

Musical Ornaments

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Outline

- A look toward the future
- Music Tools
- Equivalences and Ornaments

Sources:

- <https://github.com/halfaya/MusicTools>

Eventually all the arbitrary programming languages are going to be just swept away with the oceans, and we will have the permanence of constructive, intuitionistic type theory as the master theory of computation—without doubt, in my mind, no question. So, from my point of view—this is a personal statement—working in anything else is a waste of time.

CMU Homotopy Type Theory lecture 1, 52:56–53:20.

What will programming look like in 50 years?

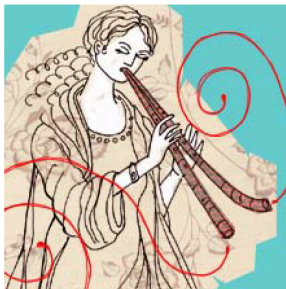
- Convergence of math and computer science
- Functional Programming, Algebra of Programming
- Dependent Types or a successor (Cubical?)
- Who does the programming?

How do we get there from here?

- Add dependent types to an industrial-strength language (Haskell)
- Make a dependently typed language (Agda, Idris) practical to use
- Learn how to program using dependent types
- Many theoretical and practical advances are still needed

The Haskell School of Music

— From Signals to Symphonies —



Paul Hudak

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Music Tools

- Collection of composable tools for synthesis and analysis of music
- Originally written in Haskell
- Converted to Agda, using Haskell for MIDI interface
- Explore programming using dependent types in a circumscribed yet rich domain
- Use math, including transport of equivalences (from HoTT) and Ornaments

Look vs Time (1997)

A musical score for a piece titled "Look vs Time (1997)". The score is written for four staves, all in 4/4 time. The first three staves are for a piano (treble and bass clefs), and the fourth staff is for a percussion instrument (square clef). The piano part features a melody in the right hand and a complex, multi-voiced accompaniment in the left hand. The percussion part consists of a series of rhythmic patterns represented by 'x' marks on a staff.

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Music Representation à la Euterpea

```
data Pitch : Set where  
  pitch :  $\mathbb{Z} \rightarrow$  Pitch
```

```
data Duration : Set where  
  duration :  $\mathbb{N} \rightarrow$  Duration
```

```
data Note : Set where  
  note : Duration  $\rightarrow$  Pitch  $\rightarrow$  Note  
  rest : Duration  $\rightarrow$  Note
```

```
data Music : Set where  
  note : Note  $\rightarrow$  Music  
  _::_ : Music  $\rightarrow$  Music  $\rightarrow$  Music -- sequential  
  _||_ : Music  $\rightarrow$  Music  $\rightarrow$  Music -- parallel
```

Equivalent Representations of Pitch

`data Pitch : Set where`
`pitch : $\mathbb{Z} \rightarrow$ Pitch`

`chromaticScaleSize : \mathbb{N}`
`chromaticScaleSize = 12`

`data RelativePitch : Set where`
`relativePitch : Fin chromaticScaleSize \rightarrow RelativePitch`

`data Octave : Set where`
`octave : $\mathbb{Z} \rightarrow$ Octave`

`PitchOctave : Set`
`PitchOctave = RelativePitch \times Octave`

Equivalences

- Define an equivalence between `Pitch` and `PitchOctave`
- Using HoTT techniques, automatically lift this equivalence to functions defined using `Pitch`
- See *Equivalences for Free!* (Tabareau, Tanter, Sozeau)
- Challenge: Defining base equivalences. Can this be automated?

Ornaments

```
data Music a = ...  
  | Modify Control (Music a)  
  
data Control = ...  
  | Phrase [PhraseAttribute]  
  
data PhraseAttribute = ...  
  | Orn Ornament  
  
data Ornament =  
  Trill | Mordent | InvMordent | DoubleMordent |  
  Turn | TrilledTurn | ShortTrill ...
```

Ornaments

- Functions on a base `Music` structure can be automatically lifted to operate on `Music` ornamented with additional information
- See works on Ornaments by McBride, Dagand and others
- Challenge: Shallow embedding of ornaments in Agda

Conclusion

- Music is a good domain in which to explore practical application of dependent types
- Using math can be more work at first, but should be a big win in the long term
- Figure out how to minimize the work and maximize the reward