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Control signal selection for damping oscillations with wind power plants based on fundamental limitations

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***Abstract*—Transmission system operators are increasingly demanding that wind power plants (WPPs) contribute to enhance stability of power systems. When WPPs act as power system stabilizers (PSSs), the control and measured signals used to damp the oscillations may be located far away from the point where the oscillations are originated, typically at the synchronous generators. The use of only local measurements would imply a more robust system not requiring fast communication systems. The output signal used by the PSS in a WPP can be active or reactive power. The optimal combination of input and output to improve the system performance requires a detailed analysis. In this article, a novel criterion to select input-output signals used by a PSS based on WPPs is presented. Unlike previous results, the proposed criterion considers explicitly local and remote signals in the analysis. Using fundamental design limitations and controllability and observability concepts, the criterion is able to identify the most adequate pair of input-output local signals without designing the controller. The application of the proposed selection criterion is illustrated in a three synchronous machine system with a WPP connected.**

***Index Terms*—Control design limitations, disturbance response, geometric measures, input-output signal selection, power system oscillation damping, wind power plants (WPPs)**

I. Introduction

THE penetration of renewable energies is rapidly increasing as a result of global environmental concern [?]. Within these new clean energy sources, wind power is becoming one of the most important in electricity share [?]. Under this scenario, wind power plants (WPPs) are now required not only to capture energy from the wind but also to support power systems in maintaining the complete system stable. For example, some expected contributions from WPPs include: fault-ride through capability [?], frequency response [?], reactive power regulation [?] and damping of power system oscillations [?], [?].

The location of WPPs is a relevant factor in the capability of power system stabilizers (PSSs) to damp oscillations [?]. However, the location of wind power cannot be defined by the requirement of the power system contributions. The location is based mainly on energy capture possibilities and other economic aspects. As a consequence, WPPs may be far away from synchronous generators in which power system oscillations arise. For this reason, signals measured at the connection point of the WPP (local signals) may not be the most adequate ones to detect oscillations at remote locations where the synchronous generators are placed [?]. Recently, wide-area measurement systems (WAMSs) have emerged again allowing the use of remote signals in damping system based on WPPs and thus providing a solution to the previous issues [?]. However, WAMSs present some drawbacks in terms of signal delays and reliability [?], [?], aspects that can produce negative effects on oscillation damping.

In recent years, several papers have proposed different methodologies to determine the most adequate control signals and location for a PSS; they are mainly focused on either FACTS, HVDC-links, or both at the same time [?], [?], [?], [?]. However, these analyses cannot be used for WPPs, since their location is not determined for oscillation damping purposes. Furthermore, in WPPs, the analysis must consider the fact that the disturbances and the remote signals to be damped might not be available, making local signals preferred. Therefore, the plant to be considered may be understood as the general control configuration (shown in Fig. 1), in which the oscillating signal cannot be fed back into the PSS. This control scheme, which imposes additional constraints on the controller design, results in limitations in the capability to attenuate disturbances [?], [?].

The aim of this paper is to propose an input-output selection criterion for the design of PSS for WPPs. The criterion especially considers the control limitations imposed by the impossibility of direct measurement of the signals to be controlled. Unlike previous works, the proposed criterion takes into account the controllability and observability of the local input-output pairs and the lower bound on the disturbance attenuation caused by the use of local signals in the PSS scheme as described in Fig. 1.

The paper is organized as follows. Section II introduces the problem statement with some mathematical background. Section III presents different methods to predict control design limitations: a classical method to analyze the controllability and observability of a determined mode is presented in Section III-A and a methodology to determine the fundamental design limitations of a controller is presented in Section III-B. In Section IV, a general criterion for input-output pair selection

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is proposed. The application to a case of study is presented in Section V. Finally, the conclusions are summarized in Section VI.

II. Power System Stabilization based on WPPs

The dynamic behavior of a power system is described by a set of nonlinear differential and algebraic equations [**?**]. For small signal analyses, the nonlinear system is linearized around an operating point. Then, after some mathematical manipulations, the dynamics around the operating point is given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_w w(t) + B_u u(t),\\ z(t) &= C_z x(t) + D_{zw} w(t) + D_{zu} u(t),\\ y(t) &= C_y x(t) + D_{yw} w(t) + D_{yu} u(t),\end{aligned} \tag{1}$$

where $A, \ldots, D_{yu}$ are matrices of adequate dimensions, x denotes the state vector, w the disturbance and u the control input. The outputs z and y are signals in which the oscillations can be observed. The signal z represents, for example, the angle of the synchronous machines and w a disturbance in the mechanical torque reference of the synchronous generator. These are remote signals that cannot be used by the PSS in order to avoid delays and reliability issues. Therefore, the local signal y (*e.g.* voltage or angle at the connection point) is the only information about the oscillations that the PSS can use to compensate them by injecting the correction signal u (*e.g.* active or reactive power).

The transfer function $G(s)$ corresponding to (1) can be partitioned as

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = G(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} G_{zw}(s) & G_{zu}(s) \\ G_{yw}(s) & G_{yu}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix}. \tag{2}$$

From the previous definitions, the problem of designing a PSS based on WPP consists in computing a controller K to damp the oscillations in the signal z, as shown in Fig. 1. From (2), the closed loop transfer

$$T_{zw} = G_{zw} + G_{zu} K \left(1 - G_{yu} K\right)^{-1} G_{yw} \tag{3}$$

should exhibit a more damped behavior than the open loop transfer G_{zw}.

This control problem is known as general control configuration and presents more limitations than the case in which the signal z is measured (*i.e.* when $z = y$) [**?**]. The performance achieved by any controller is limited by the need of guaranteeing stability of the closed loop system. These fundamental limitations are only dependent on the properties of the open loop system and not on the particular controller. The main purpose of the paper is to provide a criterion that using the fundamental limitations of the system permits to select the most adequate input-output pairs (u, y) to achieve the best oscillation damping.

III. Fundamental limitations in control design

The idea in the study of fundamental limitations of control design is to predict the performance of the closed loop system without computing the controller. Before presenting the input-output selection criterion, some background concepts are presented in this section.

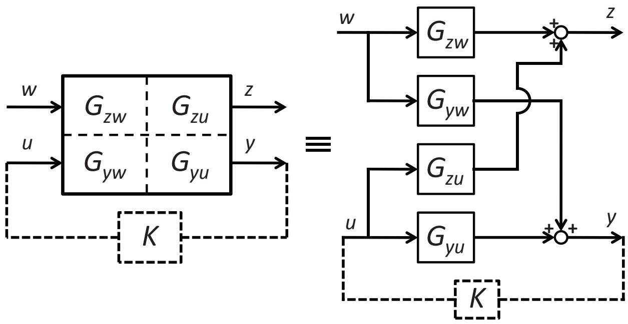

Fig. 1. General control configuration

A. Controllability and Observability

Controllability gives a measure of the possibility of modifying the location of an oscillation mode by state feedback. Observability provides a measure of the visibility of a particular oscillation mode on a given output. To select the best input-output pair, both properties are important since the control is assumed based on output feedback. The residues or the geometric joint controllabilty/observability analyses are the commonly used methods in an input-output selection procedure. Both analyses are close connected and basically state that for good controllability/observability characteristics, the matrices B_u and C_y should not be orthogonal to the eigenvectors corresponding to the oscillation mode [**?**].

In particular, the geometric approach allows evaluating the controllability/observability of each eigenvalue of the system. In addition, the measures are normalized which is an important factor to compare transfer functions with different units. The geometric joint controllability/observability measure is given by

$$\mathrm{m}_{\mathrm{co},k} = \mathrm{m}_{\mathrm{c},k} \mathrm{m}_{\mathrm{o},k} \tag{4}$$

where

$$\begin{aligned}\mathrm{m}_{\mathrm{c},k} &= \frac{|B_u^T f_k|}{\|f_k\| \|B_u\|},\\ \mathrm{m}_{\mathrm{o},k} &= \frac{|C_y e_k|}{\|e_k\| \|C_y\|},\end{aligned}$$

are the controllability and observability measures, respectively, and e_k and f_k are the right and left eigenvectors, respectively, associated to the eigenvalue λ_k. A value near zero indicates that the eigenvalue is poorly controllable or observable. On the other hand, a value close to one corresponds to good controllability and observability features [**?**].

The input-output pair more controllable and observable will be the pair that allows to damp the oscillation mode with lower control effort. Controllability and observability are common criterion in the selection of the most adequate input-output pair. However, in the general control configuration in Fig. 1, controllability and observability are not the only limitations on the achievable damping.

B. Design limitations caused by feedback

The capability of attenuating a signal by feedback is limited by the necessity of ensuring closed loop stability. In the case of the general control configuration, these limitations depend not only on the transfer G_{yu} but also on the transfers G_{yw} and G_{zu}. The last transfer functions are not in the loop but they may affect the control effectiveness to attenuate the disturbance w effects on the output of interest z. The term fundamental design limitations refers to the fact that the value of T_{zw} at certain frequencies is fixed independently of the stabilizing controller. These limitations arise when G_{yu}, G_{zu} and G_{yw} have non-minimum phase (NMP) zeros (*i.e.*, in the closed right hand plane (CRHP)) or G has poles in CRHP. These limitations have been extensively studied, the case of standard feedback problem ($z = y$) can be found in [**?**]. The general control configuration case is analyzed in [**?**]. Next, a brief summary is presented to ease the understanding of the proposed selection criterion.

To solve the ideal disturbance attenuation problem means to make arbitrarily small the magnitude of the frequency response of T_{zw}. If $G_{zu} \not\equiv 0$, $G_{yw} \not\equiv 0$ and the determinant $\det G = G_{zw}G_{yu} - G_{zu}G_{yw} \not\equiv 0$, then the controller

$$K^C(s) = \frac{G_{zw}(s)}{\det G(s)} \tag{5}$$

leads to a T_{zw} equal to zero for all s, where the symbol $f \not\equiv 0$ means that f is not identical to zero (it can be zero for a set of s but not for all s). The controller K^C is not useful in practice because of its strong dependence on the plant model can yield a closed loop system sensitive to modeling errors. The importance of K^C is that shows the existence or not of theoretical limitations to achieve ideal disturbance attenuation. In case of G_{yu}, G_{zu} and G_{yw} with zeros in CRHP or unstable poles in G, the controller K^C is not always able to ensure stability and thus ideal disturbance attenuation. In these cases, independently of controller choice, the magnitude of T_{zw} will not be arbitrary small.

1) Systems reducible to a feedback loop: In case of $\det G \equiv 0$, the system is said to be reducible to a feedback loop and

$$T_{zw} = G_{zw}S,$$

where $S = (1 - G_{yu}K)^{-1}$ is the sensitivity function. It is well known that if ξ in CRHP is a zero of G_{yu} then $S(\xi) = 1$ [**?**]. Therefore, $T_{zw}(\xi) = G_{zw}(\xi)$ and

$$\|T_{zw}\|_\infty = \max_\omega |T_{zw}(j\omega)| \geq |G_{zw}(\xi)|.$$

This is a fundamental limitation and does not depend on a particular controller.

2) Case $\det G \not\equiv 0$: If the system is not reducible to a feedback loop, the fundamental limitations are caused by the presence of zeros of G_{zu} or G_{yw} in CRHP or by unstable poles of G.

a) CRHP zeros of G_{zu} and G_{yw}: If $\det G \not\equiv 0$ and ξ in CRHP is a zero of G_{zu} or G_{yw} but it is not a pole of G and the following condition

$$m_{zw}(\xi) < m_{zu}(\xi) + m_{yw}(\xi) \tag{6}$$

is satisfied, where m_{zw}, m_{zu} and m_{yw} are the multiplicities of ξ as a zero of G_{zw}, G_{zu} and G_{yw}, respectively, then

$$||T_{zw}||_\infty \geq |G^0_{zw}(\xi)| > 0, \tag{7}$$

where G^0_{zw} is a transfer function such that $G_{zw} = G^0_{zw}\mathcal{B}_\xi$ where $\mathcal{B}_\xi$ is a Blaschke product with $m_{zw}(\xi)$ zeros at ξ and, if ξ is complex, at its complex conjugate, *i.e.*

$$\mathcal{B}_\xi = \prod_{l=1}^{m_{zw}} \frac{(\xi - s)}{(\xi + s)}.$$

The lower bound (7) implies that no matter the controller, the disturbance attenuation cannot be lower than $|G^0_{zw}(\xi)|$. Moreover, independently of the controller the closed loop transfer T_{zw} will pass through the point $G^0_{zw}(\xi)$. This can be a serious limitation if the oscillation mode is close to the zero ξ and $|G^0_{zw}(\xi)|$ is not small.

b) Unstable poles of G: Assuming that T_{zw} is stable and that $\det G \not\equiv 0$ and let p be an open RHP pole of G with multiplicity $\gamma_G(p) \geq 1$, if $\gamma_G(p) > \gamma_{zw}(p)$ and p is a transmission zero of G with multiplicity $m_G(p) > 0$ and either $G_{zw} \equiv 0$ or

$$m_G(p) < m_{zw}(p) + \gamma_G(p) - \gamma_{zw}(p), \tag{8}$$

then

$$||T_{zw}||_\infty \geq \lim_{s \to p} \left| \frac{\det G(s)\mathcal{B}_p^{-1}(s)}{G_{yu}(s)} \right| > 0, \tag{9}$$

where $\mathcal{B}_p$ is a Blaschke product with $m_G(p)$ zeros at p. As in the previous case, the lower bound (9) imposes a lower bound on the achievable attenuation at frequencies close to the unstable pole p.

3) Ideal disturbance attenuation: Previous results allow stating the necessary and sufficient conditions for solving the ideal disturbance attenuation problem. Removing any CRHP pole-zero cancelation, the controller (5) results in

$$K^C = \frac{G_{zw}}{\det G}. \tag{10}$$

Then, assuming that $\det G \not\equiv 0$, that each zero ξ of G_{zw} and G_{yw} is not a pole of G and does not satisfy the condition (6), and that each unstable pole of G does not satisfy the condition (8), then the controller (10) stabilizes the closed loop system. Otherwise, a stabilizing controller that achieves a perfect disturbance attenuation does not exist, and the closed loop system cannot be arbitrary small.

IV. Input-Output Selection Criterion

In the light of the concepts summarized in the previous section, a criterion for the selection of the most adequate input-output pair is proposed. The idea is to find the local output y in which the oscillation to be damped is more visible and the local input u more effective to cancel the oscillations. The proposed criterion is based on small signal analysis therefore the first step consists in linearizing the model of the power system. With the linear model (1), the criterion consists in evaluating the controllability/observability of G_{yu} and the fundamental limitations imposed by G. The first test permits to determine the input-output pair that would demand less control effort to damp the oscillations. In general, if the oscillation mode is clearly visible from y and the input u has a strong effect on the controlled variable, less correction terms would be necessary to damp the oscillation.

The fundamental limitations provide a criterion to determine the lower limits in the achievable attenuation. As mentioned in the previous section, the presence of zeros of G_{zu} and G_{yw} in the CRHP or unstable poles impose a lower limit on the magnitude of the frequency response of the closed loop system. If the zeros or poles are close to an oscillation mode, such analysis reveals that no matter the controller the mode cannot be arbitrarily damped.

The proposed input-output selection criterion can be summarized as follows.

1) Obtain the linear description of the power systems (1). This can be done by an analytic procedure if the mathematical expressions are available or by numerical linearization if the nonlinear model is provided by an electrical simulation software (*e.g.* SimPowerSystem Toolbox or DigSilent, etc.). Besides the disturbance w and the signal z located where the oscillation must be damped, the model must include all the signals that can be used as control action u and local measure y.
2) Check the controllability and observability for the mode λ_k to be damped in order to determine the input-output pair in which the control is more effective to increase the damping. The best pair will be the one with the higher geometric measure $\mathrm{m_{co}}$.
3) Determine if the system is reducible to a feedback loop and find the zeros of G_{yu}, G_{yw} and G_{zu} and poles of G. If there exist zeros or poles in the CRHP, use the formulae in Section III-B to compute the lower limits in the attenuation. The best input-output pair will be the one with a lower limit in $||T_{zw}||_\infty$.
4) The most adequate pair will be the one with better controllability/observability characteristic and the lower limit in the attenuation.

The criterion is based on well mature numeric procedures, therefore it is applicable to power systems of considerable complexity. Although the analysis of all oscillation modes may result cumbersome, it must be reminded that the analysis should be limited to those oscillation modes considered critical. Moreover, the remote signals considered will depend on the synchronous generator with larger participation factor on such oscillations.

It is interesting to mention that the criterion can be extended to consider $\mathcal{D}$-stability regions. These regions are subsets of the left half plane with additional stability conditions, such as a cone in which all poles present a damping coefficient greater than a given value. In case of including $\mathcal{D}$-stability regions, in the previous analysis, the CRHP should be replaced by the complement of the $\mathcal{D}$-stability region.

V. Case of Study: Three machines system with wind power plant generation

In order to illustrate the application of the proposed input-output selection criterion, the power system shown in Fig. 2 is analyzed. The system presented in Fig. 2 is an electrical network that can be used to show the interaction between conventional generation and wind power plants. The electrical network is assumed to be lossless (only inductances). The nonlinear expression for this system can be found by means of synchronous generator and standard power flow equations. The synchronous generator is described with a standard one-axis model without controllers. It is assumed that the WPP with converter-based wind turbines behaves as a negative load, since the converter dynamics is much faster than the electromechanical dynamics [**?**]. The power delivery by the WPP is assumed saturated in $\pm 25\%$ of its nominal power. The equations and parameters used in the power system modeling can be found in [**?**].

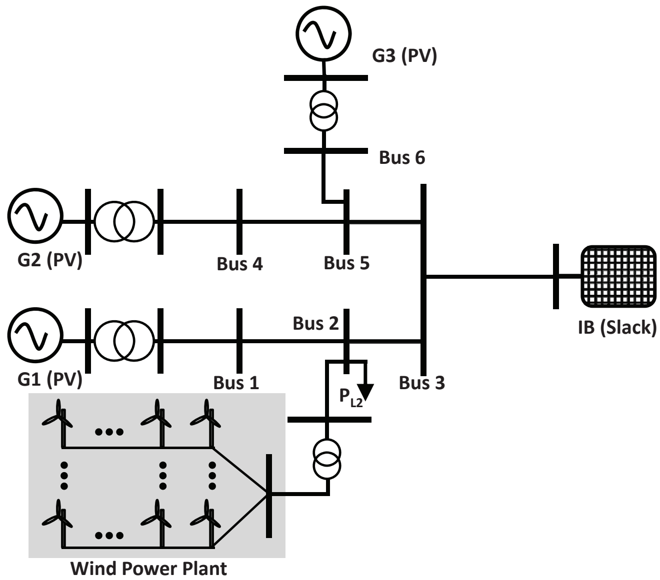

Fig. 2. Three machines system with a wind power plant connected

The nonlinear power system is governed by the equations of the synchronous generators connected into the power system [**?**]

$$\begin{aligned}
\dot{\delta}_i &= \omega_i - \omega_s, \\
\dot{\omega}_i &= \frac{\omega_s}{2H_i}\left(P_{mi} - \frac{E_i' V_k}{X_i'}\sin(\delta_i - \theta_k) - D_i\omega_i\right), \\
\dot{E}_i' &= \frac{1}{T_{0i}}\left(E_{fi} - \frac{X_i}{X_i'}E_i' + \frac{X_i - X_i'}{X_i'}V_k\cos(\delta - \theta_k)\right),
\end{aligned} \tag{11}$$

where $i = 1, 2, 3$ refer to the synchronous generators, and δ_i, ω_i and E_i' are the rotor angle, the rotor frequency and the internal electromagnetic field (EMF) of all the synchronous generators, respectively. The transient reactance and the rotor reactance of the synchronous generators are given by X_i' and X_i, respectively, T_{0i} is the open-circuit time constant, P_{mi} is the mechanical input power, ω_s denotes the synchronous reference, D_i is the mechanical damping, H_i refers to the inertia constant and E_{fi} excitation voltage, for each generator implemented in the system. Parameter values are in Appendix A and power flow equations in Appendix B.

The mechanical power reference of the synchronous generator T_{m1} is considered as the disturbance w that causes the oscillations. The signal to be controlled z, the signal on which the oscillation must be damped, is the angle of the synchronous machine δ_1. The active power P_{wt} and the reactive power Q_{wt}

are considered as possible control signals u, and the voltage magnitude measured at the WPP connection point V_{wt} and the phase angle of the voltage WPP bus θ_{wt} as the local measure y.

A. Selection of the most adequate input-output pair

Linearization: After linearizing the nonlinear model compound by (11) and the power flow equations, the following state equations are obtained

$$\begin{aligned}\dot{x} &= Ax + B_w w + B_u u,\\ z &= C_z x,\\ y &= C_y x + D_{yu} u,\end{aligned} \tag{12}$$

where $x = [\Delta\delta_1\ \Delta\omega_1\ \Delta E_1'\ \Delta E_2'\ \Delta\delta_2\ \Delta E_3'\ \Delta\delta_3\ \Delta\omega_2\ \Delta\omega_3]^T$, with Δ denoting the incremental values. The power system has been linearized by numerical computation.

The eigenvalues of A are listed in Table I. It can be observed two lightly-damped local oscillation modes at $\lambda_{2,3} = -0.0776 \pm j4.7011$ and $\lambda_{4,5} = -0.0633 \pm j4.4687$, these will be considered as the oscillation modes to be damped. Notice that the eigenvalues in Table I are all associated with synchronous generators dynamics, since the WPP dynamics is assumed much faster than the electromechanical dynamics.

TABLE I
EIGENVALUES OF THE MATRIX A

Eigenvalue	Value	Freq. (Hz)	Damping (%)
$\lambda_{1,2}$	$-0.3244 \pm j6.5644$	1.0448	4.9358
$\boldsymbol{\lambda_{3,4}}$	$\mathbf{-0.0776 \pm j4.7011}$	**0.7482**	**1.6505**
$\boldsymbol{\lambda_{5,6}}$	$\mathbf{-0.0633 \pm j4.4687}$	**0.7112**	**1.4164**
λ_7	-0.2180	0	1
λ_8	-0.3062	0	1
λ_9	-0.4511	0	1

Controllability and observability test: There are four input-output pairs to be analyzed according to the possible configurations mentioned before. Since the modes to be damped are $\lambda_{3,4}$ and $\lambda_{5,6}$, the controllability and observability test is applied only to these cases. The geometric measures of controllability and observability are listed in Table II. It can be seen that the four pairs exhibit good controllability characteristics for $\lambda_{5,6}$; however, they are slightly lower in the case of $\lambda_{3,4}$. Nevertheless, all the values are similar. The main differences are in the observability. The measures in Table II suggest that the best measured signal y for $\lambda_{3,4}$ is the voltage magnitude at the connection point V_{wt}. On the other hand, considering the mode $\lambda_{5,6}$, the best measured signal y is the phase angle of the voltage at the connection point θ_{wt}. Therefore, the pair (Q_{wt}, θ_{wt}) is the best option in the $\lambda_{5,6}$ case; and (Q_{wt}, V_{wt}) in the $\lambda_{3,4}$ case. Remember that it is expected that the oscillation will be more visible and the controller will need less control effort to increase the damping when the pair with higher $\mathrm{m_{co}}$ is chosen. Then, from the controllability/observability point of view, the input-output pairs depending on the oscillation mode can be ordered as follows.

- For $\lambda_{3,4}$ case
  1) (Q_{wt}, V_{wt}),
  2) (P_{wt}, V_{wt}),
  3) (Q_{wt}, θ_{wt}),
  4) (P_{wt}, θ_{wt}).
- For $\lambda_{5,6}$ case
  1) (Q_{wt}, θ_{wt}),
  2) (P_{wt}, θ_{wt}),
  3) (Q_{wt}, V_{wt}),
  4) (P_{wt}, V_{wt}).

TABLE II
GEOMETRIC MEASURES OF THE CONTROLLABILITY AND OBSERVABILITY OF $\lambda_{3,4}$ AND $\lambda_{5,6}$ MODES FOR THE FOUR I/O PAIRS

Case	I/O pairs	$\mathrm{m_c}$	$\mathrm{m_o}$	$\mathrm{m_{co}}$
$\lambda_{3,4}$	(P_{wt}, V_{wt})	0.1148	0.0859	0.0099
	(Q_{wt}, V_{wt})	0.1212	0.0859	0.0104
	(P_{wt}, θ_{wt})	0.1148	0.0559	0.0064
	(Q_{wt}, θ_{wt})	0.1212	0.0559	0.0068
$\lambda_{5,6}$	(P_{wt}, V_{wt})	0.3807	0.0942	0.0359
	(Q_{wt}, V_{wt})	0.4483	0.0942	0.0422
	(P_{wt}, θ_{wt})	0.3807	0.1161	0.0442
	(Q_{wt}, θ_{wt})	0.4483	0.1161	0.0521

Fundamental limitations: In the four cases analyzed, $\det G \not\equiv 0$ therefore the system is not reducible to a feedback loop. The transfer function G has not unstable poles but there exist NMP zeros in all cases except for the pair (P_{wt}, V_{wt}). The zeros of G_{yw} and G_{zu} are listed in Table III for each case. As all NMP zeros have multiplicity 1, the three cases satisfy condition (6) and they are not zeros of G_{zw}. Therefore, except for the pair (P_{wt}, V_{wt}), independently on the controller, the closed loop transfer T_{zw} will not be arbitrary zero at the frequencies of the zeros. These lower limits can be seen in the last column in Table III. From these values, it can be concluded that the best pair from the limitation point of view is (P_{wt}, V_{wt}). Nevertheless, the NMP zero in the pair (Q_{wt}, V_{wt}) does not impose a strong limitation since $|G^0_{zw}(\xi)|$ is small and it is distant from the oscillation modes $\lambda_{3,4}$ and $\lambda_{5,6}$. Hence, under the analysis of fundamental limitations, the input-output pairs result ordered as follows.

1) (P_{wt}, V_{wt})
2) (Q_{wt}, V_{wt}),
3) (P_{wt}, θ_{wt}) and (Q_{wt}, θ_{wt}).

Input/Output signal selection: According to the previous analysis and considering the oscillation modes $\lambda_{3,4}$ and $\lambda_{5,6}$ simultaneously, the pairs (P_{wt}, V_{wt}) and (Q_{wt}, V_{wt}) present similar controllability and observability characteristics (slightly lower than the other cases). The pair (P_{wt}, V_{wt}) presents no limitations associated to NMP zeros pointing out the existence of a controller capable of achieving the ideal disturbance attenuation of the oscillation modes $\lambda_{3,4}$ and $\lambda_{5,6}$. Since the zero ξ in the CRHP appearing in the (Q_{wt}, V_{wt}) pair is not close to the oscillation modes, the attenuation is not seriously affected by the presence of this NMP zero.

The NMP zeros in the pairs (P_{wt}, θ_{wt}) and (Q_{wt}, θ_{wt}) impose a stronger limitation to increase the damping due

TABLE III
ZEROS OF G_{yw} AND G_{zu} AND LOWER LIMITS OF $\|T_{zw}\|_{\infty}$

I/O Pairs	Zeros of G_{zu}	Zeros of G_{yw}	$\lvert G^0_{zw}(\xi)\rvert$
(P_{wt}, V_{wt})	$-0.3198 \pm j6.617$	$-0.316 \pm j6.577$	–
	$-0.0067 \pm j4.654$	$-0.0014 \pm j4.561$	–
	-0.2028	-0.2285	–
	-0.3011	-0.2517	–
	-0.4510	-0.4508	–
(Q_{wt}, V_{wt})	$-0.3218 \pm j6.607$	$-0.316 \pm j6.577$	–
	$-0.0076 \pm j4.610$	$-0.0014 \pm j4.561$	–
	0.0262	-0.2285	**0.7868**
	-0.3029	-0.2517	–
	-0.4509	-0.4508	–
(P_{wt}, θ_{wt})	$-0.3198 \pm j6.617$	$-0.31 \pm j6.585$	–
	$-0.0067 \pm j4.654$	$\mathbf{0.008 \pm j4.614}$	**16.074**
	-0.2028	-0.2176	–
	-0.3011	-0.3176	–
	-0.4510	-0.4516	–
(Q_{wt}, θ_{wt})	$-0.3218 \pm j6.607$	$-0.31 \pm j6.585$	–
	$-0.0076 \pm j4.610$	$\mathbf{0.008 \pm j4.614}$	**16.074**
	0.0262	-0.2176	**0.7868**
	-0.3029	-0.3176	–
	-0.4509	-0.4516	–

to the proximity to the modes $\lambda_{3,4}$ and $\lambda_{5,6}$. This aspect prevails over the controllability/observability measures. The choice between (P_{wt}, θ_{wt}) and (Q_{wt}, θ_{wt}) is clear since the NMP zero in (Q_{wt}, θ_{wt}) is distant from the oscillation modes. The geometric measures can be used to obtain the final order of the pair. Based on the previous arguments, the following order is suggested.

1) (Q_{wt}, V_{wt}),
2) (P_{wt}, V_{wt}),
3) (Q_{wt}, θ_{wt}),
4) (P_{wt}, θ_{wt}).

B. Frequency and transient responses

In order to illustrate the meaning of the previous results, four controllers were designed using H_{∞} optimal control tools [?]. The objectives were to enforce the damping of both modes $\lambda_{3,4}$ and $\lambda_{5,6}$ therefore the signal z was weighted with

$$W_z = \frac{\kappa \omega_{osc}^2 s}{s^2 + 2\xi_{damp}\omega_{osc} s + \omega_{osc}^2},$$

with $\kappa = 0.125$, $\omega_{osc} = 4.6$ and $\xi_{damp} = 0.3$. The control u was also penalized with

$$W_u = \frac{0.15s^2 + 1.65s + 1.5}{0.1s^2 + 10.01s + 1}$$

to limit the control effort and thus avoiding numerical issues. The proposed criterion does not require the computation of the controller, here they are computed only to illustrate the connections between the previous predictions and frequency and transient responses. For this reason the controller design is not explained in detail, a more extensive description of the application of H_{∞} optimal control tools in power systems can be found in [?]. Notice that the cancelation controller K^C (10) only could be used in the pair (P_{wt}, V_{wt}) since it would not guarantee closed loop stability due to the presence of NMP zeros.

The magnitude of the frequency responses of the open loop system G_{zw} and the closed loop system with the four controllers corresponding to each input-output pair can be seen in Fig. 3. At $s = 0.008 \pm j4.614$ the magnitude of T_{zw} is 16.074 in the case of pairs (P_{wt}, θ_{wt}) and (Q_{wt}, θ_{wt}) and 0.7868 at $s = 0.0262$ in the case of pairs (Q_{wt}, V_{wt}), as predicted by the fundamental limitation analysis. It can be observed that the attenuation achieved in the case of the pairs (P_{wt}, θ_{wt}) and (Q_{wt}, θ_{wt}) is rather poor as a consequence of the proximity of the NMP zeros to the oscillation modes $\lambda_{3,4}$ and $\lambda_{5,6}$. On the other hand, with the pair (Q_{wt}, V_{wt}), with NMP zeros more distant from oscillation modes, the limitation is not a serious constraint to achieve the attenuation of the oscillation modes. In the case (P_{wt}, V_{wt}), there are no limitations caused by NMP zeros. Fig. 3 clearly shows that from the fundamental limitations view, the best measured signal y is V_{wt}.

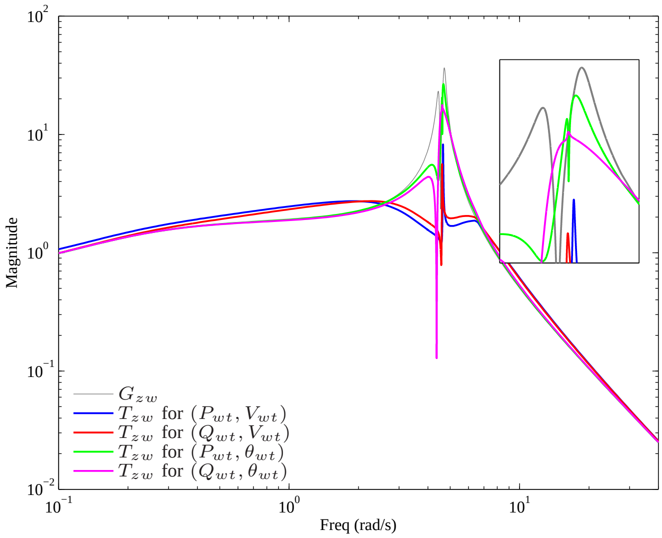

Fig. 3. Magnitude of the open loop transfer G_{zw} and the closed loop transfer T_{zw} for the four possible input-output pairs with H_{∞} controllers

Fig. 4 shows the transient responses of the open loop and the closed loop for the pairs (P_{wt}, θ_{wt}) and (P_{wt}, V_{wt}). The simulations were performed using the linear expressions of the power system and the corresponding H_{∞} controllers. The responses correspond to an impulse on the mechanical power reference of the synchronous generator T_{m1}. As predicted by the selection criterion, the choice of the voltage V_{wt} as measured signal y is a better choice than the angle θ_{wt}. The control using the pair (P_{wt}, V_{wt}) is clearly more effective in damping the oscillations and the upper and lower saturation limits imposed on the active power delivered by the WPP are not reached during the entire simulation.

In Fig. 5, it can be observed the transient responses of the open loop system and the closed loop systems for the pairs (Q_{wt}, θ_{wt}) and (Q_{wt}, V_{wt}). The control configuration using the voltage magnitude at the connection point V_{wt} as measured

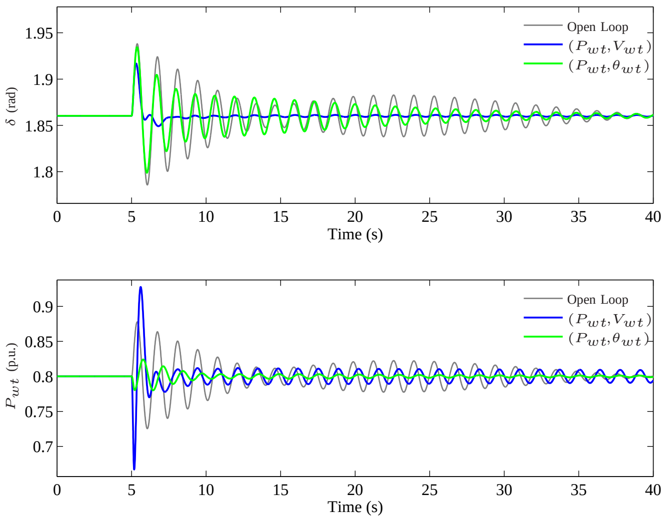

Fig. 4. Transient responses of the open loop and closed loop for the pairs (P_{wt}, V_{wt}) and (P_{wt}, θ_{wt})

signal y is the best choice. Clearly, the damping achieved with the pair (Q_{wt}, V_{wt}) is better than the one obtained with the pair (Q_{wt}, θ_{wt}). Again it can be seen that the upper and lower saturation limits imposed on the reactive power delivered by the WPP are not reached. Comparing Figs. 4 and 5, it can be observed a similar response to damp the oscillation modes in the case of the pair (P_{wt}, V_{wt}) with respect to (Q_{wt}, V_{wt}). This also fits with the proposed selection criterion.

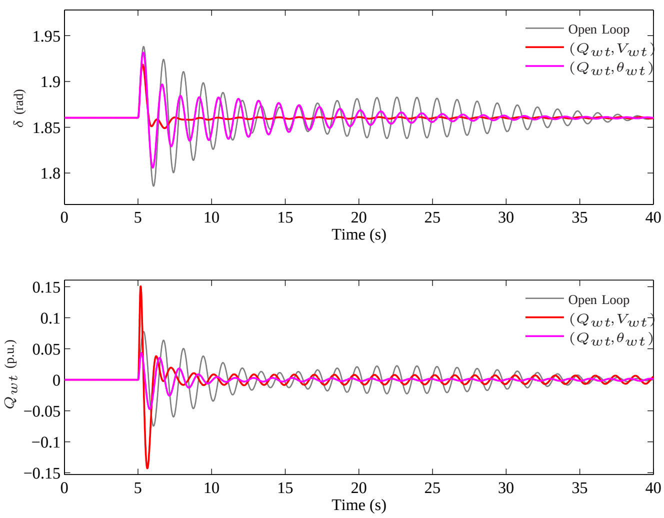

Fig. 5. Transient responses of the open loop and closed loop for the pairs (Q_{wt}, V_{wt}) and (Q_{wt}, θ_{wt})

VI. Conclusion

A criterion to select the most adequate input-output pair for the design of PSS for WPP has been proposed. The main difference with previous criteria is in considering the use of local signals to damp remote oscillations. When WPP are employed to assist in the enhancement of the power system stability, the PSS might be far away from the location where the rotor oscillations of the synchronous generators arise. The proposed selection criterion analyzes fundamental limitations of the system to predict, without designing the controller, the capability of a particular choice of local signals to damp an oscillation mode. The study of the controllability and observability characteristics allows to determine the input-output pair that will require less control effort to achieve the oscillation damping. The analysis of the fundamental limitations permits to identify frequencies at which is not possible to reduce the magnitude of the frequency response of the closed loop system. If these frequencies are close to the oscillation modes to be damped, it will be difficult, no matter the controller choice, to achieve a noticeable increase of damping. The application of the selection criterion to a set of candidate input-output pairs produces a ranking of the input-output local signals to achieve the best oscillation damping according to open loop characteristics of the system.

In order to illustrate the use of the selection criterion, a three synchronous machine system with a WPP has been analyzed. Frequency responses and simulation results have been presented to connect the fundamental limitation with the more intuitive transient responses of the power system to an impulse disturbance.

Appendix

A. Power System Parameters

The parameter values used in the power system modeling (synchronous generator, lines and reference values of power and voltage) are as follows.

- **Power System:** P_{gs1} = 0.8 p.u., P_{gs2} = 0.8 p.u., P_{gs3} = 0.6 p.u., P_{wt} = 0.8 p.u., P_{L2} = 2 p.u., Q_{L2} = 0.5 p.u., E'_1 = E'_2 = 1.03 p.u., E'_1 = 1.01 p.u. , V_{wt} = 1 p.u., ω_s = 100π rad/s, X_{12} = X_{wt2} = X_{23} = X_{53} = X_{65} = X_{45} = 0.1 p.u. and $X_{3\infty}$ = 0.2 p.u.
- **Synchronous Machines:** T_{01} = 6 p.u., T_{02} = T_{03} = 3 p.u., H_1 = 4 s, H_2 = 2 s, H_3 = 4.6 s, D_1 = 0.01 p.u., D_2 = 0.0125 p.u., D_3 = 0.005125 p.u, X_{ta} = 0.1 p.u. and X'_1 = X'_2 = X'_3 = 1.15 + X_{ta} p.u.

B. Power flow equations

The power flow equations are given by

$$\begin{aligned} P_{gi} &= \frac{E'_i V_j}{X'_i}\sin(\delta_i - \theta_j), \\ Q_{gi} &= \frac{E'^2_i}{X'_i} - \frac{E'_i V_j}{X'_i}\cos(\delta_i - \theta_j), \\ P_{kl} &= \frac{V_k V_l}{X_{kl}}\sin(\theta_k - \theta_l), \\ Q_{kl} &= \frac{V_k^2}{X_{kl}} - \frac{V_k V_l}{X_{kl}}\cos(\theta_k - \theta_l), \end{aligned} \tag{13}$$

where P_{gi} and Q_{gi} $(i \in \{1,2,3\})$ are the active and reactive power transferred from the synchronous generators through the power transformer, P_{kl} and Q_{kl} $(k \in \{1,2,3,4,5,6,wt\}$ and $l \in \{2,3,5,\infty\})$ are the active and reactive power transferred through the power lines, V_j and θ_j $(j \in \{1,4,6\})$ represent the voltage magnitude and phase angle of the busses after the power transformers, V_k, V_l, θ_k and θ_l $(k \in \{1,2,3,4,5,6,wt\}$

and $l \in \{2,3,5,\infty\}$) represent the voltage magnitude and phase angle of the power system busses, X'_i ($i \in \{1,2,3\}$) are the reactances of the power transformers, and X_{kl} ($k \in \{1,2,3,4,5,6,wt\}$ and $l \in \{2,3,5,\infty\}$) are the reactances of the lines. The sum of all the active and reactive power at each bus must be equal to zero.

REFERENCES

[1] United Nations Framework Convention on Climate Change, "Kyoto protocol to the united nations framework convention on climate change," United Nations, 1998. [Online]. Available: http://unfccc.int/resource/docs/convkp/kpeng.pdf

[2] J. Wilkes, J. Moccia, and M. Dragan, "Wind in power: 2011 european stadistics," EWEA, Tech. Rep., 2012.

[3] O. Gomis-Bellmunt, A. Junyent-Ferre, A. Sumper, and J. Bergas-Jane, "Ride-through control of a doubly fed induction generator under unbalanced voltage sags," *IEEE Transactions on Energy Conversion*, vol. 23, pp. 1036–1045, 2008.

[4] A. Sumper, O. Gomis-Bellmunt, A. Sudri-Andreu, R. Villafafila-Robles, and J. Rull-Duran, "Response of fixed speed wind turbines to system frequency disturbances," *IEEE Transactions on Power Systems*, vol. 24, pp. 181–192, 2009.

[5] M. Martinez-Rojas, A. Sumper, O. Gomis-Bellmunt, and A. Sudri-Andreu, "Reactive power dispatch in wind farms using particle swarm optimization technique and feasible solutions search," *Applied Energy*, vol. 88, pp. 4678–4686, 2011.

[6] M. de Industria Turismo y Comercio: Secretaria de Estado de Energía, "Instalaciones conectadas a la Red de Transporte y equipo generador: requisitos minimos de diseño, equipamiento, funcionamiento, puesta en servicio y seguridad," Red Eléctrica, Tech. Rep., October 2008.

[7] J. L. Domínguez-García, O. Gomis-Bellmunt, F. Bianchi, and A. Sumper, "Power oscillation damping supported by wind power: A review," *Renewable and Sustainable Energy Reviews*, vol. 16, no. 7, pp. 4994–5006, 2012.

[8] J. L. Domínguez-García, F. Bianchi, and O. Gomis-Bellmunt, "Analysis of the damping contribution of power system stabilizers driving wind power plants," *Wind Energy*, vol. In Early View, 2012.

[9] A. Heniche and I. Kamwa, "Assessment of two methods to select wide-area signals for power system damping control," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 572–581, 2008.

[10] H. Wu, K. Tsakalis, and G. Heydt, "Evaluation of time delay effects to wide-area power system stabilizer design," *IEEE Transactions on Power Systems*, vol. 19, no. 4, pp. 1935–1941, 2004.

[11] F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour, and A. Safdarian, "Impact of WAMS malfunction on power system reliability assessment," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1302–1309, 2012.

[12] E. Larsen, J. Sanchez-Gasca, and J. Chow, "Concepts for design of FACTS controllers to damp power swings," *IEEE Transactions on Power Systems*, vol. 10, no. 2, pp. 948–956, 1995.

[13] M. Farsangi, Y. Song, and K. Lee, "Choice of facts device control inputs for damping interarea oscillations," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 1135–1143, 2004.

[14] Y. Li, C. Rehtanz, S. Rüberg, L. Luo, and Y. Cao, "Assessment and choice of input signals for multiple hvdc and facts wide-area damping controllers," *IEEE Transactions on Power Systems*, 2012.

[15] Y. Zhang and A. Bose, "Design of wide-area damping controllers for interarea oscillations," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1136–1143, 2008.

[16] J. Freudenberg, C. Hollot, R. Middleton, and V. Toochinda, "Fundamental design limitations of the general control configuration," *IEEE Transactions on Automatic Control*, vol. 48, no. 8, pp. 1355–1370, 2003.

[17] V. Wertz, E. Silva, G. Goodwin, and B. Codrons, "Performance limitations arising in the control of power plants," in *17th IFAC World Congress*, 2008.

[18] P. M. Anderson and A. A. Fouad, *Power System Control and Stability*. Iowa State University Press., 1977.

[19] C.-T. Chen, *Linear System Theory and Design*, 3rd ed. New York, NY, USA: Oxford University Press, Inc., 1998.

[20] A. Hamdan and A. Elabdalla, "Geometric measures of modal controllability and observability of power system models," *Electric Power System Research*, vol. 15, no. 2, pp. 147–155, 1988.

[21] R. Sánchez-Peña and M. Sznaider, *Robust Systems: Theory and Applications*. Wiley, 1998.

[22] E. Hagstrøm, I. Norheim, and K. Uhlen, "Large-scale wind power integration in Norway and impact on damping in the Nordic grid," *Wind Energy*, vol. 8, pp. 375–384, 2005.

[23] K. Elkington, V. Knazkins, and M. Ghandhari, "On the stability of power systems containing doubly fed induction generator-based generation," *Electric Power System Research*, vol. 78, pp. 1477–1484, 2008.

[24] J. Chow, L. Harrist, O. H. Kale, M.A., J. Sanchez-Gasca, and G. Terwilliger, "Robust control design of power system stabilizer using multivariable frequency domain techniques," in *Proc. of the 29th Conference on Decision and Control*, 1990.

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