LPV wind turbine control with anti-windup features covering the complete wind speed range

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***Abstract*—This work addresses the control of a variable-speed variable-pitch wind turbine in the whole wind speed range. To this end, a linear parameter varying (LPV) anti-windup (AW) controller is proposed as part of a control structure focused on improving the transition between low and high wind speed operation. The control structure is similar to classical PI controls used in commercial wind turbines. However, a more advanced gain-scheduled controller and AW compensation is proposed. As a consequence, the new control scheme is capable of improving the behavior of the wind turbine in the transition zone and provides better stability margins. The proposed control was evaluated in a 5 MW wind turbine benchmark and compared with a classical control scheme. To this end, very demanding and realistic testing scenarios were built using the FAST aeroelastic wind turbine simulator as well as standardized wind speed profiles.**

***Index Terms*—Control of wind turbines; Gain-scheduling control; Anti-windup; Linear parameter varying systems.**

I. INTRODUCTION

OPERATION and control of wind turbines change significantly along the wind speed range. Figure 1 shows a typical power–wind speed curve where three operating regions are identified [1]. In the low wind speed region (region 1, also called partial load region) the main objective is the energy capture maximization. This objective is generally fulfilled by imposing a generator torque proportional to the squared rotational speed. In high wind speeds (region 3, also called full load region) the control goal is to regulate the turbine at rated power while maintaining rotational speed within safety limits. This is typically achieved by keeping constant the generator torque and acting on the pitch angle of the blades to limit the input power. Region 2 is a transition region between partial and full load, in which the objectives and even the variables to be controlled are changing. This region is critical because the limitation of the rotational speed increases the mechanical loads. Furthermore, the low sensitivity of the aerodynamic torque w.r.t. pitch actions leads to low controllability conditions that impose severe performance constraints on the controller design.

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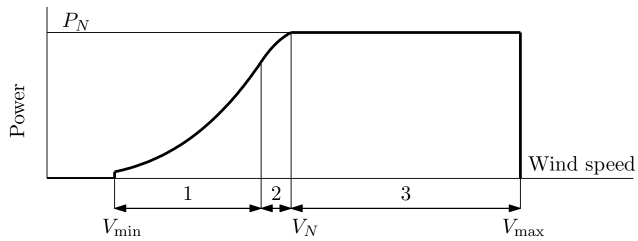

Fig. 1. Typical power-wind speed curve

Basically, the design of wind turbine control strategies valid for the entire operation envelope can be addressed in two ways. A single MIMO controller valid for all wind speeds can be designed. This approach provides more systematic procedures with stability and performance guaranties. However, the strong changes in the control structure and objectives impose serious constraints on the global performance, especially the low controllability exhibited in the transition region. Consequently, the design of a single controller valid for the whole operation range is quite cumbersome and usually results in conservative performance. Alternatively, two different controllers can be designed to achieve the control goals under partial and full load operation. This approach demands the use of some bumpless or AW compensation to avoid undesirable responses after controller switching. The control topology preferred by the wind industry is the two-controller one, the optimization of their transition being an open problem.

Wind turbine control is being intensively treated in the specialized literature. The most basic approach uses a look-up-table (L.U.T.) in low wind speeds to achieve maximum energy capture, a PI controller in high speeds to regulate the rotational speed and classical AW compensation [2]. However, a large number of advanced control techniques and tools are being exploited to design high performance wind turbine controllers. For instance, nonlinear, optimal, adaptive and sliding mode control strategies have been recently reported in the literature [3], [4], [5], [6], [7]. Most of the proposals are confined to either partial [6], [7] or full load [8] operation, while only a few cover a wide operating range [3], [4].

Among the most advanced wind turbine control techniques, the LPV gain scheduling approach is receiving great attention. In fact, after its first application in the wind energy field one decade ago [9], LPV techniques have been widely accepted and refined by the wind energy research community. LPV controllers following the two topologies mentioned above have been developed. For instance, [10], [11], [12] propose LPV controllers valid along the whole operating envelope. Bianchi *et al.* [10] describes an LPV speed control of a fixed-pitch wind turbine. In [11] a MIMO LPV controller for partial and full load operation considering also mechanical loads is presented. In [12] a gain scheduling PI controller is used for pitch control, whereas an LPV control is used for torque control. Other proposals are based on the two-controller topology. Some of them develop several controllers specifically designed for the different operating regions [13], [14]. In [13] three LPV controllers are designed for the different regions (below rated, transition and above rated). To avoid undesirable responses after controller switching, additional constraints on the controller matrices are imposed. Also, parameter dependent piecewise-affine Lyapunov functions are used, leading to less conservative designs at the cost of more difficult on-line implementation. In [14] two different LPV controllers are designed for the low and high wind speeds range, but operation and performance in the transition region is not treated in detail. Other proposals focus on the full load operating region [15], [16], [17]. In [15] a robust MIMO LPV control for the high wind speed region is designed including model uncertainties. Constant Lyapunov functions are used, leading to an easily implementable controller. In [16] a robust and fault tolerant LPV control above rated wind speed is proposed. In [17] an LPV controller for high wind speeds integrating the design of the structural parameters is presented. The use of a parameter-dependent Lyapunov function provides less conservatism at the cost of some more complex on-line implementation.

In this paper, a control strategy for a variable-speed variable-pitch wind turbine operating along the whole wind speed range is presented. The main contribution is to provide a framework to optimally combine the partial- and full-load controllers in a two-controller topology. To this end, an LPV AW algorithm is developed, which combined with an LPV pitch controller for full-load operation and a (constant-pitch) maximum power point tracking controller for partial-load operation provides formal guaranties of stability and performance along the whole operating region. Furthermore, the main LPV controller and LPV AW compensation can be combined into a unique controller rendering a simple implementation. The partial-load controller considered here is the classical quadratic torque-speed law, whereas the full-load controller consists of an LPV pitch controller designed using constant Lyapunov functions. Of course, more sophisticated control laws for high and low wind speeds can be designed and still inserted in the proposed scheme to obtain the control for the entire operating region. Moreover, the proposed LPV AW algorithm could be incorporated to pre-existent controllers like the gain-scheduling PI classically used in wind industry.

The proposed control has been thoroughly evaluated by numerical simulation and compared with a classical control scheme, focusing on the performance in the transition region. The control strategy has been applied to a 5 MW wind turbine large-scale model available in the FAST (Fatigue, Aerodynamics, Structures, and Turbulence) code developed by the National Renewable Energy Laboratory (NREL) [18]. The controller performance has been evaluated perturbing the wind turbine with very demanding wind speed profiles established in IEC standards.

II. Problem Description

The energy captured by a wind turbine is often modeled by

$$P_r(V,\beta,\Omega_r) = \frac{\pi\rho R^2}{2} C_P(\lambda,\beta)V^3, \tag{1}$$

where V is the wind speed, β is the pitch angle, Ω_r is the shaft speed, R is the rotor radius and ρ is the air density. The power coefficient C_P characterizes the conversion efficiency of the wind rotor and depends on the pitch angle and the tip-speed-ratio $\lambda = \Omega_r R/V$. Consequently, the energy captured by a wind turbine can be controlled by acting on β and Ω_r, where Ω_r is driven indirectly by the reaction torque T_g of the electrical machine.

For a matter of clarity and with the aim of standing out the main attributes of the proposal, just the first drive train mode is considered in the control-oriented model. That is, the wind turbine dynamics is modeled as the two-mass system [1], [19]

$$\begin{aligned} \dot{\Theta} &= \Omega_r - \Omega_g/N_g, \\ J_r\dot{\Omega}_r &= T_r - T_{sh}, \\ J_g\dot{\Omega}_g &= T_{sh}/N_g - T_g, \end{aligned}$$

where Ω_g is the generator speed, J_r and J_g are the inertia of the rotor and generator, respectively, N_g is the gear box ratio, $T_{sh} = K_s\Theta + B_s\Omega_r - B_s\Omega_g$ is the shaft torque, K_s the stiffness coefficient and B_s the friction. The rotor torque is obtained from the power extracted by the wind rotor as

$$T_r(V,\beta,\Omega_r) = P_r(V,\beta,\Omega_r)/\Omega_r. \tag{2}$$

In variable speed wind turbines, the electrical machine is interfaced by a full or partial power converter that controls T_g and decouples the rotational speed from the grid. Since the electrical dynamics are much faster than the mechanical subsystem, it can be assumed for the purpose of this work that T_g coincides with the torque reference.

Generally, the blades are collectively pitched. For control-oriented purposes, the pitch actuators are usually modeled as first-order low-pass filters with amplitude and rate saturation. In their linear operating mode, the pitch actuator dynamics are

$$\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta_r, \tag{3}$$

where τ is the time constant and β_r the pitch angle command.

For the design of LPV controllers, the highly nonlinear expression of T_r is linearized around the operating locus yielding

$$\hat{T}_r = B_r(\cdot)\hat{\Omega}_r + k_V(\cdot)\hat{V} + k_\beta(\cdot)\hat{\beta}, \tag{4}$$

where

$$\begin{aligned} B_r(\bar{V},\bar{\beta},\bar{\Omega}_r) &= \left.\frac{\partial T_r}{\partial \Omega_r}\right|_{(\bar{V},\bar{\beta},\bar{\Omega}_r)} \\ k_V(\bar{V},\bar{\beta},\bar{\Omega}_r) &= \left.\frac{\partial T_r}{\partial V}\right|_{(\bar{V},\bar{\beta},\bar{\Omega}_r)} \\ k_\beta(\bar{V},\bar{\beta},\bar{\Omega}_r) &= \left.\frac{\partial T_r}{\partial \beta}\right|_{(\bar{V},\bar{\beta},\bar{\Omega}_r)} \end{aligned}$$

The bar and hat over the variables denote values at the operating point and deviations w.r.t. it, respectively.

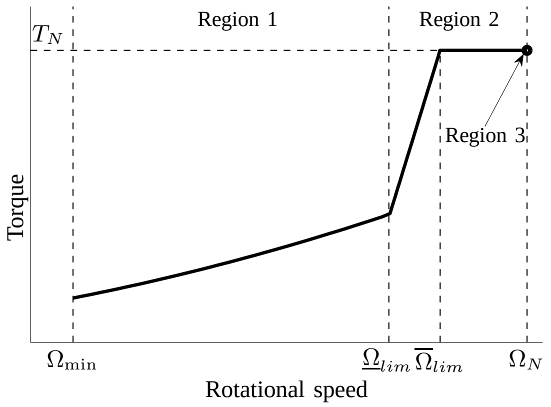

Fig. 2. Wind turbine operating locus

Combining the previous expressions, the following linear model describing the local dynamics is found:

$$\dot{x} = \begin{bmatrix} 0 & 1 & -1/N_g & 0 \\ -K_s/J_r & (B_r - B_s)/J_r & B_s/J_r & k_\beta/J_r \\ K_s/J_gN_g & B_s/J_gN_g & -B_s/J_gN_g^2 & 0 \\ 0 & 0 & 0 & -1/\tau \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ k_V/J_r & 0 & 0 \\ 0 & -1/J_g & 0 \\ 0 & 0 & 1/\tau \end{bmatrix} \begin{bmatrix} \widehat{V} \\ u \end{bmatrix}, \tag{5}$$

where $x = [\hat{\Theta}\ \hat{\Omega}_r\ \hat{\Omega}_g\ \hat{\beta}]^T$ is the state, $u = [T_g\ \beta_r]^T$ the control input and $\hat{V}$ the wind speed disturbance.

The desired operating locus is plotted on the torque-speed plane in Figure 2, where the three regions previously mentioned can also be identified.

- Region 1: Recall that the objective in this region is to maximize the energy extraction, which implies keeping the pitch angle and the tip-speed-ratio at their optimum values β_o and λ_o, respectively, so that $C_P(\lambda_o, \beta_o) = C_{P\max}$. The classical control in this region drives the generator currents so that the reaction torque and the rotational speed are related by the quadratic law [20]

$$T_g = \left(\frac{\pi\rho R^5}{2\lambda_o^3} C_{P\max}\right)\Omega_g^2 = k_t\,\Omega_g^2, \tag{6}$$

- Region 2: The aim of this region is to avoid detrimental interactions between the partial- and full-load controllers. There are several proposals for this transition region. The strategy implemented in commercial wind turbines consists of a linear region where torque increases in proportion to speed (also called region 1-1/2), and a constant torque region once rated torque is reached.
- Region 3: The objective in this region is to keep the turbine working at its nominal operating point, *i.e.* at rated power and rated rotational speed. With this aim, the electrical torque is kept constant at rated value while the rotational speed is regulated using the pitch actuator.

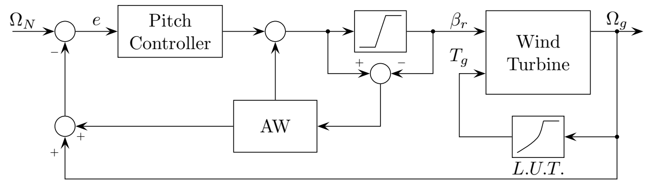

Fig. 3. Proposed control scheme

III. Proposed LPV control with LPV AW strategy

Figure 3 sketches the two-loop control structure considered in this paper. The rotational speed is controlled by means of the generator torque under partial load conditions and by means of the pitch angle under full load operation. For wind speeds below rated, a L.U.T. builds the static torque-speed reference curve for maximum energy capture (Figure 2) as in commercial wind turbine control systems. In high wind speeds, the pitch controller regulates the rotational speed at its rated value Ω_N. This actuator is only active in high wind speeds since it is saturated at its lower limit in low wind speeds. To avoid undesirable behavior in the transition wind speed range, an AW compensation is incorporated to the control system.

In this paper, we propose a systematic approach to design an LPV AW algorithm. This algorithm can not only be combined with an LPV pitch controller but with different pitch and torque controllers. Moreover, it can be incorporated to already existing gain-scheduled control systems. This proposal has several interesting properties. As can be seen in Figure 3, the AW algorithm shapes both the input and output of the pitch controller providing more degrees of freedom that can be exploited to improve the performance of the classical AW design. Since the AW algorithm is designed in the LPV framework, it results from an optimization procedure to accomplish the transition region specifications. Furthermore, the combination with an LPV pitch controller strengthen its attractive features. In fact, both AW and pitch controllers can be scheduled using the same variable, rendering a simple LPV controller implementation with stability and performance guaranties.

A. LPV description of the wind turbine

The first step to design an LPV controller is to find an LPV description of the nonlinear model. In the case of the two-mass wind turbine model, the LPV description is

$$G(\theta) : \begin{cases} \dot{x}(t) = A(\theta)x(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{7}$$

where

$$A(\theta) = \begin{bmatrix} 0 & 1 & -1/N_g & 0 \\ -K_s/J_r & -B_s/J_r & -B_s/J_rN_g & 0 \\ K_s/N_gJ_g & -B_s/N_gJ_g & -B_s/J_gN_g^2 & 0 \\ 0 & 0 & 0 & -1/\tau \end{bmatrix} + B_r(\theta)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1/J_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + k_{r,\beta}(\theta)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/J_r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1/Jg & 0 \\ 0 & 1/\tau \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.$$

This model is obtained by linearizing the nonlinear system as described in Section II. The term associated to the wind speed (*i.e.*, K_V) is not included in (7) since it does not affect stability. Considering the control strategy of Figure 2, there exists in region 3 a one-to-one correspondence among the values $\bar{V}$ and $\bar{\Omega}_r$, and the pitch angle $\bar{\beta}$. Therefore, the LPV model (7) can be parameterized by only one variable $\theta = \bar{\beta}$.

B. LPV pitch controller

The pitch controller $K(\theta)$ is an LPV system of the form

$$\begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} = \sum_{j=0}^{2} f_j(\theta(t)) \begin{bmatrix} A_{c,j} & B_{c,j} \\ C_{c,j} & D_{c,j} \end{bmatrix} \begin{bmatrix} x_c(t) \\ e(t) \end{bmatrix}, \qquad (8)$$

where $f_0 = 1$, $f_1(\theta) = B_r(\theta)$ and $f_2(\theta) = k_{r,\beta}(\theta)$. The design of the LPV gain-scheduling controller (8) is similar to H_∞ optimal control, *i.e.*, the control specifications are expressed as the minimization of the induced $\mathcal{L}_2$ norm of the operator $T_{zw} : w \to z$, mapping the disturbance w to the output z,

$$\|T_{zw}\|_{\mathcal{L}_2} = \sup_{\substack{w \neq 0 \\ \theta \in \Theta}} \frac{\|z\|_2}{\|w\|_2} < \gamma \qquad (9)$$

where $\|z\|_2 = \sqrt{\int z^T z dt}$ and $\gamma > 0$ [21].

Consequently, previous to the design of an LPV controller, it is necessary to define the performance signal z, the disturbance w and the interconnection between the plant and controller. The control setup is shown in Figure 4. The design can be stated as a mixed sensitivity problem where a compromise between rotational speed deviations and pitch activity needs to be fulfilled. Therefore, the disturbance w is the rotational speed set-point and the performance signal is given by $z = \begin{bmatrix} \tilde{e} & \tilde{u} \end{bmatrix}^T$, where

$$\tilde{e} = W_e(\Omega_N - \Omega_g), \qquad \tilde{u} = W_u u.$$

The weighting functions W_e and W_u penalize the speed error in low frequencies and the high frequency components of the control action, respectively. Since it appears in the same way as the additive uncertainty, W_u allows also to cover the model uncertainty. Integral action is included to ensure zero steady-state error. For stabilizability reasons, the controller is factorized as $K(\theta) = (1/s) \cdot \tilde{K}(\theta)$ [22]. Once the control setup is defined, the controller (8) is obtained by solving a convex optimization problem with infinite number of Linear Matrix Inequalities (LMIs) constraints. To circumvent this issue, the parameter space Θ is sampled at a set of points $\theta \in \Theta_g \triangleq \{\theta = \theta_i, i = 1, \ldots, n_p\}$ (see the Appendix for more details).

C. AW compensation

The AW compensation scheme proposed in this paper is inspired on the theoretical framework introduced in [23] and sketched in Figure 5. As mentioned above, the AW compensation proposed here comprises two compensation terms: one (u_d) acting on the controller output u and the other y_d on the controller input e. Defining

$$\begin{bmatrix} u_d(t) \\ y_d(t) \end{bmatrix} = T_{aw}(\theta(t)) * \check{u}(t) = \begin{bmatrix} M(\theta(t)) - I \\ N(\theta(t)) \end{bmatrix} * \check{u}(t),$$

where $N(\theta) = G(\theta) \cdot M(\theta)$ and $*$ denotes the input-output mapping, it can be proved after some system manipulations that the compensation scheme in Figure 5 is equivalent to the block diagram of Figure 6. It can be inferred from this figure that $M(\theta)$ must be designed to ensure stability of the closed loop comprising $M(\theta) - I$ and the nonlinear operator, as well as to minimize the effect of y_d on the controlled variable. Moreover, factorizing the LPV system $G(\theta)$ in coprime factors [24], the AW compensator design comes down to the design of a parameter varying state feedback gain fulfilling an induced $\mathcal{L}_2$ norm condition. More precisely, let

$$\begin{bmatrix} \dot{x}_{aw}(t) \\ u_d(t) \\ y_d(t) \end{bmatrix} = \sum_{j=0}^{2} f_j(\theta) \begin{bmatrix} A_j + B_2 H_j & B_2 \\ H_j & 0 \\ C_2 & 0 \end{bmatrix} \begin{bmatrix} x_{aw}(t) \\ \check{u}(t) \end{bmatrix}$$

be the state-space realization of $T_{aw}(\theta)$, where $H(\theta) = \sum_{j=0}^{2} f_j(\theta) H_j$ is a state-feedback gain such that $T_{aw}(\theta)$ is quadratically stable for $\theta \in \Theta$. Then, using the Small Gain Theorem, the AW compensator will ensure quadratic stability during saturation if $\|M(\theta) - I\|_{\mathcal{L}_2} < 1$. The minimization of the effect on the controlled variable can similarly be expressed

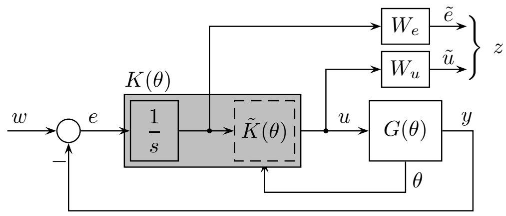

Fig. 4. Setup for the design of the LPV pitch controller

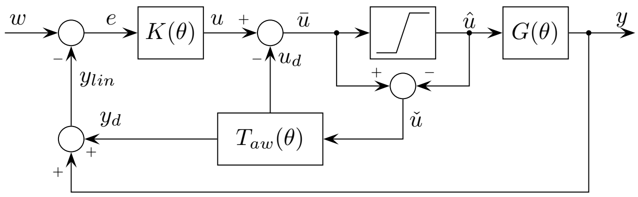

Fig. 5. Anti-windup compensation scheme

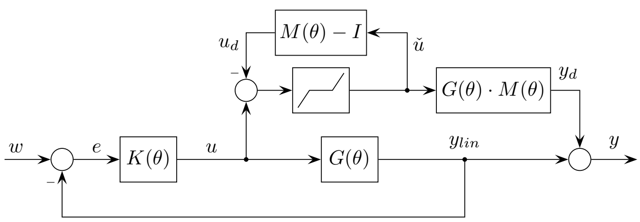

Fig. 6. Equivalent representation of the AW compensation scheme in Figure 5

as $\|N(\theta)\|_{\mathcal{L}_2} < \nu$. Both conditions will be satisfied if

$$\left\| \begin{matrix} M(\theta) - I \\ N(\theta) \end{matrix} \right\|_{\mathcal{L}_2} < \nu, \quad (10)$$

with $\nu < 1$. Therefore, using standard results of LPV theory [21], [25], the design of the AW compensation consists in solving the following optimization procedure

minimize $\nu(Q, W(\theta))$,
subject to

$$\begin{bmatrix} QA(\theta) + B_2W(\theta) + (\star) & \star & \star & \star \\ B_2^T & -\nu I_{n_u} & \star & \star \\ W(\theta) & 0 & -\nu I_{n_u} & \star \\ C_2Q & 0 & 0 & -\nu I_{n_y} \end{bmatrix} <0,$$
$$Q = Q^T > 0, \quad \nu <1.$$

for all $\theta \in \Theta$ with $\star$ induced by symmetry and $W(\theta) = \sum_{j=0}^{2} f_j(\theta)W_j$. The state feedback is then computed as $H(\theta) = Q^{-1}W(\theta)$. Like in the LPV controller design, the parameter space Θ is sampled at a set of points $\theta \in \Theta_g$.

Note that the AW compensation only depends on the non saturated system $G(\theta)$.

D. LPV controller with AW compensation

The LPV AW algorithm can be easily embedded to the LPV pitch controller using the same scheduling variable. So, both main pitch controller and AW compensation can be implemented jointly as a single LPV controller. Effectively, the LPV model of the pitch controller with AW compensation is given by

$$\begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} = \sum_{j=0}^{2} f_j(\theta) \begin{bmatrix} \bar{A}_{c,j} & \bar{B}_{c1,j} & \bar{B}_{c2,j} \\ \bar{C}_{c,j} & D_{c,j} & 0 \end{bmatrix} \begin{bmatrix} x_c(t) \\ e(t) \\ \check{u}(t) \end{bmatrix}, \quad (11)$$

$$\bar{A}_{c,j} = \begin{bmatrix} A_{c,j} & -B_{c,j}C_2 \\ 0 & A_j + B_2H_j \end{bmatrix}, \; \bar{B}_{c1,j} = \begin{bmatrix} B_{c,j} \\ 0 \end{bmatrix},$$
$$\bar{B}_{c2,j} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \; \bar{C}_{c,j} = \begin{bmatrix} C_{c,j} & -(D_{c,j}C_2 + H_j) \end{bmatrix}.$$

where the parameter dependence of the main controller is preserved.

IV. SIMULATION RESULTS

The proposed control strategy was assessed on a 5 MW three-bladed variable-speed variable-pitch wind turbine benchmark [18]. The wind turbine parameters are listed in Table I. The operating locus for the generator torque is given in Figure 2, with $\underline{\Omega}_{lim} = 1079$ rpm and $\overline{\Omega}_{lim} = 1115$ rpm. The performance of the proposed control scheme was evaluated numerically using a large-scale 16 degrees-of-freedom model running on FAST [26], so that the robustness against unmodeled dynamics can be checked (recall that the controller was designed using a control-oriented third order model).

The proposed LPV control scheme was computed according to Sections III and Appendix, with

$$W_e(s) = 3, \qquad W_u(s) = 0.02\frac{s/2.5 + 1}{s/250 + 1}.$$

TABLE I
WIND TURBINE PARAMETERS

Parameter Value	Description
$P_N = 5.5967$ MW	Rated Power
$N_p = 3$	Blades Number
$R = 63$	Rotor Radius
$N_g = 97$	Gear Box Ratio
$B_s = 6210$ KNm/(r/s)	Damping
$J_r = 38759227$ kgm^2	Rotor inertia
$J_g = 534.2$ kgm^2	Generator inertia
$K_s = 867637$ KN/r	Stifness
$V_{min} = 3$ m/s	Cut-in Wind Speed
$V_{max} = 25$ m/s	Cut-out Wind Speed
$\beta_{min} = 0$	Minimum pitch angle
$\beta_{max} = 30$	Maximum pitch angle
$\lvert\dot{\beta}\rvert_{max} = 10\ ^o/s$	Maximum pitch rate
$\Omega_N = 1,173.7$ rpm	Rated speed
$T_N = 43093.55$ Nm	Rated torque

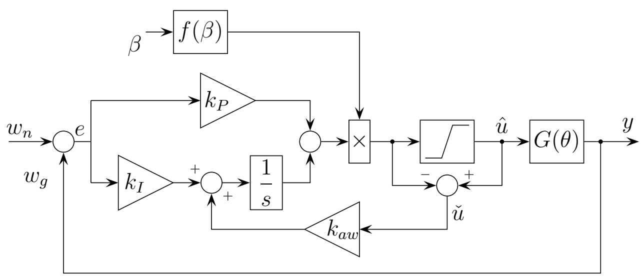

Fig. 7. PI control configuration to compare with

This choice ensures good speed regulation with low pitch activity. It also provides robustness against the unmodeled dynamics of the FAST wind turbine. The parameter space Θ was sampled at 13 points along the interval ranging from 0 to $30°$. The optimization problems to obtain the LPV controller and the AW compensation were solved using Sedumi [27] and YALMIP [28].

With the aim of comparing the proposed control scheme with a baseline controller, the responses obtained using a typical gain-scheduled PI control with back-calculation AW compensation depicted in Figure 7 are also presented. The gain-scheduled PI controller, in combination with the benchmark wind turbine under analysis, is broadly used to assess the performance of new control strategies This controller structure and details on its design can be found in [18]. The scheduling function and tuning parameters are selected here as $f(\beta) = 1/(1 + \beta/0.11$ deg), $k_P = 18.8 \times 10^{-3}$ seconds and $k_I = 8.07 \times 10^{-3}$. This tuning leads to a response with a damping close to 0.7 and a natural frequency of 0.6 rad/s, which has been reported as the optimum response for the benchmark wind turbine [29]. Finally, the classical back-calculation AW gain was chosen as $k_{aw} = 0.5$ after several attempts since higher values did not improve AW compensation.

Three demanding scenarios have been built to test the

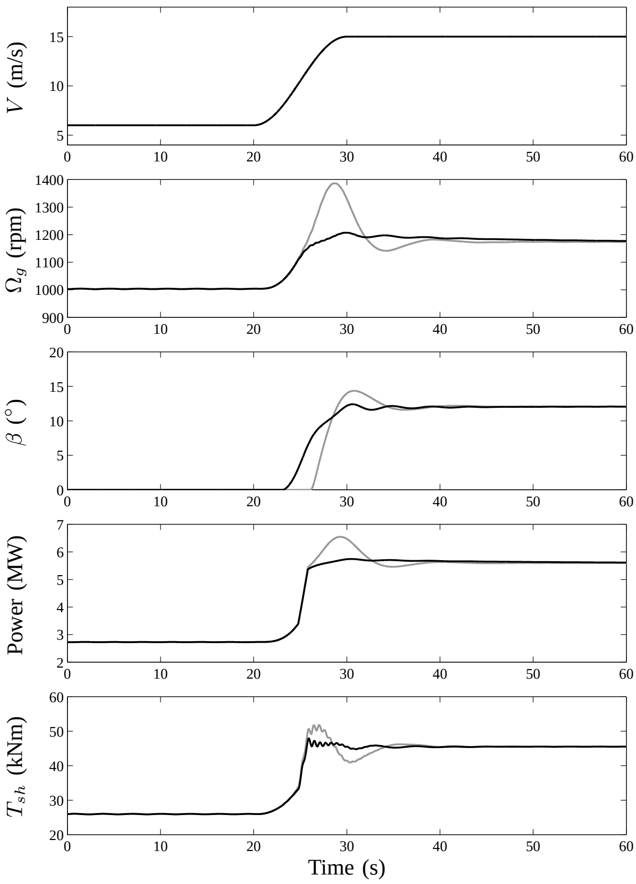

Fig. 8. Closed loop responses to a wind rise profile with the LPV control (black line) and the classical PI control (gray line)

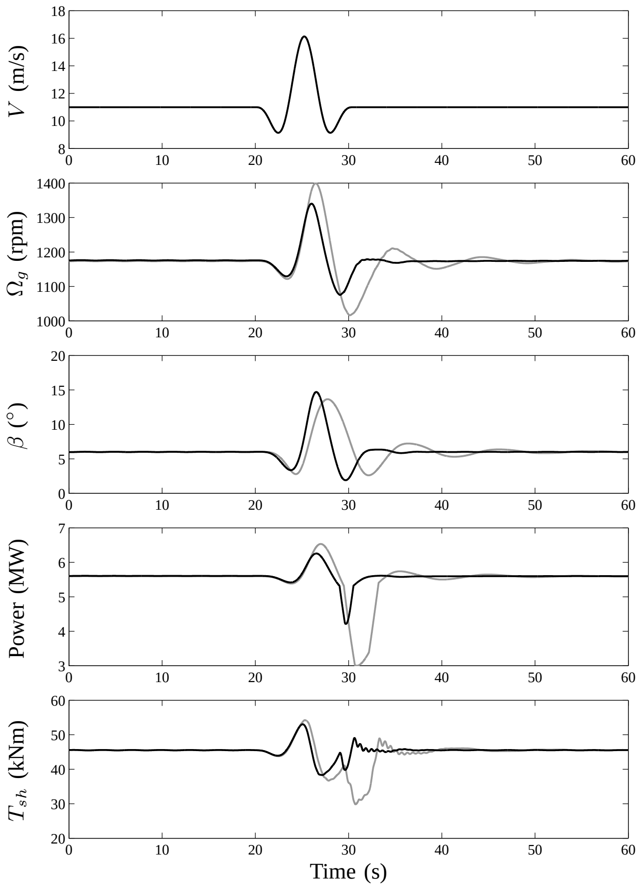

Fig. 9. Closed loop responses to a wind gust above rated with the LPV control (black line) and the classical PI control (gray line)

proposed control. The first scenario corresponds to a wind rise established in the standard IEC 61400-1. This wind profile allows evaluating the behavior under extreme conditions in which the wind turbine goes through the three operating regions. As can be seen in Figure 8, the wind speed rises from 6 m/s to 15 m/s in 10 seconds. Recall that the rated wind speed for the NREL 5 MW wind turbine is 11.4 m/s. It can be observed that the proposed control achieves more effective regulation of the rotational speed. In fact, the overshoot in the speed response is very low as the maximum speed exceeds Ω_N in just 2.55 %. On the contrary, the PI control with classical AW produces a large overshoot leading to an over speed of 17.9 % Ω_N. It can also be observed that the improvement in regulation is achieved with a smoother pitch signal, which exposes the wind turbine to less mechanical stresses. The bottom plots in Figure 8 show the power and shaft torque T_{sh} profiles. The latter provides an idea of the mechanical stress supported by the drive-train. Clearly, T_{sh} excursions are lower for the proposed control.

The second scenario corresponds to a wind gust also included in the standard IEC 61400-1. In this case, the wind turbine remains in region 3 most of the time. Therefore, the responses observed are not affected by the AW compensation. In Figure 9, it can be seen that the proposed control also achieves a tight regulation of the rotational speed, especially in the falling edge. This has a significant effect on the power and the shaft torque as can be seen in the bottom plots of Figure 9. Better regulation is achieved at the cost of faster pitch angle changes. Nevertheless, the pitch activity in both control approaches are similar. A more marked improvement can be seen in the power and the shaft torque at the bottom of Figure 9.

The last scenario considers a 10-minutes realistic three-dimensional wind speed field covering the whole swept area. The wind speed field was generated with Turbsim [30]. The 8 m/s mean wind speed was selected so that the turbine works most of the time in the transition region. The responses obtained with both controllers can be observed in Figure 10. The top plot depicts the wind speed at hub height. Note that the aerodynamic torque produced by the wind speed field is not the same as the corresponding to the wind speed at the hub. This explains, together with the the system inertia and control dynamics, that during some time intervals the pitch controller is active while wind speed at the hub is well below rated. The Figure 10 corroborates that the proposed controller performs significantly better than the baseline controller. In fact, it exhibits better speed regulation and smoother power output with lower pitch activity, particularly of high frequencies.

V. CONCLUSIONS

This contribution presents a novel control design for wind turbines, extending the widely accepted two-step design

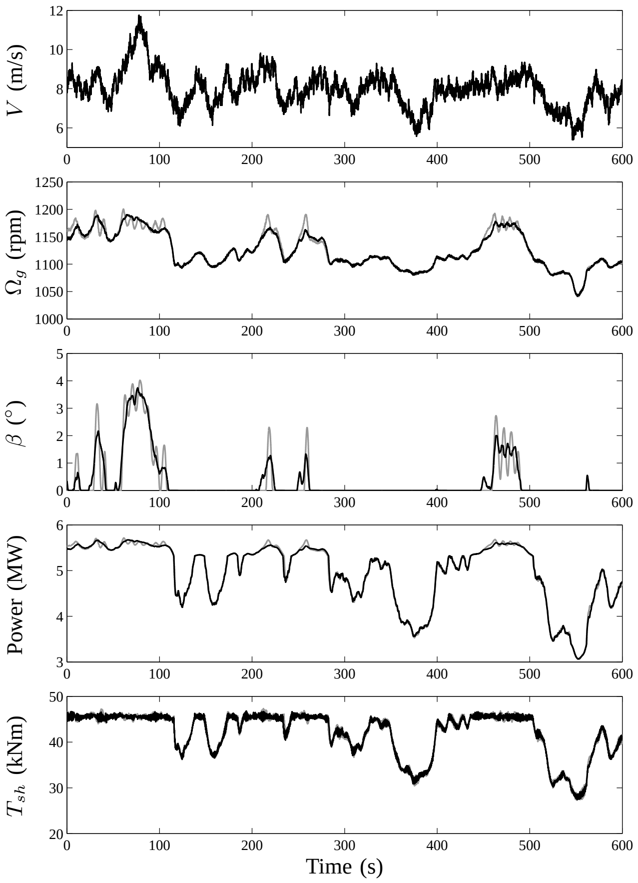

Fig. 10. Closed loop responses to a realistic wind profile with the LPV control (black line) and the classical PI control (gray line)

paradigm to more advanced control techniques. The proposed AW compensation designed within the LPV framework can indistinctly be combined with LPV or other gain scheduling controllers. The extra degrees of freedom of the proposed AW scheme are exploited to optimize the transition between partial and full load operation, providing also better stability margins. When combined with an LPV pitch controller, stability and performance guaranties along the whole operating envelope are achieved. Moreover, since both controllers can be scheduled by a single parameter, a simple controller implementation results. The control strategy was assessed by numerical simulation of a high-order wind turbine benchmark under very demanding scenarios. The tests showed that the proposed LPV controller improves appreciably the transition performance obtained with the classical gain-scheduled PI control with back-calculated AW compensation. In fact, tighter speed regulation and smoother torque and power transitions between partial and full load operation were corroborated. These improvements were obtained with lower pitch activity, particularly of high frequencies, what is crucial to extend the useful life of the pitch servos and drive train components of large wind turbines.

APPENDIX

This appendix presents a brief summary of LPV techniques [31], [32].

An LPV system can be expressed in the general form

$$\begin{bmatrix}\dot{x}(t)\\ z(t)\\ e(t)\end{bmatrix} = \left(\begin{bmatrix}A_0 & B_{1,0} & B_2\\ C_{1,0} & D_{11,0} & D_{12}\\ C_2 & D_{21} & 0\end{bmatrix} + \sum_{j=1}^{m} f_j(\theta(t))\begin{bmatrix}A_j & B_{1,j} & 0\\ C_{1,j} & D_{11,j} & 0\\ 0 & 0 & 0\end{bmatrix}\right)\begin{bmatrix}x(t)\\ w(t)\\ u(t)\end{bmatrix} \tag{12}$$

with $x \in \mathbb{R}^{n_s}$ being the state, $z \in \mathbb{R}^{n_z}$ the performance output, $y \in \mathbb{R}^{n_y}$ the measured variable, $w \in \mathbb{R}^{n_w}$ the disturbance and $u \in \mathbb{R}^{n_u}$ the control action. The parameter $\theta \in \mathbb{R}^{n_p}$ lies in a compact set Θ and f_j $(j = 1, \ldots, m)$ are continuous functions.

The synthesis problem consists in finding an stabilizing LPV gain-scheduled controller (8) so that the performance constraint (9) is satisfied. This controller is computed by solving the following optimization problem with LMIs constraints.

minimize $\gamma(X, Y, \hat{A}_c(\theta), \hat{B}_c(\theta), \hat{C}_c(\theta), D_c(\theta))$,

subject to (13) and

$$\begin{bmatrix}X & I\\ I & Y\end{bmatrix} > 0,\ X = X^T > 0,\ Y = Y^T > 0,$$

for all $\theta \in \Theta$ and with

$$\hat{A}_c(\theta) = \sum_{j=0}^{m} f_j(\theta(t))\hat{A}_{c,j}, \quad \hat{B}_c(\theta) = \sum_{j=0}^{m} f_j(\theta(t))\hat{B}_{c,j},$$
$$\hat{C}_c(\theta) = \sum_{j=0}^{m} f_j(\theta(t))\hat{C}_{c,j}, \quad \hat{D}_c(\theta) = \sum_{j=0}^{m} f_j(\theta(t))\hat{D}_{c,j}.$$

The controller matrices are given by

$$\begin{aligned}A_c(\theta) &= N^{-1}(\hat{A}_c(\theta) - X(A(\theta) - B_2 D_c(\theta) C_2)Y\\ &\qquad - \hat{B}_c(\theta) C_2 Y - X B_2 \hat{C}_c(\theta))M^{-T},\\ B_c(\theta) &= N^{-1}(\hat{B}_c(\theta) - X B_2 D_c(\theta)),\\ C_c(\theta) &= (\hat{C}_c(\theta) - D_c(\theta) C_2 Y)M^{-T},\end{aligned}$$

where $I - XY = NM^T$ [31]. This is a convex optimisation problem with infinite number of constraints. To reduce the problem to a finite number of LMIs, the parameter space Θ is sampled at a set of points $\Theta_g = \{\theta_j,\, l = 1, \ldots, n_p\}$. Then, the constraint (13) is evaluated at every point in the grid. If the grid is dense enough, then the solution is a good approximation to the infinite dimensional problem.

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$$\begin{bmatrix} XA(\theta)+\hat{B}_c(\theta)C_2+(\star) & \star & \star & \star \\ \hat{A}_c(\theta)^T+A(\theta)+B_2D_c(\theta)C_2 & A_c(\theta)Y+B_2\hat{C}_c(\theta)+(\star) & \star & \star \\ (XB_1(\theta)+\hat{B}_c(\theta)D_{12})^T & (B_1(\theta)+B_2D_c(\theta)D_{21})^T & -\gamma I_{n_w} & \star \\ C_1(\theta)+D_{12}D_c(\theta)C_2 & C_1(\theta)Y+D_{12}\hat{C}_c(\theta) & D_{11}(\theta)+D_{12}\hat{D}_c(\theta)D_{21} & -\gamma I_{n_z} \end{bmatrix} < 0. \tag{13}$$

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