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Awesome title

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Abstract: Great abstract highlighting your hard work

Keywords: attitude control, asymptotic stability, special orthogonal group, constraint

1. INTRODUCTION

Rigid body attitude control is an important problem for aerospace vehicles, ground and underwater vehicles, as well as robotic systems [1,2]. One distinctive feature of the attitude dynamics of rigid bodies is that it evolves on a nonlinear manifold. The three-dimensional special orthogonal group, or $\mathsf{SO}(3)$, is the set of 3×3 orthogonal matrices whose determinant is one. This configuration space is non-Euclidean and yields unique stability properties which are not observable on a linear space. For example, it is impossible to achieve global attitude stabilization using continuous time-invariant feedback [3].

2. PROBLEM FORMULATION

2.1. Attitude Dynamics

Consider the attitude dynamics of a rigid body. We define an inertial reference frame and a body-fixed frame, whose origin is at the center of mass and aligned with the principle directions of the body. The configuration manifold of the attitude dynamics is the special orthogonal group:

$$\mathsf{SO}(3) = \{R \in \mathbb{R}^{3\times 3} \,|\, R^T R = I,\ \det[R] = 1\},$$

where a rotation matrix $R \in \mathsf{SO}(3)$ represents the transformation of the representation of a vector from the body-fixed frame to the inertial reference frame. The equations of motion are given by

$$J\dot{\Omega} + \Omega \times J\Omega = u + W(R,\Omega)\Delta, \tag{1}$$

$$\dot{R} = R\hat{\Omega}, \tag{2}$$

where $J \in \mathbb{R}^{3\times 3}$ is the inertia matrix, and $\Omega \in \mathbb{R}^3$ is the angular velocity represented with respect to the body-fixed frame. The control moment is denoted by $u \in \mathbb{R}^3$, and it is expressed with respect to the body-fixed frame. We assume that the external disturbance is expressed by $W(R,\Omega)\Delta$, where $W(R,\Omega) : \mathsf{SO}(3) \times \mathbb{R}^3 \to \mathbb{R}^{3\times p}$ is a known function of the attitude and the angular velocity. The disturbance is represented by $\Delta \in \mathbb{R}^p$ and is an unknown, but fixed uncertain parameter. In addition, we assume that a bound on $W(R,\Omega)$ and Δ is known and given by

$$\|W\| \leq B_W, \quad \|\Delta\| \leq B_\Delta, \tag{3}$$

for positive constants B_W, B_Δ.

Proposition 1 (Attitude Error Function)**:** Define an attitude error function $\Psi : \mathsf{SO}(3) \to \mathbb{R}$, an attitude error vector $e_R \in \mathbb{R}^3$, and an angular velocity error vector $e_\Omega \in \mathbb{R}^3$ as follows:

$$\Psi(R) = A(R)B(R), \tag{4}$$

$$e_R = e_{R_A} B(R) + A(R) e_{R_B}, \tag{5}$$

$$e_\Omega = \Omega, \tag{6}$$

with

$$A(R) = \frac{1}{2}\operatorname{tr}\!\left[G\left(I - R_d^T R\right)\right], \tag{7}$$

$$B(R) = 1 - \frac{1}{\alpha}\ln\left(\frac{\cos\theta - r^T R^T v}{1 + \cos\theta}\right). \tag{8}$$

$$e_{R_A} = \frac{1}{2}\left(G R_d^T R - R^T R_d G\right)^\vee, \tag{9}$$

$$e_{R_B} = \frac{\left(R^T v\right)^\wedge r}{\alpha\left(r^T R^T v - \cos\theta\right)}. \tag{10}$$

where $\alpha \in \mathbb{R}$ is defined as a positive constant and the matrix $G \in \mathbb{R}^{3\times 3}$ is defined as a diagonal matrix matrix for distinct, positive constants $g_1, g_2, g_3 \in \mathbb{R}$. Then, the following properties hold

(i) Ψ is positive definite about $R = R_d$ on $\mathsf{SO}(3)$.

(ii) The variation of $A(R)$ with respect to a variation of $\delta R = R\hat{\eta}$ for $\eta \in \mathbb{R}^3$ is given by

$$\mathbf{D}_R A \cdot \delta R = \eta \cdot e_{R_A}. \tag{11}$$

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(iii) The variation of $B(R)$ with respect to a variation of $\delta R = R\hat{\eta}$ for $\eta \in \mathbb{R}^3$ is given by

$$\mathbf{D}_R B \cdot \delta R = \eta \cdot e_{R_B}. \tag{12}$$

(iv) An upper bound of $\|e_{R_A}\|$ is given as:

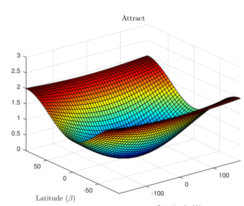
$$\|e_{R_A}\|^2 \leq \frac{A(R)}{b_1}, \tag{13}$$

where the constant b_1 is given by $b_1 = \frac{h_1}{h_2+h_3}$ for

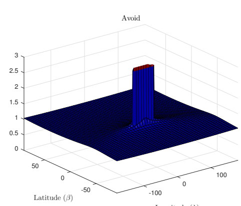
$$h_1 = \min\{g_1+g_2, g_2+g_3, g_3+g_1\},$$
$$h_2 = \min\left\{(g_1-g_2)^2, (g_2-g_3)^2, (g_3-g_1)^2\right\},$$
$$h_3 = \min\left\{(g_1+g_2)^2, (g_2+g_3)^2, (g_3+g_1)^2\right\}.$$

Proof: See Appendix 1. $\square$

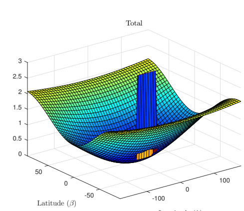
Equation (4) is composed of an attractive term, $A(R)$ toward the desired attitude, and a repulsive term, $B(R)$ away from the undesired direction $R^T v$. In order to visualize the attitude error function on $\mathsf{SO}(3)$, we utilize a spherical coordinate representation. Recall, that the spherical coordinate system represents the position of a point relative to an origin in terms of a radial distance, azimuth, and elevation. This coordinate system is commonly used to define locations on the Earth in terms of a latitude and longitude. Similarly, the positions of celestial objects are defined on the celestial sphere in terms of right ascension and declination.



(a) Attractive $A(R)$



(b) Repulsive $B(R)$



(c) Configuration Ψ

Fig. 1. Configuration error function visualization

2.2. Attitude Control without Disturbance

2.3. Adaptive Control

3. CONCLUSIONS

Awesome conclusion

APPENDIX A

1. Proof of Proposition 1

To prove (i), we note that (7) is a positive definite function about $R = R_d$ [4]. The constraint angle is assumed $0^\circ \leq \theta \leq 90^\circ$ such that $0 \leq \cos\theta$. The term $r^T R^T v$ represents the cosine of the angle between the body fixed vector r and the inertial vector v. It follows that

$$0 \leq \frac{\cos\theta - r^T R^T v}{1+\cos\theta} \leq 1,$$

for all $R \in \mathsf{SO}(3)$. As a result, its negative logarithm is always positive and from (8), $1 < B$. The error function $\Psi = AB$ is composed of two positive terms and is therefore also positive definite.

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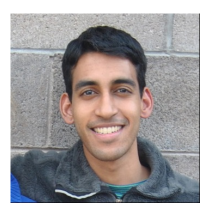

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