Fatemeh Darbehani [redacted]

ENSC474 – SFU – Spring 2017

Assignment 7

Periodic Noise

- In this assignment I created a periodic noise function that takes u0and v0 and implements the following function.

```
cos_noise = cos(2*pi*(((u0*x)/M) + ((v0*y)/N))) + 1; % To keep values +ve
```

- The result of the periodic functions and their discrete Fourier Transforms are shown in figure 1 to 4. Note that DC component (+1 in the above formula) appears at the origin (0, 0) of DFT.

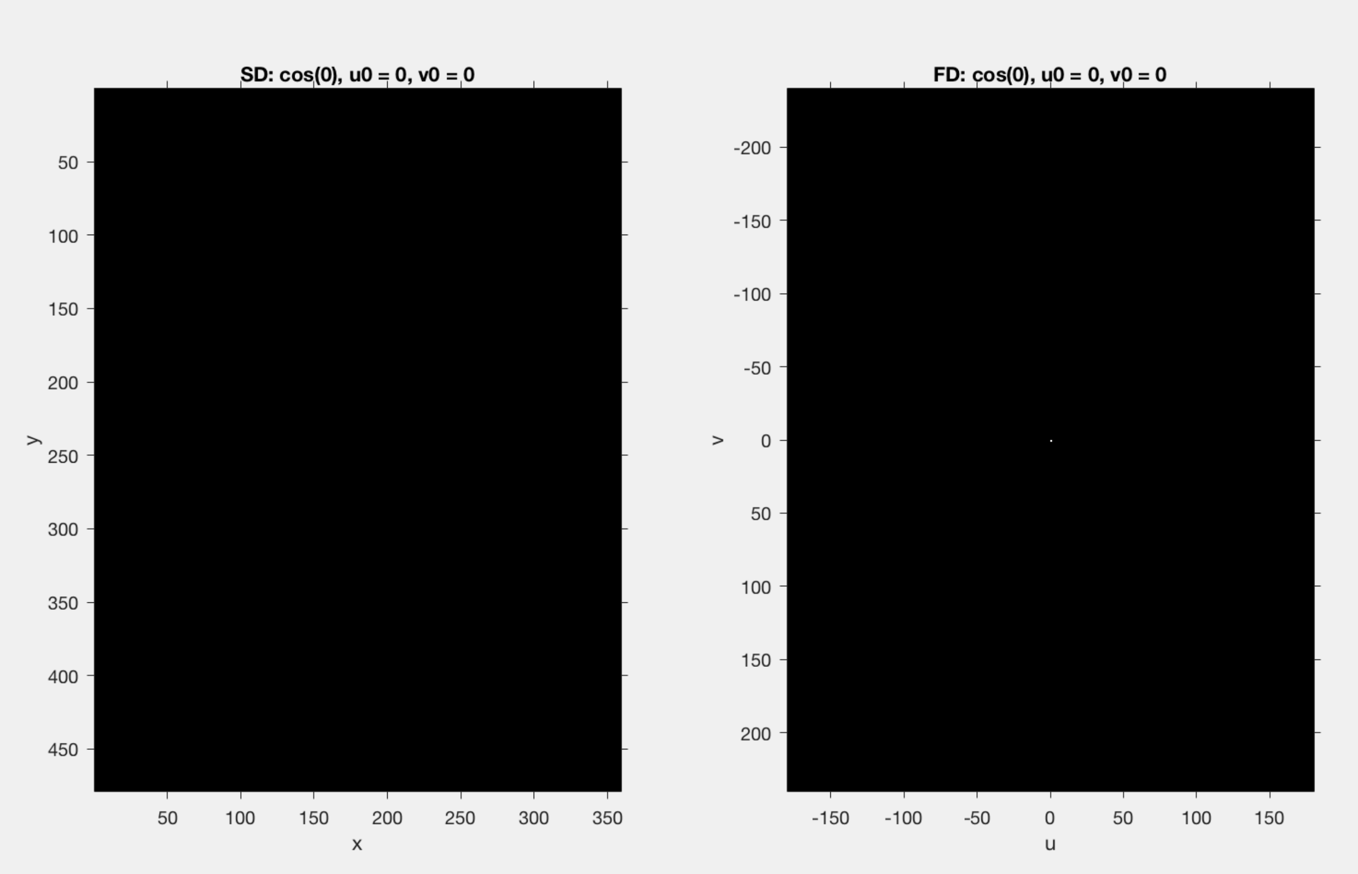
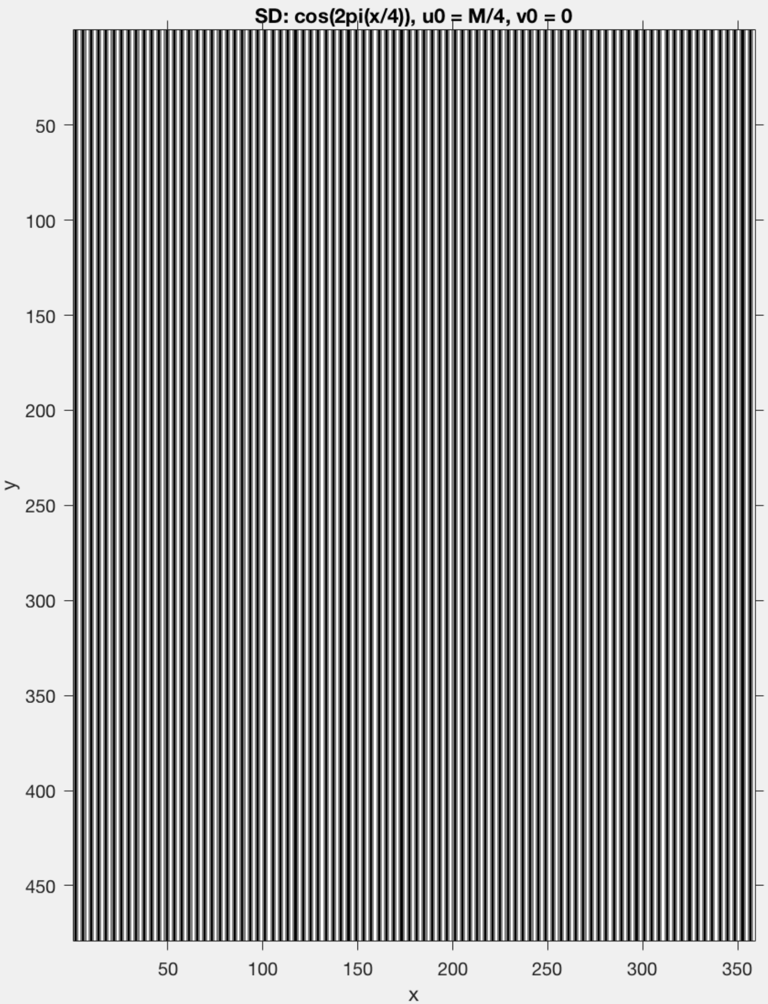


Figure1: Periodic Noise With u0 = 0 and v0 = 0. DFT has peak at (0, 0).

Periodic Noise



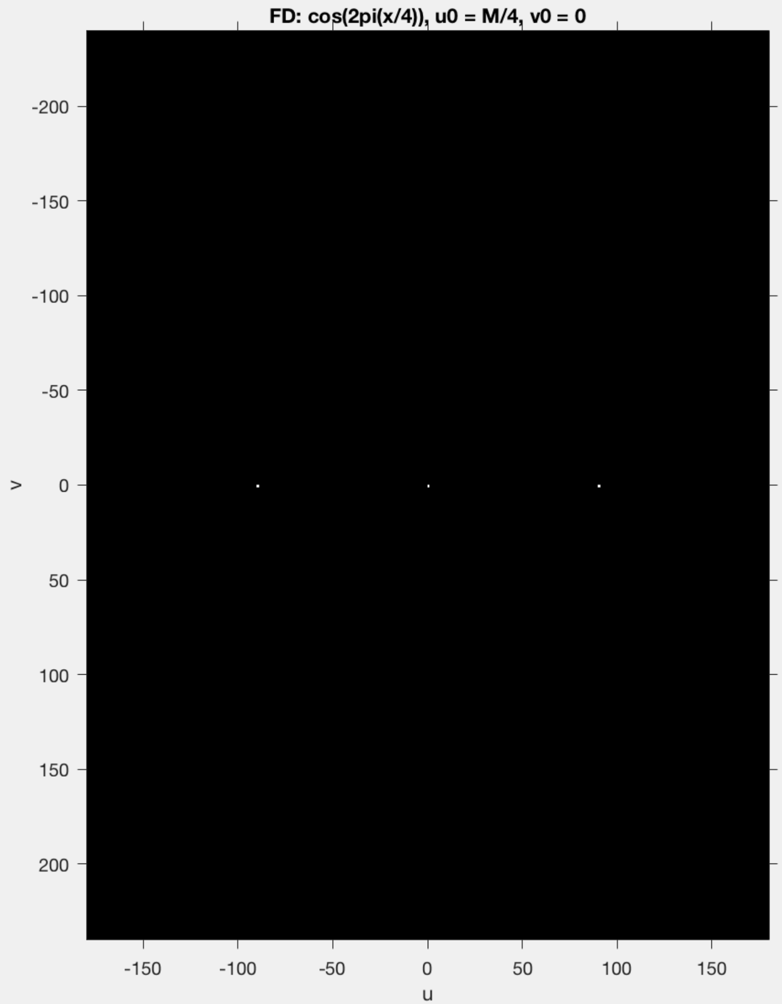


Figure2: Periodic Noise with u0 = M/4 and v0 = 0.
DFT has peaks at (0, 0), (M/4, 0), and (-M/4, 0).

Periodic Noise

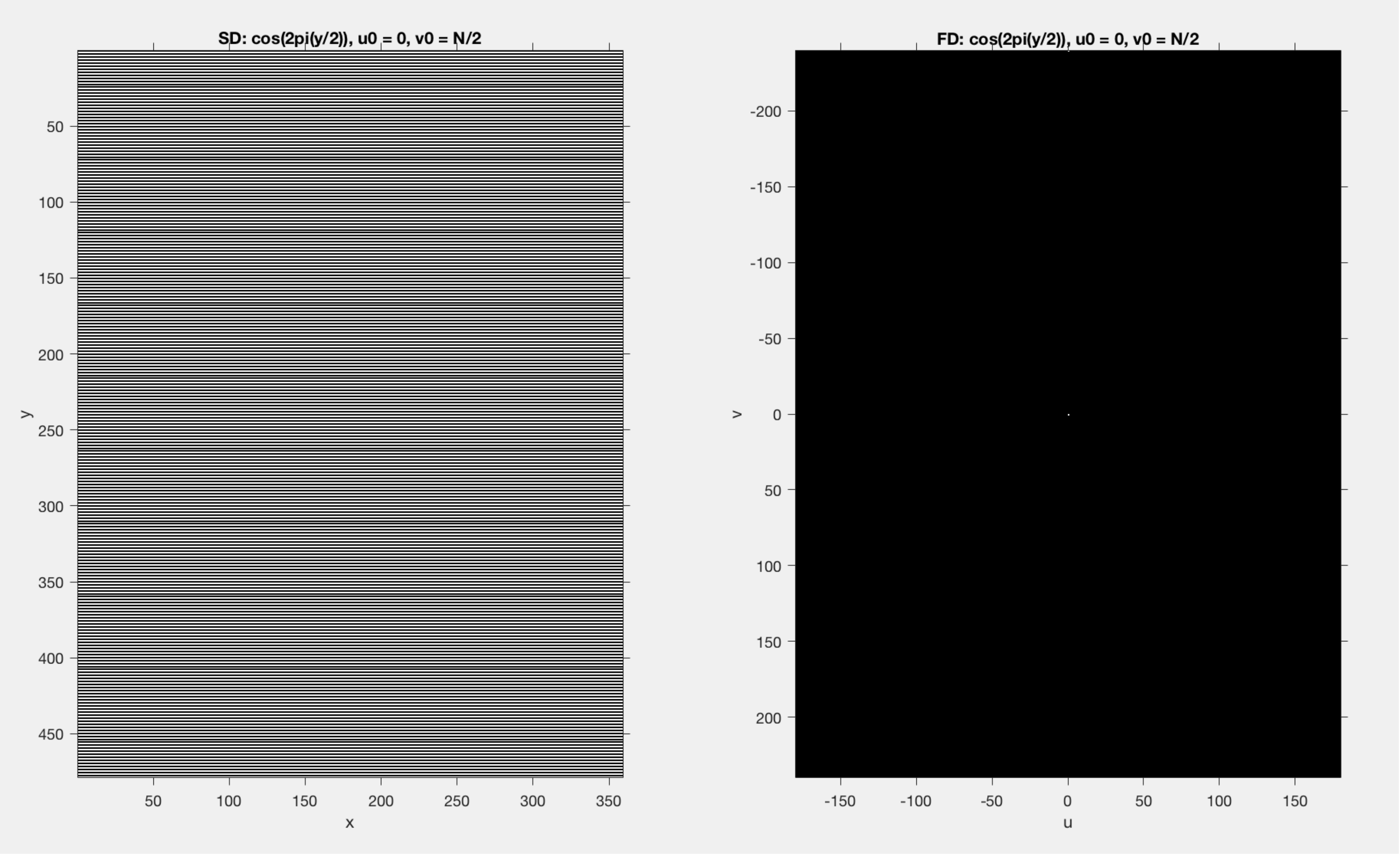


Figure3: Periodic Noise with u0 = 0 and v0 = N/2.
DFT has peaks at (0, 0), (0, N/2), and (0, -N/2).

Periodic Noise

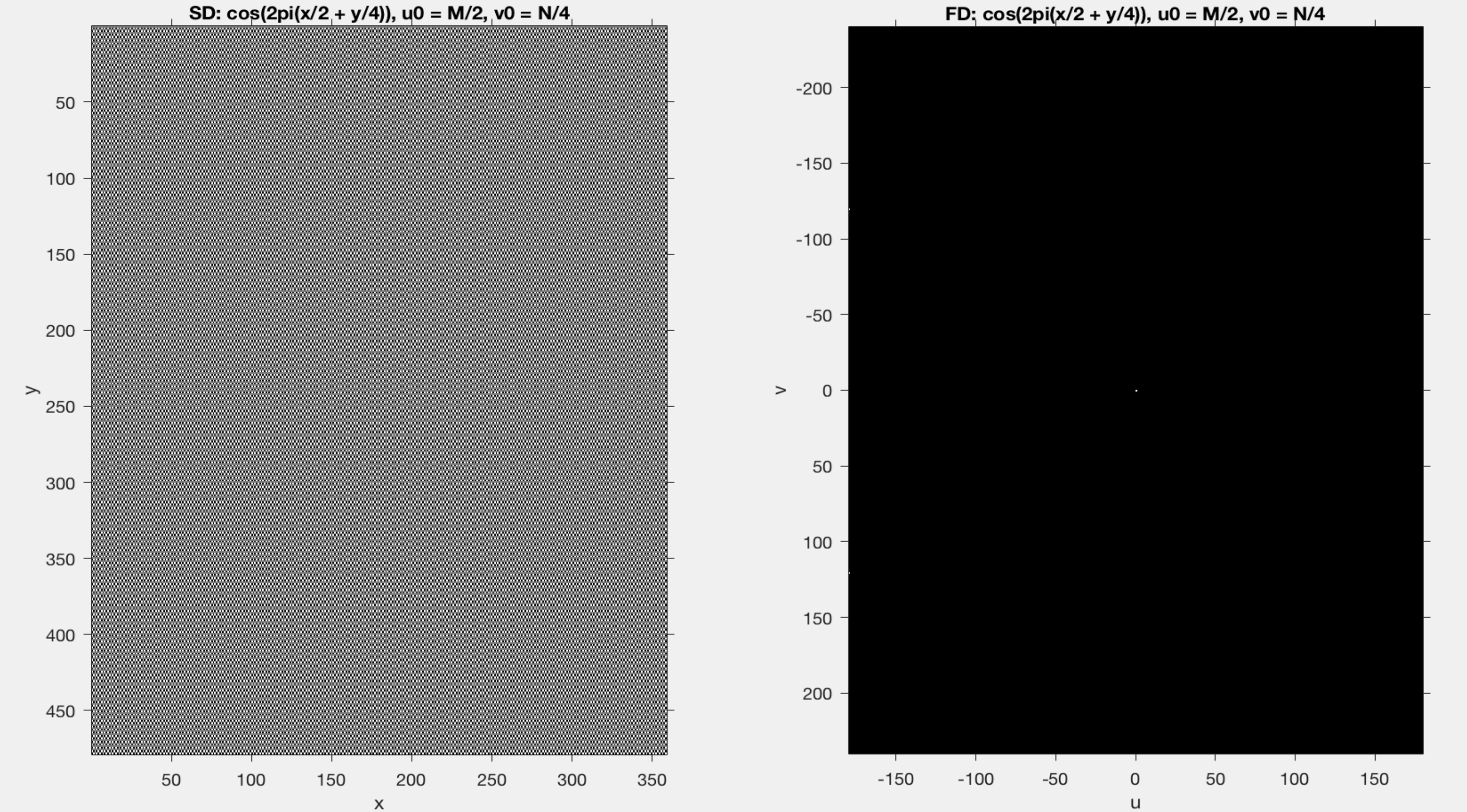


Figure4: Periodic Noise with u0 = M/2 and v0 = N/4.
DFT has peaks at (0, 0), (M/2, N/4), and (-M/2, -N/4).

Discrete Fourier Transform of Cos

- DFT of my cosine periodic noise should theoretically match the following formula and figures.

$$\cos(2\pi f_x x + 2\pi f_y y) \quad \Leftrightarrow \quad \frac{1}{2}\Big(\delta(u - f_x, v - f_y) + \delta(u + f_x, v + f_y)\Big)$$

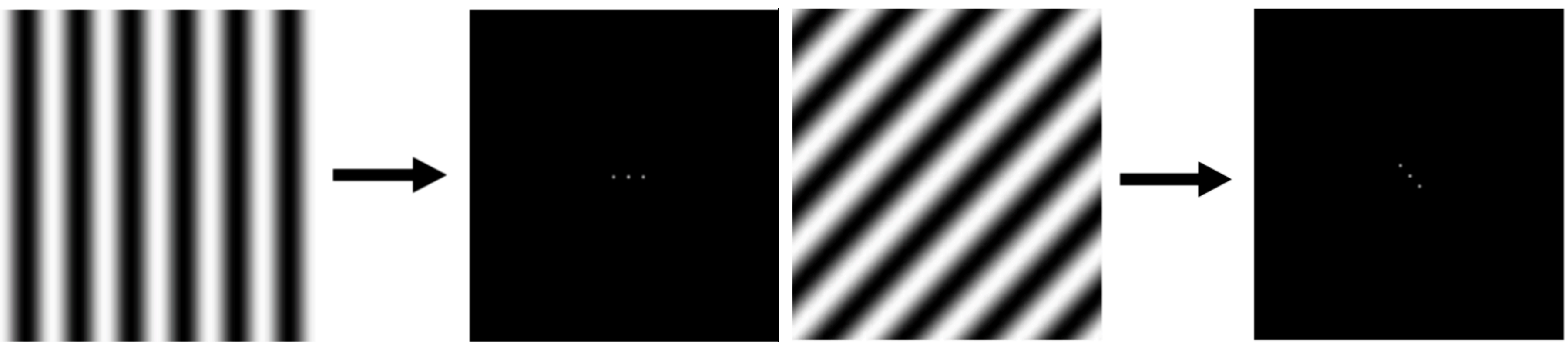
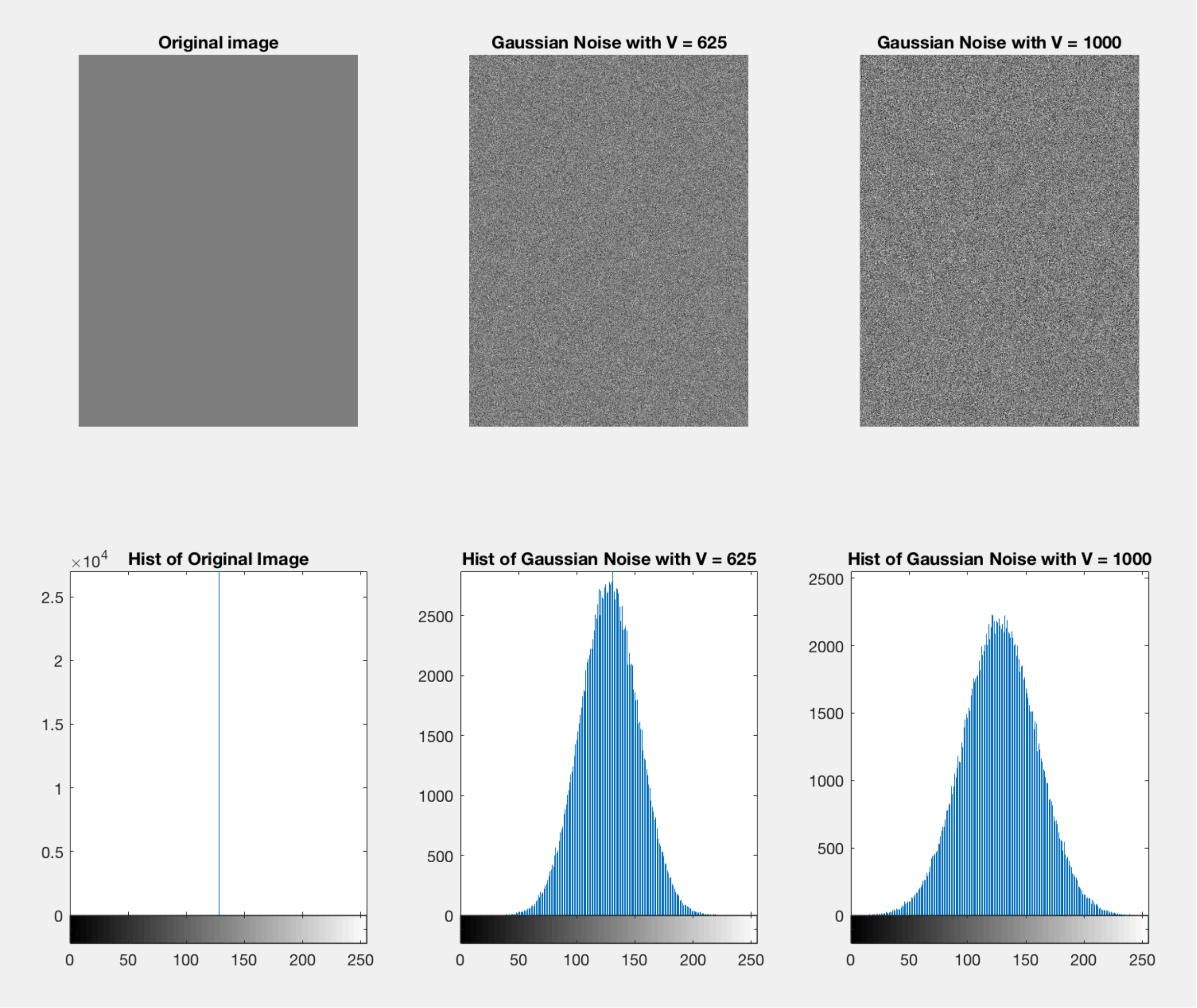


Figure5: 2D DFT of cosine image (source: http://www.serc.iisc.ernet.in/~venky/SE263/slides/FreqDomain.pdf)

- While cos(2pi (x/4)) matches this, cos(2pi (y/2)) and cos(2pi(x/2+y/4)) seem to show the peaks at other locations. This happens because these peaks lie on a boundary and moved to wrong locations by fftshift.

Gaussian Noise

- In this part I created a constant image that has value of 128 and added Gaussian and Salt and Pepper noises to it using 'imnoise' function.



- We can observe that histogram of constant image only has one intensity contributing to it. Also, shape of Gaussian noise changes with variance.

Figure6: Constant image and Gaussian noises added to it.

Salt & Pepper Noise

- As density of Salt & Pepper Noise increases, intensities of black(0) and white(255) increase in the histogram.

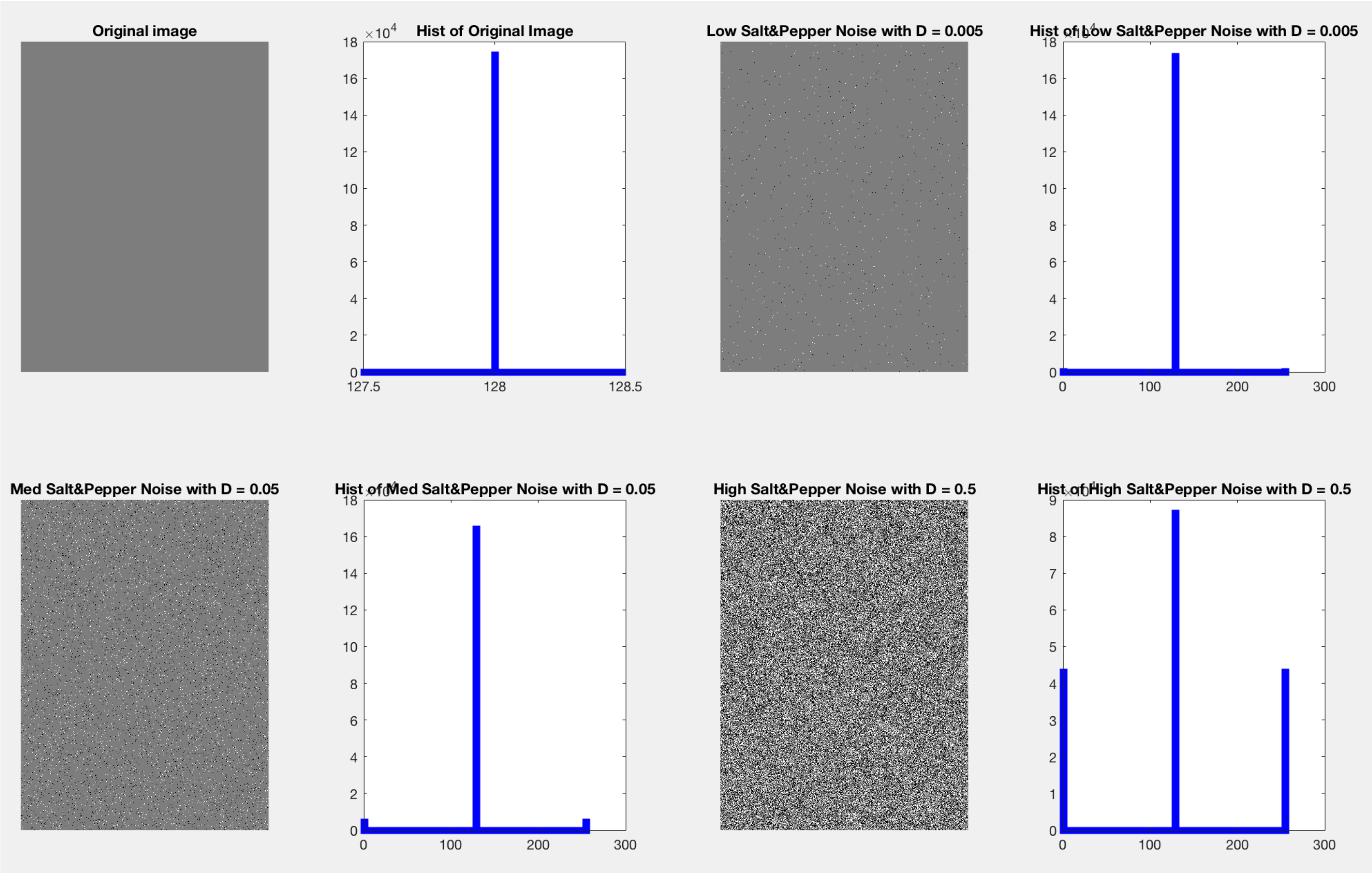


Figure7: Constant image and Salt&Pepper noises added to it.

MATLAB Camera + Noise

- ‘fspecial’ function was used to approximate linear motion of a camera by length of 15 and angle of 10 degrees. Periodic and Random Noises then were added to the image.

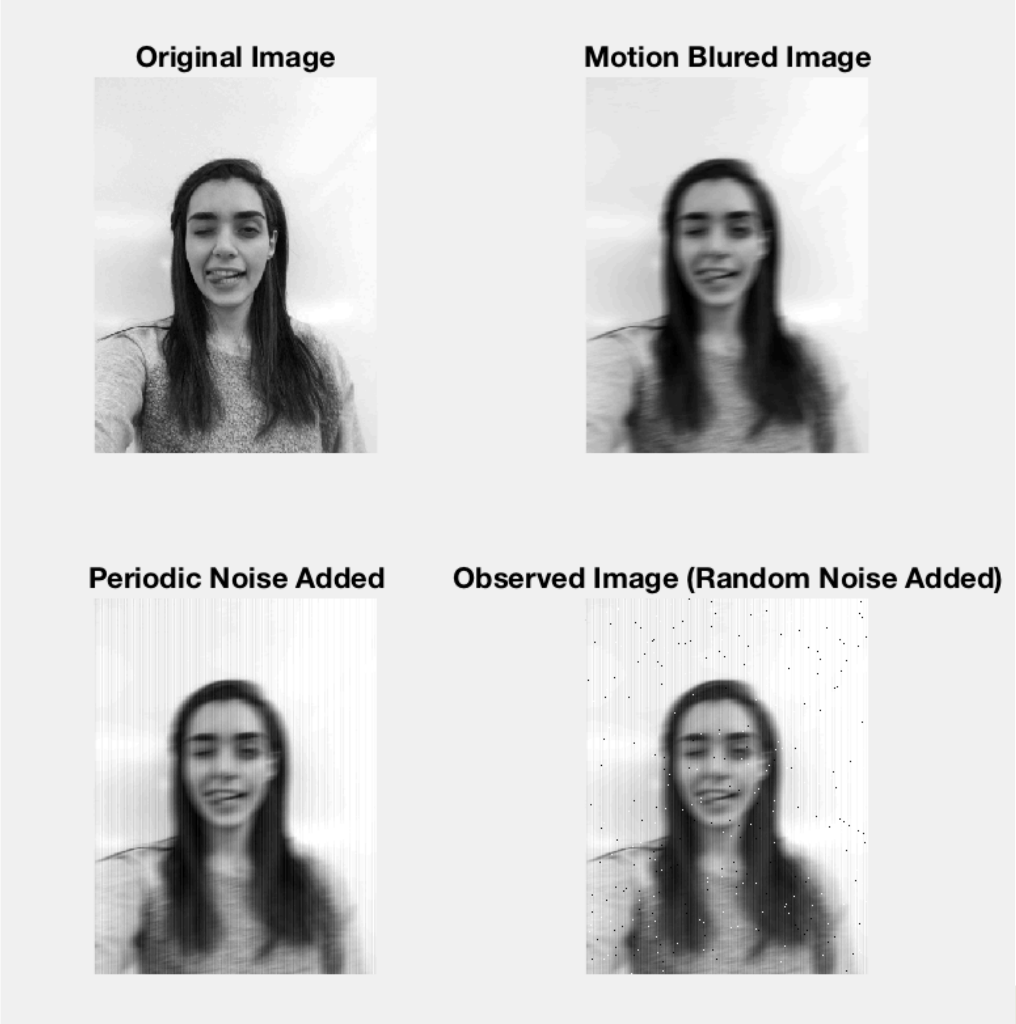
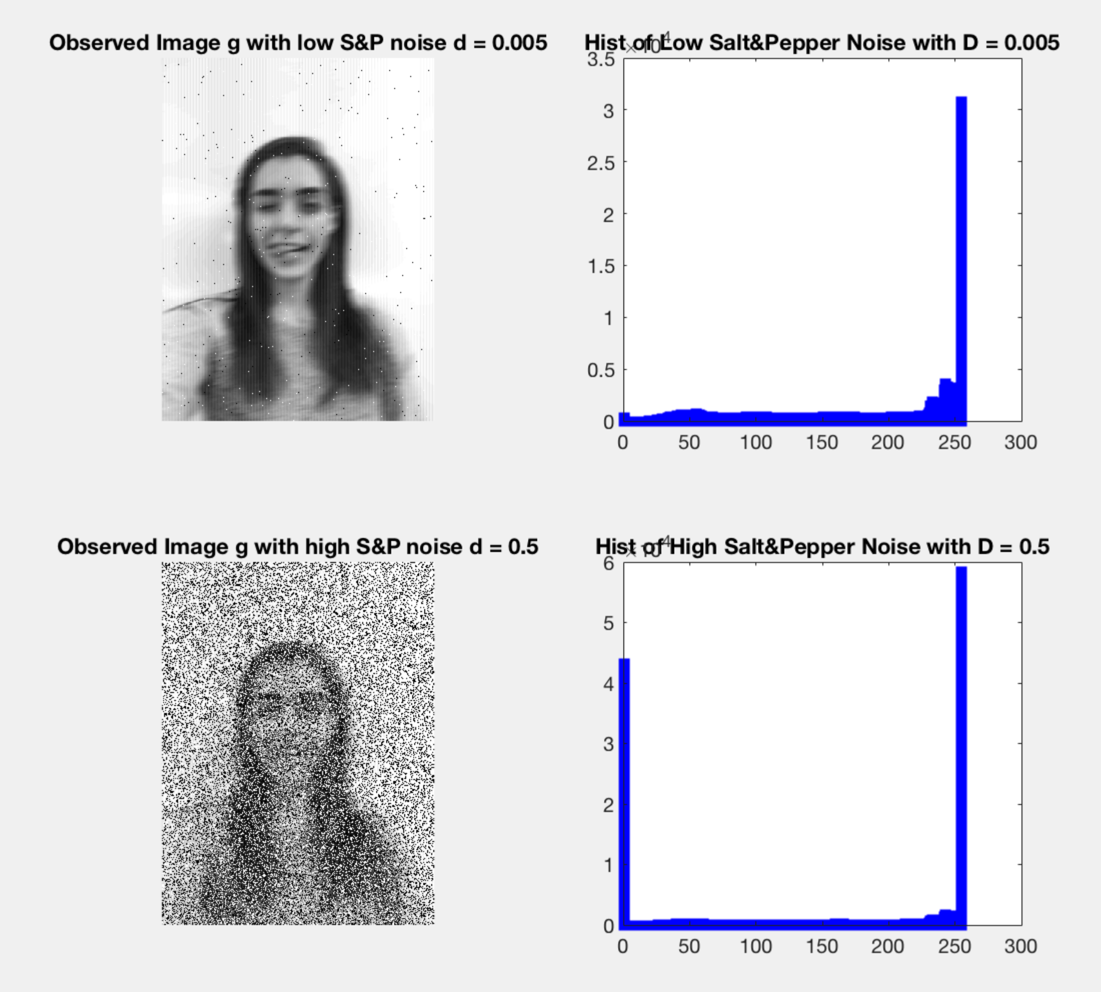


Figure8: Motion Blurred, periodic and Random Noise added to the mugshot.

Restoration

- To reconstruct the image, I tried to remove Random Noise first. By looking at the image and its Histogram I could observe that image included salt and pepper noise.

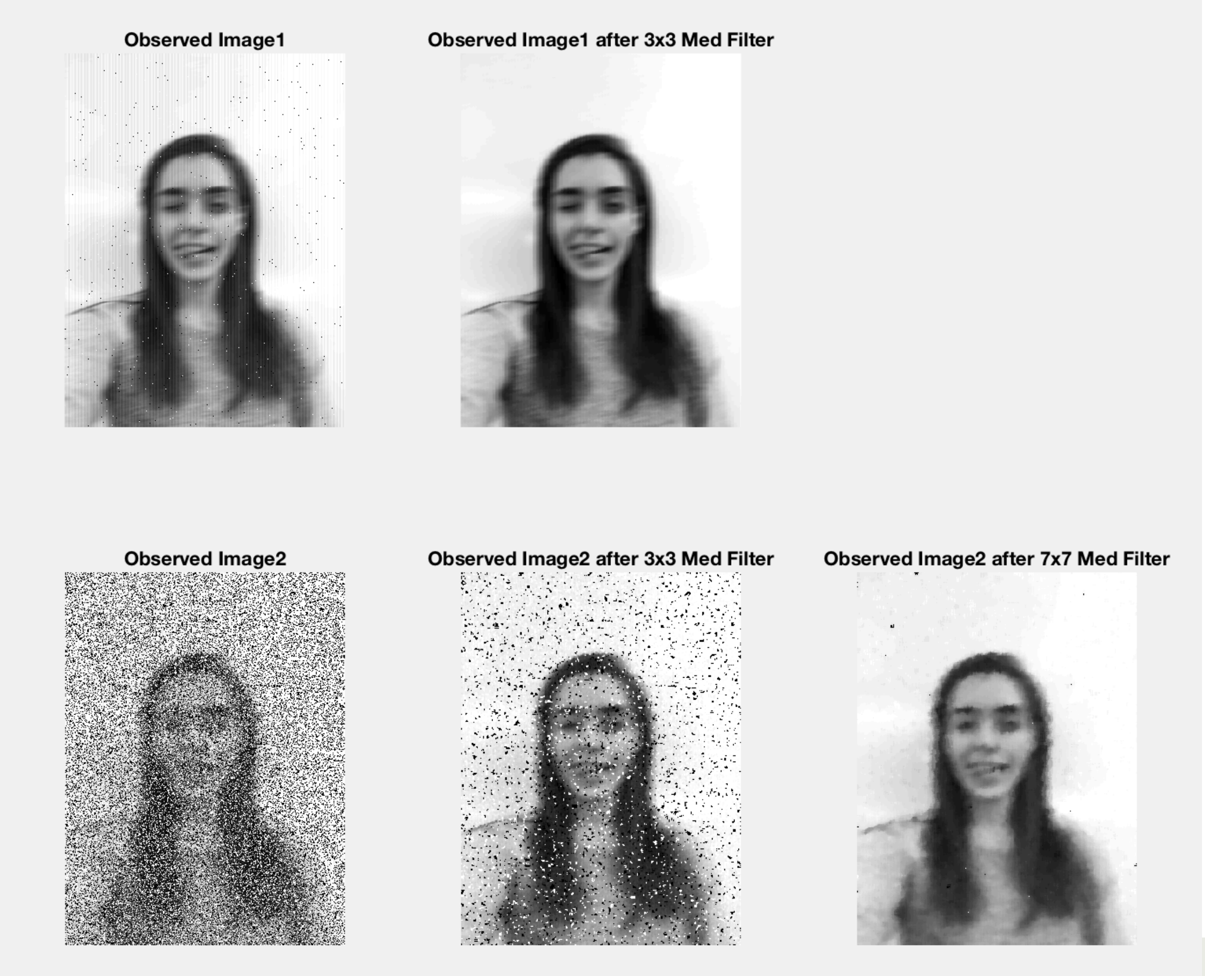


- S&P noise was easier to identify when its intensity was high. However, I was still able to identify low intensity S&P noise from both the image and its Histogram.

Figure9: Observed images with their Histograms

Restoration

- After identifying the random noise, I decided to use Median filter to reduce the effect of this noise. Median filters are a good choice for S&P noise because they provide effective noise reduction with less blurring compared to other linear smoothing filters such as ‘mean’ filters.



- To remove the effect of S&P noise with high intensity, I had to use a larger Median filter. 7x7 filter reduced the noise noticeably but caused more blurring.

Figure 10: Image with S&P noise and result of Median filter

Restoration

- Periodic noise was analyzed and filtered using the DFT of the image. The basic idea is that periodic noises appear as concentrated bursts of energy in the Fourier transform, at locations corresponding to the frequencies of the periodic interference.

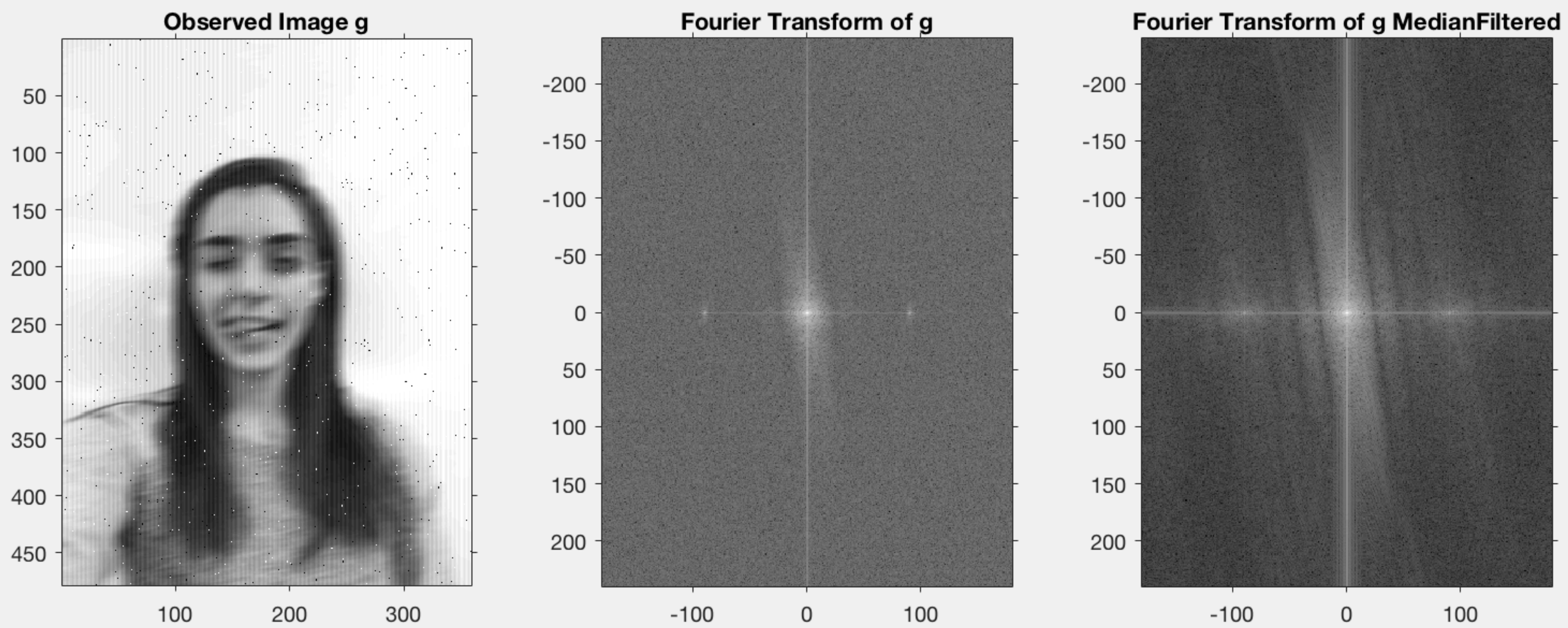


Figure 11: Observed image and its DFT before and after Med Filter. Note that the two bright peaks at +ve and –ve horizontal frequencies are caused by periodic noise.

Restoration

- To remove periodic noise I created 3 following Notch filters with different cutoffs at observed frequencies (u = -90 and 90) and compared their effects.

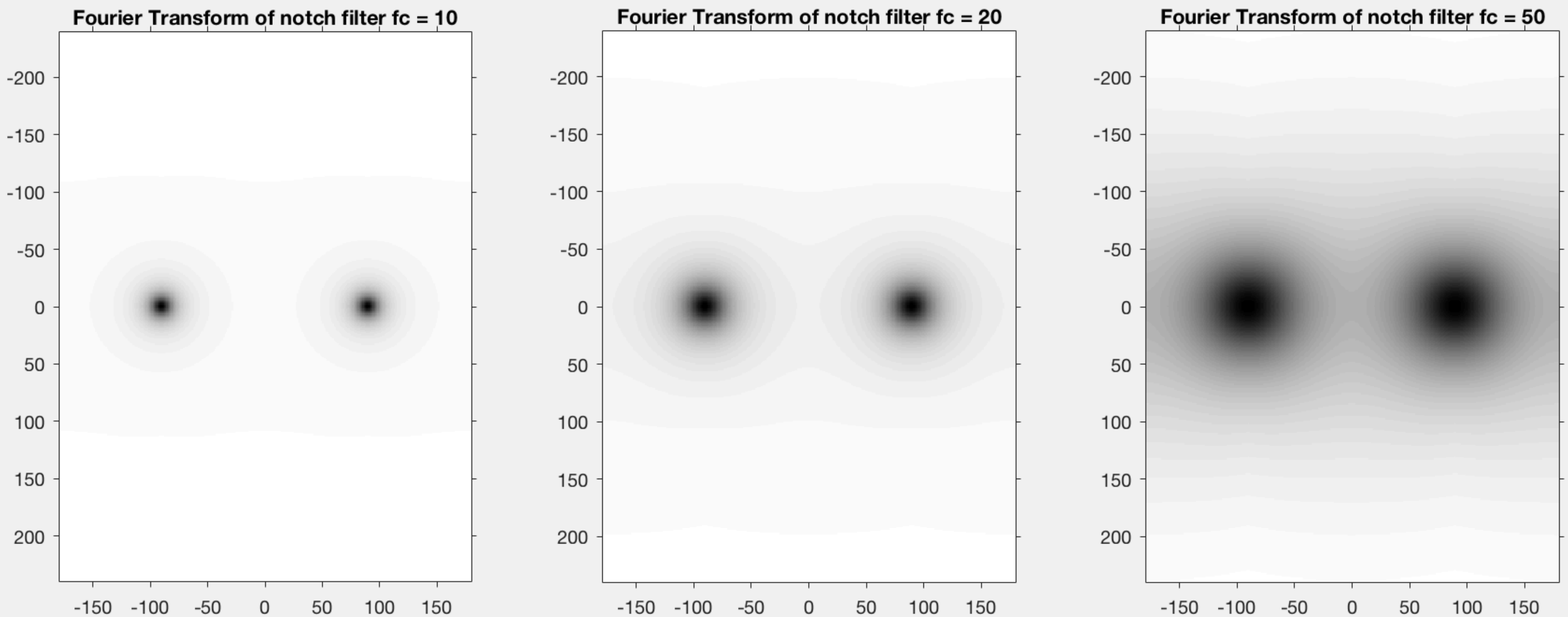
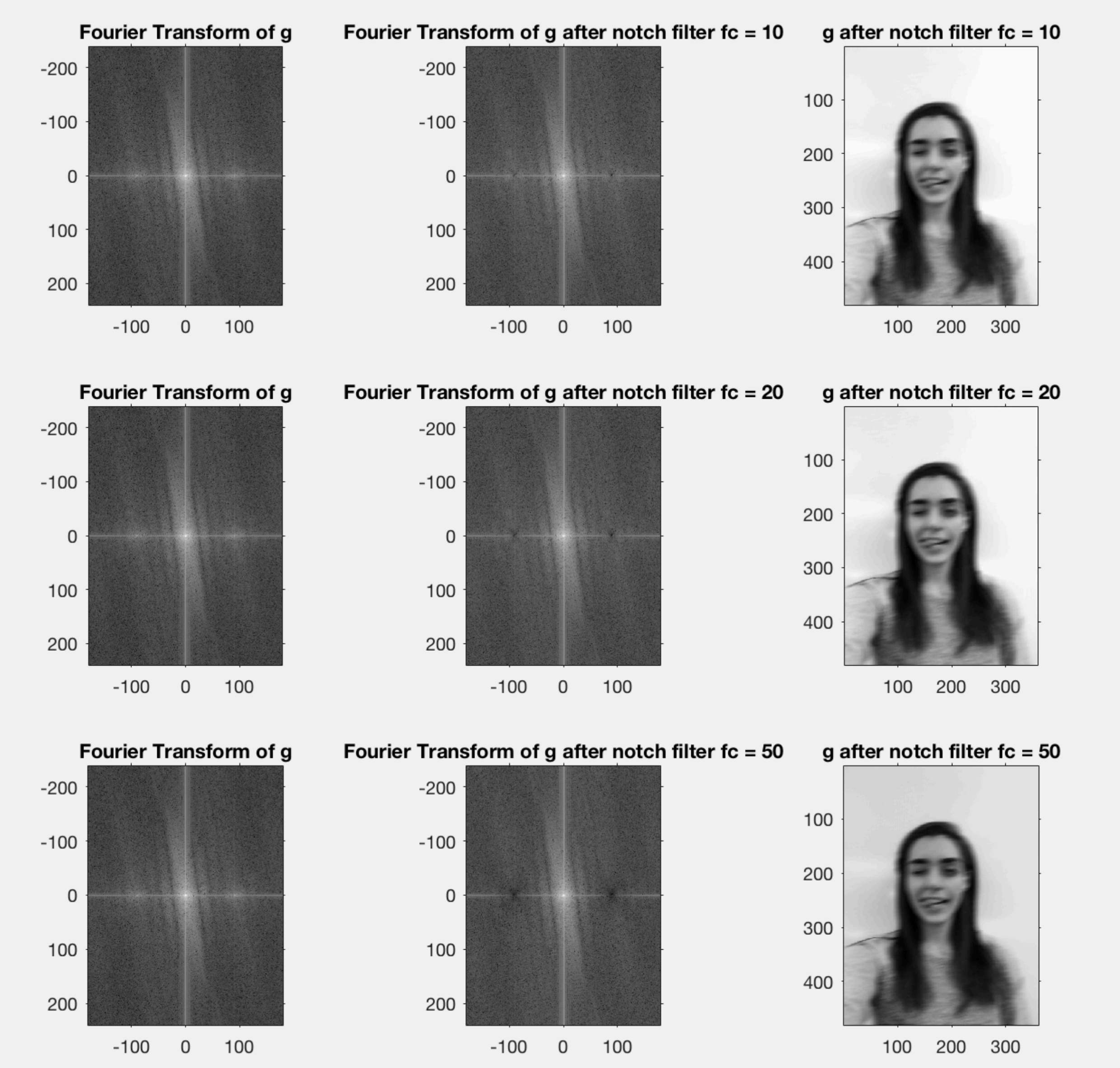


Figure 12: 3 Notch filters at u = -90 and 90

Restoration



- As we can see, cutoff of 10 didn't completely remove the periodic noise while cutoff of 50 removed the periodic noise but also changed the histogram of the image (image appears darker). Therefore I used the second Notch filter (cutoff = 20) for restoration of the image

Figure 13: Removing periodic noise using notch filter

Restoration

- I also tried removing the random and periodic noises in a different orders. Their results were very similar so I just used the first one for restoration of my image.

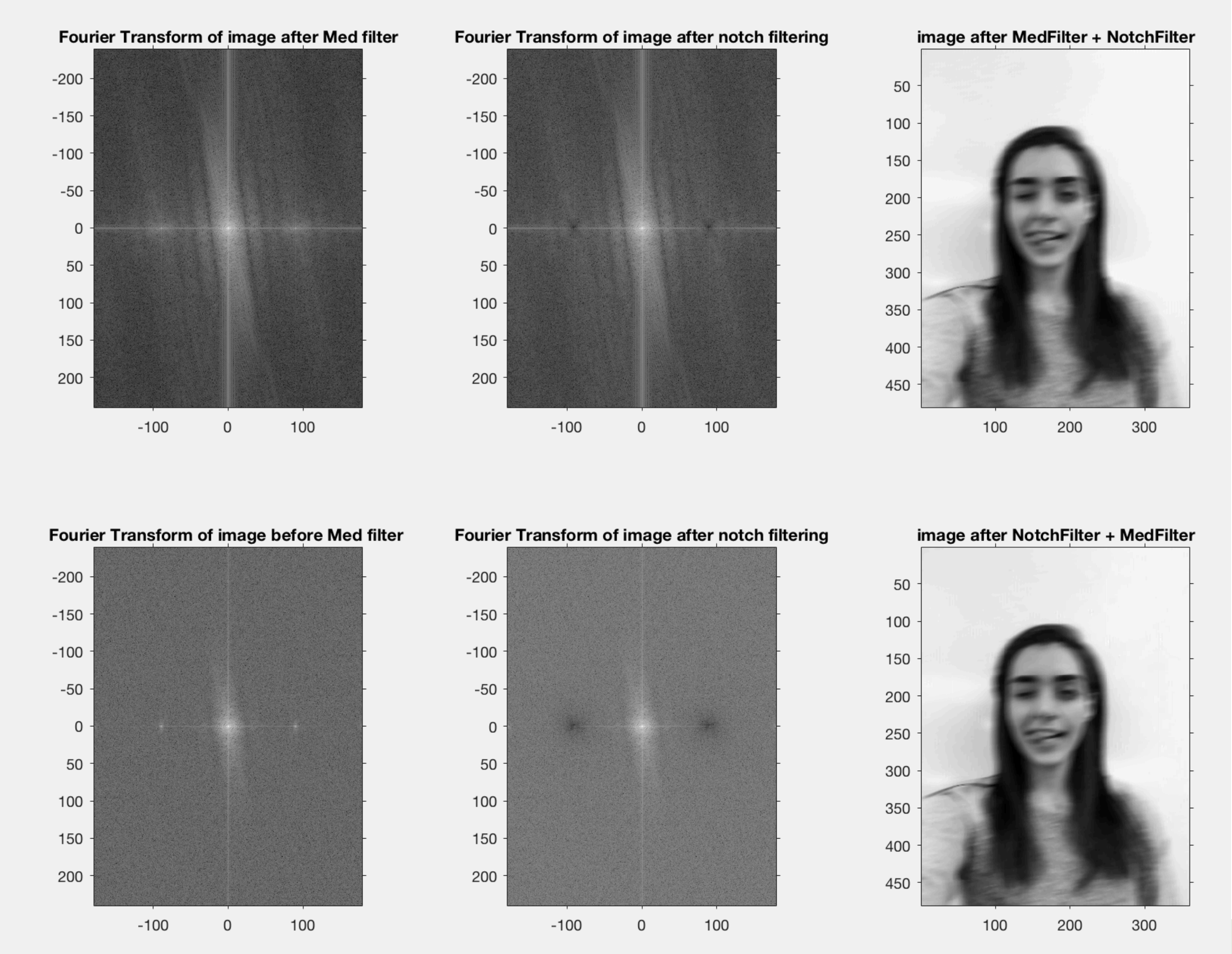


Figure 14: Reducing Random and Periodic noises in different orders.

Restoration

- I used MATLAB’s Wiener filter ‘deconvwnr ’ to de-convolve f and g. This method considers images and noises as random variables, and finds an estimate of the uncorrupted image *f* such that the mean square error between them is minimized. It uses the following formula:

$$
\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v) \\
&= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)
\end{aligned}
$$

Where,

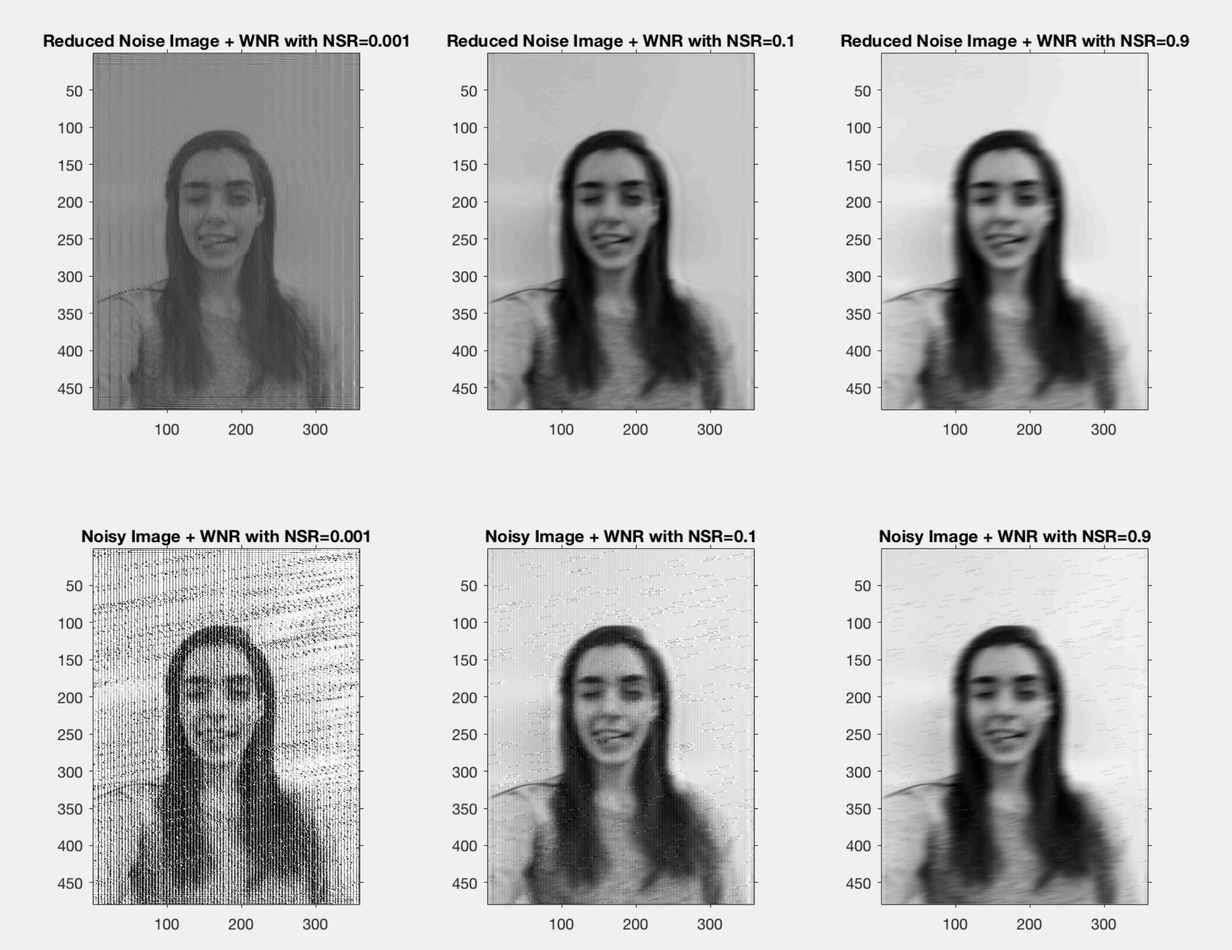
$$
\begin{aligned}
&H(u,v) = \text{degradation function} \\
&H^*(u,v) = \text{complex conjugate of } H(u,v) \\
&|H(u,v)|^2 = H^*(u,v)H(u,v) \\
&S_\eta(u,v) = |N(u,v)|^2 = \text{power spectrum of the noise} \\
&S_f(u,v) = |F(u,v)|^2 = \text{power spectrum of the undegraded image}
\end{aligned}
$$

- Note that if the noise is zero, then the noise power spectrum vanishes and the Wiener filter reduces to the inverse filter.

Source: R. C. Gonzalez, R. E. Woods, & S. L. Eddins. Digital Image Processing Using MATLAB

Restoration

- I tried the Wiener Filter with different noise to signal ratios on the observed image g both before using the previous noise reduction steps and after noise reduction steps to find the resorted image closest to the original image. I could observe that the final result is better when using Wiener of noise-reduced image.

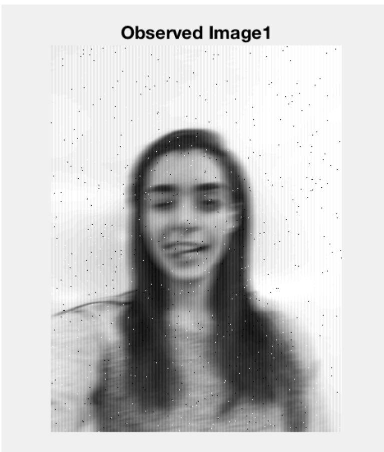


- NSR = 0.01 didn't restore the image noticeably while NSR=0.9 reduced noise but also made the image blurry. So I chose NSR = 0.1 for restoration of my mugshot.

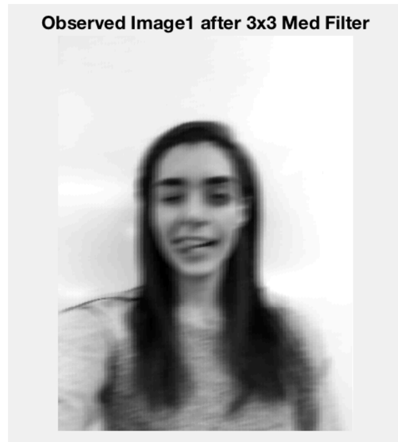
Figure 15: Wiener Filter with different NSR before and after noise reduction

Conclusion

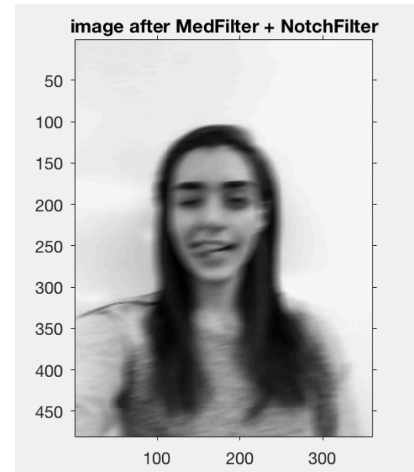
- The following steps were taken in this assignment to improve an observed image taken by a camera with random and periodic noises.



Median Filter



Notch Filter



Wiener Filter