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Spacecraft Trajectory Design Near Asteroid 4769 Castalia

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Introduction

- Asteroids and comets are of significant interest
  - Science - Insight into early solar system formation
  - Mining - vast quantities of useful materials
  - Impact - high risk from hazardous Near-Earth asteroids
- Near-Earth asteroids (NEAs) are especially interesting
  - Orbit close to the Earth and are easily accessible
  - Many asteroids hold vast quantities of useful materials
  - Asteroid mining: Precious metals, propulsion fuels, semiconductors
  - Commercialization is feasible with huge amounts of possible profit
- High probability of future asteroid impacts

Asteroid Mining

Asteroid Impact

Technical Challenges

- Low-thrust propulsion systems offer innovative options
  - Electric propulsion offers much greater efficiency
  - Allows for greater velocity change with a reduced mass cost
  - Key component for long duration missions with frequent thrusting
  - Requires new methods of design
- Optimal trajectory design is complicated
  - Highly nonlinear and chaotic dynamics requires intuition by designer
  - Using low-thrust propulsion adds additional difficulties in accurately capturing the small perturbations
- Astrodynamic trajectory design typically uses direct optimal control
  - Large nonlinear programming problem inherently approximates the true optimal solution
  - High dimensionality of the solution makes it extremely computationally intensive

Gravitational Modeling

- Asteroids are extended bodies - not point masses
  - Gravity is the key force in orbital mechanics
  - An accurate representation of gravity is critical to accurate and realistic analysis
- Spherical Harmonic approach is popular but not ideal
  - Model is only valid outside of circumscribing sphere
  - Composed of an infinite series - always results in an approximation
  - Model will diverge when close to the surface and is not ideal for landing missions
- Polyhedron Gravitational model used to represent the asteroid
  - Gravity is a function of the shape model
  - Globally valid and closed-form analytical solution for gravity
  - Exact potential assumes a constant density assumption
  - Accuracy is only dependent on the shape

$$U(r) = \frac{1}{2} G\sigma \sum_{e \in \text{edges}} r_e \cdot E_e \cdot r_e \cdot L_e - \frac{1}{2} G\sigma \sum_{f \in \text{faces}} r_f \cdot F_f \cdot r_f \cdot \omega_f$$

Dynamics about the asteroid 4769 Castalia

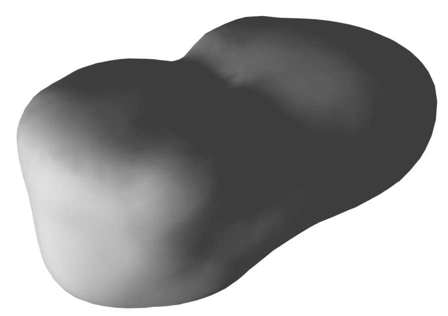
- Dynamics are very similar to the famous three-body problem

$$\begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ g(r) + h(v) + u \end{bmatrix}$$

- Huge history of analytical tools allow for great insight into the dynamics
- Analytical insight is critical to understanding the free motion around an asteroid
  - We require an accurate understanding of the motion under the influence of gravity alone
  - Efficient use of the limited oboard fuel is dependent on exploiting the natural dynamics of the asteroid environment
- Jacobi Integral - single constant of motion which bounds the feasible regions in terms of "energy"

$$J(r, v) = \frac{1}{2}\omega^2 \left(x^2 + y^2\right) + U(r) - \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

- Spacecraft is operating around 4769 Castalia
  - Discovered in 1989, Castalia is a potentially hazardous asteroid and passes close to the Earth
  - In 1989, Castalia passed close enough to allow for high resolution radar imagery
  - High resolution shape is used in polyhedral gravity model



Asteroid 4769 Castalia

Simulation Results

- Transfer between two periodic orbits of 4769 Castalia
  - Thruster represents a current electric propulsion $\approx 600\,\mathrm{mN}$
  - Combining multiple iterations of the rechability computation allows for general transfers
- Combining four iterations of the reachability set
  - Each iteration of the reachability set enlarges the achievable states
  - We choose a direction on the reachability set which lies closest to the target

$$d = \sqrt{k_x\left(x_f - x_t\right)^2 + k_z\left(z_f - z_t\right)^2 + k_{\dot{x}}\left(\dot{x}_f - \dot{x}_t\right)^2 + k_{\dot{z}}\left(\dot{z}_f - \dot{z}_t\right)^2}$$

  - This iterative approach avoids the difficulty in choosing accurate initial guesses for optimization

- Optimal Control is used to calculate the reachability set

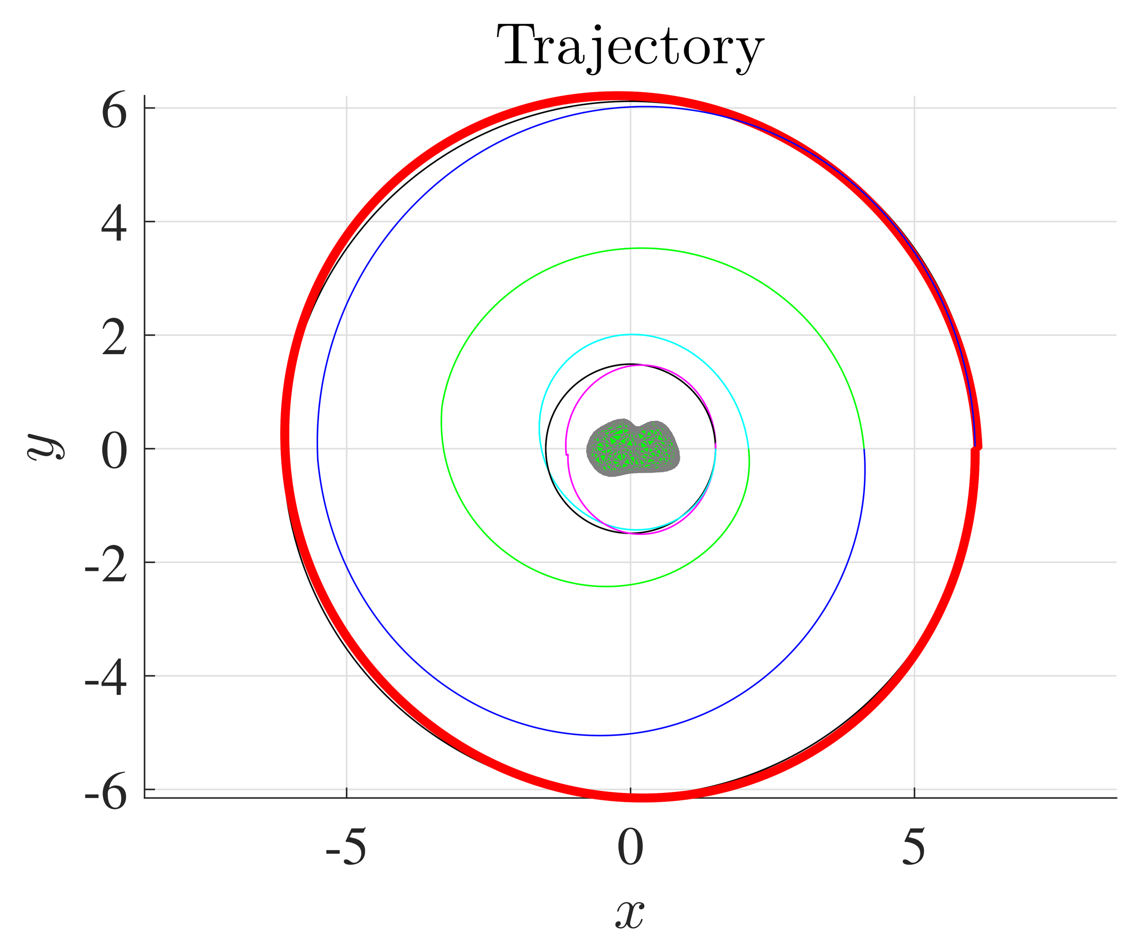
$$J = -\frac{1}{2}\left(x(t_f) - x_n(t_f)\right)^T Q \left(x(t_f) - x_n(t_f)\right)$$

- Maximize the distance on the Poincaré section using the low thrust propulsion
- Thruster magnitude is limited by physical system

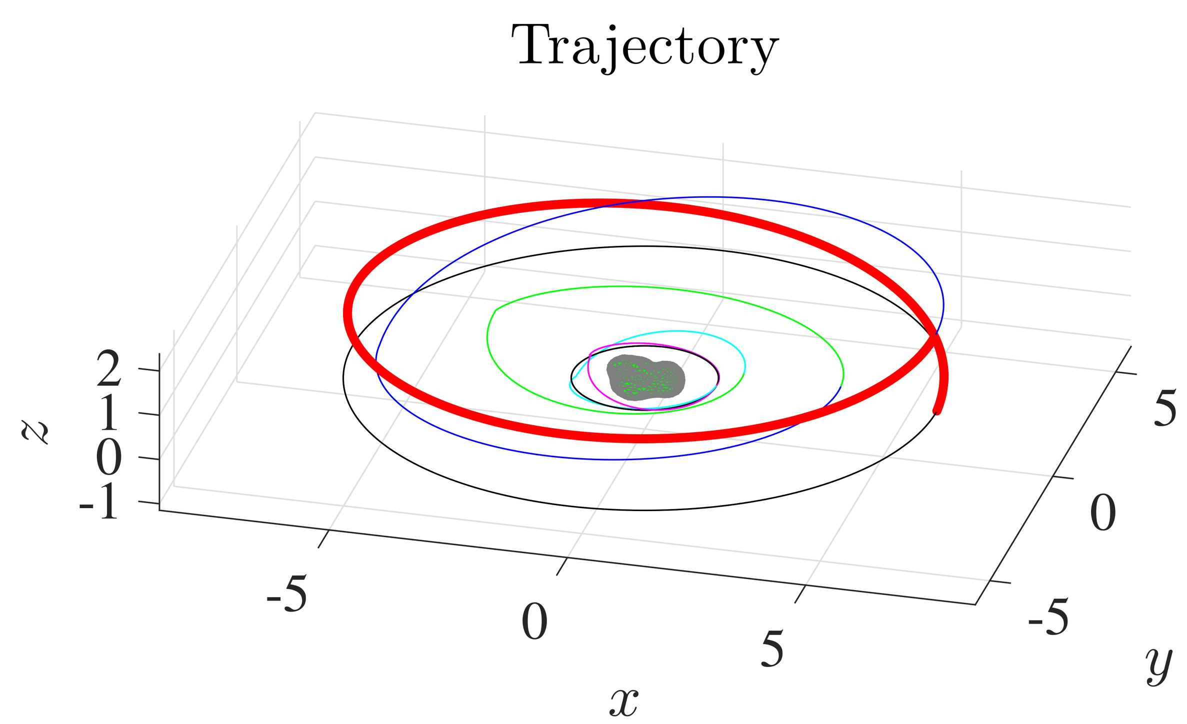
$$c(u) = u^T u - u_m^2 \leq 0$$

- Terminal constraints ensure intersection with Poincaré section

$$m_1 = y = 0$$
$$m_2 = (\sin\phi_{1_d})\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right) - x_1^2 = 0$$
$$m_3 = (\sin\phi_{2_d})\left(x_2^2 + x_3^2 + x_4^2\right) - x_2^2 = 0$$
$$m_4 = (\sin\phi_{3_d})\left(2x_3^2 + 2x_3\sqrt{x_4^2 + 2x_4^2}\right) - x_3 - \sqrt{x_4^2 + x_3^2} = 0$$

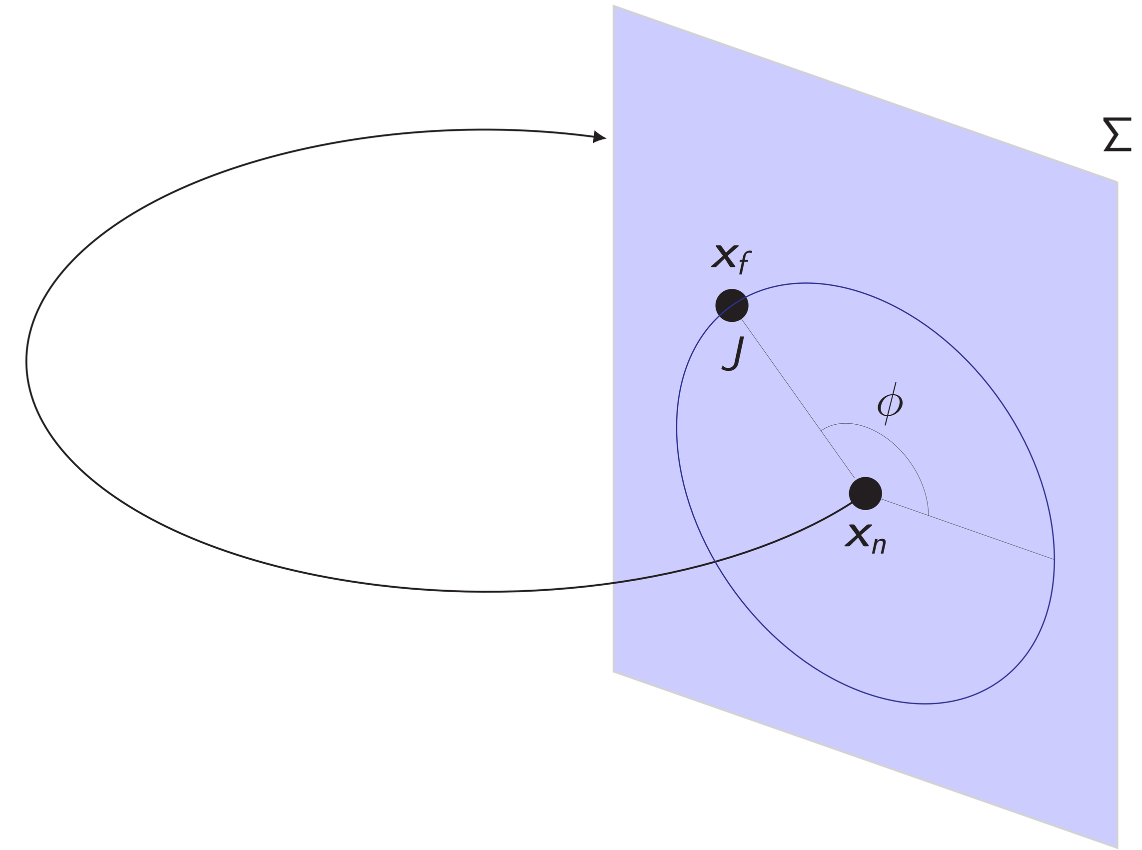


(a) Equatorial Transfer Trajectory



3D Transfer Trajectory

Reachability on the Poincaré section



Reachability set on a Poincaré section

- Poincaré map is a useful tool in the analysis of dynamical systems
  - Enables visualization of complicated systems - intrinsic structure becomes visible to the engineer
  - Rather than considering the entire state (6D position and velocity) we simply investigate the intersections with a lower dimensional space
  - This reduces the complexity of analyzing the dynamics and allows for visualization of highly complex dynamic interactions
- A periodic orbit on the Poincaré map is identified by fixed points x_n
- Using the low-thrust propulsion system of the spacecraft we can enlarge the space that is achievable
  - Reachability Set - the set of states which are attainable subject to the constraints of the system
  - The thruster of the spacecraft is used to design a transfer trajectory by repeatedly maximizing the reachability set
  - Thruster allows us to depart from the fixed orbit and intersect at a new state x_f
- Reachability Set is computed on the Poincaré section and provides additional insight
  - Spacecraft can only move to areas inside of the reachable set

Conclusions

- Demonstrate a transfer around an asteroid using multiple reachability sets
  - Each reachability set moves the spacecraft towards the target
- Alleviates the need for selecting accurate initial guesses
- Automatically gain insight into the feasible region of motion for the spacecraft
- Future work will extend this principle to landing trajectories on asteroids
  - Irregular shape of asteroids requires innovative techniques for controlling both position and orientation
  - Nonlinear control allows for the exploitation of the coupled dynamics
  - Complex dynamics requires accurate integration schemes - Variational Integrators
- Successful extension of previous work in the circular restricted three-body problem