# Geometric Control for Autonomous Landing on Asteroid Itokawa using Visual Localization

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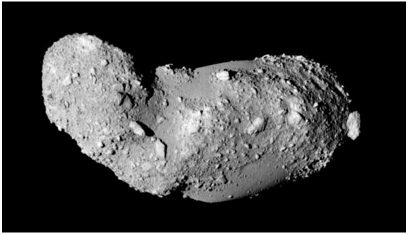
WASHINGTON, DC

August 20, 2017

# Asteroid Missions

- Science - insight into the early formation of the solar system
- Mining - vast quantities of useful materials
- Impact - high risk from hazardous near-Earth asteroids



# Asteroid Mining

- Useful materials can be extracted from asteroids to support:
  - Propulsion, construction, life support, agriculture, and precious/strategic metals
- Commercialization of near-Earth asteroids is feasible

| Element | Price ($\$\,\mathrm{kg}^{-1}$) | Sales ($\$\,$M/yr) |
|---|---:|---:|
| Phosphorous (P) | 0.08 | 2167 |
| Gallium (Ga) | 300.00 | 1544 |
| Germanium (Ge) | 745.00 | 6145 |
| Platinum (Pt) | 12 394.00 | 1705 |
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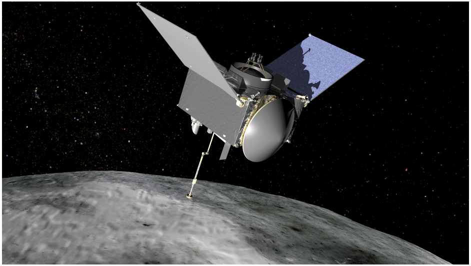
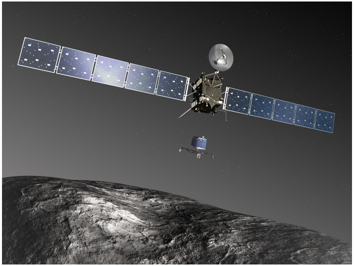
- Long history of asteroid/planetary missions
  - NEAR, Hayabusa, OSIRIS-REX, Rosetta
- Asteroid landing is particularly challenging:
  - Challenging dynamics - low gravity and spinning
  - Poor model - ground based observation of dim bodies
  - Attitude coupling - pertubations are large





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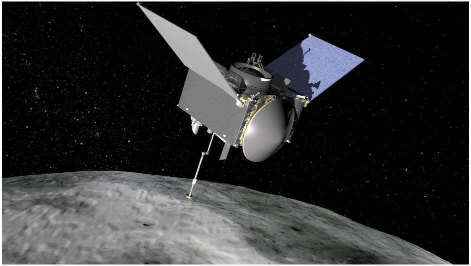

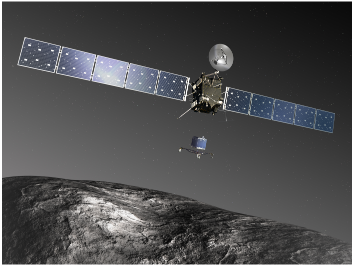

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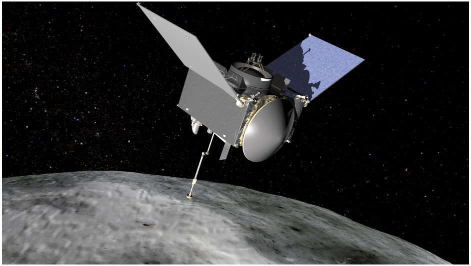

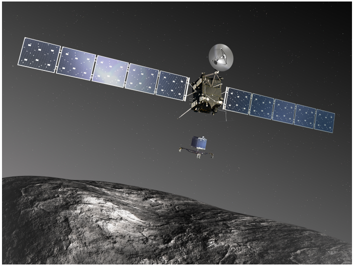

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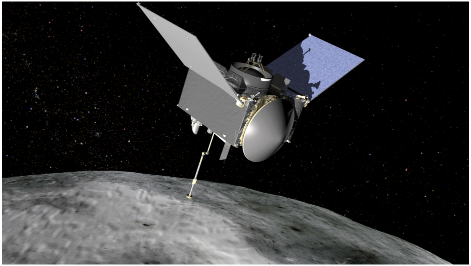

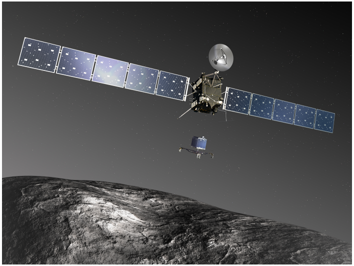

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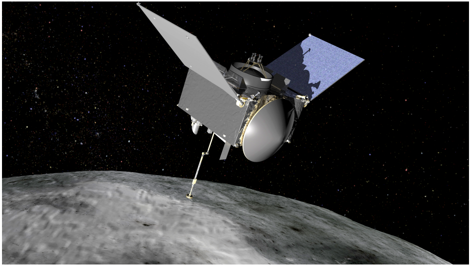

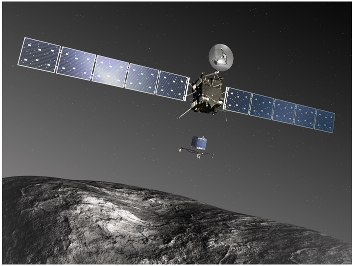

## Problem Statement

1. Develop copuled equations of motion for a spacecraft
2. EOMs on $\mathsf{SE}(3)$ to derive landing control law
   - $\mathsf{SE}(3)$ - special euclidean group describes the general motion of rigid bodies
3. Implement monocular localization to provide state estimates
4. Alleviates many of the issues of past approaches:
   - Explicitly consider rotational dynamics
   - Geometric control defined directly on $\mathsf{SE}(3)$
   - Onboard state measurements for "autonomy"

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# Attitude Parameterizations

- Euler Angles
  - Minimal representation used for small attitude changes.
  - Singularities exist for large angle slews: requires switching between 24 sequences
  - Complicated trigonometric functions
- Quaternion
  - No singularities
  - Two anti-podal quaternions for the same attitude
  - Unwinding behavior for control systems
- Geometric control
  - Globally and uniquely characterize attitude: $R \in \mathsf{SO}(3)$
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# Gravitational Modeling

- Asteroids are extended bodies not point masses
- Spherical Harmonic - only valid outside of circumscribing sphere

$$U(\boldsymbol{r}) = \frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\frac{R}{r}\right)^{n} P_{n,m}(\sin\phi)\left\{C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\right\}$$

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$$U(\boldsymbol{r}) = \frac{1}{2} G\sigma \sum_{e \in \text{edges}} \boldsymbol{r}_e \cdot \boldsymbol{E}_e \cdot \boldsymbol{r}_e \cdot L_e - \frac{1}{2} G\sigma \sum_{f \in \text{faces}} \boldsymbol{r}_f \cdot \boldsymbol{F}_f \cdot \boldsymbol{r}_f \cdot \omega_f$$

# Kinematics of Dumbbell

- Dynamics derived on SE(3), Special Euclidean group
- Rigid dumbbell model - captures attitude coupling
  - $\boldsymbol{x} \in \mathbb{R}^3$ - inertial position of COM
  - $R \in \mathsf{SO}(3)$ - transforms from body to inertial frame
  - $\boldsymbol{\Omega} \in \mathbb{R}^3$ - the angular velocity of the spacecraft
  - $R_A \in \mathsf{SO}(3)$ - transforms from asteroid to inertial frame
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  - Models the mass distribution of spacecraft
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# Equations of Motion

- Polyhedron potential in translational and rotational dynamics

$$
\begin{aligned}
\dot{\boldsymbol{x}} &= \boldsymbol{v}, \\
(m_1 + m_2)\,\dot{\boldsymbol{v}} &= m_1 R_A \frac{\partial U}{\partial \boldsymbol{z}_1} + m_2 R_A \frac{\partial U}{\partial \boldsymbol{z}_2} + \boldsymbol{u}_f, \\
\dot{R} &= R S(\boldsymbol{\Omega}), \\
J\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times J\boldsymbol{\Omega} &= \boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{u}_m.
\end{aligned}
$$

- Moment on dumbbell

$$
\boldsymbol{M}_i = m_i \left( S(R_A^T \boldsymbol{\rho}_i) R^T \frac{\partial U}{\partial \boldsymbol{z}_i} \right).
$$

# Nonlinear Control

- Geometric control used to track a desired trajectory
- Developed directly on the nonlinear manifold
  - Avoids chattering issues of sliding mode control
  - Incorporates attitude dynamics
  - Asymptotic trajectory tracking stability

$$\begin{aligned} \boldsymbol{u}_m &= -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega - J\left(\hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d\right) \\ &\quad - \boldsymbol{M}_1 - \boldsymbol{M}_{\boldsymbol{2}}, \\ \boldsymbol{u}_f &= -k_x e_x - k_v e_v + (m_1 + m_2)\ddot{x}_d - \boldsymbol{F}_1 - \boldsymbol{F}_2 \end{aligned}$$

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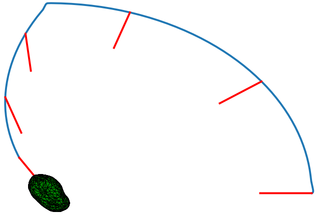
# Landing Trajectory - Attitude

- Goal: transition from orbit about Itokawa to vertical descent
- Attitude controlled to point camera at surface
  - $\boldsymbol{b}_1$ - body axis points at surface
  - $\boldsymbol{b}_3$ - orthogonal projection in $\boldsymbol{e}_3, \boldsymbol{b}_1$ plane

$$
\begin{aligned}
\boldsymbol{b}_{1d} &= -\frac{\boldsymbol{x}}{\|\boldsymbol{x}\|}, \\
\boldsymbol{b}_{3d} &= \frac{\boldsymbol{f}_3 - (\boldsymbol{f}_3 \cdot \boldsymbol{b}_{1d})\, \boldsymbol{b}_{1d}}{\|\boldsymbol{f}_3 - (\boldsymbol{f}_3 \cdot \boldsymbol{b}_{1d})\, \boldsymbol{b}_{1d}\|}, \\
\boldsymbol{b}_{2d} &= \boldsymbol{b}_{3d} \times \boldsymbol{b}_{1d}, \\
R_d &= \begin{bmatrix} \boldsymbol{b}_{1d} & \boldsymbol{b}_{2d} & \boldsymbol{b}_{3d} \end{bmatrix}.
\end{aligned}
$$



# Landing Trajectory - Position

- Transition from horizontal motion to vertical descent
  - Move from inertial $\boldsymbol{e}_2$ to asteroid $\boldsymbol{f}_1$ axis
  - Remain in the equatorial plane ( $\boldsymbol{f}_1, \boldsymbol{f}_2$ plane)
- Command is divided into two segments
  - Move to $\boldsymbol{f}_1$ along a circular path
  - Vertical descent along $\boldsymbol{f}_1$

$$\boldsymbol{x}_d = \begin{cases} 2.550 \begin{bmatrix} \sin \omega t & -\cos \omega t & 0 \end{bmatrix}, & t \leq t_d \\ R_A \begin{bmatrix} \frac{2}{t_d}(t - t_d) + 2.550 & 0 & 0 \end{bmatrix}, & t > t_d, \end{cases}$$

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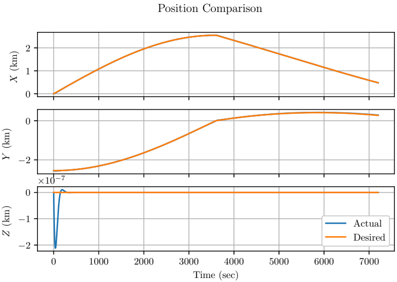
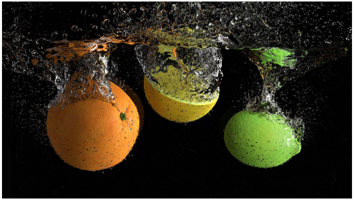


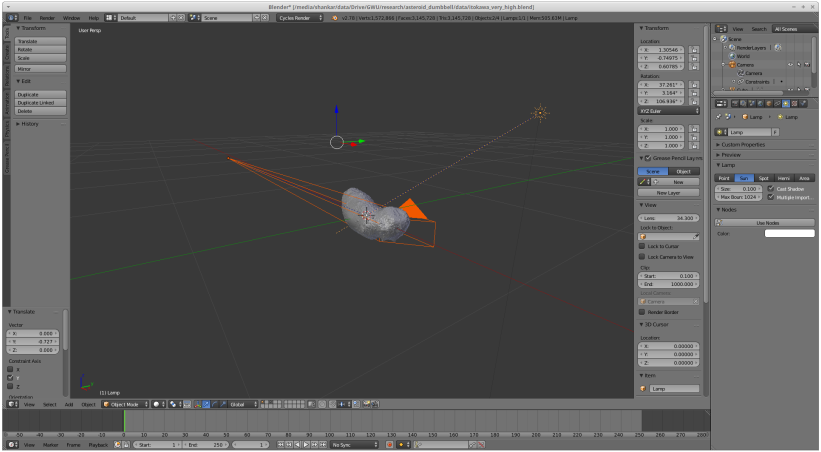
# Blender Simulation

- Open-source computer graphics rendering
  - Photorealistic Rendering
  - Realistic material and lighting
  - Physics based simulation and game engine
- Additional capability by utilizing a Python API
  - Load a asteroid shape model
  - Define camera properties/location
  - Define light source and position
  - Render image



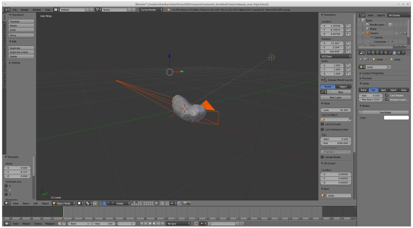
# Blender Simulation

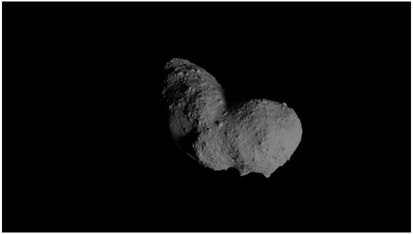
- Can render scenes using a graphical interface



# Blender Simulation

- Python used to generate imagery - incorporate in larger code

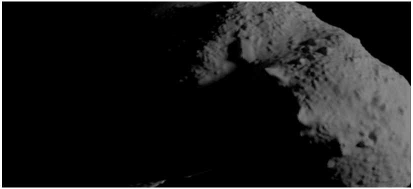
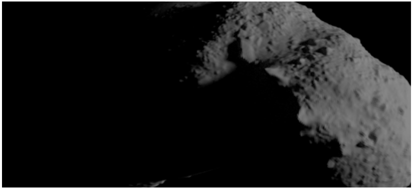




```
import bpy

scene = bpy.context.scene
bpy.ops.import_scene.obj('ast.obj')
ast = bpy.data.objects[asteroid]

ast.scale = [1, 1, 1]
ast.location = (0, 0, 0)

scene.render.filepath = 'out.png'
bpy.ops.render.render(write_still=True)
```

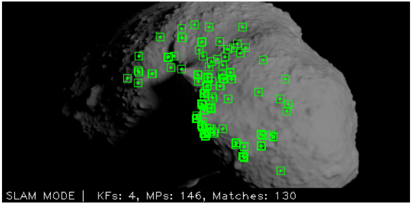
# Simulated imagery using Blender

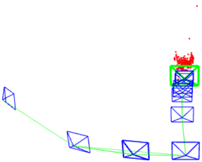
- Free & open-source 3D computer graphics program
- Offer programmatic interface through Python
- Images of Itokawa emulated to match NEAR MSI camera

# Monocular Localization

- ORB-SLAM2 - Open-source mapping and localization framework
- Allows for real-time monocular based SLAM
- Utilized here to demonstrate localization from imagery



# Conclusions

- Nonlinear controller for landing on an asteroid
- Coupled dynamics derived and considered on $\mathsf{SE}(3)$
- Localization performed using simulated monocular imagery
- Future Research Goals:
  - Utilize state estimates directly in controller
  - Update shape and gravity model in real-time

# Thank you

**Flight Dynamics & Control Lab**
Mechanical & Aerospace Engineering
School of Engineering & Applied Science

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