Mis-Classified, Binary, Endogenous Regressors: Identification and Inference

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October 11th, 2018

Binary Regressors

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- Thank you for inviting me. Joint work with Camilo Garcia-Jimeno.
- Intro. 'metrics students learn that a valid IV serves double duty: correct for endogeneity and classical measurement error
- Classical measurement error is a special case: requires true value of regressor indep. of or at least uncorrelated with measurement error
- Applied work often involves endogenous binary regressor: smoker/non-smoker or union/non-union. Binary $\implies$ non-classical error. True 0 $\implies$ can only mis-measure *upwards* as 1; true 1 $\implies$ can only mis-measure *downwards* as 0. Error *negatively correlated* with truth.
- To accommodate this, consider *non-diff* error. Say more later, but roughly non-diff means *conditionally classical*: condition on truth and controls, remaining component of error unrelated to everything else.
- Today pose simple question: binary, endog. regressor subject to non-diff. error. Can valid IV correct for *both* measurement error and endog?

What is the effect of T^*?

$$y = c(\mathbf{x}) + \beta(\mathbf{x})T^* + \varepsilon$$

- y – Outcome of interest
- T^* – Unobserved, endogenous binary regressor
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- z – Discrete (typically binary) instrumental variable

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- Here is the specific model I will focus on today. Additively separable model, want to learn the causal effect of binary regressor T^* on y. Unfortunately T^* is unobserved. Observe only mis-measured binary surrogate T. To make matters worse, T^* is endogenous, but we have a discrete instrument z.
- Additive separability is an assumption. Allow very general forms of observed heterogeneity through $\mathbf{x}$ but restricts unobserved heterogeneity.
- Conditionally linear model. This is without loss of generality since the model is additively separable and T^* is binary.
- Mainly focus on additively separable case today, but will also discuss implications of our results for a LATE model.

Using a discrete IV to learn about $\beta(\mathbf{x})$

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Contributions of This Paper

1. Show that only existing point identification result for mis-classified, endogenous T^* is incorrect.
2. Sharp identified set for β under standard assumptions.
3. Point identification of β under slightly stronger assumptions.
4. Point out problem of weak identification in mis-classification models, develop identification-robust inference for β.

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- Here are the main contributions of paper that I will discuss today.
- Many papers consider using IV to identify effect of exog. mis-measured binary regressor, but little work on endog. case. First: show only point identification result for this case incorrect: ident. is an open question.
- Next: use standard assumptions to derive the “sharp identified set” for β. This means *fully* exploit all information in the data and our assumptions to derive tightest possible bounds for β. If bounds contain a single point, β is point identified. Otherwise partially identified.
- Novel and informative bounds for β, but not point identified. Then consider slightly stronger assumptions that allow us to exploit additional features of the data and show that these suffice to point identify β.
- Next consider inference. Show that mis-classification models, suffer from potential weak identification. Propose procedure for robust inference.
- Now a motivating example...

Example: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with pregnant smokers in England: half given nicotine patches, the rest given placebo patches. Some given nicotine fail to quit; some given placebo quit.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Indicator of nicotine patch

Baseline Assumptions I – Model & Instrument

Additively Separable Model

$$y = c(\mathbf{x}) + \beta(\mathbf{x})T^* + \varepsilon, \quad \mathbb{E}[\varepsilon] = 0$$

Valid & Relevant Instrument: $z \in \{0, 1\}$

- $\mathbb{P}(T^* = 1|\mathbf{x}, z = 1) \neq \mathbb{P}(T^* = 1|\mathbf{x}, z = 0)$
- $\mathbb{E}[\varepsilon|\mathbf{x}, z] = 0$
- $0 < \mathbb{P}(z = 1|\mathbf{x}) < 1$

Baseline Assumptions II – Measurement Error

Notation

- $\alpha_0(\mathbf{x}, z) \equiv \mathbb{P}(T = 1 | T^* = 0, \mathbf{x}, z)$
- $\alpha_1(\mathbf{x}, z) \equiv \mathbb{P}(T = 0 | T^* = 1, \mathbf{x}, z)$

Mis-classification unaffected by z

$$\alpha_0(\mathbf{x}, z) = \alpha_0(\mathbf{x}), \quad \alpha_1(\mathbf{x}, z) = \alpha_1(\mathbf{x})$$

Extent of Mis-classification

$$\alpha_0(\mathbf{x}) + \alpha_1(\mathbf{x}) < 1 \quad (T \text{ is positively correlated with } T^*)$$

Non-differential Mis-classification

$$\mathbb{E}[\varepsilon | \mathbf{x}, z, T^*, T] = \mathbb{E}[\varepsilon | \mathbf{x}, z, T^*]$$

Existing Results

Correct: Exogenous T^*

- Mahajan (2006), Frazis & Loewenstein (2003)
- $\mathbb{E}[\varepsilon|\mathbf{x}, z, T^*] = 0$ + "Baseline" $\Rightarrow \beta(\mathbf{x})$ identified.

Incorrect: Endogenous T^*

- Mahajan (2006) A.2
- $\mathbb{E}[\varepsilon|\mathbf{x}, z, T^*, T] = \mathbb{E}[\varepsilon|\mathbf{x}, T^*]$ + "Baseline" $\Rightarrow \beta(\mathbf{x})$ identified.

We show: Mahajan's assumptions imply that the instrument z is uncorrelated with T^* unless T^* is in fact *exogenous*.

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We show: Mahajan's assumptions imply that the instrument z is uncorrelated with T^* unless T^* is in fact *exogenous*.

- Point out that the FL estimator is a nonlinear GMM rather than IV and note that they require*joint* exogeneity of T^* and z.
- 1st contribution: show that only existing point identification result for mis-measured, binary, endog. regressor is false
- As mentioned a few minutes ago, main result from Mahajan (2006; Ecta) is for T^*, but paper also contains a result for the endogenous case [READ THE RESULT]
- Exotic–looking assumption is needed to leverage Mahajan's result for the exogenous case. Unfortunately we show that it leads to a contradiction. [READ THE RESULT]
- Identification in this model is an open question: though Mahajan's proof fails, this does not establish that β is unidentified under the baseline assumptions.
- Next step show you two known results: simple bounds for α_0, α_1, and relationship between IV estimator and α_0, α_1, yielding bounds for β
- Then our 2nd contribution: sharp identified set for β under baseline

"Weak" Bounds

First-Stage

$$p_k(\mathbf{x}) \equiv \mathbb{P}(T = 1|\mathbf{x}, z = k)$$

IV Estimand

$$\frac{\mathbb{E}[y|\mathbf{x}, z = 1] - \mathbb{E}[y|\mathbf{x}, z = 0]}{p_1(\mathbf{x}) - p_0(\mathbf{x})} = \frac{\beta(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}$$

Bounds for (α_0, α_1)

$\alpha_0(\mathbf{x}) \leq \min_k \{p_k(\mathbf{x})\}, \quad \alpha_1(\mathbf{x}) \leq \min_k \{1 - p_k(\mathbf{x})\}$ more

Bounds for β

$\beta(\mathbf{x})$ is between IV and Reduced form; same sign as IV. more

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Bounds for β

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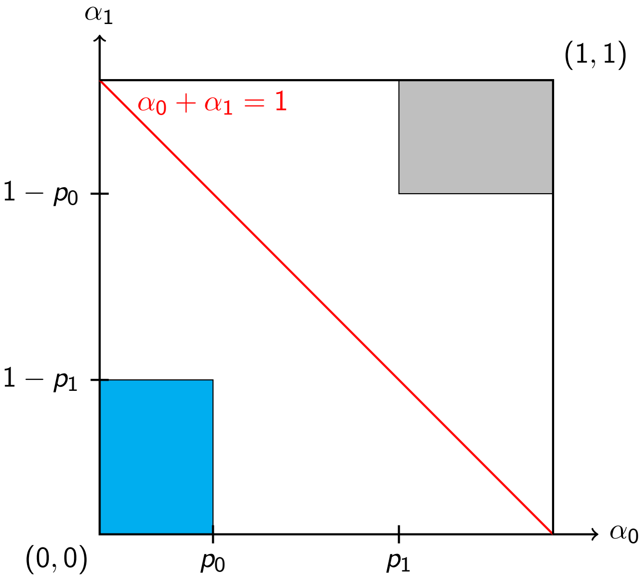
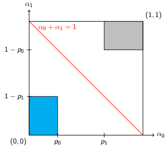
This doesn't rely on non-diff assumption or additive separability. Mention F&L (2003) and Ura (2016). But the point identification results from the literature rely on non-diff, and these bounds do not in fact impose that. Do these contain any additional information about β? Perhaps they even point identify it!



α_1
$(1,1)$
$\alpha_0 + \alpha_1 = 1$
$1 - p_0$
$1 - p_1$
α_0
$(0,0)$
p_0
p_1

Binary Regressors



- Describe the picture. Explain how the upper-right corner of the blue rectangle corresponds to the reduced form estimator and the lower-left to the IV estimator. Explain $\alpha_0 + \alpha_1 < 1$ in relation to the gray rectangle and red line.
- Weak bounds for $(\alpha_0, \alpha_1, \beta)$ simple and informative. Others have used related idea: Frazis & Loewenstein (2003) and Ura (forthcoming). But weak bounds don't use non-diff assump. Know that non-diff is powerful: point identifies effect of an exog T^*. Can we improve upon weak bounds for endog. T^*?
- To answer this, derive sharp identified set under baseline assumptions: new to the literature. Important even if our main concern is point identification: while we showed a flaw in Mahajan's proof, we did *not* demonstrate
- How to derive sharp set? Question: for what values of unknown params can we construct valid joint dist. for (y, T, T^*, z) compatible with observed joint for (y, T, z) under our assumptions? Factorize: joint for (T, T^*, z) & conditional for $y|T, T^*, z$. Turns out that weak bounds for (α_0, α_1) ensure valid joint for (T, T^*, z) so suffices to look at conditional: $y|T, T^*, z$.

Restrictions from Non-differential Mis-classification?

(Suppress **x** for simplicity)

Notation

- $r_{tk} \equiv \mathbb{P}(T^* = 1 | T = t, z = k)$
- z_k is shorthand for $z = k$

Iterated Expectations over T^*

$$\mathbb{E}(y|T = 0, z_k) = (1 - r_{0k})\mathbb{E}(y|T^* = 0, T = 0, z_k) + r_{0k}\mathbb{E}(y|T^* = 1, T = 0, z_k)$$

$$\mathbb{E}(y|T = 1, z_k) = (1 - r_{1k})\mathbb{E}(y|T^* = 0, T = 1, z_k) + r_{1k}\mathbb{E}(y|T^* = 1, T = 1, z_k)$$

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Adding Non-differential Assumption

$$\mathbb{E}(y|T = 0, z_k) = (1 - r_{0k})\mathbb{E}(y|T^* = 0, z_k) \qquad + r_{0k}\mathbb{E}(y|T^* = 1, z_k)$$

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$\boxed{\text{2 equations in 2 unknowns} \Rightarrow \text{solve for } \mathbb{E}(y|T^* = t^*, z = k) \text{ given } (r_{0k}, r_{1k}).}$

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$\boxed{\text{2 equations in 2 unknowns} \Rightarrow \text{solve for } \mathbb{E}(y|T^*=t^*, z=k) \text{ given } (r_{0k}, r_{1k}).}$

- Suppress dependence on **x**. Study conditional dist of $y|T, T^*, z$. Unobserved but related to dist of $y|T, z$ via a mixture model. Mixing probs are r_{tk}. These turn out to be a function of (α_0, α_1) and observables only. Shorthand: z_k denotes $z = k$.
- First look at means. For each value k that the IV takes on, there are two observed means $\mathbb{E}[y|T = (0,1), z_k]$ and four unobserved means $\mathbb{E}[y|T = (0,1), T^* = (0,1), z_k]$. But the non-diff assumption restricts the four unobserved means: we can *drop* T from the conditioning set after conditioning on T^*, z. Hence, only two unknown means: color-coded to show common unknowns across equations.
- Remember: r_{tk} is known given (α_0, α_1), so we see that the non-diff. assumption lets us solve for the two unknown means at any specified pair (α_0, α_1): we simply have two linear equations in two unknowns.

Restrictions from Non-differential Mis-classification?

Mixture Representation

$$F_{tk} = (1 - r_{tk})F_{tk}^{0} + r_{tk}F_{tk}^{1}$$

$$F_{tk} \equiv y|(T = t, z = k)$$
$$F_{tk}^{t^*} \equiv y|(T^* = t^*, T = t, z = k)$$

Restrictions

- $\mathbb{E}(y|T^*, T, z) = \mathbb{E}(y|T^*, z)$ observable given (α_0, α_1)
- r_{tk} observable given (α_0, α_1)

Question

Given (α_0, α_1) can we always find (F_{tk}^{0}, F_{tk}^{1}) to satisfy the mixture model?

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Question

Given (α_0, α_1) can we always find (F_{tk}^0, F_{tk}^1) to satisfy the mixture model?

- Looked at means, now look at distributions. Observe F_{tk} the distribution of $y|T, z$. This is a mixture of two unobserved distributions: F_{tk}^0 and F_{tk}^1.
- Although (F_{tk}^0, F_{tk}^1) are unobserved, they're constrained. First, they need to "integrate" to F_{tk} which is observed. Second, the mixing probability r_{tk} is a *known* function of (α_0, α_1) given observables. Third, as we saw on the preceding slide, non-differential measurement error implies that the means of F_{tk}^0 and F_{tk}^1 are *known* functions of (α_0, α_1).
- Given these constraints, can we find valid distributions (F_{tk}^0, F_{tk}^1) to satisfy the mixture representation for *any pair* (α_0, α_1)? Or are there some values for the mis-classification probabilities that are incompatible with the mixture model?

Restrictions from Non-differential Mis-classification?

Equivalent Problem

Given a specified CDF F, for what values of p and μ do there exist valid CDFs (G, H) with $F = (1-p)G + pH$ and $\mu = \text{mean}(H)$?

Necessary and Sufficient Condition if F is Continuous

$$\underline{\mu}(F,p) \leq \ \mu \ \leq \overline{\mu}(F,p)$$

$$\underline{\mu}(F,p) \equiv \int_{-\infty}^{\infty} x \left[p^{-1} f(x) \mathbf{1}\{x < F^{-1}(p)\}\right] dx = \int_{-\infty}^{\infty} x \underline{h}(x)\, dx$$

$$\overline{\mu}(F,p) \equiv \int_{-\infty}^{\infty} x \left[p^{-1} f(x) \mathbf{1}\{x > F^{-1}(1-p)\}\right] dx = \int_{-\infty}^{\infty} x \bar{h}(x)\, dx$$

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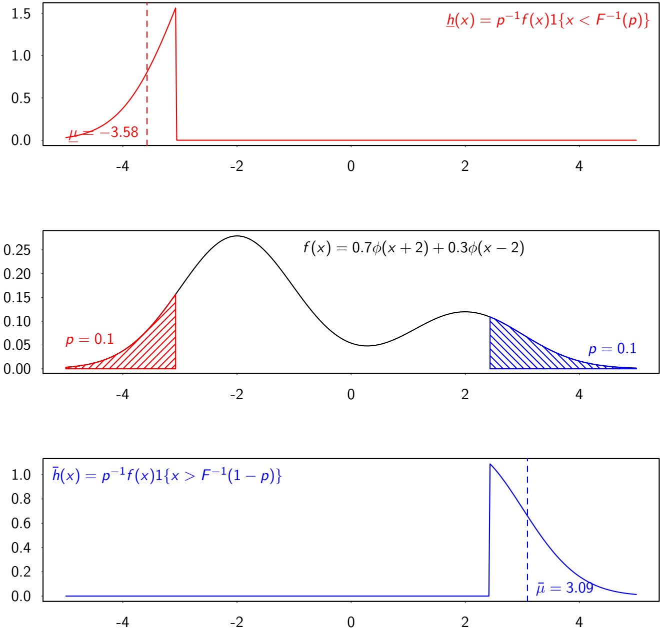
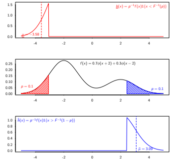
- To answer this question, we need to answer a more abstract question about mixture distributions. In particular, suppose that we observe a distribution F. Can we construct valid distributions (G, H) such that F ia s mixture of G and H in which H has mixing weight p and mean μ?
- To be clear: in this exercise F is fixed. The question is: if I postulate a mixing probability p and a mean μ for one of the mixture components, can this ever lead to a contradiction? Are we free to pick any pair (p, μ) or does the observed distribution F tie our hands?
- It turns out that if y is continuously distributed, one can derive relatively simple necessary and sufficient conditions using a first-order stochastic dominance argument.
- In particular: for any fixed (F, p) there is a lower bound $\underline{\mu}$ and an upper bound $\overline{\mu}$ within which the postulated mean μ *must* lie, for it to be possible to construct a valid mixture. These lower and upper bounds are in fact expectations taken with respect to densities constructed by *truncating* F.
- Rather than staring at these integrals, let's look at a simple example.

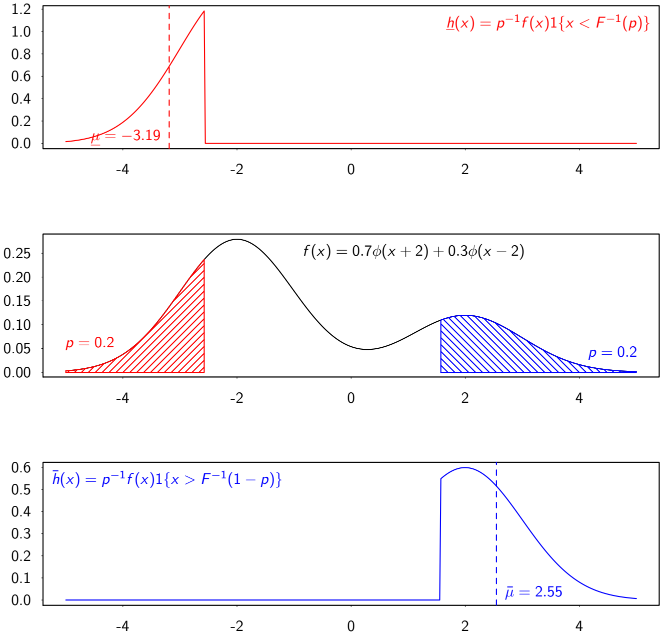


$\underline{h}(x) = p^{-1}f(x)1\{x < F^{-1}(p)\}$
$\underline{\mu} = -3.58$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.1$
$p = 0.1$
$\bar{h}(x) = p^{-1}f(x)1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 3.09$

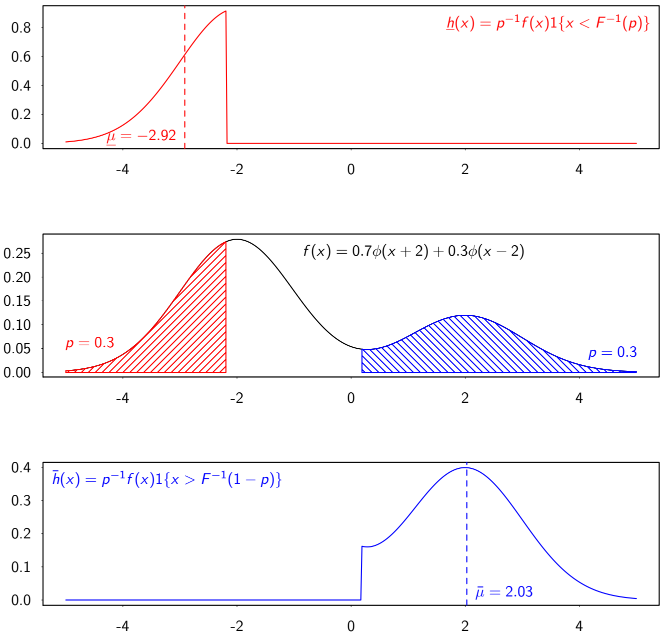
Binary Regressors

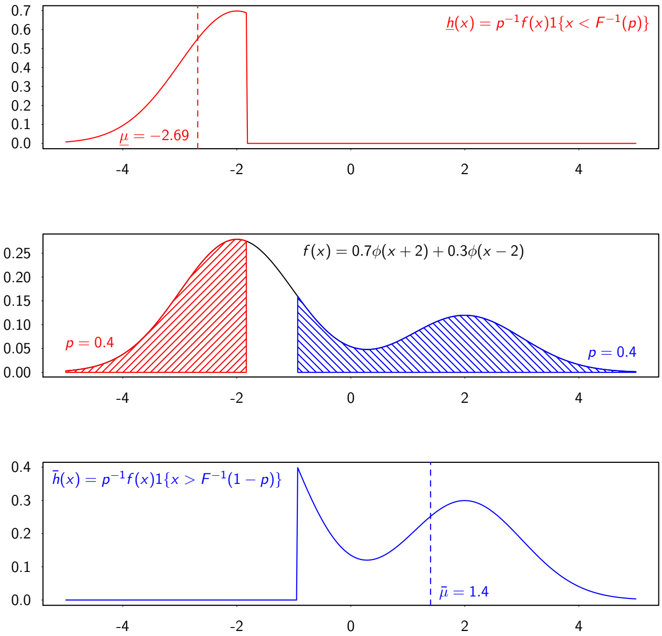


- This picture has three panels. The middle panel shows the observed distribution f. I have chosen a simple mixture of normals with variance equal to one: 70% of the weight is assigned to the one with a mean of -2 and 30% to the one with a mean of $+2$.
- The top panel depicts the "lower bound" density $\underline{h}$. This density takes its shape from the *lower tail* of f. In is simply f *truncated* to take on values below its pth quantile.
- The bottom panel depicts the "upper bound" density $\overline{h}$. This density takes its shape from the *upper tail* of f. It is simply f *trucated* to take on values above its $(1-p)$th quantile.
- For this particular choice of observed distribution f, the figure shows how a particular postulated value of p, in this instance 0.1, constrains μ: it is bounded below by $\underline{\mu} = -3.58$ and bounded above by $\overline{\mu} = 3.09$. This means that if $p = 0.1$, then μ must lie between -3.58 and 3.09 for it to be possible to construct a valid mixture that "integrates" to f. As we increase p, these bounds tighten, so we have less freedom in our choice of μ.

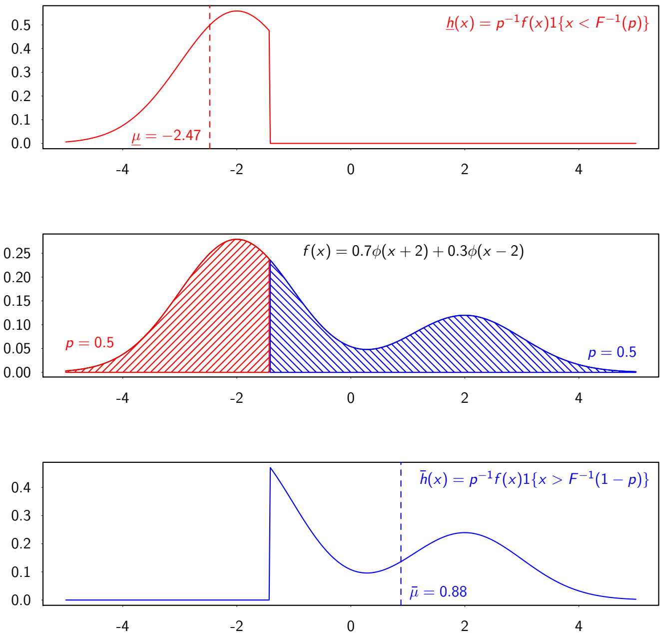


$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -3.19$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.2$
$p = 0.2$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 2.55$

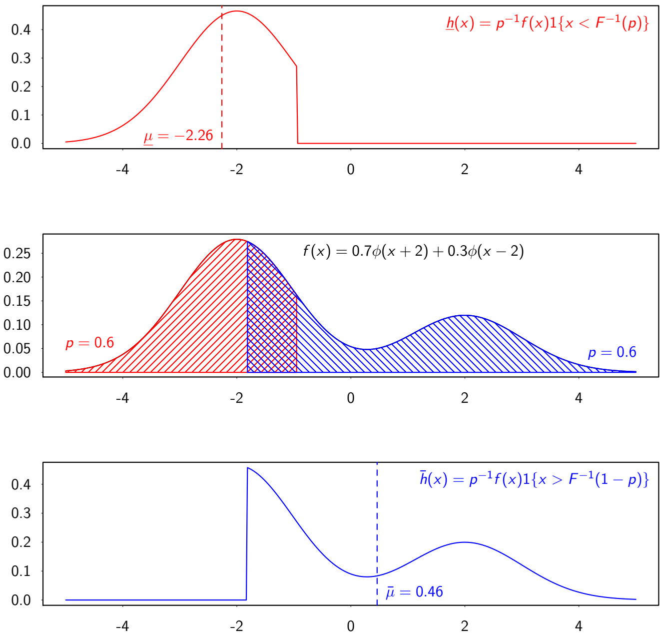

$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.92$
0.8
0.6
0.4
0.2
0.0
-4
-2
0
2
4
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.3$
$p = 0.3$
0.25
0.20
0.15
0.10
0.05
0.00
-4
-2
0
2
4
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 2.03$
0.4
0.3
0.2
0.1
0.0
-4
-2
0
2
4



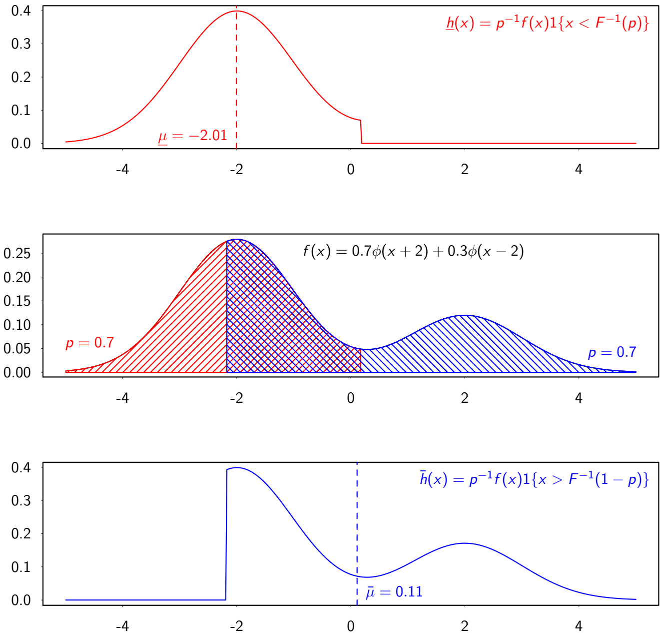
$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.69$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.4$
$p = 0.4$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 1.4$



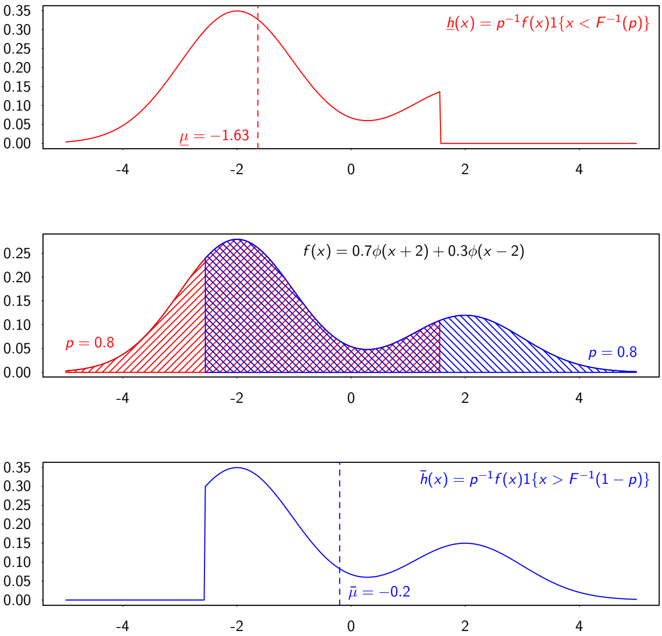
$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.47$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.5$
$p = 0.5$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 0.88$



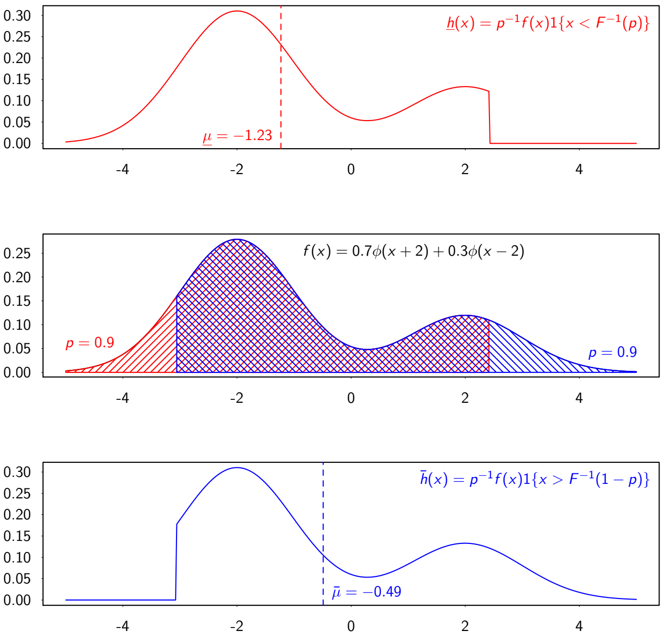
$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.26$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.6$
$p = 0.6$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 0.46$

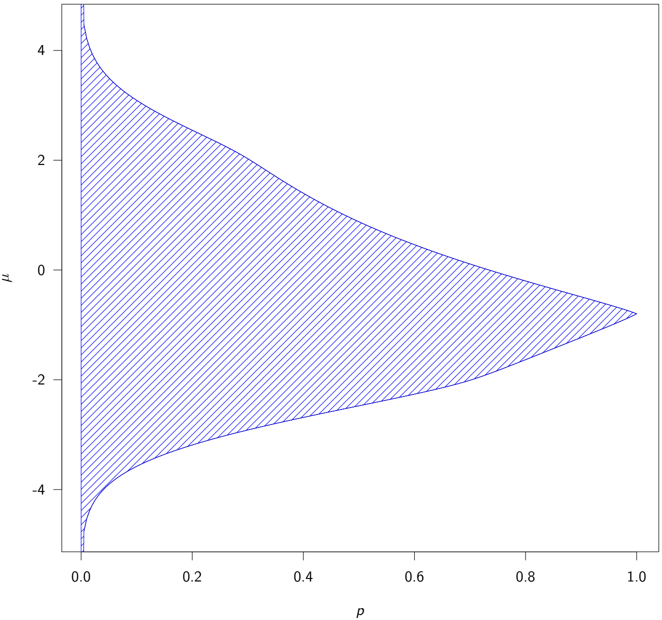


$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -2.01$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.7$
$p = 0.7$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = 0.11$



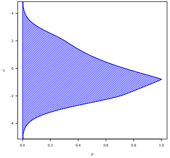
$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -1.63$
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.8$
$p = 0.8$
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = -0.2$


$\underline{h}(x) = p^{-1} f(x) 1\{x < F^{-1}(p)\}$
$\underline{\mu} = -1.23$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-4
-2
0
2
4
$f(x) = 0.7\phi(x+2) + 0.3\phi(x-2)$
$p = 0.9$
$p = 0.9$
0.25
0.20
0.15
0.10
0.05
0.00
-4
-2
0
2
4
$\bar{h}(x) = p^{-1} f(x) 1\{x > F^{-1}(1-p)\}$
$\bar{\mu} = -0.49$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-4
-2
0
2
4



4
2
0
-2
-4
μ
0.0
0.2
0.4
0.6
0.8
1.0
p

Binary Regressors



- For this particular choice of f, a mixture of normals, the blue shaded region shows all pairs (p, μ) that are compatible with the mixture.
- If $p = 0$, μ is unconstrained. This makes sense: in this case H can have any mean because it contributes nothing to the mixture that generates F.
- In contrast, if $p = 1$ then μ must *equal* the mean of the observed distribution F, in this case -0.8, since this corresponds to a degenerate mixture in which $F = H$.
- So how does this relate to our original problem? Remember that we observe the distribution of $y|T, z$ which is related to the unobserved distribution of $y|T, T^*, z$ via a mixture model. The mixing probability depends only on observables and (α_0, α_1) as do the means of the mixture components. Hence, some values of (α_0, α_1) are incompatible with the mixture model. This in turn restricts β through its relationship to the IV estimand. In fact we have *joint* restrictions for all (t, k) so the book-keeping is complicated, but the basic intuition is exactly as I've shown you in this simple mixture of normals example.

Sharp Identified Set under Baseline Assumptions

Theorem

(i) If $\mathbb{E}[y|\mathbf{x}, T = 0, z = k] \neq \mathbb{E}[y|\mathbf{x}, T = 1, z = k]$ for some k, non-differential assump. strictly improves upon weak bounds.

(ii) Under the baseline assumptions, β is not point identified, regardless of how many (discrete) values z takes on.

Corollary

Bounds for α_0, α_1, and β remain valid in a LATE model. They may not be sharp, however, sharp, since they do not incorporate the testable implications of the LATE assumptions.

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- Second main contribution: sharp identified set for $(\alpha_0, \alpha_1, \beta)$ under the baseline assumptions. The description of the set is fairly complicated, so I'm not going to show it on the slide. But the form that this set takes leads to two important results. First, the non-differential measurement error assumption *generically* improves upon the weak bounds. Second, under the baseline assumptions β is *never* point identified, regardless of how many different (discrete) values z takes.
- Some intuition: the true β always lies within the identified set by definition. It turns out that $\alpha_0 = \alpha_1 = 0$ implies that the mixing probabilities r_{tk} are all either zero or one. But in this case the mixtures are trivial, so we can simply set $F = H$. Hence, the IV estimand always lies in the sharp identified set.
- Corollary: everything I've said so far concerns an additively separable model. But in fact, bounds we derive under the baseline assumptions remain valid if we re-state our assumptions so that they involve a LATE model. These bounds may not be sharp in a LATE model, however, because the LATE assumptions themselves have testable implications. We don't impose these since we're mainly interested in the additively separable case.
- What now? Sharp bounds quite informative in practice, but they do not point identify β. Baseline assumptions aren't enough. Are there slightly stronger but still plausible assumptions that allow us to point identify β? Yes!

Point Identification: 1st Ingredient

Reparameterization

$$\theta_1(\mathbf{x}) = \beta(\mathbf{x}) / \left[1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})\right]$$

$$\theta_2(\mathbf{x}) = \left[\theta_1(\mathbf{x})\right]^2 \left[1 + \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})\right]$$

$$\theta_3(\mathbf{x}) = \left[\theta_1(\mathbf{x})\right]^3 \left[\left\{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})\right\}^2 + 6\alpha_0(\mathbf{x})\left\{1 - \alpha_1(\mathbf{x})\right\}\right]$$

Lemma

Baseline Assumptions $\implies$ $\text{Cov}(y, z|\mathbf{x}) = \theta_1(\mathbf{x})\text{Cov}(z, T|\mathbf{x})$.

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Lemma

Baseline Assumptions $\implies \mathrm{Cov}(y, z|\mathbf{x}) = \theta_1(\mathbf{x})\mathrm{Cov}(z, T|\mathbf{x})$.

Note that $\beta = 0$ iff $\theta_1 = \theta_2 = \theta_3 = 0$.

Point Identification: 2nd Ingredient

Assumption (II)

$\mathbb{E}[\varepsilon^2|\mathbf{x}, z] = \mathbb{E}[\varepsilon^2|\mathbf{x}]$

Lemma

(Baseline) + (II) $\implies$

$$\mathrm{Cov}(y^2, z|\mathbf{x}) = 2\mathrm{Cov}(yT, z|\mathbf{x})\theta_1(\mathbf{x}) - \mathrm{Cov}(T, z|\mathbf{x})\theta_2(\mathbf{x})$$

Corollary

(Baseline) + (II) + $[\beta(\mathbf{x}) \neq 0] \implies [\alpha_1(\mathbf{x}) - \alpha_0(\mathbf{x})]$ is identified.

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Notice that the corollary implies that β is point identified if mis-classification is one-sided, as it might well be in the smoking example.

Point Identification: 3rd Ingredient

Assumption (III)

(i) $\mathbb{E}[\varepsilon^2|\mathbf{x}, z, T^*, T] = \mathbb{E}[\varepsilon^2|\mathbf{x}, z, T^*]$

(ii) $\mathbb{E}[\varepsilon^3|\mathbf{x}, z] = \mathbb{E}[\varepsilon^3|\mathbf{x}]$

Lemma

(Baseline) + (II) + (III) $\implies$

$$\text{Cov}(y^3, z|\mathbf{x}) = 3\text{Cov}(y^2 T, z|\mathbf{x})\theta_1(\mathbf{x}) - 3\text{Cov}(yT, z|\mathbf{x})\theta_2(\mathbf{x}) + \text{Cov}(T, z|\mathbf{x})\theta_3(\mathbf{x})$$

Point Identification Result

Theorem

(Baseline) + (II) + (III) $\implies \beta(\mathbf{x})$ is point identified. If $\beta(\mathbf{x}) \neq 0$, then $\alpha_0(\mathbf{x})$ and $\alpha_1(\mathbf{x})$ are likewise point identified.

Explicit Solution

$$\beta(\mathbf{x}) = \text{sign}\left[\theta_1(\mathbf{x})\right]\sqrt{3\left[\theta_2(\mathbf{x})/\theta_1(\mathbf{x})\right]^2 - 2\left[\theta_3(\mathbf{x})/\theta_1(\mathbf{x})\right]}$$

Sufficient for (II) and (III)

(a) T is conditionally independent of (ε, z) given $(T^*, \mathbf{x})$

(b) z is conditionally independent of ε given $\mathbf{x}$

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Comment on the sufficient conditions: say that we really think these are what people have in mind in a natural experiment setting. Explain about reporting results in both logs and levels.

Inference for a Mis-classified Regressor

Weak Identification

- β small $\Rightarrow$ moment equalities uninformative about (α_0, α_1) more
- (α_0, α_1) could be on the boundary of the parameter space
- Also true of existing estimators that assume T^* exogenous

Our Approach

- Sharp identified set yields *inequality* moment restrictions that remain informative even if $\beta \approx 0$. more
- Identification-robust inference with equality and inequality MCs.

Inference with Moment Equalities and Inequalities

Moment Conditions

$\mathbb{E}\left[m_j(\mathbf{w}_i, \vartheta_0)\right] \geq 0, \quad j = 1, \cdots, J$

$\mathbb{E}\left[m_j(\mathbf{w}_i, \vartheta_0)\right] = 0, \quad j = J+1, \cdots, J+K$

Test Statistic

$$T_n(\vartheta) = \sum_{j=1}^{J} \left[\frac{\sqrt{n}\ \bar{m}_{n,j}(\vartheta)}{\widehat{\sigma}_{n,j}(\vartheta)}\right]_-^2 + \sum_{j=J+1}^{J+K} \left[\frac{\sqrt{n}\ \bar{m}_{n,j}(\vartheta)}{\widehat{\sigma}_{n,j}(\vartheta)}\right]^2$$

Critical Value

- $\sqrt{n}\,\bar{m}_n(\vartheta_0) \to_d$ normal limit with covariance matrix $\Sigma(\vartheta_0)$
- Use this to bootstrap the limit dist. of $T_n(\vartheta)$ under $H_0 \colon \vartheta = \vartheta_0$

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Explain about the meaning of the m-var, the sigma-hat and the "minus" subscript

Generalized Moment Selection

Andrews & Soares (2010)

- Inequalities that don't bind reduce power of test, so eliminate those that are "far from binding" before calculating critical value:

$$\boxed{\text{Drop inequality } j \text{ if } \frac{\sqrt{n}\,\bar{m}_{n,j}(\vartheta_0)}{\widehat{\sigma}_{n,j}(\vartheta_0)} > \sqrt{\log n}}$$

- Uniformly valid test of $H_0\colon \vartheta = \vartheta_0$ even if ϑ_0 is not point identified.
- Not asymptotically conservative.

Problem

Joint test for the whole parameter vector but we're only interested in β. Projection is conservative and computationally intensive.

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- Not asymptotically conservative.

Problem

Joint test for the whole parameter vector but we're only interested in β. Projection is conservative and computationally intensive.

Explain what not asymptotically conservative means. Explain what projection is and why it's conservative and computationally intensive.

Our Solution: Bonferroni-Based Inference

Special Structure

- β only enters MCs through $\theta_1 = \beta/(1 - \alpha_0 - \alpha_1)$
- Strong instrument $\Rightarrow$ inference for θ_1 is standard.
- Nuisance pars $\boldsymbol{\gamma}$ strongly identified under null for (α_0, α_1)

Procedure

1. Concentrate out $(\theta_1, \boldsymbol{\gamma}) \Rightarrow$ joint GMS test for (α_0, α_1)
2. Invert test $\Rightarrow (1 - \delta_1) \times 100\%$ confidence set for (α_0, α_1)
3. Project $\Rightarrow$ CI for $(1 - \alpha_0 - \alpha_1)$
4. Construct standard $(1 - \delta_2) \times 100\%$ IV CI for θ_1
5. Bonferroni $\Rightarrow (1 - \delta_1 - \delta_2) \times 100\%$ CI for β

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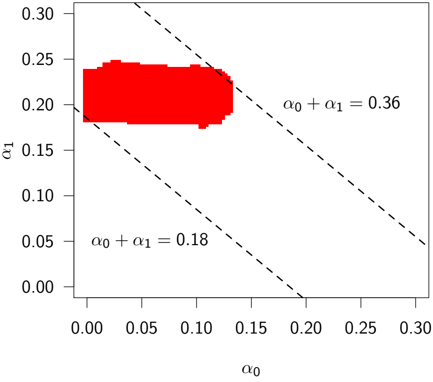
Explain that the procedure works well in simulations etc. Possibly add link to simulation here.

Example (sim data: $\beta = 1, \alpha_0 = 0.1, \alpha_1 = 0.2, n = 5000$)

97.5% GMS Confidence Region for (α_0, α_1)



Bonferroni Interval

1. 97.5% CI for $(1 - \alpha_0 - \alpha_1) = (0.64, 0.82)$
2. 97.5% CI for $\theta_1 = (1.20, 1.47)$
3. $>$ 95% CI for β:
   $(0.64 \times 1.20, 0.82 \times 1.47) = (0.77, 1.21)$

Comparisons

- (0.88, 1.04) for IV if T^* were observed
- (1.22,1.45) for naive IV interval using T

Conclusion

This Paper

- Partial and point identification results for effect of binary, endogenous regressor using a valid instrument.
- Identification-robust inference in models with mis-classification

Related Work

- Relaxing Instrument Validity: "A Framework for Eliticing, Incorporating, and Disciplining Identification Beliefs in Linear Models" (with Camilo Garcia-Jimeno)
- Relaxing Non-differential Measurement Error: "Estimating the Returns to Lying" (with Arthur Lewbel)

Simple Bounds for Mis-classification from First-stage

Unobserved	Observed
$p_k^*(\mathbf{x}) \equiv \mathbb{P}(T^* = 1 \mid \mathbf{x}, z = k)$	$p_k(\mathbf{x}) \equiv \mathbb{P}(T = 1 \mid \mathbf{x}, z = k)$

Relationship

$$p_k^*(\mathbf{x}) = \frac{p_k(\mathbf{x}) - \alpha_0(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}, \quad k = 0, 1$$

z does not affect (α_0, α_1); denominator $\neq 0$

Bounds for Mis-classification

$$\alpha_0(\mathbf{x}) \leq p_k(\mathbf{x}) \leq 1 - \alpha_1(\mathbf{x}), \quad k = 0, 1$$

$\alpha_0(\mathbf{x}) + \alpha_1(\mathbf{x}) < 1$

back

What does IV estimate under mis-classification?

Unobserved

$$\beta(\mathbf{x}) = \frac{\mathbb{E}[y|\mathbf{x}, z=1] - \mathbb{E}[y|\mathbf{x}, z=0]}{p_1^*(\mathbf{x}) - p_0^*(\mathbf{x})}$$

Wald (Observed)

$$\frac{\mathbb{E}[y|\mathbf{x}, z=1] - \mathbb{E}[y|\mathbf{x}, z=0]}{p_1(\mathbf{x}) - p_0(\mathbf{x})} = \beta(\mathbf{x}) \left[\frac{p_1^*(\mathbf{x}) - p_0^*(\mathbf{x})}{p_1(\mathbf{x}) - p_0(\mathbf{x})}\right] = \frac{\beta(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}$$

$$\boxed{p_1^*(\mathbf{x}) - p_0^*(\mathbf{x}) = \frac{p_1(\mathbf{x}) - \alpha_0(\mathbf{x})}{1 - \alpha_0 - \alpha_1(\mathbf{x})} - \frac{p_0(\mathbf{x}) - \alpha_0(\mathbf{x})}{1 - \alpha_0 - \alpha_1(\mathbf{x})} = \frac{p_1(\mathbf{x}) - p_0(\mathbf{x})}{1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})}}$$

back

Partial Identification Bounds for $\beta(\mathbf{x})$

$$\boxed{\beta(\mathbf{x}) = [1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x})]\left[\frac{\mathbb{E}\left[y|\mathbf{x}, z=1\right] - \mathbb{E}\left[y|\mathbf{x}, z=0\right]}{p_1(\mathbf{x}) - p_0(\mathbf{x})}\right]}$$

$$\boxed{0 \leq \alpha_0 \leq \min_k\{p_k(\mathbf{x})\}, \quad 0 \leq \alpha_1 \leq \min_k\{1 - p_k(\mathbf{x})\}}$$

No Mis-classification

$$\alpha_0(\mathbf{x}) = \alpha_1(\mathbf{x}) = 0 \implies \beta(\mathbf{x}) = \text{Wald}$$

Maximum Mis-classification

$$\alpha_0(\mathbf{x}) = p_{\min}(\mathbf{x}),\ \alpha_1(\mathbf{x}) = 1 - p_{\max}(\mathbf{x})$$

$$\implies 1 - \alpha_0(\mathbf{x}) - \alpha_1(\mathbf{x}) = p_{\max}(\mathbf{x}) - p_{\min}(\mathbf{x}) = |p_1(\mathbf{x}) - p_0(\mathbf{x})|$$

$$\implies \beta(\mathbf{x}) = \text{sign}\left\{p_1(\mathbf{x}) - p_0(\mathbf{x})\right\} \times (\text{Reduced Form})$$

back

Just-Identified System of Moment Equalities

Suppress dependence on $\mathbf{x}$...

$$\mathbb{E}\left[\left\{\boldsymbol{\Psi}(\boldsymbol{\theta})\mathbf{w}_i - \boldsymbol{\kappa}\right\} \otimes \begin{pmatrix} 1 \\ z \end{pmatrix}\right] = \mathbf{0}$$

$$\boldsymbol{\Psi}(\boldsymbol{\theta}) \equiv \begin{bmatrix} -\theta_1 & 1 & 0 & 0 & 0 & 0 \\ \theta_2 & 0 & -2\theta_1 & 1 & 0 & 0 \\ -\theta_3 & 0 & 3\theta_2 & 0 & -3\theta_1 & 1 \end{bmatrix}$$

$$\mathbf{w}_i = (T_i, y_i, y_i T_i, y_i^2, y_i^2 T_i, y_i^3)' \qquad \theta_1 = \beta/(1 - \alpha_0 - \alpha_1)$$

$$\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \kappa_3)' \qquad \theta_2 = \theta_1^2(1 + \alpha_0 - \alpha_1)$$

$$\theta_3 = \theta_1^3\left[(1 - \alpha_0 - \alpha_1)^2 + 6\alpha_0(1 - \alpha_1)\right]$$

back

Moment Inequalities I – First-stage Probabilities

$\alpha_0 \leq p_k \leq 1 - \alpha_1$ becomes $\mathbb{E}\left[m(\mathbf{w}_i, \boldsymbol{\vartheta})\right] \geq \mathbf{0}$ for all k where

$$m(\mathbf{w}_i, \boldsymbol{\vartheta}) \equiv \left[\begin{array}{l} \mathbf{1}(z_i = k)(T - \alpha_0) \\ \mathbf{1}(z_i = k)(1 - T_i - \alpha_1) \end{array} \right]$$

Moment Inequalities II – Non-differential Assumption

For all k, we have $\mathbb{E}[m\big(\mathbf{w}_i, \vartheta, \mathbf{q}_k)] \geq 0$ where

$$m\Big(\mathbf{w}_i, \vartheta, \mathbf{q}_k\Big) \equiv \left[\begin{array}{r} y_i \mathbf{1}\left(z_i = k\right)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i \leq \underline{q}_{0k})(1 - T_i)\left(\frac{1-\alpha_0-\alpha_1}{\alpha_1}\right)\right\} \\ -y_i \mathbf{1}(z_i = k)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i > \overline{q}_{0k})(1 - T_i)\left(\frac{1-\alpha_0-\alpha_1}{\alpha_1}\right)\right\} \\ y_i \mathbf{1}\left(z_i = k\right)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i \leq \underline{q}_{1k}) T_i\left(\frac{1-\alpha_0-\alpha_1}{1-\alpha_1}\right)\right\} \\ -y_i \mathbf{1}(z_i = k)\left\{(T_i - \alpha_0) - \mathbf{1}(y_i > \overline{q}_{1k}) T_i\left(\frac{1-\alpha_0-\alpha_1}{1-\alpha_1}\right)\right\} \end{array}\right]$$

and $\mathbf{q}_k \equiv (\underline{q}_{0k},\ \overline{q}_{0k},\ \underline{q}_{1k},\ \overline{q}_{1k})'$ defined by $\mathbb{E}[h(\mathbf{w}_i, \vartheta, \mathbf{q}_k)] = 0$ with

$$h(\mathbf{w}_i, \vartheta, \mathbf{q}_k) = \left[\begin{array}{l} \mathbf{1}(y_i \leq \underline{q}_{0k})\mathbf{1}(z_i = k)(1 - T_i) - \left(\frac{\alpha_1}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(T_i - \alpha_0) \\ \mathbf{1}(y_i \leq \overline{q}_{0k})\mathbf{1}(z_i = k)(1 - T_i) - \left(\frac{1-\alpha_0}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(1 - T_i - \alpha_1) \\ \mathbf{1}(y_i \leq \underline{q}_{1k})\mathbf{1}(z_i = k) T_i - \left(\frac{1-\alpha_1}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(T_i - \alpha_0) \\ \mathbf{1}(y_i \leq \overline{q}_{1k})\mathbf{1}(z_i = k) T_i - \left(\frac{\alpha_0}{1-\alpha_0-\alpha_1}\right)\mathbf{1}(z_i = k)(1 - T_i - \alpha_1) \end{array}\right]$$

back