Mis-Classified, Binary, Endogenous Regressors: Identification and Inference

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What is the effect of T^*?

$$y = c(\mathbf{x}) + \beta(\mathbf{x})T^* + \varepsilon$$

- y – Outcome of interest
- T^* – Unobserved, endogenous binary regressor
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- z – Discrete (typically binary) instrumental variable

Using a discrete IV to learn about $\beta(\mathbf{x})$

$$y = c(\mathbf{x}) + \beta(\mathbf{x})T^* + \varepsilon$$

Constributions of This Paper

1. Show that only existing point identification result for mis-classified, endogenous T^* is incorrect.
2. Sharp identified set for β under standard assumptions.
3. Point identification of β under slightly stronger assumptions.
4. Point out problem of weak identification in mis-classification models, develop identification-robust inference for β.

Example: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with pregnant smokers in England: half given nicotine patches, the rest given placebo patches. Some given nicotine fail to quit; some given placebo quit.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Indicator of nicotine patch

Baseline Assumptions I – Model & Instrument

Additively Separable Model

$$y = c(\mathbf{x}) + \beta(\mathbf{x})T^* + \varepsilon, \quad \mathbb{E}[\varepsilon] = 0$$

Valid & Relevant Instrument: $z \in \{0, 1\}$

- $\mathbb{P}(T^* = 1|\mathbf{x}, z = 1) \neq \mathbb{P}(T^* = 1|\mathbf{x}, z = 0)$
- $\mathbb{E}[\varepsilon|\mathbf{x}, z] = 0$
- $0 < \mathbb{P}(z = 1|\mathbf{x}) < 1$

Baseline Assumptions II – Measurement Error

Notation

- $\alpha_0(\mathbf{x}, z) \equiv \mathbb{P}\left(T = 1 | T^* = 0, \mathbf{x}, z\right)$
- $\alpha_1(\mathbf{x}, z) \equiv \mathbb{P}\left(T = 0 | T^* = 1, \mathbf{x}, z\right)$

Mis-classification unaffected by z

$$\alpha_0(\mathbf{x}, z) = \alpha_0(\mathbf{x}), \quad \alpha_1(\mathbf{x}, z) = \alpha_1(\mathbf{x})$$

Extent of Mis-classification

$$\alpha_0(\mathbf{x}) + \alpha_1(\mathbf{x}) < 1 \quad (T \text{ is positively correlated with } T^*)$$

Non-differential Mis-classification

$$\mathbb{E}[\varepsilon|\mathbf{x}, z, T^*, T] = \mathbb{E}[\varepsilon|\mathbf{x}, z, T^*]$$

Existing Results

Correct Result – Exogenous T^*

- Mahajan (2006) Theorem 1, Frazis & Loewenstein (2003),...
- $\mathbb{E}[\varepsilon|\mathbf{x}, z, T^*] = 0$ + "Baseline" $\Rightarrow \beta(\mathbf{x})$ identified.

Incorrect Result – Endogenous T^*

- Mahajan (2006) A.2
- $\mathbb{E}[\varepsilon|\mathbf{x}, z, T^*, T] = \mathbb{E}[\varepsilon|\mathbf{x}, T^*]$ + "Baseline" $\Rightarrow \beta(\mathbf{x})$ identified.

We show: Mahajan's assumptions imply that the instrument z is uncorrelated with T^* unless T^* is in fact *exogenous*.

Simple Bounds for Mis-classification from First-stage

Unobserved	Observed
$p_k^*(\mathbf{x}) \equiv \mathbb{P}(T^* = 1 \mid \mathbf{x}, z = k)$	$p_k(\mathbf{x}) \equiv \mathbb{P}(T = 1 \mid \mathbf{x}, z = k)$

Relationship

$$p_k^*(\mathbf{x}) = \frac{p_k(\mathbf{x}) - \alpha_0(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}, \quad k = 0, 1$$

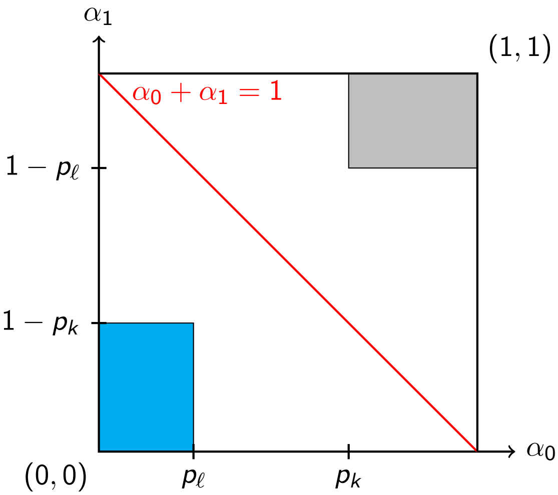
z does not affect (α_0, α_1); denominator $\neq 0$

Bounds for Mis-classification

$$\alpha_0(\mathbf{x}) \leq p_k(\mathbf{x}) \leq 1 - \alpha_1(\mathbf{x}), \quad k = 0, 1$$

$\alpha_0(\mathbf{x}) + \alpha_1(\mathbf{x}) < 1$

$\alpha_0 \leq \min_k \{p_k\}, \quad \alpha_1 \leq \min_k \{1 - p_k\}$



What does IV estimate under mis-classification?

Unobserved

$$\beta(\mathbf{x}) = \frac{\mathbb{E}[y|\mathbf{x}, z=1] - \mathbb{E}[y|\mathbf{x}, z=0]}{p_1^*(\mathbf{x}) - p_0^*(\mathbf{x})}$$

Wald (Observed)

$$\frac{\mathbb{E}[y|\mathbf{x}, z=1] - \mathbb{E}[y|\mathbf{x}, z=0]}{p_1(\mathbf{x}) - p_0(\mathbf{x})} = \beta(\mathbf{x}) \left[\frac{p_1^*(\mathbf{x}) - p_0^*(\mathbf{x})}{p_1(\mathbf{x}) - p_0(\mathbf{x})}\right] = \frac{\beta(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}$$

$$\boxed{p_1^*(\mathbf{x}) - p_0^*(\mathbf{x}) = \frac{p_1(\mathbf{x}) - \alpha_0(\mathbf{x})}{1 - \alpha_0 - \alpha_1(\mathbf{x})} - \frac{p_0(\mathbf{x}) - \alpha_0(\mathbf{x})}{1 - \alpha_0 - \alpha_1(\mathbf{x})} = \frac{p_1(\mathbf{x}) - p_0(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}}$$

Partial Identification Bounds for $\beta(\mathbf{x})$

"Weak Bounds"

- $\beta(\mathbf{x})$ is between Wald and Reduced form; same sign as Wald.
- Doesn't rely on non-differential assumption or additive sep.
- Frazis & Loewenstein (2003), Ura (2016), ...

Non-differential Assumption

- $\mathbb{E}[\varepsilon|\mathbf{x}, T^*, T, z] = \mathbb{E}[\varepsilon|\mathbf{x}, T^*, z]$
- Used in literature to identify $\beta(\mathbf{x})$ when T^* is exogenous.
- Does it restrict the identified set when T^* is endogenous?

Restrictions from Non-differential Mis-classification?

(Suppress $\mathbf{x}$ for simplicity)

Notation

- $r_{tk} \equiv \mathbb{P}(T^* = 1 | T = t, z = k)$
- z_k is shorthand for $z = k$

Iterated Expectations over T^*

$$\mathbb{E}(y|T=0,z_k) = (1-r_{0k})\mathbb{E}(y|T^*=0,\, T=0,z_k) + r_{0k}\mathbb{E}(y|T^*=1,\, T=0,z_k)$$

$$\mathbb{E}(y|T=1,z_k) = (1-r_{1k})\mathbb{E}(y|T^*=0,\, T=1,z_k) + r_{1k}\mathbb{E}(y|T^*=1,\, T=1,z_k)$$

Restrictions from Non-differential Mis-classification?

(Suppress $\mathbf{x}$ for simplicity)

Notation

- $r_{tk} \equiv \mathbb{P}(T^* = 1 | T = t, z = k)$
- z_k is shorthand for $z = k$

Adding Non-differential Assumption

$$\mathbb{E}(y|T = 0, z_k) = (1 - r_{0k})\mathbb{E}(y|T^* = 0, z_k) \qquad + r_{0k}\mathbb{E}(y|T^* = 1, z_k)$$

$$\mathbb{E}(y|T = 1, z_k) = (1 - r_{1k})\mathbb{E}(y|T^* = 0, z_k) \qquad + r_{1k}\mathbb{E}(y|T^* = 1, z_k)$$

$\boxed{\text{2 equations in 2 unknowns} \Rightarrow \text{solve for } \mathbb{E}(y|T^* = t^*, z = k) \text{ given } (r_{0k}, r_{1k}).}$

Restrictions from Non-differential Mis-classification?

Law of Total Probability

$$\boxed{F_{tk} = (1 - r_{tk})F_{tk}^{0} + r_{tk}F_{tk}^{1}}$$

$F_{tk} \equiv$ Observed CDF: $y|(T = t, z = k)$

$F_{tk}^{t^*} \equiv$ Unobserved CDF: $y|(T^* = t^*, T = t, z = k)$

Previous Slide

- r_{tk} observable given (α_0, α_1)
- $\mathbb{E}(y|T^*, T, z) = \mathbb{E}(y|T^*, z)$ observable given (α_0, α_1)

Key Question

Given (α_0, α_1) can we always find (F_{tk}^0, F_{tk}^1) to satisfy the mixture model?

Restrictions from Non-differential Mis-classification?

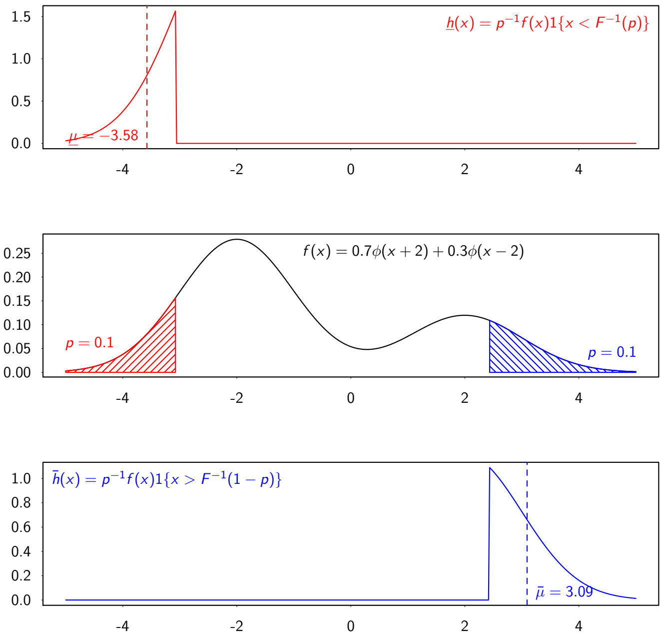
Equivalent Problem

Given a specified CDF F, for what values of p and μ do there exist valid CDFs (G, H) with $F = (1-p)G + pH$ and $\mu = \text{mean}(H)$?

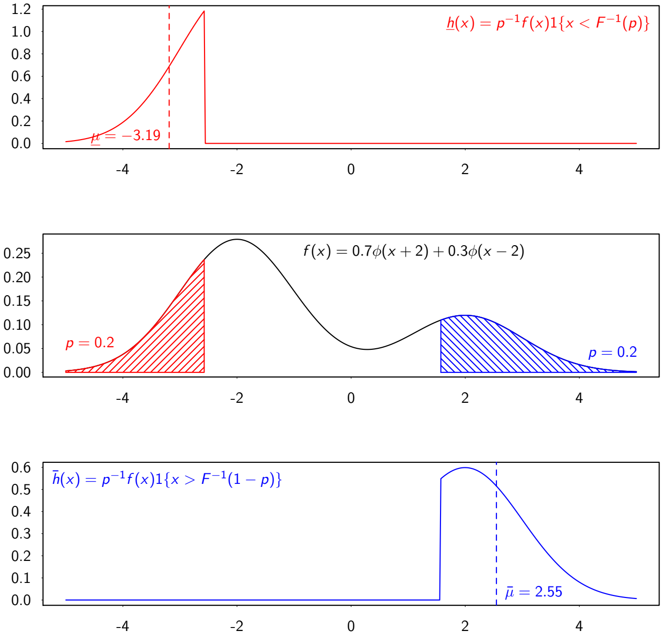
Necessary and Sufficient Condition if F is Continuous

$$\underline{\mu}(F,p) \leq \ \mu \ \leq \overline{\mu}(F,p)$$

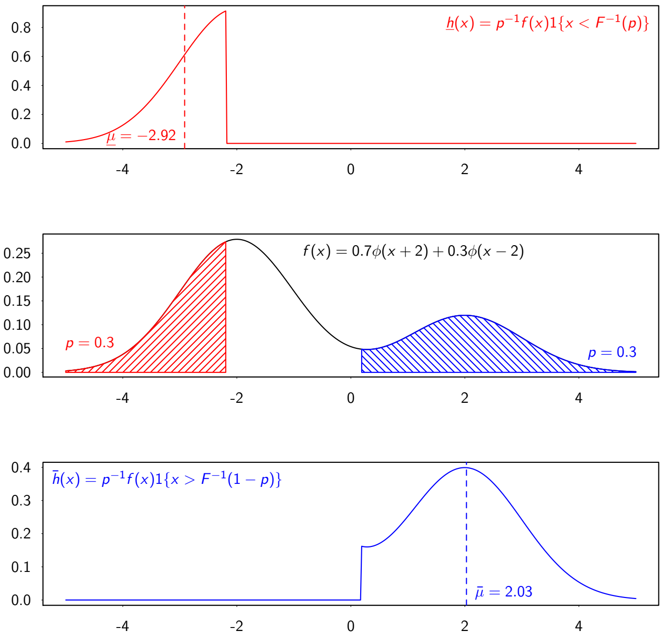
$$\underline{\mu}(F,p) \equiv \int_{-\infty}^{F^{-1}(p)} \frac{x}{p}\, f(x)\, dx, \qquad \overline{\mu}(F,p) \equiv \int_{F^{-1}(1-p)}^{+\infty} \frac{x}{p}\, f(x)\, dx$$

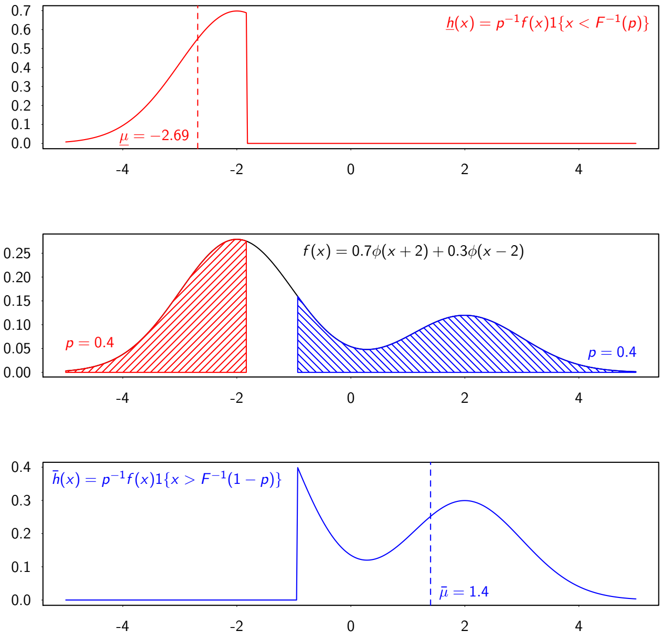


$\underline{h}(x) = p^{-1}f(x)1\{x < F^{-1}(p)\}$
$\underline{\mu} = -3.58$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.1$
$p = 0.1$
$\bar{h}(x) = p^{-1}f(x)1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 3.09$

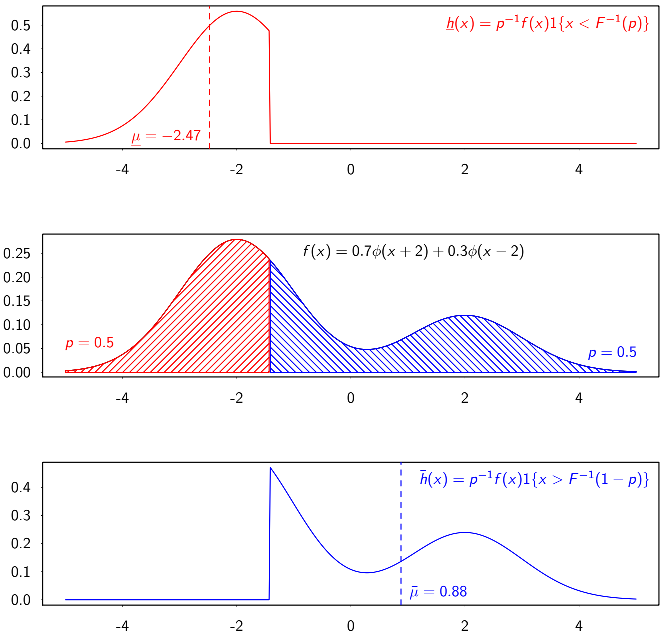


$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -3.19$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.2$
$p = 0.2$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 2.55$

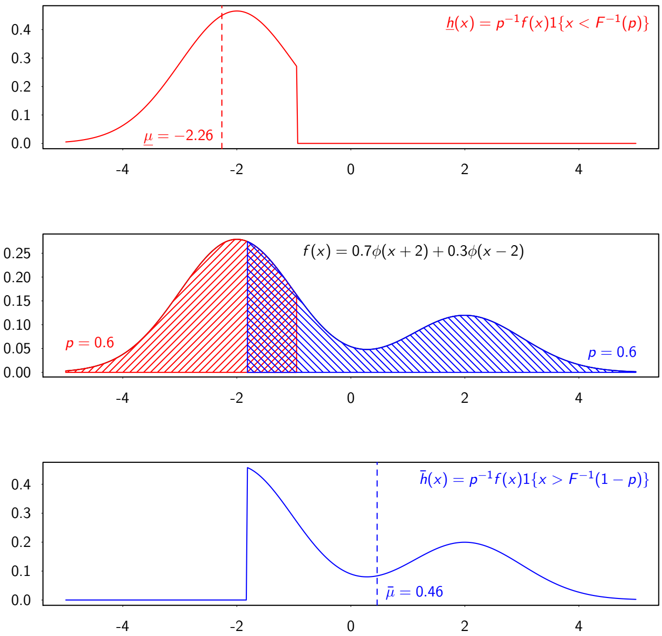

$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.92$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.3$
$p = 0.3$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 2.03$



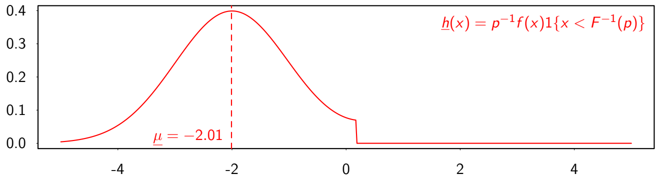
$\underline{h}(x) = p^{-1}f(x)1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.69$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.4$
$p = 0.4$
$\bar{h}(x) = p^{-1}f(x)1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 1.4$



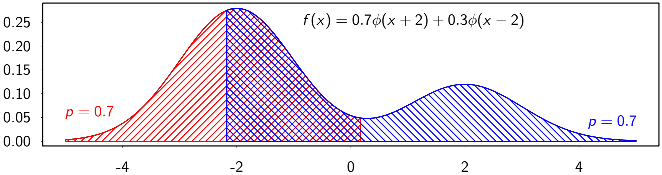
$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.47$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.5$
$p = 0.5$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 0.88$



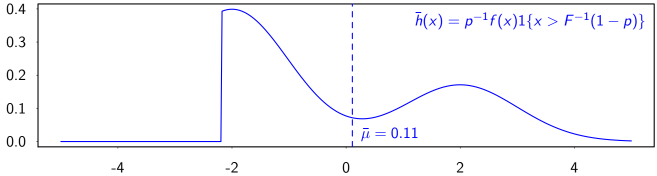
$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.26$
0.4
0.3
0.2
0.1
0.0
-4
-2
0
2
4
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.6$
$p = 0.6$
0.25
0.20
0.15
0.10
0.05
0.00
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 0.46$



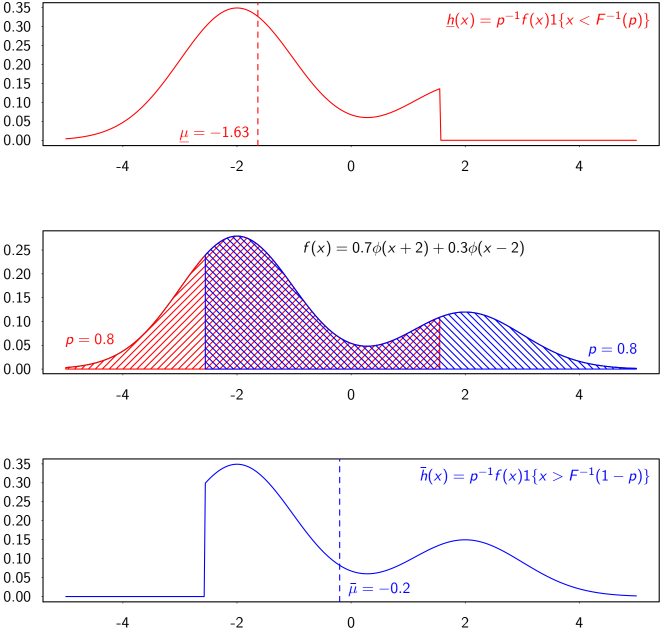
$\underline{h}(x) = p^{-1}f(x)1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.01$
0.4
0.3
0.2
0.1
0.0
-4
-2
0
2
4



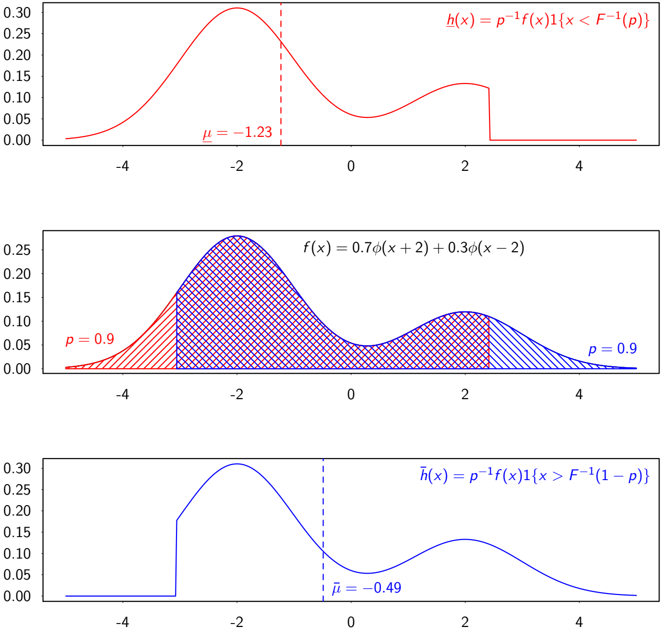
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.7$
$p = 0.7$
0.25
0.20
0.15
0.10
0.05
0.00
-4
-2
0
2
4



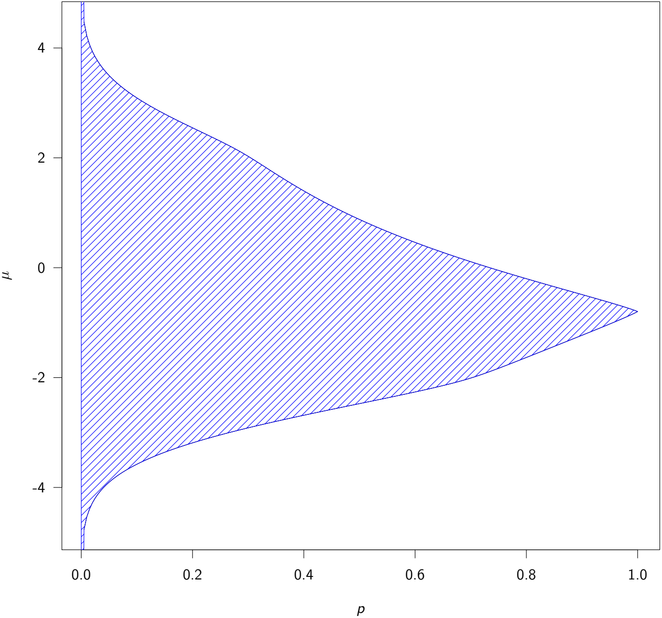
$\bar{h}(x) = p^{-1}f(x)1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 0.11$
0.4
0.3
0.2
0.1
0.0
-4
-2
0
2
4



$\underline{h}(x) = p^{-1}f(x)1\{x < F^{-1}(p)\}$
$\underline{\mu} = -1.63$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.8$
$p = 0.8$
$\bar{h}(x) = p^{-1}f(x)1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = -0.2$



$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -1.23$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.9$
$p = 0.9$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = -0.49$



4
2
0
-2
-4
μ
0.0
0.2
0.4
0.6
0.8
1.0
p

Back to Our Original Problem

- Observe F_{tk} for all (t, k)
- r_{tk} pinned down by (α_0, α_1)
- Can we find $F_{tk}^{t^*}$ so that $F_{tk} = (1 - r_{tk})F_{tk}^0 + r_{tk}F_{tk}^1$?
- Non-diff. assumption $\Rightarrow$ mean of F_{tk}^1 pinned down by (α_0, α_1).
- Implies joint restrictions on (α_0, α_1), hence β.

Sharp Identified Set under Baseline Assumptions

Theorem

Under baseline assumptions, sharp identified set for $\beta(\mathbf{x})$ is never a singleton, regardless of how many (discrete) values z takes on.

Intuition

No mis-classification $\Rightarrow r_{tk} = 0$ or 1 and we can always form a valid mixture in this case. Show that Wald estimand always lies within the sharp identified set for β.

Point Identification: 1st Ingredient

Reparameterization

$$\theta_1(\mathbf{x}) = \beta(\mathbf{x}) / \left[1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})\right]$$
$$\theta_2(\mathbf{x}) = \left[\theta_1(\mathbf{x})\right]^2 \left[1 + \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})\right]$$
$$\theta_3(\mathbf{x}) = \left[\theta_1(\mathbf{x})\right]^3 \left[\left\{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})\right\}^2 + 6\alpha_0(\mathbf{x})\left\{1 - \alpha_1(\mathbf{x})\right\}\right]$$

$$\boxed{\beta(\mathbf{x}) = 0 \iff \theta_1(\mathbf{x}) = \theta_2(\mathbf{x}) = \theta_3(\mathbf{x}) = 0}$$

Lemma

Baseline Assumptions $\implies$ $\text{Cov}(y, z|\mathbf{x}) = \theta_1(\mathbf{x})\text{Cov}(z, T|\mathbf{x})$.

Point Identification: 2nd Ingredient

Assumption (II)

$\mathbb{E}[\varepsilon^2|\mathbf{x}, z] = \mathbb{E}[\varepsilon^2|\mathbf{x}]$

Lemma

(Baseline) + (II) $\implies$

$$\mathrm{Cov}(y^2, z|\mathbf{x}) = 2\mathrm{Cov}(yT, z|\mathbf{x})\theta_1(\mathbf{x}) - \mathrm{Cov}(T, z|\mathbf{x})\theta_2(\mathbf{x})$$

Corollary

(Baseline) + (II) + $[\beta(\mathbf{x}) \neq 0] \implies [\alpha_1(\mathbf{x}) - \alpha_0(\mathbf{x})]$ is identified.

Point Identification: 3rd Ingredient

Assumption (III)

(i) $\mathbb{E}[\varepsilon^2|\mathbf{x}, z, T^*, T] = \mathbb{E}[\varepsilon^2|\mathbf{x}, z, T^*]$

(ii) $\mathbb{E}[\varepsilon^3|\mathbf{x}, z] = \mathbb{E}[\varepsilon^3|\mathbf{x}]$

Lemma

(Baseline) + (II) + (III) $\implies$

$$\mathrm{Cov}(y^3, z|\mathbf{x}) = 3\mathrm{Cov}(y^2 T, z|\mathbf{x})\theta_1(\mathbf{x}) - 3\mathrm{Cov}(yT, z|\mathbf{x})\theta_2(\mathbf{x}) + \mathrm{Cov}(T, z|\mathbf{x})\theta_3(\mathbf{x})$$

Point Identification Result

Theorem

(Baseline) + (II) + (III) $\implies \beta(\mathbf{x})$ is point identified. If $\beta(\mathbf{x}) \neq 0$, then $\alpha_0(\mathbf{x})$ and $\alpha_1(\mathbf{x})$ are likewise point identified.

Sufficient Conditions for (II) and (III)

(i) T is conditionally independent of (ε, z) given $(T^*, \mathbf{x})$

(ii) z is conditionally independent of ε given $\mathbf{x}$

Inference for a Mis-classified Regressor

Weak Identification

- β small $\Rightarrow$ moment equalities uninformative about (α_0, α_1) ▸ more
- (α_0, α_1) could be on the boundary of the parameter space
- Also true of existing estimators that assume T^* exogenous

Our Approach

- Sharp identified set yields *inequality* moment restrictions that remain informative even if $\beta \approx 0$. ▸ more
- Identification-robust inference with equality and inequality MCs.

Inference with Moment Equalities and Inequalities

Moment Conditions

$\mathbb{E}\left[m_j(\mathbf{w}_i, \vartheta_0)\right] \geq 0, \quad j = 1, \cdots, J$

$\mathbb{E}\left[m_j(\mathbf{w}_i, \vartheta_0)\right] = 0, \quad j = J+1, \cdots, J+K$

Test Statistic

$$T_n(\vartheta) = \sum_{j=1}^{J} \left[\frac{\sqrt{n}\ \bar{m}_{n,j}(\vartheta)}{\widehat{\sigma}_{n,j}(\vartheta)}\right]_{-}^{2} + \sum_{j=J+1}^{J+K} \left[\frac{\sqrt{n}\ \bar{m}_{n,j}(\vartheta)}{\widehat{\sigma}_{n,j}(\vartheta)}\right]^{2}$$

Critical Value

- $\sqrt{n}\,\bar{m}_n(\vartheta_0) \to_d$ normal limit with covariance matrix $\Sigma(\vartheta_0)$
- Use this to bootstrap the limit dist. of $T_n(\vartheta)$ under $H_0\colon \vartheta = \vartheta_0$

Generalized Moment Selection

Andrews & Soares (2010)

- Inequalities that don't bind reduce power of test, so eliminate those that are "far from binding" before calculating critical value.
- Uniformly valid test of $H_0\colon \vartheta = \vartheta_0$ even if ϑ_0 is not point identified.
- Not asymptotically conservative.

Problem

Joint test for the whole parameter vector but we're only interested in β.
Projection is conservative and computationally intensive.

Our Solution: Bonferroni-Based Inference

Special Structure

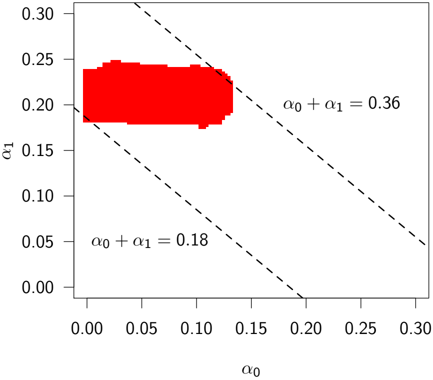
- β only enters MCs through $\theta_1 = \beta/(1 - \alpha_0 - \alpha_1)$
- Strong instrument $\Rightarrow$ inference for θ_1 is standard.
- Nuisance pars $\boldsymbol{\gamma}$ strongly identified under null for (α_0, α_1)

Procedure

1. Concentrate out $(\theta_1, \boldsymbol{\gamma}) \Rightarrow$ joint GMS test for (α_0, α_1)
2. Invert test $\Rightarrow (1 - \delta_1) \times 100\%$ confidence set for (α_0, α_1)
3. Project $\Rightarrow$ CI for $(1 - \alpha_0 - \alpha_1)$
4. Construct standard $(1 - \delta_2) \times 100\%$ IV CI for θ_1
5. Bonferroni $\Rightarrow (1 - \delta_1 - \delta_2) \times 100\%$ CI for β

Example (sim data: $\beta = 1, \alpha_0 = 0.1, \alpha_1 = 0.2, n = 5000$)

97.5% GMS Confidence Region for (α_0, α_1)



Bonferroni Interval

1. 97.5% CI for $(1 - \alpha_0 - \alpha_1) = (0.64, 0.82)$
2. 97.5% CI for $\theta_1 = (1.20, 1.47)$
3. $>$ 95% CI for β:
   $(0.64 \times 1.20, 0.82 \times 1.47) = (0.77, 1.21)$

Comparisons

- (0.88, 1.04) for IV if T^* were observed
- (1.22,1.45) for naive IV interval using T

Conclusion

- Identification and inference for effect of binary, mis-classified, endogenous regressor.
- Only existing point identification result is incorrect.
- Sharp identified set for $\beta(\mathbf{x})$ under standard assumptions.
- Point identification of $\beta(\mathbf{x})$ under slightly stronger assumptions.
- Point out weak identification problem in mis-classification models, develop identification-robust inference for $\beta(\mathbf{x})$.

Just-Identified System of Moment Equalities

Suppress dependence on $\mathbf{x}$...

$$\mathbb{E}\left[\{\boldsymbol{\Psi}(\boldsymbol{\theta})\mathbf{w}_i - \boldsymbol{\kappa}\} \otimes \begin{pmatrix} 1 \\ z \end{pmatrix}\right] = \mathbf{0}$$

$$\boldsymbol{\Psi}(\boldsymbol{\theta}) \equiv \begin{bmatrix} -\theta_1 & 1 & 0 & 0 & 0 & 0 \\ \theta_2 & 0 & -2\theta_1 & 1 & 0 & 0 \\ -\theta_3 & 0 & 3\theta_2 & 0 & -3\theta_1 & 1 \end{bmatrix}$$

$$\mathbf{w}_i = (T_i, y_i, y_i T_i, y_i^2, y_i^2 T_i, y_i^3)' \qquad \theta_1 = \beta/(1-\alpha_0-\alpha_1)$$

$$\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \kappa_3)' \qquad \theta_2 = \theta_1^2(1+\alpha_0-\alpha_1)$$

$$\theta_3 = \theta_1^3\left[(1-\alpha_0-\alpha_1)^2 + 6\alpha_0(1-\alpha_1)\right]$$

back

Moment Inequalities I – First-stage Probabilities

$\alpha_0 \leq p_k \leq 1 - \alpha_1$ becomes $\mathbb{E}\left[m(\mathbf{w}_i, \vartheta)\right] \geq \mathbf{0}$ for all k where

$$m(\mathbf{w}_i, \vartheta) \equiv \left[\begin{array}{l} \mathbf{1}(z_i = k)(T - \alpha_0) \\ \mathbf{1}(z_i = k)(1 - T_i - \alpha_1) \end{array} \right]$$

Moment Inequalities II – Non-differential Assumption

For all k, we have $\mathbb{E}[m\Big(\mathbf{w}_i, \vartheta, \mathbf{q}_k)] \geq 0$ where

$$m\Big(\mathbf{w}_i, \vartheta, \mathbf{q}_k\Big) \equiv \left[\begin{array}{r} y_i \mathbf{1}\left(z_i = k\right)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i \leq \underline{q}_{0k})(1 - T_i)\left(\frac{1-\alpha_0-\alpha_1}{\alpha_1}\right)\right\} \\ -y_i \mathbf{1}(z_i = k)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i > \overline{q}_{0k})(1 - T_i)\left(\frac{1-\alpha_0-\alpha_1}{\alpha_1}\right)\right\} \\ y_i \mathbf{1}\left(z_i = k\right)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i \leq \underline{q}_{1k}) T_i \left(\frac{1-\alpha_0-\alpha_1}{1-\alpha_1}\right)\right\} \\ -y_i \mathbf{1}(z_i = k)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i > \overline{q}_{1k}) T_i \left(\frac{1-\alpha_0-\alpha_1}{1-\alpha_1}\right)\right\} \end{array}\right]$$

and $\mathbf{q}_k \equiv (\underline{q}_{0k},\, \overline{q}_{0k},\, \underline{q}_{1k},\, \overline{q}_{1k})'$ defined by $\mathbb{E}[h(\mathbf{w}_i, \vartheta, \mathbf{q}_k)] = 0$ with

$$h(\mathbf{w}_i, \vartheta, \mathbf{q}_k) = \left[\begin{array}{l} \mathbf{1}(y_i \leq \underline{q}_{0k})\mathbf{1}(z_i = k)(1 - T_i) - \left(\frac{\alpha_1}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(T_i - \alpha_0) \\ \mathbf{1}(y_i \leq \overline{q}_{0k})\mathbf{1}(z_i = k)(1 - T_i) - \left(\frac{1-\alpha_0}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(1 - T_i - \alpha_1) \\ \mathbf{1}(y_i \leq \underline{q}_{1k})\mathbf{1}(z_i = k) T_i - \left(\frac{1-\alpha_1}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(T_i - \alpha_0) \\ \mathbf{1}(y_i \leq \overline{q}_{1k})\mathbf{1}(z_i = k) T_i - \left(\frac{\alpha_0}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(1 - T_i - \alpha_1) \end{array}\right]$$

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