Linear Regression - Part 1 - Edx Analytical Edge

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Contents

This document is inspired and use the *EDx course - Analytical Edge* resources.

Linear regression is one of the easiest tool in the predicitve analytics field. This paper intends to show by example how to make it works with R using the great libraries of dplyr, ggplot2 whenever necessary.

Single variable regression.

The general equation for a linear regression model

$$y^i = \beta_0 + \beta_1 x^i + \epsilon^i$$

where:

- y^i is the i^{th} observation of the dependent variable
- β_0 is the intercept coefficient
- β_1 is the regression coefficient for the dependent variable
- x^i is the i^{th} observation of the independent variable
- ϵ^i is the error term for the i^{th} observation. It basically is the difference in therm of y between the observed value and the estimated value. It is also called the residuals. A good model minimize these errors. [^ Remember that the error term, ϵ^i, in the simple linear regression model is independent of x, and is normally distributed, with zero mean and constant variance.]

Some ways to assess how good our model is to:

1. compute the SSE (the sum of squared error)
    - SSE = $(\epsilon^1)^2 + (\epsilon^2)^2 + \ldots + (\epsilon^n)^2 = \sum_{i=1}^N \epsilon^i$
    - A good model will minimize SSE
    - problem: SSE is dependent of N. SSE will naturally increase as N increase
2. compute the RMSE (the root mean squared error)
    - RMSE = $\sqrt{\frac{SSE}{N}}$
    - Also a good model will minimize SSE
    - It depends of the unit of the dependent variable. It is like the average error the model is making (in term of the unit of the dependent variable)

3. compute R^2
    - It compare the models to a baseline model
    - R^2 is **unitless** and **universaly** interpretable
    - SST is the sum of the squared of the difference between the observed value and the mean of all the observed value

$R^2 = 1 - \frac{SSE}{SST}$

First example. Predicting wine price.

The wine.csv file is used in the class.

Let's load it and then have a quick look at its structure.

```
wine = read.csv("wine.csv")
str(wine)
```

```
## 'data.frame':    25 obs. of  7 variables:
##  $ Year       : int  1952 1953 1955 1957 1958 1959 1960 1961 1962 1963 ...
##  $ Price      : num  7.5 8.04 7.69 6.98 6.78 ...
##  $ WinterRain : int  600 690 502 420 582 485 763 830 697 608 ...
##  $ AGST       : num  17.1 16.7 17.1 16.1 16.4 ...
##  $ HarvestRain: int  160 80 130 110 187 187 290 38 52 155 ...
##  $ Age        : int  31 30 28 26 25 24 23 22 21 20 ...
##  $ FrancePop  : num  43184 43495 44218 45152 45654 ...
```

```
head(wine)
```

```
##   Year  Price WinterRain    AGST HarvestRain Age FrancePop
## 1 1952 7.4950        600 17.1167         160  31  43183.57
## 2 1953 8.0393        690 16.7333          80  30  43495.03
## 3 1955 7.6858        502 17.1500         130  28  44217.86
## 4 1957 6.9845        420 16.1333         110  26  45152.25
## 5 1958 6.7772        582 16.4167         187  25  45653.81
## 6 1959 8.0757        485 17.4833         187  24  46128.64
```

We use the `lm` function to find our linear regression model. We use $AGST$ as the independent variable while the *price* is the dependent variable.

```
model1 = lm(Price ~ AGST, data = wine)
summary(model1)
```

```
##
## Call:
## lm(formula = Price ~ AGST, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.78450 -0.23882 -0.03727  0.38992  0.90318
##
## Coefficients:
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -3.4178     2.4935  -1.371 0.183710
## AGST          0.6351     0.1509   4.208 0.000335 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4993 on 23 degrees of freedom
## Multiple R-squared:  0.435,  Adjusted R-squared:  0.4105
## F-statistic: 17.71 on 1 and 23 DF,  p-value: 0.000335
```

The `summary` function applied on the model is giving us a bunch of important information

- the stars next to the predictor variable indicated how significant the variable is for our regression model
- it also gives us the value of the R coefficient

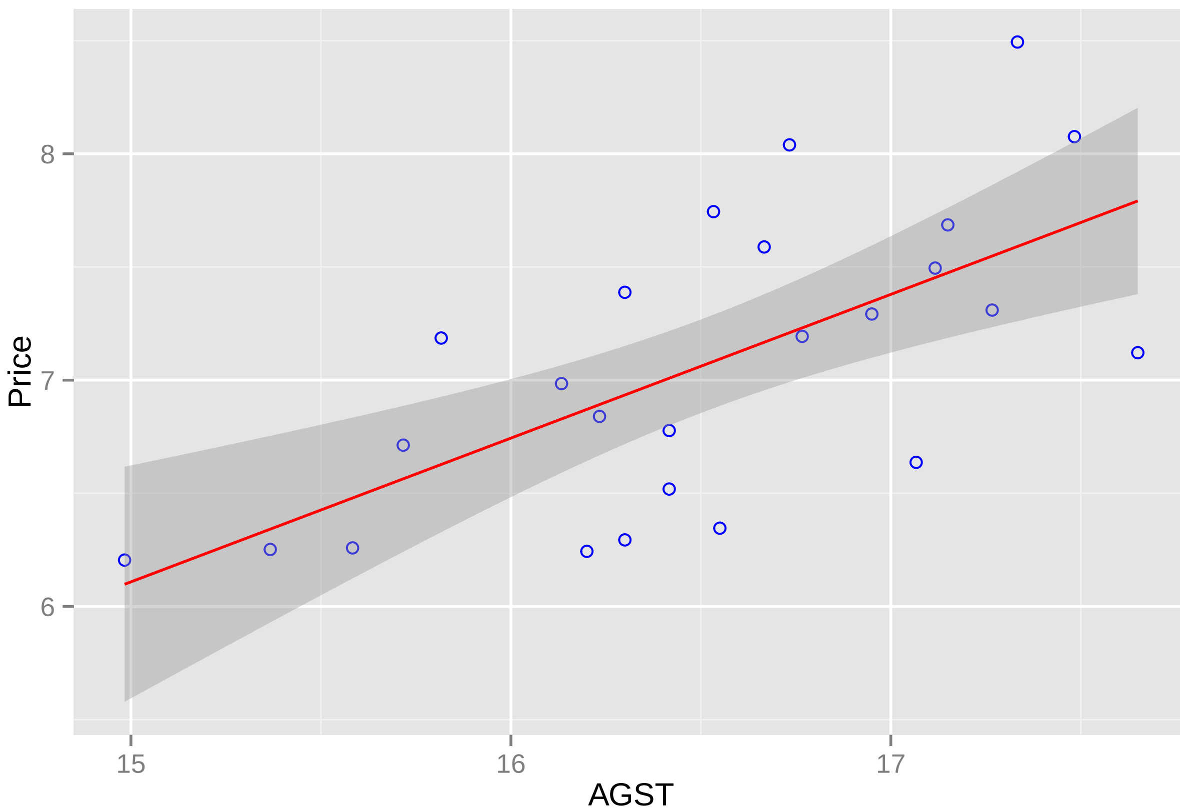
We could have calculated the R value ourselves:

```
SSE = sum(model1$residuals^2)
SST = sum((wine$Price - mean(wine$Price))^2)
r_squared = 1 - SSE/SST
r_squared
```

```
## [1] 0.4350232
```

We can now plot the observations and the line of regression; and see how the linear model fits the data.

```
library(ggplot2)
ggplot(wine, aes(AGST, Price)) + geom_point(shape = 1, col = "blue") + geom_smooth(method = "lm", col =
```

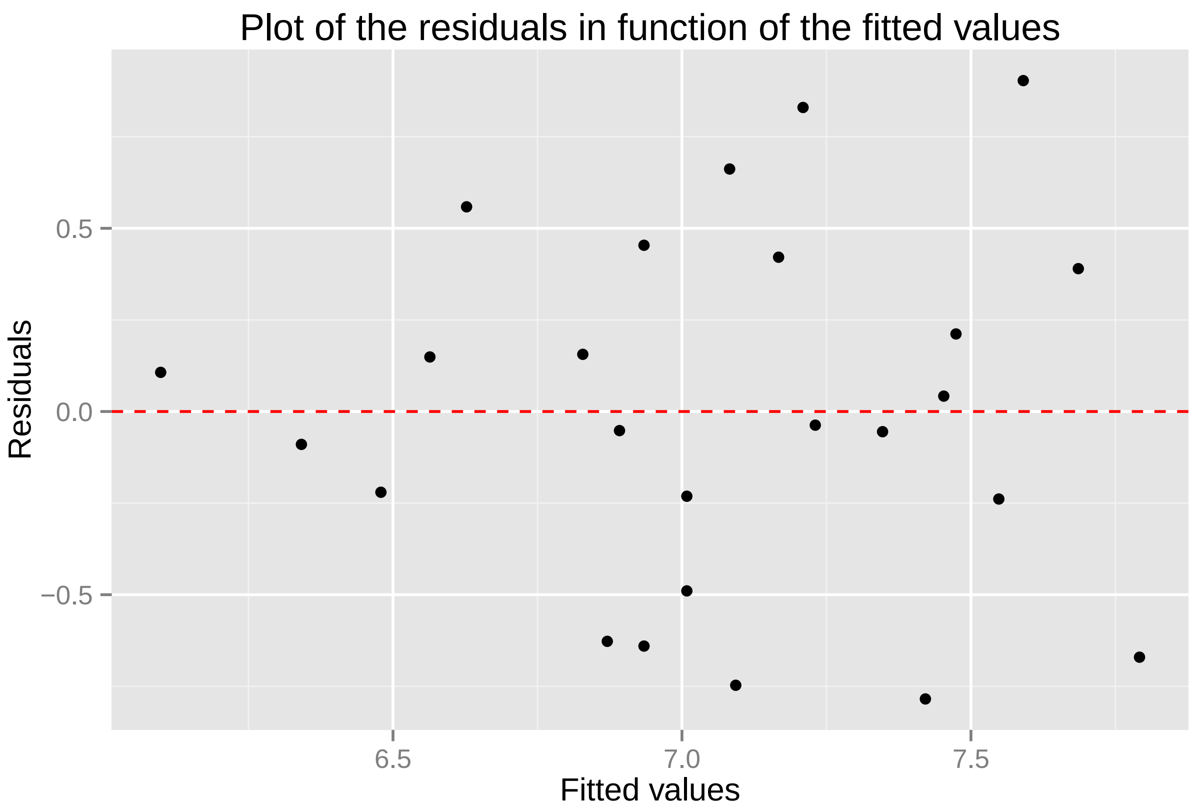


By

default, the `geom_smooth()` will use a 95% confidence interval (which is the grey-er area on the graph). There are 95% chance the line of regression will be within that zone for the whole population.

It is always nice to see how our residuals are distributed.
We use the `ggplot2` library and the `fortify` function which transform the `summary(model1)` into a data frame usable for plotting.

```
model1 <- fortify(model1)
p <- ggplot(model1, aes(.fitted, .resid)) + geom_point()
p <- p + geom_hline(yintercept = 0, col = "red", linetype = "dashed")
p <- p + xlab("Fitted values") + ylab("Residuals") + ggtitle("Plot of the residuals in function of the f
p
```



Multi-variables regression.

Instead of just considering one variable as predictor, we'll add a few more variables to our model with the idea to increase its predictive ability.

We have to be cautious in adding more variables. Too many variable might give a high R^2 on our training data, but this not be the case as we switch to our testing data.

The general equations can be expressed as

$$y^i = \beta_0 + \beta_1 x_1^i + \beta_2 x_2^i + \ldots + \beta_k x_k^i + \epsilon^i$$

when there are k predictors variables.

There are a bit of trials and errors to make while trying to fit mutliple variables into a model, but a rule of thumb would be to include most of the variable (all these that would make sense) and then take out the ones that are not very significant using the `summary(modelx)`

First example. Predicting wine price.

We continue here with the same dataset, *wine.csv*.
First, we can see how each variable is correlated with each other ones, using

```r
cor(wine)
```

```
##                    Year      Price   WinterRain        AGST HarvestRain
## Year         1.00000000 -0.4477679  0.016970024 -0.24691585  0.02800907
## Price       -0.44776786  1.0000000  0.136650547  0.65956286 -0.56332190
## WinterRain   0.01697002  0.1366505  1.000000000 -0.32109061 -0.27544085
## AGST        -0.24691585  0.6595629 -0.321090611  1.00000000 -0.06449593
## HarvestRain  0.02800907 -0.5633219 -0.275440854 -0.06449593  1.00000000
## Age         -1.00000000  0.4477679 -0.016970024  0.24691585 -0.02800907
## FrancePop    0.99448510 -0.4668616 -0.001621627 -0.25916227  0.04126439
##                     Age    FrancePop
## Year        -1.00000000  0.994485097
## Price        0.44776786 -0.466861641
## WinterRain  -0.01697002 -0.001621627
## AGST         0.24691585 -0.259162274
## HarvestRain -0.02800907  0.041264394
## Age          1.00000000 -0.994485097
## FrancePop   -0.99448510  1.000000000
```

by default, R uses the Pearson coefficient of correlation.
So let's start by using all variables.

```r
model2 <- lm(Price ~ Year + WinterRain + AGST + HarvestRain + Age + FrancePop, data = wine)
summary(model2)
```

```
##
## Call:
## lm(formula = Price ~ Year + WinterRain + AGST + HarvestRain +
##     Age + FrancePop, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.48179 -0.24662 -0.00726  0.22012  0.51987
##
## Coefficients: (1 not defined because of singularities)
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.092e-01  1.467e+02   0.005 0.996194
## Year        -5.847e-04  7.900e-02  -0.007 0.994172
## WinterRain   1.043e-03  5.310e-04   1.963 0.064416 .
## AGST         6.012e-01  1.030e-01   5.836 1.27e-05 ***
## HarvestRain -3.958e-03  8.751e-04  -4.523 0.000233 ***
## Age                 NA         NA      NA       NA
## FrancePop   -4.953e-05  1.667e-04  -0.297 0.769578
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3019 on 19 degrees of freedom
## Multiple R-squared:  0.8294, Adjusted R-squared:  0.7845
## F-statistic: 18.47 on 5 and 19 DF,  p-value: 1.044e-06
```

While doing so, we notice that the variable *Age* has NA (issues wiht missing data?) and that the variable *FrancePop* isn't very predictive of the price of wine. So we can refine our models, by taking out these 2 variables, and as we'll see, it won't affect much our R^2 value. Note that with multiple variables regression, it is important to look at the **Adjusted R-squared** as it take into consideration the amount of variables in the model.

```
model3 <- lm(Price ~ Year + WinterRain + AGST + HarvestRain, data = wine)
summary(model3)
```

```
##
## Call:
## lm(formula = Price ~ Year + WinterRain + AGST + HarvestRain,
##     data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.45470 -0.24273  0.00752  0.19773  0.53637
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 44.0248601 16.4434570   2.677 0.014477 *
## Year        -0.0239308  0.0080969  -2.956 0.007819 **
## WinterRain   0.0010755  0.0005073   2.120 0.046694 *
## AGST         0.6072093  0.0987022   6.152  5.2e-06 ***
## HarvestRain -0.0039715  0.0008538  -4.652 0.000154 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.295 on 20 degrees of freedom
## Multiple R-squared:  0.8286, Adjusted R-squared:  0.7943
## F-statistic: 24.17 on 4 and 20 DF,  p-value: 2.036e-07
```

Although it isn't now feasible to graph in 2D the *Price* in function of the other variables, we can still graph our residuals.

```
model3 <- fortify(model3)
p <- ggplot(model3, aes(.fitted, .resid)) + geom_point()
p <- p + geom_hline(yintercept = 0, col = "red", linetype = "dashed") + xlab("Fitted values")
p <- p + ylab("Residuals") + ggtitle("Plot of the residuals in function of the fitted values (multiple
```