Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

Francis J. DiTraglia

Camilo Garcia-Jimeno

University of Pennsylvania

November 19th, 2015

What is the causal effect of T^*?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Unknown function that *does not depend on* i
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete instrumental variable

Example 1: Smoking and Birthweight

RCT with 612 pregnant smokers in Glasgow, Scotland: 306 are offered financial incentives to attend smoking cessation program.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Offer of financial incentive

Example 2: Schooling and Test Scores

RCT in Afghanistan: a school is built in 6 out of 11 villages.

- y – Score on math and language test
- T^* – True school attendance
- T – Self-reported school attendance
- $\mathbf{x}$ – Household characteristics
- z – School built in village

Non-classical Measurement Error: Binary T^*

- Many applications of linear model have *binary* treatment
- Binary $T^* \implies \mathbb{E}[T^* w] \leq 0$
- Misclassification Probabilities:

$$\alpha_0 = \mathbb{P}(T = 1 | T^* = 0)$$
$$\alpha_1 = \mathbb{P}(T = 0 | T^* = 1)$$

- Non-Differential Measurement Error: $T \perp (z, u) | T^*$
- $\sigma^2_{T^*} \nless \sigma^2_T$ so work with α_0, α_1 rather than κ
- *Four-dimensional* Problem...

Results for a Mis-classified Binary Regressor

Aigner (1973), Bollinger (1996)...

- Even if $\rho_{T^*u} = 0$, OLS is biased and inconsistent: typically attenuated towards zero *but could flip signs!*

Kane et al. (1999), Black et al. (2000), Frazis et al. (2003)...

- $\rho_{zu} = 0 \implies$ IV solves endogenous regressor problem if there is no mis-classification
- $\rho_{T^*u} = 0$ and $\rho_{zu} = 0 \implies$ non-linear GMM estimator can solve the mis-classification problem

OLS and IV Probability Limits: Binary T^*

$$
\begin{aligned}
\text{plim}\left(\widehat{\beta}_{OLS}\right) &= \frac{\sigma^2_{T^*}}{\sigma^2_T}\left[\beta\left(1-\alpha_0-\alpha_1\right)+\frac{\sigma_{T^*u}}{\sigma^2_{T^*}}\right] \\
\text{plim}\left(\widehat{\beta}_{IV}\right) &= \frac{\beta}{1-\alpha_0-\alpha_1}+\frac{\sigma_{zu}}{\sigma_{zT}} \\
\sigma^2_{T^*} &= \frac{\left(p-\alpha_0\right)\left(1-p-\alpha_1\right)}{\left(1-\alpha_0-\alpha_1\right)^2}
\end{aligned}
$$

Where $p=\mathbb{P}(T=1)$

What About Endogenous, Mis-measured T^*, Valid z?

$$y = \beta T^* + u$$
$$u = c + \varepsilon$$

- No results in the literature for this case
- Important setting in applied work: e.g. RCTs
- Discrete Instrument: $z \in \{z_1, \cdots, z_K\}$
- Non-parametric First Stage: $p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$
- What does $E[\varepsilon | z] = 0$ buy us in this case?

Observable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 1$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\dots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\dots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 1$	$\dots$	$z = K$
$T^* = 0$	m^*_{01} p^*_{01}	m^*_{02} p^*_{02}	$\dots$	m^*_{0K} p^*_{0K}
$T^* = 1$	m^*_{11} p^*_{11}	m^*_{12} p^*_{12}	$\dots$	m^*_{1K} p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u | T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$p^*_k = \mathbb{P}(T^* = 1 | z = z_k) = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}$$

Possible Assumptions On m^*_{tk}

Joint Exogeneity: $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m^*_{tk} = c \quad \text{for all } t, k$$

Exogenous Treatment: $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p^*_{tk} m^*_{tk} = c \quad \text{for all } t$$

Exogenous Instrument: $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p^*_k) m^*_{0k} + p^*_k m^*_{1k} = c \quad \text{for all } k$$

Moment Conditions Imposing $\mathbb{E}[\varepsilon|z] = 0$

One pair of equations for each $k = 1, \ldots, K$

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) + (1-\alpha_0)c - (p_k - \alpha_0)m_{1k}^*$$

$$\hat{y}_{1k} = (1-\alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m_{1k}^*$$

where $\hat{y}_{0k} = (1-p_k)\bar{y}_{0k}$ and $\hat{y}_{0k} = p_k \bar{y}_{1k}$

$\boxed{2K \text{ Equations in } K+4 \text{ Unknowns}}$

Mahajan

Proposition: β is Undentified Regardless of K

Proof Sketch

(1) Show that $\mathcal{W} = \beta/(1 - \alpha_0 - \alpha_1)$ is identified.

(2) Show that $\mathcal{Q} = c + \beta(1 - \alpha_0)/(1 - \alpha_0 - \alpha_1)$ is identified.

(3) $(1) + (2) \implies (\mathcal{Q}, \mathcal{W})$ are *fixed*

(4) Use (3) to rewrite equations in terms of $(\mathcal{Q}, \mathcal{W})$.

(5) Discover that there is only *one* equation per k! Rearranging:

$$m_{1k}^* = \frac{\mathcal{W}(\hat{y}_{0k} - \alpha_1 \mathcal{Q}) - \beta(\mathcal{Q} - \beta - \mathcal{W}\alpha_1) + \mathcal{W}^2(1 - p_k)\alpha_1}{\mathcal{W}(1 - p_k - \alpha_1) - \beta}$$

Special Case of Prev Proof: $\alpha_0 = 0$

$$\hat{y}_{0k} = c + p_k \beta \left(\frac{\alpha_1}{1 - \alpha_1} \right) - p_k m^*_{1k}$$

$$\hat{y}_{1k} = p_k \beta + p_k m^*_{1k}$$

Identification by Conditional Variances?

New Assumption

Homoskedastic errors w.r.t. the *instrument*: $E[\varepsilon^2|z] = E[\varepsilon^2]$

Not Crazy!

Holds in an RCT or a *true* natural experiment.

New Moment Conditions

For each pair (k, ℓ)

$$\begin{aligned} s_k^2 - s_\ell^2 = \mathcal{W}^2 \left[p_k(1-p_k) - p_\ell(1-p_\ell) + (\alpha_0 - \alpha_1)(p_k - p_\ell) \right] \\ +2\mathcal{W} \left[(p_k - \alpha_0)(m_{1k}^* - c) - (p_\ell - \alpha_0)(m_{1\ell}^* - c) \right] \end{aligned}$$

Where $s_k^2 = Var(y|z = z_k)$, and $\mathcal{W}$ is the Wald IV estimator.

Proposition: $(\alpha_0 - \alpha_1)$ is Identified

Define

$$\widetilde{\mathcal{W}}_{k\ell} = \frac{\mathbb{E}[yT|z_k] - \mathbb{E}[yT|z_\ell]}{p_k - p_\ell}$$

Show that:

$$\begin{aligned}(p_k - \alpha_0)(m^*_{1k} - c) - (p_\ell - \alpha_0)(m^*_{1\ell} - c) = \\ (p_k - p_\ell)\left[\widetilde{W}_{k\ell} - \mathbb{E}[y] - \mathcal{W}\left\{(1-p) + (\alpha_0 - \alpha_1)\right\}\right]\end{aligned}$$

Substituting and rearranging:

$$\alpha_0 - \alpha_1 = (2p - 1 - p_k - p_\ell) + \frac{2(\widetilde{W}_{k\ell} - \mathbb{E}[y])}{\mathcal{W}} - \frac{s_k^2 - s_\ell^2}{(p_k - p_\ell)\mathcal{W}^2}$$

What Good is $(\alpha_0 - \alpha_1)$?

- Test a necessary condition for *no mis-classification*: $\alpha_0 = \alpha_1$
- Simple, tighter partial identification bounds for β
- In some settings, one of the mis-classification probabilities is known to be zero $\implies$ β point identified

Identification from Third Moments

Simulation Study

$y = \beta T^* + \varepsilon$

$T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$

$\gamma_0 = \Phi^{-1}(\delta)$, $\gamma_1 = \Phi^{-1}(1-\delta) - \Phi(\delta)$ so that δ

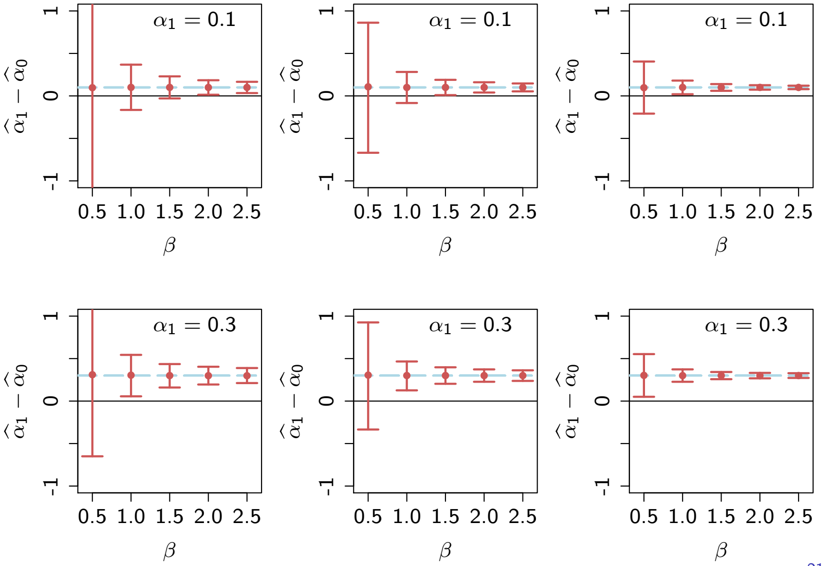
E.g. if $\delta = 0.1$ then 10% of those *not* offered treatment get it anyway, and 10% of those offered treatment don't take it up.

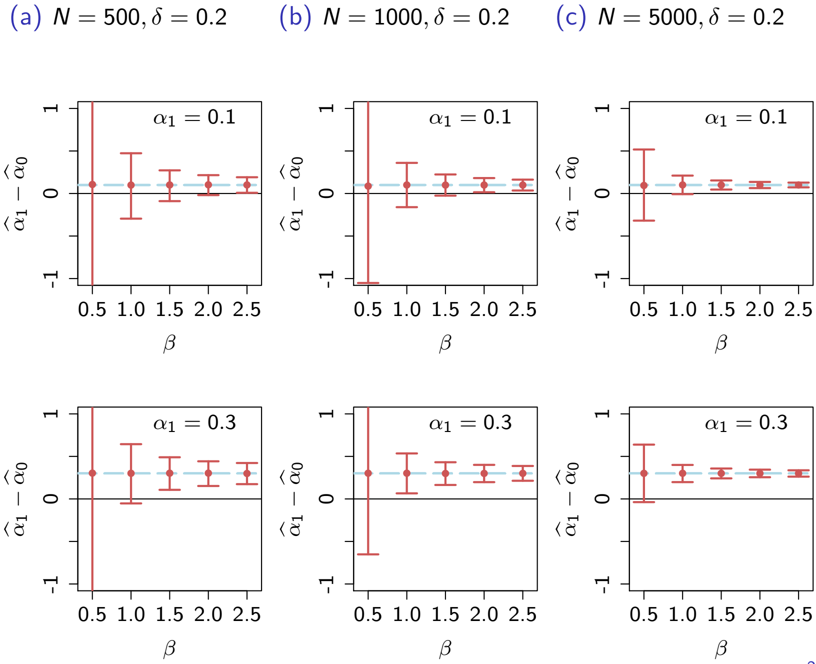
If $T^* = 0$ then $T = 0$ (E.g. Birthweight and smoking)

$T|T^* = 1 \sim$ Bernoulli(?)

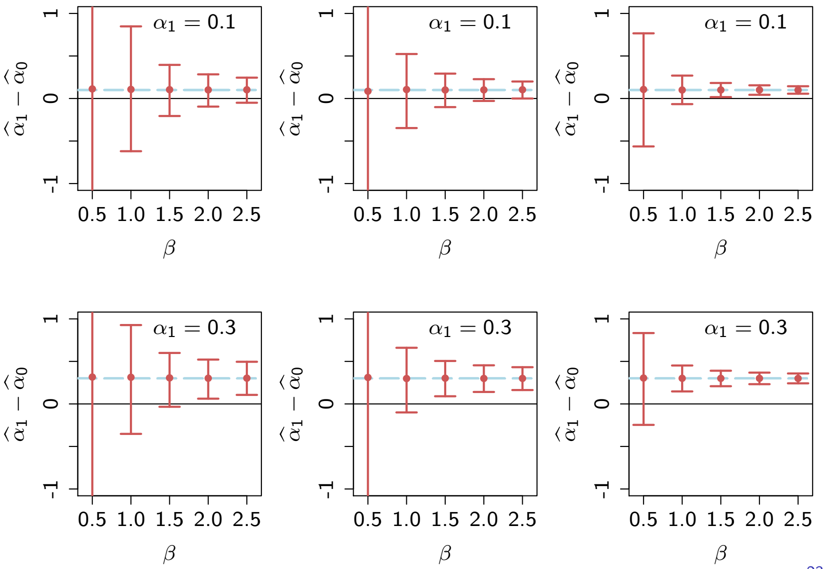
$$\begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix} \right)$$

(a) $N = 500, \delta = 0.1$ (b) $N = 1000, \delta = 0.1$ (c) $N = 5000, \delta = 0.1$

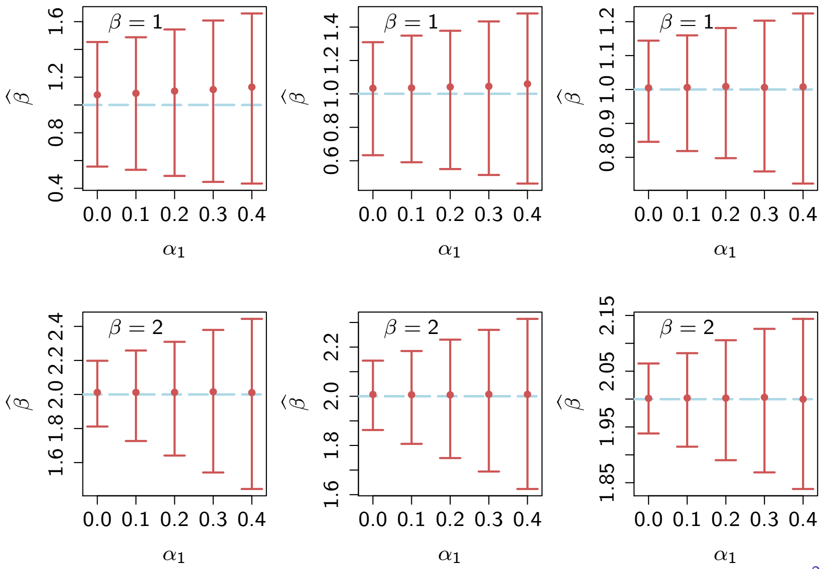


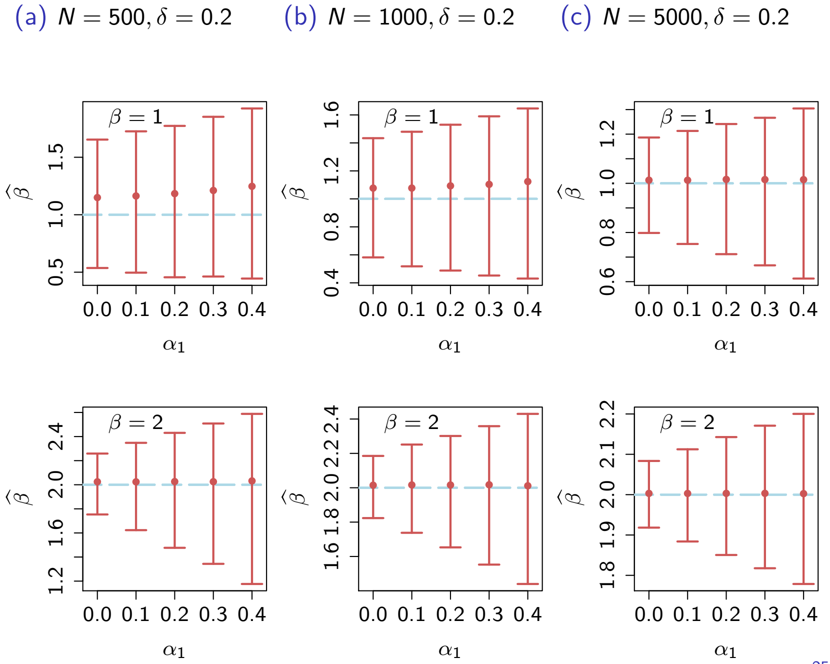

(a) $N = 500, \delta = 0.2$
(b) $N = 1000, \delta = 0.2$
(c) $N = 5000, \delta = 0.2$
$\alpha_1 = 0.1$
$\alpha_1 = 0.3$
$\widehat{\alpha}_1 - \widehat{\alpha}_0$
β
0.5 1.0 1.5 2.0 2.5
-1 0 1

(a) $N = 500, \delta = 0.3$ (b) $N = 1000, \delta = 0.3$ (c) $N = 5000, \delta = 0.3$



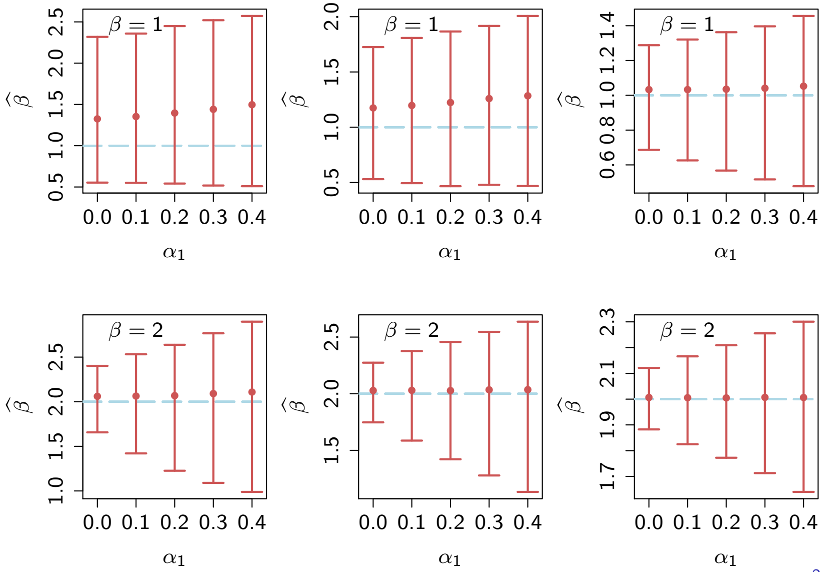
(a) $N = 500, \delta = 0.1$ (b) $N = 1000, \delta = 0.1$ (c) $N = 5000, \delta = 0.1$





(a) $N = 500, \delta = 0.2$
(b) $N = 1000, \delta = 0.2$
(c) $N = 5000, \delta = 0.2$
$\beta = 1$
$\beta = 2$
$\widehat{\beta}$
α_1
0.0 0.1 0.2 0.3 0.4
0.5 1.0 1.5
0.4 0.8 1.2 1.6
0.6 0.8 1.0 1.2
1.2 1.6 2.0 2.4
1.6 1.8 2.0 2.2 2.4
1.8 1.9 2.0 2.1 2.2

(a) $N = 500, \delta = 0.3$ (b) $N = 1000, \delta = 0.3$ (c) $N = 5000, \delta = 0.3$



Empirical Illustration: Schooling and Test Scores