Demo: Counterpoint by Construction

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Western music of the common practice period tends to loosely follow sets of rules, which were developed over time to ensure the aesthetic quality of the composition. Among these rules, those for harmony [Piston and DeVoto 1987] and counterpoint (harmonically interdependent melodies) [Fux 1965] are particularly fundamental and continue to be taught to music students, not only as a means to understand the music of that period, but also as a foundation for modern art and popular music.

To help analyze and synthesize tonal music, it is worthwhile to encode these rules into a programming language. As shown by recent studies, functional programming languages are particularly suited to this task. In the past decade, Haskell has been extensively used to encode the rules of harmony [De Haas et al. 2011, 2013; Koops et al. 2013; Magalhães and de Haas 2011; Magalhães and Koops 2014] as well as counterpoint [Szamozvancev and Gale 2017]. Haskell's static type sytem is used to encode these rules, so that "well-typed music does not sound wrong". Unfortunately the type system of plain Haskell is not powerful enough to guarantee these properties. Previous studies have thus incorporated language extensions such as GADTs [Cheney and Hinze 2002] and singleton types [Eisenberg and Weirich 2013] to approximate dependently-typed programming.

In this demonstration of work in progress, we present Music Tools [Cong and Leo 2019], a library of small tools that can be combined functionally to help analyze and synthesize music. To allow simple and natural encoding of rules, we built the library in Agda [Norell 2007], which is a functional language with full dependent types. As an application of the library, we demonstrate an implementation of species counterpoint, based on the rules given by Fux [1965]. Thanks to Agda's rich type system, we can express these rules naturally, and thus ensure by construction that well-typed counterpoint satisfies all the required rules.

Let us briefly explain how to implement the rule system of first-species counterpoint[1]. In first-species counterpoint, one starts with a base melody (the *cantus firmus*), and contructs a counterpoint melody note-by-note in the same rhythm. The two voices are represented as a list of pitch-interval pairs, where intervals must not be dissonant (2nds, 7ths, or 4ths).

```
data IntervalQuality : Set where
  min3  : IntervalQuality
  maj3  : IntervalQuality
  per5  : IntervalQuality
  min6  : IntervalQuality
  maj6  : IntervalQuality
  per8  : IntervalQuality
  min10 : IntervalQuality
  maj10 : IntervalQuality

PitchInterval : Set
PitchInterval = Pitch × IntervalQuality
```

In addition, it is prohibited to move from any interval to a perfect interval (5th or octave) via parallel or similar motion. Therefore, we define a predicate that checks whether a motion is allowed or not.

```
motionOk : (i1 : Interval)
           (i2 : Interval) → Set
motionOk i1 i2 with motion i1 i2
         | isPerfectInterval i2
motionOk i1 i2 | contrary | _     =
motionOk i1 i2 | oblique  | _     =
motionOk i1 i2 | parallel | false =
motionOk i1 i2 | parallel | true  =
motionOk i1 i2 | similar  | false =
motionOk i1 i2 | similar  | true  =
```

The last requirement is that the music must end with a cadence, which is a final motion from the 2nd or 7th degree to the tonic (1st degree). We impose this requirement by declaring two cadence constructors as the base cases of counterpoint. Thus, we arrive at the following datatype for well-typed counterpoint[2].

```
data FirstSpecies : PitchInterval →
                    Set where
  cadence2 : (p : Pitch) →
    FirstSpecies
      (transpose (+ 2) p , maj6)
  cadence7 : (p : Pitch) →
    FirstSpecies
      (transpose -[1+ 0 ] p , min10)
  _::_ : (pi : PitchInterval) →
         {pj : PitchInterval} →
         {_ : motionOk pi pj} →
```

[1] The code is available at https://github.com/halfaya/MusicTools/blob/master/agda/Counterpoint.agda.

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[2] For readability, we have ommited explicit conversions from PitchInterval (which ensures the interval is not dissonant) to the general Interval.

```
        FirstSpecies pj →
        FirstSpecies pi
```

Observe that motionOk is an implicit argument of the _::_ constructor. The argument can be resolved automatically by the type checker, hence there is no need to manually supply this proof.

Now we can write valid first-species counterpoint as in the example below.

```
example : FirstSpecies (g 4 , per8)
example =
  (g 4 , per8) :: (c 5 , maj10) ::
  (c 5 , per8) :: (c 5 , maj10) ::
  (e 5 , min10) :: (g 5 , per8) ::
  (cadence2 (c 6))
```

At the FARM workshop, we intend to give a gentle introduction to counterpoint, and describe our Agda implementation, showing how the type-based approach both aids human composition and allows for computer-generated creation of correct counterpoint. We then contrast our work to a recent study on generating natural-sounding counterpoint by machine learning [Huang et al. 2017], which does not provide correctness guarantees. Finally, we discuss further applications of our library, including representation of functional harmony.

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