Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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What is the causal effect of T^*?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Unknown function that *does not depend on i*
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete instrumental variable

Example 1: Smoking and Birthweight

RCT with 612 pregnant smokers in Glasgow, Scotland: 306 are offered financial incentives to attend smoking cessation program.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Offer of financial incentive

Example 2: Schooling and Test Scores

RCT in Afghanistan: a school is built in 6 out of 11 villages.

- y – Score on math and language test
- T^* – True school attendance
- T – Self-reported school attendance
- $\mathbf{x}$ – Household characteristics
- z – School built in village

Non-classical Measurement Error: Binary T^*

- Many applications of linear model have *binary* treatment
- Binary $T^* \implies \mathbb{E}[T^* w] \leq 0$
- Misclassification Probabilities:

$$\alpha_0 = \mathbb{P}(T = 1 | T^* = 0)$$
$$\alpha_1 = \mathbb{P}(T = 0 | T^* = 1)$$

- Non-Differential Measurement Error: $T \perp (z, u) | T^*$
- $\sigma^2_{T^*} \nless \sigma^2_T$ so work with α_0, α_1 rather than κ
- *Four-dimensional* Problem...

Results for a Mis-classified Binary Regressor

Aigner (1973), Bollinger (1996)...

- Even if $\rho_{T^*u} = 0$, OLS is biased and inconsistent: typically attenuated towards zero *but could flip signs!*

Kane et al. (1999), Black et al. (2000), Frazis et al. (2003)...

- $\rho_{zu} = 0 \implies$ IV solves endogenous regressor problem if there is no mis-classification
- $\rho_{T^*u} = 0$ and $\rho_{zu} = 0 \implies$ non-linear GMM estimator can solve the mis-classification problem

OLS and IV Probability Limits: Binary T^*

$$
\begin{aligned}
\text{plim}\left(\widehat{\beta}_{OLS}\right) &= \frac{\sigma^2_{T^*}}{\sigma^2_T}\left[\beta\left(1-\alpha_0-\alpha_1\right)+\frac{\sigma_{T^*u}}{\sigma^2_{T^*}}\right] \\
\text{plim}\left(\widehat{\beta}_{IV}\right) &= \frac{\beta}{1-\alpha_0-\alpha_1}+\frac{\sigma_{zu}}{\sigma_{zT}} \\
\sigma^2_{T^*} &= \frac{\left(p-\alpha_0\right)\left(1-p-\alpha_1\right)}{\left(1-\alpha_0-\alpha_1\right)^2}
\end{aligned}
$$

Where $p = \mathbb{P}(T = 1)$

What About Endogenous, Mis-measured T^*, Valid z?

$$y = \beta T^* + u$$
$$u = c + \varepsilon$$

- No results in the literature for this case
- Important setting in applied work: e.g. RCTs
- Discrete Instrument: $z \in \{z_1, \cdots, z_K\}$
- Non-parametric First Stage: $p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$
- What does $E[\varepsilon | z] = 0$ buy us in this case?

Observable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 1$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\dots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\dots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 1$	$\ldots$	$z = K$
$T^* = 0$	m^*_{01} p^*_{01}	m^*_{02} p^*_{02}	$\ldots$	m^*_{0K} p^*_{0K}
$T^* = 1$	m^*_{11} p^*_{11}	m^*_{12} p^*_{12}	$\ldots$	m^*_{1K} p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u|\,T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$p^*_k = \mathbb{P}(T^* = 1|z = z_k) = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}$$

Possible Assumptions On m^*_{tk}

Joint Exogeneity: $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m^*_{tk} = c \quad \text{for all } t, k$$

Exogenous Treatment: $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p^*_{tk} m^*_{tk} = c \quad \text{for all } t$$

Exogenous Instrument: $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p^*_k) m^*_{0k} + p^*_k m^*_{1k} = c \quad \text{for all } k$$

Moment Conditions Imposing $\mathbb{E}[\varepsilon|z] = 0$

One pair of equations for each $k = 1, \ldots, K$

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) + (1-\alpha_0)c - (p_k - \alpha_0)m^*_{1k}$$

$$\hat{y}_{1k} = (1-\alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m^*_{1k}$$

where $\hat{y}_{0k} = (1-p_k)\bar{y}_{0k}$ and $\hat{y}_{0k} = p_k\bar{y}_{1k}$

$\boxed{2K \text{ Equations in } K+4 \text{ Unknowns}}$

Mahajan

Proposition: β is Undentified Regardless of K

Proof Sketch

(1) Show that $\mathcal{W} = \beta/(1 - \alpha_0 - \alpha_1)$ is identified.

(2) Show that $\mathcal{Q} = c + \beta(1 - \alpha_0)/(1 - \alpha_0 - \alpha_1)$ is identified.

(3) $(1) + (2) \implies (\mathcal{Q}, \mathcal{W})$ are *fixed*

(4) Use (3) to rewrite equations in terms of $(\mathcal{Q}, \mathcal{W})$.

(5) Discover that there is only *one* equation per k! Rearranging:

$$m_{1k}^* = \frac{\mathcal{W}(\hat{y}_{0k} - \alpha_1 \mathcal{Q}) - \beta(\mathcal{Q} - \beta - \mathcal{W}\alpha_1) + \mathcal{W}^2(1 - p_k)\alpha_1}{\mathcal{W}(1 - p_k - \alpha_1) - \beta}$$

Special Case of Prev Proof: $\alpha_0 = 0$

$$\hat{y}_{0k} = c + p_k \beta \left(\frac{\alpha_1}{1 - \alpha_1} \right) - p_k m^*_{1k}$$

$$\hat{y}_{1k} = p_k \beta + p_k m^*_{1k}$$

Identification by Conditional Variances?

New Assumption

Homoskedastic errors w.r.t. the *instrument*: $E[\varepsilon^2|z] = E[\varepsilon^2]$

Not Crazy!

Holds in an RCT or a *true* natural experiment.

New Moment Conditions

For each pair (k, ℓ)

$$\begin{aligned} s_k^2 - s_\ell^2 = \mathcal{W}^2 \left[p_k(1-p_k) - p_\ell(1-p_\ell) + (\alpha_0 - \alpha_1)(p_k - p_\ell) \right] \\ +2\mathcal{W} \left[(p_k - \alpha_0)(m_{1k}^* - c) - (p_\ell - \alpha_0)(m_{1\ell}^* - c) \right] \end{aligned}$$

Where $s_k^2 = Var(y|z = z_k)$, and $\mathcal{W}$ is the Wald IV estimator.

Proposition: $(\alpha_0 - \alpha_1)$ is Identified

Define

$$\widetilde{\mathcal{W}}_{k\ell} = \frac{\mathbb{E}[yT|z_k] - \mathbb{E}[yT|z_\ell]}{p_k - p_\ell}$$

Show that:

$$\begin{aligned}(p_k - \alpha_0)(m^*_{1k} - c) - (p_\ell - \alpha_0)(m^*_{1\ell} - c) = \\ (p_k - p_\ell)\left[\widetilde{W}_{k\ell} - \mathbb{E}[y] - \mathcal{W}\left\{(1-p) + (\alpha_0 - \alpha_1)\right\}\right]\end{aligned}$$

Substituting and rearranging:

$$\alpha_0 - \alpha_1 = (2p - 1 - p_k - p_\ell) + \frac{2(\widetilde{W}_{k\ell} - \mathbb{E}[y])}{\mathcal{W}} - \frac{s_k^2 - s_\ell^2}{(p_k - p_\ell)\mathcal{W}^2}$$

What Good is $(\alpha_0 - \alpha_1)$?

- Test a necessary condition for *no mis-classification*: $\alpha_0 = \alpha_1$
- Simple, tighter partial identification bounds for β
- In some settings, one of the mis-classification probabilities is known to be zero $\implies \beta$ point identified

Identification from Third Moments

Simulation Study

$y = \beta T^* + \varepsilon$

$T^* = \mathbf{1}\left\{\gamma_0 + \gamma_1 z + \eta > 0\right\}$

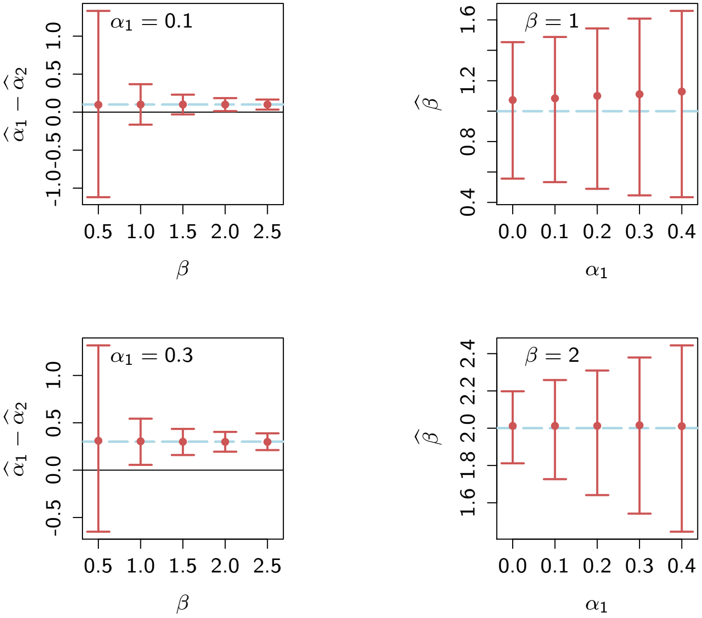
$\gamma_0 = \Phi^{-1}(\delta)$, $\gamma_1 = \Phi^{-1}(1-\delta) - \Phi(\delta)$ so that δ

E.g. if $\delta = 0.1$ then 10% of those *not* offered treatment get it anyway, and 10% of those offered treatment don't take it up.

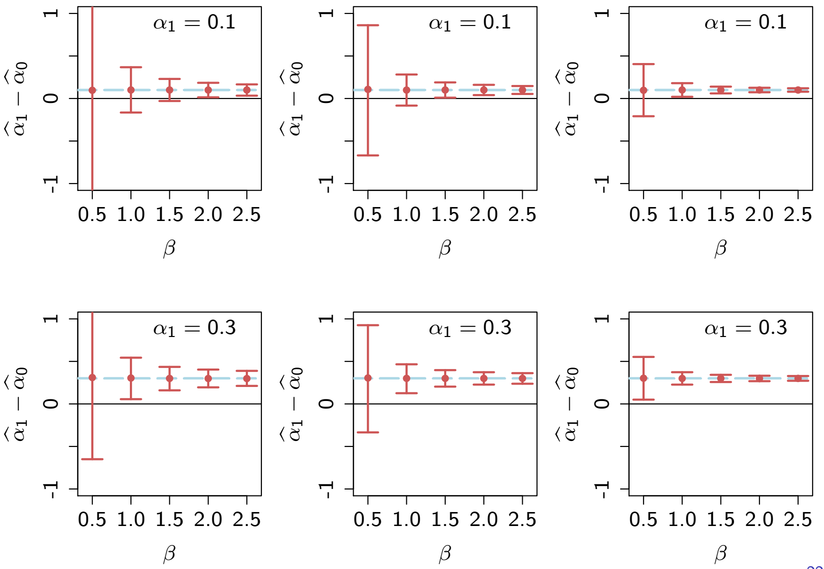
If $T^* = 0$ then $T = 0$ (E.g. Birthweight and smoking)

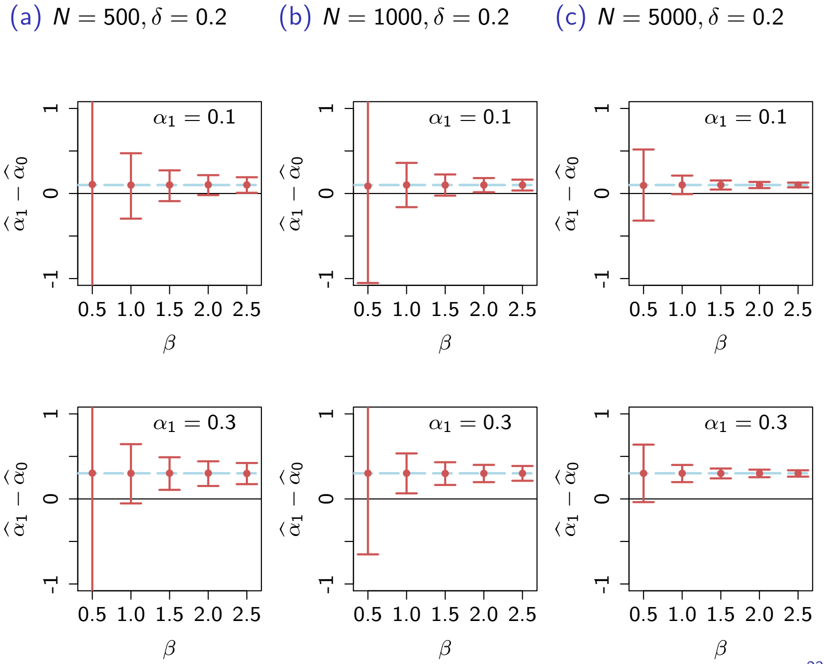
$T | T^* = 1 \sim$ Bernoulli(?)

$$\left[\begin{array}{c} \varepsilon \\ \eta \end{array}\right] \sim N\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} 1 & 0.3 \\ 0.3 & 1 \end{array}\right]\right)$$


$\alpha_1 = 0.1$
$\widehat{\alpha}_1 - \widehat{\alpha}_2$
-1.0 -0.5 0.0 0.5 1.0
0.5 1.0 1.5 2.0 2.5
β
$\beta = 1$
$\widehat{\beta}$
0.4 0.8 1.2 1.6
0.0 0.1 0.2 0.3 0.4
α_1
$\alpha_1 = 0.3$
$\widehat{\alpha}_1 - \widehat{\alpha}_2$
-0.5 0.0 0.5 1.0
0.5 1.0 1.5 2.0 2.5
β
$\beta = 2$
$\widehat{\beta}$
1.6 1.8 2.0 2.2 2.4
0.0 0.1 0.2 0.3 0.4
α_1

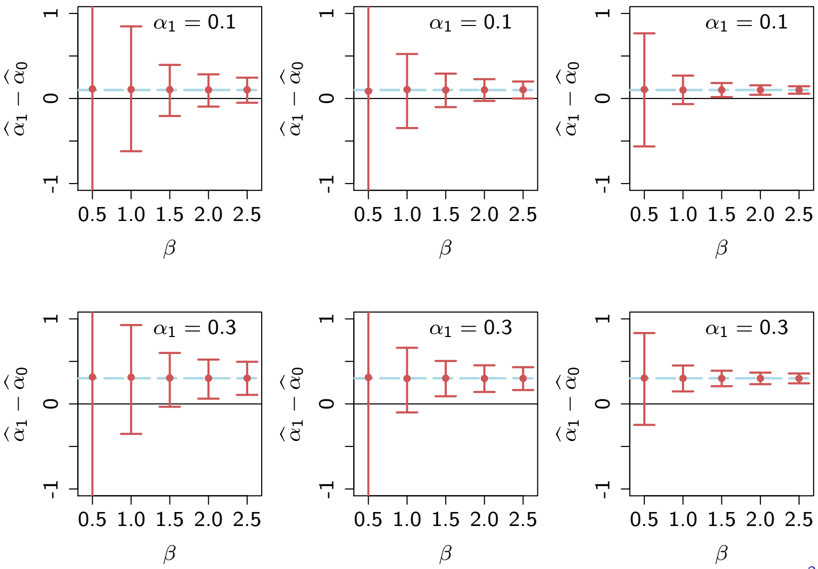
(a) $N = 500, \delta = 0.1$ (b) $N = 1000, \delta = 0.1$ (c) $N = 5000, \delta = 0.1$





(a) $N = 500, \delta = 0.2$
(b) $N = 1000, \delta = 0.2$
(c) $N = 5000, \delta = 0.2$
$\alpha_1 = 0.1$
$\alpha_1 = 0.3$
$\hat{\alpha}_1 - \hat{\alpha}_0$
β
0.5 1.0 1.5 2.0 2.5
-1 0 1

(a) $N = 500, \delta = 0.3$ (b) $N = 1000, \delta = 0.3$ (c) $N = 5000, \delta = 0.3$



Empirical Illustration: Schooling and Test Scores