

Geometric Control for Autonomous Landing on Asteroid Itokawa using Visual Imagery

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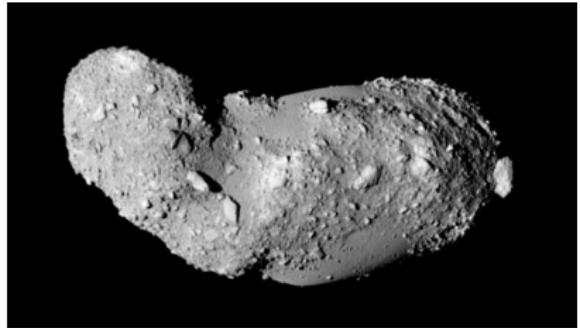
THE GEORGE WASHINGTON UNIVERSITY

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Asteroid Missions

- Science - insight into the early formation of the solar system
- Mining - vast quantities of useful materials
- Impact - high risk from hazardous near-Earth asteroids



Asteroid Mining

- Useful materials can be extracted from asteroids to support:
 - Propulsion, construction, life support, agriculture, and precious/strategic metals
- Commercialization of near-Earth asteroids is feasible

Element	Price (\$ kg ⁻¹)	Sales (\$ M/yr)
Phosphorous (P)	0.08	2167
Gallium (Ga)	300.00	1544
Germanium (Ge)	745.00	6145
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Gold (Au)	12 346.00	49
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Asteroid Landing

- Long history of asteroid/planetary missions
 - NEAR, Hayabusa, OSIRIS-REX, Rosetta
- Asteroid landing is particularly challenging:
 - Challenging dynamics - low gravity and spinning
 - Poor model - ground based observation of dim bodies
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Problem Statement

- ➊ Develop coupled equations of motion for a spacecraft
- ➋ Use EOMs on $\text{SE}(3)$ to derive landing control law
- ➌ Implement monocular localization to provide state estimates
- ➍ Alleviates many of the issues of past approaches:
 - Explicitly consider rotational dynamics
 - $\text{SE}(3)$ geometric control instead of sliding mode
 - Onboard state measurements for “autonomy”

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Gravitational Modeling

- Asteroids are extended bodies not point masses
- Spherical Harmonic - only valid outside of circumscribing sphere

$$U(r) = \frac{\mu}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left(\frac{R}{r}\right)^n P_{n,m}(\sin \phi) \{C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)\}$$

- Infinite series is an approximation and adds complexity
 - Model switching at circumscribing sphere
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Polyhedron Gravitation Model

- Potential is a function of only the shape model
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- Exact potential assumes a constant density
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$$U(\mathbf{r}) = \frac{1}{2}G\sigma \sum_{e \in \text{edges}} \mathbf{r}_e \cdot \mathbf{E}_e \cdot \mathbf{r}_e \cdot \mathbf{L}_e - \frac{1}{2}G\sigma \sum_{f \in \text{faces}} \mathbf{r}_f \cdot \mathbf{F}_f \cdot \mathbf{r}_f \cdot \omega_f$$

Kinematics of Dumbbell

- Dynamics derived on $\text{SE}(3)$, Special Euclidean group
- Rigid dumbbell model - captures attitude coupling
 - $x \in \mathbb{R}^3$ - inertial position of COM
 - $R \in \text{SO}(3)$ - transforms from body to inertial frame
 - $\Omega \in \mathbb{R}^3$ - the angular velocity of the spacecraft
 - $R_A \in \text{SO}(3)$ - transforms from asteroid to inertial frame
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Equations of Motion

- Polyhedron potential in translational and rotational dynamics

$$\dot{\boldsymbol{x}} = \boldsymbol{v},$$

$$(m_1 + m_2) \dot{\boldsymbol{v}} = m_1 R_A \frac{\partial U}{\partial \boldsymbol{z}_1} + m_2 R_A \frac{\partial U}{\partial \boldsymbol{z}_2} + \boldsymbol{u}_f,$$

$$\dot{\boldsymbol{R}} = \boldsymbol{R} S(\boldsymbol{\Omega}),$$

$$J \dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times J \boldsymbol{\Omega} = \boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{u}_m.$$

- Moment on dumbbell

$$\boldsymbol{M}_i = m_i \left(S(R_A^T \boldsymbol{\rho}_i) R^T \frac{\partial U}{\partial \boldsymbol{z}_i} \right).$$

Nonlinear Control

- Geometric control used to track a desired trajectory
- Developed directly on the nonlinear manifold
 - Avoids chattering issues of sliding mode control
 - Incorporates attitude dynamics
 - Stability guarantee

$$\begin{aligned} \mathbf{u}_m = & -k_R e_R - k_\Omega e_\Omega + \boldsymbol{\Omega} \times \boldsymbol{J}\boldsymbol{\Omega} - \boldsymbol{J} \left(\hat{\boldsymbol{\Omega}} \boldsymbol{R}^T \boldsymbol{R}_d \boldsymbol{\Omega}_d - \boldsymbol{R}^T \boldsymbol{R}_d \dot{\boldsymbol{\Omega}}_d \right) \\ & - \boldsymbol{M}_1 - \boldsymbol{M}_2, \end{aligned}$$

$$\mathbf{u}_f = -k_x e_x - k_v e_v + (m_1 + m_2) \ddot{x}_d - \boldsymbol{F}_1 - \boldsymbol{F}_2$$

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Landing trajectory

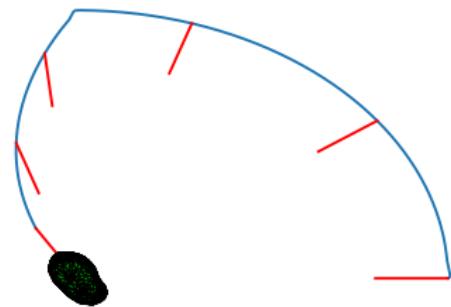
- Goal: transition from orbit about Itokawa to vertical descent
- Attitude controlled to point camera at surface
 - \mathbf{b}_1 - body axis points at surface
 - \mathbf{b}_3 - orthogonal projection in $\mathbf{e}_3, \mathbf{b}_1$ plane

$$\mathbf{b}_{1d} = -\frac{\mathbf{x}}{\|\mathbf{x}\|},$$

$$\mathbf{b}_{3d} = \frac{\mathbf{f}_3 - (\mathbf{f}_3 \cdot \mathbf{b}_{1d}) \mathbf{b}_{1d}}{\|\mathbf{f}_3 - (\mathbf{f}_3 \cdot \mathbf{b}_{1d}) \mathbf{b}_{1d}\|},$$

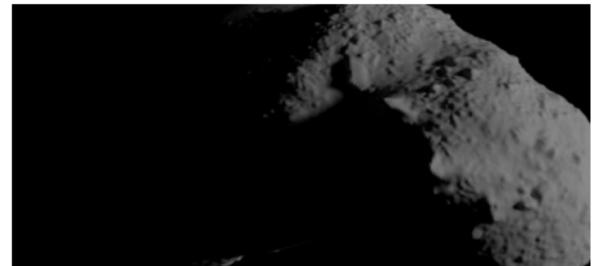
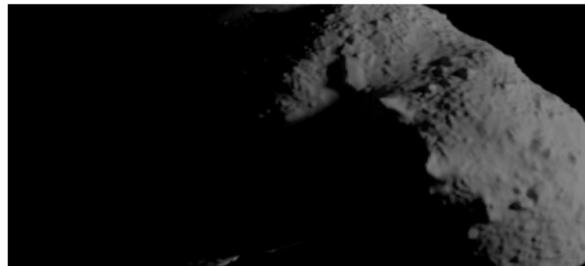
$$\mathbf{b}_{2d} = \mathbf{b}_{3d} \times \mathbf{b}_{1d},$$

$$R_d = [\mathbf{b}_{1d} \quad \mathbf{b}_{2d} \quad \mathbf{b}_{3d}].$$



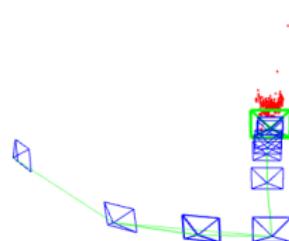
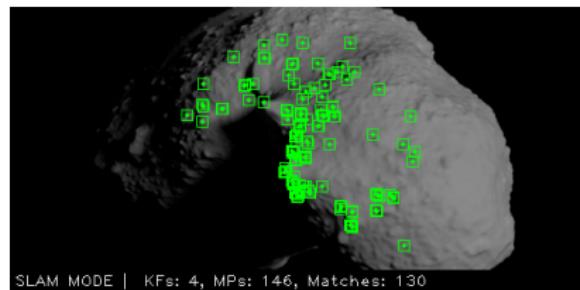
Simulated imagery using Blender

- Free & open-source 3D computer graphics program
- Offer programmatic interface through Python
- Images of Itokawa emulated to match NEAR MSI camera



Monocular Localization

- ORB-SLAM2 - Open-source mapping and localization framework
- Allows for real-time monocular based SLAM
- Utilized here to demonstrate localization from imagery



Conclusions

- Nonlinear controller for landing on an asteroid
- Coupled dynamics derived and considered on $\text{SE}(3)$
- Localization performed using simulated monocular imagery
- Future Research Goals:
 - Utilize state estimates directly in controller
 - Update shape and gravity model in real-time

Thank you

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