數位影像處理
Digital Image Processing

Chapter 3
Spatial Filtering

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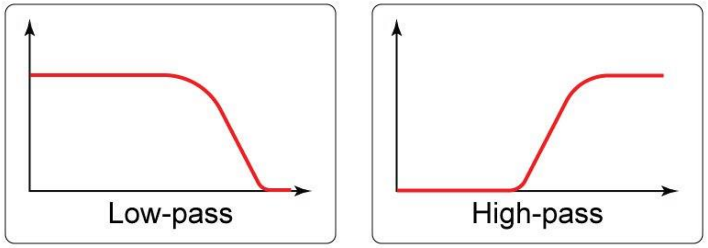
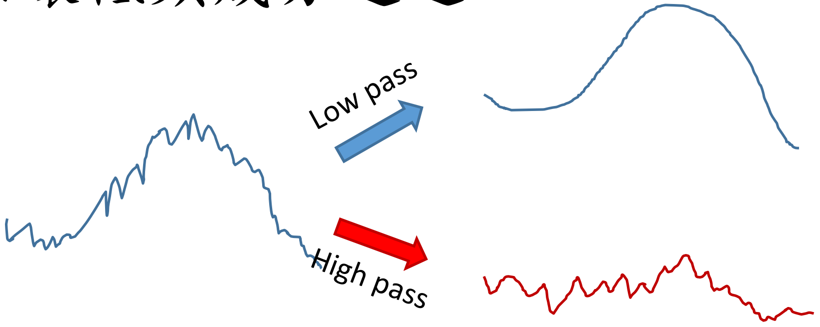
3 空間濾波 Spatial Filtering

- 1. 簡介
- 2. 空間濾波技術
- 3. 空間相關性與迴旋積
- 4. 平滑空間濾波器

1. 簡介

- 濾波器(filter)一詞是從頻域(frequency domain)發展而來。
- 濾波：指接受(通過)或拒絕某些頻率成分
  - 高通濾波器(highpass filter)只讓高頻成分通過
  - 低通濾波器(lowpass filter)只讓低頻成分通過





- 空間濾波器(spatial filter)：亦稱遮罩(mask)、核心(kernel)、模板(template)、視窗(window)…

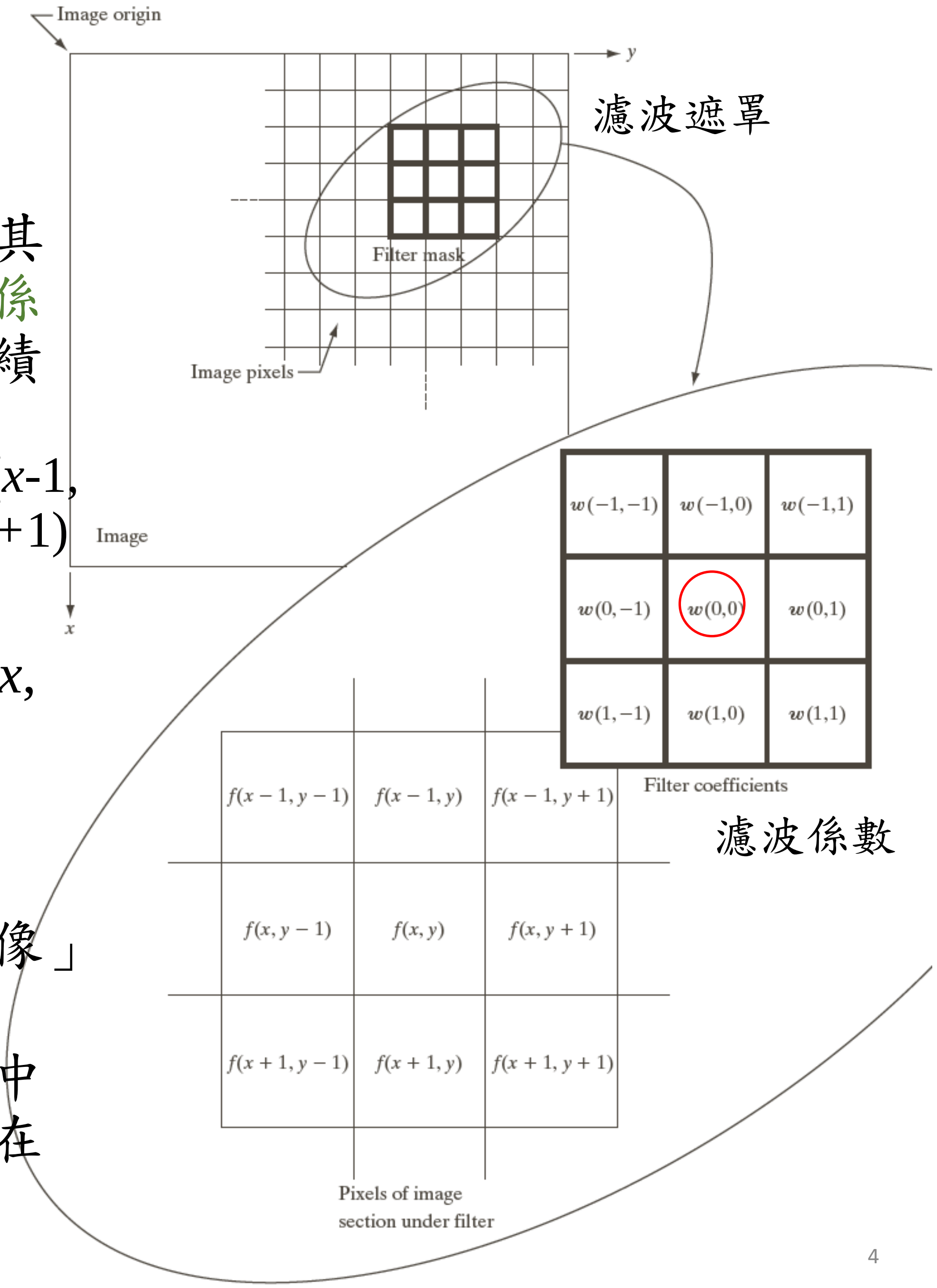
2. 空間濾波技術

- 濾波處理：影像中的一點(x, y)，其濾波響應(response) $g(x, y)$是濾波係數與濾波器所圍繞之影像像素乘積的總和:
  $g(x, y) = w(-1,-1)\, f(x-1, y-1)+ w(-1,0)\, f(x-1, y)+\ldots +w(0,0)\, f(x, y)+ w(1, 1)\, f(x+1, y+1)$

- 濾波器的中央係數$w(0, 0)$與像素(x, y)對齊。

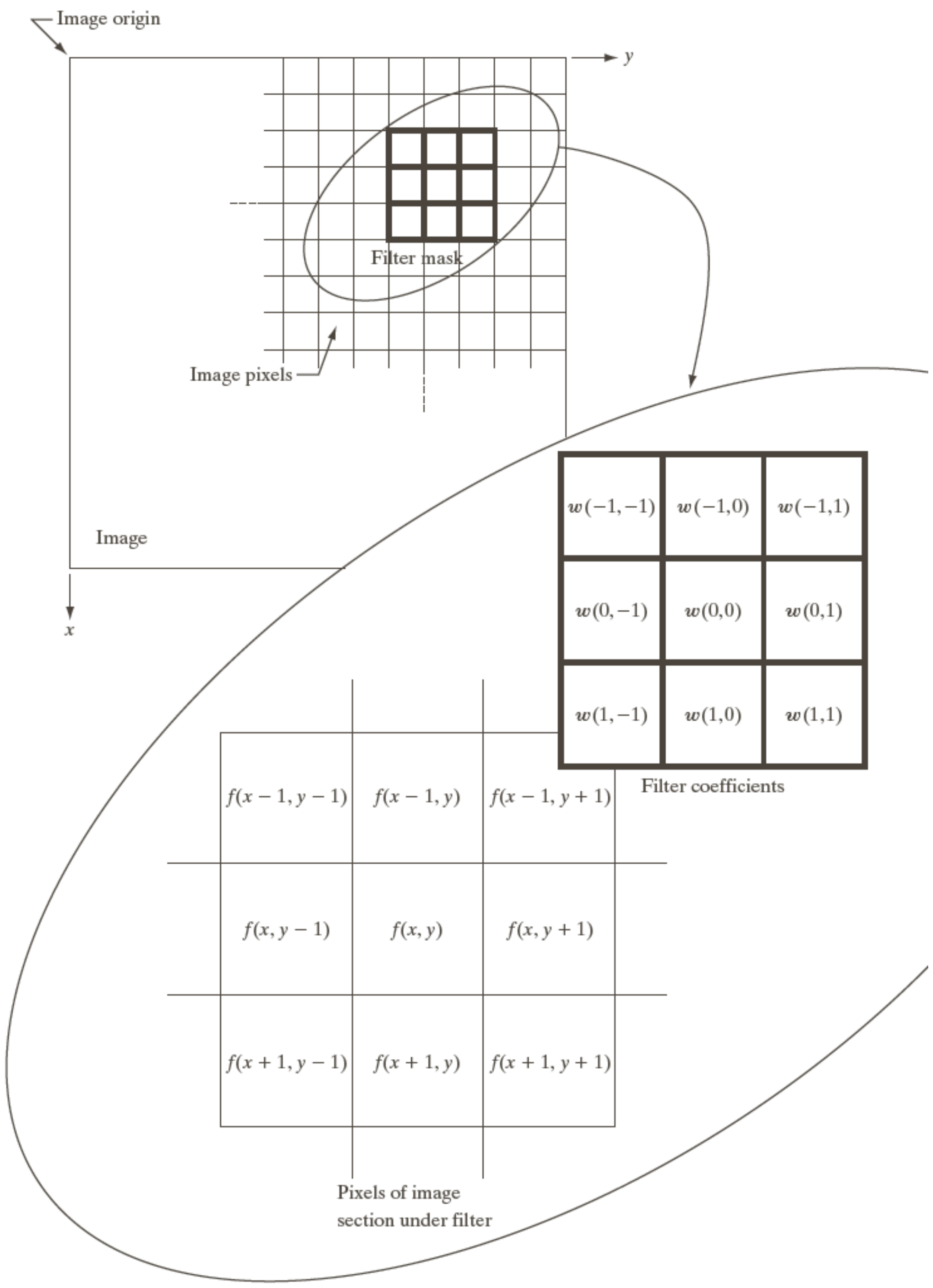
注意

- 濾波計算過程需有一張「新的影像」儲存結果。
- 很少把濾波後的像素取代原影像中對應位置上的值，因為當濾波還在執行時，這會改變影像的內容。



3. 空間相關性與迴旋積

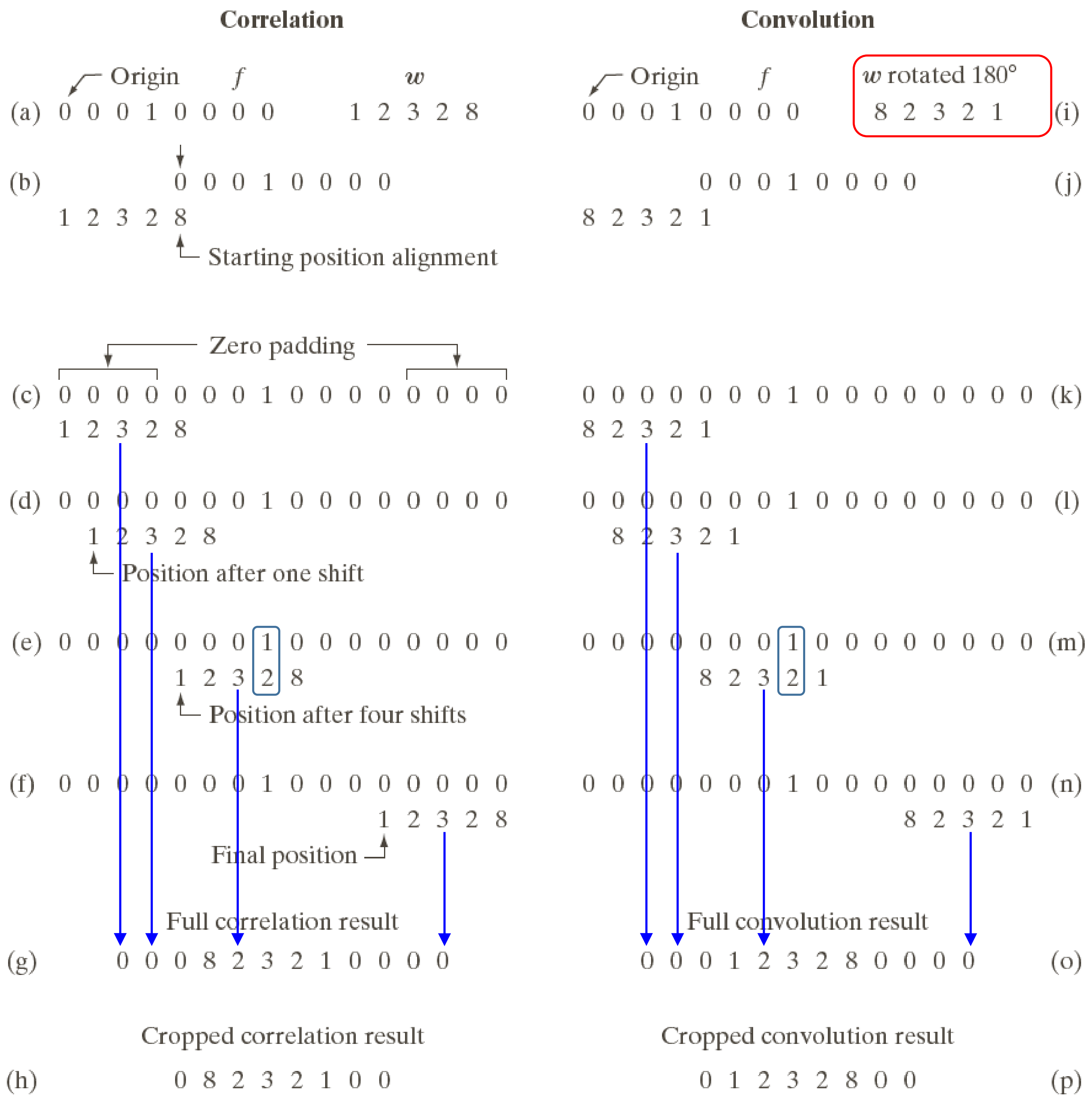
- 相關性(correlation)：如上頁之運算，把濾波遮罩在影像上移動，並在每個位置上計算乘積的和。
- $g(x, y) = w(-1,-1) f(x-1, y-1)$
  $+ w(-1,0) f(x-1, y)+\ldots$
  $+w(0,0) f(x, y)$
  $+w(1, 1) f(x+1, y+1)$
- 迴旋積(convolution)：計算方式相同，但濾波器要先旋轉180度。



一維範例

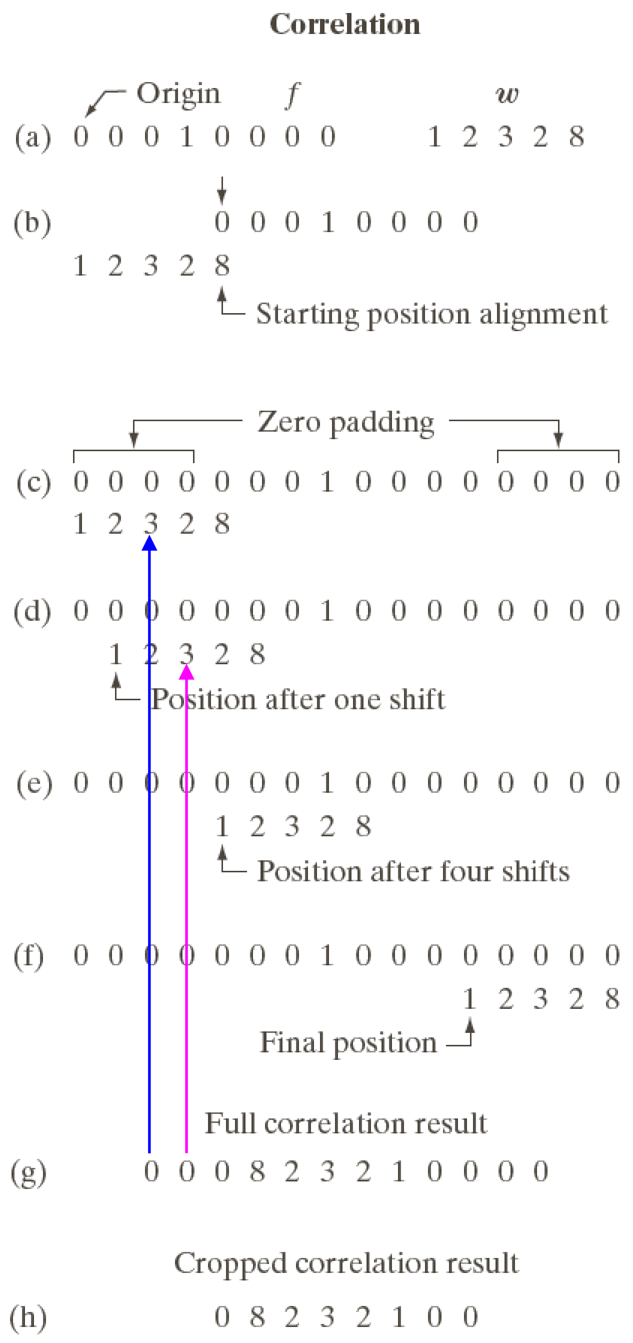
f: function

w: filter


Correlation
Convolution
Origin
f
w
(a) 0 0 0 1 0 0 0 0 1 2 3 2 8
w rotated 180°
0 0 0 1 0 0 0 0 8 2 3 2 1 (i)
(b) 0 0 0 1 0 0 0 0
1 2 3 2 8
Starting position alignment
0 0 0 1 0 0 0 0 (j)
8 2 3 2 1
Zero padding
(c) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (k)
8 2 3 2 1
(d) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after one shift
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (l)
8 2 3 2 1
(e) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after four shifts
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (m)
8 2 3 2 1
(f) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Final position
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (n)
8 2 3 2 1
Full correlation result
(g) 0 0 0 8 2 3 2 1 0 0 0 0
Full convolution result
0 0 0 1 2 3 2 8 0 0 0 0 (o)
Cropped correlation result
(h) 0 8 2 3 2 1 0 0
Cropped convolution result
0 1 2 3 2 8 0 0 (p)

3. 空間相關性與迴旋積

- 相關性是濾波器位移(displacement)的函數
  - 相關性第一個值對應濾波器的零位移
  - 第二個值對應到一個單位的位移
- 一個濾波器*w*與一個有單一個1，其它為0的函數 (稱「離散單位脈衝」discrete unit impulse)進行相關性運算後，得到一旋轉180度的*w*。
- 若將濾波器先作旋轉，則能在脈衝處產生與原函數一樣的訊號→迴旋積

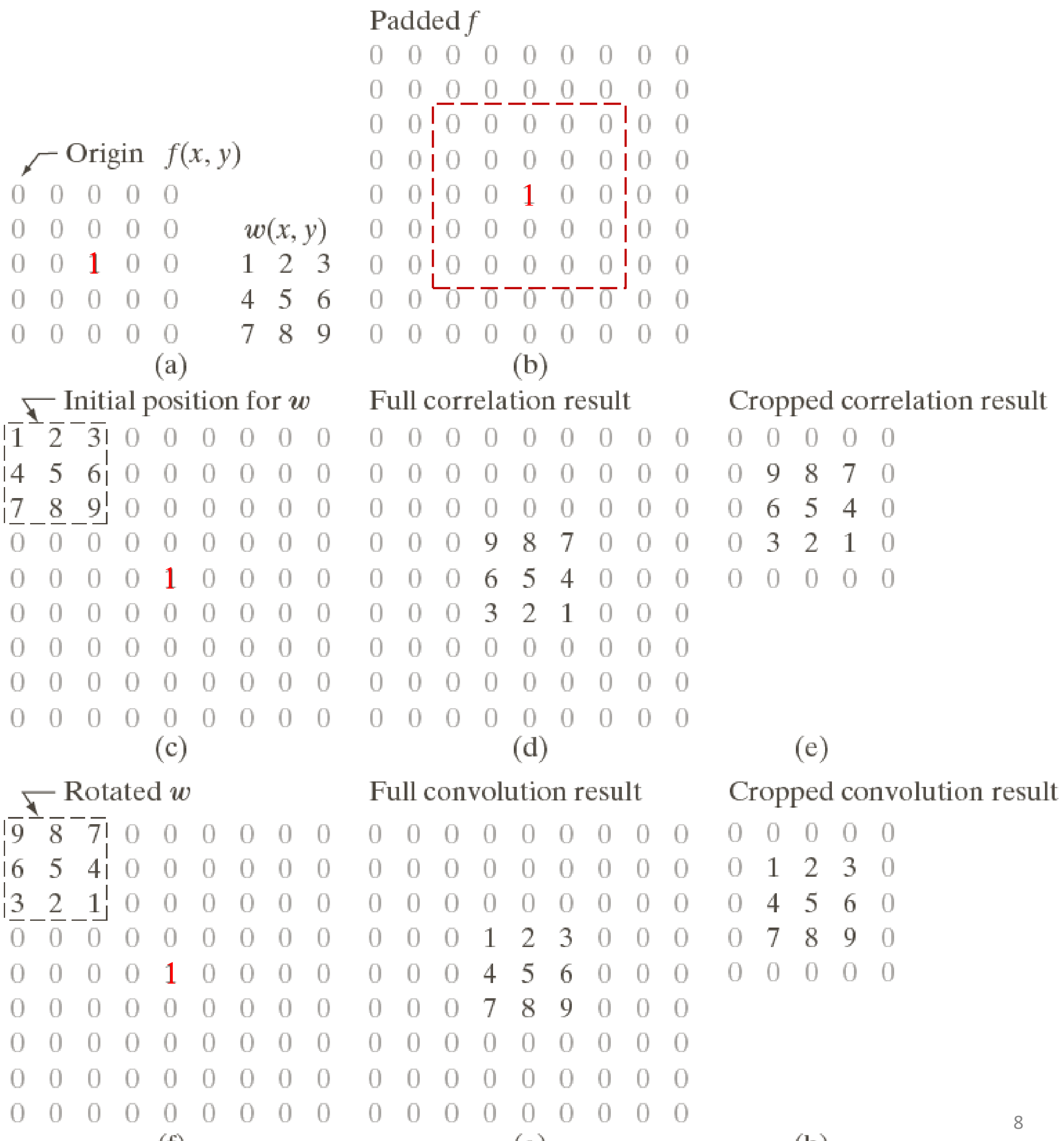


Correlation
Origin
f
w
(a) 0 0 0 1 0 0 0 0   1 2 3 2 8
(b) 0 0 0 1 0 0 0 0
1 2 3 2 8
Starting position alignment
Zero padding
(c) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
(d) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after one shift
(e) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after four shifts
(f) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Final position
Full correlation result
(g) 0 0 0 8 2 3 2 1 0 0 0 0
Cropped correlation result
(h) 0 8 2 3 2 1 0 0

二維範例

f: function

w: filter



Origin $f(x, y)$

$$\begin{matrix}0&0&0&0&0\\0&0&0&0&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&0&0\end{matrix}$$

$w(x, y)$

$$\begin{matrix}1&2&3\\4&5&6\\7&8&9\end{matrix}$$

(a)

Padded f

$$\begin{matrix}0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{matrix}$$

(b)

Initial position for w

$$\begin{matrix}1&2&3&0&0&0&0&0&0\\4&5&6&0&0&0&0&0&0\\7&8&9&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{matrix}$$

(c)

Full correlation result

$$\begin{matrix}0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&9&8&7&0&0&0\\0&0&0&6&5&4&0&0&0\\0&0&0&3&2&1&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{matrix}$$

(d)

Cropped correlation result

$$\begin{matrix}0&0&0&0&0\\0&9&8&7&0\\0&6&5&4&0\\0&3&2&1&0\\0&0&0&0&0\end{matrix}$$

(e)

Rotated w

$$\begin{matrix}9&8&7&0&0&0&0&0&0\\6&5&4&0&0&0&0&0&0\\3&2&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{matrix}$$

(f)

Full convolution result

$$\begin{matrix}0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&1&2&3&0&0&0\\0&0&0&4&5&6&0&0&0\\0&0&0&7&8&9&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{matrix}$$

(g)

Cropped convolution result

$$\begin{matrix}0&0&0&0&0\\0&1&2&3&0\\0&4&5&6&0\\0&7&8&9&0\\0&0&0&0&0\end{matrix}$$

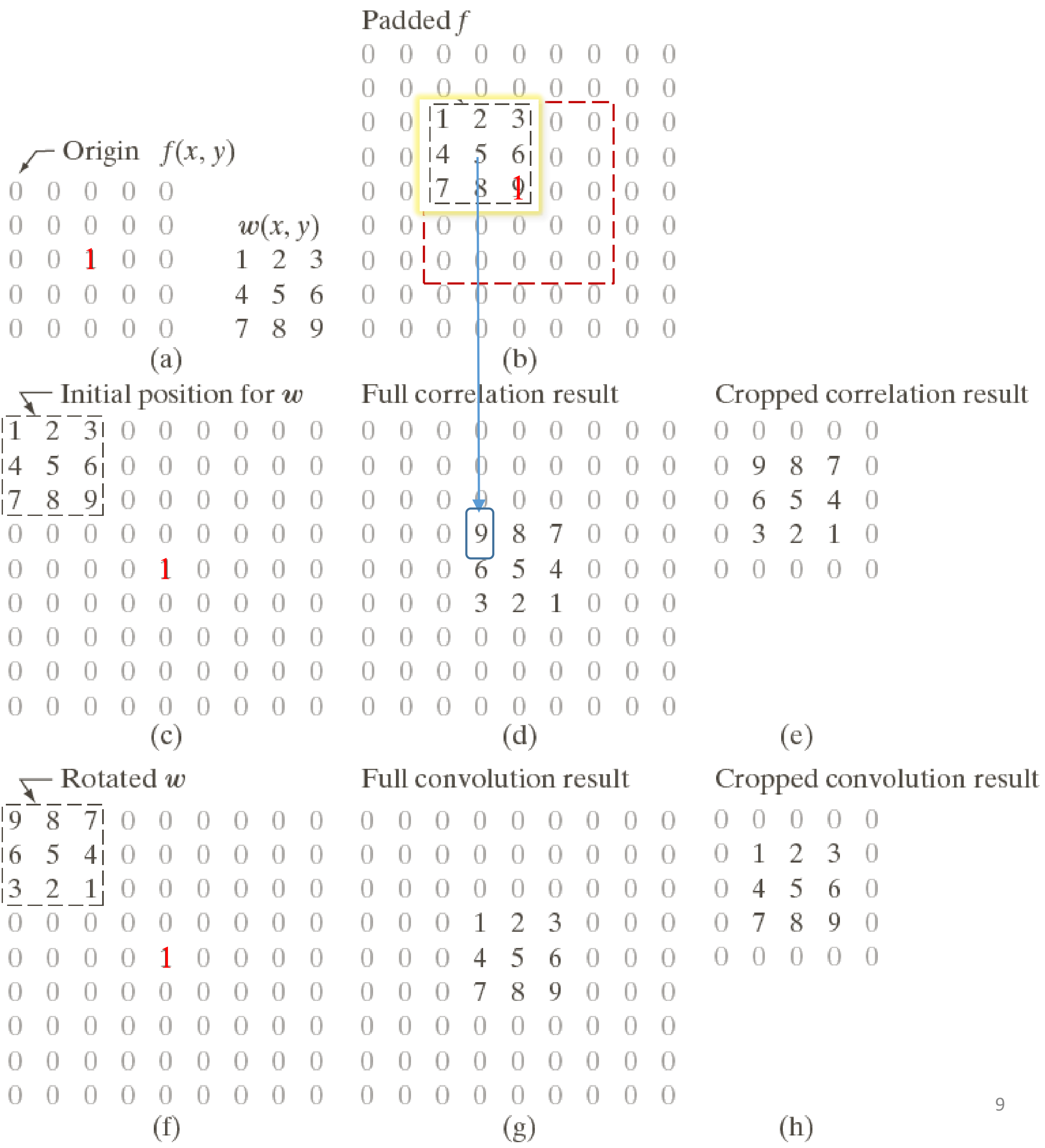
(h)

二維範例

f: function

w: filter

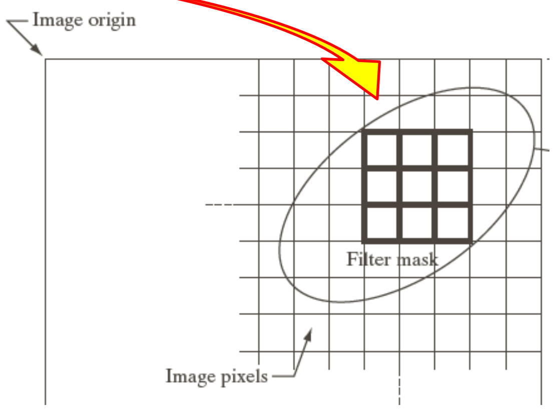
- 若濾波遮罩是對稱的，則相關性與迴旋積會有相同結果。
- 在一些文獻中可能會提到「將影像與遮罩作迴旋積」，但可能指的是前述「滑動+求積和」的過程。



4. 平滑空間濾波器

- 平滑(smoothing)濾波器用於模糊化與減少雜訊
- 平均濾波器(averaging filter)→線性濾波器

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1\\\hline 1 & 1 & 1\\\hline 1 & 1 & 1\\\hline\end{array}\qquad\frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1\\\hline 2 & 4 & 2\\\hline 1 & 2 & 1\\\hline\end{array}$$



正規化，讓總和為1。

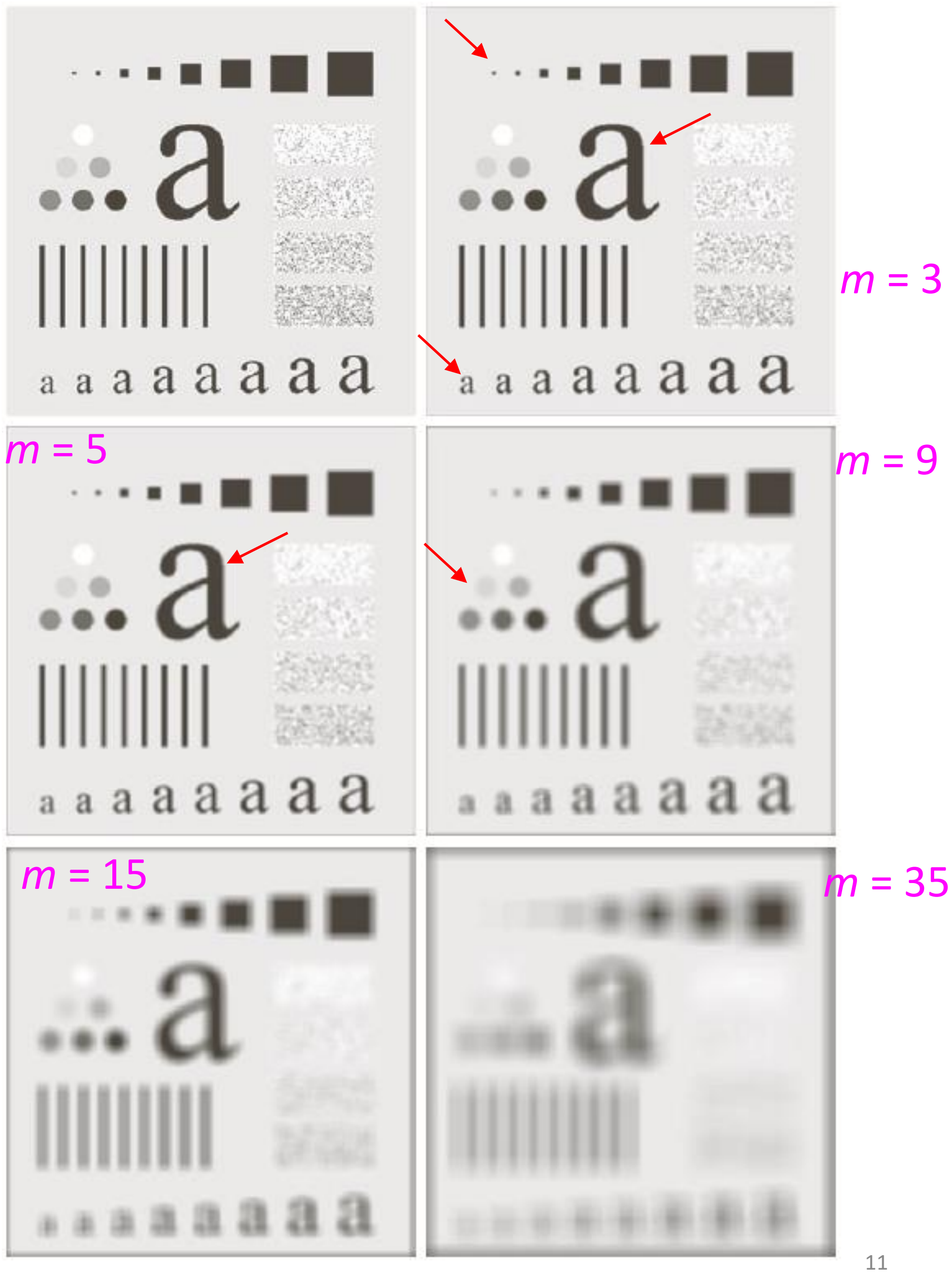
<u>**權重平均**</u>(weighted average)，給某些像素更高的重要性（權重）

4. 平滑空間濾波器

1	1	1
1	1	1
1	1	1

- 原影像：500x500
- 正方形平均濾波器
- 遮罩邊長 m = 3, 5, 9, 15, 35
- 圖上端黑正方形大小： 3, 5, 9, 25, 35, 45, 55。間隔25像素。

- m=3，輕微模糊，與遮罩同樣大小的細節影像較大。
  字元鋸齒邊緣被平滑化。
- m=5，類似，模糊略增。
- m=9，黑圈已不易與背景區分。
- m=15 and 35，小方塊、圓圈、雜訊矩形已混合到背景中。



OpenCV: Image Blur - Average

- Smooth an image by replacing each pixel by the average pixel value computed over a rectangular neighborhood.
- OutputArray **dst** = **blur**(InputArray **src**, Size **ksize**, Point **anchor**=Point(-1,-1), int **borderType**=BORDER_DEFAULT)
  - **src** – input image
  - **ksize** – blurring kernel size.
  - **anchor** – anchor point; default value Point(-1,-1) means that the anchor is at the kernel center.
  - **borderType** – border mode used to extrapolate pixels outside of the image.

➔ result=cv2.blur(image, (3,3));

$$\begin{array}{|ccc|}\hline 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ \hline\end{array}$$

Such a matrix is sometimes called a kernel or a mask.

OpenCV: Image Blur - Average

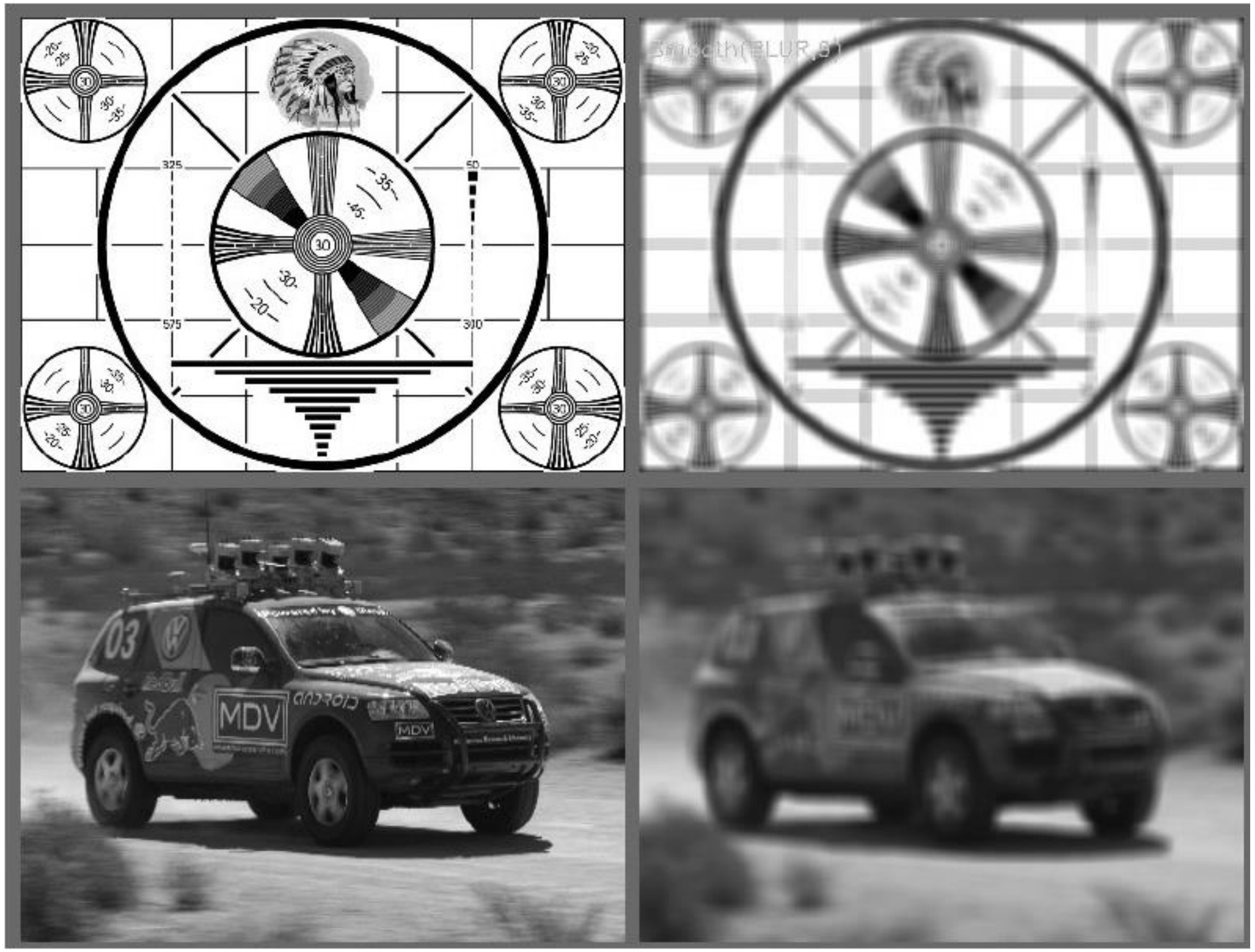
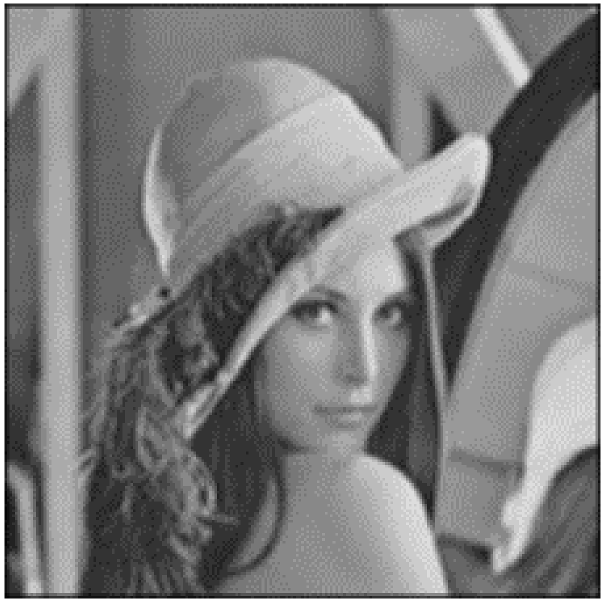


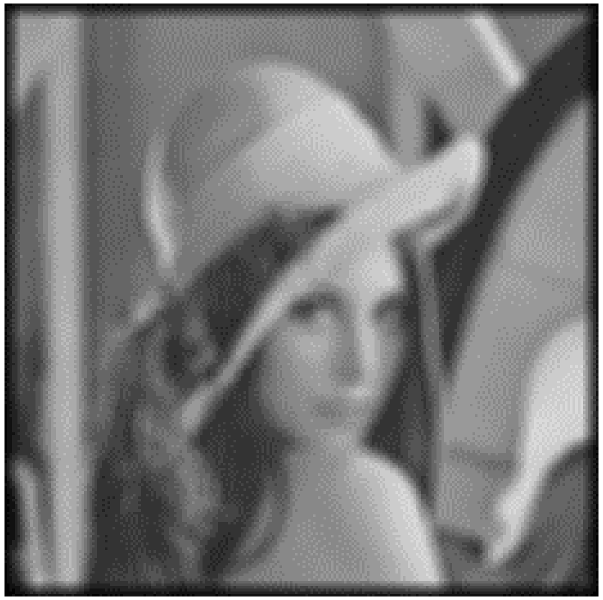
Image smoothing by block averaging

Average Blur

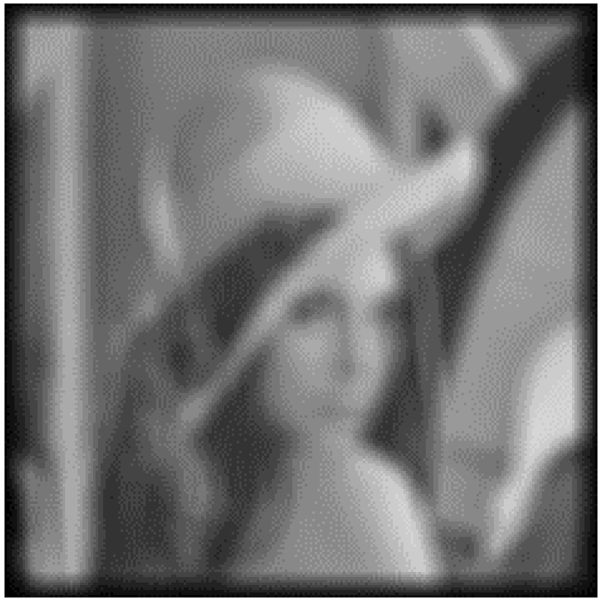
抹平!



3*3



7*7



11*11

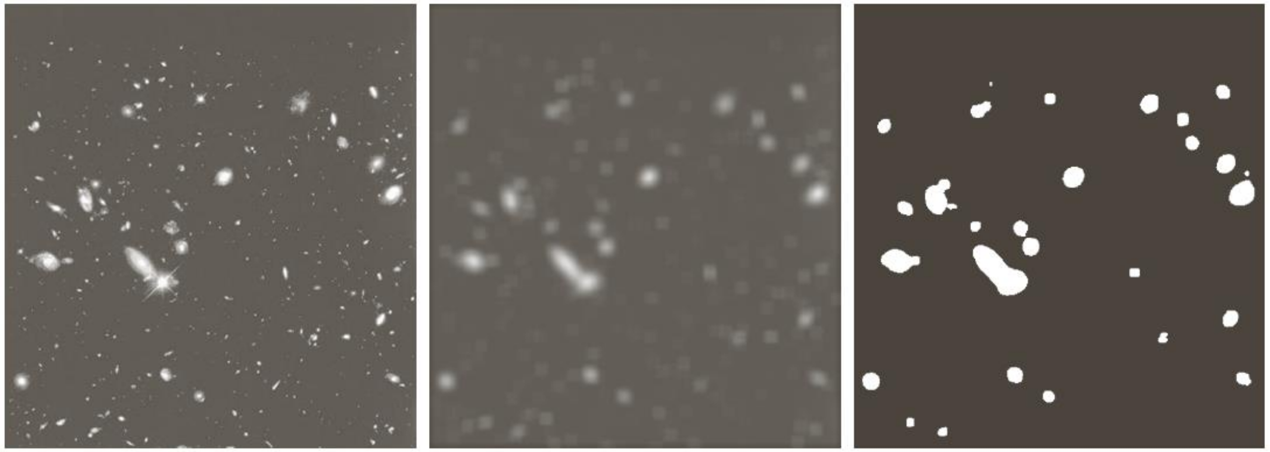
4. 平滑空間濾波器

- 空間平均的一個重要應用：為了獲得感興趣物體的整體表示而將一影像模糊，使較小的物體融入背景，較大物體則呈「斑點狀」，而較易偵測到。

門檻化 (Thresholding)

- $f(x, y)$: 像素值，$g(x, y)$: thresholding後的結果
- T: 門檻值

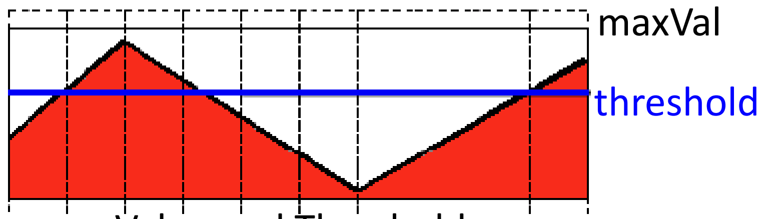
$$g(x,y)=\begin{cases}255, if\ f(x,y)>T\\ 0, if\ f(x,y)\le T\end{cases}\quad or\ g(x,y)=\begin{cases}1, if\ f(x,y)>T\\ 0, if\ f(x,y)\le T\end{cases}$$



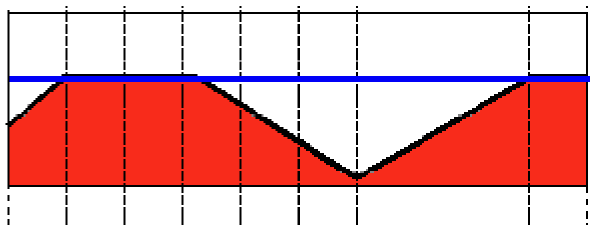
(a)哈伯望遠鏡之太空影像，528x485
(b)用15x15的平均遮罩作濾波
(c) 對(b)做門檻化

OpenCV Threshold Types

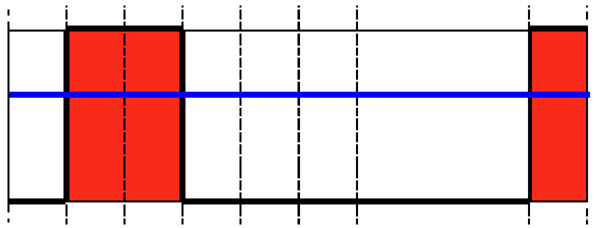
- dst = threshold (src, threshold, max_value, threshold_type);



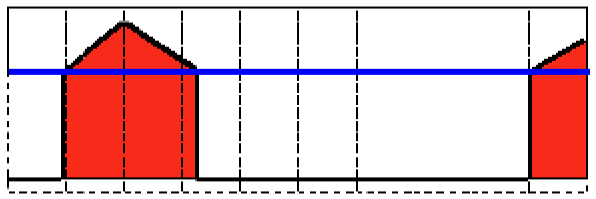
Value and Threshold



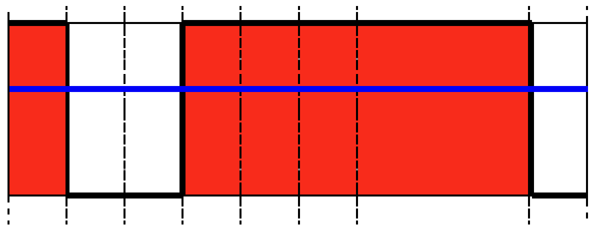
THRESH_TRUNC



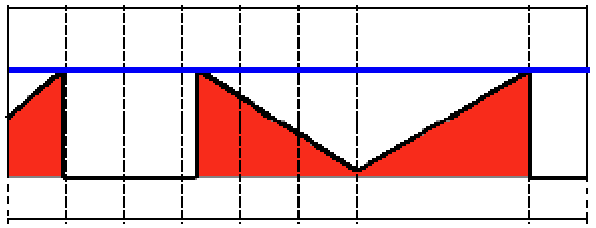
THRESH_BINARY



THRESH_TOZERO



THRESH_BINARY_INV

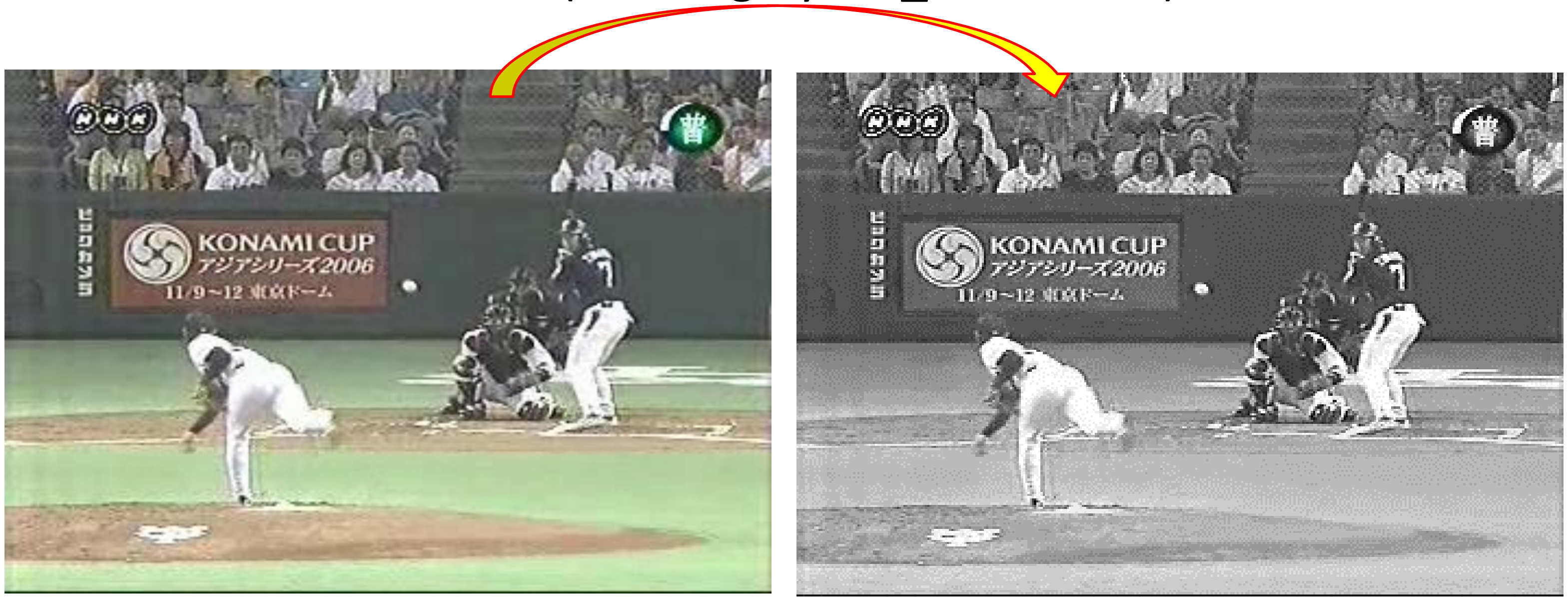


THRESH_TOZERO_INV

Thresholding 的應用

- cvtColor (src, CV_BGR2GRAY);
- threshold (src, threshold, max_value, threshold_type);

cvtColor (frame, gray, CV_BGR2GRAY);

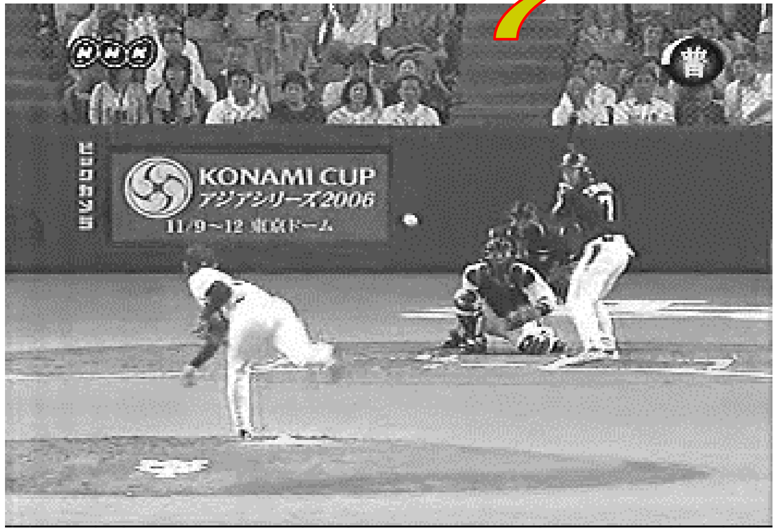


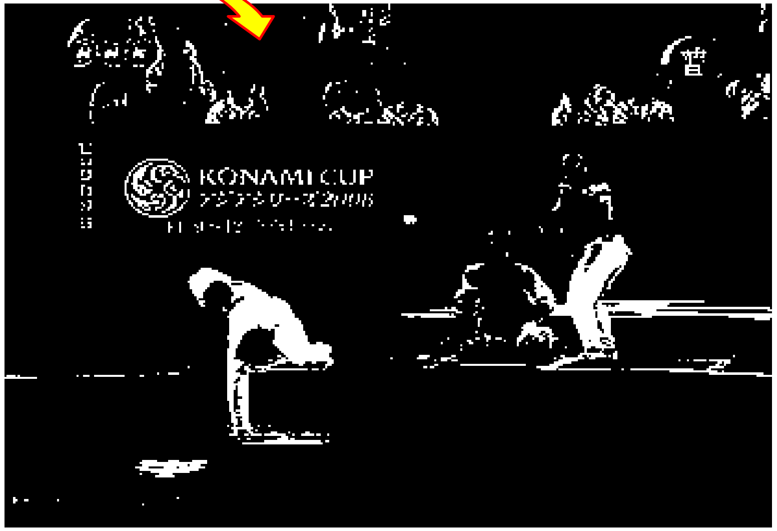
frame (3 channels) gray (1 channel)

Thresholding 的應用

- cvtColor (src, dst, CV_BGR2GRAY);
- threshold (src, dst, threshold, max_value, threshold_type);

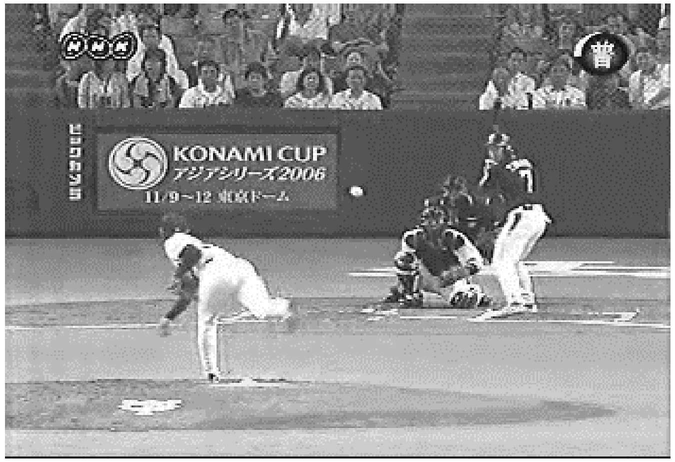
threshold (gray, white, 200, 255, **THRESH_BINARY**);

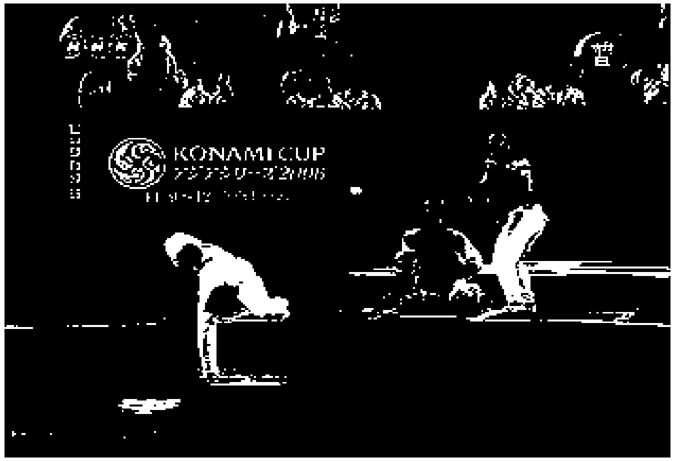

gray


White (threshold = 200)

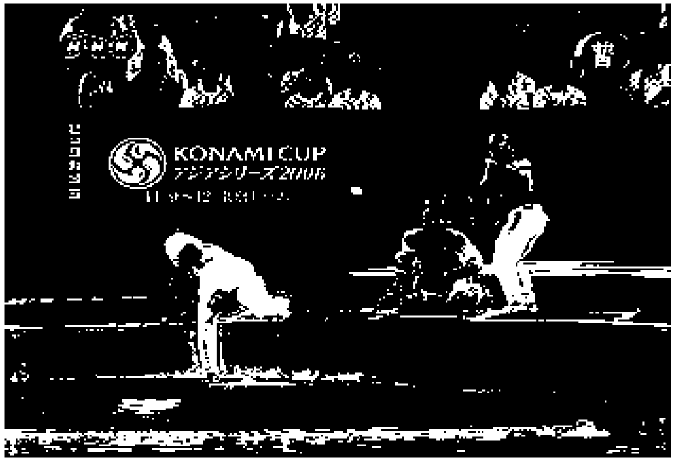
Thresholding 的應用

- Different thresholds



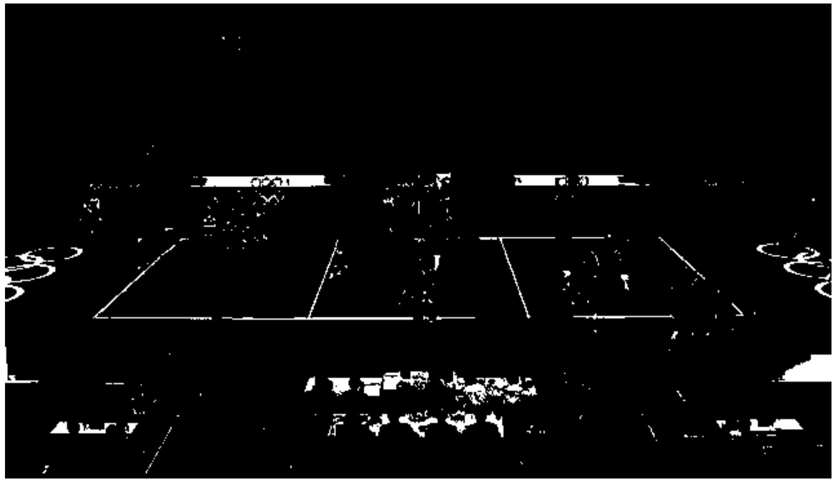


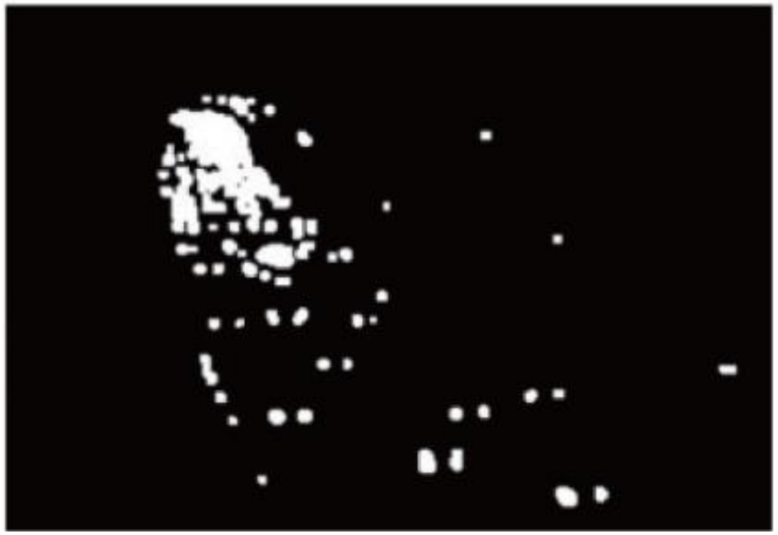
(threshold = 200)



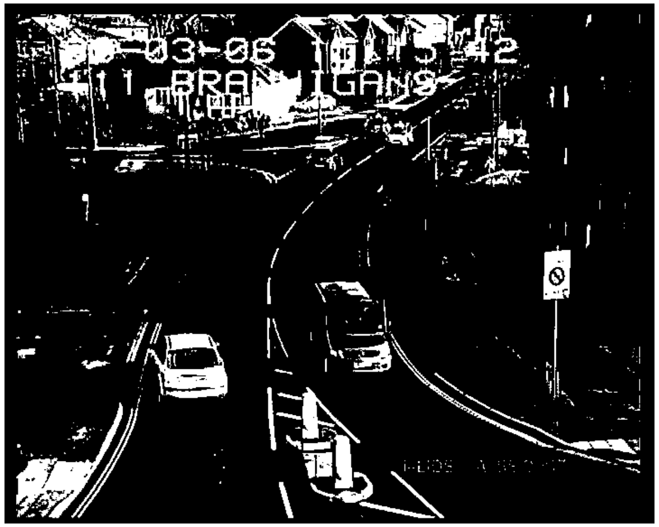
(threshold = 180)

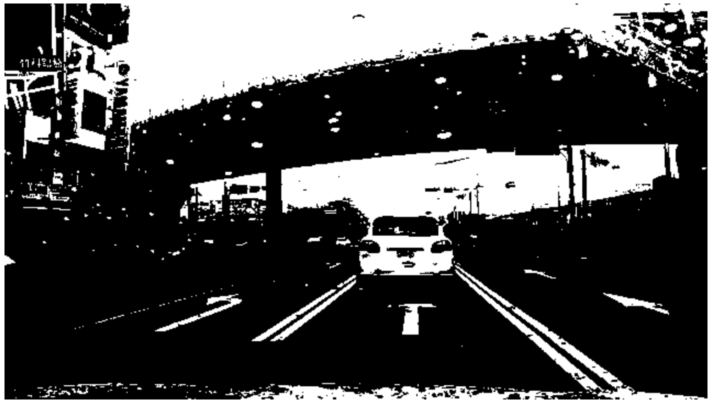
Thresholding 的應用





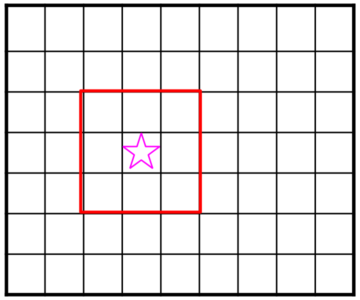
Thresholding 的應用





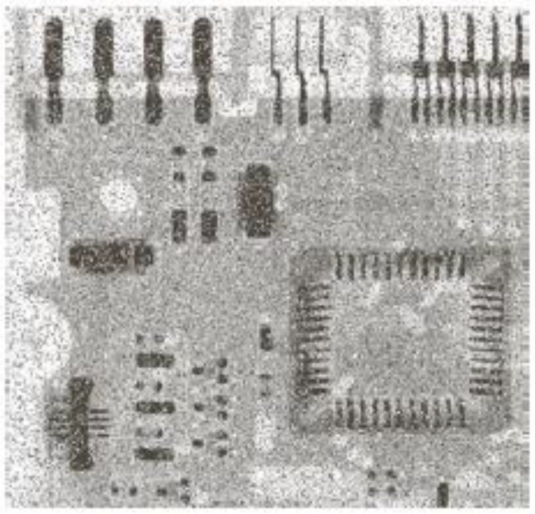
4. 平滑空間濾波器

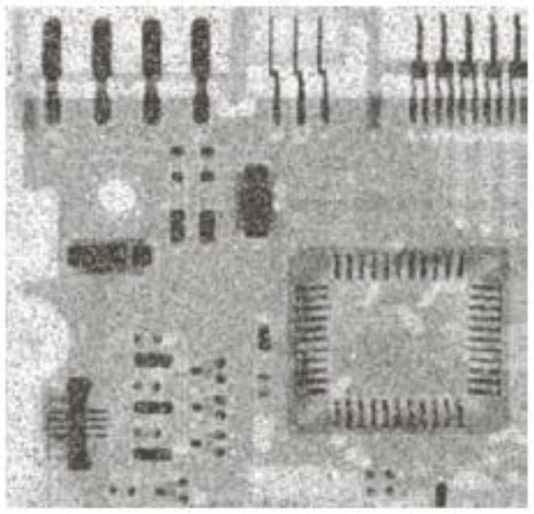
23, 23, 24, 25 ,25, 27, 27, 30, 235
Median = 25
Avg = 48.8



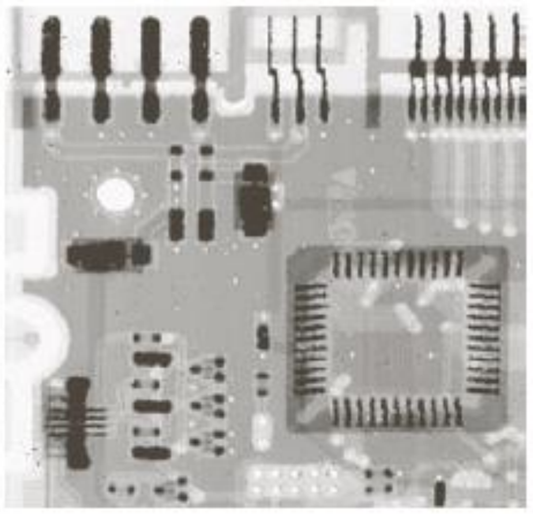
非線性濾波器

- 以中值濾波器(median filter)最常用
- 在一遮罩內，取中間值當作結果。
- 有效去除脈衝雜訊(impose noise)，因為在影像上脈衝雜訊看起來有黑點與白點，又稱椒鹽雜訊(salt-and-pepper noise)





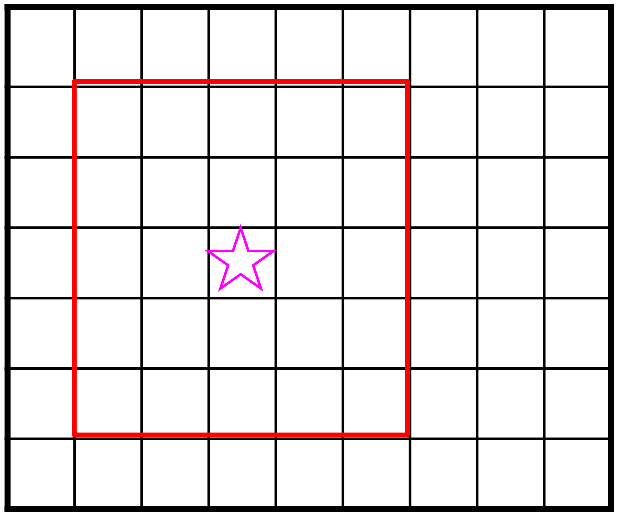
3x3 average filter



3x3 median filter

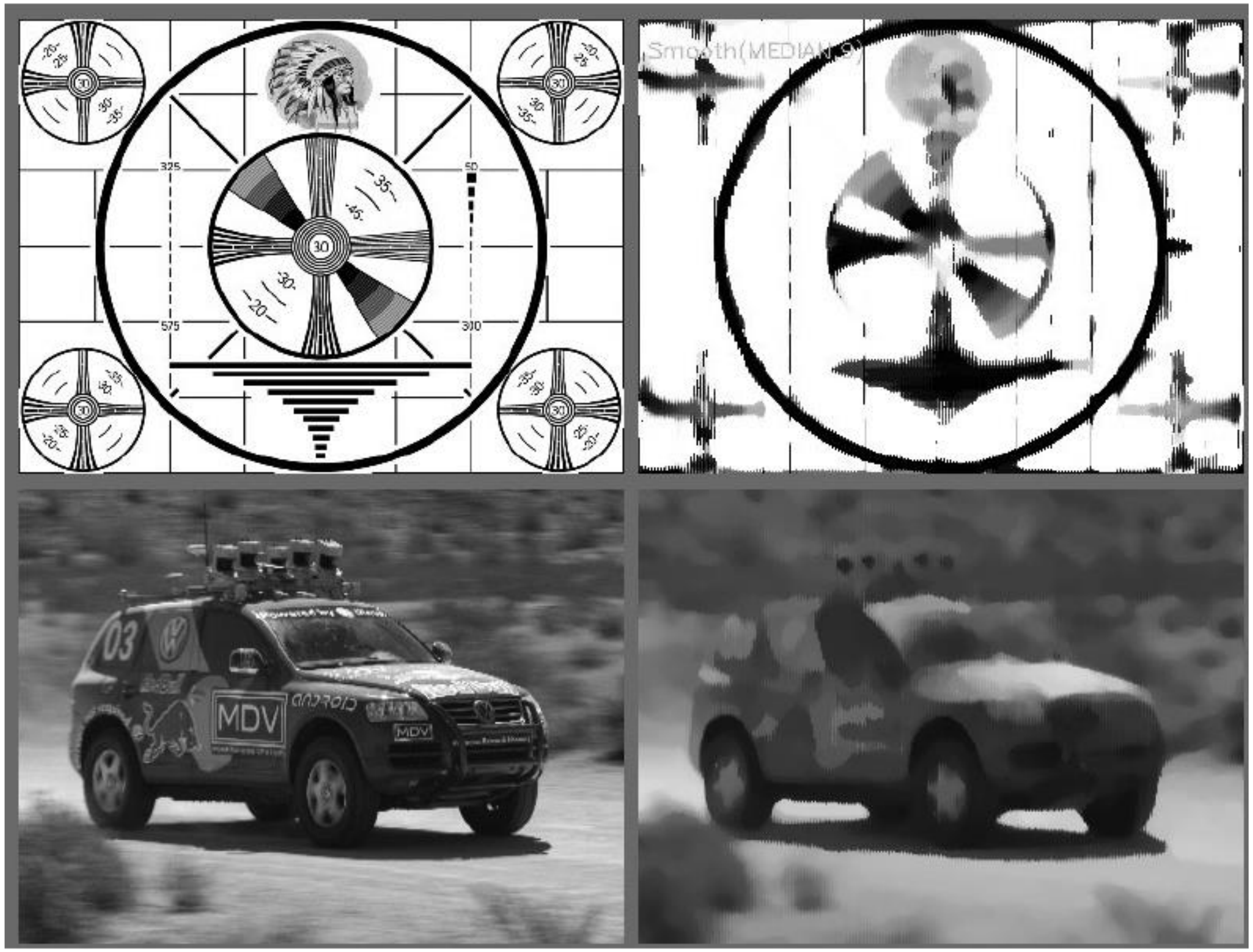
Image Blur - Median

- The median filter replaces each pixel by the median or "middle" pixel (as opposed to the mean pixel) value in a square neighborhood around the center pixel.
- OutputArray **dst medianBlur**(InputArray **src**, int **ksize**)
  - **src** – input image.
  - **dst** – output image of the same size and type as src.
  - **ksize** – aperture linear size; it must be odd and greater than 1, for example: 3, 5, 7 ...
    It is **int** → **square kernal**



→result = cv2.medianBlur(image, result, 5);

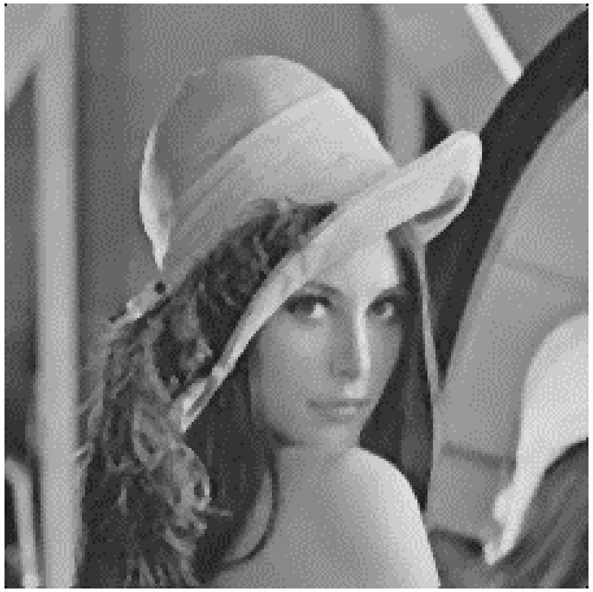
Image Blur - Median

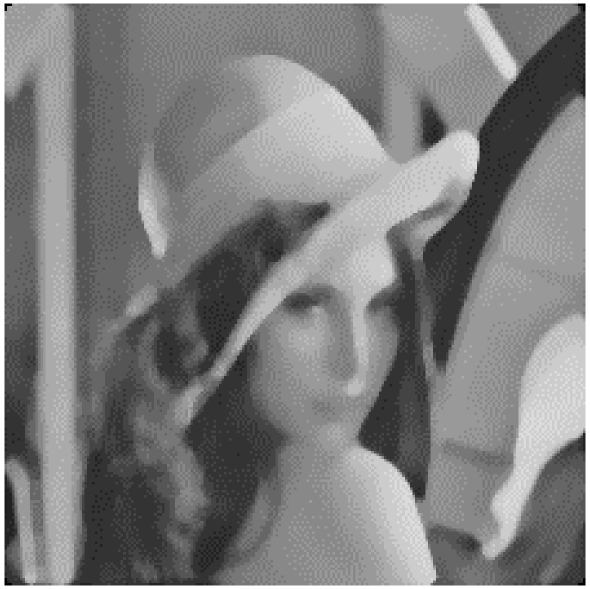


Median Blur

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 2 & 3 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \Rightarrow \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & \mathbf{1} & 3 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

挑出中位數 = 1


3*3


7*7

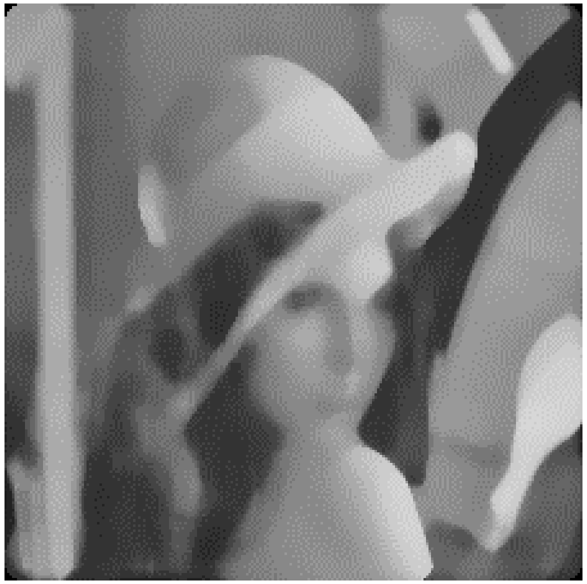
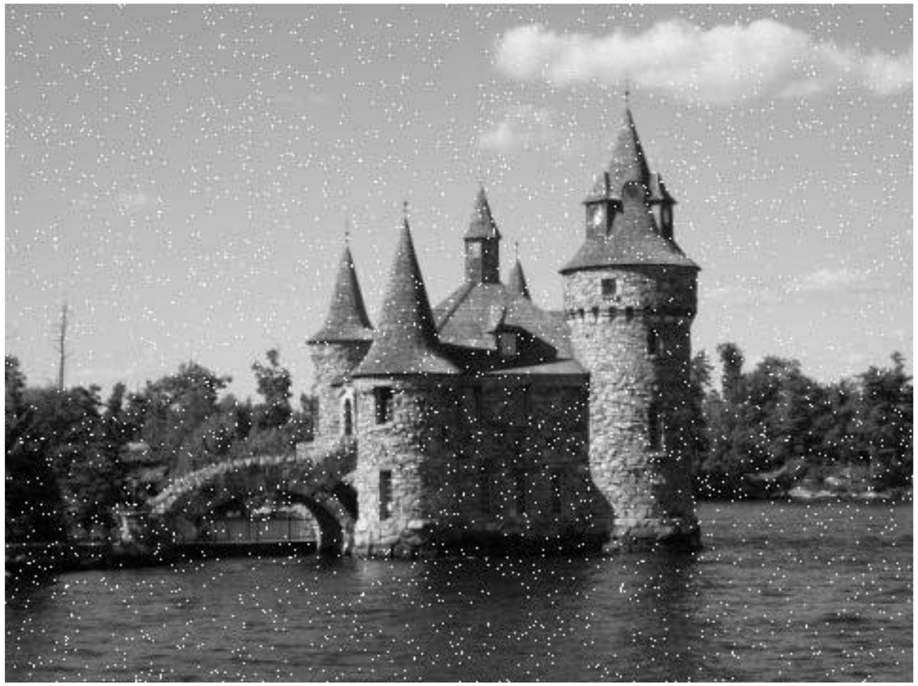
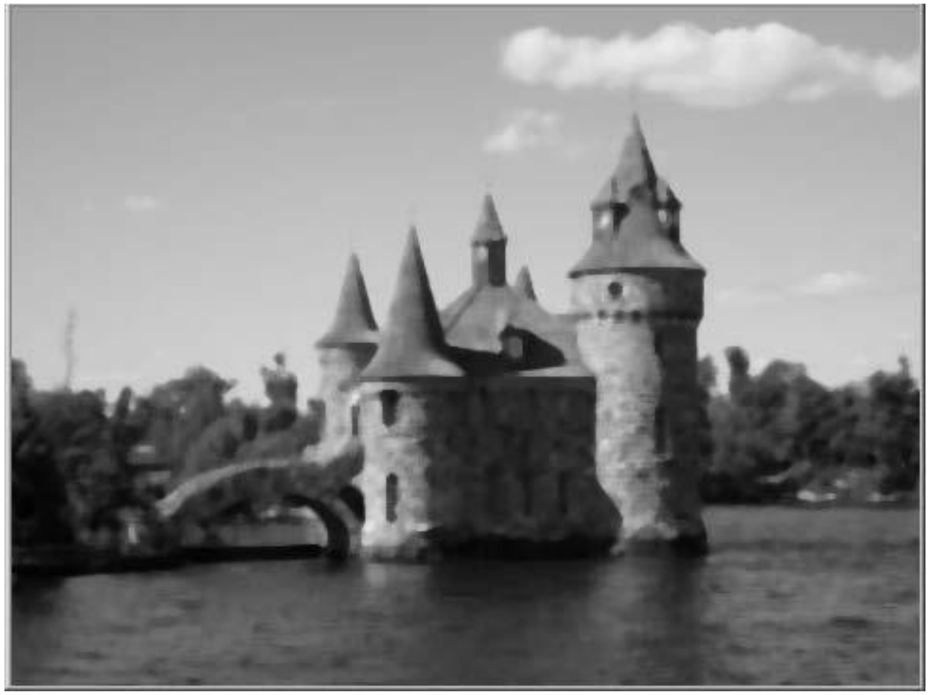

11*11

Image Blur - Median

- Simple blurring by averaging can be sensitive to image noise, especially large isolated outlier points.
- Median filtering is able to ignore the outliers by selecting the middle points.
  - particularly useful to combat salt-and-pepper noise



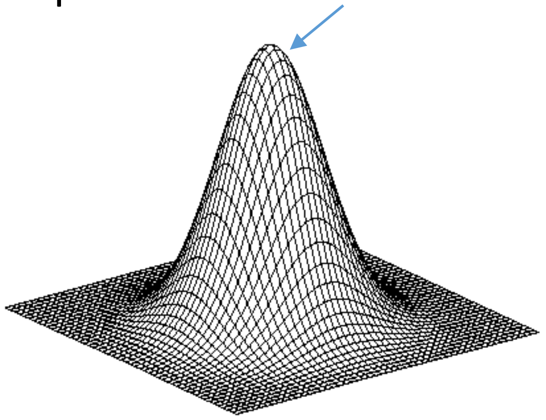


OpenCV Functions

Image Blur – Gaussian filter

- Gaussian filtering is probably the most useful.
  - convolve each point in the input array with a Gaussian kernel and then summing to produce the output array.

The pixels closer to the center are weighted more.



$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

$\frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

5x5 kernel, $\sigma_x = \sigma_y = 1$

Recall → 4. 平滑空間濾波器

- 平滑(smoothing)濾波器用於模糊化與減少雜訊
- 平均濾波器(averaging filter)→線性濾波器

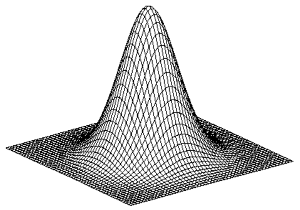
$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array} \qquad \frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

正規化，讓總和為1。

權重平均(weighted average)，
給某些像素更高的重要性（權重）

Image Blur – Gaussian filter

- void GaussianBlur(InputArray **src**, OutputArray **dst**, Size **ksize**, double **sigmaX**, double **sigmaY**=0, int **borderType**=BORDER_DEFAULT)
  - **ksize** – Gaussian kernel size.
    - ksize.width and ksize.height can differ but they both must be positive and odd.
    - Or, they can be zero's and then they are computed from sigma* .
  - **sigmaX** – Gaussian kernel standard deviation in X direction.
  - **sigmaY** – Gaussian kernel standard deviation in Y direction.
    - if sigmaY is zero, it is set to be equal to sigmaX
    - if both sigmas are zeros, they are computed from ksize.width and ksize.height, respectively.



$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

Image Blur – Gaussian filter

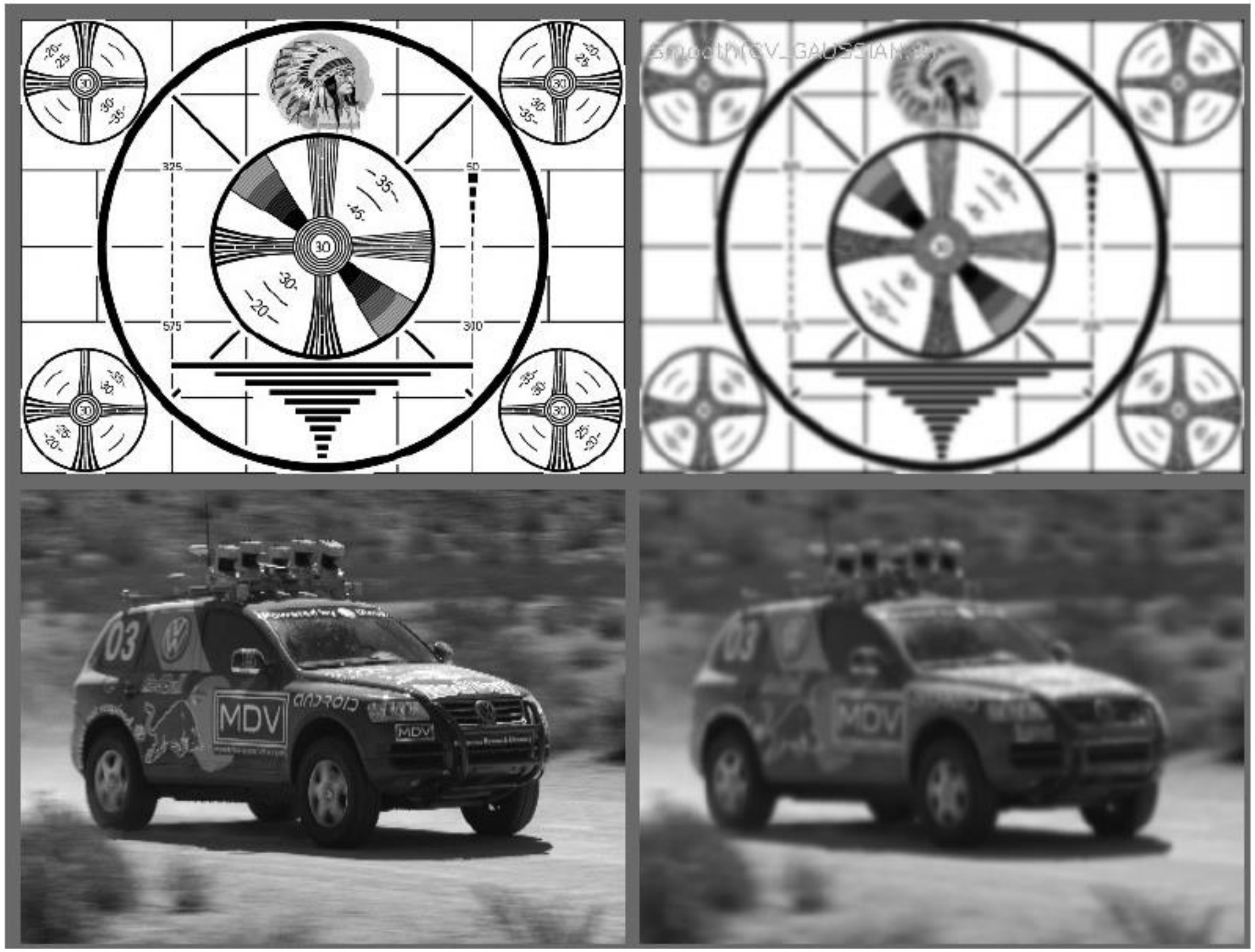
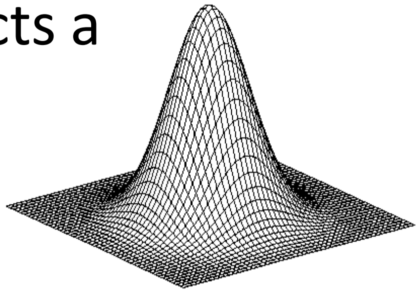


Image Blur – Bilateral filter

- Bilateral filtering →edge-preserving smoothing.
- A typical motivation for Gaussian smoothing:
  - pixels in a real image should vary slowly over space and thus be correlated to their neighbors
  - random noise can be expected to vary greatly from one pixel to the next (i.e., noise is spatially uncorrelated).
- It is in this sense that Gaussian smoothing reduces noise while preserving signal.
- Unfortunately, this method breaks down near edges, where you do expect pixels to be uncorrelated with their neighbors.

  →Gaussian smoothing smoothes away the edges.

Image Blur – Bilateral filter



- Like Gaussian smoothing, bilateral filtering constructs a weighted average of each pixel and its neighboring components.
- The weighting has two components:
  - the first is the same weighting used by Gaussian smoothing, based on the spatial distance from the center pixel
  - The second is also a Gaussian weighting but is based on the difference in intensity from the center pixel.
- You can think of bilateral filtering as Gaussian smoothing that weights more similar pixels more highly than less similar ones.
- The effect of this filter is typically to turn an image into what appears to be a watercolor painting of the same scene.
- This can be useful as an aid to segmenting the image.

http://www.dai.ed.ac.uk/CVonline/LOCAL_COPIES/MANDUCHI1/Bilateral_Filtering.html

Comparison – Gaussian filter

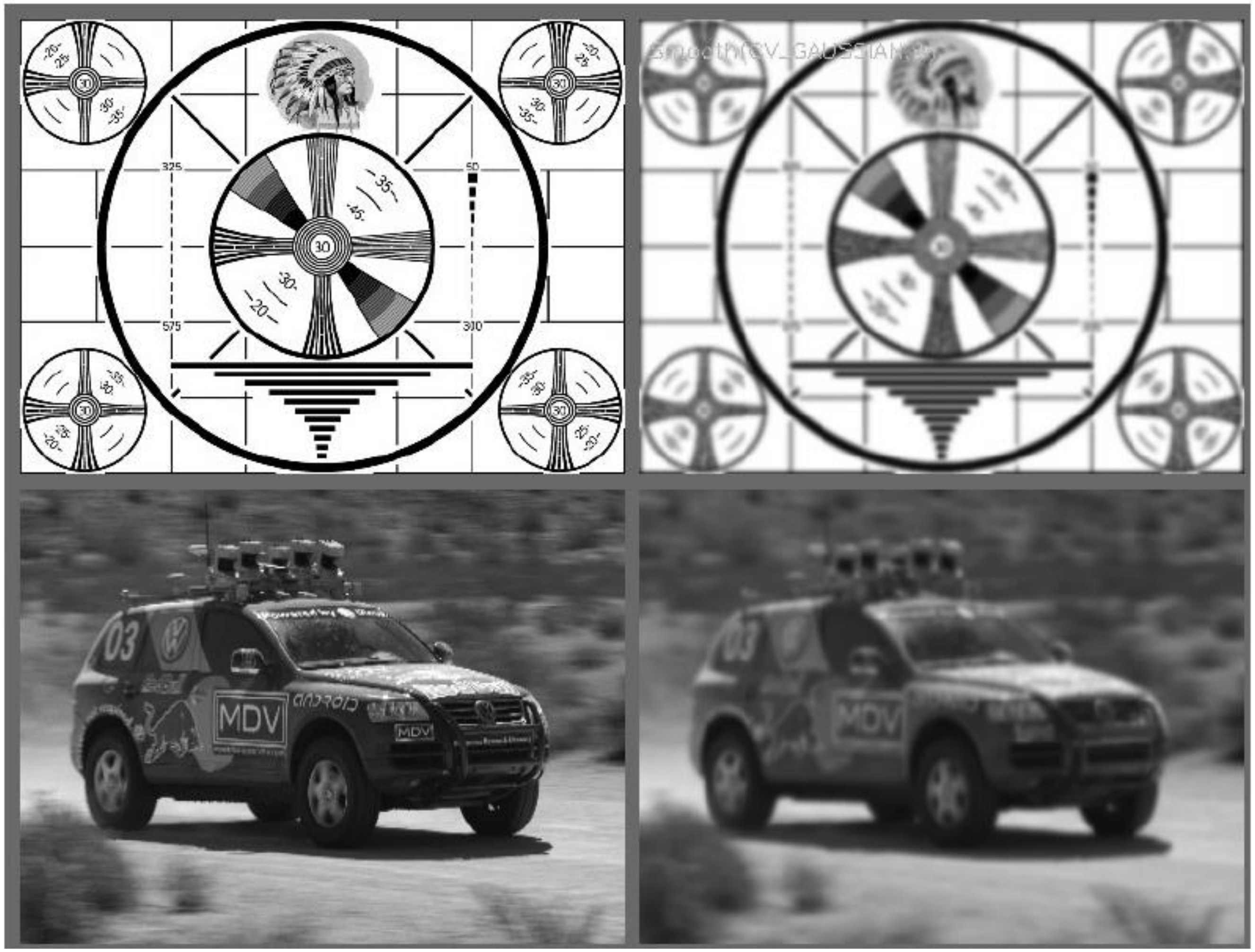


Image Blur – Bilateral filter

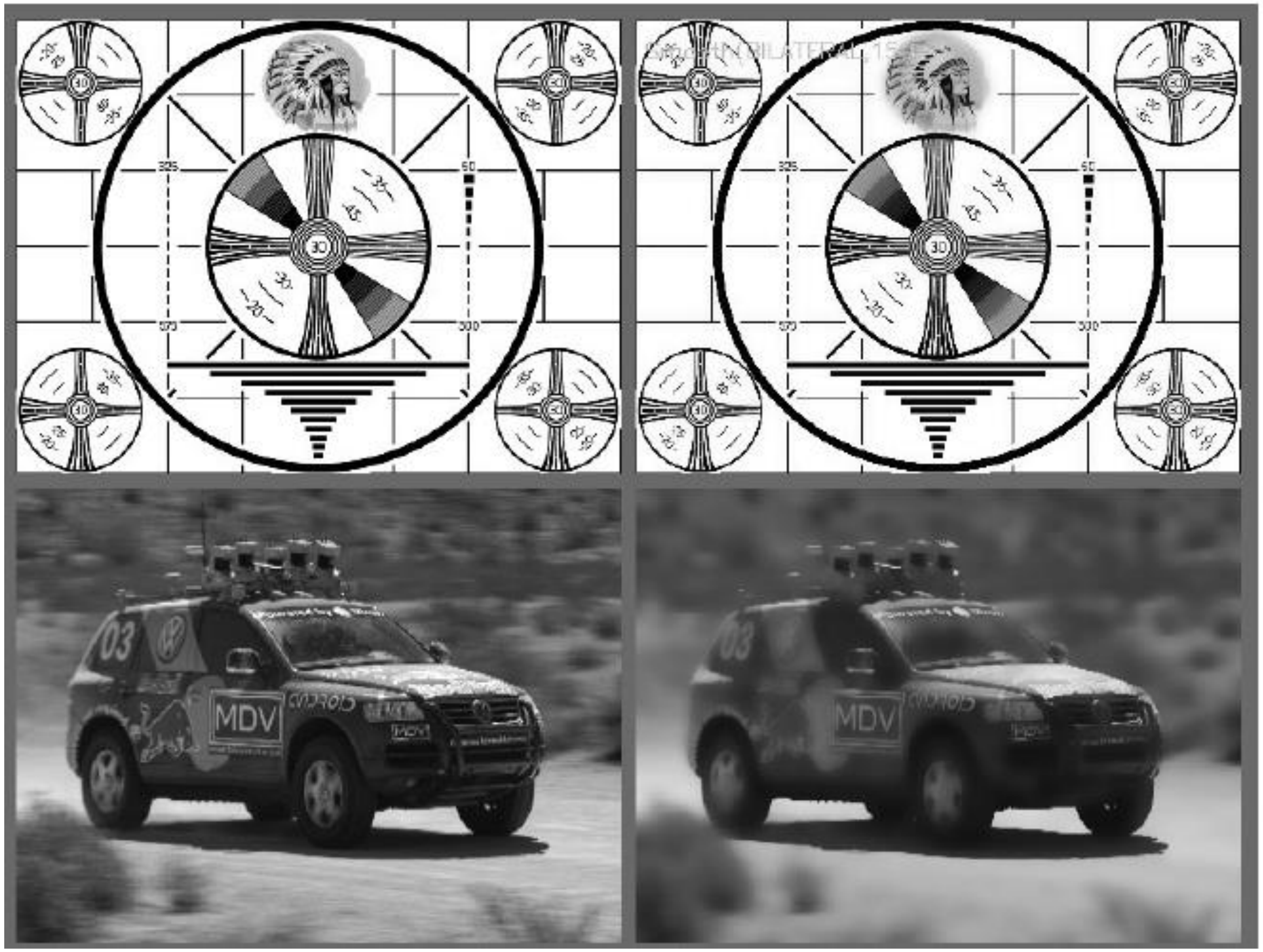


Image Blur – Bilateral filter

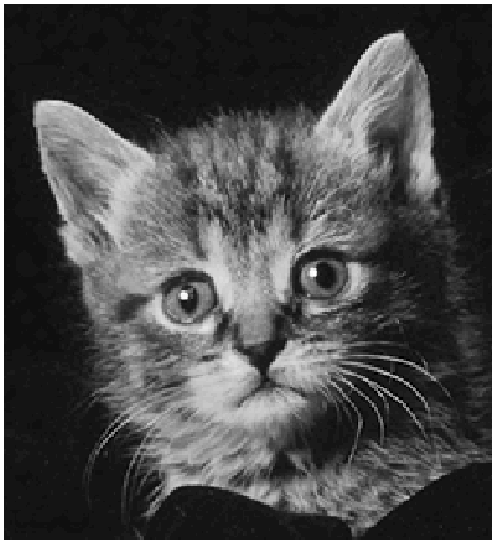
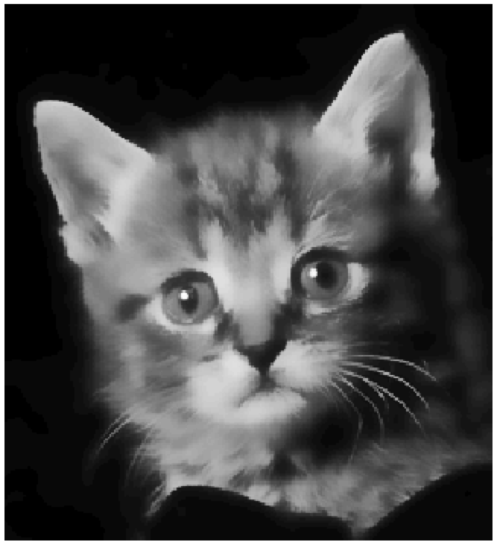
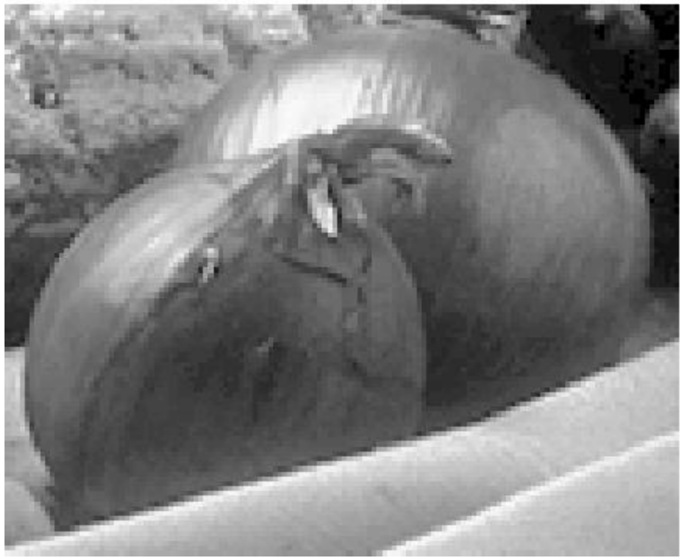
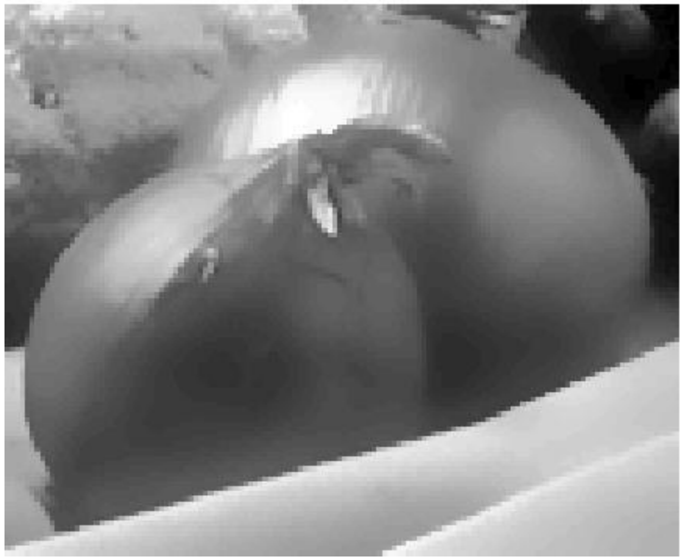





Image Blur – Bilateral filter

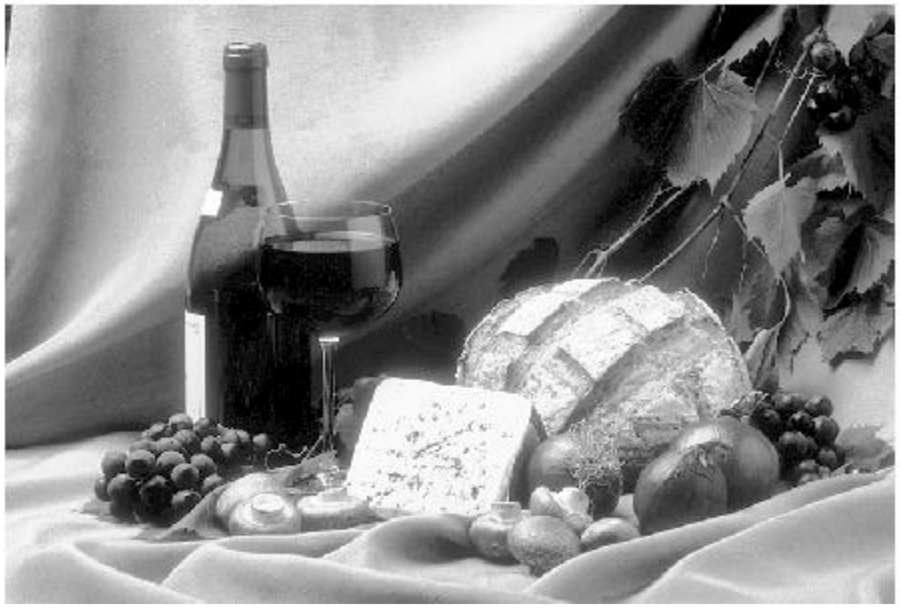
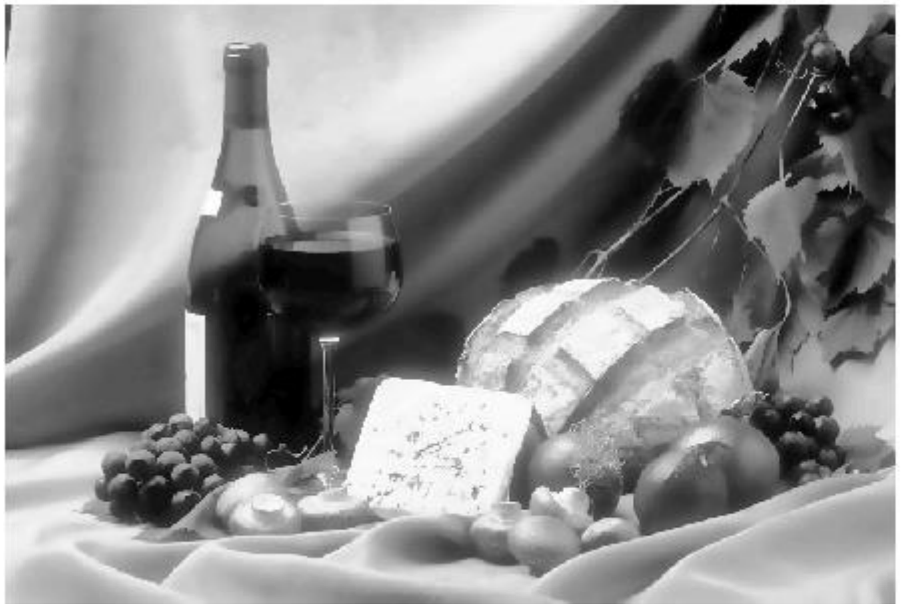
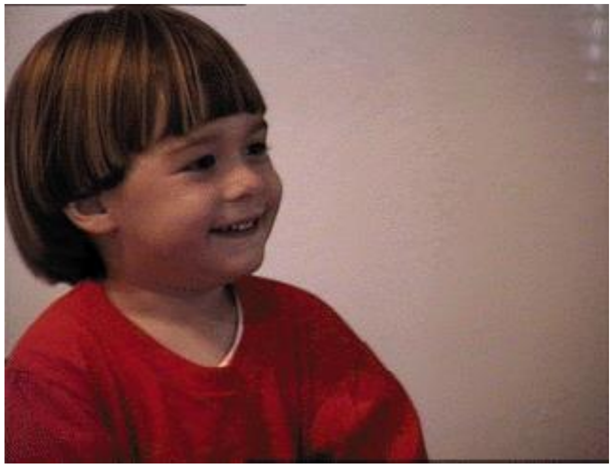
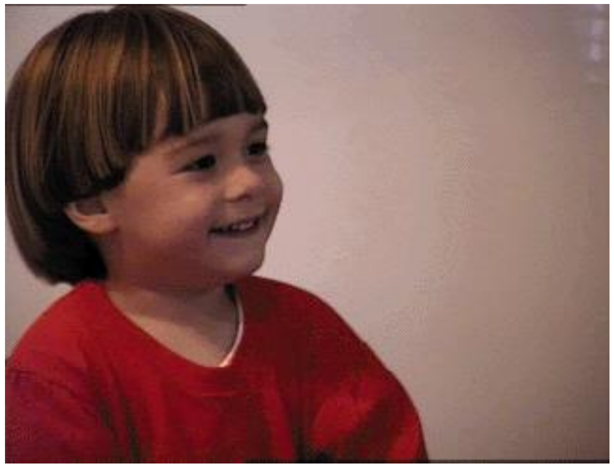
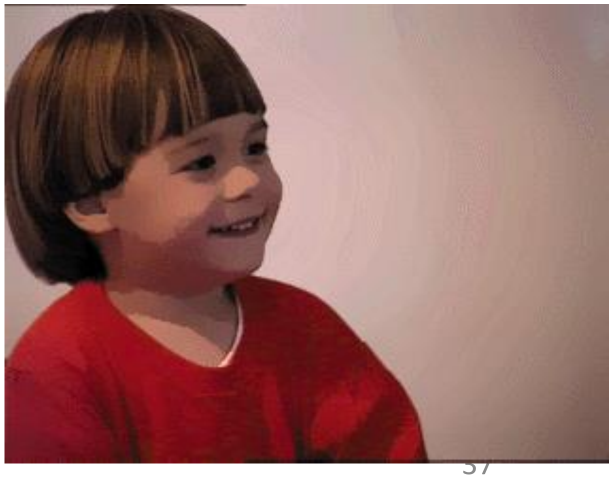
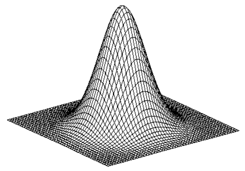






Image Blur – Bilateral filter



$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

- void **bilateralFilter**(InputArray **src**, OutputArray **dst**, int **d**, double **sigmaColor**, double **sigmaSpace**, int **borderType**=BORDER_DEFAULT)
  - **d** – Diameter of each pixel neighborhood that is used during filtering.

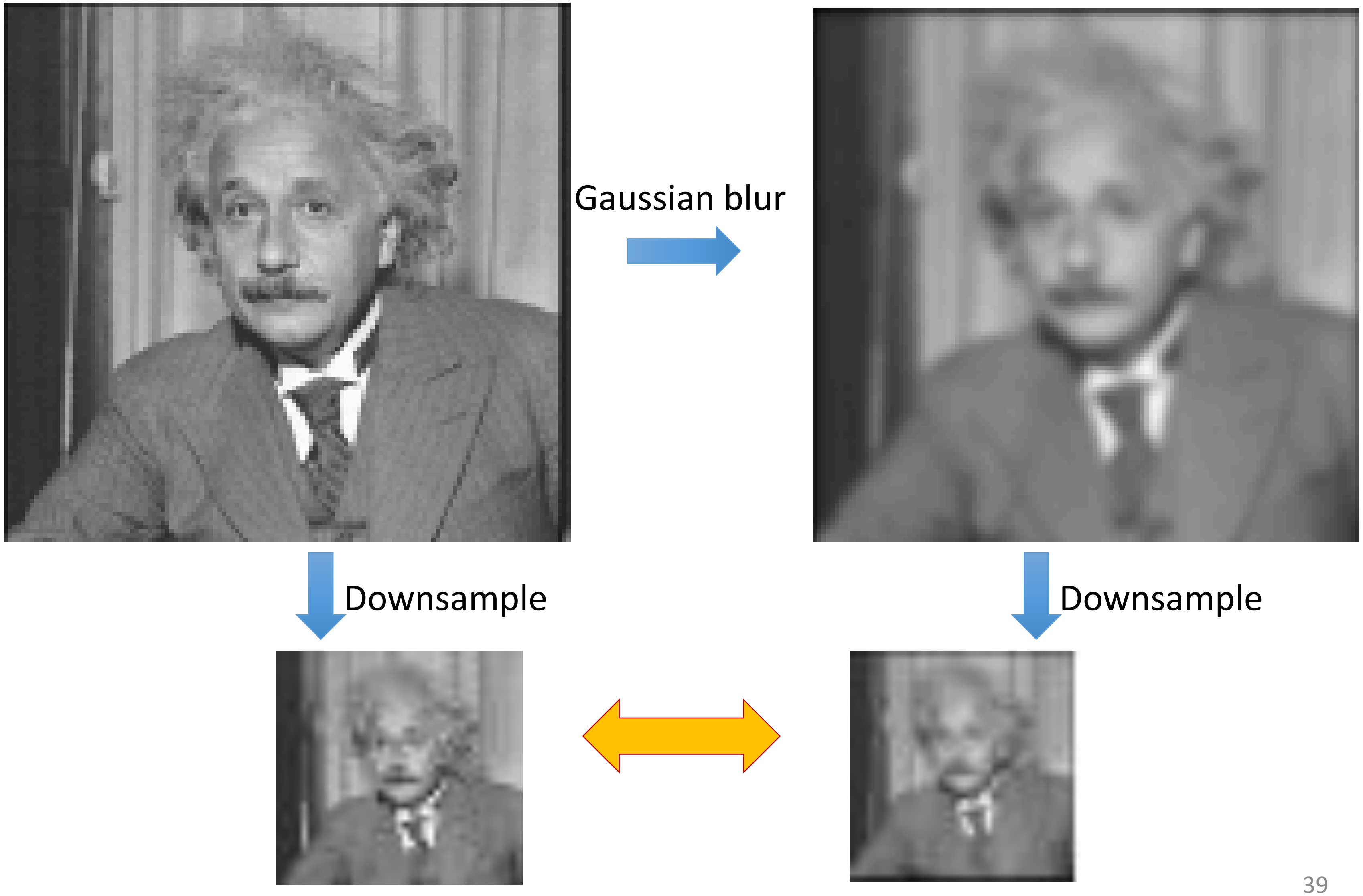
  →Large filters (d > 5) are very slow. Recommend: use d=5 for real-time applications.

  - **sigmaColor** – Filter sigma in the color space. A larger value of the parameter means that farther colors within the pixel neighborhood will be mixed together, resulting in larger areas of semi-equal color.
  - **sigmaSpace** – Filter sigma in the coordinate space. A larger value of the parameter means that farther pixels will influence each other as long as their colors are close enough.

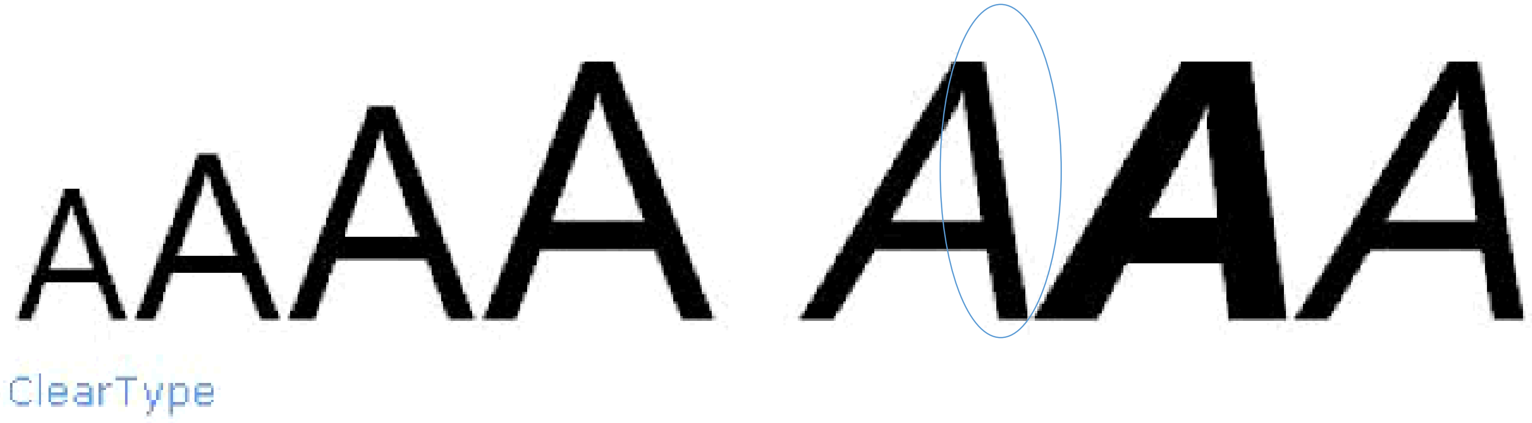
  →For simplicity, you can set the 2 sigma values to be the same.

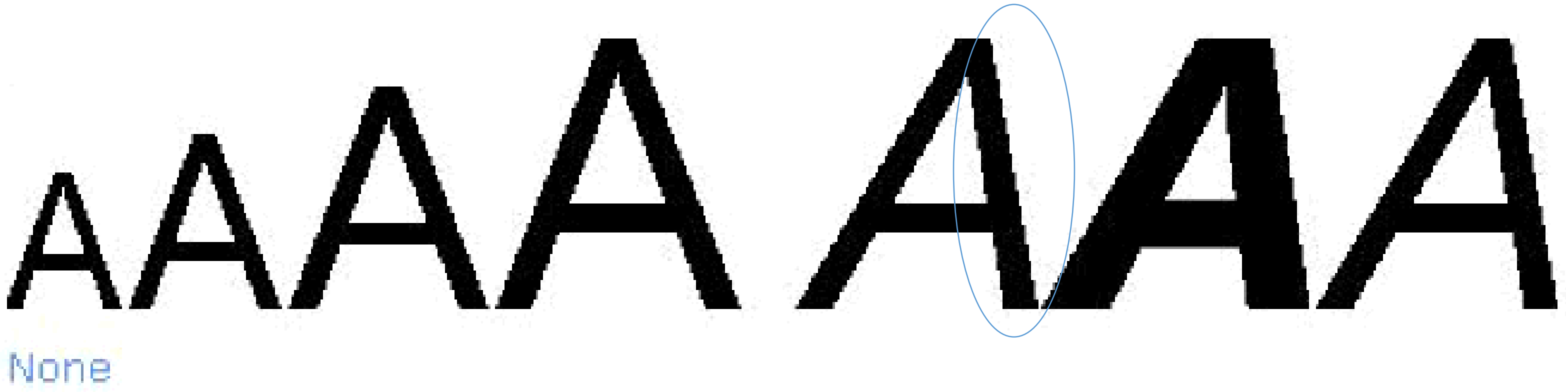
  If they are small (< 10), the filter will not have much effect, whereas if they are large (> 150), they will have a very strong effect, making the image look "cartoonish".

Some applications

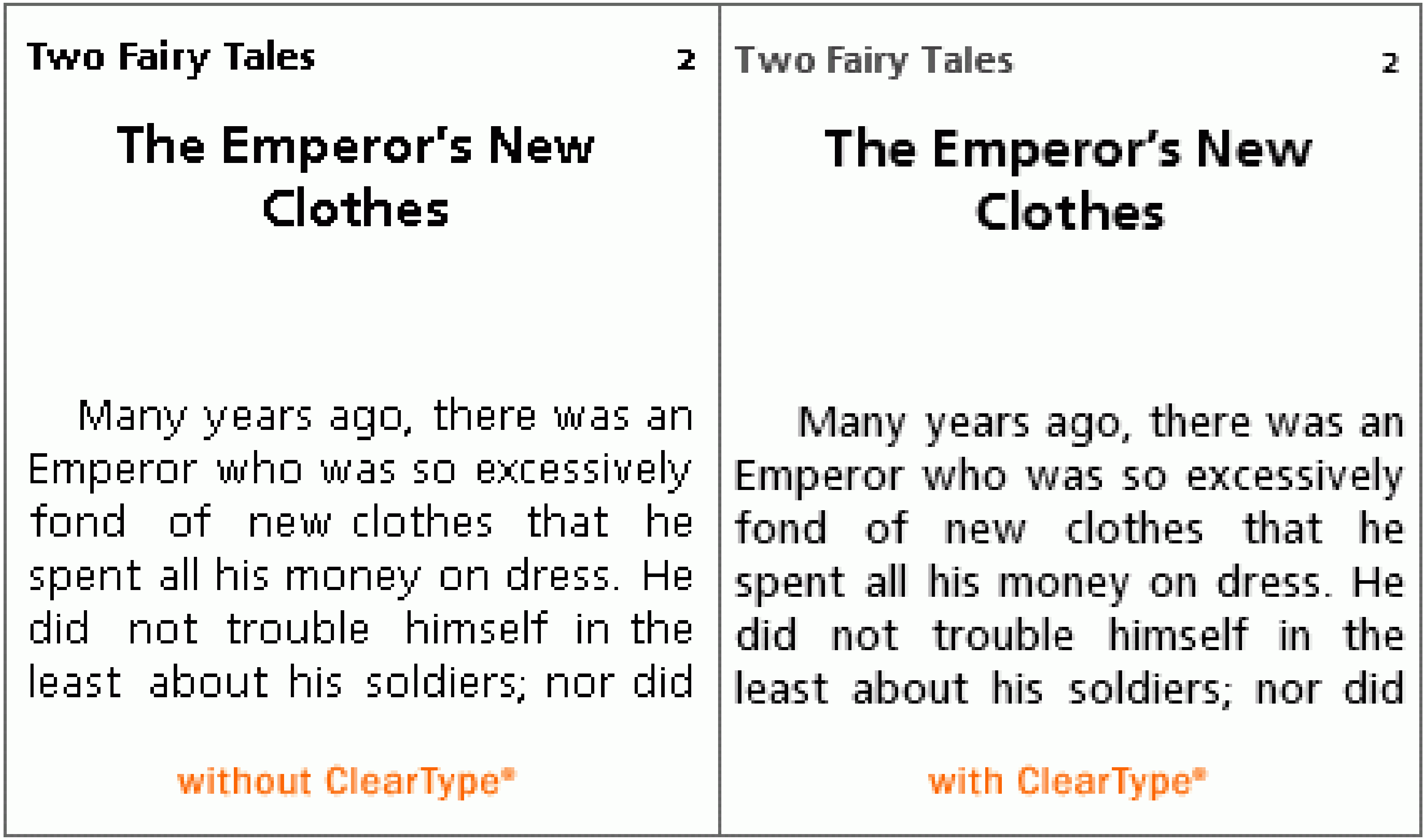


Clear Type Font


ClearType


None

Clear Type Font


Two Fairy Tales 2

The Emperor's New Clothes

Many years ago, there was an Emperor who was so excessively fond of new clothes that he spent all his money on dress. He did not trouble himself in the least about his soldiers; nor did

without ClearType*

Two Fairy Tales 2

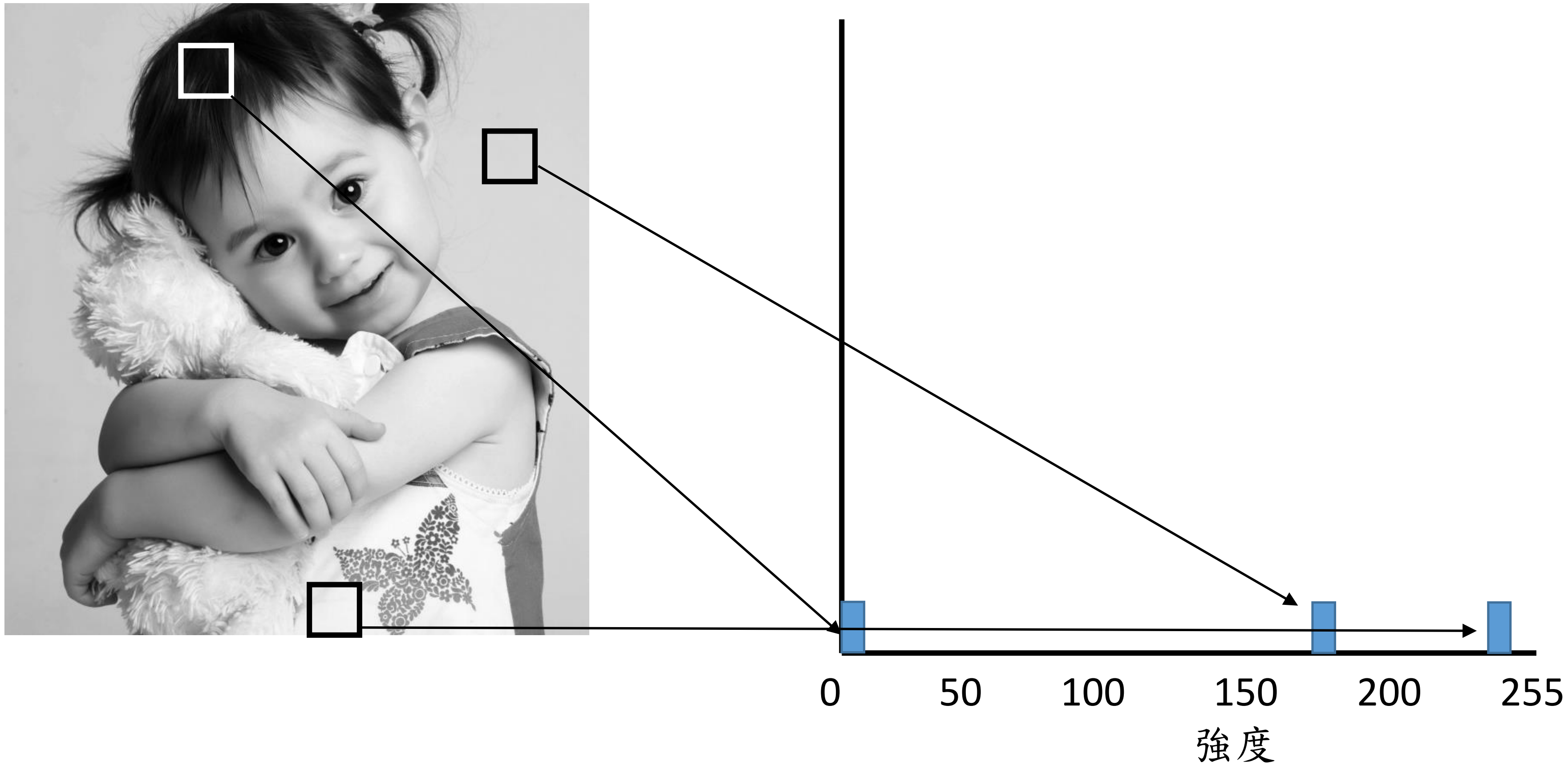
The Emperor's New Clothes

Many years ago, there was an Emperor who was so excessively fond of new clothes that he spent all his money on dress. He did not trouble himself in the least about his soldiers; nor did

with ClearType*

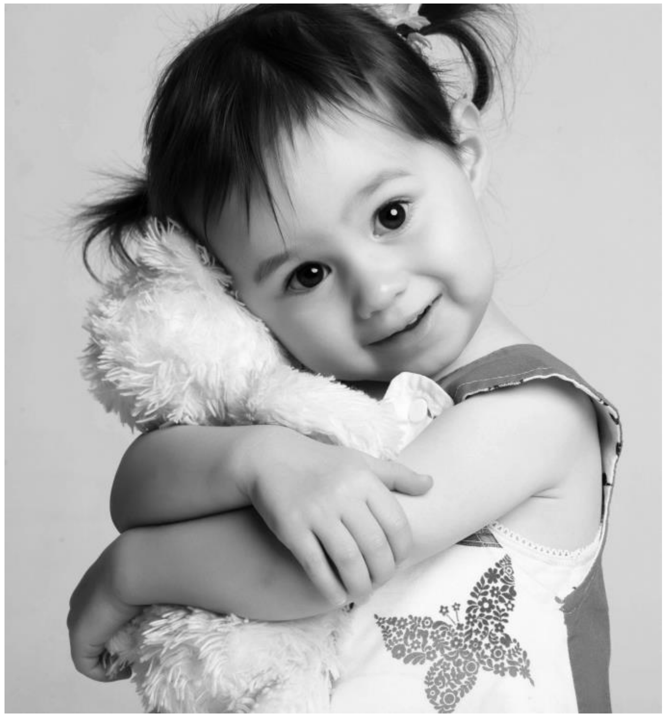
直方圖(Histogram)

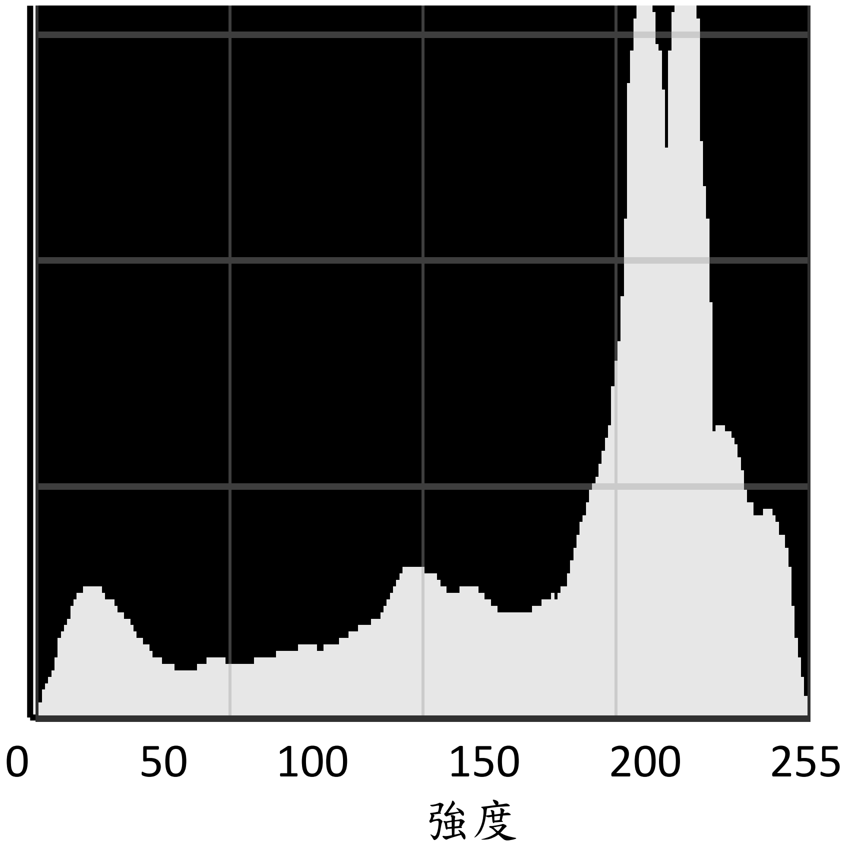
- 將影像上的強度值統計的結果



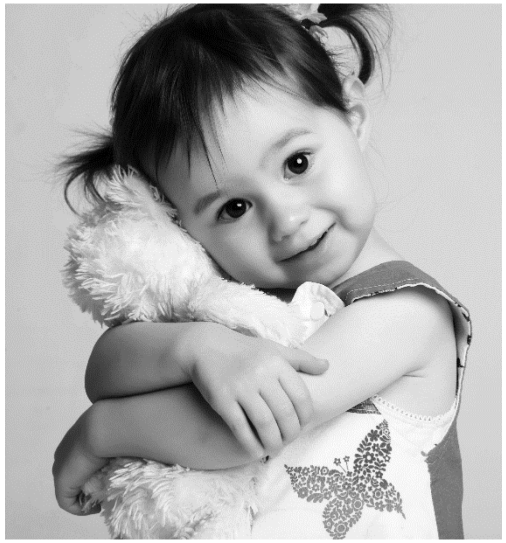
直方圖(Histogram)

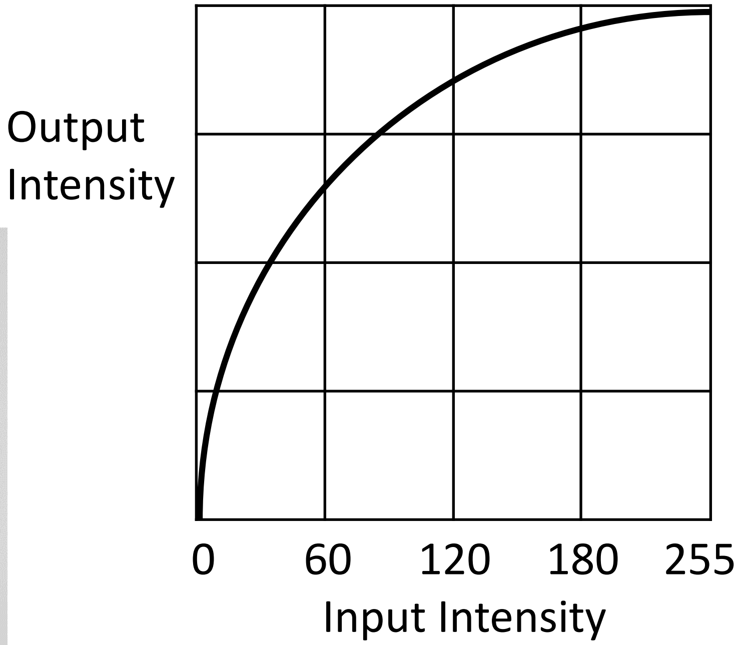
- 將影像上的強度值統計的結果

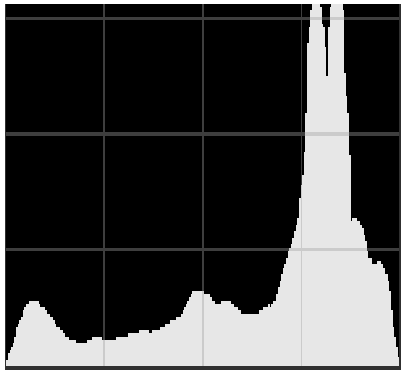


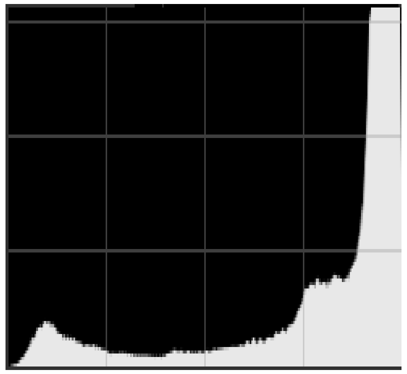


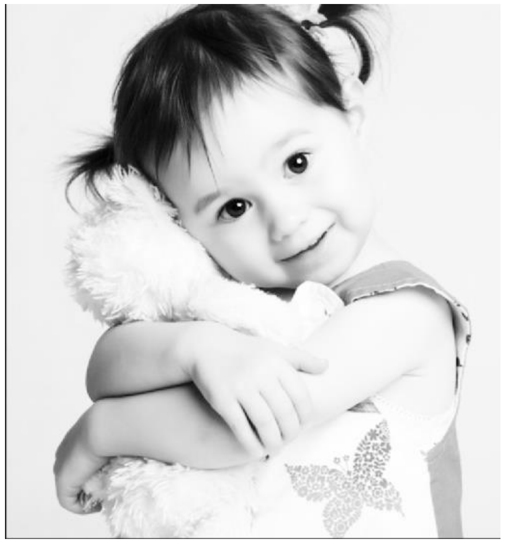
直方圖調整











彩色

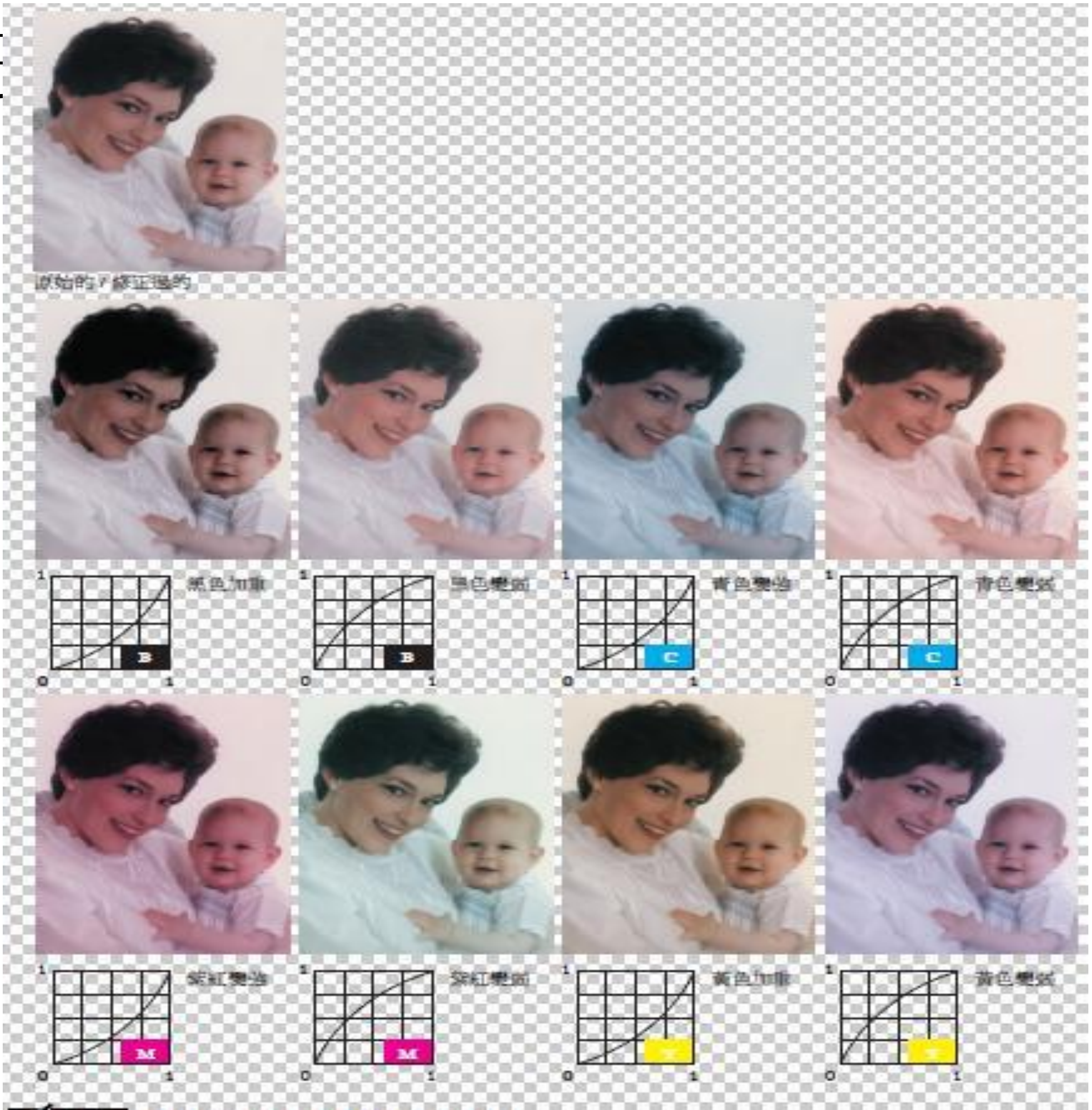
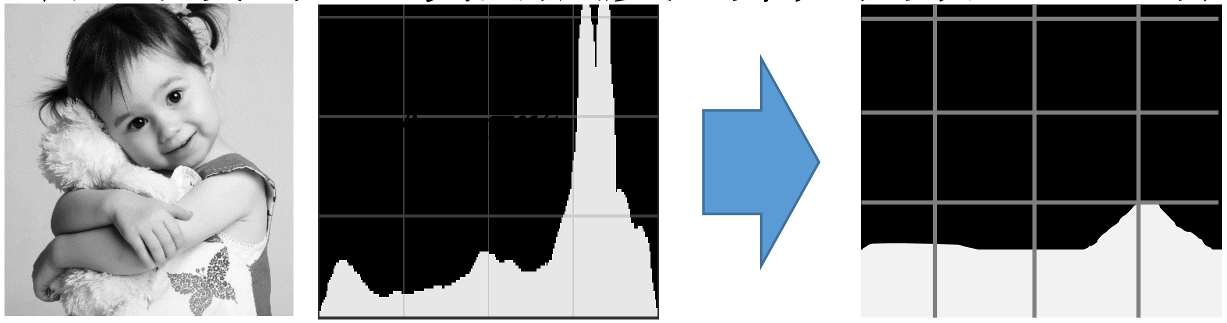


圖 6.36 對於 CMYK 彩色影像做色彩平衡修正

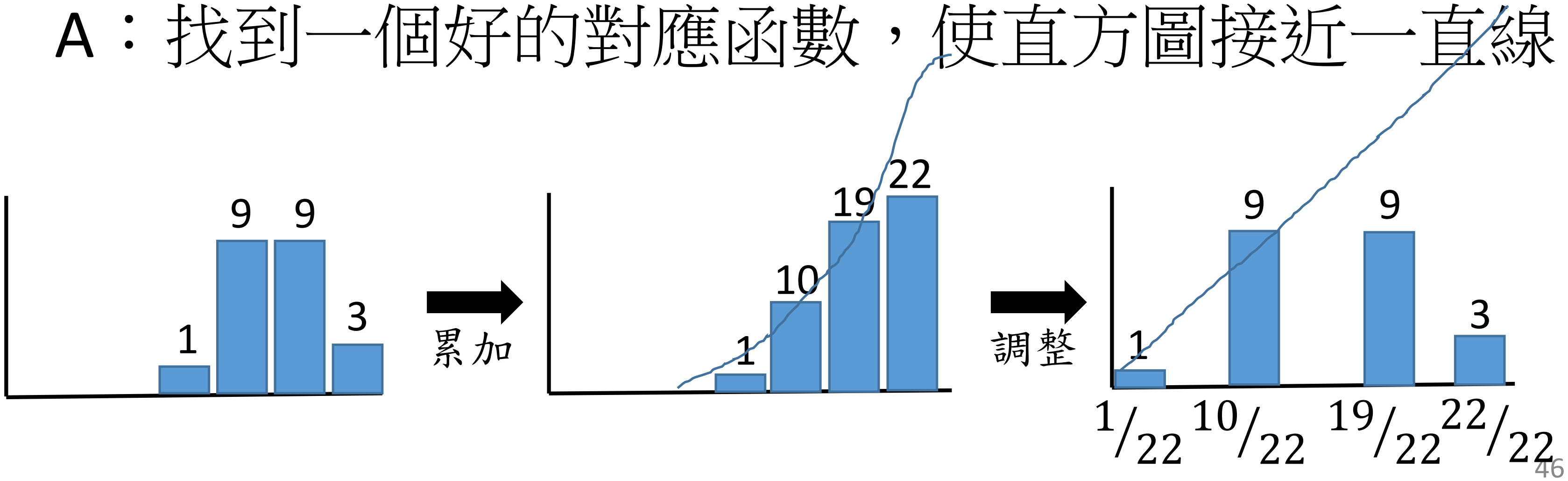
Histogram Equalization

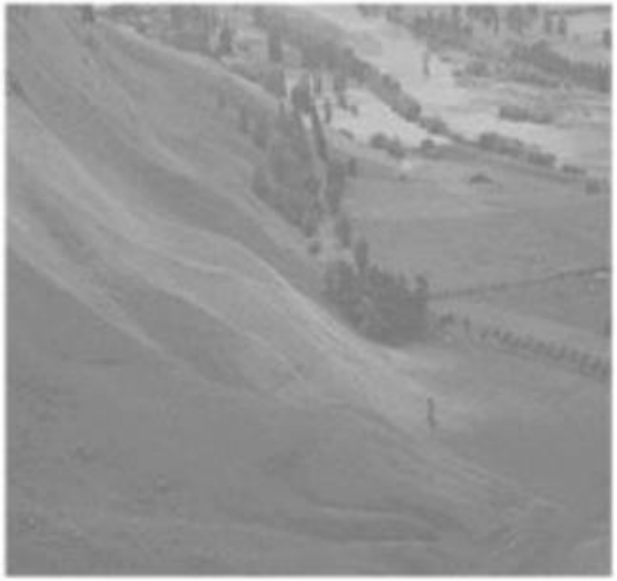
當影像強度其中在某一區(偏亮/偏暗)時我們希望讓畫面中的顏色強度較平均，較可看出細節

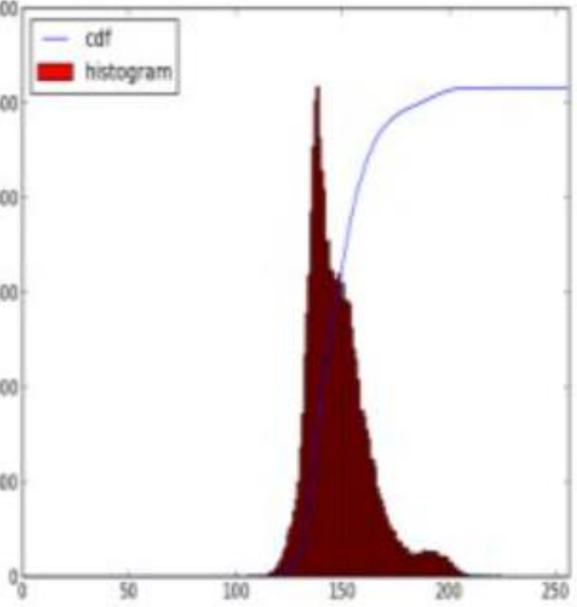
Q：如何讓直方圖上每個強度的像素數量盡可能相等？



A：找到一個好的對應函數，使直方圖接近一直線

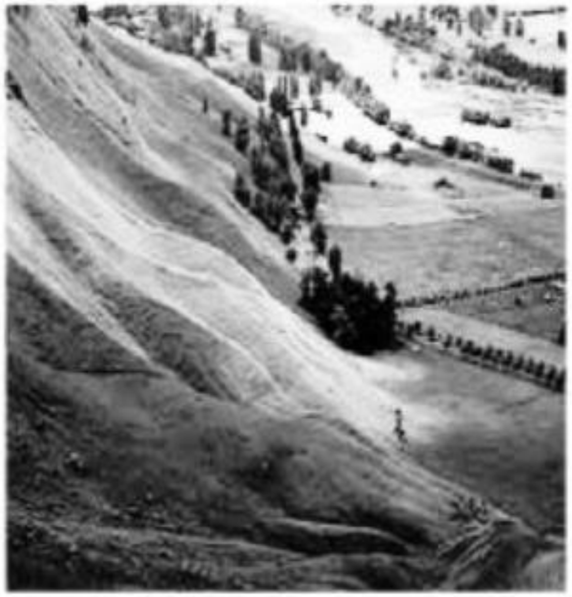


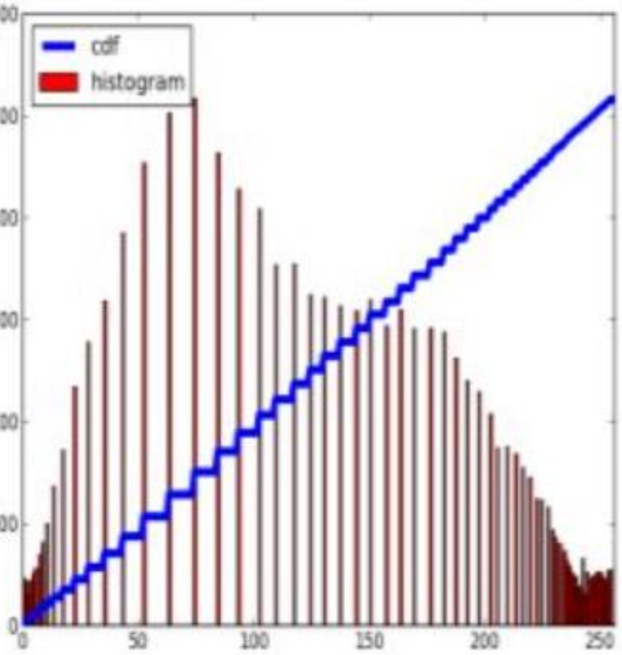




cdf
histogram
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250

image





cdf
histogram
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250

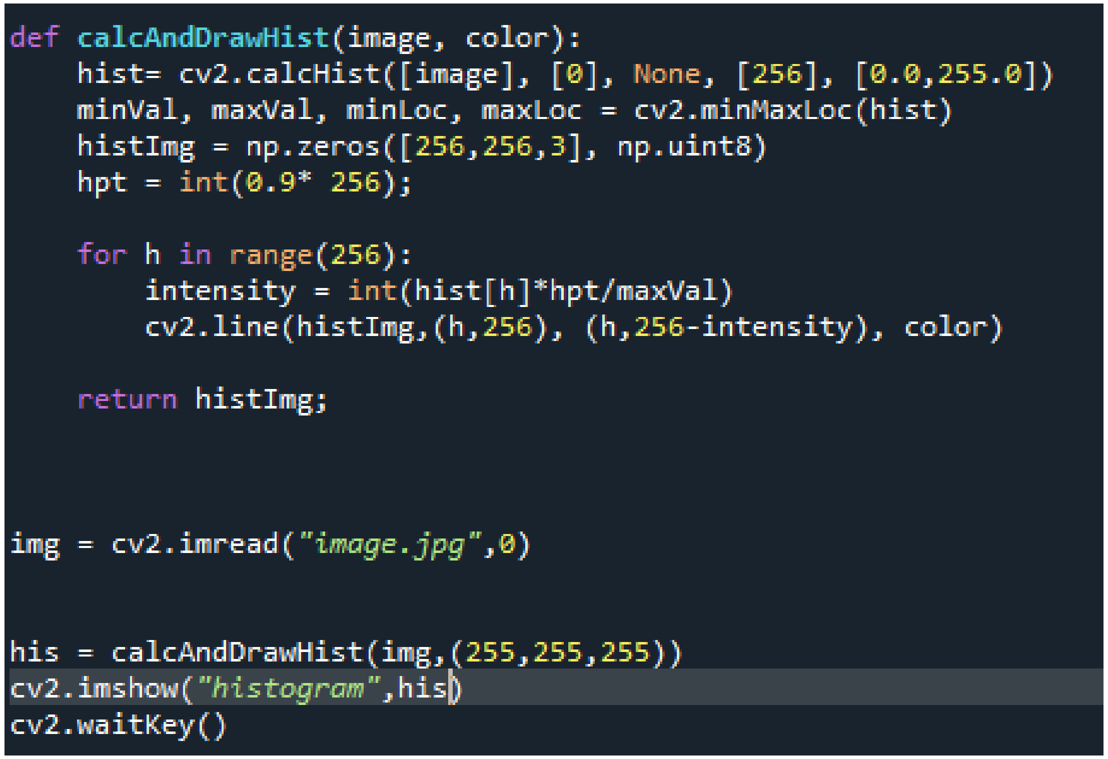
image

cv2.calcHist(images, channels, mask, histSize, ranges)

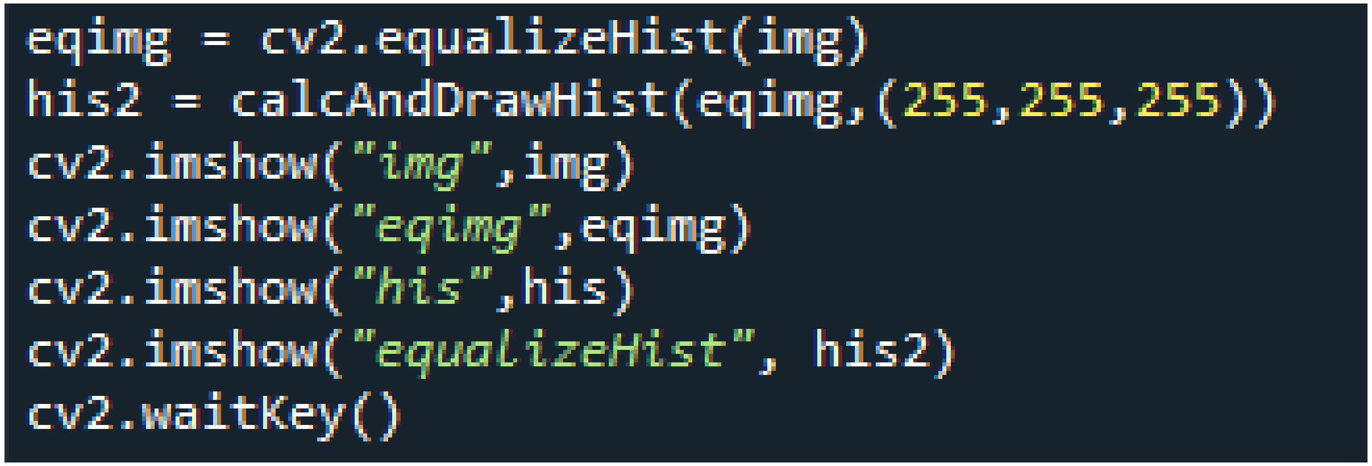
- images：要處理的圖片檔
- channels：指定產生的直方圖類型。例：[0]→灰階，[0, 1, 2]→RGB三色。
- mask：optional，只計算遮罩的部分影像。
- histSize：要切分的像素強度值範圍，預設為256。每個channel皆可指定一個範圍。例如，[32,32,32] 表示RGB三個channels皆切分為32區段。
- ranges：X軸(像素強度)的範圍，預設為[0,256]（最後那個值是表示<256）。

```
cv2.calcHist(img,[0],None,[256], [0, 256])
```

```
def calcAndDrawHist(image, color):
    hist= cv2.calcHist([image], [0], None, [256], [0.0,255.0])
    minVal, maxVal, minLoc, maxLoc = cv2.minMaxLoc(hist)
    histImg = np.zeros([256,256,3], np.uint8)
    hpt = int(0.9* 256);

    for h in range(256):
        intensity = int(hist[h]*hpt/maxVal)
        cv2.line(histImg,(h,256), (h,256-intensity), color)

    return histImg;
```



```
def calcAndDrawHist(image, color):
    hist= cv2.calcHist([image], [0], None, [256], [0.0,255.0])
    minVal, maxVal, minLoc, maxLoc = cv2.minMaxLoc(hist)
    histImg = np.zeros([256,256,3], np.uint8)
    hpt = int(0.9* 256);

    for h in range(256):
        intensity = int(hist[h]*hpt/maxVal)
        cv2.line(histImg,(h,256), (h,256-intensity), color)

    return histImg;



img = cv2.imread("image.jpg",0)


his = calcAndDrawHist(img,(255,255,255))
cv2.imshow("histogram",his)
cv2.waitKey()
```



```
eqimg = cv2.equalizeHist(img)
his2 = calcAndDrawHist(eqimg,(255,255,255))
cv2.imshow("img",img)
cv2.imshow("eqimg",eqimg)
cv2.imshow("his",his)
cv2.imshow("equalizeHist", his2)
cv2.waitKey()
```