Spacecraft Trajectory Design Near Asteroid 4769 Castalia

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Shankar Kulumani

Flight Dynamics and Controls Laboratory (Dr. Taeyoung Lee)
Department of Mechanical and Aerospace Engineering, School of Engineering and Applied Science

Introduction

- Asteroids and comets are of significant interest
  - Science - Insight into early solar system formation
  - Mining - vast quantities of useful materials
  - Impact - high risk from hazardous Near-Earth asteroids
- Near-Earth asteroids (NEAs) are especially interesting
  - Orbit close to the Earth and are easily accessible
  - Many asteroids hold vast quantities of useful materials
  - Asteroid mining: Precious metals, propulsion fuels, semiconductors
  - Commercialization is feasible with huge amounts of possible profit
- High probability of future asteroid impacts

Asteroid Mining

Asteroid Impact

Technical Challenges

- Low-thrust propulsion systems offer innovative options
  - Electric propulsion offers much greater efficiency
  - Allows for greater velocity change with a reduced mass cost
  - Key component for long duration missions with frequent thrusting
  - Requires new methods of design
- Optimal trajectory design is complicated
  - Highly nonlinear and chaotic dynamics requires intuition by designer
  - Using low-thrust propulsion adds additional difficulties in accurately capturing the small perturbations
- Astrodynamic trajectory design typically uses direct optimal control
  - Large nonlinear programming problem inherently approximates the true optimal solution
  - High dimensionality of the solution makes it extremely computationally intensive

Gravitational Modeling

- Asteroids are extended bodies - not point masses
  - Gravity is the key force in orbital mechanics
  - An accurate representation of gravity is critical to accurate and realistic analysis
- Spherical Harmonic approach is popular but not ideal
  - Model is only valid outside of circumscribing sphere
  - Composed of an infinite series - always results in an approximation
  - Model will diverge when close to the surface and is not ideal for landing missions
- Polyhedron Gravitational model used to represent the asteroid
  - Gravity is a function of the shape model
  - Globally valid and closed-form analytical solution for gravity
  - Exact potential assumes a constant density assumption
  - Accuracy is only dependent on the shape

$$U(\boldsymbol{r}) = \frac{1}{2}G\sigma \sum_{e \in \text{edges}} \boldsymbol{r}_e \cdot \boldsymbol{E}_e \cdot \boldsymbol{r}_e \cdot L_e - \frac{1}{2}G\sigma \sum_{f \in \text{faces}} \boldsymbol{r}_f \cdot \boldsymbol{F}_f \cdot \boldsymbol{r}_f \cdot \omega_f$$

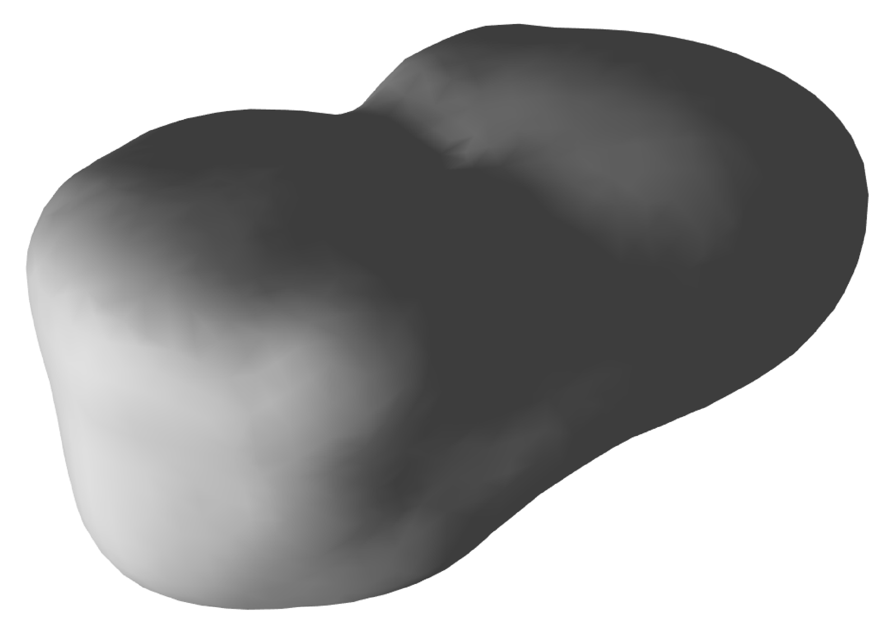
Dynamics about the asteroid 4769 Castalia

- Dynamics are very similar to the famous three-body problem

$$\begin{bmatrix} \dot{\boldsymbol{r}} \\ \dot{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v} \\ g(\boldsymbol{r}) + h(\boldsymbol{v}) + \boldsymbol{u} \end{bmatrix}$$

- Huge history of analytical tools allow for great insight into the dynamics
- Analytical insight is critical to understanding the free motion around an asteroid
  - We require an accurate understanding of the motion under the influence of gravity alone
  - Efficient use of the limited oboard fuel is dependent on exploiting the natural dynamics of the asteroid environment
- Jacobi Integral - single constant of motion which bounds the feasible regions in terms of "energy"

$$J(\boldsymbol{r}, \boldsymbol{v}) = \frac{1}{2}\omega^2\left(x^2 + y^2\right) + U(\boldsymbol{r}) - \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$



Asteroid 4769 Castalia

Reachability on the Poincaré section

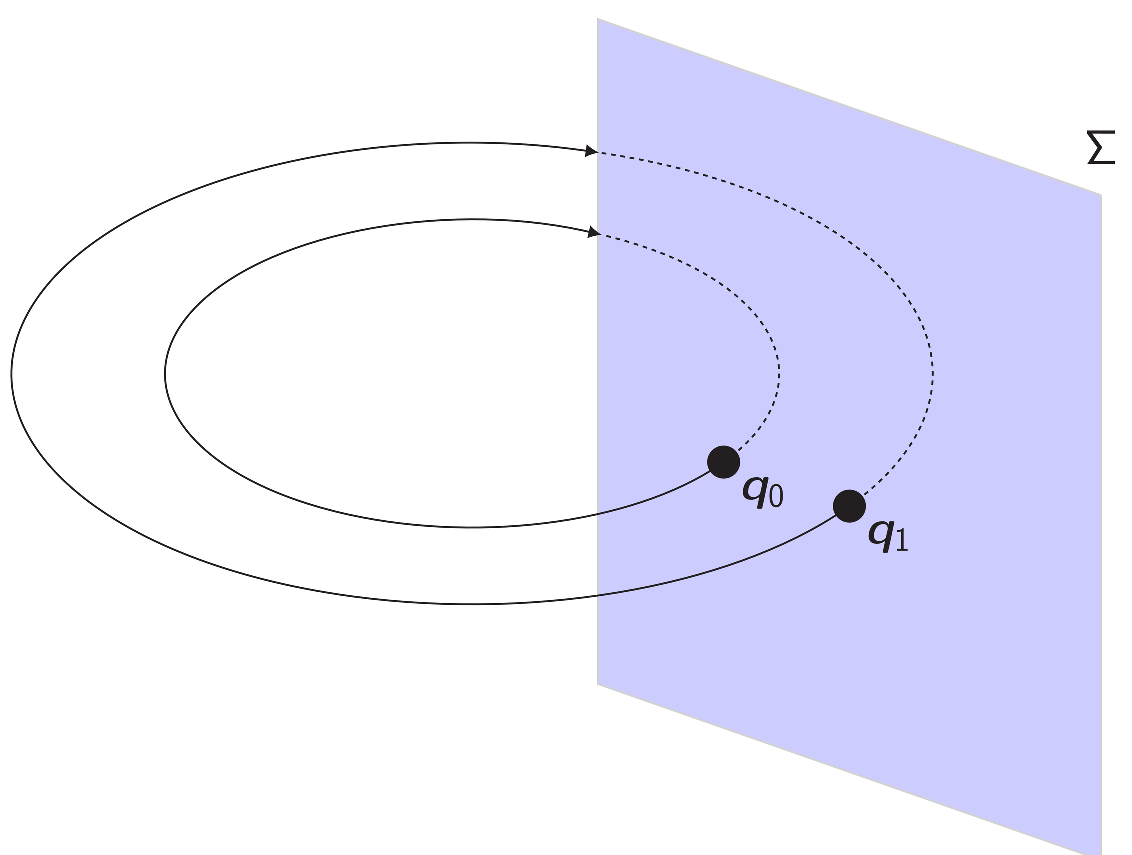
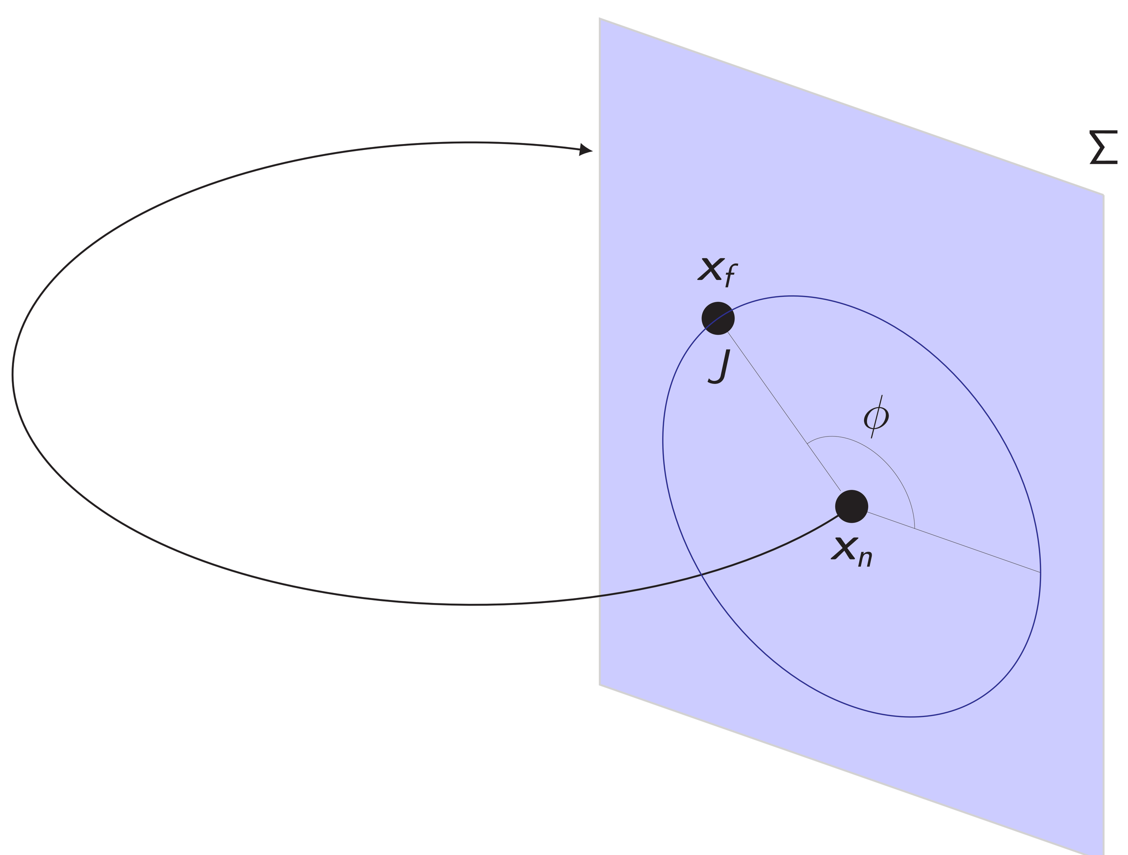


Diagram of the Poincaré map



Reachability set on a Poincaré section

- Poincaré map is a useful tool for the analysis of dynamical systems
  - Rather than considering the entire state (6D position and velocity) we simply investigate the intersections with a lower dimensional space
  - This reduces the complexity of analysing the dynamics and allows for visualization of highly complex dynamic interactions
- A periodic orbit on the Poincaré map is identified by fixed points $\boldsymbol{q}_0$ and $\boldsymbol{q}_1$
- Using the low-thrust propulsion system of the spacecraft we can enlarge the space that is achievable
  - Reachability Set - the set of states which are attainable subject to the constraints of the system
  - The thruster of the spacecraft is used to design a transfer trajectory by repeatedly maximizing the reachability set
- Optimal Control is used to calculate the reachability set

$$J = -\frac{1}{2}\left(x(t_f) - x_n(t_f)\right)^T Q\left(x(t_f) - x_n(t_f)\right)$$

- The goal is to maximize the distance on the Poincaré section using the effect of the low thrust propulsion
- The thruster is limited to ensure a realistic scenario

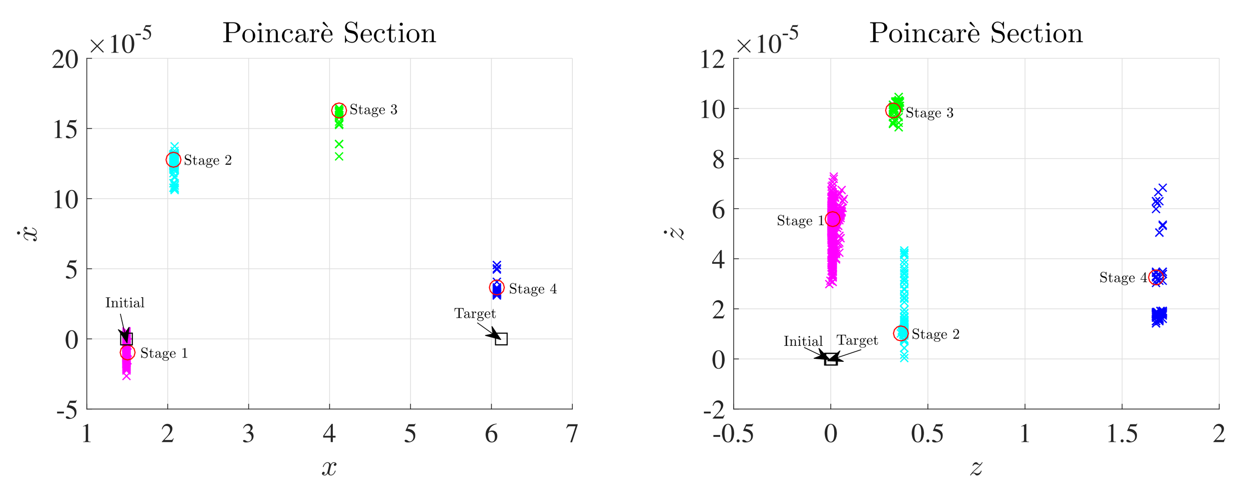
$$c(\boldsymbol{u}) = \boldsymbol{u}^T\boldsymbol{u} - u_m^2 \leq 0$$

- A series of constraints ensures that we will intersect the Poincaré section

$$m_1 = y = 0$$
$$m_2 = \left(\sin\phi_{1_d}\right)\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right) - x_1^2 = 0$$
$$m_3 = \left(\sin\phi_{2_d}\right)\left(x_2^2 + x_3^2 + x_4^2\right) - x_2^2 = 0$$
$$m_4 = \left(\sin\phi_{3_d}\right)\left(2x_3^2 + 2x_3\sqrt{x_4^2 + 2x_4^2}\right) - x_3 - \sqrt{x_4^2 + x_3^2} = 0$$

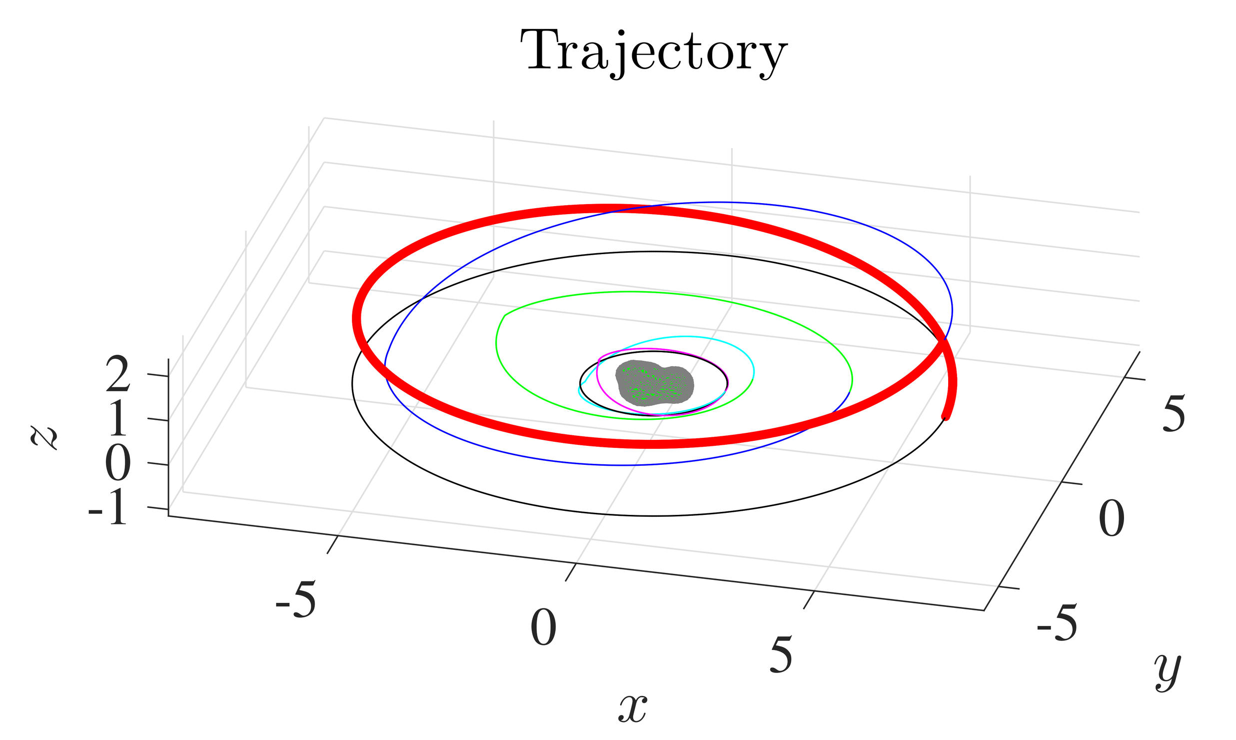
Simulation Results

- A transfer around the asteroid 4769 Castalia is completed
  - Discovered in 1989, Castalia is a potentially hazardous asteroid and passes close to the Earth
  - In 1989, Castalia passed close enough to the Earth to allow for high resolution radar imagery
- We wish to transfer a spacecraft between two periodic orbits of 4769 Castalia
  - The thruster is modeled after current state of the art low thrust propulsion systems
  - Combining multiple iterations of the rechability computation allows for general transfers
- Transfer is computed by combining four iterations of the reachability set
  - Each iteration of the reachability set enlarges the achievable states
  - From the reachability set we chose a direction which gets us closest to the target



x vs. $\dot{x}$ $\quad$ z vs. $\dot{z}$

Poincaré sections



Transfer Trajectory

Conclusions

- Orbital transfer around an asteroid using reachability sets
- Automatically gain insight into the feasible region of motion for the spacecraft
- Future work will extend this principle to landing trajectories
  - Irregular shape of asteroids requires innovative techniques for controlling both position and orientation