Response to the Reviewers' Comments for JASS-D-17-00005

I would like to thank the reviewers for their thoughtful comments, which are aimed towards improving the quality of the manuscript and the clarity of the results. In accordance with the comments and suggestions, the manuscript has been revised, and the answers to all comments are addressed as follows.

(In the revised manuscript, the citation numbers for equations, assumptions, propositions, and references are changed. This answer is written according to the new item numbers.)

Reviewer 1

Reviewer #1: I have some issues with this paper that, in my opinion necessitate a major revision. These issues are:

- *-It extensively re-covers issues that have been covered in many papers, making it difficult to disseminate what the original contributions are. For example, I don't think the addition of the thrust term into the dynamics necessitates re-doing every piece of analysis that has been done with various resources in the literature about variational integrators (including comparisons with RK methods). Things that can be found in other research should be cited, and summary comments can be used to point the readers to fill in this knowledge. The content of this paper easily be reduced considerably to maintain focus on the original contributions.*

  Thank you for the comment, and we agree that there is repetition of material previously covered in other publications. The challenge is that the proposed approach utilizes several tools and concepts integrated together to design low-thrust orbital maneuvers in a systematic way, and in order to describe the key idea in a self-contained manner, each of prior approach should be described properly.

  To address this, the manuscript has been reduced and reorganized to enhance the clarity and clearly separate previously material and the contributions of this paper. The introduction has been reduced and the repetition and discuss on the variational integrator has been removed. The second section has been reorganized to "Problem Formulation and Mathematical Background" and has subsections which cover all of the background material, namely the planar circular restricted three body problem, Jacobi integral, invariant manifolds and Poincaré maps, the and variational integrator. The section on variational integrators has been reduced and now only discusses the main developments of the integrator for the PCRTBP and omits the review of variational principles. The next sections introduce the optimal control formulation for the reachability analysis and the numerical examples, which are the main contributions of this paper.

  In short, the manuscript has been completely reorganized such that all of the background materials are summarized in Section 2. The remaining sections dealing with the proposed approach are expanded with more detailed descriptions and numerical results.

- *-It does not give a complete picture of the pros and cons of the proposed methodology. This method is full of both advantages and disadvantages. I don't think the authors need to be concerned with "selling" the work to the community, but rather showing the full and honest picture given the current state of the research. In the conclusions, the potential cons to this research can be mitigated by setting a path forward for future research to further improve the methodology.*

  These pros/cons include:

-The formulation of the problem with acceleration magnitude is a major technical challenge for mission design practitioners. The state of the art to design trajectories that depend on low-thrust engine hardware is either:

-To produce constant thrust magnitude or no thrust magnitude (typical of nuclear electric propulsion, or very simple electric propulsion systems.)

-To produce variable thrust magnitude associated with various throttle points at different power accessibility levels (solar electric propulsion). That is not to say that useful solutions cannot be extracted from the problem, as the authors have shown. However, a great many solutions in this approach will be unusable due to the fact that acceleration and thrust magnitude are different, i.e. $a=T/m$

Thus - a MAJOR focus of future work should be the introduction of a seventh state (mass). This gives rise to a more challenging optimal control problem requiring the switching function, but it is the form that is generally needed in computing optimal open-loop trajectories that can actually be flown.

Thank you for the comment and more focus has been included on the pros and cons of this proposed method. Additional detail has been added to the conclusion which discusses the advantages and the disadvantages, as well as the avenues for future research. The conclusion is copied below.

> " In this paper, an optimal transfer process which combines concepts of reachability and Poincaré section is used to generate transfer between planar periodic orbits in the three-body problem. The Poincaré section allows for trajectory design on a lower dimensional phase space and simplifies the process. The indirect optimal control formulation enables straightforward method of incorporating additional path and control constraints. Direct optimal control techniques must rely entirely on the ability of the numerical optimization routine to determine a feasible solution. Our approach leverages the benefits of the reachability set, which encompass the maximum set of states achievable by the spacecraft over a fixed time horizon. Using the reachability set on a Poincaré section reduces the dimensionality of the system and simplifies the analysis by avoiding the cost of a completely unmotivated exploration of the space. However, the use of optimal control techniques leads to open loop trajectories that are not robust to model uncertainties or disturbances. Furthermore, our approach relies heavily on the relative simplicity of the PCRTBP and this method is much more challenging for the non-planar case.
>
> There is additional research to extend these results to more general transfer scenarios in future work. The incorporation of fourth body perturbations, such as the Sun in the Earth-Moon system, offers an additional method of increasing the reachable set with the combined use of the solar perturbation and low-thrust propulsion. In addition, the assumed acceleration magnitude is currently beyond the capabilities of current electric propulsion systems. Furthermore, this analysis did not consider the effect of variable mass on the optimal control solution. This will result in a more complicated optimal control problem and is a focus of future research. Finally, Lyapunov control theory, which has previously been applied to the two-body problem, is being investigated in the hope of designing closed loop control schemes for this three-body scenario [1]. The addition of attitude dynamics and realistic pointing constraints would significantly improve the applicability of this work. "

- *The pros and cons of the PCRTBP*

  -The PCRTB gives rise to exploiting Poincare sections in a systematic manner. 2D solutions might be close enough for qualitative understanding of the solution space for many problems (as long as large plane changes aren't needed).

  -The PCRTB is limiting for many pragmatic departure orbits (e.g. LEO and GTO, the most common departure orbits for low-thrust transfers).

  Thank you for the comment and these characteristics of our choice of the PCRTBP. The authors would like to point out that the key idea of designing low-thrust orbits through the reachability analysis on the Poincaré section may be applied to other dynamic system. This manuscript illustrates such idea through PCRTB.

  However, according to the reviewers suggestion, additional details have been stated in the problem formulation section, including discussions on the choice of the PCRTBP and the relative strengths/weaknesses of this approach as copied below.

  > " We utilize the planar circular restricted three body problem (PCRTBP) as the basis of our system definition. It is a popular model in the preliminary analysis of multibody spacecraft trajectories. In the context of Earth based mission, the PCRTBP affords a relatively simple dynamic model while still capturing the major third body perturbation of the Moon. In addition, this model allows for a systematic process to define and exploit Poincaré sections. Furthermore, the 2D solutions afforded by the PCRTBP are frequently use to gain a qualitative understanding of the trade space of transfers in the Earth-Moon system. The PCRTBP approach offers insight into the fundamental dynamical structure while capturing the major dynamic properties of the planar motion. As a result, this approach is best suited for trajectories which do not require large plane changes. "

- *The pros and cons of variational integrators should really be acknowledged rather than completely dismissing the RK methods which are still the current state of the art in mission design (using higher-orders and suitable tolerances)*

  -Pros = boundedness in the energy behavior. Very long term integration will not deviate from the known dynamical structure.

  -Cons = you don't know where you are on the energy surface, just that you are on it somewhere. This is still a problem in actual trajectory design. Knowing the error is on an energy surface does not mean that you don't have error.

  Thank you for the comment and the detail on the benefits/drawbacks of variational integrators. The authors agree that the general-purpose numerical integrators such as higher-order RK methods are suitable in many cases. The proposed approach utilizes the variational integrator, particularly due to its numerical structural stability of capturing long-term behaviors of small effects accurately and efficiently, which is critical in low-thrust orbital maneuvers.

  While the structural stability of geometric numerical integrators is often illustrated by the conserved quantities such as energy or momentum, it has been shown that variational integrators are also effective in computing the rate of energy change for non-conservative systems [13].

  To clarify these, additional discussion has been included in the paper in the subsection which introduces the variational integrator for the PCRTBP. Much of the discussion of variational integrators in the introduction has also been removed. The additional content, which discusses the relative benefits of variational integrators is copied below.

> " Both integration schemes result in trajectories that are initially nearly identical. Fig. 3b shows the mean Jacobi integral deviation over the entire simulation time as a function of computation time. For a given computational effort, in the form of simulation run time, the variational integrator will provide a smaller energy deviation as compared to the conventional integration scheme. Over long simulation horizons or with the addition of small control inputs the inability of conventional integration schemes to accurately track the system energy limits the applicability of conventional techniques in which energy conservation is mandatory for characterizing the solution space. In spite of this improved energy behavior, the order and design of the variational integrator still play an important role in the accuracy of the state vector. This work uses a second order integrator which has improved energy behavior but potentially lower state accuracy as compared to high order Runge Kutta methods. However, for this preliminary analysis the variational integrator provides reduced computational demands and is useful to characterize and explore the trade space in preliminary mission design scenarios. In addition, it has been shown that variable step integrators, such as `ODE45` in Matlab, tend to degrade first-order gradient based methods, such as those used in Section 3.2 [16]. In summary, this paper utilizes the variational integrator to capture the long-term effects of low-thrust devices accurately and efficiently. The structural stability during numerical integration is critical for numerical optimization in the subsequent developments. "

From here on out, comments specific to various sections of the paper follow:

1. *In the statement "In addition, non-Keplerian orbits and multi-body dynamics have been shown to allow for a much greater range of potential missions at a reduced energy cost [15]" - you may want to reference, for example, the Lunar Ice Cube mission paper which absolutely requires low-thrust propulsion and non-keplerian dynamics to satisfy its mission objectives.*

   Folta, D., Dichmann, D., Clark, P., Haapala, A., and Howell, K., "LunarCube Transfer Trajectory Options," AAS/AIAA Space Flight Mechanics Meeting, Williamsburg, Virginia, January 11-15, 2015.

   Thank you for the comment and the additional reference. In addition to this change we have made other modifications to the introductory paragraphs to improve the organization and clarity. Specifically, the first paragraph has been divided, which motivates the use of electric propulsion and the advantages of small satellites which use electric propulsion. In addition, the suggested reference has been included in the paper. The modified paragraphs are duplicated below.

   > " Designing spacecraft trajectories is a classic and ongoing topic of research. There has been significant research into the design of orbital transfers for space vehicles. Optimal expenditure of onboard propellant is critical to allowing a mission to continue for a longer period of time or to enable the launch of a less massive spacecraft. Electric propulsion systems offer a much greater specific impulse than chemical systems. As a result, the greatly increased efficiency allows for greater

payload mass or extended duration missions. However, these electric propulsion systems typically have much less thrust than their chemical counterparts and therefore orbital maneuvers have a much longer time of flight. In spite of this drawback, a wide variety of missions, such as communication and deep space probes, have utilized the unique benefits of low thrust electric propulsion to great effect [2].

With reduced development intervals and decreased launch costs, small satellites have gained increased attention as a cost effective means of scientific and technologic development. Furthermore, these cubesats are easily launched as secondary payloads rather than requiring a dedicated launch. The merger of small satellites with miniature electric propulsion enables inexpensive and responsive missions requiring large changes in orbital energy or extended mission lifespan. Recent developments in miniature electric propulsion offer the potential for new research opportunities for small spacecraft [7]. The advancements in miniature electric propulsion allows for greater flexibility in the deployment of cubesats as secondary payloads. Without the explicit requirement of a dedicated launch trajectory, cubesats are able to exploit a much wider range of possible mission designs [4]. With the potential for more demanding missions, even greater importance is placed on the mission design to ensure that optimal trajectories satisfy mission requirements. In addition, non-Keplerian orbits and multi-body dynamics have been shown to allow for a much greater range of potential missions at a reduced energy cost [4, 10]. Future space missions are increasing in complexity and will require new classes of orbits that are not possible via the traditional patched-conic approach [18, 5]. Optimally combining the structure of the dynamic environment with low-thrust propulsion systems is vital for future mission success. ”

2. *"Since there is an insufficient number of analytical constants, or integral of motion, numerical methods must be used to investigate solutions to the three-body problem." This isn't completely true as there are also approximations to the (uncontrolled) three-body motion, such such as the Richardson-Cary approximation. Saying numerical methods are required may be a bit strong. May want to say "often required".*

   Thank you for the comment. The sentence has been modified as follows.

   > “ Finally, there is no closed form analytical solution for the three-body problem and there are an insufficient number of analytical constants [22]. As a result numerical methods are often required to investigate solutions to the three body problem. ”

3. *"These techniques suffer from numerical instability and energy drift behaviors which make them ill-suited for long-term propagation. These dissipative effects are even more detrimental with the addition of low-thrust propulsion to the dynamic equations of motion." It really depends on what you mean by "long term". If transfers are on the order of hundreds of days, then high precision runge-kutte integrator, for example a Runge-Kutte Verner 8(9), can maintain suitable accuracy such that the energy drift in the Hamiltonian is very small, as can be seen in some of the references that the authors have cited. If, on the other hand you mean 10-100's of years or more, i.e., when you create your Poincare maps, then this drift is more apparent. I think that this statement needs to be quantified better because it gives a dishonest portrayal of high-precision Runge-Kutte integrators without numbers attached to it.*

Thank you for the comment. Those numerical dissipations would depend on several parameters. But, for the particular numerical comparison illustrated by Fig. 3b, such difference in conservation properties becomes noticeable in the simulation for 15 years. As suggested, the claims about the variational integrator have been qualified with some additional detail. The correction is copied below.

> " As a result, these methods suffer from numerical instability and energy drift behaviors which make them ill-suited for the long-term propagation, e.g. on the order of decades to centuries, which is typical of the three-body problem. "

4. *"Conventional integration techniques fail to capture the physical laws and geometric properties of the dynamic system [8]. As a result, the long term effects of low-thrust on the spacecraft trajectory are not accurately captured." Again, I think situationally I would disagree. Transfers on the order of 100's of days can maintain accuracies of 1e-10 (non-dimensional units) in the Hamiltonian, which is generally considered a "small" amount of energy drift.*

   These sentences have been modified and combined to enhance clarity and reduce some of the unnecessary bloat. In addition, some additional detail has been added to explicitly define the notion of "long-term" in the context of numerical integration. As shown by Fig. 3b, the variational integrator exhibit smaller conservation errors than ODE45 at the similar computational efforts, for the particular numerical case considered. The modifications are copied below.

   > " Conventional Runge-Kutta integration techniques, such as those implemented in [14, 6], fail to capture the physical laws and geometric properties of the dynamic system. As a result, these methods suffer from numerical instability and energy drift behaviors which make them ill-suited for the long-term propagation, e.g. on the order of decades to centuries, which is typical of the three-body problem. "

5. *- "[15,25] have illustrated the rich structure that exists in the three-body problem." - "[23,6] implement the solutions using conventional Runge-Kutta integration techniques." I don't think you can grammatically begin a sentence with reference numbers. Please fix all instances of this.*

   Thank you for the comment. Evidently in some styles, such as APA, it is acceptable to start with a reference. We agree that it is not the best with the numeric style citations. The sentences have been reworded to avoid starting with a reference number. The corrections are shown below.

   > "
   >
   > - Conventional Runge-Kutta integration techniques, such as those implemented in [14, 6], fail to capture the physical laws and geometric properties of the dynamic system.
   > - There exists a rich structure in the three-body problem [10, 18].
   >
   > "

6. *"It has been shown that there exist multi-dimensional tubes, or invariant manifolds, of constant energy trajectories that span the state space." The "tube" statement is really only true for some libration point orbits at certain Jacobi Constants. For some orbits these structures are*

more chaotic and don't resemble tubes. May want to soften this language or change "tubes" to "structures".

Thank you for the clarification. The sentence has been modified as follows.

> " It has been shown that there exist multi-dimensional structures, termed invariant manifolds, of constant energy trajectories that span the state space. "

7. *-"In this paper, we propose a systematic design method which enables low-thrust transfers in the planar circular restricted three-body problem." Why planar? For simplicity? Almost all parking orbits departing the Earth are likely going to have some inclination with respect to the rotating reference plane, and will likely require some three-dimensional thrusting to target the manifold. You may want to indicate that this is a first step in the research to exploit the Poincare sections more easily, which is why I'm assuming it was done. It doesn't hurt to be straightforward about this, as it is a first-step towards characterizing the design space.*

   Thank you for the comment. The PCRTBP was primarily chosen as a simple dynamic system which still captures the primary structure of the three-body problem. The introduction has been modified with an additional sentence indicating our use of the planar three body problem

   > " Utilizing the planar assumption greatly simplifies the problem and allows for the use of established methods in determining periodic orbits and Poincaré sections. "

8. *-"In addition, through the use of geometric integrators we accurately capture the effects of low thrust on the system dynamics in the numerical simulation." A variational integrator will accurately capture the qualitative effects of the transfer (e.g. propellant use), however it will not necessarily accurately capture accuracy in the final state vector - it is guaranteed to be on an energy surface, but it is not guaranteed to be at exactly the right location on the energy surface. I would soften this claim or reword.*

   While the geometric integrator will not exactly characterize the terminal energy or state of a dynamic system, it will better preserve the total energy variation as compared to conventional integration schemes. In addition, the total energy deviation will remain bounded and less than that of conventional schemes which experience unbounded energy deviation. As a result, the terminal state/energy will be more accurately characterized using a variational integrator as compared to conventional schemes. Also this difference becomes more evident over increasing time horizons. Some additional detail has been added stating that the increased accuracy is most evident over long time periods.

   > " In addition, through the use of geometric integrators we more accurately capture the effects of low thrust on the system dynamics in the numerical simulation over extended periods which are typical of electric propulsion systems. "

9. *-"The Poincare section reduces the dimensionality of the system dynamics to the study of a related discrete update map." It does, but at the expense if a large up-front computational run that must be run well in advance to capture the problem characteristics. Also, it is more challenging to execute in 3D, which should be noted.*

   Thank you for the comment. Some additional detail has been included mentioning the difficulties in applying the Poincaré section approach to the 3D case.

> “ While the Poincaré section is well defined for the planar case, as a 2D representation, it is more challenging to implement and visualize for the general 3D three-body problem. ”

10. -*”This discrete update map, or variational integrator, shares the same geometric properties of the continuous time system and exhibits much better energy behavior than the traditional integration methods, especially over long transfer durations with small magnitude control inputs.” Yes, but the order of the variational integrator still matters. The drift in the state vector accuracy is still important to a mission designer, even if it is guaranteed to lie on an energy surface. I think you should indicate that this is more for preliminary analysis to characterize the transfers, when this drift in state vector accuracy is tolerable. I.E., it’s useful for the introductory phases of mission design when trying to globally characterize the design space. Characterizing the scope of the analysis in this way would be helpful to the audience.*

    Thank you for the comment. The section on the variational integrator has been reduced and this specific sentence has been removed. However, additional detail has been provided in the variational integrator subsection which discusses this drawback.

    > “
    >
    > Over long simulation horizons or with the addition of small control inputs the inability of conventional integration schemes to accurately track the system energy limits the applicability of conventional techniques in which energy conservation is mandatory for characterizing the solution space. In spite of this improved energy behavior, the order and design of the variational integrator still play an important role in the accuracy of the state vector. This work uses a second order integrator which has improved energy behavior but potentially lower state accuracy as compared to high order Runge Kutta methods. However, for this preliminary analysis the variational integrator provides reduced computational demands, as compared with general-purpose numerical integrators with a similar level of numerical error, and is useful to characterize and explore the trade space in preliminary mission design scenarios.
    >
    > ”

11. *”Optimal solutions are generally sensitive to small variations in the initial multipliers. As a result, the numerical stability of sensitivity derivatives is critical to accuracy and computational performance.” I don’t agree that this implies that variational integrators are better for gradient-based methods. The primary issue is that gradient-based methods that are solving optimal control problems are using first-order approximations to iterate on the problem. The first-order approximation is what is poor, not necessarily the gradients. Even if these gradients are of exact accuracy (and I agree that they should always be as accurate as possible), the first-order methods in the chaotic problem are what give the difficulties. One could use a variational or standard integrator in a first-order gradient based method and nothing will change the fact that the method will be highly sensitive, save for partitioning the sensitivities as you did with your multiple shooting approach. At best, you can change the statement to indicate that the accuracy in the gradients may improve, which is always helpful to gradient-based methods, but it’s no guarantee that it eliminates major problem sensitivity.*

    It has been shown that the structure-preserving properties of geometric numerical integrators are also useful to numerical optimizations for improved computational efficiency and stability. For example, see the following reference.

- Ober-Blbaum, Sina, Oliver Junge, and Jerrold E. Marsden. "Discrete mechanics and optimal control: an analysis." ESAIM: Control, Optimisation and Calculus of Variations 17.2 (2011): 322-352.

The proposed approach utilizes desirable numerical properties of the variational integrator. However, the main contribution is systematic design of low-thrust orbital maneuvers through reachability set analysis on the Poincaré section. The above sentences may be confusing, and the authors decided to remove them from the introduction of the revised manuscript.

12. *Authors may also want to point out the loss of accuracy in variable step integrators that can also degrade first-order gradient based methods. I didn't see this anywhere and it's something that should be acknowledged. Basically, variable step integrators will introduce artificial "noise" into the gradients:*

    Pellegrini, E.P., Russell, R.P., On the Accuracy of Trajectory State Transition Matrices, Paper AAS-15-785, AAS/AIAA Astrodynamics Specialist Conference, Vail, CO, Aug 2015

    Thank you for the comment and helpful reference.

    It is well known that variable step size may break conservational properties of a geometric numerical integrator. Variational integrators are no exception, and there has been a multisymplectic framework to address it. See

    - Lew, Adrian, et al. "Asynchronous variational integrators." Archive for Rational Mechanics and Analysis 167.2 (2003): 85-146.

    However, the above formulation for structure-preserving, adaptive time-step variational integrators is beyond the scope of this paper, and we use a fixed time step throughout the presented numerical results.

    Additional detail has been included in the manuscript discussing this potential drawback of variable step integrators.

    > "
    >
    > In addition, it has been shown that variable step integrators, such as `ODE45` in Matlab, tend to degrade first-order gradient based methods, such as those used in Section 3.2 [16]. The presented results are based on the variational integrator with a fixed time step for numerical simulation and gradient computation.
    >
    > "

13. *"This results in an optimal solution rather than the suboptimal solution typical of direct optimal control methods." I disagree that suboptimality is typical of direct methods. A direct method will typically solve the KKT conditions, which can be directly transcribed into the costates in the optimal control problem. A direct method fully satisfying the KKT conditions can map to the indirect problem. If you want to strengthen this argument, then you should argue that it applies to direct methods which rely on coarse or suboptimal representations of the control. For example, a hybrid direct method could use continuous optimal control theory control law, but directly minimize the cost function instead of solving the full TPBVP.*

    Thank you for the comment. The suboptimal claims of direct optimization has been reduced and modified. Additional detail has been included on the merits of indirect optimal control vs. direct optimal control methods.

> “ This results in an optimal solution rather than the suboptimal solution typical of direct optimal control methods which rely on coarse or suboptimal control parameterizations. For example, a hybrid direct method may use a continuous control law developed from indirect optimal control theory while directly minimizing a cost function instead of solving the full two-point boundary value problem [15]. ”

In addition, the introduction has been modified to explicitly mention that direct methods which use suboptimal control parameterizations will result in suboptimal solutions.

> “ The indirect approach results in a lower dimensioned problem than the direct approach and provides algebraic conditions that guarantee local optimality in contrast to direct methods which generally rely on suboptimal representations of the control. ”

14. *”Application of the Euler-Lagrange equations results in following equations of motion defined in the rotating reference frame” Add a reference after this statement for those who want to see the derivation. One of the given references probably does already does this, so just pick one*

    Thank you for the comment. An additional comment has been included in the problem formulation section with a reference to a source of the detailed derivation.

    > “ Following a straightforward application of the Euler-Lagrange equations, a more detailed derivation is provided in [22], results in the following equations of motion defined in the rotating reference frame ”

15. *It appears from equation 5 and your control input definition that you control is unbounded? Later on in reading the paper I found that you do add bounds on the control. You may want to indicate earlier on in the paper (closer to eq. 5) that bounds will be added to the control.*

    Thank you for the comment. An additional comment has been included mentioning that the control is assumed to be bounded in magnitude.

    > “ . . . and the control inputs is defined as $\boldsymbol{u} = \begin{bmatrix} u_x & u_y \end{bmatrix}^T \in \mathbb{R}^{2\times1}$ and assumed to be continuously variable but bounded in magnitude, i.e. $\boldsymbol{u}^T\boldsymbol{u} \leq u_{max}^2$. ”

16. *Figure 1 should probably have ”axis equal” applied to it. For that matter, please check all trajectory plots (those which plot the position state), and ensure that all plots are using ”axis equal”. If not, please regenerate those plots.*

    Thank you for the comment. All of the figures showing position states have been modified to use `axis equal`.

17. *Section 3.1 has been discussed in many references, the reader may want to direct them to this material. (Choose one of Marsden or West’s papers for example.)*

    Thank you for the comment. The discussion on the variational integrators has been reduced. Much of the background material on Lagrangian mechanics is removed and an additional paragraph is included to summarize the key ideas of variational integrators. In addition, the variational integrator details are moved into a titled subsection. The additional paragraph is shown below.

" Geometric numerical integration deals with numerical integration methods which preserve the geometric properties of the flow of a differential equation, such as invariant properties and symplecticity. Variational integrators are constructed by discretizing Hamilton's principle rather than the continuous Euler-Lagrange equations. As a result, integrators developed in this manner have the desirable properties that they are symplectic and momentum preserving. In addition, they exhibit improved energy behavior over long integration periods. A thorough discussion of variational integrators is provided in [23, 13]. We use this approach to construct a variational integrator for the PCRTBP with the inclusion of low-thrust propulsion.

Consider the autonomous mechanical system described by the Lagrangian, $L(q, \dot{q})$, for the generalized coordinates, q and velocities $\dot{q}$. The integration of the continuous Lagrangian along a path, $q(t)$, followed by the system must satisfy Hamilton's principle, which results in the well-known Euler-Lagrange equations [12]. In the discrete time scenario, the continuous path $q(t), \dot{q}(t)$, is replaced by a finite difference approximation over a fixed time step, e.g. $q_0, \frac{q_1 - q_0}{h}$, which converges to the true velocity as h tends towards zero. In this manner, we define a discrete Lagrangian, $L_d(q_0, q_1, h)$ which approximates the integral of the true Lagrangian over the time interval h between q_0 and q_1 [13]. "

18. *The second-order trapezoidal rule will give significantly poorer state vector accuracy with respect to a high-precision RK integrator, even if the state is guaranteed to lie on an energy surface. Why didn't the authors consider going to a higher-order variational integrator, or even a Taylor Integrator? It is very likely that the integration accuracy will introduce some errors in the accuracy of the partial derivatives and in the states.*

    We chose the trapezoidal rule as it provides a second order quadrature approximation. Furthermore, instead of using Taylor methods or higher order options, the trapezoidal rule provides an explicit second order accurate integration rule. This greatly reduces the complexity of the optimization routine. In addition, the background material on the variational integrators have been removed from the manuscript. Only the specific details covering the development of the variational integrator for the PCRTBP are discussed.

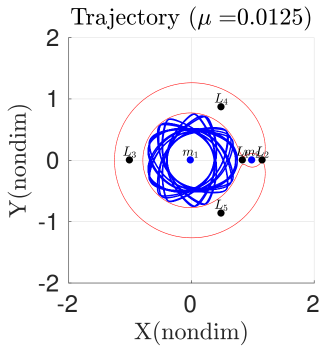
19. *Section 3.2 - please cite a reference as to where the selected quadrature rules presented in Table 1 have been derived.*

    In the interest of conciseness and to avoid repeating material that has already been covered in the literature, the discussion on variational integrators has been substantially reduced. The background material on the variational integrators has been removed and this table has also been removed from the manuscript.

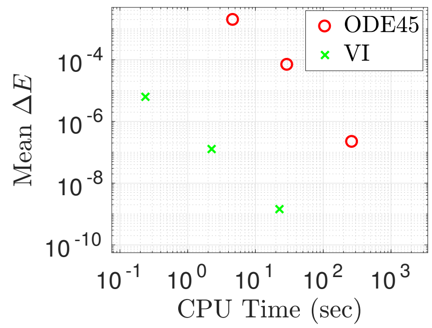
20. *Figure 2 - x-axis has no label but I think it's non-dimensional time, please indicate. Label all axes even if they are non-dimensional units.*

    Thank you for the comment. All of the plots have been updated to include all axes labels. Furthermore, the specific plot in questions has been replaced with another plot more explicitly showing the benefits of variational integrators over conventional integration schemes. This modified plot is shown below.

"



(a) Earth-Moon three-body trajectory used for integrator comparison: A stable trajectory is used to test the variational and conventional integrators. The energy level is high enough to enter the vicinity of the Moon but not escape the three body system.



(b) Mean Jacobi integral deviation between variational integrator and Matlab `ODE45`: A range of fixed step sizes are used for the variational integrator to approximately match the computational time of `ODE45`

”

21. *I recommend removing appendix B. Giving the equations and stating that the Matrix inversion is from Gauss-Jordan elimination is probably sufficient, unless there is some compelling algebraic manipulation that is very unique to the problem.*

    Thank you for the comment. Appendix B has been removed from the revised manuscript. The manuscript has been modified to mention that Gauss Jordan elimination is used to avoid the use of the matrix inversion.

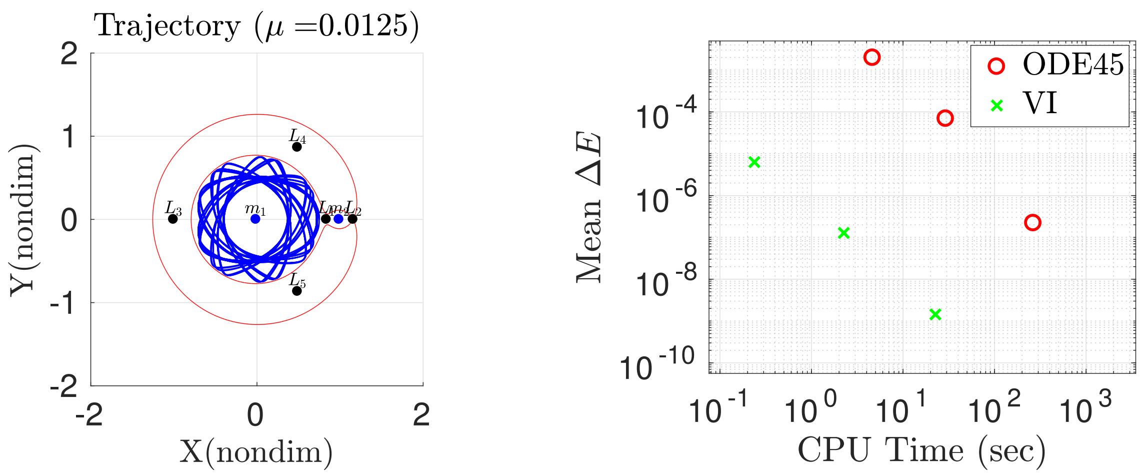
    > “ The derivation of (18) is given in Appendix A. In addition, the computation of (18) requires inversion of the Jacobian matrix. This is a computationally expensive operation that is prone to numerical error and instability. We use Gauss-Jordan elimination to avoid this inversion in (18) and determine an explicit update map $\boldsymbol{\lambda}_k \to \boldsymbol{\lambda}_{k+1}$. ”

22. *In line 14 (page 12), you define the state transition matrix over one orbital period, and then on line 18 you start referring to it as the monodromy matrix. I understand what you're doing, but you may want to make the definition more explicit. Some of this section can be substantially reduced and other source material could be referenced anyway.*

    Thank you for the comment. The discussion about the invariant manifolds and Poincaré sections has been reduced. This discussion has been removed from the revised manuscript and replaced with a general summary and some relevant citations. This specific discussion no longer exists in the manuscript but rather cites several references with the details on determining the invariant manifolds.

23. *Comparison like that of Figure 3.4 are very common in the well-known literature concerning variational integrators. Moreover, the proper way to present such a comparison is with different delta-T's in the integration step, showing when the Runge-Kutta methods actually break down in terms of maintaining an accurate portrayal of the Energy/Hamiltonian conservation.*

    For example -



(a) Earth-Moon three-body trajectory used for integrator comparison

(b) Mean Jacobi integral deviation between variational integrator and Matlab `ODE45`

Figure 2: Integrator Comparison

Figure 2.4 Lew, A., Marsden, J., Ortiz, M., West, M., "An Overview of Variational Integrators," FINITE ELEMENT METHODS: 1970's AND BEYOND L.P. Franca (Ed.) c CIMNE, Barcelona, Spain 2003.

Online copy: http://lagrange.mechse.illinois.edu/pubs/LeMaOrWe2004/LeMaOrWe2004.pdf

Thank you for the comment. The comparison plot has been modified to follow the precedent in the literature. The plot has been modified to show the energy deviation as a function of computational time for both the variational integrator and `ODE45`. The modified content is copied below.

> " A simulation comparing the variational integrator to a conventional Runge-Kutta method is given in Section 2.3. A particle is simulated from an initial condition of $\boldsymbol{x}_0 = \begin{bmatrix}0.75 & 0 & 0.2 & 0\end{bmatrix}^T$ for $t_f = 200 \approx 15$ years in the Earth-Moon system. The variational integrator uses a range of step sizes between 47.2 s to 4720 s while the Runge-Kutta method uses a variable step size implemented via `ODE45` in Matlab. The step size of the variable integrator is varied to approximately match the run time required by the conventional `ODE45` integrator. Fig. 3a shows the trajectory of the spacecraft in the rotating reference for this comparison. Both integration schemes result in trajectories that are initially nearly identical. Fig. 3b shows the mean Jacobi integral deviation over the entire simulation time as a function of computation time. For a given computational effort, in the form of simulation run time, the variational integrator will provide a smaller energy deviation as compared to the conventional integration scheme. Over long simulation horizons or with the addition of small control inputs the inability of conventional integration schemes to accurately track the system energy limits the applicability of conventional techniques in which energy conservation is mandatory for characterizing the solution space. "

24. *It may also be generally worth acknowledging the benefits of Taylor's method within the context of variational integrators: Schmitt, J., Shingel, T., Leok, M., "LAGRANGIAN AND*

HAMILTONIAN TAYLOR VARIATIONAL INTEGRATORS," eprint arXiv:1703.06599, 2017

Online copy: https://arxiv.org/pdf/1703.06599.pdf

Thank you for the comment. However, this manuscript is not really focused on variational integrators but rather the design of transfer trajectories using reachability sets. There is a wide range of topic under the umbrella of variational integrators that are not directly applicable to this manuscript and work. Interested readers can find a wide variety of literature on the relative benefits of all manner of variational integrators in papers focused on that topic specifically. As a result, we do not feel it is applicable to include additional discussion on the merits of taylor integration methods.

25. *"Over long simulation horizons or with the addition of small control inputs this poor energy behavior limits the applicability of conventional techniques". I think you need to add "in which energy conservation is mandatory for characterizing the solution space."*

    Thank you for the comment. The additional clarification has been added to the manuscript and copied below.

    > " Over long simulation horizons or with the addition of small control inputs the inability of conventional integration schemes to accurately track the system energy limits the applicability of conventional techniques in which energy conservation is mandatory for characterizing the solution space. "

26. *Section 5 - This method, while very useful in its given context is highly dependent on the use of the PCRTB in the framework of the current analysis. This needs to somehow be conveyed. 3D transfers are significantly more challenging when it comes to exploiting Poincare sections. It can be done, but more subplots, dimensions, and data storing are necessary to view the subspace - save to say, it is a more challenging endeavor that must be acknowledged for general purpose low-thrust mission design.*

    Thank you for the comment. An additional paragraph has been added to the beginning of the mathematical background section which discusses our reachability based transfer approach. This additional content is copied below.

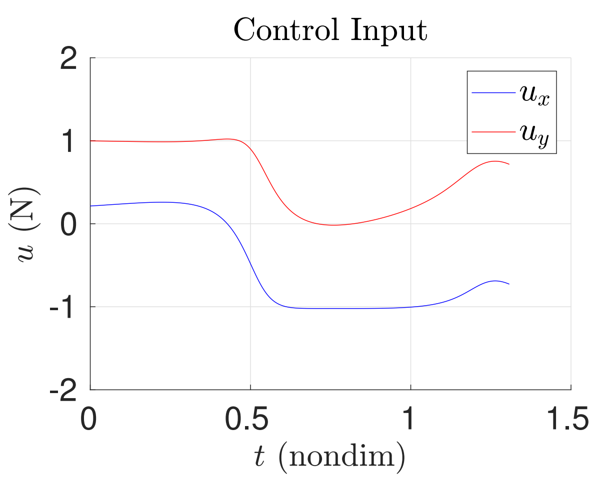
    > " The numerical examples presented in this section are designed in the context of the PCRTBP. The dynamic environment has a four dimensional state space and offers a convenient integration constant in the form of the Jacobi integral. As a result, there are well defined methods to define and exploit Poincaré sections, which result in straightforward two-dimensional subspaces of the system. Our approach uses the Poincaré section to approximate the reachability set on this reduced subspace. As a result, this approach is more difficult to apply to three-dimensional transfers in the general three body problem. Poincaré sections in the case of the general six dimensional state space are significantly more challenging and typically require more complicated visualization techniques. However, this is an area of active research and some of the authors future research is aimed at implementing this approach for non-planar transfer trajectories [11]. "

27. *Figures 10D and 13D - the authors need to present these plots in dimensional coordinates so that they may be reconciled with what existing technology can produce. I recommend Newtons or milliNewtons. For example, the NEXT (NASA evolutionary xenon thruster) can produce*

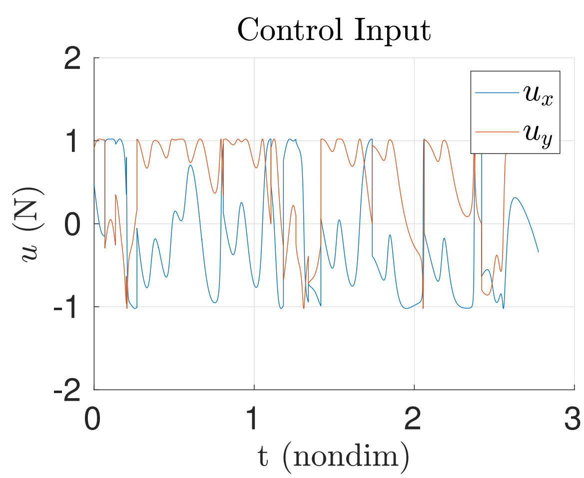
0.25 Newtons of thrust at maximum power (7 kW). The XR5 Hall thruster can produce comparable thrust at 4.5 kW of power. These are currently the highest-power single engines other than the ARM Hall thruster which is not flight ready yet. For these examples to be reconciled against the capability of existing technology, we need to see the required forces that are being demanded from the optimal control problem. It may be that the results are beyond the scope of current technology - that's fine - just say that they are examples and that it's ongoing work to formulate example problems that capture existing technology better.

Thank you for the comment. The mentioned plots of the control input have been modified to show the data in dimensional units (Newtons). Some additional detail has been provided in the manuscript which discusses the choice of $u = 0.75$ and it's relationship to the current state of the art thruster systems. The modified plots are not included here but the additional content is copied below in response to a different but related comment.

> " Next, we determine the reachability set with the addition of a low thrust control input. We define a maximum magnitude of the thrust as $u_{max} = 0.75$ and assume we can point the thrust in any direction within the plane. This model is representative of many spacecraft which have a body fixed thruster and attitude control system. We assume a fully actuated spacecraft model which decouples the translational and rotational dynamics. This acceleration limit is approximately $u_{max} = 2\,\mathrm{mm\,s^{-2}}$ in the Earth-Moon system. Assuming a fixed spacecraft mass of 500 kg, this model defines a maximum thrust of 1 N. Currently, the NASA NEXT xenon thruster is able to provide approximately 0.25 N of thrust, and a cluster of such engines could be used to provide the desired thrust used in this work [19]. The trajectories are generated from a fixed initial state of $\boldsymbol{x}_0 = \begin{bmatrix} 0.8156 & 0 & 0 & 0.1922 \end{bmatrix}^T$ over a fixed time span of $t_f = 1.4$. This initial state lies on the initial periodic orbit and the time of flight is equivalent to one half period of the initial periodic orbit.



(a) Control input for the selected transfer trajectory



(b) Control Input: Combined control history for each stage of the reachability analysis. The control always remains within the maximum bound.

> "

28. *"The multiple shooting method is implemented in this paper which alleviates many of the*

issues associated with single shooting approaches." Did the authors attempt single shooting for their example problems before coming to this conclusion? With proper scaling, and problem formulation, there are examples in the literature of single shooting yielding robust results.

Example: Ranieri, C. and Ocampo,C.. "Indirect Optimization of Spiral Trajectories", Journal of Guidance, Control, and Dynamics, Vol. 29, No. 6 (2006), pp. 1360-1366.

So, justification for its selection may be warranted. You could always find a reference in which this trade space was really explored thoroughly to show why you made the choice.

Our original attempt at solving the optimal control problem did rely on single shooting. However, during our investigation it was found that it was quite difficult for solutions to converge. As a result, we decided to implement the multiple shooting approach in hopes of generating more consistent and robust solutions. While it is possible to use single shooting, such as the provided reference, it is known that the single shooting approach will have greater sensitivity to initial costate errors as compared to a multiple shooting method. This can be alleviated, or entirely mitigated in the case of [17], it is highly case specific and dependent on a large variety of case specific techniques. For example, the work in [15] used both a single shooting and multiple shooting approach to solve the optimal control problem. The large sensitivity of the single shooting approach was mitigated through the use of the multiple shooting method, at the expense of additional design variables.

In the manuscript, additional detail has been added in this section which describes the difficulties of the single shooting approach and our decision to use the multiple shooting method. The modified portion of the manuscript is copied here.

> " Rather than numerical integration over the entire time interval, multiple shooting segments the interval into several smaller sub-arcs [20]. This multiple shooting approach incorporates additional interior constraints but reduces the sensitivity of the costates along each sub-arc. The use of the multiple shooting method reduces the sensitivity of changes in the initial costate at the expense of additional design variables, but has been shown to provide more stable and robust solutions [15]. "

29. *"In this work, we use the Matlab nonlinear solver fsolve to solve the system of nonlinear equations defined by the multiple shooting algorithm." Can authors give some additional details? Which fsolve algorithm? trust region dogleg, trust region, or Levenberg-Marquardt? Which additional options? E.g. various tolerances. For users of this method it's also useful to see the iteration history (more comments on this later).*

Thank you for the comment. Additional detail has been incorporated which discusses the specific solver used in computing the solutions. The description does not specifically mention all of the possible options in `fsolve` but rather gives the key components used in our solution. This approach seems to be consistent with the literature, e.g. [15, 21]. The modified portion is copied below.

> " In this work, we use the Matlab nonlinear solver `fsolve` to solve the system of nonlinear equations defined by the multiple shooting algorithm. Within `fsolve`, we use the trust-region dogleg solver which makes use of the Powell dogleg procedure for computing a step direction and magnitude to minimize successive iterations of the solver. All numerical integration is performed using the discrete variational integrator described in Section 2.2. "

30. *Did the authors experiment with the number of multiple shooting stages in their example problems? There must be some sort of "sweet spot" in number of stages for the example problems attempted. If I missed it, I apologize, but I didn't see it.*

    We did spend considerable time in determining the appropriate number of stages. One key issue was that our implementation in software depended on dividing the total number of integration steps by the number of interior stages. For simplicity, this reduced the possible number of stages to an even number. Additionally, we discovered that adding many stages tended to degrade the solution and cause difficulties in convergence. The solver would tend to ensure a continuous trajectory, by satisfying the interior stages, at the expense of the terminal stage. Some additional detail has been provided that mentions our choice of the number of stages and has been copied below.

    > " Based on experimentation, we use four stages in our multiple shooting method. This provided the best performance and convergence stability while minimizing the difficulties in additional interior constraints. "

31. *Section 6.1 - I don't think you need to explain how you created the three-body orbits. This can be found in many references - You can just cite a reference and give the orbit specifications. (Starting with line 49, page 18, up to line 21 page 19.)*

    Thank you for the comment. The discussion of generating periodic orbits in the three body problem has been removed. It is replaced with a small introduction mentioning the differential correction process and a citation to a relevant resource. The revised paragraph is copied below.

    > " The first objective is to design a transfer trajectory from a planar periodic orbit about the L_1 Lagrange point to a bounded orbit in the vicinity of the Moon. The target region is created by choosing an initial condition of $x_0 = \begin{bmatrix}1.05 & 0 & 0 & 0.35\end{bmatrix}^T$ with $\mu = 0.0125$. The target set is propagated over a period of $t = 20$ in non-dimensional units which corresponds to approximately 1.5 years. Fig. 6 shows that the target set remains in the vicinity of the Moon, or m_2, in the rotating reference frame. This type of orbit would be useful for a variety of mission scenarios. The bounded trajectories of the vehicles and constant line of sight to both the Moon and the Earth would allow for constant communication for future manned missions and potential habitats. The initial set is a planar periodic orbit about L_1, which is generated using the process of differential correction of a linear approximation [10]. "

32. *"We define a maximum magnitude of the thrust as umax = 0.75 and assume we can point the thrust in any direction within the plane." And "we again assume a upper bound on the thrust magnitude of umax = 0.75." What was the reason behind this? Isn't this a pretty large value of dimensional acceleration for your problem of interest? Even if it's chosen for demonstrative purposes, there should be some sort of explanation as to why. In the context of what electric propulsion systems can deliver, it could be arguable that this value isn't really "low-thrust" but "finite-thrust".*

    The maximum thrust magnitude was chosen to be approximately 1 N for an assumed 500 kg spacecraft. While larger than the NEXT thruster mentioned earlier, a cluster of similar engines could provide the required thrust (assuming adequate available power). Additional detail has been included in the manuscript describing this thrust limit in dimensional units.

> " Next, we determine the reachability set with the addition of a low thrust control input. We define a maximum magnitude of the thrust as $u_{max} = 0.75$ and assume we can point the thrust in any direction within the plane. This model is representative of many spacecraft which have a body fixed thruster and attitude control system. We assume a fully actuated spacecraft model which decouples the translational and rotational dynamics. This acceleration limit is approximately $u_{max} = 2\,\mathrm{mm\,s^{-2}}$ in the Earth-Moon system. Assuming a fixed spacecraft mass of 500 kg, this model defines a maximum thrust of 1 N. Currently, the NASA NEXT xenon thruster is able to provide approximately 0.25 N of thrust, and a cluster of such engines could be used to provide the desired thrust used in this work [19]. The trajectories are generated from a fixed initial state of $\boldsymbol{x}_0 = \begin{bmatrix} 0.8156 & 0 & 0 & 0.1922 \end{bmatrix}^T$ over a fixed time span of $t_f = 1.4$. This initial state lies on the initial periodic orbit and the time of flight is equivalent to one half period of the initial periodic orbit. "

33. *Is this method exceptionally difficult to adapt to create a numerical example from LEO or GTO? These would be the most pragmatically interesting departure orbits to the mission design community. Section 6.2 presents an example from GEO, but it is unlikely that GEO represents a "typical" parking orbit, since there is no reason to insert a spacecraft into GEO before it transfers to the destination orbit. This should be called out in the conclusions or future work.*

This method is not difficult to adapt for other scenarios. However, there is a certain amount of experimentation to ensure that trajectories converge. In addition, the details in this paper were designed in the context of the planar three body problem and as a result and extension to the nonplanar GTO is non trivial. The examples presented are primarily aimed at introducing and demonstrating the feasibility of this approach, rather than designing operational mission profiles for spacecraft. However, this is a valuable comment and a goal of future research. Additional detail has been included in the conclusion discussing this topic. The modified conclusion is copied below.

> " In this paper, an optimal transfer process which combines concepts of reachability and Poincaré section is used to generate transfer between planar periodic orbits in the three-body problem. The Poincaré section allows for trajectory design on a lower dimensional phase space and simplifies the process. The indirect optimal control formulation enables straightforward method of incorporating additional path and control constraints. Direct optimal control techniques must rely entirely on the ability of the numerical optimization routine to determine a feasible solution. Our approach leverages the benefits of the reachability set, which encompass the maximum set of states achievable by the spacecraft over a fixed time horizon. Using the reachability set on a Poincaré section reduces the dimensionality of the system and simplifies the analysis by avoiding the cost of a completely unmotivated exploration of the space. However, the use of optimal control techniques leads to open loop trajectories that are not robust to model uncertainties or disturbances. Furthermore, our approach relies heavily on the relative simplicity of the PCRTBP and this method is much more challenging for the non-planar case.

There is additional research to extend these results to more general transfer scenarios in future work. The incorporation of fourth body perturbations, such as the Sun in the Earth-Moon system, offers an additional method of increasing the reachable set with the combined use of the solar perturbation and low-thrust propulsion. In addition, the assumed acceleration magnitude is currently beyond the capabilities of current electric propulsion systems. Furthermore, this analysis did not consider the effect of variable mass on the optimal control solution. This will result in a more complicated optimal control problem and is a focus of future research. Finally, Lyapunov control theory, which has previously been applied to the two-body problem, is being investigated in the hope of designing closed loop control schemes for this three-body scenario [1]. The addition of attitude dynamics and realistic pointing constraints would significantly improve the applicability of this work. ”

34. *The authors should discuss some statistics of the convergence of their multiple shooting TPBVP - number of iterations, compute time, etc.*

    Thank you for the comment, however it seems to be rare for the research community to give detailed description of the numerical optimization method. Most of the literature tends to not spend time discussing the low level details of the computational procedure, e.g. [6, 21]. In addition, the number of iterations, computation time and even the eventual cost function value are highly dependent on the specific system and more specifically the implementation of the algorithm in software. There are a wide variety of works which are focused more exclusively on the computational implementation of various optimal control techniques which do spend significant time on the computational characteristics. However, our manuscript is primarily focused on demonstrating the feasibility of using reachability sets to design transfer trajectories. As a result, there is not much benefit to detailed statistics of the implementation of this methodology, and we have omitted it from the manuscript. However, we have added some rough detail on the computation time and computer hardware used in the numerical optimization. This additional detail is copied below.

    > “ Each optimal control solution, corresponding to a discrete value of θ_d, is solved using `fsolve` as described earlier. Each solution on the reachability set is computed in approximately 2 min on a desktop computer using an 3.4 GHz Intel i7-3700. ”

35. *Can any intuition be gained from reporting the optimal converged cost function values in section 6.1 and 6.2? One could at least compare them to compare the reachability in the two different examples.*

    The cost function only parameterizes the distance between the controlled and uncontrolled terminal states on the Poincaré section. In addition, each state which lies on the reachable set would typically have a different distance and therefore a different converged cost function value. Furthermore, it would be difficult to characterize a single parameter which characterizes the reachable set, in the form of the cost function, for all states of the reachable set. Also, the cost function is highly dependent on the transit time of trajectory, i.e. a longer transit time corresponds to a larger reachable set and a larger magnitude of the cost function. However, this is an interesting suggestions for future research. For example, the cost function gives some indication of area/volume of the reachable set. It may be possible to use this value as another way to characterize the subspace on the Poincaré section and is an intriguing avenue of future research.

36. *"Low energy transfers from the Earth to the Lagrange points are necessary for future missions." One must be careful not to confuse these with traditional "low-energy transfers" to the moon, which use Sun third-body perturbations to transfer into lunar orbit. These are what many mission designers define as "low-energy" lunar transfers. This is actually a four-body problem (Sun, Earth, moon, spacecraft), and represents additional delta-V savings (as was done with NASA's GRAIL mission.)*

    Example:

    Parker, J. and Anderson, R., "Low-Energy Lunar Trajectory Design," DEEP SPACE COMMUNICATIONS AND NAVIGATION SERIES, Jet Propulsion Laboratory Pasadena, California, July 2013.

    Online Copy: https://descanso.jpl.nasa.gov/monograph/series12/LunarTraj–Overall.pdf

    Thank you for the comment. This sentence has been modified to make it more clear that we're discussing low thrust propulsion rather than low-energy transfers. The modified sentence is copied below.

    > " Transfers from the Earth to the Lagrange points, through the use of low-thrust electric propulsion, offer an additional and potentially shorter time of flight in comparison to the low-energy transfers which utilize solar perturbations. "

37. *For what it's worth, exploiting sun perturbations with low-thrust in the Earth-moon RTBP would be highly beneficial and represent a worthy avenue of future research (more on this below.)*

    Thank you for the comment. This is a very interesting suggestion and something that we will take into consideration for future research.

38. *The conclusions of the paper should acknowledge other potentially limiting factors of the presented analysis*

    -Omits advantage of exploiting fourth body perturbation (Sun in Earth/moon RTBP)

    -Transfers are 2D only, method is more challenging to adapt to 3D

    -Presented control magnitudes are dimensionally large, and more work is needed to determine feasibility of approach for lower accelerations

    -Thrust and mass are completely decoupled from the analysis, which is limiting in terms of finding suitable hardware that can deliver the controls needed, as currently presented.

    Thank you for the comment. The conclusion has been expanded with some discussion of the drawbacks of this research and avenues of future work. The modified conclusion is copied below.

    > " In this paper, an optimal transfer process which combines concepts of reachability and Poincaré section is used to generate transfer between planar periodic orbits in the three-body problem. The Poincaré section allows for trajectory design on a lower dimensional phase space and simplifies the process. The indirect optimal control formulation enables straightforward method of incorporating additional path and control constraints. However, the use of optimal control techniques leads to open loop trajectories that are not robust to model uncertainties or disturbances.

Furthermore, our approach relies heavily on the relative simplicity of the PCRTBP and this method is much more challenging for the non-planar case.

There is additional research to extend these results to more general transfer scenarios in future work. The incorporation of fourth body perturbations, such as the Sun in the Earth-Moon system, offers an additional method of increasing the reachable set with the combined use of the solar perturbation and low-thrust propulsion. In addition, the assumed acceleration magnitude is currently beyond the capabilities of current electric propulsion systems. Furthermore, this analysis did not consider the effect of variable mass on the optimal control solution. This will result in a more complicated optimal control problem and is a focus of future research. Finally, Lyapunov control theory, which has previously been applied to the two-body problem, is being investigated in the hope of designing closed loop control schemes for this three-body scenario [1]. The addition of attitude dynamics and realistic pointing constraints would significantly improve the applicability of this work. "

Reviewer 2

Reviewer #2:

PAPER SUMMARY: The contribution of the authors is the synthesis of reachability theory with traditional low-energy mission design approaches to ease the study of the low-thrust mission design space in the CR3BP. A crucial point is that reachability analysis in non-integrable systems is typically limited by the dimensionality of the control parameterization, which can be extremely large for continuous-thrust problems, depending on the time discretization used. The authors numerically probe the boundaries of the reachable space by optimizing a distance metric on the Poincare section, avoiding the extreme cost of unmotivated exploration. Numerical experiments show how the method could be applied in two CR3BP orbit transfer scenarios. For a multi-rev solution, the method is applied once per rev and the reachable end state with minimal distance to the target manifold is used as the basis for the next rev.

ASSESSMENT:

The central premise of the paper - using optimization of Poincare distance as a guiding motivation to make low-thrust reachability analysis more tractable - is interesting, useful, and to my knowledge reasonably novel. Furthermore, the quality of the writing is good (at local scales) and there is an appropriate degree of literature review. However, there are two major problems with this paper.

- *First, in terms of composition, large portions of the paper are spent on lengthy development of foundational work that is well-known within the astrodynamics community and available in textbooks.*

  Thank you for the comment, and we agree that there is significant repetition of material previously covered in other sources. The manuscript has been reduced and reorganized to enhance the clarity and clearly separate previously material and the contributions of this paper. The introduction has been reduced and the repetition and discuss on the variational integrator has been removed. The second section has been retitled to "Problem Formulation and Mathematical Background" and has subsections which cover all of the background material, namely the planar circular restricted three body problem, Jacobi integral, invariant manifolds and Poincaré maps, the variational integrator. The next sections introduce the optimal control formulation for the reachability analysis and the numerical examples, followed by conclusions.

The section on variational integrators has been reduced and now only discusses the main developments of the integrator for the PCRTBP and omits the review of variational principles.

- *Secondly, the contributions and findings of the authors feel highly preliminary and do not seem to evaluate their method at significant depth.*

  Thank you for the comment. The numerical examples are meant to demonstrate the use of this approach and the feasibility of using the reachability set on a Poincaré section to determine transfer trajectories. The examples give two demonstrations of finding a transfer trajectory in the PCRTBP and demonstrate combining several iterations of the reachability set for a transfer.

- *While there are the beginnings of a good article within this manuscript, it is not sufficiently close to publishable form to be accepted. I recommend that the paper be rejected and that the authors push their investigation a bit further. The eventual publication should focus less on background and should provide more insight into the fundamental properties of their method, using more rigorously constructed and analyzed examples.*

  The authors believe that the given two nontrivial numerical examples illustrate the efficacy of the proposed approach in designing low-thrust maneuvers through reachability set analysis on the Poincaré section. Furthermore, the manuscript has been completely reorganized to emphasize the contributions and clearly separate the background material. In particular, Section 4 includes additional paragraphs and figures for the geostationary to periodic orbit transfer. These additional figures demonstrate the ability to chain together multiple iterations of the reachability set computation to generate a nontrivial transfer trajectory. Combined together, the numerical examples demonstrate the ability to depart from the vicinity of the Earth to the vicinity of the Moon using our proposed method of coupling low-thrust maneuvers with reachability set analysis.

- *I can sympathize that there was quite a long road to get to the point of understanding this topic and generating results in it. I hope that the authors will not find this verdict too discouraging, as I believe that stronger results are well within their reach.*

SPECIFIC FEEDBACK:

1. **** Bloat In the introduction, the exposition of the problem could be considerably condensed, given the venue of publication. Additionally, the somewhat sprawling discussion jumps back and forth, and many key points seem to be repeated more than necessary. It needs to be organized into titled subsections with specific scope.*

   Thank you for the comment. The introduction is reduced and many of the paragraphs have been reworded and combined. In addition, the extended discussion of the benefits of variational integrators is removed from the introduction. The mathematical background has been condensed into a single section with separate subsections which discuss some of the mathematical details. The background section is focused on reviewing the relevant material rather than reintroducing topics. In the interest of space the reorganized section is omitted in this response. In short, the manuscript is completely reorganized and all of the background materials are summarized in Section 2.

2. *The development of the CR3BP, invariant manifolds and Poincare maps, all go on a bit too long. These should be refreshers to the audience that expose the specific equations and concepts*

that you'll be leveraging, not full-on introductions to well-established areas. Spending several pages on variational principles is way too much.

Thank you for the comment. The mathematical background has been reduced and condensed into a single section with multiple subsections. The discussion of the PCRTBP, invariant manifolds and Poincaré maps are combined into a single subsection. Furthermore, the variational integrator discussion has been reduced and the background material on the derivation of variational mechanics has been omitted. Instead, only the specific discussion of the development of the variational integrator is included.

3. *Section 4 ends on page 14 with essentially a thesis statement ("In this paper, we..."). Strive to push the boundary between background and new developments to a much earlier point in the paper; it shouldn't happen so late that you need to remind the audience what the paper is for.*

   Thank you for the comment. The introductory detail has been reduced and combined into a single section. As a result, the third section begins with the new contributions which describe the optimal control formulation of the reachability set.

4. *A missing relevant reference: "On target for Venus a set oriented computation of energy efficient low thrust trajectories" (Dellnitz, Junge, Post, and Thiere, 2006)*

   Thank you for the comment. The reference has been noted in the discussion of reachability sets.

   > " More recently, reachability theory has recently been applied to space systems [8, 9, 3]. "

5. **** Variational Integrators Many of the specific claims regarding the use of a variational integrator are unsubstantiated and, I believe, incorrect. Why is energy drift "even more detrimental" with regards to "the long term effects of low-thrust"? I would need to be convinced that a non-geometric integrator's energy drift relative to a constant energy baseline is significantly different from its drift relative to a gradually shifting baseline corresponding to low thrust. The specific claims about the numerical stability of derivatives also don't seem related to energy preservation.*

   The main contribution is systematic design of low-thrust orbital maneuvers through reachability set analysis on the Poincaré section. The authors decided to utilize the variational integrator as it is more computationally efficient in simulating a complex dynamic system accurately.

   More explicitly, while the structural stability of geometric numerical integrators is often illustrated by the conserved quantities such as energy or momentum, it has been shown that variational integrators are also effective in computing the rate of energy change for non-conservative systems accurately [13]. This is particularly useful for low-thrust orbital maneuvers where the energy dissipate or accumulates slowly. Furthermore, it has been shown that the structure-preserving properties of geometric numerical integrators are also useful to numerical optimizations for improved computational efficiency and stability. For example, see the following reference.

   - Ober-Blbaum, Sina, Oliver Junge, and Jerrold E. Marsden. "Discrete mechanics and optimal control: an analysis." ESAIM: Control, Optimisation and Calculus of Variations 17.2 (2011): 322-352.

In the revised manuscript, all of the comments for the variational integrator are re-examined and reorganized to Section 2.2, as also discussed in the response to the first reviewer.

6. *Apart from these claims, I question the need for a variational integrator in the first place. The example in section 3.4 is not convincing because there is no apples-to-apples basis for the comparison; a variable-step integrator with unspecified tolerance is compared to a fixed-step integrator with an arbitrary step size. Integrator comparisons should consider error in proportion to compute cost, which can be tuned by step size or tolerance. Also, fixed step seems inappropriate for a path through diverse dynamic regimes, and it's not clear that the problem scope calls specifically for energy preservation (especially since the idea is to use low thrust to mitigate some time cost of exploiting the problem structure).*

   In the revised manuscript, the computational properties of the variational integrator is explicitly compared with a general purpose integrator in Fig. 3b. According to the reviewers suggestion, the error in energy conservation is illustrated for several computational costs. It is shown that the variational integrator exhibits lower conservation error against a Runge-Kutta method for the similar level of CPU time.

   With regards to the comments on the adaptive step size, the variational integrator can be generalized according to the multisymplectic framework, as in the following reference.

   - Lew, Adrian, et al. "Asynchronous variational integrators." Archive for Rational Mechanics and Analysis 167.2 (2003): 85-146.

   However, the above formulation for structure-preserving, adaptive time-step variational integrators is beyond the scope of this paper, and we use a fixed time step throughout the presented numerical results.

   More importantly, it has been shown variational integrators are also effective in computing the rate of energy change for non-conservative systems accurately [13], even for the fixed-step sizes.

7. **** Figures Altogether, the figures need more descriptive captions.*

   Thank you for the comment, and we agree that the figures will benefit from more descriptive captions. We've added additional descriptions for all of the figures. The figure captions are shown below, with the figures reduced in size to save some space.

"

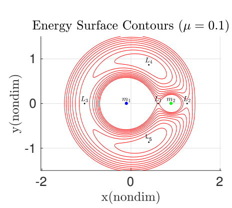


Figure 4: Contour plot of Jacobi integral: Zero velocity curves of constant Jacobi integral. A particle with fixed energy level cannot cross the contour lines and is therefore limited to specific regions of the phase space.

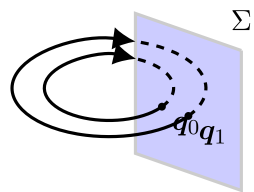
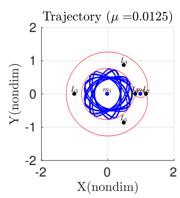
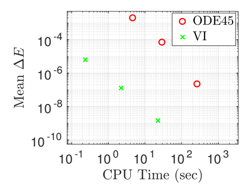


Figure 5: Diagram of the Poincaré map: Periodic orbits will appear as fixed points on the Poincaré section Σ. Stability of periodic orbits is clearly visible on the section as successive intersections approach or depart a fixed point.



(a) Earth-Moon three-body trajectory used for integrator comparison: A stable trajectory is used to test the variational and conventional integrators. The energy level is high enough to enter the vicinity of the Moon but not escape the three body system.



(b) Mean Jacobi integral deviation between variational integrator and Matlab `ODE45`: A range of fixed step sizes are used for the variational integrator to approximately match the computational time of `ODE45`

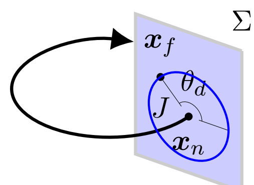


Figure 7: Reachability set on a Poincaré section: Pictorial representation of the reachability set (blue circle) on the Poincaré section, Σ. The terminal state, x_n, is the intersection without any control input. Adding a control input allows for the terminal state, x_f, to be displaced by some distance/cost J as measured on the section. We parameterize a specific direction on the section with the angles θ_d and seek to maximize the distance between x_f and x_n. Computation of the maximum distance, or reachability, for a variety angles gives a discrete approximation of the reachability set.

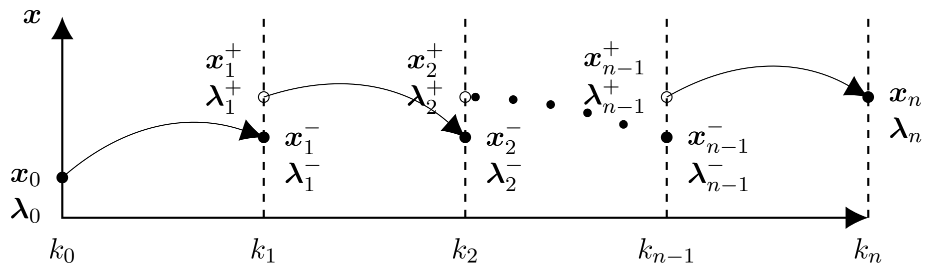


Figure 8: Schematic diagram of the multiple shooting method: The complete trajectory is split into a number of sub-segments, and additional interior constraints are included to ensure state and costate continuity. Splitting the optimal trajectory into short segments reduces the sensitivity of terminal states to variations of the initial states.

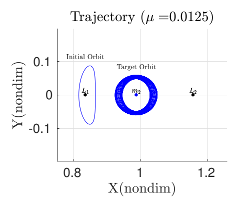
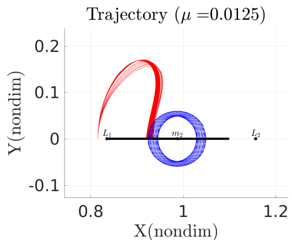


Figure 9: L_1 periodic orbit transfer to orbit of the Moon: Example scenario demonstrating the initial and target orbit. Without the low-thrust propulsion, the spacecraft is constrained to the initial periodic orbit. We determine the reachability set to find a transfer trajectory to the target orbit about the moon.

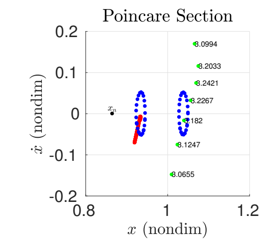
(a) Position space view of invariant manifold

(b) Poincaré intersection of invariant manifold (green), target orbit (blue) and the initial orbit (black)

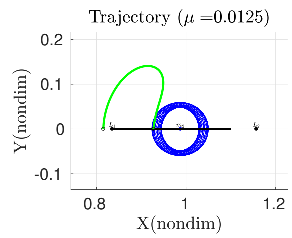
Figure 10: Invariant manifold transfer: An example transfer using the invariant manifolds is shown in both the position and Poincaré spaces. The control free transfer from the initial periodic orbit to the target orbit result in a long time of flight. In addition, the manifold only intersects the target orbit on the descending or far side of the moon.



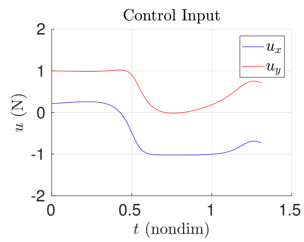
(a) Position space view of reachability set



(b) Poincaré section view of reachability set



(c) Transfer trajectory selected from reachability set viewed in the position space



(d) Control input for the selected transfer trajectory

Figure 11: L_1 Reachability set transfer: The low thrust propulsion is used to approximate the reachability set starting from the initial periodic orbit over a fixed time horizon. The reachability set is shown in the upper two figures in both the position and Poincaré space. From this reachability set we chose a trajectory which intersects the target orbit and it is shown in the lower left figure. The optimal control to achieve this transfer is shown in the lower right figure.

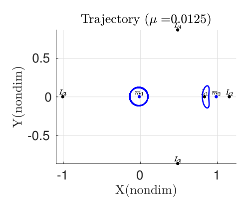
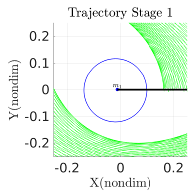
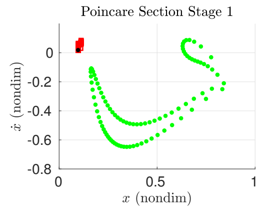


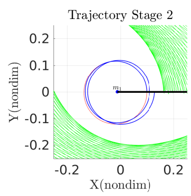
Figure 12: Geostationary to L_1 transfer: Representation of the initial and target orbits for the reachability transfer. Vehicle is assumed to begin on the geostationary orbit about m_1 and will transfer to the periodic orbit about L_1



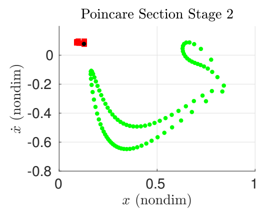
(a) Stage 1 Trajectory



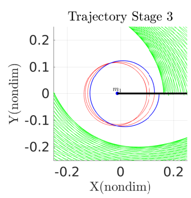
(b) Stage 1 Poincaré section



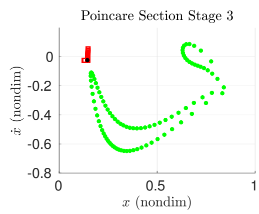
(c) Stage 2 Trajectory



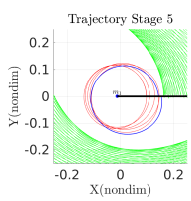
(d) Stage 2 Poincaré section



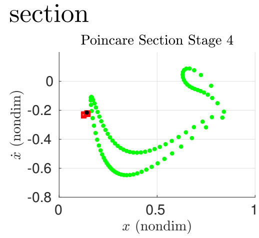
(e) Stage 3 Trajectory



(f) Stage 3 Poincaré section

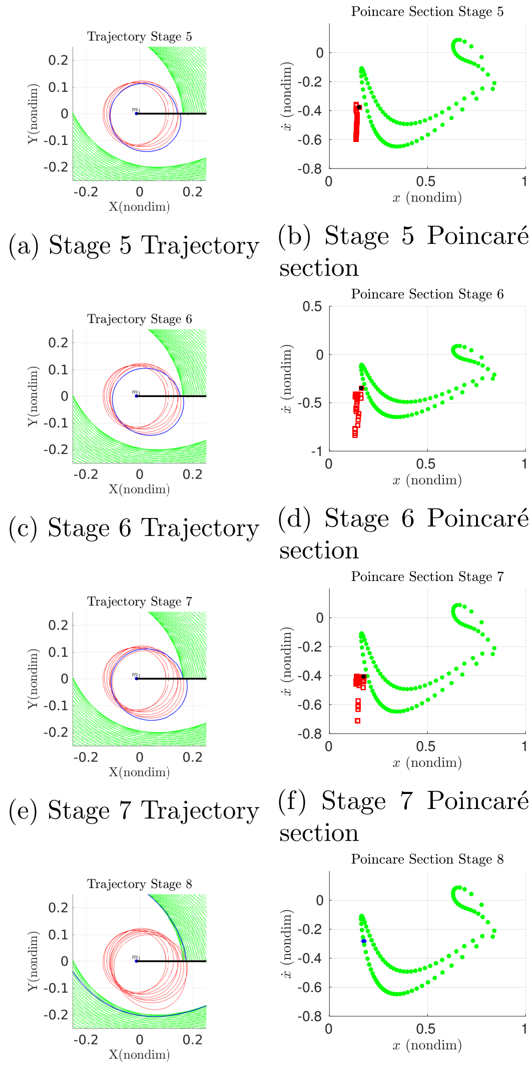


(g) Stage 4 Trajectory



(h) Stage 4 Poincaré section

Figure 13: Stage 1-4 reachability sets: The first four reachability sets visualized in both the position (left) and Poincaré space (right). The minimum trajectories from the preceding stages are shown in red, while the next stage is shown in blue. The transfer goal is to generate a complete trajectory from the initial geostationary orbit to the L_1 stable manifold, with an eventual arrival at the L_1 periodic orbit



(a) Stage 5 Trajectory (b) Stage 5 Poincaré section

(c) Stage 6 Trajectory (d) Stage 6 Poincaré section

(e) Stage 7 Trajectory (f) Stage 7 Poincaré section

(g) Stage 8 Trajectory (h) Stage 8 Poincaré section

Figure 14: Stage 5-8 reachability sets: The last four reachability sets visualized in both the position (left) and Poincaré space (right). The minimum trajectories from the preceding stages are shown in red, while the next stage is shown in blue. The transfer goal is to generate a complete trajectory from the initial geostationary orbit to the L_1 stable manifold, with an eventual arrival at the L_1 periodic orbit

”

8. *Figure 4 does not show what I think you intend it to show; it looks like two periodic circular orbits.*

   Thank you for the comment, however the figure is drawn to show a pictorial representation of a Poincaré section. The periodic orbits intersect the section at fixed points, as a result it becomes simple to visualize and determine these fixed solutions by searching for fixed points on the section Conversely, stable/unstable trajectories will also appear on the section with a distinctive signature, and by tracking successive intersections of the trajectory with the Poincaré section.

In order to improve the clarity and reduce confusion, we have added some additional detail to the discussion of the Poincaré map to the manuscript. This addition has been copied below.

> " Poincaré maps are a useful tool in the analysis of the flow near periodic orbits in the three-body problem. We let Σ define a hypersurface of section chosen such that all trajectories in the vicinity of a state $\boldsymbol{q} \in \Sigma$ cross Σ transversely and in the same direction. A Poincaré map, $P(\boldsymbol{q}) = \phi(T; \boldsymbol{q})$, maps the state of a trajectory from one intersection to the next. Choosing a section in this manner results in a Poincaré section as shown in Fig. 2. In Fig. 2 we show two examples of periodic trajectories intersecting the Poincaré section. Periodic solutions will appear as fixed points on the section, such as q_0, q_1 in Fig. 2, while stable or unstable trajectories become clearly visible by viewing successive intersections of the section. This allows for greater insight into the stability and dynamics of periodic solutions of a dynamic system as a fixed point on the Poincaré section corresponds to a periodic orbit while movement on the section is associate with the stability of neighboring trajectories. "

9. *Figure 6 is also very unclear. Is phi supposed to be* **theta_d**? *Is the bounding curve specifically circular, or should it be a more general shape?*

   Thank you for the comment. The angle in the figure is meant to match the terminal constraint θ_d. In addition, the reachable set is only circular for ease of design in the figure. In reality the figure will be a general shape, rather than specifically circular.

   The manuscript has been modified to correct the figure and also incorporate some more discussion of the figure. The additions are included below as well as the updated figure.

> " Fig. 4 illustrates how, without any control input, trajectories will intersect with the Poincaré section at $\boldsymbol{x}_n$. However, the addition of low thrust propulsion allows the spacecraft to depart from the natural dynamics and intersect the Poincaré section at a different location. We use a cost function to define a distance metric on the Poincaré section from the control-free intersection to an intersection under the influence of the control input. Maximization of this distance along varying directions enables us to generate the largest reachability set under the bounded control input. In Fig. 4 the reachable set is shown as a circular region on the Poincaré section. In practice, the reachable set will be of a general shape and also higher dimensional in the nonplanar case.

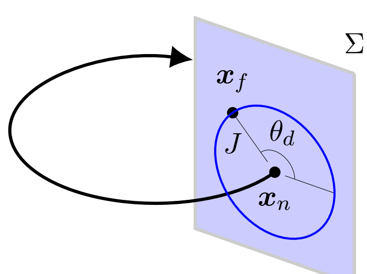


Figure 15: Reachability set on a Poincaré section: Pictorial representation of the reachability set (blue circle) on the Poincaré section, Σ.

”

10. *Consider plotting the intersections of manifolds with the Poincare section as continuous curves.*

    Thank you for the comment. The manifold intersections with the Poincaré section are plotted as discrete points as the curves were generated in a discrete manner. A series of trajectories were generated by perturbing the trajectory at various points along the periodic orbit. As a result, we show that the intersection is a discrete approximation of the true intersection of the manifold with the section by using the discrete points rather than a continuous curve.

11. *Also consider zooming/scaling as necessary to show the details you discuss.*

    Thank you for the comment. The figure have been revised to adequately show the desired information in the revised manuscript. The figures are centered around the various intersections of the reachable set and the Poincaré section. Furthermore, in the case of the much larger geostationary transfer requires it becomes more difficult to zoom in on the fine details. However, a detailed view is presented which shows the various iterations of the reachability set computations.

12. **** Method/Examples I urge a revisiting of the claims with regards to optimality. The optimization method is being initialized with *something*, and stationary points are *local* extrema. More precise claims and assurances about solution quality (and therefore the validity of the computed reachable set boundary) are needed.*

    The presented numerical results satisfy the necessary conditions for optimality. As common in gradient-based numerical optimizations for non-convex systems, it guarantees local optimality. Applying various techniques to obtain the globally optimal solution is beyond the scope of this paper that is focused on systematic design of low-thrust orbital maneuvers thorough reachability set analysis on the Poincaré section.

    To clarify these, the following comment is added at the first paragraph of Section 4.

    > “ The numerical examples presented in this section satisfy necessary conditions for local optimality, and obtaining a globally optimal solution is considered beyond the scope of this paper. ”

13. *How damaging is the assumption of a fixed travel time? And how about the specification of a specific departure point on the initial orbit?*

    The fixed travel time is primarily used at this stage to reduce the complexity of the optimal control formulation. Furthermore, the reachability set is defined in terms of a fixed terminal time. A variable terminal travel time would add an additional dimension, time, to the reachable space and add additional complexity to the reachability set visualization. The fixed travel time does not adversely affect the optimality of the computed trajectories on the reachable set, as the reachable set is defined in terms of a fixed travel time.

    The choice of starting from a fixed point on the initial periodic orbit is again mostly used for convenience. In this case of the periodic orbit the departing trajectories are all along the unstable invariant manifold associated with the periodic orbit. The uncontrolled trajectories associated with the invariant manifolds serve as the “center” of each reachability set computation. The low-thrust propulsion system is then used to enlarge the distance between the uncontrolled and controlled states. Starting from a different location on the periodic orbit will not create a qualitatively different situation as the entire invariant manifold also intersects the Poincaré section. However, different locations along the periodic orbit may have vastly different travel times before intersecting the Poincaré section. This will require modification of the optimal control problem, with a larger terminal time, but will result in a similar behavior regardless of the location on the initial periodic orbit.

14. *In Fig 10b, it is hard to see, but appears that the low-thrust-reachable set is actually *centered* on part of the target set. Is this related to the existence of the uncontrolled transfer opportunity? There must be some more clear way to show how the low thrust set expands upon the zero thrust set. Perhaps beginning with the time horizon of the zero-thrust opportunity, and successively lowering it, would help.*

    The reachability set is closely spaced in terms of both the position space as well as on the Poincaré section. This is due to the fact that there is a very low-thrust control input on the spacecraft. As a result, the trajectories associated with the reachability set are all closely spaced, but more widely separated from the terminal state without any control input. In the figure, there is a fixed point labeled x_n which is the terminal state without any control input, which will also lie on the periodic orbit. The reachable set, in red, is widely separated from the uncontrolled state due the effect of the low thrust propulsion, however the subsequent spread of the entire reachability set is much more constrained. The low thrust expands the reachability set from a fixed point to the red elliptical region. This is also explained in the manuscript with a similar description of the figure.

15. *Perhaps there is also a better way to show what the control is doing, e.g., in some alternative parametrization? This could help provide assurances that the solution is doing something meaningful and is not just stuck in a weird local minimum.*

    Thank you for the comment. The control plots show the planar components of the control thrust. Furthermore, the scale has been modified to show the thrust in terms of Newtons for an assumed 500 kg satellite. The control plots are meant to demonstrate the control input required by the trajectories which lie on the maximum reachability set. As a result, the control input is determined through optimization to maximize the reachability set in a particular direction on the Poincaré section. This is in contrast to typical optimal control applications where the control input is used directly in the trajectory design process. Our method seeks to approximate the reachability set, and from this set trajectories are chosen in a systematic method which drives the spacecraft towards a target.

16. *In the second example, what do you lose by chaining together multiple reachability-based steps? Aren't you imposing that the trajectory routinely returns to the energy level of the Poincare section? How feasible would it be to conduct a single reachability analysis covering multiple revolutions? And can you use knowledge about directionality of movement on the Poincare section to limit the scope of your reachability search?*

    While the trajectories do return to the energy level of the Poincaré section, there is a transfer of "energy" between the states of the vehicle. It is in this manner that the reachability set is enlarged to eventually include the target state.

    Conducting a single reachability analysis becomes more complicated as the larger time of flight increases the sensitivity of the terminal states to variations in the initial costates. Furthermore, the larger time of flight creates a much larger and more chaotic reachable set for the analysis. Through this work it was found that shorter trajectories tended to have more stable and consistent solutions.

17. *In general, please devote more depth to probing the strengths, weaknesses, and fundamental properties of the method. Considering extreme cases can help with this.*

    The authors believe that the given two nontrivial numerical examples illustrate the efficacy of the proposed approach in designing low-thrust maneuvers through reachability set analysis on the Poincaré section. The examples demonstrate the ability to generate transfer trajectories in the PCRTBP and can be easily extended to non-planar transfers [11]. The proposed approach provides for a systematic design process for low-thrust orbital transfers, in contrast to other methods which must rely entirely on numerical optimization to determine trajectories. This design process leverages the reachability set on a Poincaré section to greatly reduce the computational complexity of an unmotivated search of the reachable set. Through the use of the reachability set, our proposed approach provides a methodical process to determine transfers, using single or multiple iterations.

    There are several drawbacks of this approach. First, it relies heavily on the PCRTBP, which limits the applicability to planar trajectories. Furthermore, this reduces it's benefit to operational missions which are often non-planar, e.g. LEO to GTO missions. Second, the method does not consider the effect of variable mass on the system dynamics. This is a focus of future work to incorporate the effect of variable mass on the optimal control problem. Finally, the variational integrator provides for a bounded energy behavior but does not ensure exact energy over the simulation. As a result, there is still error in the variational integrator with respect to energy deviation.

    The manuscript has been completely reorganized to emphasize the contributions and clearly separate the background material. In particular, Section 4 includes additional paragraphs and figures for the geostationary to periodic orbit transfer. These additional figures demonstrate the ability to chain together multiple iterations of the reachability set computation to generate a nontrivial transfer trajectory. Combined together, the numerical examples demonstrate the ability to depart from the vicinity of the Earth to the vicinity of the Moon using our proposed method of coupling low-thrust maneuvers with reachability set analysis.

    In addition, the conclusion has been modified to better demonstrate the strengths and weaknesses of our approach as well as avenues for future research. The conclusion is copied below.

    > “
    >
    > In this paper, an optimal transfer process which combines concepts of reachability and Poincaré section is used to generate transfer between planar periodic orbits in the three-body problem. The Poincaré section allows for trajectory design on a lower dimensional phase space and simplifies the process. The indirect optimal control formulation enables straightforward method of incorporating additional path and control constraints. Direct optimal control techniques must rely entirely on the ability of the numerical optimization routine to determine a feasible solution. Our approach leverages the benefits of the reachability set, which encompass the maximum set of states achievable by the spacecraft over a fixed time horizon. Using the reachability set on a Poincaré section reduces the dimensionality of the system and simplifies the analysis by avoiding the cost of a completely unmotivated exploration of the space. However, the use of optimal control techniques leads to open loop trajectories that are not robust to model uncertainties or disturbances. Furthermore, our approach relies heavily on the relative simplicity of the PCRTBP and this method is much more challenging for the non-planar case.

There is additional research to extend these results to more general transfer scenarios in future work. The incorporation of fourth body perturbations, such as the Sun in the Earth-Moon system, offers an additional method of increasing the reachable set with the combined use of the solar perturbation and low-thrust propulsion. In addition, the assumed acceleration magnitude is currently beyond the capabilities of current electric propulsion systems and future research is aimed at investigating smaller magnitude control inputs. Furthermore, this analysis did not consider the effect of variable mass on the optimal control solution. This will result in a more complicated optimal control problem and is a focus of future research. Finally, Lyapunov control theory, which has previously been applied to the two-body problem, is being investigated in the hope of designing closed loop control schemes for this three-body scenario [1]. The addition of attitude dynamics and realistic pointing constraints would also significantly improve the applicability of this work. ”