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Automatica 41 (2005) 1099–1103
www.elsevier.com/locate/automatica

Technical communique

VSS global performance improvement based on AW concepts☆

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Received 22 March 2004; received in revised form 17 November 2004; accepted 20 January 2005
Available online 16 March 2005

Abstract

The influence of the reaching mode on the global performance of variable structure systems (VSS) undergoing sliding regimes is stressed. A comparative analysis between the behaviour during this reaching mode of operation and the problem of windup is realized. Based on the similarities between both control problems, some tools of the control theory of constrained linear systems are exploited to improve the reaching mode of VSS.
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Keywords: Variable structure control; Sliding mode; Reaching mode; Windup; Constrained systems

1. Introduction

Variable structure systems (VSS) undergoing sliding motions (SM) have many attractive properties such as robustness to matched disturbances and reduced closed-loop dynamics (Utkin, Guldner, & Shi, 1999). Actually, a reaching phase (RM) precedes the establishment of the desired SM. Even though the latter has been more discussed in the literature, the former is not less important when the global performance is considered (Ryan & Corless, 1984). In fact, a long RM may seriously deteriorate the transient response. In the survey (Hung, Gao, & Hung, 1993), different approaches to the RM problem are summarized. Despite some interesting properties, these approaches focus on the surface coordinate dynamics instead of on the system dynamics, do not take into consideration the limits of the actuators, are not applicable when the control signal can only take some discrete values (such as in power electronics where the control signal represents the state of a switch) and are particular or intuition-based solutions (Hung et al., 1993; Mantz, De Battista, & Puleston, 2001).

The goal of this note is to draw a parallel between the RM and another control problem extensively discussed in recent years: reset-windup (RW). The significance of this correlation lies in the possibility of applying the strong theory of constrained systems to the RM problem. The basic idea is to shape the controller state, thus facilitating the establishment of the SM. In this context, a pair of compensation strategies to improve the RM in VSS is derived from classical anti-windup (AW) algorithms.

2. Problem formulation and main results

Fig. 1 sketches a VSS with the proposed RM compensation. P is the process to be controlled. Δ depicts the parametric uncertainties. Sw, that switches between the input signals u^+ and u^-, is driven by the output σ of the LTI controller $K(s) = C(sI - A)^{-1}B + D$. Let us assume for a moment that the RM compensation Λ of K is inactive, i.e. $\hat{K} = K$. The input to K is $v = \mathrm{col}(r, y, x_p)$, where r is the set-point, and x_p and y are the state and output of P (some nonlinear outputs $y = f(x_p)$ can be deliberately defined as inputs to K to address the case of nonlinear processes). The state of

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate editor Wei Lin under the direction of Editor P. Van den Hof. This work was supported by ANPCyT, CICpBA, CONICET and UNLP.

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0005-1098/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2005.01.007

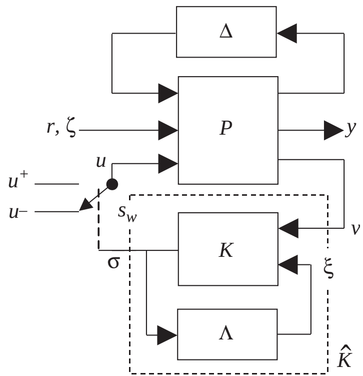

Fig. 1. VSS with proposed RM compensation.

$K(x_k)$ may include dynamic expansions to reject steady-state disturbances ζ (Bühler, 1986; Mantz, Puleston, & De Battista, 1999). On the contrary, to reduce chattering (Sira-Ramírez, 1993) the dynamic expansion must be inserted at the input u to P. Hereinafter, the following conventional notation is used (where only D has meaning in the case of static feedback):

$$K = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c|c|c} A & B_r & B_y & B_p \\ \hline C & D_r & D_y & D_p \end{array}\right]. \tag{1}$$

2.1. Similarity between RM and RW problems

VSS design is carried out in two steps. On the one hand, the sliding surface $\sigma = 0$ is chosen to fulfil control specifications such as dynamic behaviour, robustness to state disturbances and model uncertainties, etc. On the other hand, the inputs (u^+, u^-) and the switching logic are selected to enforce the state convergence towards $\sigma = 0$ (Ryan & Corless, 1984).

During ideal SM, u switches at infinite frequency between u^+ and u^-. This discontinuous action produces the same dynamic behaviour as a fictitious continuous input signal, the so-called equivalent control $u_{\mathrm{eq}}(v, x_k)$ derived from the invariance condition ($\sigma = 0,\ \dot{\sigma} = 0$). During SM, the equivalent process input u_{eq} is constrained between the limit signals $u^- < u_{\mathrm{eq}} < u^+$ (Utkin et al., 1999). On the contrary, during RM the switch is fixed at one position and the process input is either u^+ or u^-. This means that the P–K loop is open and the process dynamics evolves independently of the controller. This lack of correspondence during RM can degrade the global performance of VSS. This degradation is worse when the state reaches $\sigma = 0$ outside the sliding domain. When this occurs, the state trajectory cannot be confined to the surface and crosses it. Hence, the RM, i.e., the open-loop operation, is prolonged.

From the previous discussion, a noticeable correspondence arises between causes and effects of RM and RW problems. Actually, RW is an undesirable transient behaviour caused by the inconsistency between the process input and the controller state of continuous feedback systems subjected to restrictions.[1] Effectively, the system performs in open loop because of this inconsistency, possibly leading to large overshoots and long settling time (Peng, Vrančić, & Hanus, 1996; Kothare, Campo, Morari, & Nett, 1994). Moreover, similar to what happens in VSS, the controller design for constrained linear systems is commonly developed following a two-step procedure (Peng et al., 1996; Kothare et al., 1994). Firstly, a controller to guarantee the control requirements is designed ignoring the physical limitations. Then, the AW compensation is incorporated to the previous controller satisfying the following specifications:

(1) stability,
(2) correction only when the limitation is active,
(3) graceful degradation with respect to the unrestricted control system.

Based on the close connection between these problems, classical solutions and more recent progresses in the control theory of constrained linear systems can potentially be extended to improve the RM of VSS.

2.2. RM compensation scheme

In this note, the feedback correction block Λ drawn in Fig. 1 is proposed to improve the RM. This compensation approach closely follows the AW scheme encompassing most of the existing AW methods (see Kothare et al., 1994). Analogously, different RM algorithms obtained as a generalization of AW methods can be seen as particular designs of Λ. To assure the compensated controller $\hat{K}$ can also be realized as an LTI system, Λ is assumed causal and LTI. Based on the understanding that the sliding surface has been designed according to the control specifications, the correction of the state and output of K (ξ_1 and ξ_2, respectively) must only be active during RM, i.e.

$$\sigma(t) = 0 \quad \Rightarrow \quad \xi(t) = 0. \tag{2}$$

The sliding dynamics obtained by the discontinuous action, as well as by u_{eq}, can also be accomplished by a saturated actuator with gain $k \to \infty$ (Utkin et al., 1999). This allows defining, in the context of RM, a saturation error equivalent to the one used in all AW methods (Peng et al., 1996; Kothare et al., 1994)

$$e = \lim_{k \to \infty} \left(\sigma - \frac{u}{k}\right) = \sigma. \tag{3}$$

[1] Although RW is usually associated with the controller dynamics and actuator constraints, it may also be caused by slow or unstable dynamics of the process and its own constraints even when static controllers are used (Hippe & Wurmthaler, 1999).

Then

$$\xi = \Lambda \lim_{k\to\infty}\left(\sigma - \frac{u}{k}\right) = \begin{bmatrix}\Lambda_1\\ \Lambda_2\end{bmatrix}\sigma, \tag{4}$$

where according to (2), Λ_1 and Λ_2 are static gains. The loop K–Λ results in $\hat{K}(s) = [I - V(s) \;\; U(s)]$, where

$$V = \left[\begin{array}{c|c} A - H_1C & -H_1 \\ \hline H_2C & H_2 \end{array}\right], \; U = \left[\begin{array}{c|c} A - H_1C & B - H_1D \\ \hline H_2C & H_2D \end{array}\right]$$

are the left coprime factors of $K(s) = V^{-1}(s)U(s)$. In addition, $H_1 = \Lambda_1(I + \Lambda_2)^{-1}$, $H_2 = (I + \Lambda_2)^{-1}$.

Hereinafter, illustrative examples of RM design are derived from conventional AW strategies. They exploit the well-known concepts of observers (Åström & Rundqwist, 1989), and of the conditioning technique (Hanus, Kinnært, & Henrotte, 1987; Walgama, Rönnbäck, & Sternby, 1992). These RM methods fit within the proposed correction scheme (Fig. 1) and can be seen as particular selections of parameters H_1 and H_2 of Λ.

2.3. Observer-based RM compensation

Here, it is proposed to improve the conventional RM by estimating the state of K as suggested by Åström and Rundqwist in the observer-based AW method (Åström & Rundqwist, 1989). Then, this state estimation is used in $\hat{K}$:

$$\begin{aligned}\dot{\hat{x}}_k &= A\hat{x}_k + B_r r + B_y y + B_p x_p + L\left(\lim_{k\to\infty}\frac{u}{k} - \sigma\right),\\ \sigma &= C\hat{x}_k + Dv = C\hat{x}_k + D_r r + D_y y + D_p x_p.\end{aligned} \tag{5}$$

Consequently, the compensated controller $\hat{K}$ results in

$$\hat{K} = \left[\begin{array}{c|c|c|c} A - LC & B_r - LD_r & B_y - LD_y & B_p - LD_p \\ \hline C & D_r & D_y & D_p \end{array}\right].$$

After trivial algebra, it is verified that this RM approach is characterized by parameters $H_1 = L$ and $H_2 = I$.

In order to evaluate the effects of this compensation, the dynamics of σ for the conventional RM

$$\dot{\sigma} = C(A\hat{x}_k + Bv) + D\dot{v}, \tag{6}$$

and for the proposed observer-based RM

$$\dot{\sigma} = -CL\sigma + C(A\hat{x}_k + Bv) + D\dot{v} \tag{7}$$

are compared. It is seen that the proposed correction introduces the term $-CL\sigma$ that improves the convergence of the RM provided $CL > 0$. In fact, the time derivative of the Lyapunov function candidate $V = \sigma^2$, has a negative term $-CL\sigma^2$ that is absent in conventional RM.

Example 1. Consider the system

$$\begin{cases}\dot{x}_{p1} = x_{p2} + \zeta,\\ \dot{x}_{p2} = 25u.\end{cases}$$

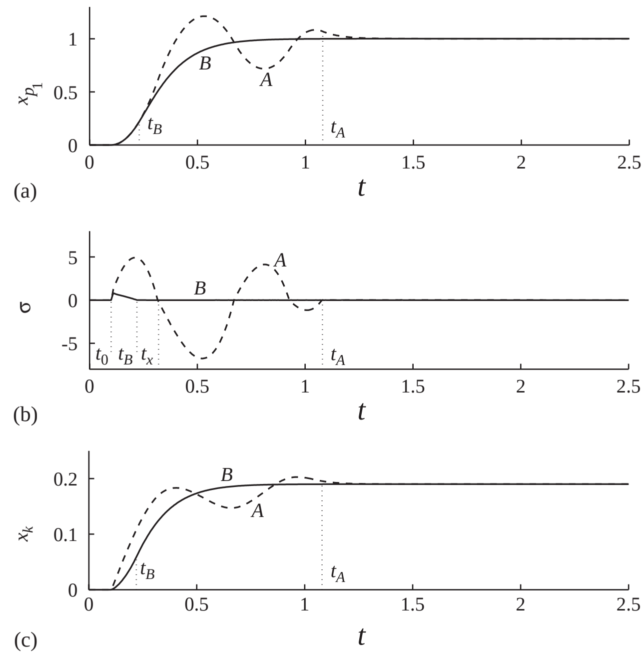

Fig. 2. Example 1 with conventional (dashed) and observer-based (solid) RM. (a) Step response of the controlled variable x_{p_1} (b) Surface coordinate σ (c) Integral state x_k.

To reject a constant ζ, an integral state x_k is included in K. Furthermore, the following switching law is proposed:

$$\sigma = r - k_p^\top x_p + k_k x_k \begin{cases} u = u^+ = +1 & \text{if } \sigma > 0\\ u = u^- = -1 & \text{if } \sigma \leqslant 0,\end{cases} \tag{8}$$

where the sliding gains k_p and k_k are selected to assign both the SM closed-loop eigenvalues at -10.

Fig. 2 shows the performance of the VSS using conventional RM (Case A) and observer-based RM compensation (Case B). The notably dissimilar responses can be exclusively ascribed to the difference between both RM approaches.

Case A: Let us first analyze the low proficiency achieved with conventional RM (Fig. 2, dashed line).

(i) After a set-point step at $t = t_0$, $\sigma(x)$ becomes positive (Fig. 2b). Then, the actuator provides the maximum control effort u^+. This control is insufficient to maintain the SM. So, the P–K loop opens. It is observed that x_k grows (Fig. 2c) and σ increases further in consequence. That is, the actuator oversaturates as in typical windup behaviour, and the trajectory moves further away from $\sigma = 0$ (Fig. 2b).

(ii) At $t_x = 0.32\,\mathrm{s}$ the state reaches again $\sigma = 0$, but a SM is not established. Evidently, there is no correlation among the state variables to verify the SM existence condition $-1 < u_{\mathrm{eq}} < 1$. Hence, the state trajectory crosses and moves away from the surface (Fig. 2b).

(iii) The previous situation is repeated several times until the SM is at last established at $t_A = 1.08$ s. Clearly, the RM dominates the global performance of the VSS. Hence the undesirable transient response (Fig. 2a).

Moreover, it can be verified that the RM may be unstable if the control effort is reduced.

Case B: A significant improvement of the VSS response is accomplished when the RM compensation is applied (Fig. 2, solid line). In contrast with conventional RM, the proposed approach reduces σ (Fig. 2b) despite the increasing x_k (Fig. 2c). This allows a fast convergence towards $\sigma = 0$ where the SM existence condition $-1 < u_{\text{eq}} < 1$ holds. Hence, the sliding regime is established at $t_B = 0.23$ s. Now, the SM dominates the transient response. Then, the VSS response (Fig. 2a) is practically in accordance with the prescribed SM dynamics (i.e. with the pair of eigenvalues at -10).

2.4. RM based on the concept of realizable reference

The concept of realizable reference, introduced by Hanus et al. in the context of windup (Hanus et al., 1987; Peng et al., 1996), is used here to derive RM algorithms. The basic idea is to shape the reference with the aim of restoring consistency between K and the input to P. The modified reference is called realizable reference (r_{r}). The proposed conditioning is such that if r_{r} had been applied to K, the system would have always operated in SM. Thus, exciting K described by (1) with r_{r} yields

$$\begin{aligned} \dot{x}_k &= Ax_k + B_{\text{r}}r_{\text{r}} + B_y y + B_p x_p, \\ 0 &= Cx_k + D_{\text{r}}r_{\text{r}} + D_y y + D_p x_p. \end{aligned} \tag{9}$$

From (1) and (9), r_{r} verifies $\sigma = D_{\text{r}}(r - r_{\text{r}})$. Hence, $r_{\text{r}} = r - D_{\text{r}}^{-1}\sigma$.

In the following, two RM approaches are deduced. The first one uses r_{r} instead of r in the state equation of K (as in Hanus et al., 1987). In the other one, the reference is filtered before performing the conditioning technique (Walgama et al., 1992).

RM based on the conditioning technique: After replacing r in the state equation of K (1) by $r_{\text{r}} = r - D_{\text{r}}^{-1}\sigma$, the compensated controller results in

$$\hat{K} = \left[\begin{array}{c|c|c|c} A - B_r D_r^{-1} C & 0 & B_y - B_r D_r^{-1} D_y & B_p - B_r D_r^{-1} D_p \\ \hline C & D_r & D_y & D_p \end{array}\right],$$

where the parameters $H_1 = B_{\text{r}} D_{\text{r}}^{-1}$ and $H_2 = I$ characterizing the RM correction block Λ can be recognized.

Evaluating the dynamics of σ for the conventional RM (6) and for this approach

$$\dot{\sigma} = -CBD_{\text{r}}^{-1}\sigma + C(A\hat{x}_k + Bv) + D\dot{v}, \tag{10}$$

it is seen that an additional term appears as in the case of the observer-based RM. Comparing the corresponding parameters H_1 and H_2, or alternatively (7) and (10), it follows that this RM strategy can be seen (in the context of the compensation scheme of Fig. 1) as a particular solution of the observer-based RM with $L = B_{\text{r}} D_{\text{r}}^{-1}$.

Case C (*RM based on the generalized conditioning technique. Application to a statick*): Walgama et al. (1992) suggest a generalization of the AW method proposed by Hanus et al. In this technique, conditioning is performed on a filtered reference signal instead of on the reference itself. Based on this idea, another RM compensation is proposed. In contrast with the previous methods which adapt x_k, this approach allows one to address the case of plant windup with static controllers (Hippe & Wurmthaler, 1999). Hence, attention is focused here on sliding controllers comprising static feedback of the process state. The extension to dynamic controllers is immediate. In the context of VSS, the AW Walgama's proposal consists in (1) the inclusion of a filter F to smooth the reference signal (this is sometimes used in VSS), and (2) the conditioning of the filter state x_{f} as function of σ (this allows avoiding conservative designs of F).

Then, considering static feedback of x_p, K reduces to

$$\sigma = r_{\text{f}} + D_h \begin{bmatrix} y \\ x_p \end{bmatrix} \tag{11}$$

where $D_h = D_{\text{r}}^{-1}[D_y \ D_p]$ and r_{f} is the output of F (with matrices A_{f}, B_{f}, C_{f} and D_{f}). Applying realizable reference concepts, r is replaced by $r_{\text{r}} = r - D_{\text{f}}^{-1}\sigma$ in the state equation of F. It then follows the controller

$$\hat{K} = \left[\begin{array}{c|c|c} A_f - B_f D_f^{-1} C_f & 0 & -B_f D_f^{-1} D_h \\ \hline C_f & D_f & D_h \end{array}\right],$$

where $H_1 = B_{\text{f}} D_{\text{f}}^{-1}$ and $H_2 = I$ can be recognized.

Example 2. Consider the system

$$\begin{cases} \dot{x}_1 = \dfrac{1}{2}(1 + x_1^2)x_2, \\ \dot{x}_2 = \dfrac{1}{J}u_c = u \end{cases}$$

representing the kinematics and dynamics of a single axis jet-controlled spacecraft with respect to a skewed axis (Dwyer & Sira-Ramírez, 1988; Mantz et al., 2001). A desirable reduced-order dynamics for the attitude error would be an LTI system with exponential rate of decay

$$\dot{x}_1 = -\lambda(x_1 - \theta), \quad \lambda = 1 \tag{12}$$

with θ being the desired orientation. The sliding surface is designed accordingly as

$$\sigma = -y - \lambda(x_1 - \theta) = 0 \tag{13}$$

with $y = \frac{1}{2}(1 + x_1^2)x_2$, which corresponds to a static K. The bang–bang controller $u = 1.5 \cdot \text{sign}(\sigma)$ is used for the RM. To apply the RM compensation discussed above, a reference filter is included. Although a faster filter could be chosen,

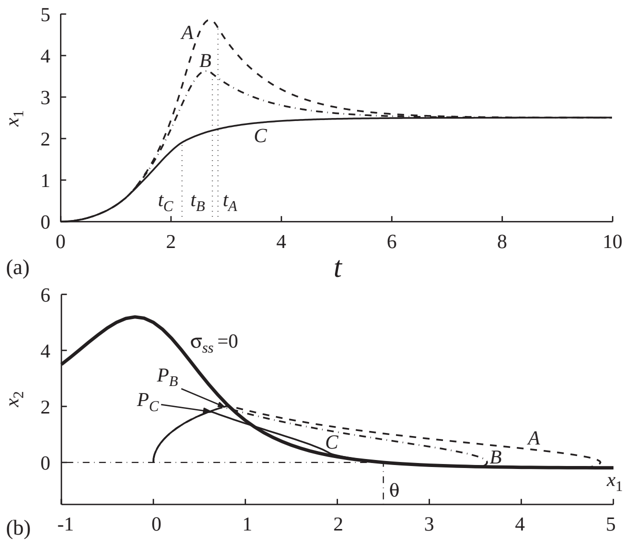

Fig. 3. (a) Controlled variable and (b) state space trajectories of the VSS with conventional RM (dashed), with conventional RM and filtered reference (dot–dashed) and with RM based on the generalized conditioning technique (solid).

a first order one with eigenvalue $A_f = -2$ has been used here for illustrative purposes.

Fig. 3a displays the step response of the VSS using conventional RM (Case A), conventional RM with filtered reference (Case B), and RM based on the generalized conditioning technique (Case C). The reaching time is indicated for each case. A 100% overshoot is observed when conventional RM is used (dashed line). This overshoot is slightly reduced by filtering the reference (dot–dashed line). Finally, a remarkable improvement is noted when conditioning is applied to x_f (solid line).

Fig. 3b portrays the trajectories in the state space for the three cases. Surface (13), labelled $\sigma_{ss}=0$, is drawn with thick trace. Actually, for Cases B and C, (13) depicts the surface in steady state. In fact, $\theta = 2.5$ should be replaced by the output of F which is changing. This explains the anticipated breakpoints P_B and P_C. They are the points where trajectories B and C reach the corresponding time varying surfaces. Then, the proposed RM compensation may be interpreted as a transient change in $\sigma = 0$ that helps the trajectories to reach it.

3. Conclusions

In this work, it was shown that causes and effects of RM problems have common aspects with windup. The close relation between these problems allows the extension of the results in AW compensation to solve RM problems. Based on this new RM interpretation, some RM strategies were derived from conventional AW algorithms.

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