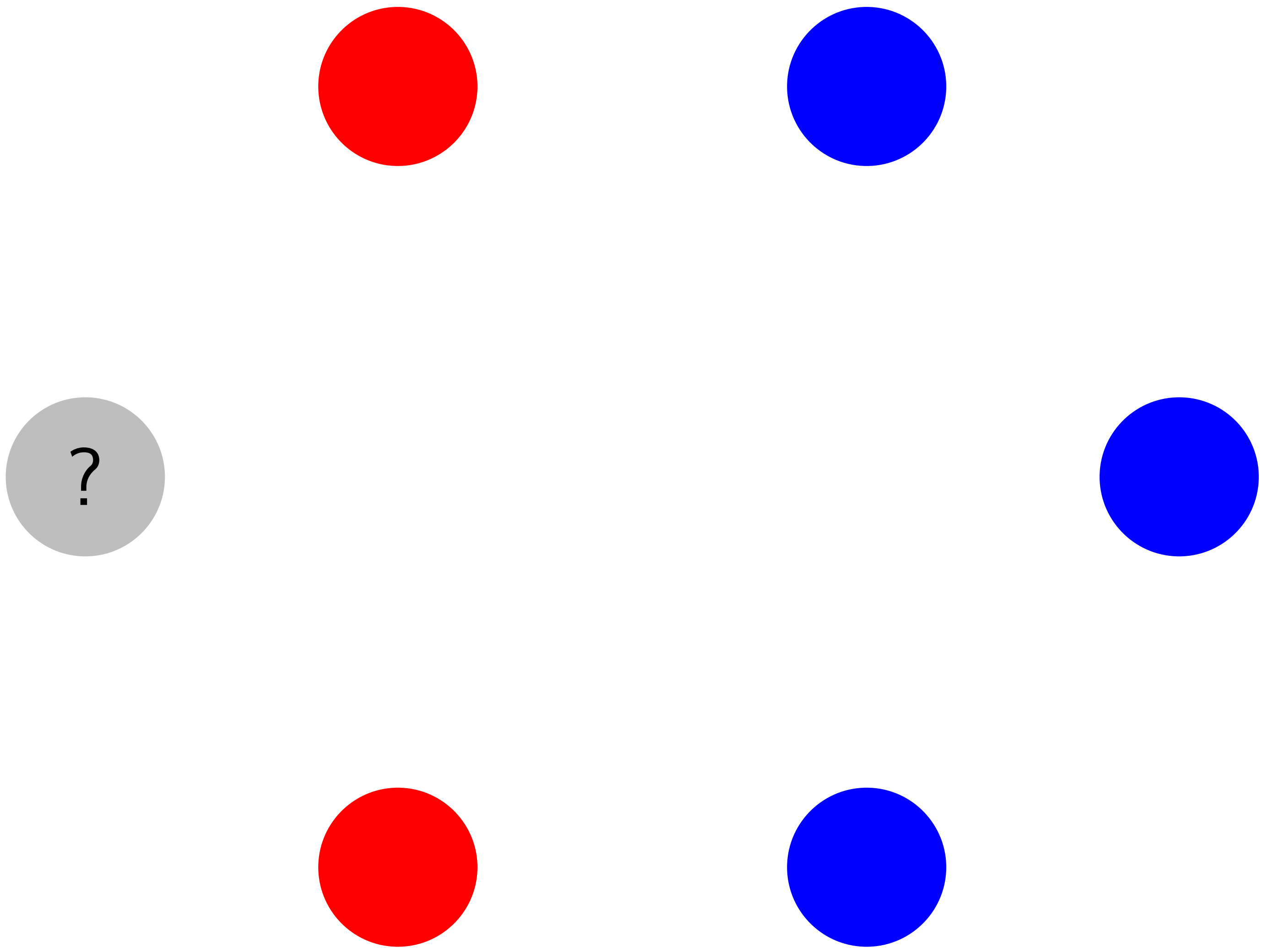

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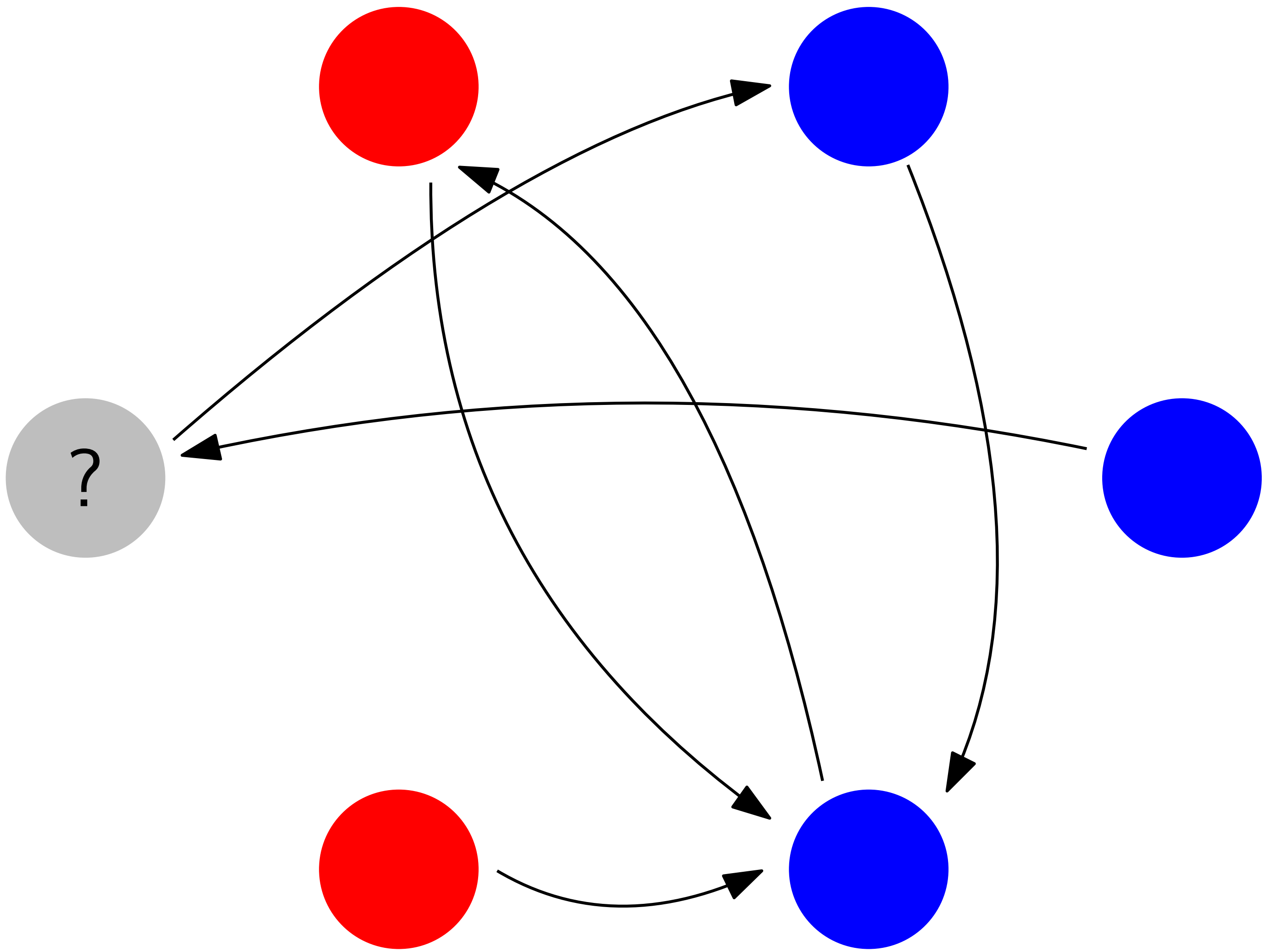

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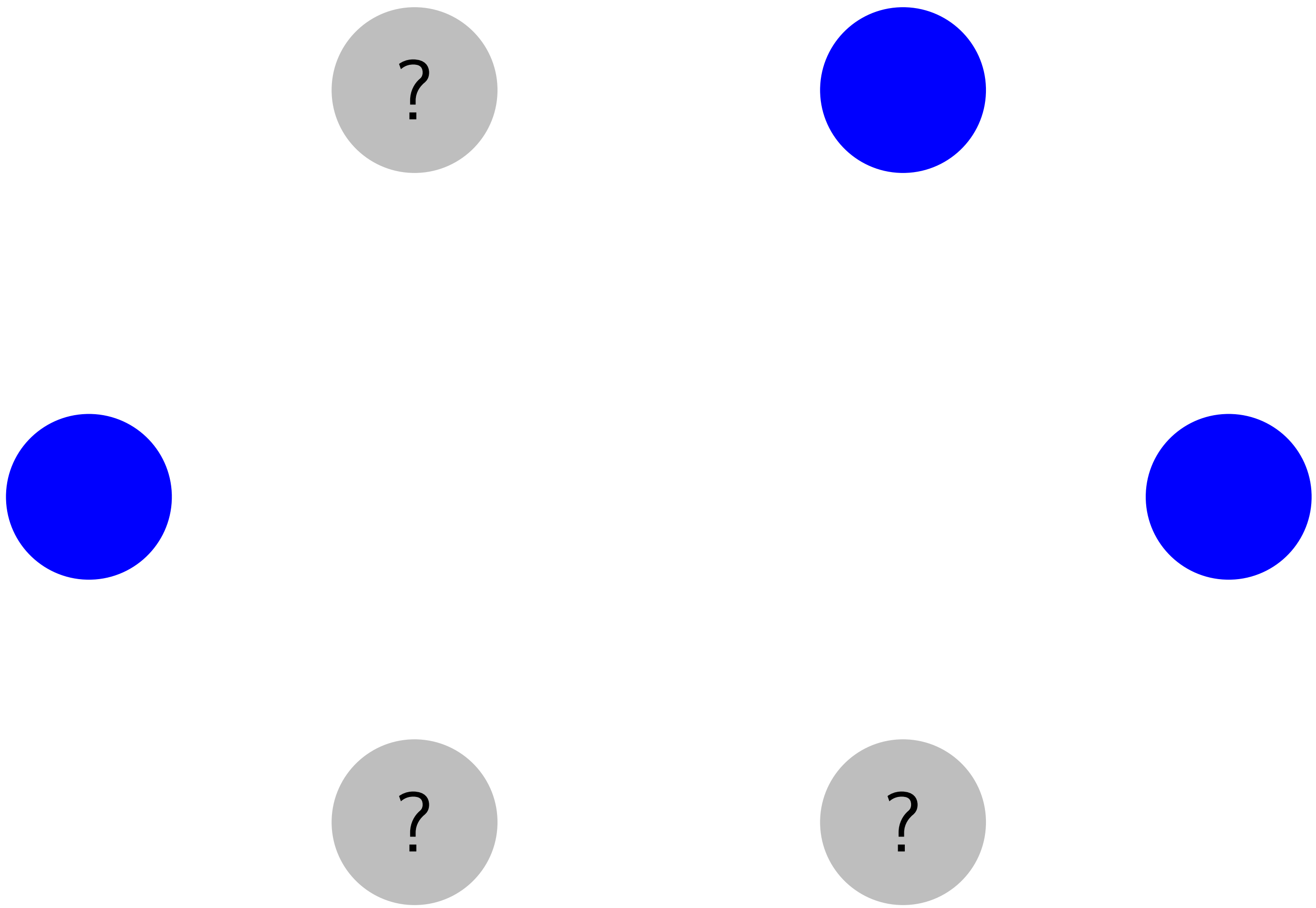

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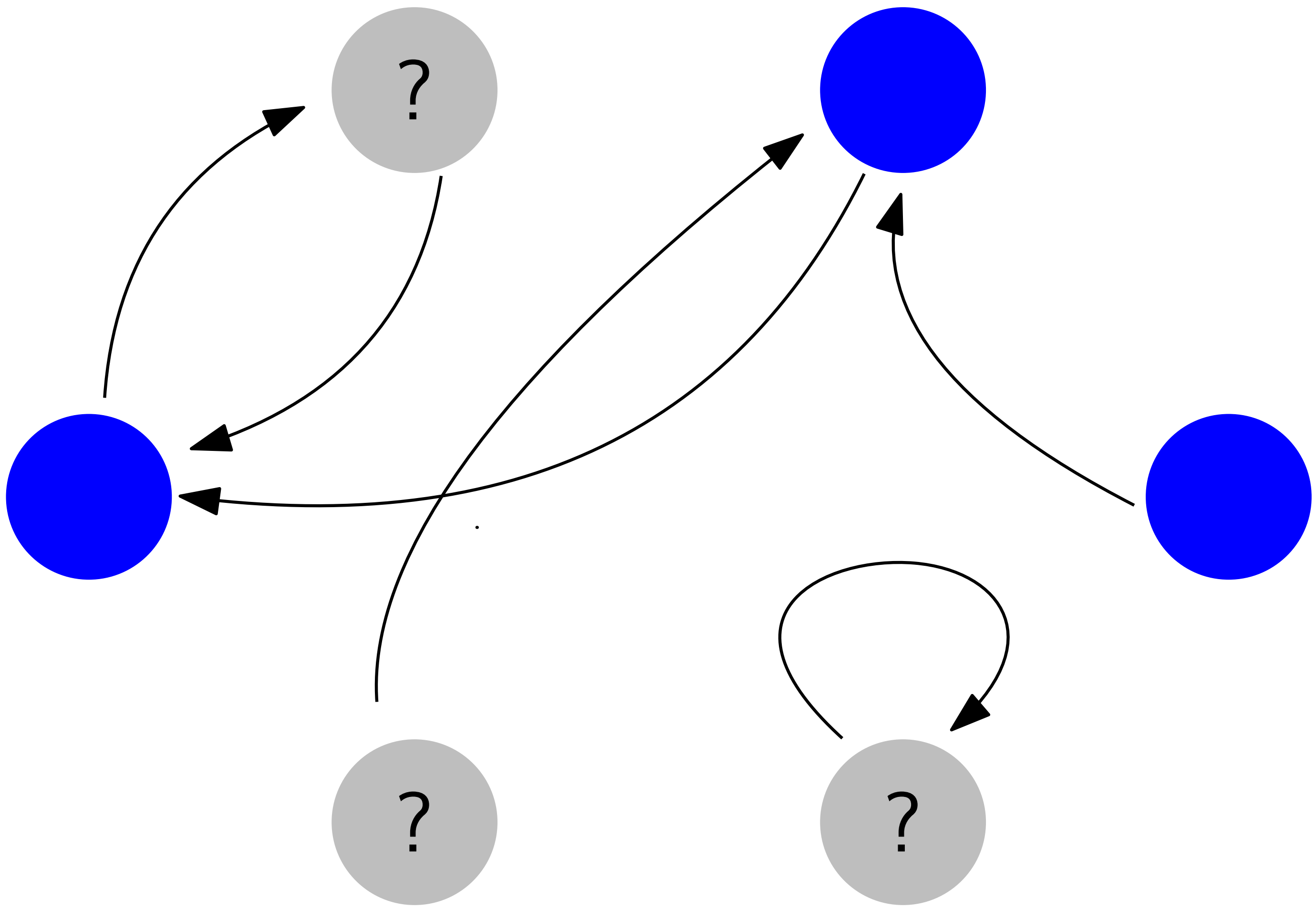

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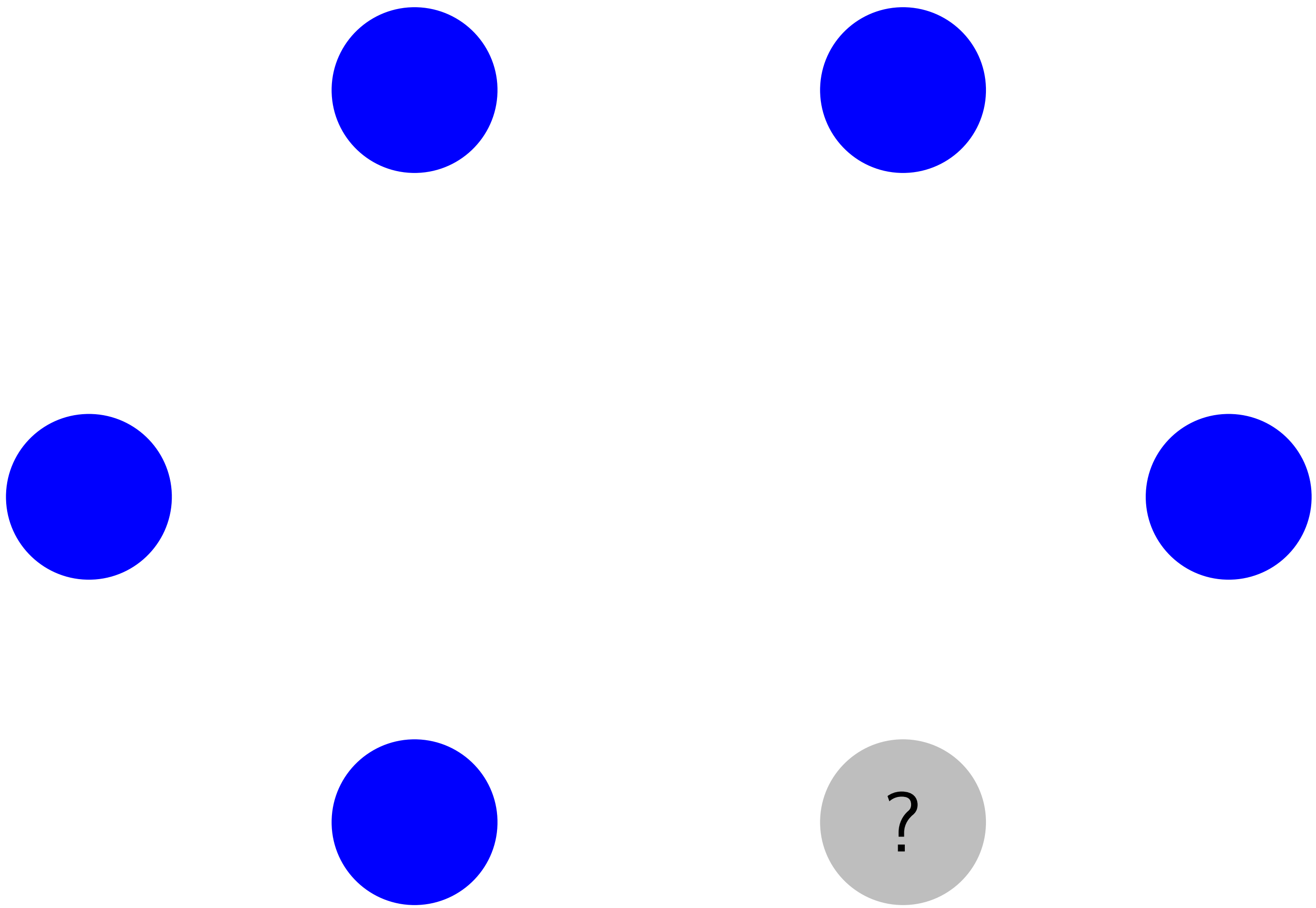

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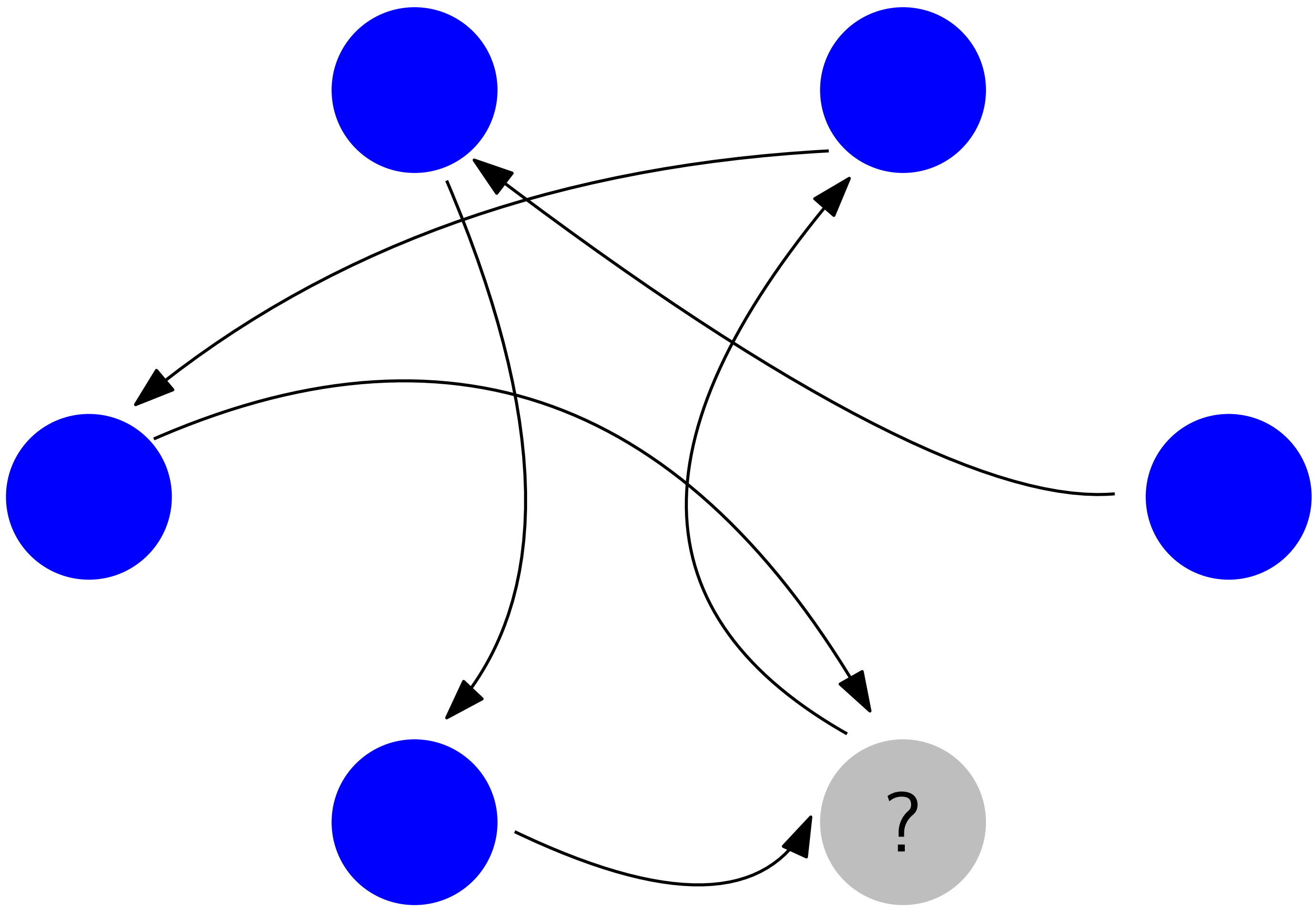

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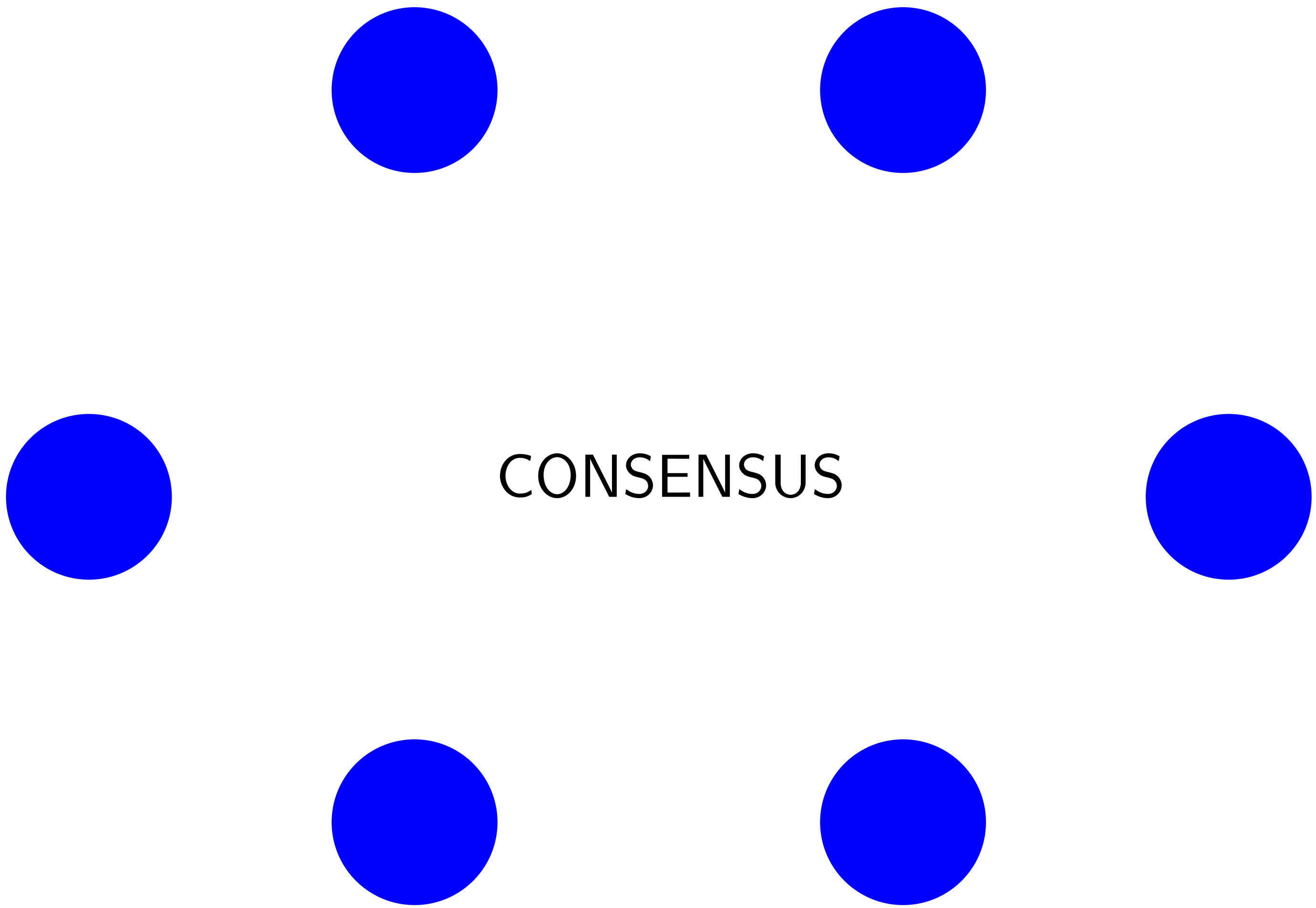

The U-Dynamics: Motivations

It can be derived from the **best nest site selection process** in honeybees [Reina et al. '17] by

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Simple, lightweight and efficient non-linear dynamics for the (Majority) Consensus Problem

- agent memory $= \log|\Sigma| + 1$
- n exchanged messages each round
- time-homogeneus

Previous Works

[Becchetti et al. '15] analyzes the case of $k = o(n)$ colors in K_n

- assumes initial majority size to be at least $(1 + \epsilon)$ times any other community size
- proves the convergence to majority consensus within time $\mathcal{O}\left(k \log n\right)$, w.h.p.

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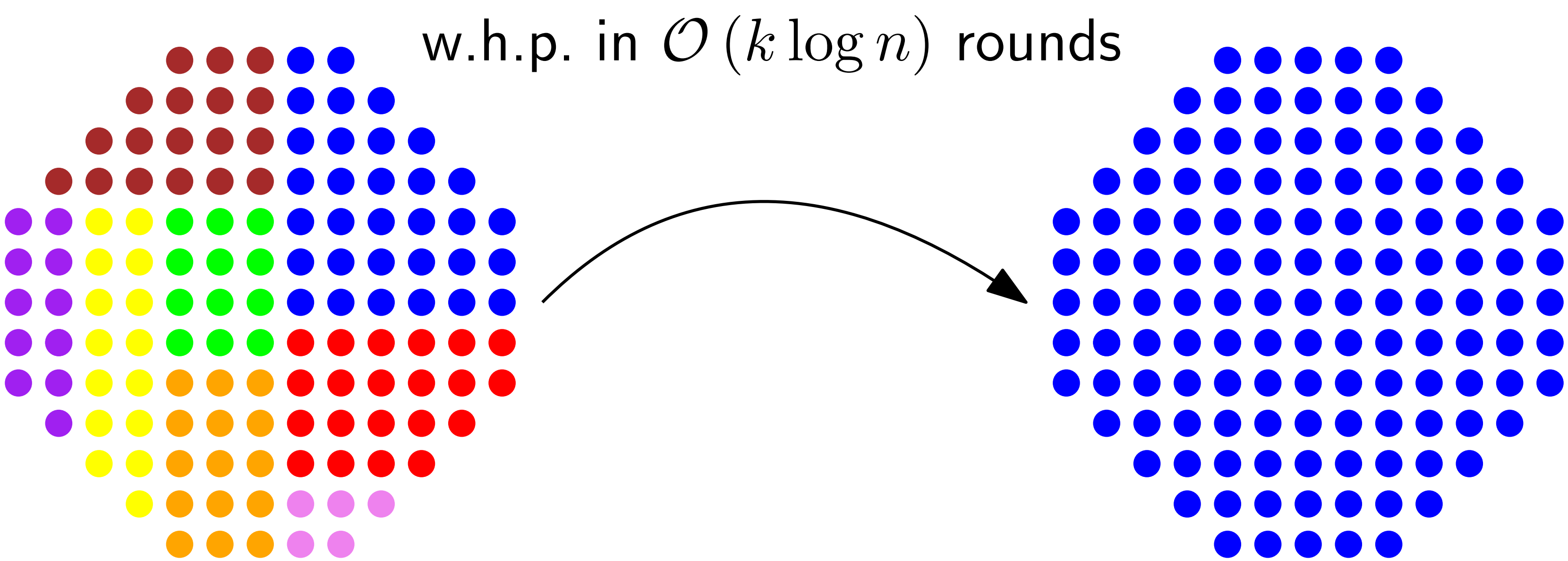

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2. from any balanced initial configuration, symmetry is broken, i.e. bias $\Omega\left(\sqrt{n \log n}\right)$ is reached, within $\mathcal{O}(\log n)$ rounds, w.h.p.
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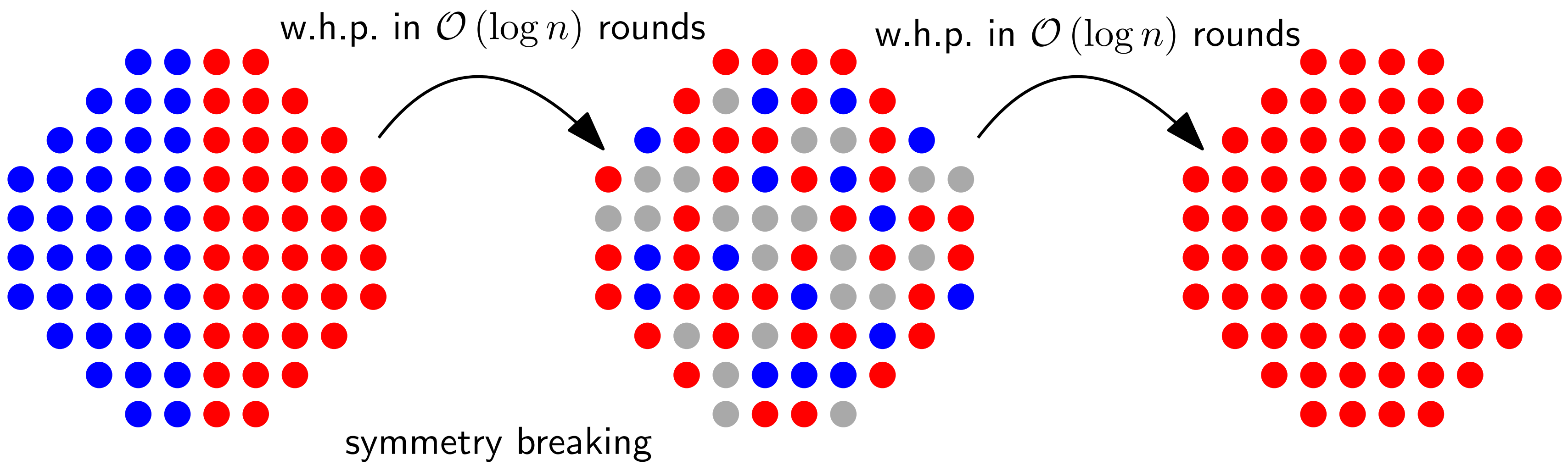

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Natural question: what if communication is subject to noise?

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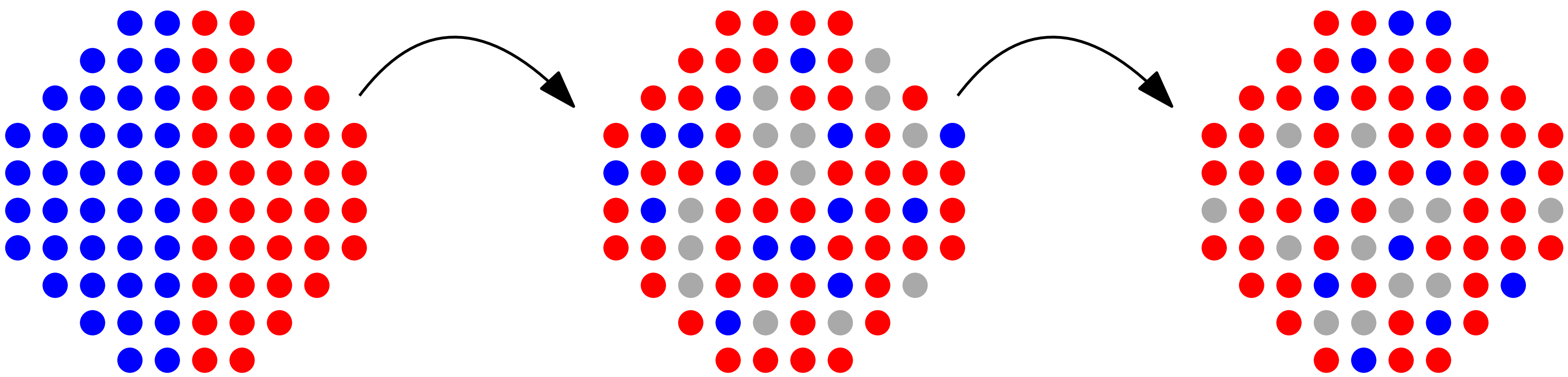

w.h.p. in finite time   w.h.p. for $poly(n)$ rounds

Our Work

U-dynamics in K_n with two colors ($\Sigma = \{\text{red}, \text{blue}\}$)

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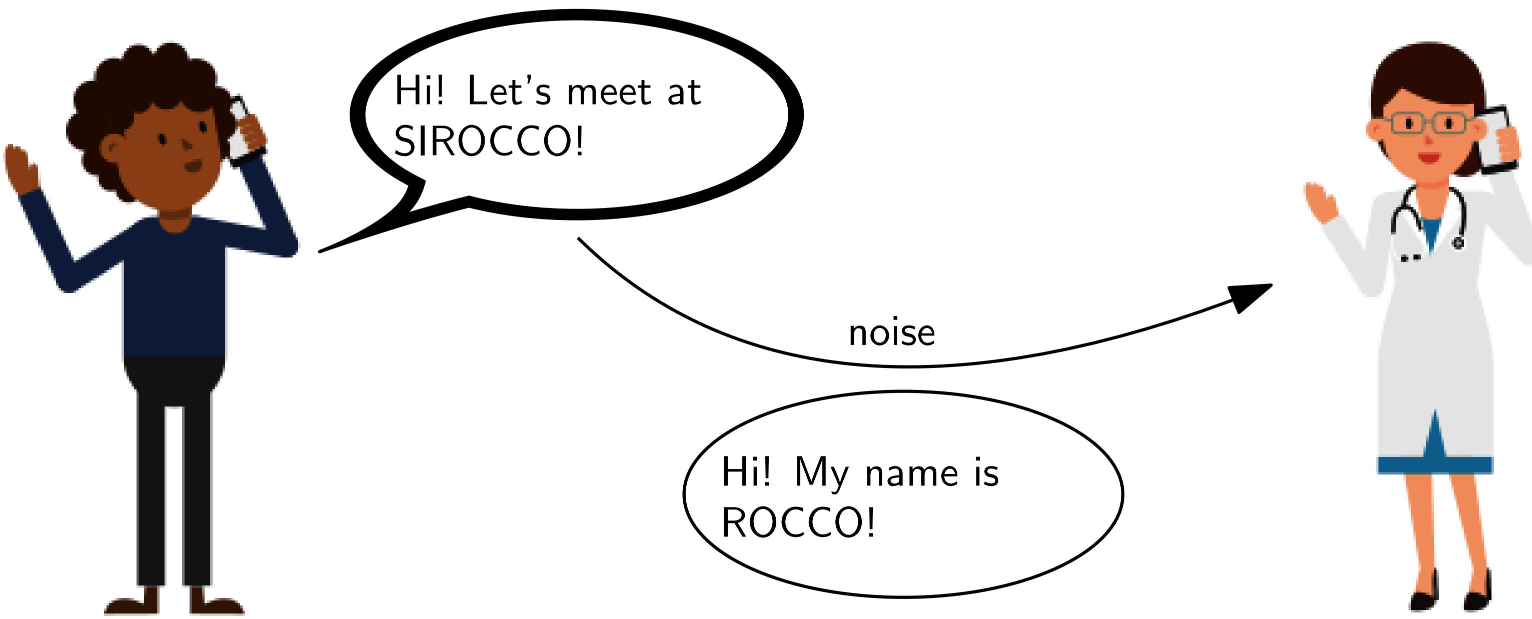

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Inspired by [Feinerman et al. '17], [Freignaud and Natale '18]

Let $p \in (0, 1/3)$ be a constant

Let u pull v's state x

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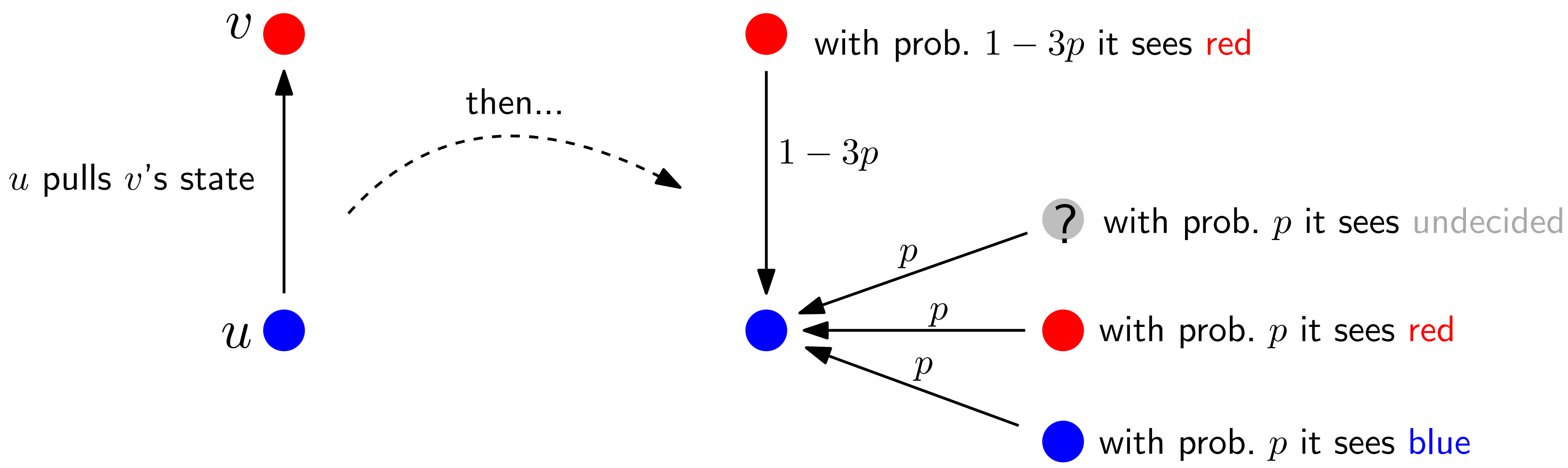

Equivalence with the Stubborn Model

Idea from [Yildiz et al. '13]

Definition (stubborn): a stubborn agent never changes color

Let
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- $n_{\text{stub}}/3$ nodes are stubborn red agents
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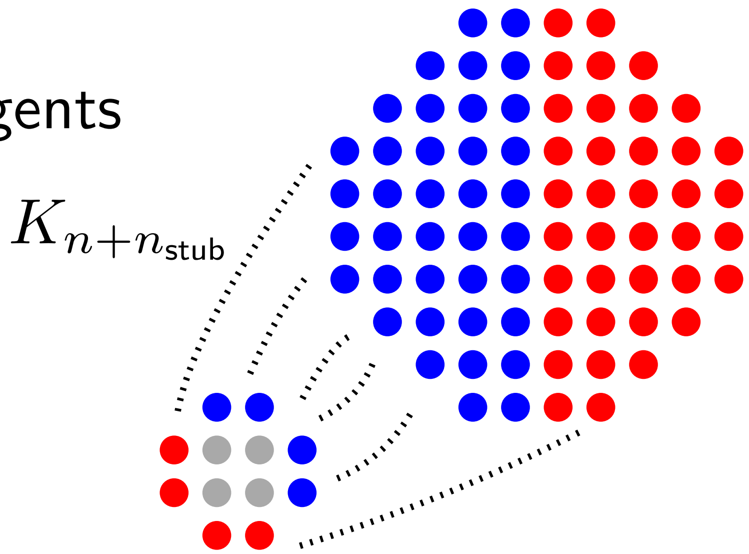

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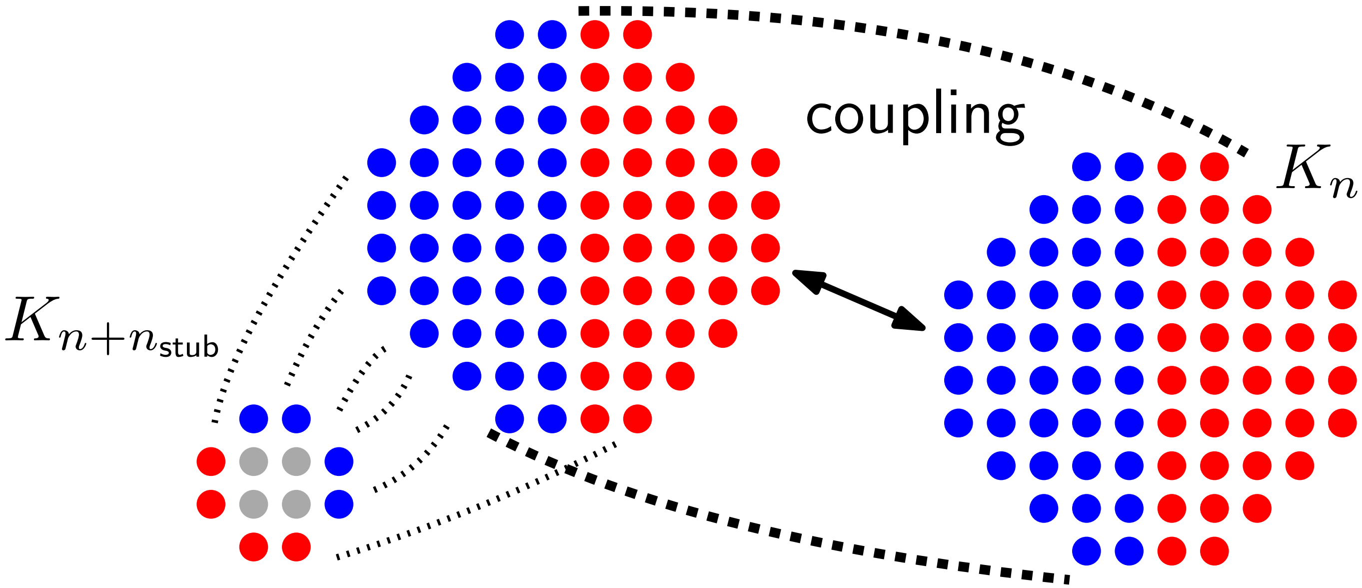

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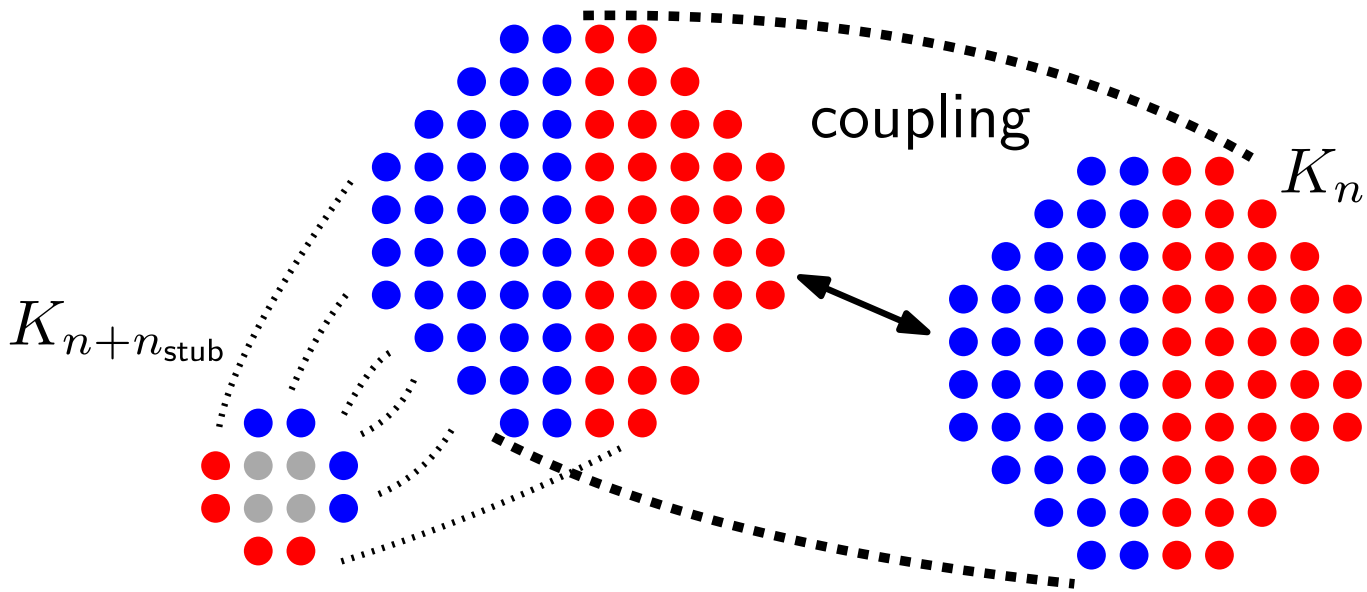

Fact: each result stated for the noisy U-process in K_n has an analogous statement for the former U-process in $K_{n+n_{\mathsf{stub}}}$, and vice versa

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We **prove** that starting from

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2. any configuration with bias $\Omega\left(\sqrt{n \log n}\right)$, the system
   - reaches bias $\Theta\left(n\right)$ towards the majority color within $\mathcal{O}\left(\log n\right)$ rounds, w.h.p.
   - enters a meta-stable phase of length $poly(n)$ rounds, in which the bias keeps $\Theta\left(n\right)$ towards the majority color, w.h.p.

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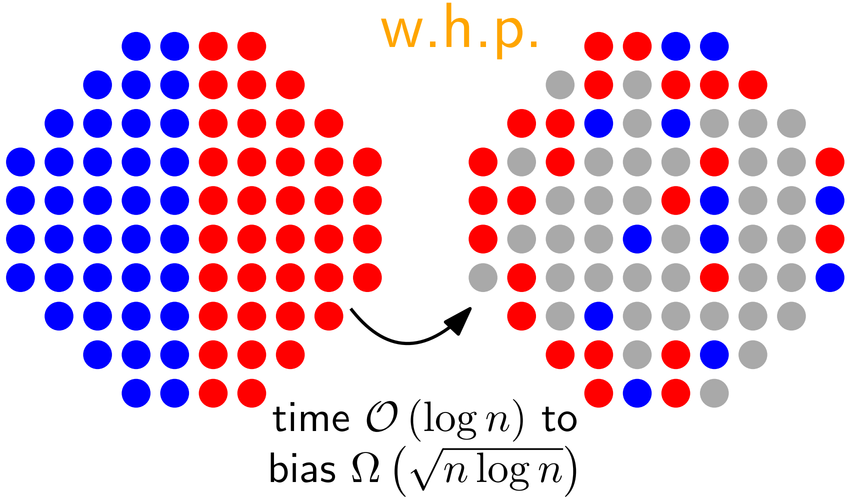

Results (Noisy Model)

Let
- $p < 1/6$ be a constant
- $3p$ be the noise probability

We **prove** that starting from

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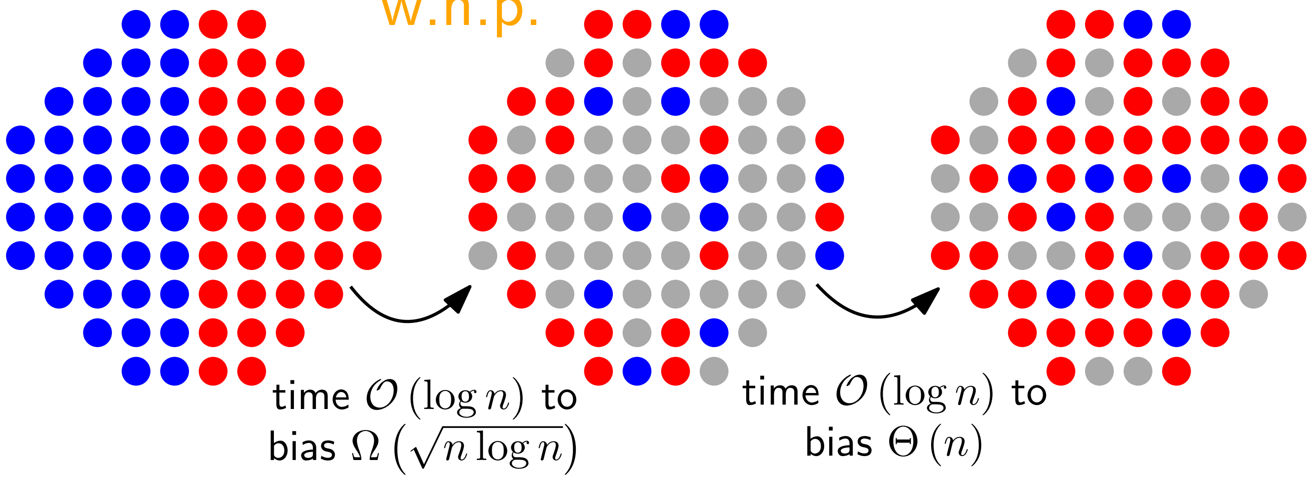

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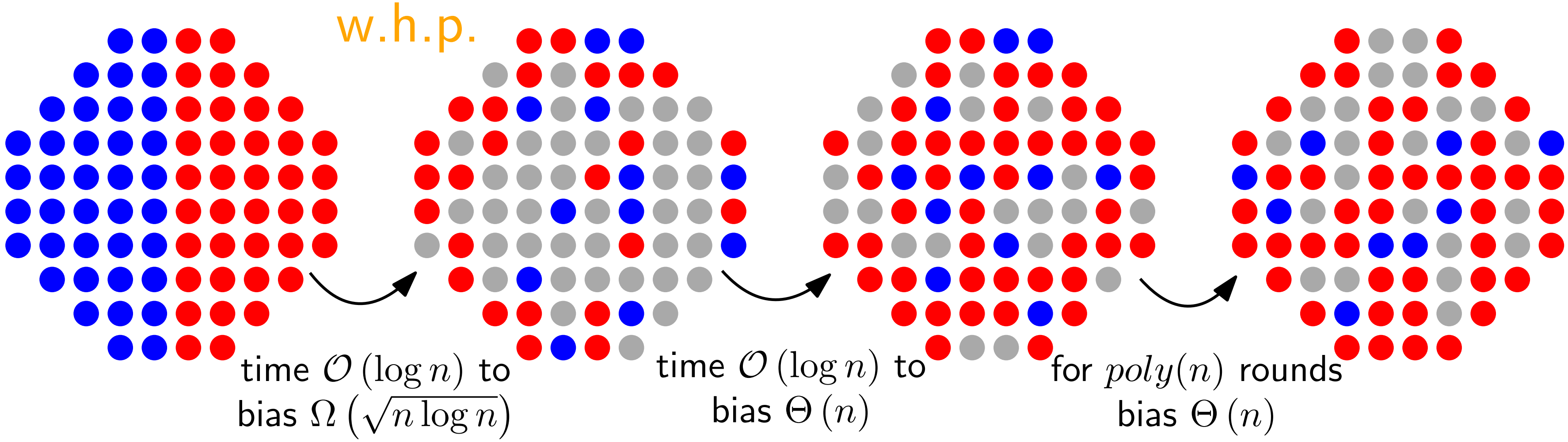

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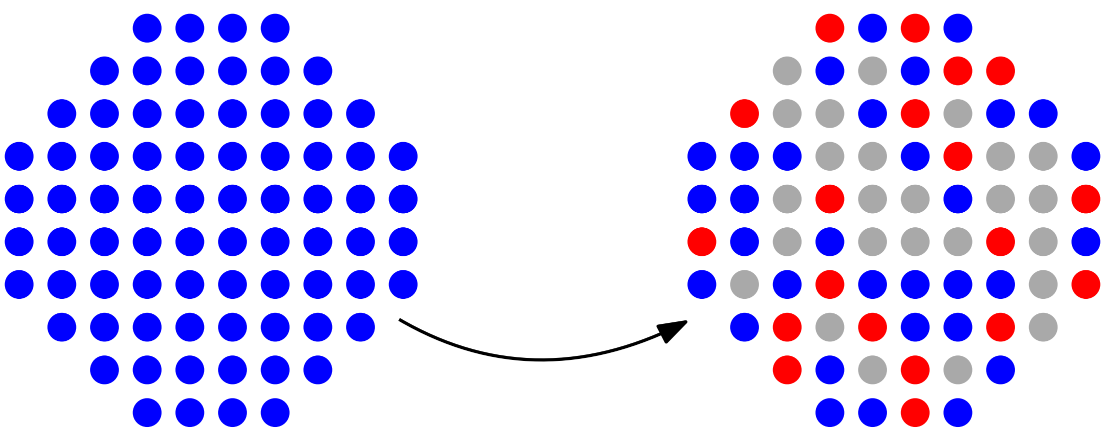

time $\mathcal{O}\left(\log n\right)$ to reach bias $\mathcal{O}\left(\sqrt{n \log n}\right)$

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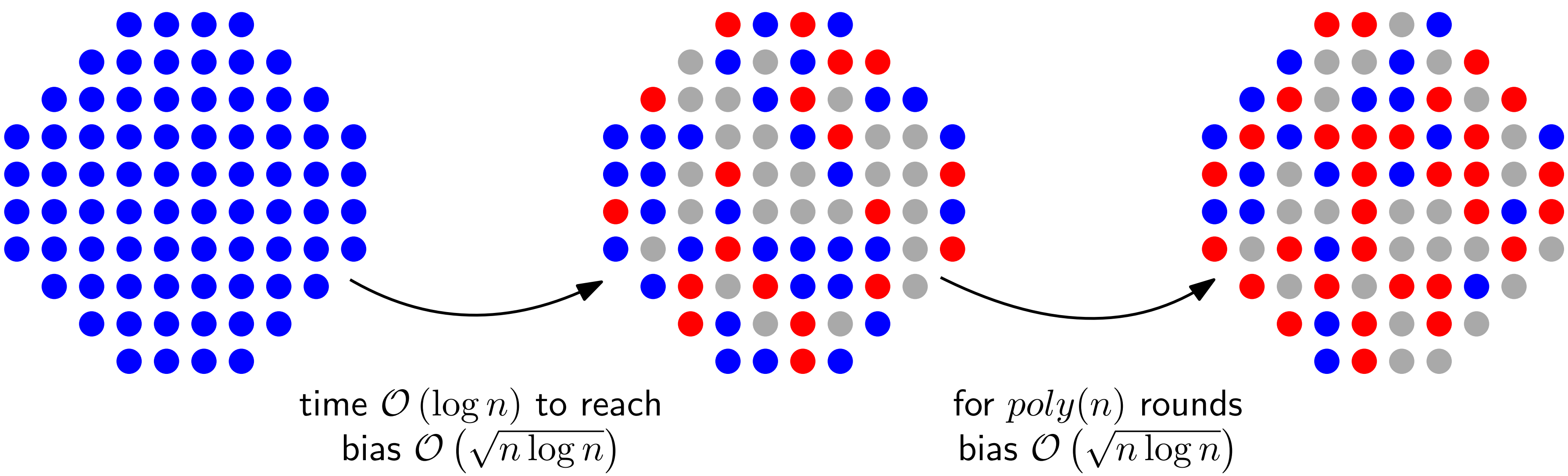

Proofs Idea

For $p < 1/6$, just the majority consensus

Let
- S_t be the r.v. yielding the bias of the configuration at time t
- $S_0 = \Omega\left(\sqrt{n \log n}\right)$
- Q_t be the r.v. yielding number of undecided nodes at time t

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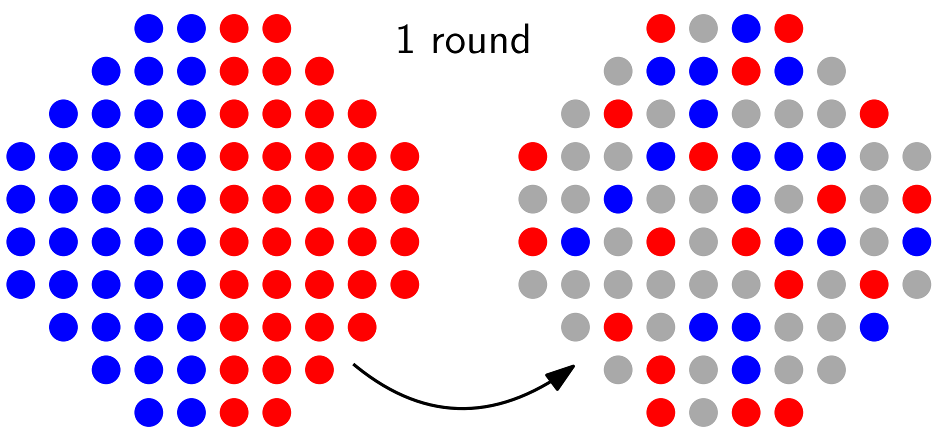

From zero bias we get almost $n/2$ undecided nodes

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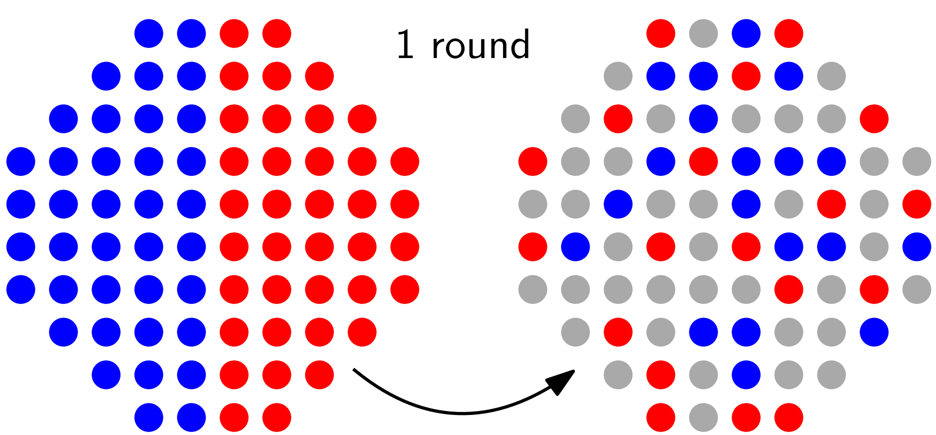

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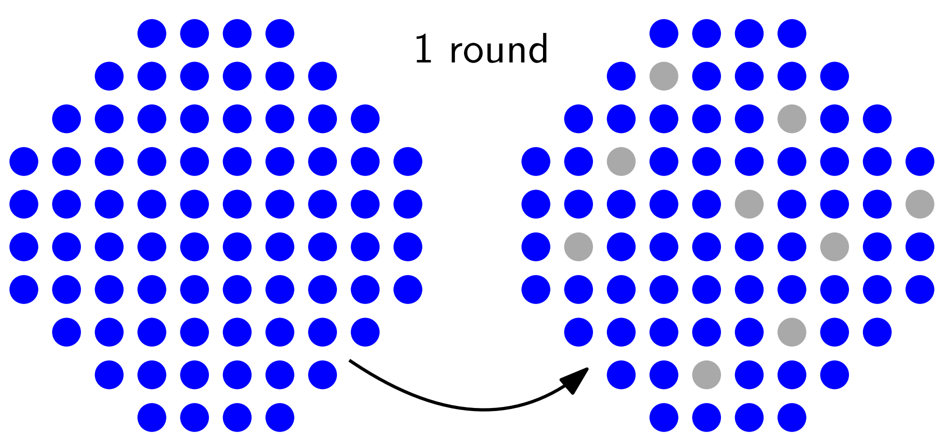

From max bias we get a little constant factor of undecided nodes

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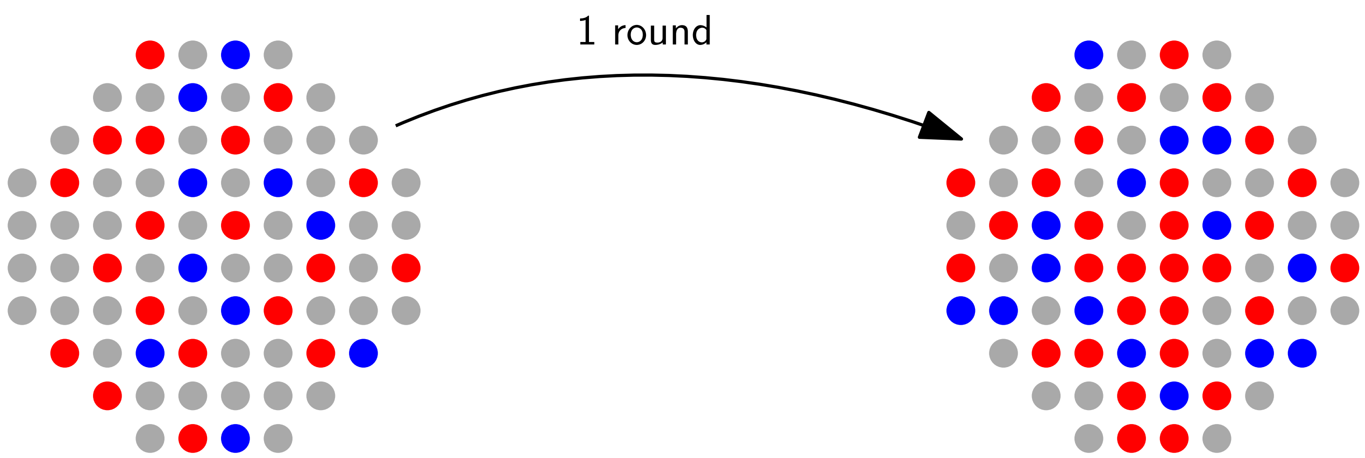

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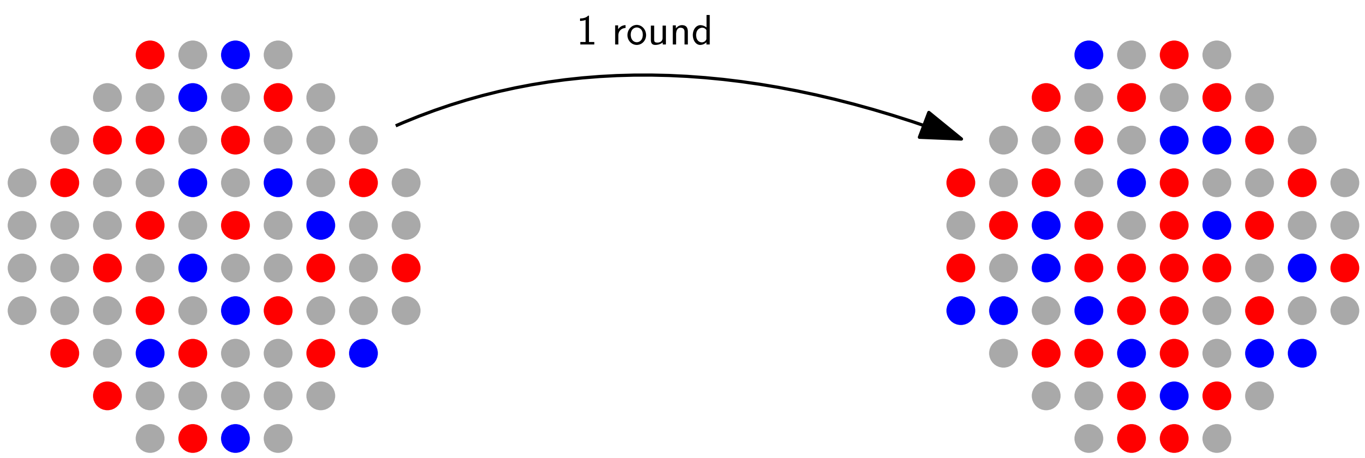

We identify these treshold quantities and concentrate with Chernoff bounds

Proofs Idea

For some $0 < \beta, c < 1$, some small enough $\epsilon > 0$, and some $\delta > 0$, we **prove** that

a) if $S_t = \Omega\left(\sqrt{n \log n}\right)$, then $S_{t+1} \geq (1-\epsilon)S_t$, w.h.p.
b) if $\Omega\left(\sqrt{n \log n}\right) = S_t < \beta n$ and $Q_t > cn$, then $S_{t+1} \geq (1+\delta)S_t$, w.h.p.
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By combining (a) + (b) + (c) we get that the system

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- enters a meta-stable phase of length $poly(n)$ rounds in which the bias keeps $\Theta(n)$, w.h.p.

Conclusions

- first step towards investigation of noise in non-linear opinion dynamics
- better comprehension of plausible models for biological systems

Honey bee

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Questions

- what about sparser topologies (e.g., expanders)?
- what about other non-linear opinion dynamics?

THANK YOU FOR YOUR ATTENTION

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