Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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What is the causal effect of T^*?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Unknown function that *does not depend on i*
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete instrumental variable

Example: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with 1050 pregnant smokers in England: 521 given nicotine patches, the rest given placebo patches.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Indicator of nicotine patch

Example: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters. Randomly choose 6 and build a school in each village of these clusters.

- y – Child's score on math and language test
- T^* – Child's true school attendance
- T – Parent's report of child's school attendance
- $\mathbf{x}$ – Child and household characteristics
- z – School built in village

Related Literature

Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015)

Model: $y = c + \beta T^* + \varepsilon$

First-stage

$$p_k^* \equiv \mathbb{P}(T^* = 1|z = z_k) \neq \mathbb{P}(T^* = 1|z = z_\ell) \equiv p_\ell^*,\ k \neq \ell$$

Measurement Error

- Non-differential: $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- Does not depend on z:

$$\begin{aligned} \alpha_0 &= \mathbb{P}(T = 1|T^* = 0, z) \\ \alpha_1 &= \mathbb{P}(T = 0|T^* = 1, z) \end{aligned}$$

Notation

Define error term that absorbs constant: $u = c + \varepsilon$

Observable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\ldots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\ldots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\ldots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\ldots$	$z = K$
$T^* = 0$	m^*_{01} p^*_{01}	m^*_{02} p^*_{02}	$\ldots$	m^*_{0K} p^*_{0K}
$T^* = 1$	m^*_{11} p^*_{11}	m^*_{12} p^*_{12}	$\ldots$	m^*_{1K} p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u | T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p^*_k = \mathbb{P}(T^* = 1 | z = z_k)$$

Unrestricted System of Equations

$$\begin{aligned}(1-p_k)\bar{y}_{0k} &\equiv \widetilde{y}_{0k} = (\beta + m^*_{1k})\alpha_1 p^*_k + (1-\alpha_0)(1-p^*_k)m^*_{0k} \\ p_k\bar{y}_{1k} &\equiv \widetilde{y}_{1k} = (\beta + m^*_{1k})(1-\alpha_1)p^*_k + \alpha_0(1-p^*_k)m^*_{0k}\end{aligned}$$

$$p^*_k = \frac{p_k - \alpha_0}{1-\alpha_0-\alpha_1}$$

Possible Restrictions On m^*_{tk}

Joint Exogeneity: $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m^*_{tk} = c \quad \text{for all } t, k$$

Exogenous Treatment: $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p^*_{tk} m^*_{tk} = c \quad \text{for all } t$$

Exogenous Instrument: $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p^*_k) m^*_{0k} + p^*_k m^*_{1k} = c \quad \text{for all } k$$

Later I'll consider relaxing the assumption that z is exogenous...

Theorem: β is undentified regardless of K.

(For general case, see paper.)

Proof of special case: $\alpha_0 = 0$

1. System of equations:

$$\widetilde{y}_{0k} = c + p_k \beta \left(\frac{\alpha_1}{1 - \alpha_1} \right) - p_k m^*_{1k}$$

$$\widetilde{y}_{1k} = p_k \beta + p_k m^*_{1k}$$

2. $\beta/(1 - \alpha_1) \equiv \mathcal{W}$ is identified and imposing this, algebra gives $\beta \alpha_1/(1 - \alpha_1) = \mathcal{W} - \beta$.

Theorem: β is undentified regardless of K.

(For general case, see paper.)

Proof of special case: $\alpha_0 = 0$ continued...

3. Substituting:

$$\begin{aligned}(c + p_k \mathcal{W} - \widetilde{y}_{0k})/p_k &= \beta + m^*_{1k} \\ \widetilde{y}_{1k}/p_k &= \beta + m^*_{1k}\end{aligned}$$

4. Linear system in (β, m^*_{1k}) – no solution or ∞ of solutions.
5. Sum original pair of equations $\implies c + p_k \mathcal{W} - \widetilde{y}_{0k} = \widetilde{y}_{1k}$ thus ∞ of solutions. The model is unidentified.

Conditional *Second* Moment Independence.

New Assumption

Homoskedastic errors w.r.t. the *instrument*: $E[\varepsilon^2|z] = E[\varepsilon^2]$

Reasonable?

Makes sense in an RCT or a true natural experiment.

New Moment Conditions

Defining $\mu^*_{k\ell} = (p_k - \alpha_0)m^*_{1k} - (p_\ell - \alpha_0)m^*_{k\ell}$,

$$\mathbb{E}(y^2|z_k) - \mathbb{E}(y^2|z_\ell) \equiv \Delta\overline{y^2} = \beta\mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu^*_{k\ell}$$

$$\mathbb{E}(yT|z_k) - \mathbb{E}(yT|z_\ell) \equiv \Delta\overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu^*_{k\ell}$$

Theorem: $(\alpha_1 - \alpha_0)$ is Identified if $E[\varepsilon^2|z] = E[\varepsilon^2]$

Requires only binary z

Solve for $\mu^*_{k\ell}$, substitute $\beta = \mathcal{W}(1 - \alpha_0 - \alpha_1)$, rearrange to find

$$\alpha_1 - \alpha_0 = 1 + \mathcal{R}/\mathcal{W}, \quad \text{where} \quad \mathcal{R} \equiv \frac{\Delta\overline{y^2} - 2\mathcal{W}\Delta\overline{yT}}{\mathcal{W}(p_k - p_\ell)}.$$

What good is $(\alpha_1 - \alpha_0)$?

- Test necessary condition for *no mis-classification*: $\alpha_0 = \alpha_1$
- Simple, tighter partial identification bounds for β
- If α_0 known, e.g. zero $\implies$ β point identified

Conditional *Third* Moment Independence

New Assumption

Third moment independence w.r.t instrument: $E[\varepsilon^3|z] = E[\varepsilon^3]$

New Moment Conditions

Define $\lambda^*_{k\ell} = (p_k - \alpha_0)v^*_{1k} - (p_\ell - \alpha_0)v^*_{1\ell}$
where $v^*_{tk} = \mathbb{E}(u^2|\, T^* = t, z_k)$. Then

$$
\begin{aligned}
\mathbb{E}(y^3|z_k) &- \mathbb{E}(y^3|z_\ell) \equiv \\
\Delta\overline{y^3} &= \beta^2\mathcal{W}(p_k - p_\ell) + 3\beta\mathcal{W}\mu^*_{k\ell} + 3\mathcal{W}\lambda^*_{k\ell} \\
\mathbb{E}(y^2T|z_k) &- \mathbb{E}(y^2T|z_\ell) \equiv \\
\Delta\overline{y^2T} &= \beta(1-\alpha_1)\mathcal{W}(p_k - p_\ell) + 2(1-\alpha_1)\mathcal{W}\mu^*_{k\ell} + \lambda^*_{k\ell}
\end{aligned}
$$

Theorem: β, α_0 and α_1 identified

Adding $E[\varepsilon^3|z] = E[\varepsilon^3]$, z need only be binary.

Solve for $\lambda^*_{k\ell}$, substitute and rearrange. After further substitutions:

$$2\mathcal{W}^2(1-\alpha_1)^2 + 2\mathcal{R}\mathcal{W}(1-\alpha_1) + (\mathcal{S} - \mathcal{R}^2) = 0$$

where

$$\mathcal{S} \equiv \frac{\Delta\overline{y^3} - 3\mathcal{W}\left[\Delta\overline{y^2 T} + \mathcal{R}\Delta\overline{yT}\right]}{\mathcal{W}(p_k - p_\ell)}$$

- Quadratic in $(1-\alpha_1)$ and observables only
- Always two real roots: one is $(1-\alpha_1)$ and the other is α_0.
- To tell which is which, need $\alpha_0 + \alpha_1 < 1$.

Recap of Results

1. Using first-moment information alone, β is unidentified regardless of how many values the instrument takes on.
2. Using second moment information $\alpha_1 - \alpha_0$ is identified
   - Partial identification bound for β
   - Identifies β if α_0 is known (e.g. smoking/birthweight example)
3. Using third moment information β, α_0 and α_1 are identified so long as $\alpha_0 + \alpha_1 < 1$.

Empirical Illustration: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters. Randomly choose 6 and build a school in each village of these clusters ($N = 1468$).

- y – Child's score on math and language test
- T^* – Child's true school attendance
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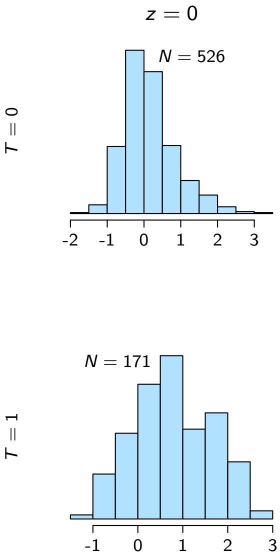
- $\mathbf{x}$ – Child and household characteristics
- z – School built in village

Empirical Illustration: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)



Empirical Illustration: Schooling and Test Scores

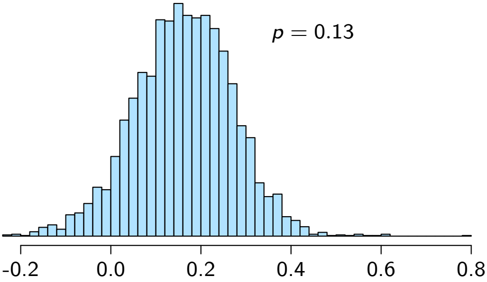
Burde & Linden (2013, AEJ Applied)

$\widehat{\beta}_{OLS} = 0.88$

$\widehat{\beta}_{IV} = 1.27$

$\widehat{\alpha}_1 - \widehat{\alpha}_0 = 0.18$

Cluster Bootstrap Distribution of $\hat{\alpha}_1 - \hat{\alpha}_0$



But what if z is endogenous?

Recall: Unrestricted System

$$
\begin{aligned}
\widetilde{y}_{0k} &= (\beta + m_{1k}^*)\alpha_1 p_k^* + (1-\alpha_0)(1-p_k^*)m_{0k}^* \\
\widetilde{y}_{1k} &= (\beta + m_{1k}^*)(1-\alpha_1) p_k^* + \alpha_0(1-p_k^*)m_{0k}^*
\end{aligned}
$$

Intelligible Quantities

$$
\begin{aligned}
\delta_{T^*} &\equiv \mathbb{E}\left[u|T^*=1\right] - \mathbb{E}\left[u|T^*=0\right] \\
\delta_z &\equiv \mathbb{E}\left[u|z=1\right] - \mathbb{E}\left[u|z=0\right]
\end{aligned}
$$

…both are linear functions of m_{tk}^*.

Identified Set for $(\alpha_0, \alpha_1, \delta_{T^*}, \delta_z)$

First Moment Information

$$\delta_z = C(\alpha_0, \alpha_1 | \mathbf{p}, \mathbf{q}, \bar{\mathbf{y}}) - \left(\frac{p_1 - p_2}{1 - \alpha_0 - \alpha_1} \right) \delta_{T^*}$$

Second Moment Information

$$\sigma_{tk}^2 = Var(y | T = t, z = k) > 0$$

$$\implies \left[\sigma_{tk}^2 - Q_{tk}(\delta_{T^*}, \delta_z, \alpha_0, \alpha_1 | \mathbf{p}, \mathbf{q}, \bar{\mathbf{y}}) \right] > 0$$

Approaches to (Partial) Identification

Identification

- $\delta_z = 0, \alpha_0 = \alpha_1 = 0 \Rightarrow$ Wald Estimator
- Joint Exogeneity ($\Rightarrow \delta_{T^*} = \delta_z = 0$)
  Kane et al. (1999), Black et al. (2000), Mahajan (2006)...

Partial Identification

- Frazis & Loewenstein (2003): $\delta_z = 0$, $(\alpha_0 + \alpha_1) \in [\ell, u]$
- Conley et al. (2012): $\delta_z \in [\underline{\delta}_z, \bar{\delta}_z]$, $\alpha_0, = \alpha_1 = 0$
- Nevo & Rosen (2012): $\delta_T^* > \delta_z$, $\delta_T^* \delta_z > 0$, $\alpha_0 = \alpha_1 = 0$

Angrist Example

Conclusion

- Effect of endogenous, mis-measured, binary treatment.
- Important in applied work but no solution in the literature.
- New partial and point identification results by exploiting higher moments of outcome variable.
- Test necessary condition for absence of measurement error.
- Next steps: use full independence of $z \rightarrow$ optimal estimator

Simulation Study

Simulation Study: $y = \beta T^* + \varepsilon$

- $(\varepsilon, \eta) \sim$ jointly normal, mean 0, variance 1, corr. 0.3.
- First stage: $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
  - Half of subjects have $z = 1$, the rest have $z = 0$.
  - $\gamma_0 = \Phi^{-1}(\delta)$
  - $\gamma_1 = \Phi^{-1}(1 - \delta) - \Phi(\delta)$
  - δ equals fraction of those offered treatment who fail to take it up *and* fraction of those not offered treatment who do.
- Generate T as follows:
  - $T^* = 0 \implies T = 0$, i.e. $\alpha_0 = 0$
  - $T | T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$
  - α_0, α_1 unknown to econometrician.

$\boxed{\text{Sampling Distribution of } \widehat{\alpha}_1 - \widehat{\alpha}_0}$