Geometric Adaptive Control of Attitude Dynamics on SO(3) with State Inequality Constraints

Shankar Kulumani, Christopher Poole, and Taeyoung Lee

Flight Dynamics & Control Lab

THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

Motivation

- Autonomous control of space vehicles is critical
  - Avoid extensive planning and interaction by operators
  - Ability to operate safely with system uncertainty
  - Independently navigate hazards and handle possible failures

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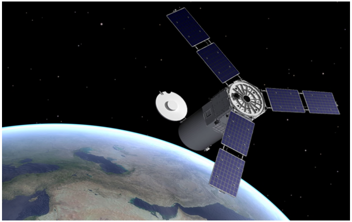

Problem Formulation

- Constrained attitude control : reorient vehicle while avoiding pointing at obstacles
  - Exclusion zones for payloads e.g infrared telescope
  - UAVs manuevering in congested locations
  - Laser/Radio emitters on spacecraft
- Previous approaches have several issues
  - Attitude parameterizations: singularities/ambiguities
  - Ad-hoc path planning: difficult to generalize to arbitrary obstacles
  - Randomized methods: lack of stability guarantees
  - Optimization based: expensive to compute and only provides open-loop control

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Attitude Parameterizations

- Euler Angles
  - Minimal representation used for small attitude changes.
  - Singularities exist for large angle slews: requires switching between 24 sequences
  - Complicated trigonometric functions
- Quaternion
  - No singularities
  - Two anti-podal quaternions for the same attitude
  - Unwinding behavior for control systems
- Geometric control
  - Globally and uniquely characterize attitude: $R \in \mathsf{SO}(3)$
  - Controller is globally valid for large angle maneuvers

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Objective

Nonlinear Control Design

Design control input u that stabilizes system from initial attitude R_0 to desired attitude R_d while avoiding obstacles

- Avoid drawbacks of other approaches
  - Geometric control - analysis is conducted directly on SO(3)
  - Barrier function - allows for arbitrary amount of constraints
  - Efficient - real time feedback control
  - Stability - Lyapunov analysis gives rigourous stability proof
  - Adaptive - handles system uncertainties

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Spacecraft Orientation

- Attitude Representation: rotation matrix from body to inertial frame

$$\mathsf{SO}(3) = \{R \in \mathbb{R}^{3\times 3} \,|\, R^T R = I,\ \det[R] = 1\}.$$

- Rigid body attitude dynamics:

$$J\dot{\Omega} + \Omega \times J\Omega = u + W(R,\Omega)\Delta, \quad \dot{R} = R\hat{\Omega}.$$

- Sensor and obstacles defined by unit vectors in $\mathbb{R}^3$
  - Body fixed sensor: $r \in \mathsf{S}^2$
  - Inertially fixed hazard: $v \in \mathsf{S}^2$
- Hard cone constraint: $r^T R^T v \leq \cos\theta$

Configuration Error Function

- Error function quantifies "distance" to desired attitude

$$\Psi(R, R_d) = A(R, R_d)B(R).$$

- Combination of attractive and repulsive terms

$$A(R, R_d) = \frac{1}{2}\mathrm{tr}\left[G\left(I - R_d^T R\right)\right].$$

$$B_i(R) = 1 - \frac{1}{\alpha_i}\ln\left(-\frac{r^T R^T v_i - \cos\theta_i}{1 + \cos\theta_i}\right).$$

Configuration Error Function

- Attractive well at the desired attitude

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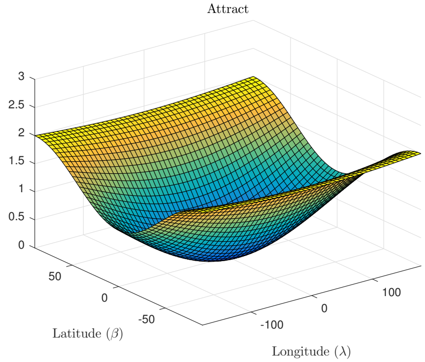
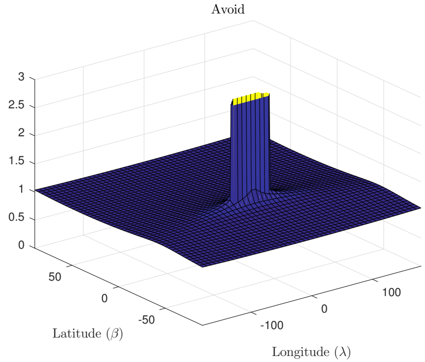


Configuration Error Function

- Define a barrier around obstacles

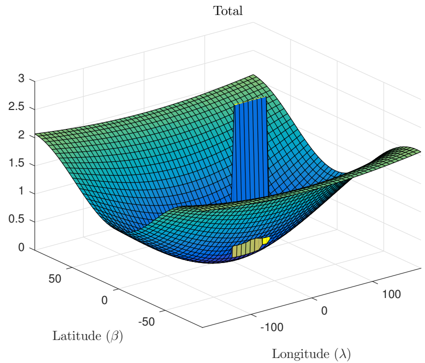
$$B_i(R) = 1 - \frac{1}{\alpha_i} \ln \left(- \frac{r^T R^T v_i - \cos\theta_i}{1 + \cos\theta_i} \right).$$



Configuration Error Function

- Configuration error: $\Psi : \mathsf{Q} \times \mathsf{Q} \to \mathbb{R}$ with control chosen to follow slope of Ψ to minimum at R_d

$$\Psi(R, R_d) = A(R, R_d)B(R).$$



Attitude Error Vectors

- Define attitude and velocity error vectors in tangent space
- Variation of Ψ expressed in terms of Lie algebra - $\delta R = R\hat{\eta}$ for $\eta \in \mathbb{R}^3$

$$\mathbf{D}_R A \cdot \delta R = \eta \cdot e_{R_A},$$
$$\mathbf{D}_R B \cdot \delta R = \eta \cdot e_{R_B}.$$

- With these error variables, we can formulate a PD-like control on $\mathsf{SO}(3)$

$$\Psi(R, R_d) = A(R, R_d) B(R),$$
$$e_R = e_{R_A} B(R) + A(R, R_d) e_{R_B},$$
$$e_\Omega = \Omega.$$

Adaptive Attitude Control

- Incorporate an adaptive control term to handle unknown disturbances, e.g. Stuck actuator or gravity gradient moment

Adaptive Attitude Controller

Zero equilibrium of error vectors are Lyapunov stable, furthermore $e_R, e_\Omega \to 0$ as $t \to \infty$

$$u = -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega - W\bar{\Delta},$$
$$\dot{\bar{\Delta}} = k_\Delta W^T (e_\Omega + c e_R).$$

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Lyapunov Analysis

- Positive definite Lyapunov function

$$\mathcal{V} = \frac{1}{2} e_\Omega \cdot J e_\Omega + k_R \Psi + c J e_\Omega \cdot e_R + \frac{1}{2k_\Delta} e_\Delta \cdot e_\Delta.$$

- Define upper bound of $e_R, \dot{e}_R$
- Upper bound of $\dot{\mathcal{V}}$ with $\zeta = [\|e_R\|, \|e_\Omega\|] \in \mathbb{R}^2$

$$\dot{\mathcal{V}} \leq -\zeta^T M \zeta.$$

- LaSalle-Yoshizawa theorem implies that $\lim_{t \to \infty} \zeta = 0$
- Estimation error e_Δ is bounded since $\mathcal{V}$ is non-increasing

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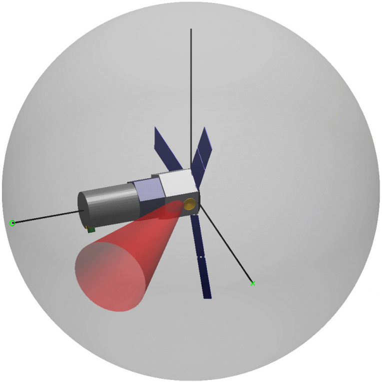
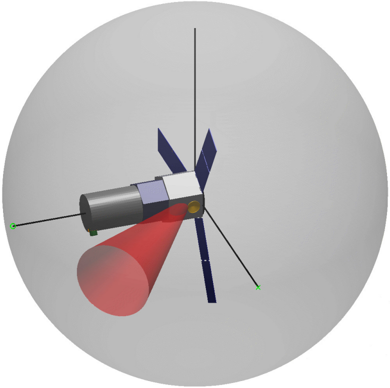
Numerical Simulation

- Simulate a S/C completing a yaw rotation
- Single obstacle in the path of sensor

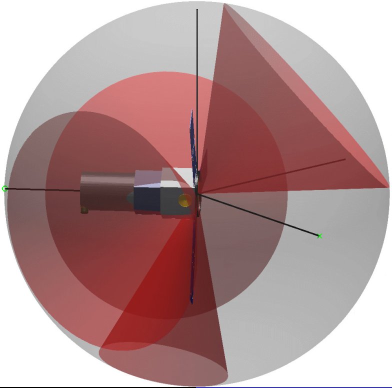




Multiple obstacles

- Easily handle multiple arbitrary constraints

$$\Psi = A(R)\left[1 + \sum_i C_i(R)\right] \qquad C_i = B(R) - 1$$

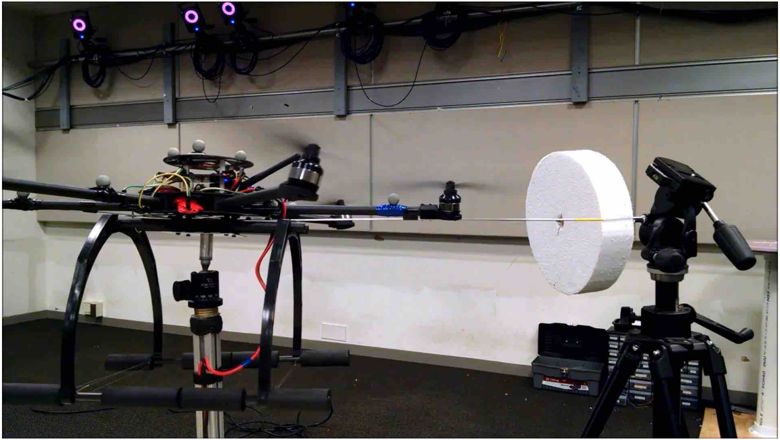


Hexrotor Design

- Three pairs of counter-rotating propellers, angled at 15°
- Multi-threaded C on Linux ODROID XU3
- Onboard IMU measures angular velocity and Vicon motion capture system for attitude
- Controlled remotely via SSH connection

Hexrotor Experiment

- Attached to spherical joint to allow only attitude dynamics



Conclusions

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  - Geometrically exact and simple attitude controller
  - Satisfies multiple attitude inequality constraints
- Obstacle avoidance computed onboard *and* in real-time
  - Ideal for aggressive large-angle rotational manuevers
  - Computationally efficient and ideal for embedded systems
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