Fourier Transform Model of Image Transformation

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Objectives

In this activity, we demonstrate the properties and applications of Fast Fourier Transform (FFT) algorithm.

Discrete FT

Familiarize with the discrete FFT and its functions by simulating the Fraunhofer diffraction patterns for different apertures.

Convolution

Use convolution to model an imaging device with varying aperture sizes

Correlation

Use the correlation technique for template matching.

Familiarization with Discrete FT

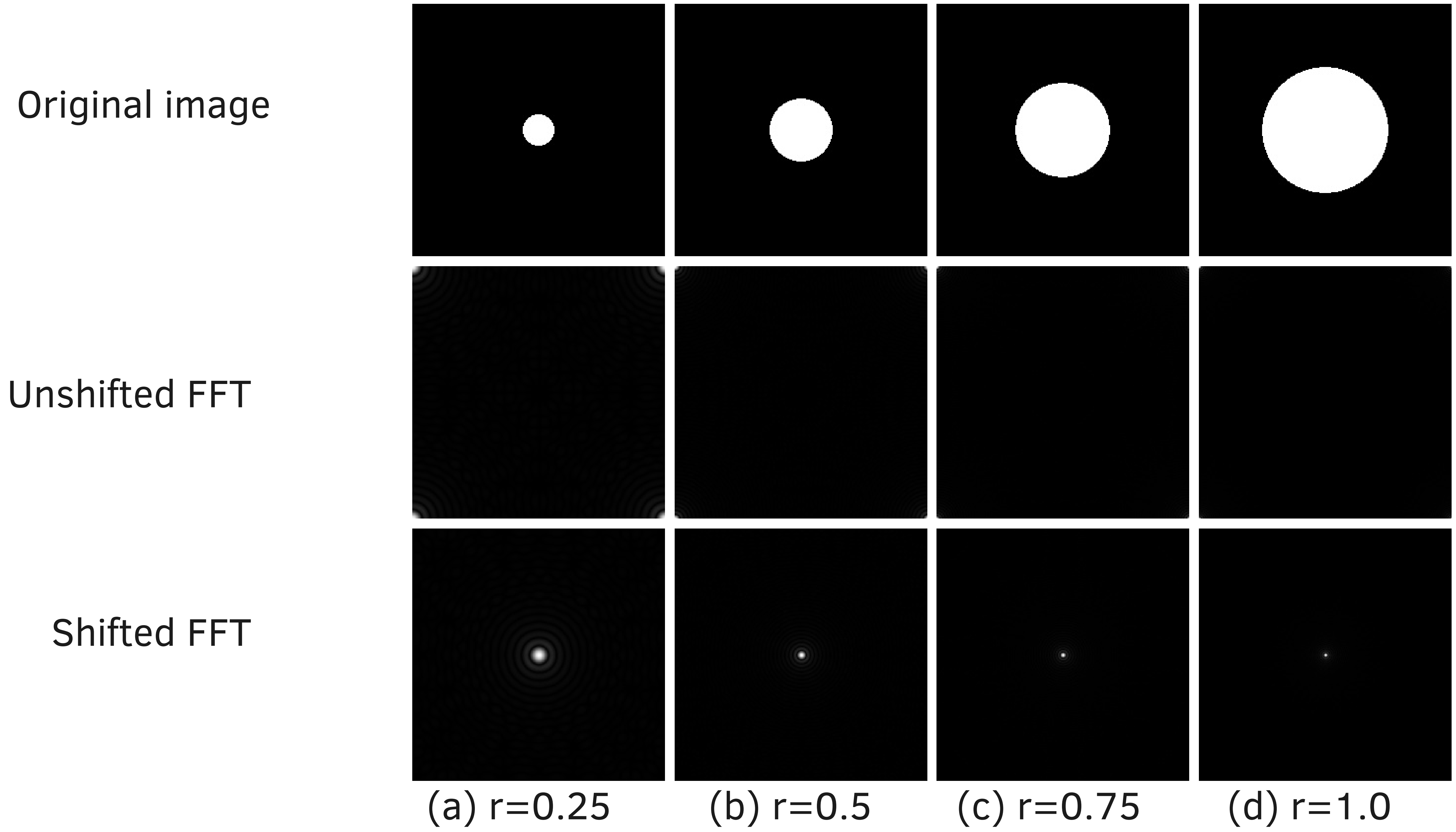


Figure 1. The FT of the circular apertures of different radius r(a.u.) are taken then shifted to finally display the diffraction pattern. The diffraction pattern for a circular aperture is known as the Airy pattern.

Familiarization with Discrete FT

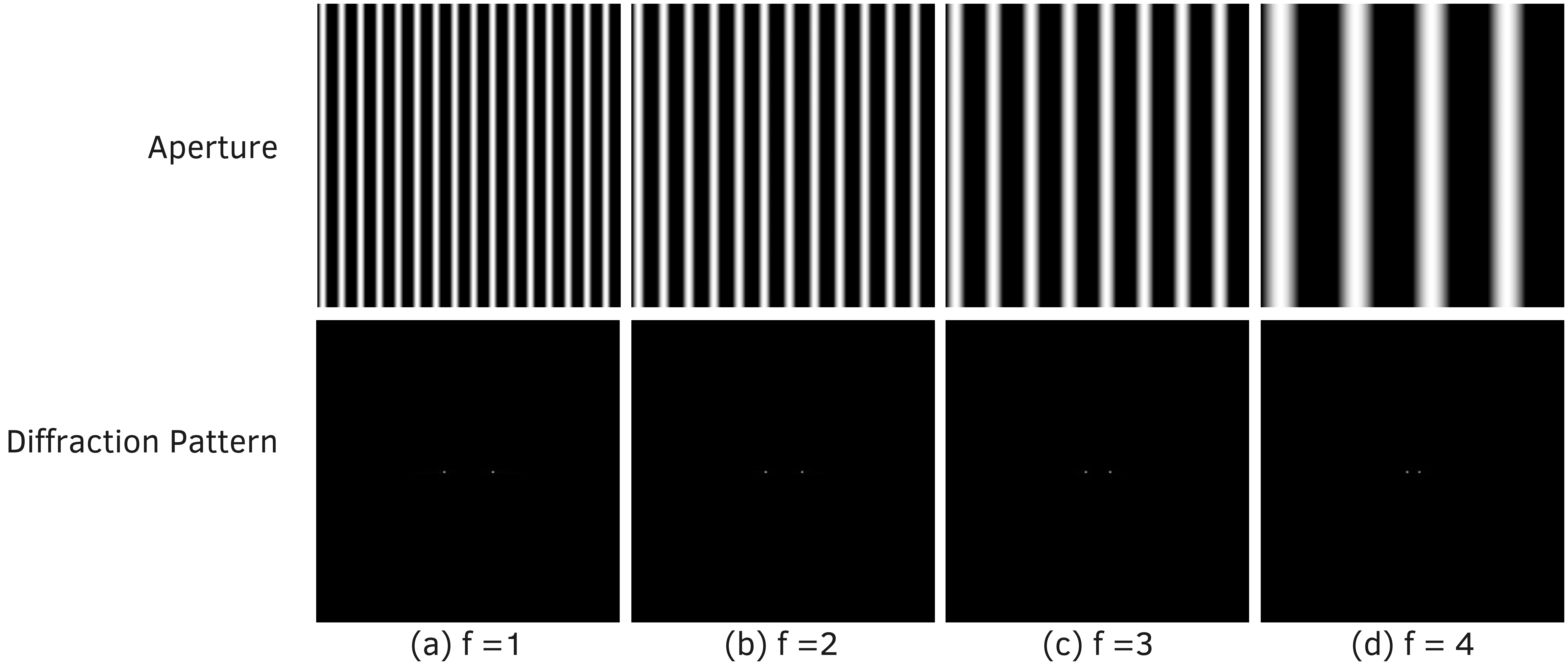


Figure 2. Sinusoids with varying frequencies f(cycles/a.u.) and their Fraunhofer diffraction patterns are simulated with FFT

Familiarization with Discrete FT

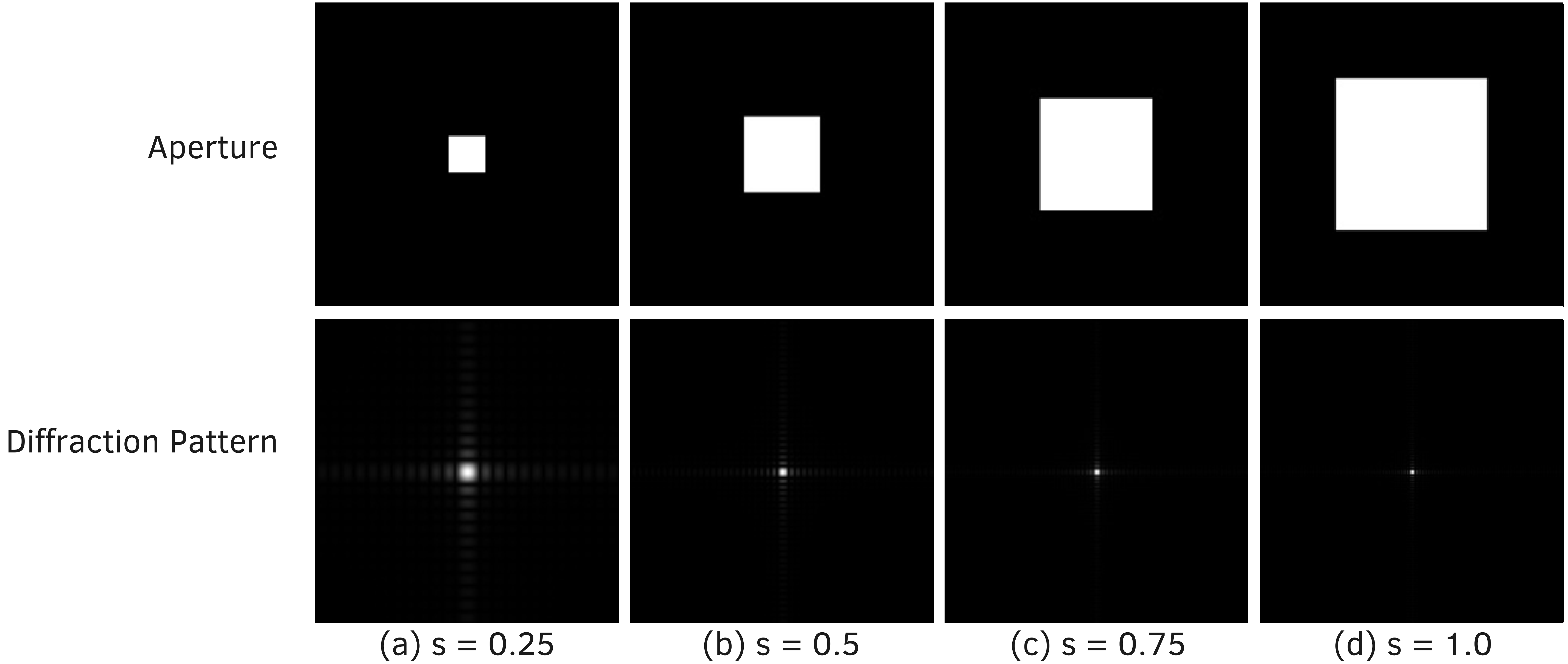


Figure 3. Square apertures with varying side lengths s(a.u.) and their Fraunhofer diffraction patterns are simulated by FFT

Familiarization with Discrete FT

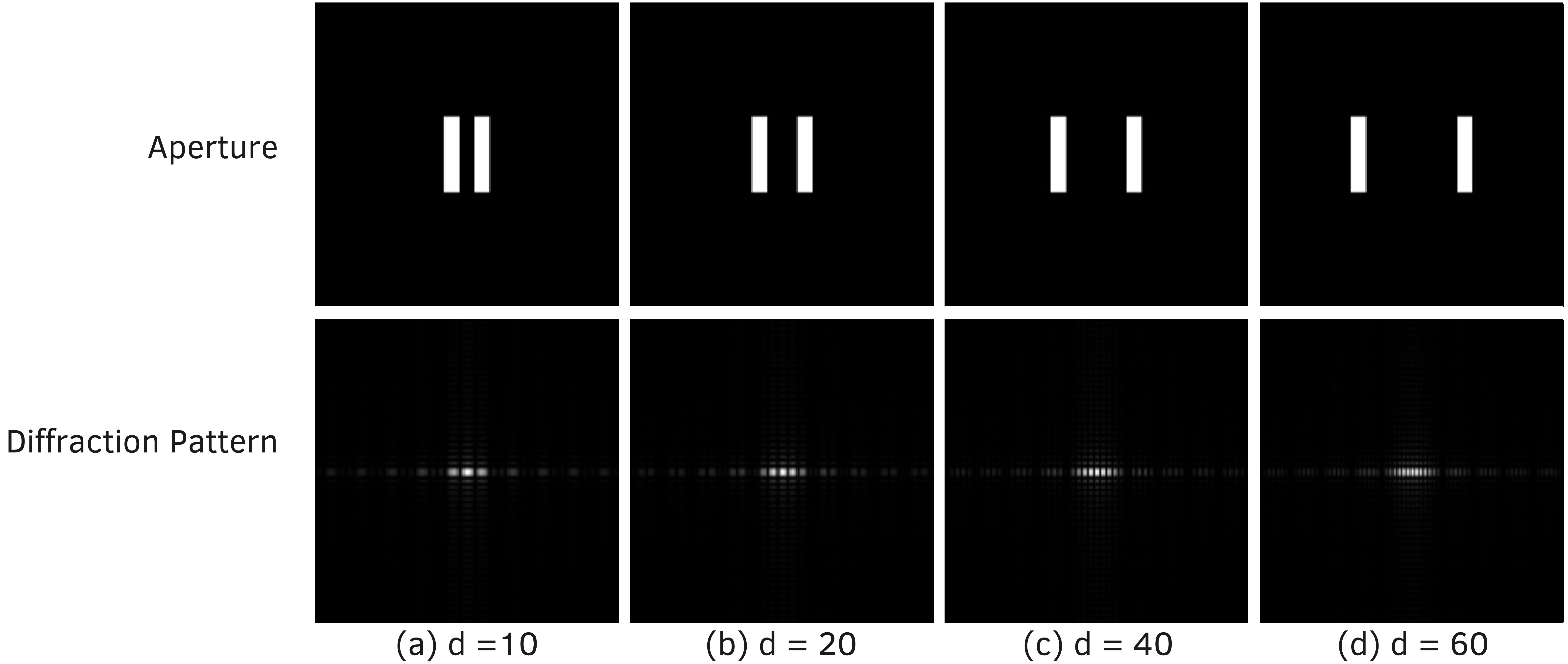


Figure 4. Double slit apertures with varying separation distances d(a.u.) and their Fraunhofer diffraction patterns simulated with FFT

Familiarization with Discrete FT

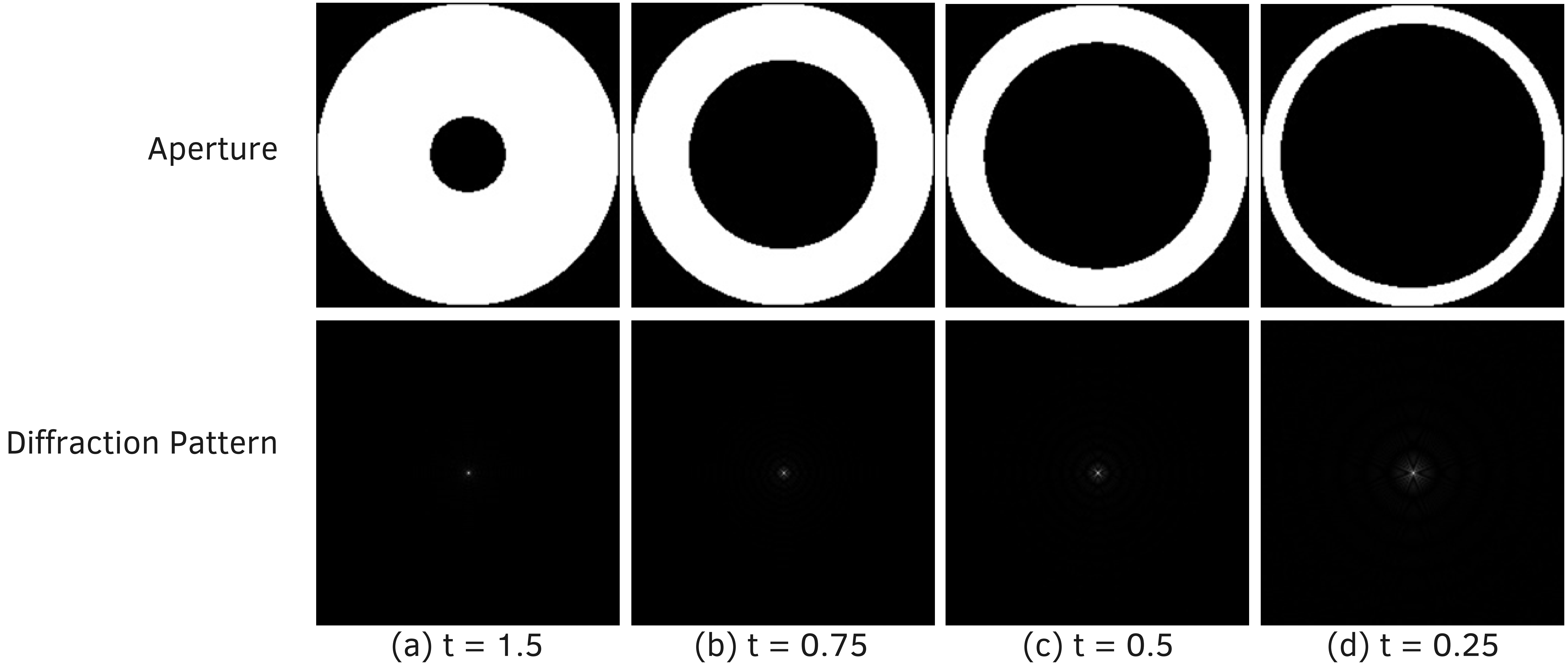


Figure 5. Annular apertures of outer radius of 2 units with varying thickness t(a.u.) and their Fraunhofer diffraction patterns simulated with FFT

Familiarization with Discrete FT

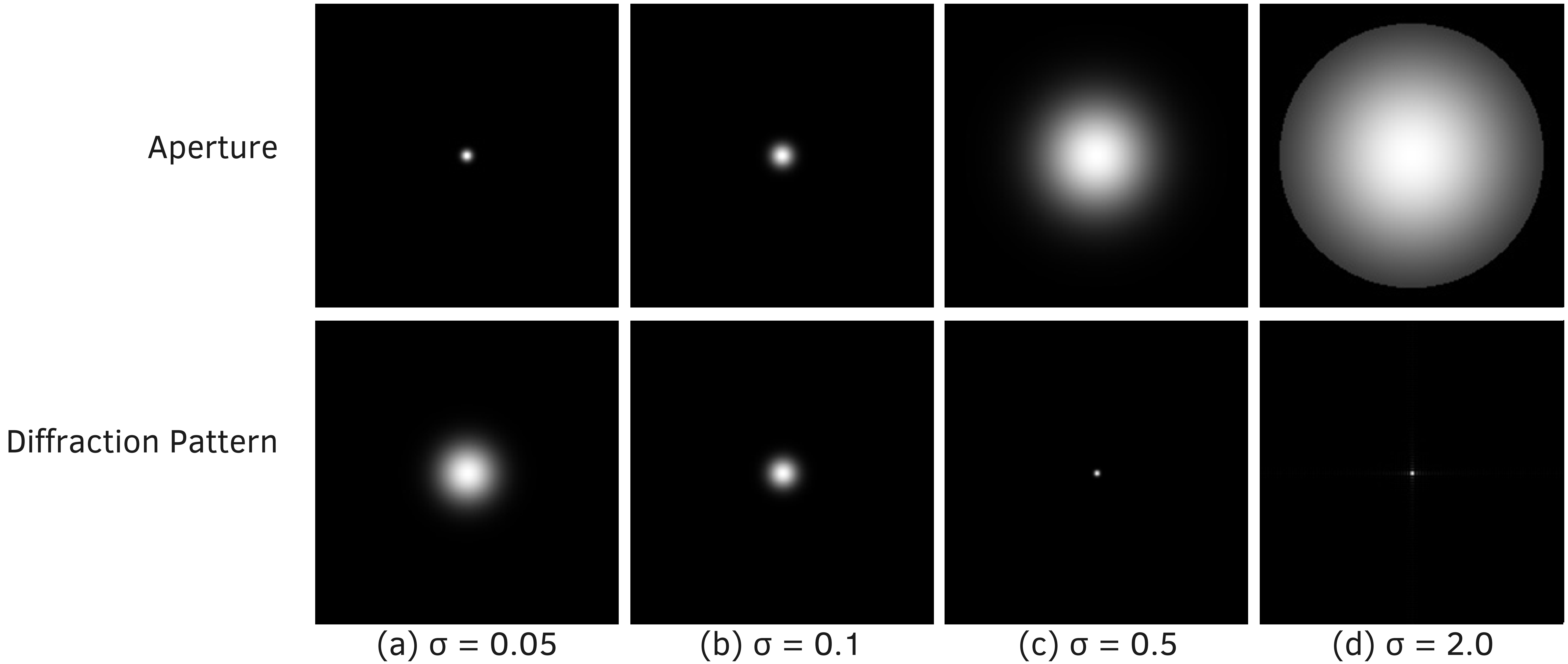


Figure 6. Gaussian aperture with radius=1.75 a.u. with varying spread σ and their Fraunhofer diffraction patterns simulated with FFT.

Analysis

Sinusoid along x-axis

The FFT of a sinusoid along the x-axis is shown to be 2 high-intensity points equidistant from the origin. That is, the FT of a sinusoid is localized at $|f|$. This demonstrates that FT decomposes an input signal to its frequency components.

Square Aperture

The Fraunhofer diffraction pattern function of a square aperture is a sinc function. Alternating rectangular fringes are observed emanating from a peak intensity at the center. The intensity profile is similar to that of an Airy pattern.

Double Slit

The diffraction pattern observed for the slit is similar to that in Young's Double-slit experiment. The maxima condition is given by:

$$y = m\lambda D/d$$

where λ is the wavelength of the light and D is the distance from the screen.
There are observed fringes in both the horizontal and vertical axis.

Analysis

Annular Aperture

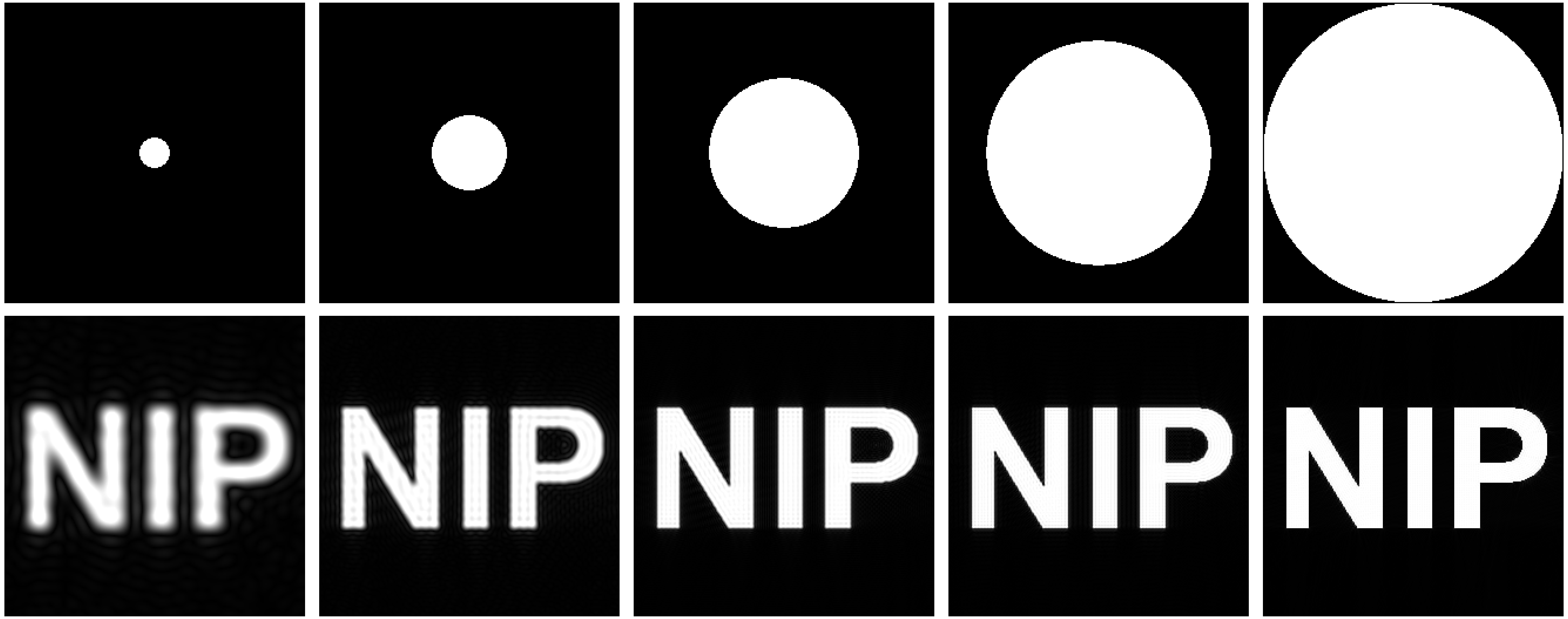
The annular aperture can be thought of as a circular aperture with a central obstruction. Its intensity profile is similar to that of an Airy pattern but dampened by a factor dictated by the radius of the central obstruction. We see in Figure 5, as the inner radius increases, the energy at the sides also increases.

Gaussian Aperture

The FT of the gaussian is also a gaussian. Hence we see the diffraction pattern that is also of a Gaussian profile.

Varying the parameters of certain apertures demonstrates the property of FT wherein the dimension of Fourier space is the inverse of the dimension of the image. Take for example the FT of the sine wave aperture, the distance between the two dots increases as the frequency decreases. Generally, we observe that increasing the size of an aperture returns a pattern of decreasing size.

Simulation of an Imaging System



(a) r =0.1 (b) r = 0.25 (c) r = 0.50 (d) r = 0.75 (e) r = 0.5

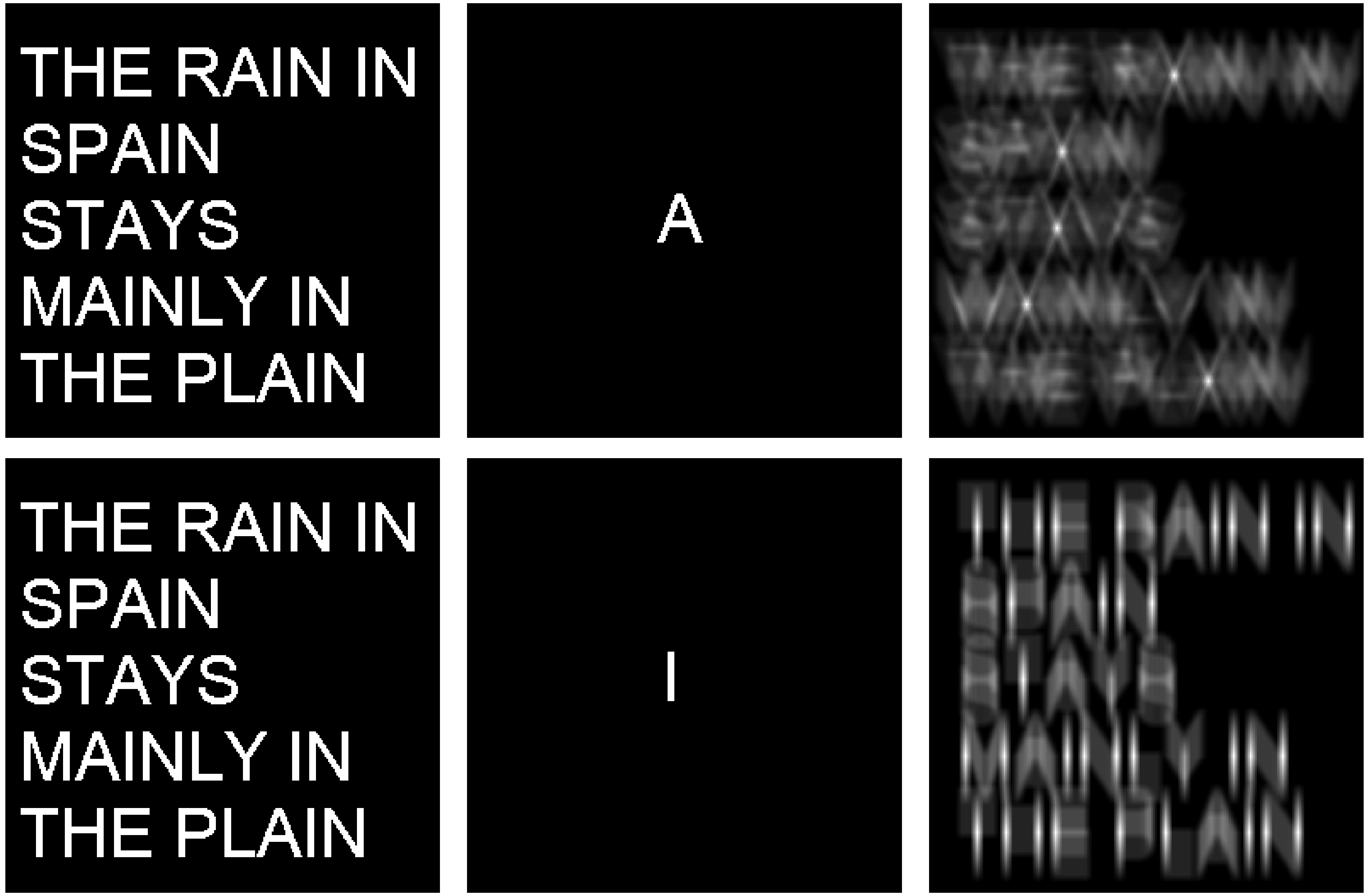
Figure 7. Imaging with different aperture sizes. The smaller the aperture size, the hazier the image.

Analysis

From Figure 7, we see that aperture size directly affects the quality of the image. Specifically, a larger aperture size renders a crisper or finer image. The image gets blurrier as the aperture size decreases. This simulates the finite radius of the lens of imaging systems and the amount of information it could gather on the object by convolution.

To get the convolved image, we take the product matrix of the FT of letter image and the shifted circular aperture image. The inverse FT of the product then produces the convolved image. This essentially can be thought of as taking the combination of the letter image an Airy pattern. Recalling the results in Figure 1, the Airy Pattern is more pronounced as the circle radius decreases.

Template Matching Using Correlation



(a) (b) (c)

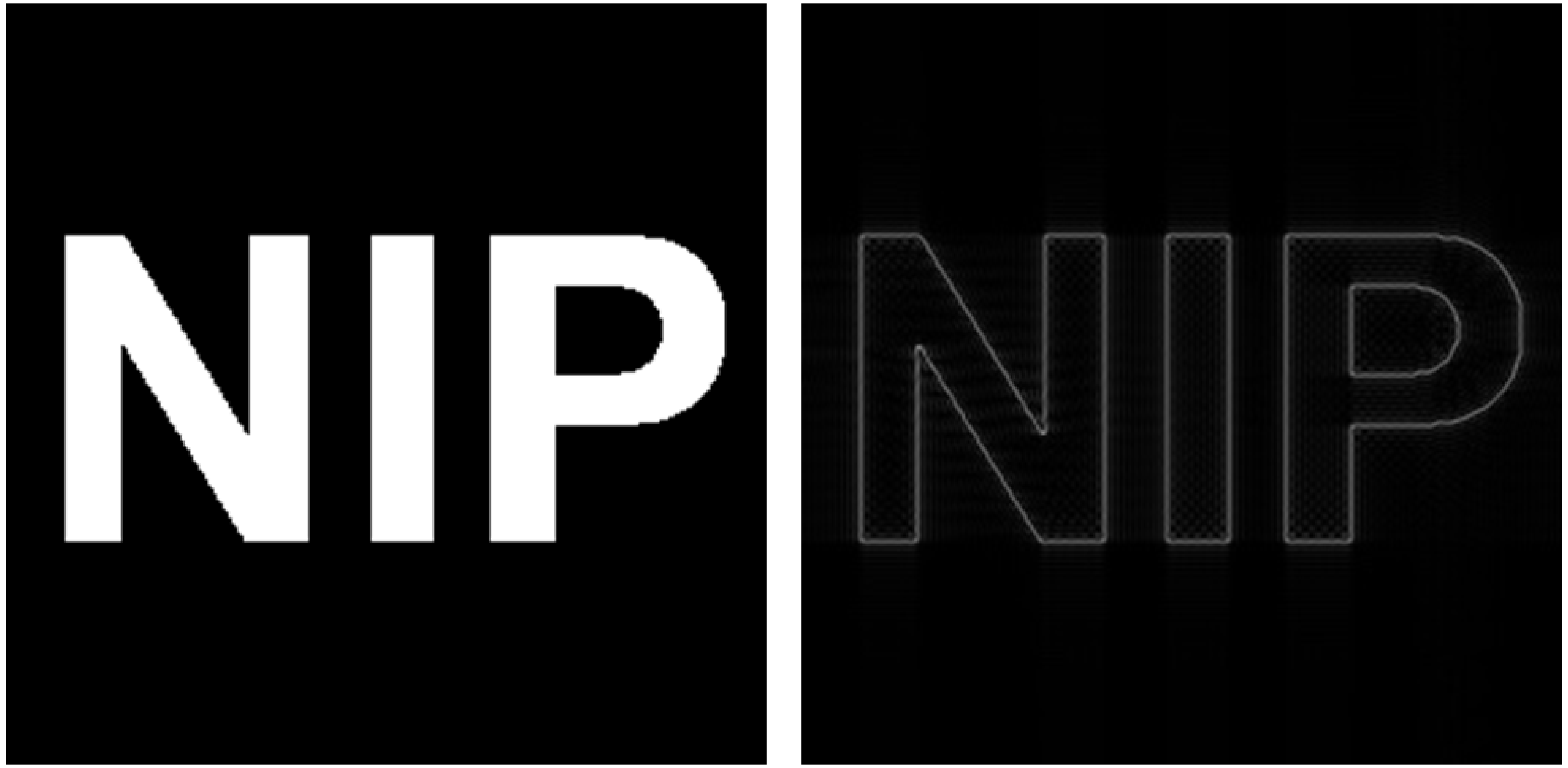
Figure 8. The (a) phrase image is correlated with the (b) template image. The (c) correlated image shows the location of the template in the target phrase image.

Analysis

Figure 8(c) shows the correlation between the template and phrase image. For template A, 5 distinct intensity points are seen from the correlated images that correspond to the location of the letter "A" in the target phrase image.

For the letter "I" template correlation, high-intensity points are not exclusive to the location of the actual letter "I" in the phrase image. This also includes the letters that have a straight line component.

Edge Detection using FT



Test image after HPF

Figure 9. Edge detection in an image using a High-Pass Filter(HPF). The edges of the letters are detected by taking the image FT and passing it through a circular HPF. This method takes advantage of the low and high frequencies in an FFT transformed image.

Reflection

The activity was really interesting as I encountered the FFT algorithm in Applied Physics 155, Fraunhofer diffraction in Physics 165, and FT in Physics 117. This activity is basically one giant cross-over of my previous and present Physics courses. The difficult part for me was not in fact the coding, but rather the analysis. I still pretty much relied on the Applied Physics 186 blogs available online.

I have tried expanding the application to other images, particularly with the template matching, however, it did not work out as expected, so I did not include it. If given the time, I will try to explore it more.

Self-Grade

I would give myself 33.3 points for technical correctness. I was able to meet all the objectives and complete all the activities in the problem. The results I got are also in line with the expected results. The codes I used for the activity are also relatively easy to follow.

I also score 33.3 for the quality of the presentation. All my figures are properly labeled and organized. I have also included the link for the codes in the appendix.

I give myself a score of 33.3 for self-reflection, as I had assessed myself by the given criteria and pointing system. I did the analysis of the results sufficiently and also cited the references I used.

I also give myself an additional point of 5 for including another application of FT, for a total of 103 points.

References

Soriano, M. (2021). Fourier Transform Model of Image Transformation

Aguinaldo, R.A. (2015, Sept 15). Activity 5: Fourier Transform Model of Image Formation https://medium.com/@aguigui17/activity-5-fourier-transform-model-of-image-formation-383198ff6ed6

Ghatak, A.K. & Thyagarajan, K. (1978). *Contemporary Optics.* Springer.

OpenCV. Fourier Transform. https://docs.opencv.org/3.4/de/dbc/tutorial_py_fourier_transform.html

Appendix

Codes and Figures

The codes and the images for the activity can be found here