Newton's Method

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Why?

- Frequently you'll encounter transcendental equations - cannot be solved analytically
- However, you can still get a numerical solution using an iterative method
- One common problem in astro - solving Kepler's problem

$$\nu_i \rightarrow E_i \rightarrow M_i \rightarrow M_f \rightarrow E_f \rightarrow \nu_f$$

Iteration

- Problem occurs in going from M_f to E_f

$$M_f \to E_f$$
$$M_f = E_f - e \sin E_f$$

- There's no way to directly solve for E_f given M_f, e
- Instead we'll use Newton's method to find an approximate solution

Newton's Method

- Find the root, or zero, of a function
- Given: $f(x)$
- Want: x_f such that $f(x_f) = 0$
- Solution approach:
  - Guess a value x_0 that's close to the true answer
  - Improve our guess $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
  - Repeat until "close" enough

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Implement for Kepler's Problem

- For our problem

$$f(E) = M - (E - e\sin E)$$
$$f'(E) = -(1 - e\cos E)$$

- So for our problem, our iteration method becomes

$$E_{n+1} = E_n + \frac{M - (E - e\sin E)}{1 - e\cos E}$$

- What is a good initial guess? - Usually $M \approx E$ so using M_f is a good guess
- Make sure you use radians or else the answers will be wrong!
- When do we stop? - look at the difference in E_{n+1} and E_n

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while np.absolute(Enext - Eprev) < 1e-8:
    loop
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Example - Write a function to solve this!

- Given: $M_f = 1.5\,\text{rad}$ and $e = 0.1$
- Find: E_f
- How should we test?