

Geometric Control for Autonomous Landing on Asteroid Itokawa using Visual Localization

**Shankar Kulumani, Kuya Takami
Taeyoung Lee**

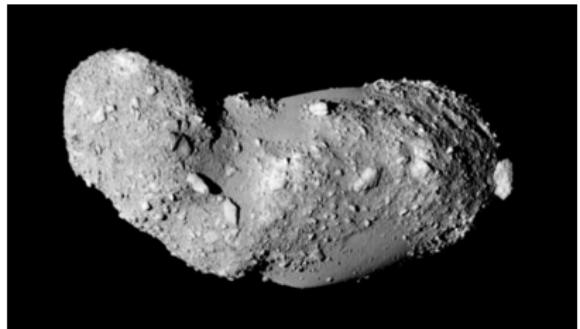
Department of Mechanical & Aerospace Engineering

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

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Asteroid Missions

- Science - insight into the early formation of the solar system
- Mining - vast quantities of useful materials
- Impact - high risk from hazardous near-Earth asteroids



Asteroid Mining

- Useful materials can be extracted from asteroids to support:
 - Propulsion, construction, life support, agriculture, and precious/strategic metals
- Commercialization of near-Earth asteroids is feasible

Element	Price (\$ kg ⁻¹)	Sales (\$ M/yr)
Phosphorous (P)	0.08	2167
Gallium (Ga)	300.00	1544
Germanium (Ge)	745.00	6145
Platinum (Pt)	12 394.00	1705
Gold (Au)	12 346.00	49
Osmium (Os)	12 860.00	307

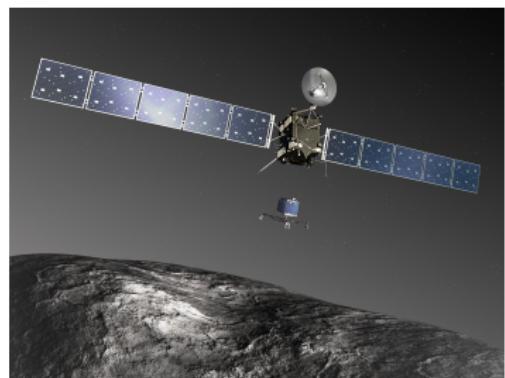
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Asteroid Landing

- Long history of asteroid/planetary missions
 - NEAR, Hayabusa, OSIRIS-REX, Rosetta
- Asteroid landing is particularly challenging:
 - Challenging dynamics - low gravity and spinning
 - Poor model - ground based observation of dim bodies
 - Attitude coupling - perturbations are large



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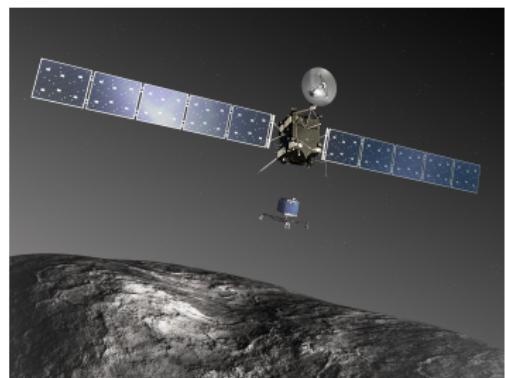
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Problem Statement

- ➊ Develop coupled equations of motion for a spacecraft
- ➋ EOMs on $\text{SE}(3)$ to derive landing control law
 - $\text{SE}(3)$ - special euclidean group describes the general motion of rigid bodies
- ➌ Implement monocular localization to provide state estimates
- ➍ Alleviates many of the issues of past approaches:
 - Explicitly consider rotational dynamics
 - Geometric control defined directly on $\text{SE}(3)$
 - Onboard state measurements for “autonomy”

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Attitude Parameterizations

- Euler Angles
 - Minimal representation used for small attitude changes.
 - Singularities exist for large angle slews: requires switching between 24 sequences
 - Complicated trigonometric functions
- Quaternion
 - No singularities
 - Two anti-podal quaternions for the same attitude
 - Unwinding behavior for control systems
- Geometric control
 - Globally and uniquely characterize attitude: $R \in \text{SO}(3)$
 - Controller is globally valid for large angle maneuvers

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Gravitational Modeling

- Asteroids are extended bodies not point masses
- Spherical Harmonic - only valid outside of circumscribing sphere

$$U(r) = \frac{\mu}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left(\frac{R}{r}\right)^n P_{n,m}(\sin \phi) \{C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)\}$$

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- Potential is a function of only the shape model
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$$U(\mathbf{r}) = \frac{1}{2}G\sigma \sum_{e \in \text{edges}} \mathbf{r}_e \cdot \mathbf{E}_e \cdot \mathbf{r}_e \cdot \mathbf{L}_e - \frac{1}{2}G\sigma \sum_{f \in \text{faces}} \mathbf{r}_f \cdot \mathbf{F}_f \cdot \mathbf{r}_f \cdot \omega_f$$

Kinematics of Dumbbell

- Dynamics derived on $\text{SE}(3)$, Special Euclidean group
- Rigid dumbbell model - captures attitude coupling
 - $x \in \mathbb{R}^3$ - inertial position of COM
 - $R \in \text{SO}(3)$ - transforms from body to inertial frame
 - $\Omega \in \mathbb{R}^3$ - the angular velocity of the spacecraft
 - $R_A \in \text{SO}(3)$ - transforms from asteroid to inertial frame
- Simple model which captures dynamic coupling
 - Models the mass distribution of spacecraft
- Preserves the geometric properties of configuration space
- Attitude and Translational motion is directly coupled

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Equations of Motion

- Polyhedron potential in translational and rotational dynamics

$$\dot{\boldsymbol{x}} = \boldsymbol{v},$$

$$(m_1 + m_2) \dot{\boldsymbol{v}} = m_1 R_A \frac{\partial U}{\partial \boldsymbol{z}_1} + m_2 R_A \frac{\partial U}{\partial \boldsymbol{z}_2} + \boldsymbol{u}_f,$$

$$\dot{\boldsymbol{R}} = \boldsymbol{R} \boldsymbol{S}(\boldsymbol{\Omega}),$$

$$J \dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times J \boldsymbol{\Omega} = \boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{u}_m.$$

- Moment on dumbbell

$$\boldsymbol{M}_i = m_i \left(S(R_A^T \boldsymbol{\rho}_i) R^T \frac{\partial U}{\partial \boldsymbol{z}_i} \right).$$

Nonlinear Control

- Geometric control used to track a desired trajectory
- Developed directly on the nonlinear manifold
 - Avoids chattering issues of sliding mode control
 - Incorporates attitude dynamics
 - Asymptotic trajectory tracking stability

$$\begin{aligned} \mathbf{u}_m = & -k_R e_R - k_\Omega e_\Omega + \boldsymbol{\Omega} \times \boldsymbol{J}\boldsymbol{\Omega} - \boldsymbol{J} \left(\hat{\boldsymbol{\Omega}} \boldsymbol{R}^T \boldsymbol{R}_d \boldsymbol{\Omega}_d - \boldsymbol{R}^T \boldsymbol{R}_d \dot{\boldsymbol{\Omega}}_d \right) \\ & - \boldsymbol{M}_1 - \boldsymbol{M}_2, \end{aligned}$$

$$\mathbf{u}_f = -k_x e_x - k_v e_v + (m_1 + m_2) \ddot{x}_d - \boldsymbol{F}_1 - \boldsymbol{F}_2$$

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Landing Trajectory - Attitude

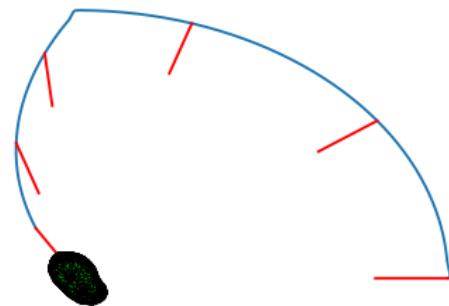
- Goal: transition from orbit about Itokawa to vertical descent
- Attitude controlled to point camera at surface
 - \mathbf{b}_1 - body axis points at surface
 - \mathbf{b}_3 - orthogonal projection in $\mathbf{e}_3, \mathbf{b}_1$ plane

$$\mathbf{b}_{1d} = -\frac{\mathbf{x}}{\|\mathbf{x}\|},$$

$$\mathbf{b}_{3d} = \frac{\mathbf{f}_3 - (\mathbf{f}_3 \cdot \mathbf{b}_{1d}) \mathbf{b}_{1d}}{\|\mathbf{f}_3 - (\mathbf{f}_3 \cdot \mathbf{b}_{1d}) \mathbf{b}_{1d}\|},$$

$$\mathbf{b}_{2d} = \mathbf{b}_{3d} \times \mathbf{b}_{1d},$$

$$R_d = [\mathbf{b}_{1d} \quad \mathbf{b}_{2d} \quad \mathbf{b}_{3d}].$$



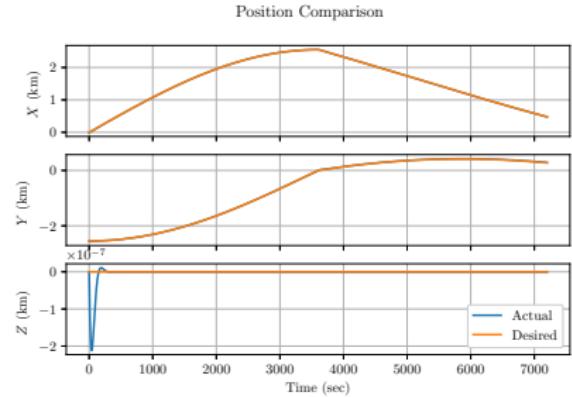
Landing Trajectory - Position

- Transition from horizontal motion to vertical descent
 - Move from inertial e_2 to asteroid f_1 axis
 - Remain in the equatorial plane (f_1, f_2 plane)
- Command is divided into two segments
 - Move to f_1 along a circular path
 - Vertical descent along f_1

$$x_d = \begin{cases} 2.550 \begin{bmatrix} \sin \omega t & -\cos \omega t & 0 \end{bmatrix}, & t \leq t_d \\ R_A \begin{bmatrix} \frac{2}{t_d}(t - t_d) + 2.550 & 0 & 0 \end{bmatrix}, & t > t_d, \end{cases}$$

Landing Trajectory - Position

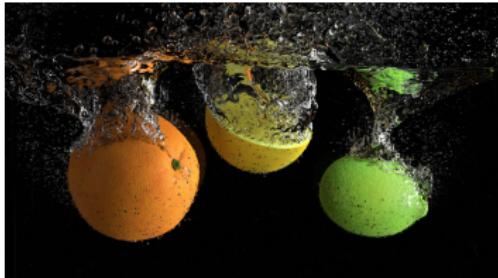
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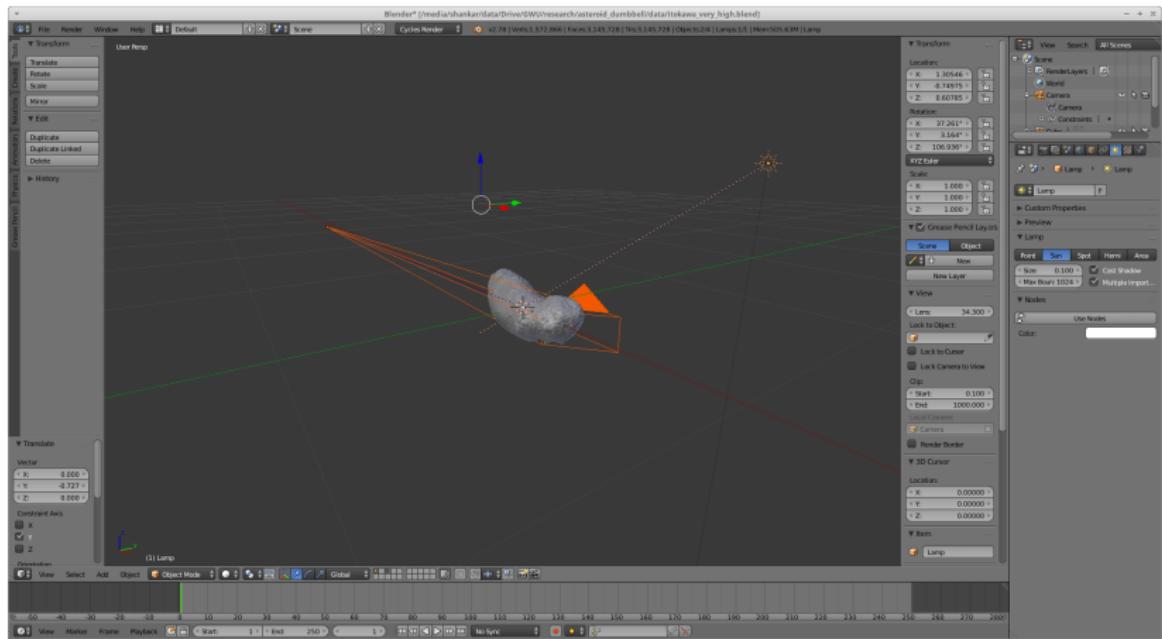
Blender Simulation

- Open-source computer graphics rendering
 - Photorealistic Rendering
 - Realistic material and lighting
 - Physics based simulation and game engine
- Additional capability by utilizing a Python API
 - Load a asteroid shape model
 - Define camera properties/location
 - Define light source and position
 - Render image



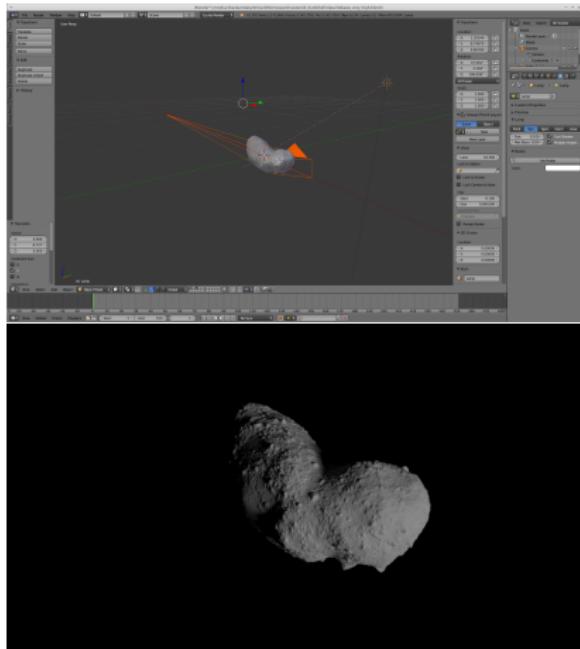
Blender Simulation

- Can render scenes using a graphical interface



Blender Simulation

- Python used to generate imagery - incorporate in larger code



```
import bpy

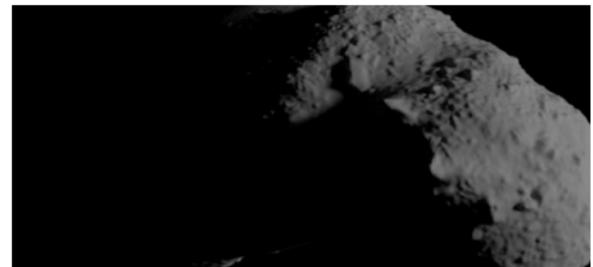
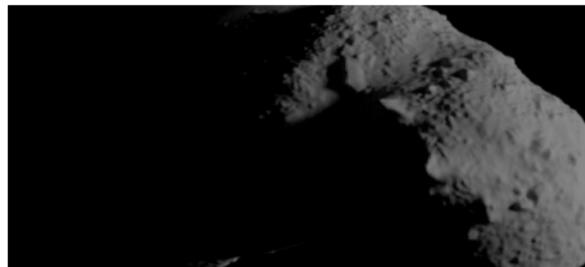
scene = bpy.context.scene
bpy.ops.import_scene.obj('ast.obj')
ast = bpy.data.objects[asteroid]

ast.scale = [1, 1, 1]
ast.location = (0, 0, 0)

scene.render.filepath = 'out.png'
bpy.ops.render.render(write_still=True)
```

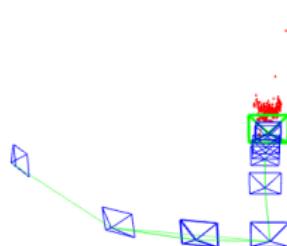
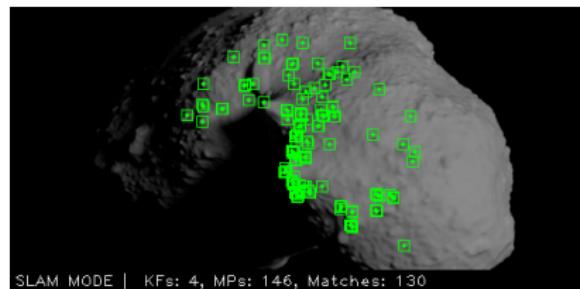
Simulated imagery using Blender

- Free & open-source 3D computer graphics program
- Offer programmatic interface through Python
- Images of Itokawa emulated to match NEAR MSI camera



Monocular Localization

- ORB-SLAM2 - Open-source mapping and localization framework
- Allows for real-time monocular based SLAM
- Utilized here to demonstrate localization from imagery



Conclusions

- Nonlinear controller for landing on an asteroid
 - Coupled dynamics derived and considered on $\text{SE}(3)$
 - Localization performed using simulated monocular imagery
 - Future Research Goals:
 - Utilize state estimates directly in controller
 - Update shape and gravity model in real-time

Thank you

Flight Dynamics & Control Lab
Mechanical & Aerospace Engineering
School of Engineering & Applied Science

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