Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

Francis J. DiTraglia

Camilo Garcia-Jimeno

University of Pennsylvania

October 17th, 2016

What is the causal effect of T^*?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Unknown function that *does not depend on i*
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete instrumental variable

Target of Inference:

ATE function: $\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$

Example: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- y – Log wage
- T^* – True training attendence
- T – Self-reported training attendance
- $\mathbf{x}$ – Individual characteristics
- z – Offer of job training

Example: Returns to Schooling

Oreopoulos (2006, AER)

Fuzzy RD: minimum school-leaving age in UK increased from 14 to 15 in 1947 but some already stayed until 15 before the law and others failed to comply after it.

- y – Log wage
- T^* – School attendance at age 15
- T – Self-report of school attendance at age 15
- $\mathbf{x}$ – Individual characteristics
- z – Indicator: born in or after 1933

Related Literature

Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015)

Model: $y = c + \beta T^* + \varepsilon$

Valid Instrument

$\mathbb{E}[\varepsilon|z] = 0.$

First-stage

$p_k^* \equiv \mathbb{P}(T^* = 1|z = z_k) \neq \mathbb{P}(T^* = 1|z = z_\ell) \equiv p_\ell^*,\ k \neq \ell$

Non-differential Measurement Error

- $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$

Notation

Define error term that absorbs constant: $u = c + \varepsilon$

Observable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\ldots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\ldots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\ldots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T^* = 0$	m^*_{01} p^*_{01}	m^*_{02} p^*_{02}	$\dots$	m^*_{0K} p^*_{0K}
$T^* = 1$	m^*_{11} p^*_{11}	m^*_{12} p^*_{12}	$\dots$	m^*_{1K} p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u | T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p^*_k = \mathbb{P}(T^* = 1 | z = z_k)$$

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Main Result (Correct) – Exogenous Treatment

Relevant binary instrument z ($p_1^* \neq p_2^*$) identifies α_0, α_1 and $\mathbb{E}[y|T^*]$ provided that $\mathbb{E}[\nu|T^*, T, z] = 0$ and $\alpha_0 + \alpha_1 < 1$.

Extension (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$, $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$ identified.

▸ Details

What if z takes on more than two values?

$$\boxed{\mathbb{E}[\varepsilon|z] = 0 \implies \textit{pair} \text{ of equations for each } k = 1, \ldots, K}$$

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + (1 - \alpha_0)c - (p_k - \alpha_0)m^*_{1k}$$

$$\hat{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m^*_{1k}$$

where $\hat{y}_{0k} = (1 - p_k)\bar{y}_{0k}$ and $\hat{y}_{0k} = p_k\bar{y}_{1k}$

$$\boxed{2K \text{ Equations in } K + 4 \text{ Unknowns}}$$

Theorem: β is undentified regardless of K.

Proof of special case: $\alpha_0 = 0$

1. System of equations:

$$\widetilde{y}_{0k} = c + p_k \left(\frac{\beta \alpha_1}{1 - \alpha_1} \right) - p_k m^*_{1k}$$

$$\widetilde{y}_{1k} = p_k \beta + p_k m^*_{1k}$$

2. $\beta/(1-\alpha_1) \equiv \mathcal{W}$ identified, $\beta\alpha_1/(1-\alpha_1) = \mathcal{W} - \beta \implies$

$$(c + p_k \mathcal{W} - \widetilde{y}_{0k})/p_k = \beta + m^*_{1k}$$

$$\widetilde{y}_{1k}/p_k = \beta + m^*_{1k}$$

3. Sum equations from 1. $\implies (c + p_k \mathcal{W} - \widetilde{y}_{0k}) = \widetilde{y}_{1k}$

What about $\alpha_0 + \alpha_1 < 1$?

$$\mathcal{W} = \frac{\beta}{1 - \alpha_0 - \alpha_1}, \quad p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}, \quad 1 - p_k^* = \frac{1 - p_k - \alpha_1}{1 - \alpha_0 - \alpha_1}$$

If $\alpha_0 + \alpha_1 > 0$, then:

- $Cor(T, T^*) > 0 \iff \alpha_0 + \alpha_1 < 1$
- β has same sign as $\mathcal{W}$
- $\alpha_0 < \min_k\{p_k\}$
- $\alpha_1 < \min_k\{1 - p_k\}$
- Two-sided bound for β

Non-differential Measurement Error Assumption

- $\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$
- $\alpha_0 + \alpha_1 < 1$
-

Weaker

$\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$

Stronger

ε conditionally independent of T given T^* and z.

Bounds From Stronger Measurement Error Assumption

Define $F_{tk}(\tau) = \mathbb{P}(Y \leq \tau | T = t, z_k)$ and $F_k(\tau) = \mathbb{P}(Y \leq \tau | z_k)$

$$\alpha_0 \leq p_k \inf_{\tau} \left\{ \left[\frac{F_{1k}(\tau)}{F_k(\tau)} \right] \wedge \left[\frac{1 - F_{1k}(\tau)}{1 - F_k(\tau)} \right] \right\} \leq p_k$$

$$\alpha_1 \leq (1 - p_k) \inf_{\tau} \left\{ \left[\frac{F_{0k}(\tau)}{F_k(\tau)} \right] \wedge \left[\frac{1 - F_{0k}(\tau)}{1 - F_k(\tau)} \right] \right\} \leq (1 - p_k)$$

Bounds for (α_0, α_1) do *not* require z to be a valid instrument!

Put diagram for Oreopoulous here and maybe Burde and Linden too.

Theorem: Identification Result

Conditional *Second* Moment Independence.

New Assumption

Homoskedastic errors w.r.t. the *instrument*: $E[\varepsilon^2|z] = E[\varepsilon^2]$

Reasonable?

Makes sense in an RCT or a true natural experiment.

New Moment Conditions

Defining $\mu^*_{k\ell} = (p_k - \alpha_0)m^*_{1k} - (p_\ell - \alpha_0)m^*_{k\ell}$,

$$\mathbb{E}(y^2|z_k) - \mathbb{E}(y^2|z_\ell) \equiv \Delta\overline{y^2} = \beta\mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu^*_{k\ell}$$

$$\mathbb{E}(yT|z_k) - \mathbb{E}(yT|z_\ell) \equiv \Delta\overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu^*_{k\ell}$$

Theorem: $(\alpha_1 - \alpha_0)$ is Identified if $E[\varepsilon^2|z] = E[\varepsilon^2]$

Requires only binary z

Solve for $\mu^*_{k\ell}$, substitute $\beta = \mathcal{W}(1 - \alpha_0 - \alpha_1)$, rearrange to find

$$\alpha_1 - \alpha_0 = 1 + \mathcal{R}/\mathcal{W}, \quad \text{where} \quad \mathcal{R} \equiv \frac{\Delta\overline{y^2} - 2\mathcal{W}\Delta\overline{yT}}{\mathcal{W}(p_k - p_\ell)}.$$

What good is $(\alpha_1 - \alpha_0)$?

- Test necessary condition for *no mis-classification*: $\alpha_0 = \alpha_1$
- Simple, tighter partial identification bounds for β
- If α_0 known, e.g. zero $\implies$ β point identified

Conditional *Third* Moment Independence

New Assumption

Third moment independence w.r.t instrument: $E[\varepsilon^3|z] = E[\varepsilon^3]$

New Moment Conditions

Define $\lambda^*_{k\ell} = (p_k - \alpha_0)v^*_{1k} - (p_\ell - \alpha_0)v^*_{1\ell}$
where $v^*_{tk} = \mathbb{E}(u^2|T^* = t, z_k)$. Then

$$
\begin{aligned}
\mathbb{E}(y^3|z_k) \quad &- \quad \mathbb{E}(y^3|z_\ell) \equiv \\
\Delta\overline{y^3} \quad &= \quad \beta^2\mathcal{W}(p_k - p_\ell) + 3\beta\mathcal{W}\mu^*_{k\ell} + 3\mathcal{W}\lambda^*_{k\ell} \\
\mathbb{E}(y^2T|z_k) \quad &- \quad \mathbb{E}(y^2T|z_\ell) \equiv \\
\Delta\overline{y^2T} \quad &= \quad \beta(1-\alpha_1)\mathcal{W}(p_k - p_\ell) + 2(1-\alpha_1)\mathcal{W}\mu^*_{k\ell} + \lambda^*_{k\ell}
\end{aligned}
$$

Theorem: β, α_0 and α_1 identified

Adding $E[\varepsilon^3|z] = E[\varepsilon^3]$, z need only be binary.

Solve for $\lambda^*_{k\ell}$, substitute and rearrange. After further substitutions:

$$2\mathcal{W}^2(1-\alpha_1)^2 + 2\mathcal{R}\mathcal{W}(1-\alpha_1) + (\mathcal{S} - \mathcal{R}^2) = 0$$

where

$$\mathcal{S} \equiv \frac{\Delta\overline{y^3} - 3\mathcal{W}\left[\Delta\overline{y^2T} + \mathcal{R}\Delta\overline{yT}\right]}{\mathcal{W}(p_k - p_\ell)}$$

- Quadratic in $(1-\alpha_1)$ and observables only
- Always two real roots: one is $(1-\alpha_1)$ and the other is α_0.
- To tell which is which, need $\alpha_0 + \alpha_1 < 1$.

Simulation Study: $y = \beta T^* + \varepsilon$

Errors

$(\varepsilon, \eta) \sim$ jointly normal, mean 0, variance 1, correlation 0.3.

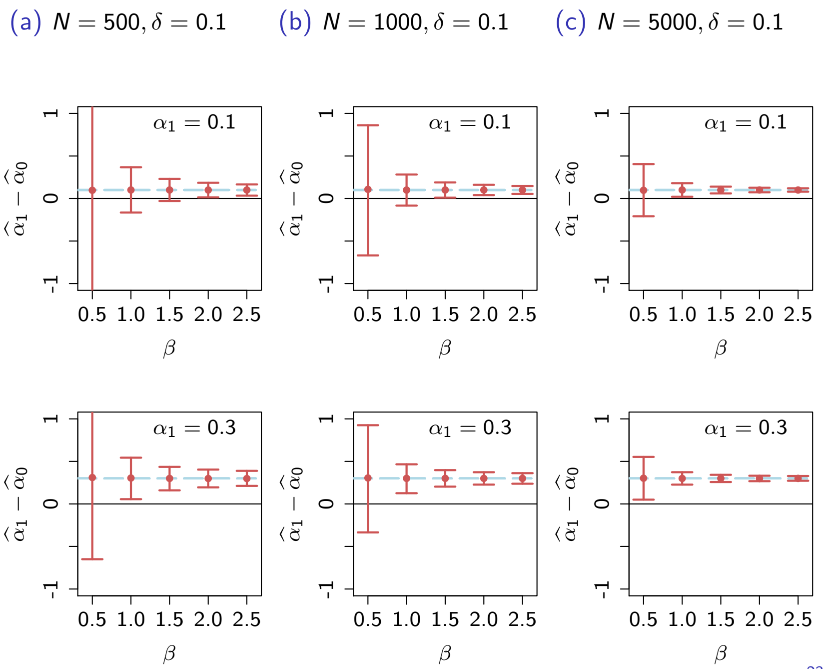
First-Stage

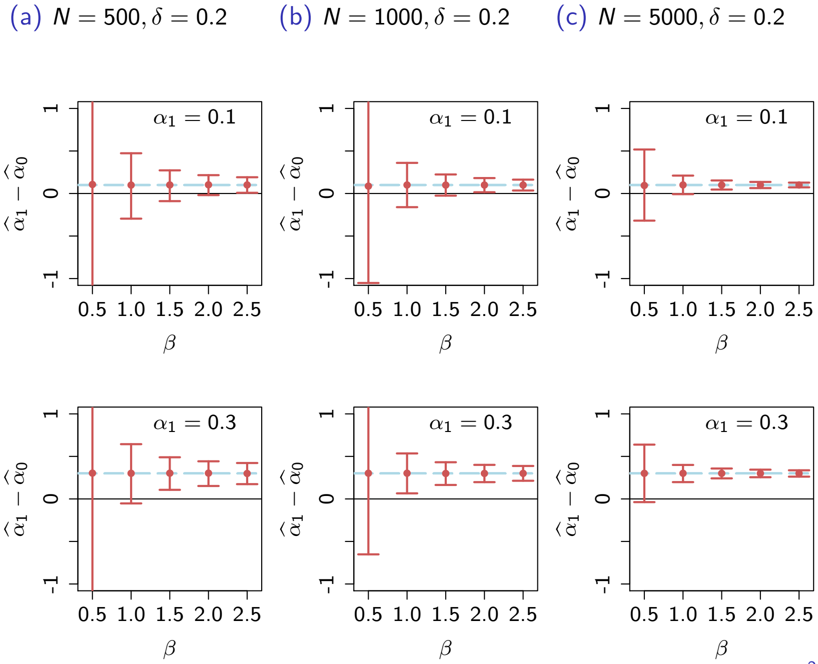
- Half of subjects have $z = 1$, the rest have $z = 0$.
- $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
- $\delta = \mathbb{P}(T^* = 0|z = 1) = \mathbb{P}(T^* = 1|z = 0)$

Mis-classification

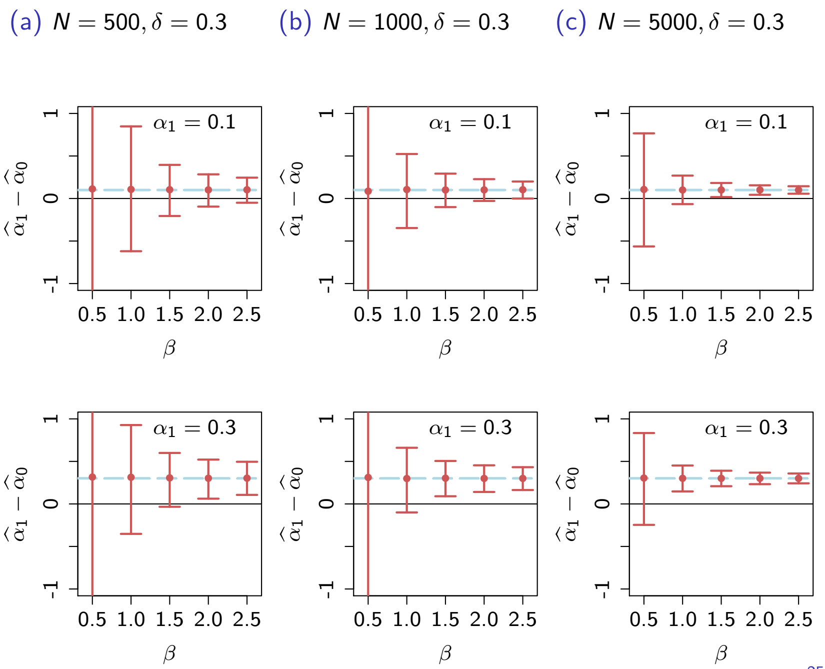
- Set $\alpha_0 = 0$ so $T^* = 0 \implies T = 0$
- $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$
- α_0, α_1 unknown to econometrician.

$$\boxed{\text{Sampling Distribution of } \widehat{\alpha}_1 - \widehat{\alpha}_0}$$


(a) $N = 500, \delta = 0.1$
(b) $N = 1000, \delta = 0.1$
(c) $N = 5000, \delta = 0.1$
$\alpha_1 = 0.1$
$\alpha_1 = 0.3$
$\hat{\alpha}_1 - \hat{\alpha}_0$
β
1
0
-1
0.5 1.0 1.5 2.0 2.5



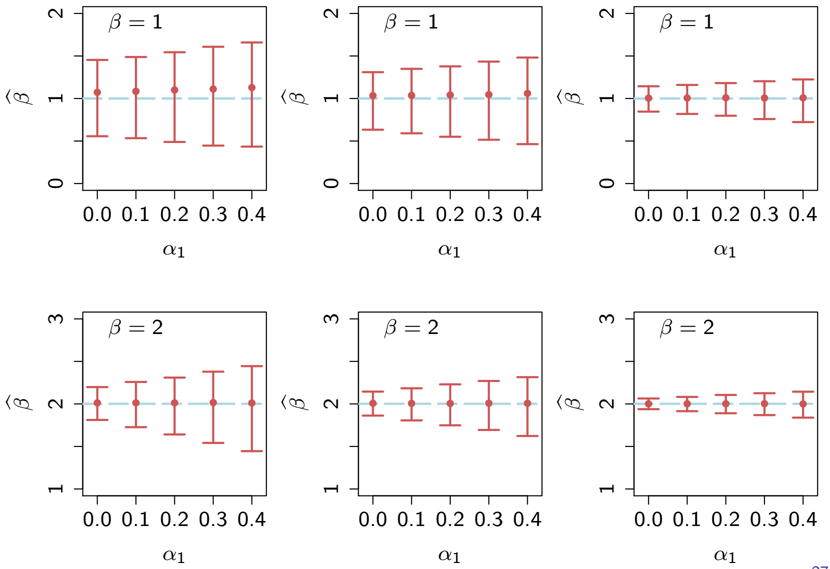
(a) $N = 500, \delta = 0.2$
(b) $N = 1000, \delta = 0.2$
(c) $N = 5000, \delta = 0.2$
$\alpha_1 = 0.1$
$\alpha_1 = 0.3$
$\hat{\alpha}_1 - \hat{\alpha}_0$
β
0.5 1.0 1.5 2.0 2.5
-1 0 1



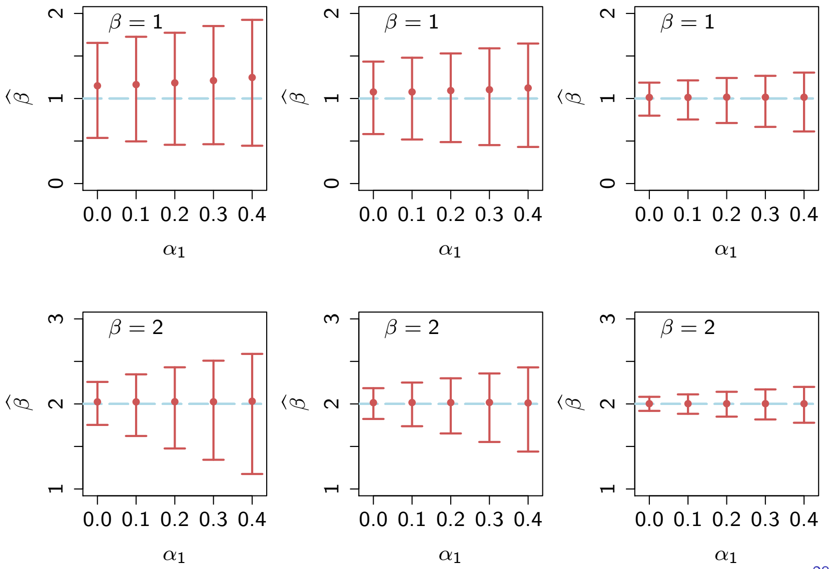
(a) $N = 500, \delta = 0.3$
(b) $N = 1000, \delta = 0.3$
(c) $N = 5000, \delta = 0.3$
$\alpha_1 = 0.1$
$\alpha_1 = 0.3$
$\hat{\alpha}_1 - \hat{\alpha}_0$
β
0.5 1.0 1.5 2.0 2.5
-1 0 1

$$\boxed{\text{Sampling Distribution of } \widehat{\beta} = (1 - \widehat{\alpha}_0 - \widehat{\alpha}_1)\widehat{\beta}_{IV}}$$

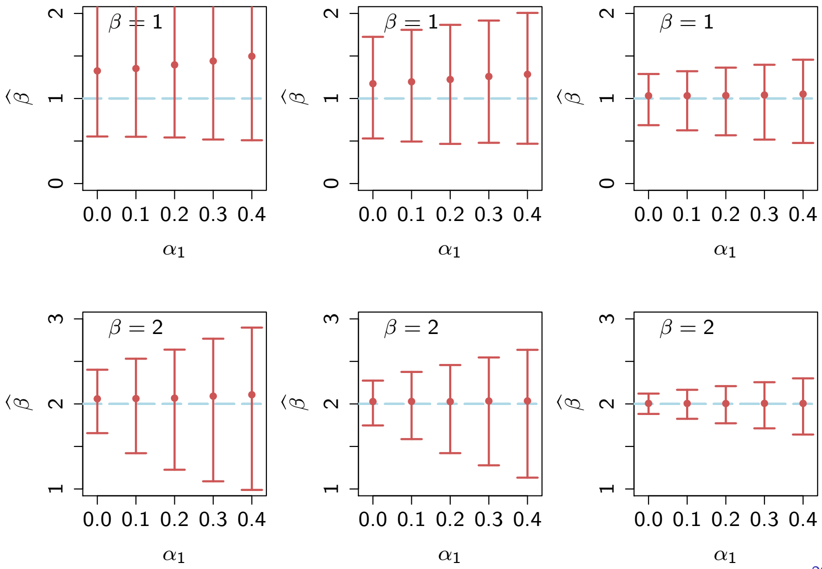
(a) $N = 500, \delta = 0.1$ (b) $N = 1000, \delta = 0.1$ (c) $N = 5000, \delta = 0.1$



(a) $N = 500, \delta = 0.2$ (b) $N = 1000, \delta = 0.2$ (c) $N = 5000, \delta = 0.2$



(a) $N = 500, \delta = 0.3$ (b) $N = 1000, \delta = 0.3$ (c) $N = 5000, \delta = 0.3$



$$\Delta(\tau) = F_k(\tau) - F_\ell(\tau)$$
$$\widetilde{\Delta}_1(\tau) = p_k F_{1k}(\tau) - p_\ell F_{1\ell}(\tau)$$

$$\widetilde{\Delta}_1(\tau+\beta) - \widetilde{\Delta}_1(\tau) = \alpha_0 \Delta(\tau+\beta) - (1-\alpha_1)\Delta(\tau)$$

$$(1-\alpha_0-\alpha_1) = \left(e^{-i\omega\beta} - 1\right)[\alpha_0 - \xi(\omega)]$$

where we define

$$\xi(\omega) \equiv \frac{\varphi_k(\omega) - \varphi_\ell(\omega)}{p_k \varphi_{1k}(\omega) - p_\ell \varphi_{1\ell}(\omega)}$$

Conclusion

- Endogenous, mis-measured binary treatment.
- Important in applied work but no solution in the literature.
- Usual (1st moment) IV assumption fails to identify β
- 2nd moment assumption identifies $\alpha_1 - \alpha_0$
- 3rd moment assumption identifies β

Mahajan's Argument

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Ingredients

1. If $p_1^* \neq p_2^*$, $\mathbb{E}[\varepsilon|z] = 0$ then, since $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$, knowledge of α_0, α_1 is sufficient to recover β. (Correct)
2. If $p_1^* \neq p_2^*$, $\mathbb{E}[\nu|T^*, T, z] = 0$, α_0, α_1 are identified. (Correct)

   How to satisfy both 1 and 2 while allowing $\mathbb{E}[\varepsilon|T^*] \neq 0$?
3. Assume that $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$

   (i.e. $m_{01}^* = m_{02}^*$ and $m_{11}^* = m_{12}^*$)

The Flaw in Mahajan's Argument

Proposition

If $\mathbb{E}[\varepsilon|T^*] \neq 0$ then $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$ combined with $\mathbb{E}[\varepsilon|z] = 0$ implies $p_1^* = p_2^*$, i.e. z is irrelevant for T^*.

Proof

$\mathbb{E}[\varepsilon|z] = 0$ implies

$$(1 - p_1^*)m_{01}^* + p_1^* m_{11}^* = c$$

$$(1 - p_2^*)m_{02}^* + p_2^* m_{12}^* = c$$

while Mahajan's assumption implies $m_{01}^* = m_{02}^*$ and $m_{11}^* = m_{12}^*$. Therefore either $m_{01}^* = m_{02}^* = m_{11}^* = m_{12}^* = c$, which is ruled out by $E[\varepsilon|T^*] = 0$, or $p_1^* = p_2^*$.

Back