

PhD Industrial Organization: Homework 2

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1 Question 1

How to run code:

To run the code, navigate to the src directory and execute:

```
julia Rust_replication.jl  
julia run_results.jl
```

The following is the output (note that it takes approximately **2248 seconds** to run):

```
=== Rust (1987) - NFXP Step 2 (fast) ===  
  
 $\beta = 0.9999$ ,  $\theta_{30} = 0.3919$ ,  $\theta_{31} = 0.5953$  ( $p_2 = 0.0128$ )  
Bins: 90,  $\Delta = 5000$  miles  
Estimates: RC = 10.0428,  $\theta_{11} = 0.456457$   
Negative log-likelihood (nll): 163.584577
```

So my estimates for RC are close but θ_{11} is not that close.

2 Question 2

(b) CCP (Hotz–Miller) two–step estimator for the Rust (1987) bus problem. Let $x \in \mathcal{X} = \{1, \dots, 90\}$ index mileage bins (5k miles), and $i \in \{0, 1\}$ index choices ($i = 0$ keep, $i = 1$ replace). The discount factor β and transition matrices $P(y \mid x, i)$ are taken as given from Step 1. Idiosyncratic shocks are i.i.d. type I extreme value. We seek $\theta = (RC, \theta_{11})$ in the linear flow cost $c(x; \theta_{11}) = 0.001 \theta_{11} x_{\text{miles}}$ so that

$$u(x, i; \theta) = \begin{cases} -RC - c(0; \theta_{11}) = -RC, & i = 1 \text{ (replace)}, \\ -c(x; \theta_{11}), & i = 0 \text{ (keep)}. \end{cases}$$

Step 1 (nonparametric CCPs). For each state x , compute smoothed empirical conditional choice probabilities (CCPs)

$$\hat{p}(i | x) = \frac{n_{xi} + \alpha}{n_x + J\alpha}, \quad J = 2, \alpha > 0,$$

where n_x is the number of observations with state x and n_{xi} the number of times choice i is taken at x (Laplace smoothing avoids $\log 0$ when actions are rare).

Step 2 (inclusive value from CCPs). With logit errors and the baseline $i_0 = 0$ (keep) normalized, the ex-ante value satisfies $EV(x) = \gamma + \log \sum_j \exp v_j(x)$ and, using $\sum_j \exp v_j(x) = 1/\hat{p}(i_0 | x)$,

$$\widetilde{EV}(x) \equiv -\log \hat{p}(i_0 | x),$$

which equals $EV(x)$ up to the constant γ ; the constant cancels in differences below.

Step 3 (Hotz–Miller inversion). For each x and each $i \neq i_0$ form

$$\Delta_{\text{HM}}(x, i) = \underbrace{\log \hat{p}(i | x) - \log \hat{p}(i_0 | x)}_{\text{current-period log-odds}} - \beta \left(\underbrace{\sum_y \widetilde{EV}(y) P(y | x, i)}_{\text{next-period ex-ante value under } i} - \underbrace{\sum_y \widetilde{EV}(y) P(y | x, i_0)}_{\text{under baseline } i_0} \right).$$

For the Rust utility, the structural difference is

$$u(x, i; \theta) - u(x, i_0; \theta) = \left\{ -RC + 0.001 \theta_{11} x_{\text{miles}}, \quad i = 1 \text{ vs } i_0 = 0. \right.$$

Step 4 (minimum distance / NLS estimation). Choose θ to minimize

$$Q(\theta) = \sum_{x \in \mathcal{X}} \sum_{i \neq i_0} w_{xi} [\Delta_{\text{HM}}(x, i) - \{u(x, i; \theta) - u(x, i_0; \theta)\}]^2,$$

with weights w_{xi} (e.g. $w_{xi} = n_x / \sum_x n_x$). If u is linear in θ , this is OLS; otherwise solve by nonlinear least squares (or GMM).

(Optional) NPL refinement (Aguirregabiria–Mira). Initialize $p^{(0)}(i | x) = \hat{p}(i | x)$. For $k = 0, 1, 2, \dots$ repeat:

1. $\widetilde{EV}^{(k)}(x) = -\log p^{(k)}(i_0 | x)$.
2. Given $\theta^{(k)}$, form model CCPs

$$p^{\text{model}}(i | x; \theta^{(k)}) \propto \exp \left(u(x, i; \theta^{(k)}) + \beta \sum_y \widetilde{EV}^{(k)}(y) P(y | x, i) \right).$$

3. Update CCPs with damping: $p^{(k+1)} := \rho p^{(k)} + (1 - \rho) p^{\text{model}}(\cdot \mid \cdot; \theta^{(k)})$.
4. Re-estimate $\theta^{(k+1)}$ by the criterion $Q(\theta)$ using $p^{(k+1)}$.
5. Stop when $\max_{x,i} |p^{(k+1)}(i \mid x) - p^{(k)}(i \mid x)| < \text{tol}$.