PhD Industrial Organization: Homework 2

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November 16, 2025

1 Question 1

How to run code:
To run the code, navigate to the `src` directory and execute:

```
julia Rust_replication.jl
julia run_results.jl
```

The following is the output (note that it takes approximately **2248 seconds** to run):

=== Rust (1987) - NFXP Step 2 (fast) ===

$\beta = 0.9999$, $\theta_{30} = 0.3919$, $\theta_{31} = 0.5953$ ($p_2 = 0.0128$)
Bins: 90, $\Delta = 5000$ miles
Estimates: RC = 10.0428, $\theta_{11} = 0.456457$
Negative log-likelihood (nll): 163.584577

So my estimates for RC are close but θ_{11} is not that close.

2 Question 2

(b) CCP (Hotz–Miller) two–step estimator for the Rust (1987) bus problem. Let $x \in \mathcal{X} = \{1, \dots, 90\}$ index mileage bins (5k miles), and $i \in \{0, 1\}$ index choices ($i = 0$ keep, $i = 1$ replace). The discount factor β and transition matrices $P(y \mid x, i)$ are taken as given from Step 1. Idiosyncratic shocks are i.i.d. type I extreme value. We seek $\theta = (RC, \theta_{11})$ in the linear flow cost $c(x; \theta_{11}) = 0.001\, \theta_{11}\, x_{\text{miles}}$ so that

$$u(x, i; \theta) = \begin{cases} -RC - c(0; \theta_{11}) = -RC, & i = 1 \text{ (replace)}, \\ -c(x; \theta_{11}), & i = 0 \text{ (keep)}. \end{cases}$$

Step 1 (nonparametric CCPs). For each state x, compute smoothed empirical conditional choice probabilities (CCPs)

$$\widehat{p}(i \mid x) = \frac{n_{xi} + \alpha}{n_x + J\alpha}, \qquad J = 2,\ \alpha > 0,$$

where n_x is the number of observations with state x and n_{xi} the number of times choice i is taken at x (Laplace smoothing avoids $\log 0$ when actions are rare).

Step 2 (inclusive value from CCPs). With logit errors and the baseline $i_0 = 0$ (keep) normalized, the ex–ante value satisfies $EV(x) = \gamma + \log \sum_j \exp v_j(x)$ and, using $\sum_j \exp v_j(x) = 1/\widehat{p}(i_0 \mid x)$,

$$\widetilde{EV}(x) \equiv -\log \widehat{p}(i_0 \mid x),$$

which equals $EV(x)$ up to the constant γ; the constant cancels in differences below.

Step 3 (Hotz–Miller inversion). For each x and each $i \neq i_0$ form

$$\Delta_{\mathrm{HM}}(x,i) = \underbrace{\log \widehat{p}(i \mid x) - \log \widehat{p}(i_0 \mid x)}_{\text{current-period log-odds}} - \beta\Bigg(\underbrace{\sum_y \widetilde{EV}(y)\, P(y \mid x, i)}_{\text{next-period ex-ante value under } i} - \underbrace{\sum_y \widetilde{EV}(y)\, P(y \mid x, i_0)}_{\text{under baseline } i_0} \Bigg).$$

For the Rust utility, the structural difference is

$$u(x,i;\theta) - u(x,i_0;\theta) = \Big\{ -RC + 0.001\,\theta_{11}\, x_{\text{miles}}, \quad i = 1 \text{ vs } i_0 = 0.$$

Step 4 (minimum distance / NLS estimation). Choose θ to minimize

$$Q(\theta) = \sum_{x \in \mathcal{X}} \sum_{i \neq i_0} w_{xi} \left[\Delta_{\mathrm{HM}}(x,i) - \{u(x,i;\theta) - u(x,i_0;\theta)\} \right]^2,$$

with weights w_{xi} (e.g. $w_{xi} = n_x / \sum_x n_x$). If u is linear in θ, this is OLS; otherwise solve by nonlinear least squares (or GMM).

NPL refinement (Aguirregabiria–Mira). Initialize $p^{(0)}(i \mid x) = \widehat{p}(i \mid x)$. For $k = 0, 1, 2, \ldots$ repeat:

1. $\widetilde{EV}^{(k)}(x) = -\log p^{(k)}(i_0 \mid x)$.
2. Given $\theta^{(k)}$, form model CCPs

$$p^{\text{model}}(i \mid x; \theta^{(k)}) \ \propto\ \exp\Bigg(u(x,i;\theta^{(k)}) + \beta \sum_y \widetilde{EV}^{(k)}(y) P(y \mid x, i) \Bigg).$$

3. Update CCPs with damping: $p^{(k+1)} := \rho\, p^{(k)} + (1-\rho)\, p^{\text{model}}(\cdot \mid \cdot; \theta^{(k)})$.
4. Re-estimate $\theta^{(k+1)}$ by the criterion $Q(\theta)$ using $p^{(k+1)}$.
5. Stop when $\max_{x,i} |p^{(k+1)}(i \mid x) - p^{(k)}(i \mid x)| < \text{tol}$.