

# Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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## What is the causal effect of $T^*$ ?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- ▶  $y$  – Outcome of interest
- ▶  $h$  – Unknown function that *does not depend on  $i$*
- ▶  $T^*$  – Unobserved, endogenous binary treatment
- ▶  $T$  – Observed, mis-measured binary surrogate for  $T^*$
- ▶  $\mathbf{x}$  – Exogenous covariates
- ▶  $\varepsilon$  – Mean-zero error term
- ▶  $z$  – Discrete (typically binary) instrumental variable

### Target of Inference:

ATE function:  $\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$

# Example: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- ▶  $y$  – Log wage
- ▶  $T^*$  – True training attendance
- ▶  $T$  – Self-reported training attendance
- ▶  $x$  – Individual characteristics
- ▶  $z$  – Offer of job training

# Example: Returns to Schooling

Oreopoulos (2006, AER)

Fuzzy RD: minimum school-leaving age in UK increased from 14 to 15 in 1947 but some already stayed until 15 before the law and others failed to comply after it.

- ▶  $y$  – Log wage
- ▶  $T^*$  – School attendance at age 15
- ▶  $T$  – Self-report of school attendance at age 15
- ▶  $x$  – Individual characteristics
- ▶  $z$  – Indicator: born in or after 1933

# Related Literature

## Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

## Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

## Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015)

Model:  $y = c + \beta T^* + \varepsilon$

Valid Instrument

$$\mathbb{E}[\varepsilon|z] = 0.$$

First-stage

$$p_k^* \equiv \mathbb{P}(T^* = 1|z = z_k) \neq \mathbb{P}(T^* = 1|z = z_\ell) \equiv p_\ell^*, \quad k \neq \ell$$

Non-differential Measurement Error

- ▶  $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- ▶  $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$
- ▶  $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$
- ▶  $\alpha_0 + \alpha_1 < 1$

Observable Moments:  $y = c + \beta T^* + \varepsilon$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ $p_{01}$	$\bar{y}_{02}$ $p_{02}$	$\dots$	$\bar{y}_{0K}$ $p_{0K}$
$T = 1$	$\bar{y}_{11}$ $p_{11}$	$\bar{y}_{12}$ $p_{12}$	$\dots$	$\bar{y}_{1K}$ $p_{1K}$

$$\bar{y}_{tk} = \mathbb{E}[y | T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1 | z = z_k)$$

## Unobservable Moments: $y = \beta T^* + u$

Define error term that absorbs constant:  $u = c + \varepsilon$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T^* = 0$	$m_{01}^*$ $p_{01}^*$	$m_{02}^*$ $p_{02}^*$	$\dots$	$m_{0K}^*$ $p_{0K}^*$
$T^* = 1$	$m_{11}^*$ $p_{11}^*$	$m_{12}^*$ $p_{12}^*$	$\dots$	$m_{1K}^*$ $p_{1K}^*$

$$m_{tk}^* = \mathbb{E}[u | T^* = t, z = z_k], \quad p_{tk}^* = q_k p_k^*$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$$



# Mahajan (2006, ECTA)

## Regression Model

$$y = \mathbb{E}[y|T^*] + \nu$$

$$\mathbb{E}[\nu|T^*] = 0 \text{ by construction}$$

## Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon|T^*] \neq 0$$

## Main Result (Correct) – Exogenous Treatment

Relevant binary instrument  $z$  ( $p_k^* \neq p_\ell^*$ ) identifies  $\alpha_0, \alpha_1$  and  $\mathbb{E}[y|T^*]$  provided that  $\mathbb{E}[\nu|T^*, T, z] = 0$  and  $\alpha_0 + \alpha_1 < 1$ .

## Extension (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$ ,  $p_k^* \neq p_\ell^*$ ,  $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$  identified.

► Details

What if  $z$  takes on more than two values?

$\mathbb{E}[\varepsilon|z] = 0 \implies$  *pair of equations for each  $k = 1, \dots, K$*

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0) \left( \frac{\beta}{1 - \alpha_0 - \alpha_1} \right) + (1 - \alpha_0)c - (p_k - \alpha_0)m_{1k}^*$$

$$\hat{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0) \left( \frac{\beta}{1 - \alpha_0 - \alpha_1} \right) + \alpha_0 c + (p_k - \alpha_0)m_{1k}^*$$

where  $\hat{y}_{0k} = (1 - p_k)\bar{y}_{0k}$  and  $\hat{y}_{1k} = p_k\bar{y}_{1k}$

*2K Equations in  $K + 4$  Unknowns*

*Theorem:*  $\beta$  is unidentified regardless of  $K$ .

Proof of special case:  $\alpha_0 = 0$

1. System of equations:

$$\tilde{y}_{0k} = c + p_k \left( \frac{\beta \alpha_1}{1 - \alpha_1} \right) - p_k m_{1k}^*$$

$$\tilde{y}_{1k} = p_k \beta + p_k m_{1k}^*$$

2.  $\beta/(1 - \alpha_1) \equiv \mathcal{W}$  identified,  $\beta \alpha_1/(1 - \alpha_1) = \mathcal{W} - \beta \implies$

$$(c + p_k \mathcal{W} - \tilde{y}_{0k})/p_k = \beta + m_{1k}^*$$

$$\tilde{y}_{1k}/p_k = \beta + m_{1k}^*$$

3. Sum equations from 1.  $\implies (c + p_k \mathcal{W} - \tilde{y}_{0k}) = \tilde{y}_{1k}$

What about  $\alpha_0 + \alpha_1 < 1$ ?

$$\mathcal{W} = \frac{\beta}{1 - \alpha_0 - \alpha_1}, \quad p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}, \quad 1 - p_k^* = \frac{1 - p_k - \alpha_1}{1 - \alpha_0 - \alpha_1}$$

If  $\alpha_0 + \alpha_1 < 1$ , then:

- ▶  $\text{Cor}(T, T^*) > 0 \iff \alpha_0 + \alpha_1 < 1$
- ▶  $\beta$  has same sign as  $\mathcal{W}$
- ▶  $\alpha_0 < \min_k \{p_k\}$
- ▶  $\alpha_1 < \min_k \{1 - p_k\}$
- ▶ Two-sided bound for  $\beta$

# Non-differential Measurement Error Assumption

- ▶  $\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$
- ▶  $\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$
- ▶  $\alpha_0 + \alpha_1 < 1$

## Weaker

$$\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$$

## Stronger

$\varepsilon$  conditionally independent of  $T$  given  $T^*$  and  $z$ .

## Bounds From Stronger Measurement Error Assumption

Define  $F_{tk}(\tau) = \mathbb{P}(Y \leq \tau | T = t, z_k)$  and  $F_k(\tau) = \mathbb{P}(Y \leq \tau | z_k)$

$$\alpha_0 \leq p_k \inf_{\tau} \left\{ \left[ \frac{F_{1k}(\tau)}{F_k(\tau)} \right] \wedge \left[ \frac{1 - F_{1k}(\tau)}{1 - F_k(\tau)} \right] \right\} \leq p_k$$

$$\alpha_1 \leq (1 - p_k) \inf_{\tau} \left\{ \left[ \frac{F_{0k}(\tau)}{F_k(\tau)} \right] \wedge \left[ \frac{1 - F_{0k}(\tau)}{1 - F_k(\tau)} \right] \right\} \leq (1 - p_k)$$

Bounds for  $(\alpha_0, \alpha_1)$  do *not* require  $z$  to be a valid instrument!

Diagram for Oreopoulous? Burde & Linden?

# Sufficient Conditions To Identify $\alpha_0, \alpha_1$ , and $\beta$

## Baseline Assumptions

- ▶  $\mathbb{E}[\varepsilon|z] = 0$
- ▶  $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- ▶  $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$ ,  $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$ ,  $\alpha_0 + \alpha_1 < 1$

## Strengthen IV Assumption

- ▶  $\mathbb{E}[\varepsilon^2|z] = \mathbb{E}[\varepsilon^2]$
- ▶  $\mathbb{E}[\varepsilon^3|z] = \mathbb{E}[\varepsilon^3]$

## Strengthen Measurement Error Assumption

- ▶  $\mathbb{E}[\varepsilon^2|T^*, T, z] = \mathbb{E}[\varepsilon^2|T^*, z]$
- ▶  $\mathbb{E}[\varepsilon^3|T^*, T, z] = \mathbb{E}[\varepsilon^3|T^*, z]$



# Identification Argument: Part I

## Impose 2nd Moment Restrictions

$$\mathbb{E}[\varepsilon^2|z] = \mathbb{E}[\varepsilon^2] \text{ and } \mathbb{E}[\varepsilon^2|T^*, T, z] = \mathbb{E}[\varepsilon^2|T^*, z]$$

## Obtain New Moment Conditions

$$\mathbb{E}(y^2|z_k) - \mathbb{E}(y^2|z_\ell) \equiv \Delta \overline{y^2} = \beta \mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu_{k\ell}^*$$

$$\mathbb{E}(yT|z_k) - \mathbb{E}(yT|z_\ell) \equiv \Delta \overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu_{k\ell}^*$$

$$\mu_{k\ell}^* = (p_k - \alpha_0)m_{1k}^* - (p_\ell - \alpha_0)m_{k\ell}^*$$

## Identify $(\alpha_1 - \alpha_0)$

$$\alpha_1 - \alpha_0 = 1 + \mathcal{R}/\mathcal{W}, \quad \text{where} \quad \mathcal{R} \equiv \frac{\Delta \overline{y^2} - 2\mathcal{W}\Delta \overline{yT}}{\mathcal{W}(p_k - p_\ell)}$$

# Identification Argument: Part II

## Impose 3rd Moment Restrictions

$$\mathbb{E}[\varepsilon^3|z] = \mathbb{E}[\varepsilon^3] \text{ and } \mathbb{E}[\varepsilon^3|T^*, T, z] = \mathbb{E}[\varepsilon^3|T^*, z]$$

## New Moment Conditions

$$\mathbb{E}(y^3|z_k) - \mathbb{E}(y^3|z_\ell) \equiv \Delta \overline{y^3} = \beta^2 \mathcal{W}(p_k - p_\ell) + 3\beta \mathcal{W}\mu_{k\ell}^* + 3\mathcal{W}\lambda_{k\ell}^*$$

$$\mathbb{E}(y^2 T|z_k) - \mathbb{E}(y^2 T|z_\ell) \equiv \Delta \overline{y^2 T} = \beta(1 - \alpha_1) \mathcal{W}(p_k - p_\ell) + 2(1 - \alpha_1) \mathcal{W}\mu_{k\ell}^* + \lambda_{k\ell}^*$$

$$\lambda_{k\ell}^* = (p_k - \alpha_0) v_{1k}^* - (p_\ell - \alpha_0) v_{1\ell}^*$$

$$v_{tk}^* = \mathbb{E}(u^2|T^* = t, z_k)$$

# Identification Argument: Part III

## Tedious Algebra

$$2\mathcal{W}^2(1 - \alpha_1)^2 + 2\mathcal{R}\mathcal{W}(1 - \alpha_1) + (\mathcal{S} - \mathcal{R}^2) = 0$$

where

$$\mathcal{S} \equiv \frac{\Delta \bar{y}^3 - 3\mathcal{W} [\Delta \bar{y}^2 \bar{T} + \mathcal{R} \Delta \bar{y} \bar{T}]}{\mathcal{W}(p_k - p_\ell)}$$

## Solve Quadratic

- ▶ Depends on  $(1 - \alpha_1)$  and observables only
- ▶ Always two real roots: one is  $(1 - \alpha_1)$  and the other is  $\alpha_0$ .
- ▶ To tell which is which, need  $\alpha_0 + \alpha_1 < 1$ .

# Simulation Study: $y = \beta T^* + \varepsilon$

## Errors

$(\varepsilon, \eta) \sim$  jointly normal, mean 0, variance 1, correlation 0.3.

## First-Stage

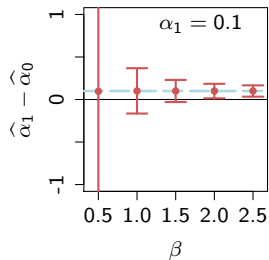
- ▶ Half of subjects have  $z = 1$ , the rest have  $z = 0$ .
- ▶  $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
- ▶  $\delta = \mathbb{P}(T^* = 0|z = 1) = \mathbb{P}(T^* = 1|z = 0)$

## Mis-classification

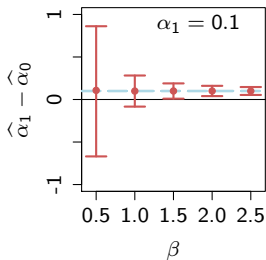
- ▶ Set  $\alpha_0 = 0$  so  $T^* = 0 \implies T = 0$
- ▶  $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$
- ▶  $\alpha_0, \alpha_1$  unknown to econometrician.

Sampling Distribution of  $\hat{\alpha}_1 - \hat{\alpha}_0$

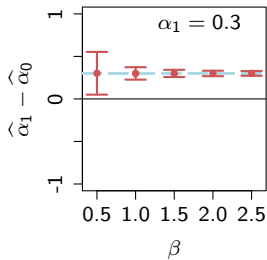
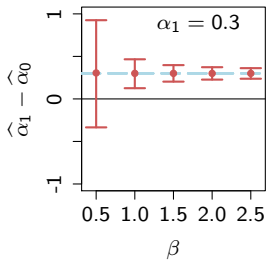
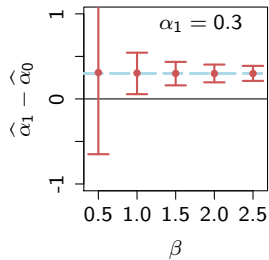
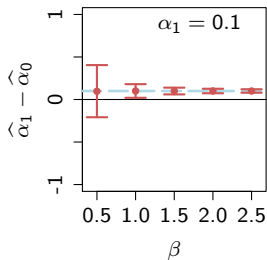
(a)  $N = 500, \delta = 0.1$



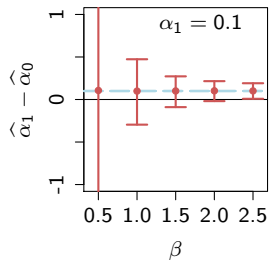
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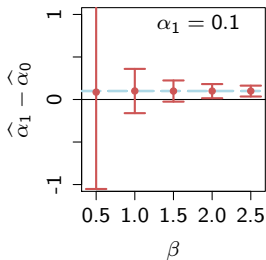
(c)  $N = 5000, \delta = 0.1$



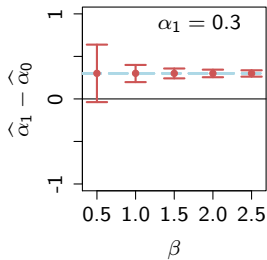
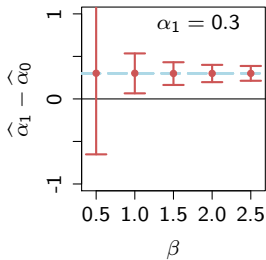
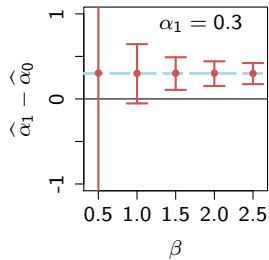
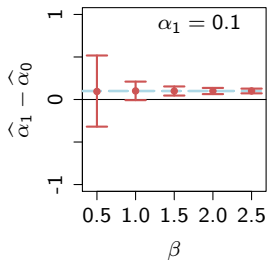
(a)  $N = 500, \delta = 0.2$



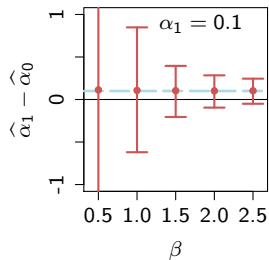
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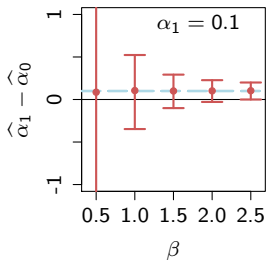
(c)  $N = 5000, \delta = 0.2$



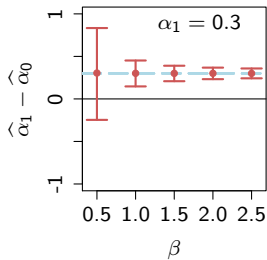
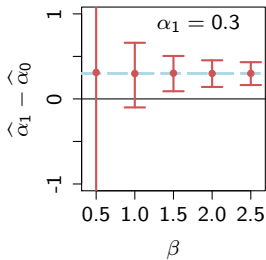
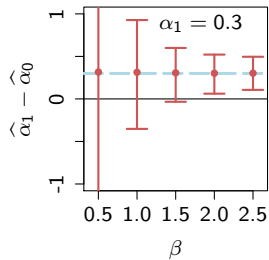
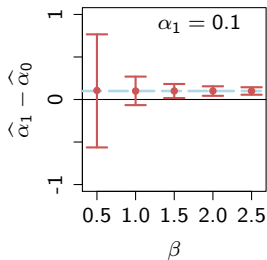
(a)  $N = 500, \delta = 0.3$



(b)  $N = 1000, \delta = 0.3$



(c)  $N = 5000, \delta = 0.3$



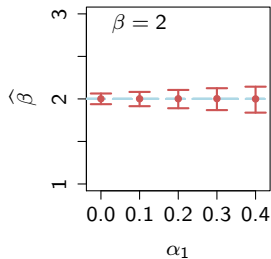
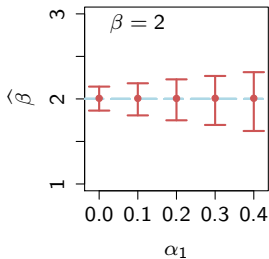
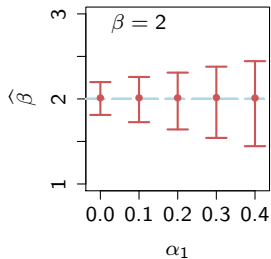
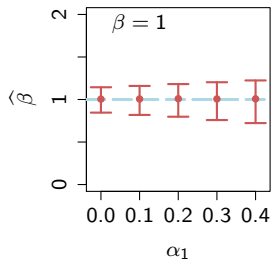
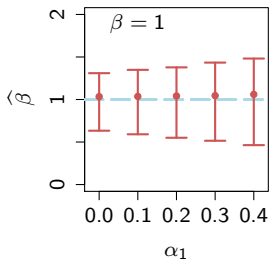
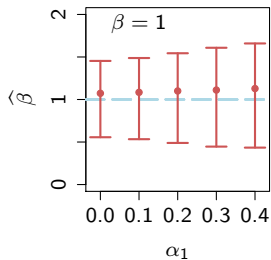


Sampling Distribution of  $\hat{\beta} = (1 - \hat{\alpha}_0 - \hat{\alpha}_1)\hat{\beta}_{IV}$

(a)  $N = 500, \delta = 0.1$

(b)  $N = 1000, \delta = 0.1$

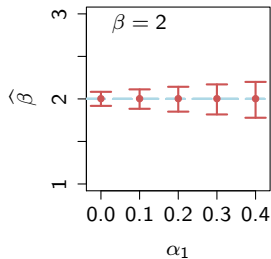
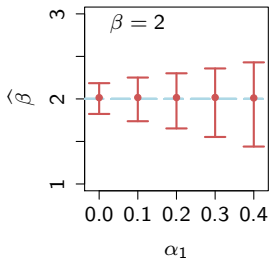
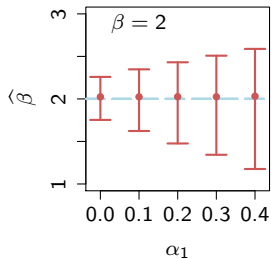
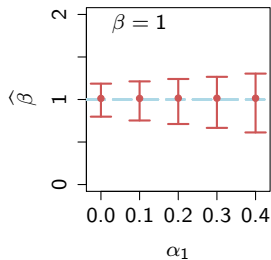
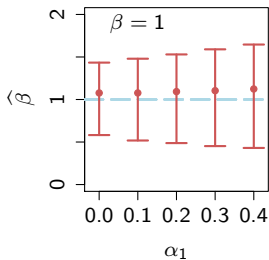
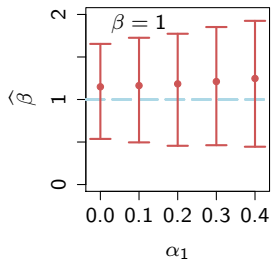
(c)  $N = 5000, \delta = 0.1$



(a)  $N = 500, \delta = 0.2$

(b)  $N = 1000, \delta = 0.2$

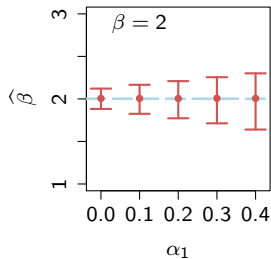
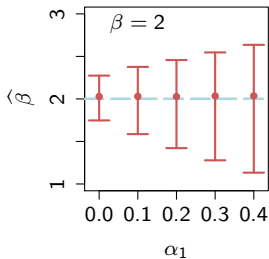
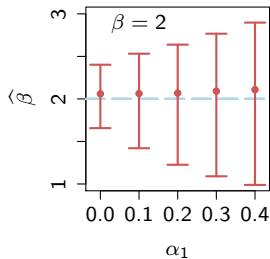
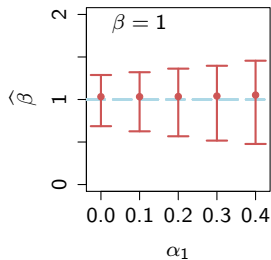
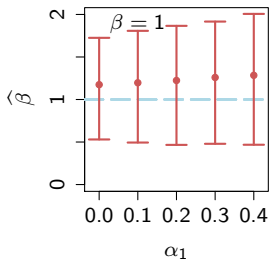
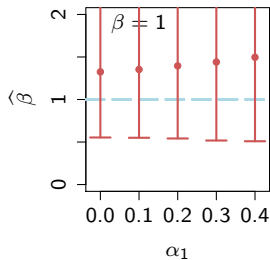
(c)  $N = 5000, \delta = 0.2$



(a)  $N = 500, \delta = 0.3$

(b)  $N = 1000, \delta = 0.3$

(c)  $N = 5000, \delta = 0.3$



$(z \perp \varepsilon)$  and  $(T \perp \varepsilon | T^*, z) \Rightarrow$  Continuum of MCs

## Distribution Functions

$$\tilde{\Delta}_1(\tau + \beta) - \tilde{\Delta}_1(\tau) = \alpha_0 \Delta(\tau + \beta) - (1 - \alpha_1) \Delta(\tau)$$

$$\Delta(\tau) = F_k(\tau) - F_\ell(\tau)$$

$$\tilde{\Delta}_1(\tau) = p_k F_{1k}(\tau) - p_\ell F_{1\ell}(\tau)$$

## Characteristic Functions

$$(1 - \alpha_0 - \alpha_1) = \left( e^{-i\omega\beta} - 1 \right) [\alpha_0 - \xi(\omega)]$$

$$\xi(\omega) \equiv \frac{\varphi_k(\omega) - \varphi_\ell(\omega)}{p_k \varphi_{1k}(\omega) - p_\ell \varphi_{1\ell}(\omega)}$$

Picture / example of using CDF conditions?

Picture / example of using CDFs to get identification?

# Conclusion

## Summary

- ▶ Endogenous, mis-measured binary treatment.
- ▶ Important in applied work but no solution in the literature.
- ▶ Usual (1st moment) IV assumption fails to identify  $\beta$
- ▶ Bounds for mis-classification probabilities and  $\beta$ .
- ▶ Higher moment / independence restrictions identify  $\beta$

## Extensions / Work in Progress

- ▶ Efficient estimation w/ continuum of MCs
- ▶ Inference / Specification Testing
- ▶ LATE interpretation



# Mahajan's Argument

## Regression Model

$$y = \mathbb{E}[y | T^*] + \nu$$

$$\mathbb{E}[\nu | T^*] = 0 \text{ by construction}$$

## Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon | T^*] \neq 0$$

## Ingredients

1. If  $p_1^* \neq p_2^*$ ,  $\mathbb{E}[\varepsilon | z] = 0$  then, since  $\beta_{IV} = \beta / (1 - \alpha_0 - \alpha_1)$ , knowledge of  $\alpha_0, \alpha_1$  is sufficient to recover  $\beta$ . (Correct)
2. If  $p_1^* \neq p_2^*$ ,  $\mathbb{E}[\nu | T^*, T, z] = 0$ ,  $\alpha_0, \alpha_1$  are identified. (Correct)

How to satisfy both 1 and 2 while allowing  $\mathbb{E}[\varepsilon | T^*] \neq 0$ ?

3. Assume that  $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$   
(i.e.  $m_{01}^* = m_{02}^*$  and  $m_{11}^* = m_{12}^*$ )

# The Flaw in Mahajan's Argument

## Proposition

If  $\mathbb{E}[\varepsilon | T^*] \neq 0$  then  $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$  combined with  $\mathbb{E}[\varepsilon | z] = 0$  implies  $p_1^* = p_2^*$ , i.e.  $z$  is irrelevant for  $T^*$ .

## Proof

$\mathbb{E}[\varepsilon | z] = 0$  implies

$$(1 - p_1^*)m_{01}^* + p_1^*m_{11}^* = c$$

$$(1 - p_2^*)m_{02}^* + p_2^*m_{12}^* = c$$

while Mahajan's assumption implies  $m_{01}^* = m_{02}^*$  and  $m_{11}^* = m_{12}^*$ .

Therefore either  $m_{01}^* = m_{02}^* = m_{11}^* = m_{12}^* = c$ , which is ruled out by  $E[\varepsilon | T^*] = 0$ , or  $p_1^* = p_2^*$ .