Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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Estimating the Effect of T^*

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Known or Unknown function
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete (typically binary) instrumental variable

Target of Inference: $\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$

Example: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- y – Log wage
- T^* – True training attendence
- T – Self-reported training attendance
- $\mathbf{x}$ – Individual characteristics
- z – Offer of job training

Related Literature

Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015), Denteh et al. (2016)

Mahajan Details

Model: $y = c + \beta T^* + \varepsilon$

Valid Instrument

$\mathbb{E}[\varepsilon|z] = 0.$

First-stage

$p_k^* \equiv \mathbb{P}(T^* = 1|z = z_k) \neq \mathbb{P}(T^* = 1|z = z_\ell) \equiv p_\ell^*,\ k \neq \ell$

Non-differential Measurement Error

- $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$
- $\alpha_0 + \alpha_1 < 1$

Observable Moments: $y = c + \beta T^* + \varepsilon$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\dots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\dots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|\,T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

Define error term that absorbs constant: $u = c + \varepsilon$

	$z = 1$	$z = 2$	$\ldots$	$z = K$
$T^* = 0$	m^*_{01} / p^*_{01}	m^*_{02} / p^*_{02}	$\ldots$	m^*_{0K} / p^*_{0K}
$T^* = 1$	m^*_{11} / p^*_{11}	m^*_{12} / p^*_{12}	$\ldots$	m^*_{1K} / p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u | T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p^*_k = \mathbb{P}(T^* = 1 | z = z_k)$$

System of Equations given $E[\varepsilon|z] = 0$

$\boxed{\mathbb{E}[\varepsilon|z] = 0 \implies \textit{pair} \text{ of equations for each } k = 1, \dots, K}$

$$(1 - p_k)\bar{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + (1 - \alpha_0)c - (p_k - \alpha_0)m^*_{1k}$$

$$p_k\bar{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m^*_{1k}$$

Theorem

$2K$ equations in $K + 4$ unknowns, but β is unidentified *regardless* of K.

Intuition

Using $E[\varepsilon|z] = 0$ to eliminate m^*_{0k} from the system "entangles" the equations such that each pair only provides one restriction.

First Moment Condition

Assumptions

- $\mathbb{E}[\varepsilon | z] = 0$
- $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$

Moment Condition

$$\mathrm{Cov}(y, z) - \left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) \mathrm{Cov}(T, z) = 0$$

MC # 1 identifies $\beta/(1 - \alpha_0 - \alpha_1)$

Second Moment Condition

Additional Assumptions

- $\mathbb{E}[\varepsilon^2 | z] = \mathbb{E}[\varepsilon^2]$
- $\mathbb{E}[\varepsilon^2 | T^*, T, z] = \mathbb{E}[\varepsilon^2 | T^*, z]$

Moment Condition

$$\text{Cov}(y^2, z) - \frac{\beta}{1-\alpha_0-\alpha_1}\left\{2\text{Cov}(yT, z) - \beta\text{Cov}(T, z)\left(\frac{1+\alpha_0-\alpha_1}{1-\alpha_0-\alpha_1}\right)\right\} = 0$$

Given MC #1, MC #2 identifies $(\alpha_1 - \alpha_0)$

Third Moment Condition

Additional Assumptions

- $\mathbb{E}[\varepsilon^3|z] = \mathbb{E}[\varepsilon^3]$
- $\mathbb{E}[\varepsilon^3|T^*, T, z] = \mathbb{E}[\varepsilon^3|T^*, z]$

Moment Condition

$$\text{Cov}(y^3, z) - \left(\frac{\beta}{1-\alpha_0-\alpha_1}\right)\left\{ \beta^2\left[1 + \frac{6\alpha_0(1-\alpha_1)}{(1-\alpha_0-\alpha_1)^2}\right]\text{Cov}(T, z) \right.$$
$$\left. -3\beta\left[\frac{1-(\alpha_1-\alpha_0)}{1-\alpha_0-\alpha_1}\right]\text{Cov}(yT, z) + 3\text{Cov}(y^2 T, z)\right\} = 0$$

Theorem

Third moment suffice to identify the model provided that $\beta \neq 0$. If $\beta = 0$, the reduced form identifies β.

GMM Estimator in Simple Special Case: $\alpha_0 = 0$

$$u(\boldsymbol{\theta}) = y - c - \frac{\beta}{1-\alpha_1} T$$

$$v(\boldsymbol{\theta}) = y^2 - \sigma_{\varepsilon\varepsilon} - c^2 - \frac{\beta}{1-\alpha_1} 2yT + \frac{\beta^2}{1-\alpha_1}$$

$$\mathbb{E}\left[g_1(\mathbf{x}, \boldsymbol{\theta})\right] = \mathbb{E}\begin{bmatrix} u(\boldsymbol{\theta}) \\ v(\boldsymbol{\theta}) \end{bmatrix} = \mathbf{0}, \quad \mathbb{E}\left[g_2(\mathbf{x}, \boldsymbol{\theta})\right] = \mathbb{E}\begin{bmatrix} u(\boldsymbol{\theta})z \\ v(\boldsymbol{\theta})z \end{bmatrix} = \mathbf{0}$$

$$\beta = \frac{2\text{Cov}(yT, z)}{\text{Cov}(T, z)} - \frac{\text{Cov}(y^2, z)}{\text{Cov}(y, z)}$$

Simulation DGP: $y = \beta T^* + \varepsilon$

Errors

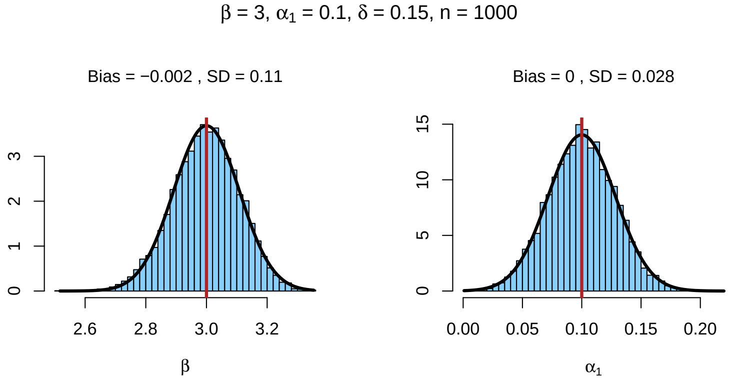
$(\varepsilon, \eta) \sim$ jointly normal, mean 0, variance 1, correlation 0.5.

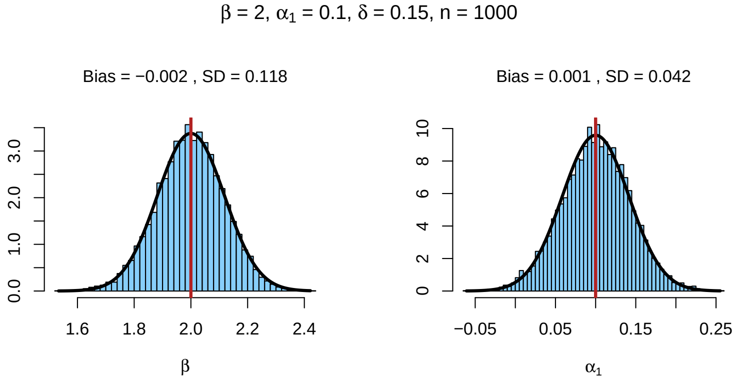
First-Stage

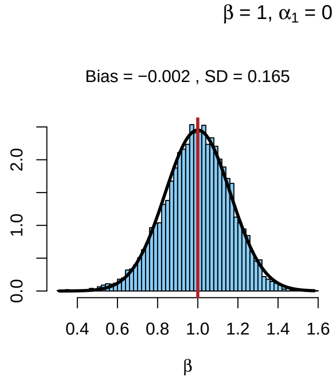
- Half of subjects have $z = 1$, the rest have $z = 0$.
- $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
- $\delta = \mathbb{P}(T^* = 0|z = 1) = \mathbb{P}(T^* = 1|z = 0)$

Mis-classification

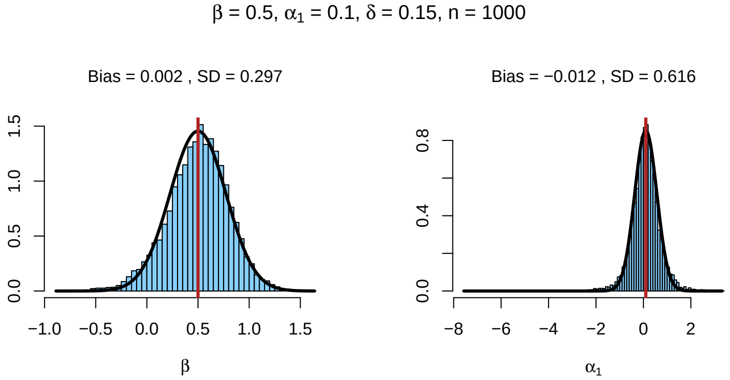
- Set $\alpha_0 = 0$
- $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$


β = 3, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.11
Bias = 0 , SD = 0.028
0
1
2
3
2.6
2.8
3.0
3.2
β
0
5
10
15
0.00
0.05
0.10
0.15
0.20
α1


β = 2, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.118
0.0
1.0
2.0
3.0
1.6
1.8
2.0
2.2
2.4
β
Bias = 0.001 , SD = 0.042
0
2
4
6
8
10
−0.05
0.05
0.15
0.25
α1



β = 1, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.165
Bias = 0.001 , SD = 0.129
0.0
1.0
2.0
0.4
0.6
0.8
1.0
1.2
1.4
1.6
β
0.0
1.0
2.0
3.0
−0.4
0.0
0.2
0.4
0.6
α1



β = 0.5, α1 = 0.1, δ = 0.15, n = 1000
Bias = 0.002 , SD = 0.297
Bias = −0.012 , SD = 0.616
0.0
0.5
1.0
1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
β
0.0
0.4
0.8
−8
−6
−4
−2
0
2
α1

Identification Failure when $\beta = 0$

Simple Special Case: $\alpha_0 = 0$

$$u(\boldsymbol{\theta}) = y - c - \frac{\beta}{1-\alpha_1} T$$

$$v(\boldsymbol{\theta}) = y^2 - \sigma_{\varepsilon\varepsilon} - c^2 - \frac{\beta}{1-\alpha_1} 2yT + \frac{\beta^2}{1-\alpha_1}$$

$$\mathbb{E}\left[g_1(\mathbf{x}, \boldsymbol{\theta})\right] = \mathbb{E}\left[\begin{array}{c} u(\boldsymbol{\theta}) \\ v(\boldsymbol{\theta}) \end{array}\right] = \mathbf{0}, \quad \mathbb{E}\left[g_2(\mathbf{x}, \boldsymbol{\theta})\right] = \mathbb{E}\left[\begin{array}{c} u(\boldsymbol{\theta})z \\ v(\boldsymbol{\theta})z \end{array}\right] = \mathbf{0}$$

- β small $\Rightarrow$ moment equalities uninformative about α_1
- $(c, \sigma_{\varepsilon\varepsilon})$ are identified at any hypothesized pair (α_1, β)

Auxiliary Moment Inequalities

General Case $\alpha_0 \neq 0$

$$\alpha_0(z) = \alpha_0, \; \alpha_1(z) = \alpha_1$$

$$\implies \quad p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}, \quad 1 - p_k^* = \frac{1 - p_k - \alpha_1}{1 - \alpha_0 - \alpha_1}$$

$$\alpha_0 + \alpha_1 < 1 \iff \mathrm{Cor}(T, T^*) > 0 \iff (1 - \alpha_0 - \alpha_1) > 0$$

Implications

- $\alpha_0 < \min_k\{p_k\}, \quad \alpha_1 < \min_k\{1 - p_k\}$
- β is between β_{RF} and β_{IV}
- β_{IV} *inflated* but has correct sign

Even Tighter Bounds for α_0, α_1 from Conditional Variances

Assume

$$\mathbb{E}[\varepsilon^2 | T^*, T, z] = \mathbb{E}[\varepsilon^2 | T^*, z]$$

Observables

$$\sigma_{tk}^2 = \text{Var}(y | T = t, z = k)$$

Constrain Unobservables

$$s_{tk}^{*2} = Var(u | T^* = t, z_k) > 0$$

$$(p_k - \alpha_0)\left[(1-\alpha_0)p_k\sigma_{1k}^2 - \alpha_0(1-p_k)\sigma_{0k}^2\right] > \alpha_0(1-\alpha_0)p_k(1-p_k)(\bar{y}_{1k} - \bar{y}_{0k})^2$$
$$(1 - p_k - \alpha_1)\left[(1-\alpha_1)(1-p_k)\sigma_{0k}^2 - \alpha_1 p_k\sigma_{1k}^2\right] > \alpha_1(1-\alpha_1)p_k(1-p_k)(\bar{y}_{1k} - \bar{y}_{0k})^2$$

Identification-Robust Joint Inference for $(\alpha_0, \alpha_1, \beta)$

- Auxiliary moment inequalities to bound (α_0, α_1)
- Joint CS for $(\alpha_0, \alpha_1, \beta)$ by inverting Anderson-Rubin Test
- Generalized Moment Selection (Andrews & Soares, 2010) for tighter confidence sets.

Conclusion

- Endogenous, mis-measured binary treatment.
- Important in applied work but no solution in the literature.
- Usual (1st moment) IV assumption fails to identify β
- Higher moment / independence restrictions identify β
- Identification-Robust Inference incorportating additional inequality moment conditions.

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Main Result (Correct) – Exogenous Treatment

Relevant binary instrument z ($p_k^* \neq p_\ell^*$) identifies α_0, α_1 and $\mathbb{E}[y|T^*]$ provided that $\mathbb{E}[\nu|T^*, T, z] = 0$ and $\alpha_0 + \alpha_1 < 1$.

Extension (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$, $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$ identified.

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Ingredients

1. If $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|z] = 0$ then, since $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$, knowledge of α_0, α_1 is sufficient to recover β. (Correct)
2. If $p_k^* \neq p_\ell^*$, $\mathbb{E}[\nu|T^*, T, z] = 0$, α_0, α_1 are identified. (Correct)

   How to satisfy both 1 and 2 while allowing $\mathbb{E}[\varepsilon|T^*] \neq 0$?

3. Assume that $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$

   (i.e. $m_{0k}^* = m_{0\ell}^*$ and $m_{1k}^* = m_{1\ell}^*$)

Flaw in the Argument

Proposition

If $\mathbb{E}[\varepsilon|T^*] \neq 0$ then $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$ combined with $\mathbb{E}[\varepsilon|z] = 0$ implies $p_k^* = p_\ell^*$, i.e. z is irrelevant for T^*.

Proof

$\mathbb{E}[\varepsilon|z] = 0$ implies

$$(1 - p_1^*)m_{0k}^* + p_1^* m_{1k}^* = c$$

$$(1 - p_2^*)m_{0\ell}^* + p_2^* m_{1\ell}^* = c$$

while Mahajan's assumption implies $m_{0k}^* = m_{0\ell}^*$ and $m_{1k}^* = m_{1\ell}^*$. Therefore either $m_{0k}^* = m_{0\ell}^* = m_{1k}^* = m_{1\ell}^* = c$, which is ruled out by $E[\varepsilon|T^*] = 0$, or $p_k^* = p_\ell^*$.

Back