Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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What is the causal effect of T^*?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Unknown function that *does not depend on* i
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete instrumental variable

Example 1: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with 1050 pregnant smokers in England: 521 given nicotine patches, the rest given placebo patches.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Indicator of nicotine patch

Example 2: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters. Randomly choose 6 and build a school in each village of these clusters.

- y – Score on math and language test
- T^* – True school attendance
- T – Self-reported school attendance
- $\mathbf{x}$ – Household characteristics
- z – School built in village

Example 3: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- y – Log wage
- T^* – True training attendence
- T – Self-reported training attendance
- $\mathbf{x}$ – Individual characteristics
- z – Offer of job training

Non-classical Measurement Error: Binary T^*

- Many applications of linear model have *binary* treatment
- Binary $T^* \implies \mathbb{E}[T^* w] \leq 0$
- Misclassification Probabilities:

$$\alpha_0 = \mathbb{P}(T = 1 | T^* = 0)$$
$$\alpha_1 = \mathbb{P}(T = 0 | T^* = 1)$$

- Non-Differential Measurement Error: $T \perp (z, u) | T^*$
- $\sigma^2_{T^*} \not< \sigma^2_T$ so work with α_0, α_1 rather than κ
- *Four-dimensional* Problem...

Results for a Mis-classified Binary Regressor

Aigner (1973), Bollinger (1996)...

- Even if $\rho_{T^*u} = 0$, OLS is biased and inconsistent: typically attenuated towards zero *but could flip signs!*

Kane et al. (1999), Black et al. (2000), Frazis et al. (2003)...

- $\rho_{zu} = 0 \implies$ IV solves endogenous regressor problem if there is no mis-classification
- $\rho_{T^*u} = 0$ and $\rho_{zu} = 0 \implies$ non-linear GMM estimator can solve the mis-classification problem

OLS and IV Probability Limits: Binary T^*

$$
\begin{aligned}
\text{plim}\left(\widehat{\beta}_{OLS}\right) &= \frac{\sigma^2_{T^*}}{\sigma^2_T}\left[\beta\left(1-\alpha_0-\alpha_1\right)+\frac{\sigma_{T^*u}}{\sigma^2_{T^*}}\right] \\
\text{plim}\left(\widehat{\beta}_{IV}\right) &= \frac{\beta}{1-\alpha_0-\alpha_1}+\frac{\sigma_{zu}}{\sigma_{zT}} \\
\sigma^2_{T^*} &= \frac{\left(p-\alpha_0\right)\left(1-p-\alpha_1\right)}{\left(1-\alpha_0-\alpha_1\right)^2}
\end{aligned}
$$

Where $p = \mathbb{P}(T=1)$

What About Endogenous, Mis-measured T^*, Valid z?

$$y = \beta T^* + u$$
$$u = c + \varepsilon$$

- No results in the literature for this case
- Important setting in applied work: e.g. RCTs
- Discrete Instrument: $z \in \{z_1, \cdots, z_K\}$
- Non-parametric First Stage: $p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$
- What does $E[\varepsilon | z] = 0$ buy us in this case?

Observable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 1$	$\ldots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\ldots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\ldots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y | T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1 | z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 1$	$\dots$	$z = K$
$T^* = 0$	m^*_{01} p^*_{01}	m^*_{02} p^*_{02}	$\dots$	m^*_{0K} p^*_{0K}
$T^* = 1$	m^*_{11} p^*_{11}	m^*_{12} p^*_{12}	$\dots$	m^*_{1K} p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u|\, T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$p^*_k = \mathbb{P}(T^* = 1|z = z_k) = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}$$

Possible Assumptions On m^*_{tk}

Joint Exogeneity: $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m^*_{tk} = c \quad \text{for all } t, k$$

Exogenous Treatment: $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p^*_{tk} m^*_{tk} = c \quad \text{for all } t$$

Exogenous Instrument: $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p^*_k) m^*_{0k} + p^*_k m^*_{1k} = c \quad \text{for all } k$$

Mahajan (2006, Econometrica)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Main Result (Correct) – Exogenous Treatment

Relevant binary instrument z ($p_1^* \neq p_2^*$) identifies α_0, α_1 and $\mathbb{E}[y|T^*]$ provided that $\mathbb{E}[\nu|T^*, T, z] = 0$.

Mahajan (2006, Econometrica)

Regression Model

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$\mathbb{E}[\varepsilon|T^*] \neq 0$

Additional Result (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$, $p_1^* \neq p_2^*$, $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$ identified.

Mahajan's Argument

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Ingredients

1. If $p_1^* \neq p_2^*$, $\mathbb{E}[\varepsilon|z] = 0$ then, since $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$, knowledge of α_0, α_1 is sufficient to recover β. (Correct)
2. If $p_1^* \neq p_2^*$, $\mathbb{E}[\nu|T^*, T, z] = 0$, α_0, α_1 are identified. (Correct)

   How to satisfy both 1 and 2 while allowing $\mathbb{E}[\varepsilon|T^*] \neq 0$?
3. Assume that $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$

   (i.e. $m_{01}^* = m_{02}^*$ and $m_{11}^* = m_{12}^*$)

Mahajan's Argument

Proposition

If $\mathbb{E}[\varepsilon | T^*] \neq 0$ then $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$ combined with $\mathbb{E}[\varepsilon|z] = 0$ implies $p_1^* = p_2^*$, i.e. z is irrelevant for T^*.

Proof

Recall that $\mathbb{E}[\varepsilon|z] = 0$ implies

$$(1 - p_1^*) m_{01}^* + p_1^* m_{11}^* = c$$

$$(1 - p_2^*) m_{02}^* + p_2^* m_{12}^* = c$$

while Mahajan's assumption implies $m_{01}^* = m_{02}^*$ and $m_{11}^* = m_{12}^*$. Therefore either $m_{01}^* = m_{02}^* = m_{11}^* = m_{12}^* = c$, which is ruled out by $E[\varepsilon | T^*] = 0$, or $p_1^* = p_2^*$.

What about increasing the support of z?

$$\boxed{\mathbb{E}[\varepsilon|z] = 0 \implies \textit{pair} \text{ of equations for each } k = 1, \ldots, K}$$

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) + (1-\alpha_0)c - (p_k - \alpha_0)m_{1k}^*$$

$$\hat{y}_{1k} = (1-\alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m_{1k}^*$$

where $\hat{y}_{0k} = (1-p_k)\bar{y}_{0k}$ and $\hat{y}_{0k} = p_k\bar{y}_{1k}$

$$\boxed{2K \text{ Equations in } K+4 \text{ Unknowns}}$$

Theorem: β is undentified regardless of K.

(For general case, see paper.)

Proof of special case: $\alpha_0 = 0$

1. System of equations simplifies to

$$\hat{y}_{0k} = c + p_k \beta \left(\frac{\alpha_1}{1 - \alpha_1} \right) - p_k m^*_{1k}$$

$$\hat{y}_{1k} = p_k \beta + p_k m^*_{1k}$$

2. $\beta/(1 - \alpha_1) \equiv \mathcal{W}$ is identified and imposing this, algebra gives $\beta\alpha_1/(1 - \alpha_1) = \mathcal{W} - \beta$.

Theorem: β is undentified regardless of K.

(For general case, see paper.)

Proof of special case: $\alpha_0 = 0$ continued...

3. Substituting:

$$(c + p_k\mathcal{W} - \hat{y}_{0k})/p_k = \beta + m^*_{1k}$$
$$\hat{y}_{1k}/p_k = \beta + m^*_{1k}$$

4. Linear system in (β, m^*_{1k}) – no solution or ∞ of solutions.
5. Sum original pair of equations $\implies c + p_k\mathcal{W} - \hat{y}_{0k} = \hat{y}_{1k}$ thus ∞ of solutions. The model is unidentified.

Identification by Conditional Variances?

New Assumption

Homoskedastic errors w.r.t. the *instrument*: $E[\varepsilon^2|z] = E[\varepsilon^2]$

Not Crazy!

Holds in an RCT or a *true* natural experiment.

New Moment Conditions

For each pair (k, ℓ)

$$\begin{aligned} s_k^2 - s_\ell^2 = \mathcal{W}^2 \left[p_k(1-p_k) - p_\ell(1-p_\ell) + (\alpha_0 - \alpha_1)(p_k - p_\ell) \right] \\ + 2\mathcal{W} \left[(p_k - \alpha_0)(m_{1k}^* - c) - (p_\ell - \alpha_0)(m_{1\ell}^* - c) \right] \end{aligned}$$

Where $s_k^2 = Var(y|z = z_k)$, and $\mathcal{W}$ is the Wald IV estimator.

Proposition: $(\alpha_0 - \alpha_1)$ is Identified

Define

$$\widetilde{\mathcal{W}}_{k\ell} = \frac{\mathbb{E}[yT|z_k] - \mathbb{E}[yT|z_\ell]}{p_k - p_\ell}$$

Show that:

$$\begin{aligned}(p_k - \alpha_0)(m^*_{1k} - c) - (p_\ell - \alpha_0)(m^*_{1\ell} - c) = \\ (p_k - p_\ell)\left[\widetilde{W}_{k\ell} - \mathbb{E}[y] - \mathcal{W}\left\{(1-p) + (\alpha_0 - \alpha_1)\right\}\right]\end{aligned}$$

Substituting and rearranging:

$$\alpha_0 - \alpha_1 = (2p - 1 - p_k - p_\ell) + \frac{2(\widetilde{W}_{k\ell} - \mathbb{E}[y])}{\mathcal{W}} - \frac{s_k^2 - s_\ell^2}{(p_k - p_\ell)\mathcal{W}^2}$$

What Good is $(\alpha_0 - \alpha_1)$?

- Test a necessary condition for *no mis-classification*: $\alpha_0 = \alpha_1$
- Simple, tighter partial identification bounds for β
- In some settings, one of the mis-classification probabilities is known to be zero $\implies$ β point identified

Identification from Third Moments

Simulation Study

$y = \beta T^* + \varepsilon$

$T^* = \mathbf{1}\left\{\gamma_0 + \gamma_1 z + \eta > 0\right\}$

$\gamma_0 = \Phi^{-1}(\delta)$, $\gamma_1 = \Phi^{-1}(1-\delta) - \Phi(\delta)$ so that δ

E.g. if $\delta = 0.1$ then 10% of those *not* offered treatment get it anyway, and 10% of those offered treatment don't take it up.

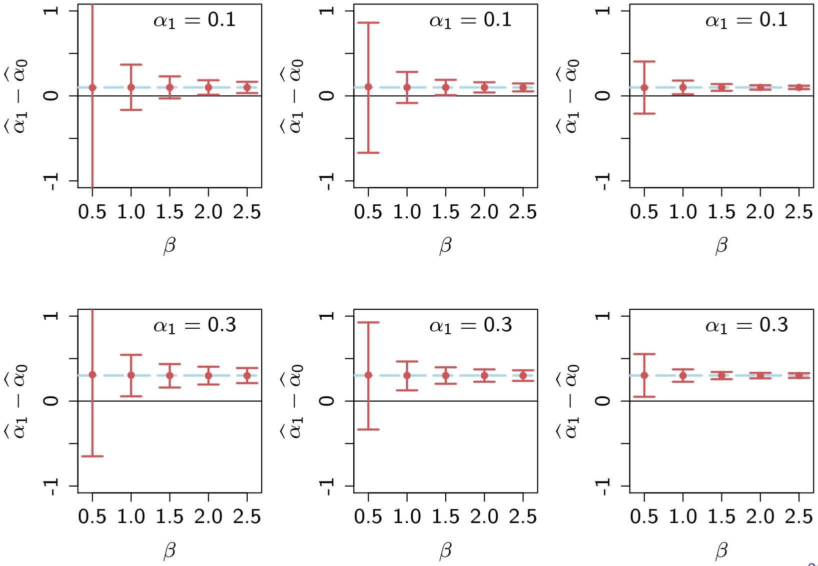
If $T^* = 0$ then $T = 0$ (E.g. Birthweight and smoking)

$T | T^* = 1 \sim$ Bernoulli(?)

$$\left[\begin{array}{c} \varepsilon \\ \eta \end{array}\right] \sim N\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} 1 & 0.3 \\ 0.3 & 1 \end{array}\right]\right)$$

$$\boxed{\text{Sampling Distribution of } \widehat{\alpha}_1 - \widehat{\alpha}_0}$$

(a) $N = 500, \delta = 0.1$ (b) $N = 1000, \delta = 0.1$ (c) $N = 5000, \delta = 0.1$



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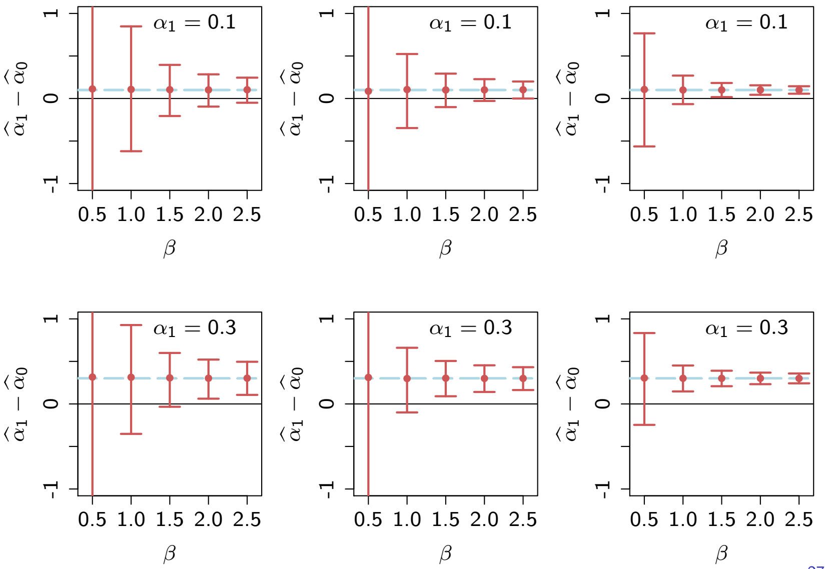
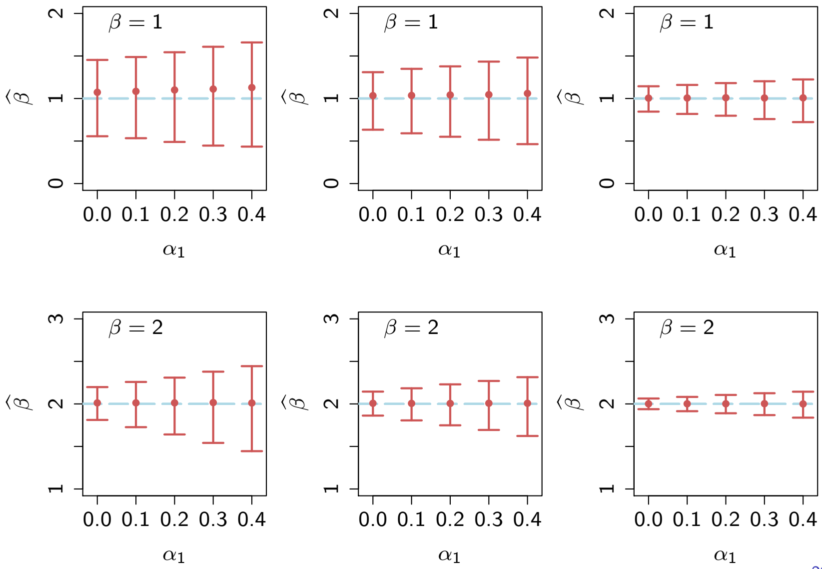
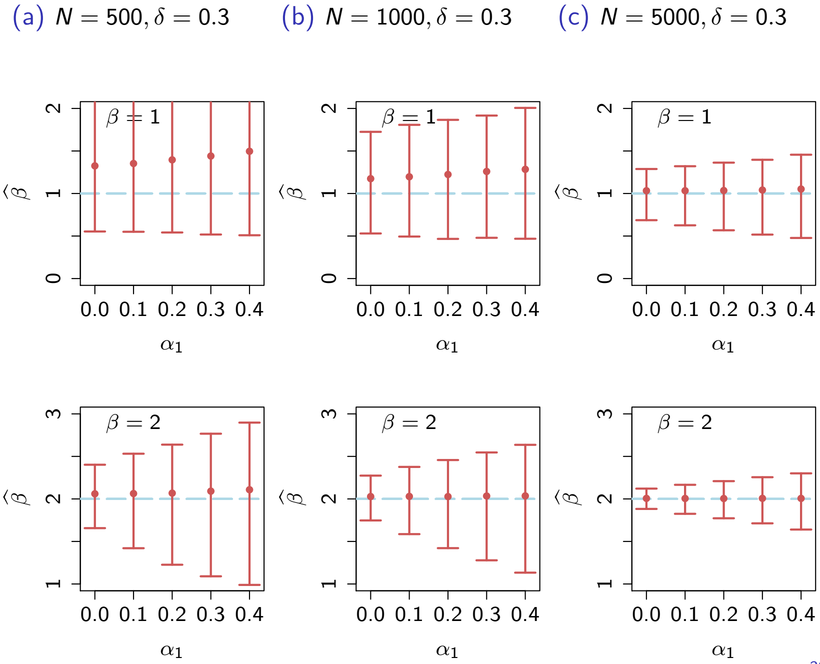


$$\boxed{\text{Sampling Distribution of } \widehat{\beta} = (1 - \widehat{\alpha}_0 - \widehat{\alpha}_1)\widehat{\beta}_{IV}}$$

(a) $N = 500, \delta = 0.1$ (b) $N = 1000, \delta = 0.1$ (c) $N = 5000, \delta = 0.1$




(a) $N = 500, \delta = 0.3$
(b) $N = 1000, \delta = 0.3$
(c) $N = 5000, \delta = 0.3$
$\beta = 1$
$\beta = 2$
$\widehat{\beta}$
α_1
0.0 0.1 0.2 0.3 0.4

Empirical Illustration: Schooling and Test Scores