

# Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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## What is the causal effect of $T^*$ ?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- ▶  $y$  – Outcome of interest
- ▶  $h$  – Unknown function that *does not depend on*  $i$
- ▶  $T^*$  – Unobserved, endogenous binary treatment
- ▶  $T$  – Observed, mis-measured binary surrogate for  $T^*$
- ▶  $\mathbf{x}$  – Exogenous covariates
- ▶  $\varepsilon$  – Mean-zero error term
- ▶  $z$  – Discrete instrumental variable

## Example: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with 1050 pregnant smokers in England: 521 given nicotine patches, the rest given placebo patches.

- ▶  $y$  – Birthweight
- ▶  $T^*$  – True smoking behavior
- ▶  $T$  – Self-reported smoking behavior
- ▶  $\mathbf{x}$  – Mother characteristics
- ▶  $z$  – Indicator of nicotine patch

## Example: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters. Randomly choose 6 and build a school in each village of these clusters.

- ▶  $y$  – Child's score on math and language test
- ▶  $T^*$  – Child's true school attendance
- ▶  $T$  – Parent's report of child's school attendance
- ▶  $\mathbf{x}$  – Child and household characteristics
- ▶  $z$  – School built in village

# Related Literature

## Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

## Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

## Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015)

Model:  $y = c + \beta T^* + \varepsilon$

### First-stage

$$p_k^* \equiv \mathbb{P}(T^* = 1 | z = z_k) \neq \mathbb{P}(T^* = 1 | z = z_\ell) \equiv p_\ell^*, k \neq \ell$$

### Measurement Error

- ▶ Non-differential:  $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$
- ▶ Does not depend on  $z$ :

$$\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$$

$$\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$$

### Notation

Define error term that absorbs constant:  $u = c + \varepsilon$

Observable Moments:  $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ $p_{01}$	$\bar{y}_{02}$ $p_{02}$	$\dots$	$\bar{y}_{0K}$ $p_{0K}$
$T = 1$	$\bar{y}_{11}$ $p_{11}$	$\bar{y}_{12}$ $p_{12}$	$\dots$	$\bar{y}_{1K}$ $p_{1K}$

$$\bar{y}_{tk} = \mathbb{E}[y | T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1 | z = z_k)$$

# Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T^* = 0$	$m_{01}^*$ $p_{01}^*$	$m_{02}^*$ $p_{02}^*$	$\dots$	$m_{0K}^*$ $p_{0K}^*$
$T^* = 1$	$m_{11}^*$ $p_{11}^*$	$m_{12}^*$ $p_{12}^*$	$\dots$	$m_{1K}^*$ $p_{1K}^*$

$$m_{tk}^* = \mathbb{E}[u | T^* = t, z = z_k], \quad p_{tk}^* = q_k p_k^*$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$$



# Unrestricted System of Equations

$$(1 - p_k)\bar{y}_{0k} \equiv \tilde{y}_{0k} = (\beta + m_{1k}^*)\alpha_1 p_k^* + (1 - \alpha_0)(1 - p_k^*)m_{0k}^*$$

$$p_k\bar{y}_{1k} \equiv \tilde{y}_{1k} = (\beta + m_{1k}^*)(1 - \alpha_1)p_k^* + \alpha_0(1 - p_k^*)m_{0k}^*$$

$$p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}$$

## Possible Restrictions On $m_{tk}^*$

Joint Exogeneity:  $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m_{tk}^* = c \quad \text{for all } t, k$$

Exogenous Treatment:  $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p_{tk}^* m_{tk}^* = c \quad \text{for all } t$$

Exogenous Instrument:  $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p_k^*) m_{0k}^* + p_k^* m_{1k}^* = c \quad \text{for all } k$$

Later I'll consider relaxing the assumption that  $z$  is exogenous. . .

*Theorem:*  $\beta$  is unidentified regardless of  $K$ .

(For general case, see paper.)

Proof of special case:  $\alpha_0 = 0$

1. System of equations:

$$\tilde{y}_{0k} = c + p_k \beta \left( \frac{\alpha_1}{1 - \alpha_1} \right) - p_k m_{1k}^*$$

$$\tilde{y}_{1k} = p_k \beta + p_k m_{1k}^*$$

2.  $\beta/(1 - \alpha_1) \equiv \mathcal{W}$  is identified and imposing this, algebra gives  
 $\beta \alpha_1 / (1 - \alpha_1) = \mathcal{W} - \beta$ .

*Theorem:*  $\beta$  is unidentified regardless of  $K$ .

(For general case, see paper.)

Proof of special case:  $\alpha_0 = 0$  continued. . .

3. Substituting:

$$(c + p_k \mathcal{W} - \tilde{y}_{0k})/p_k = \beta + m_{1k}^*$$

$$\tilde{y}_{1k}/p_k = \beta + m_{1k}^*$$

4. Linear system in  $(\beta, m_{1k}^*)$  – no solution or  $\infty$  of solutions.

5. Sum original pair of equations  $\implies c + p_k \mathcal{W} - \tilde{y}_{0k} = \tilde{y}_{1k}$   
thus  $\infty$  of solutions. The model is unidentified.

## Conditional *Second* Moment Independence.

### New Assumption

Homoskedastic errors w.r.t. the *instrument*:  $E[\varepsilon^2|z] = E[\varepsilon^2]$

### Reasonable?

Makes sense in an RCT or a true natural experiment.

### New Moment Conditions

Defining  $\mu_{k\ell}^* = (p_k - \alpha_0)m_{1k}^* - (p_\ell - \alpha_0)m_{k\ell}^*$ ,

$$\mathbb{E}(y^2|z_k) - \mathbb{E}(y^2|z_\ell) \equiv \Delta \overline{y^2} = \beta \mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu_{k\ell}^*$$

$$\mathbb{E}(yT|z_k) - \mathbb{E}(yT|z_\ell) \equiv \Delta \overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu_{k\ell}^*$$

**Theorem:**  $(\alpha_1 - \alpha_0)$  is Identified if  $E[\varepsilon^2|z] = E[\varepsilon^2]$

Requires only binary  $z$

Solve for  $\mu_{k\ell}^*$ , substitute  $\beta = \mathcal{W}(1 - \alpha_0 - \alpha_1)$ , rearrange to find

$$\alpha_1 - \alpha_0 = 1 + \mathcal{R}/\mathcal{W}, \quad \text{where} \quad \mathcal{R} \equiv \frac{\Delta \bar{y}^2 - 2\mathcal{W}\Delta \bar{y}\bar{T}}{\mathcal{W}(p_k - p_\ell)}.$$

What good is  $(\alpha_1 - \alpha_0)$ ?

- ▶ Test necessary condition for *no mis-classification*:  $\alpha_0 = \alpha_1$
- ▶ Simple, tighter partial identification bounds for  $\beta$
- ▶ If  $\alpha_0$  known, e.g. zero  $\implies \beta$  point identified

# Conditional *Third* Moment Independence

## New Assumption

Third moment independence w.r.t instrument:  $E[\varepsilon^3|z] = E[\varepsilon^3]$

## New Moment Conditions

Define  $\lambda_{k\ell}^* = (p_k - \alpha_0)v_{1k}^* - (p_\ell - \alpha_0)v_{1\ell}^*$

where  $v_{tk}^* = \mathbb{E}(u^2 | T^* = t, z_k)$ . Then

$$\mathbb{E}(y^3|z_k) - \mathbb{E}(y^3|z_\ell) \equiv$$

$$\Delta \overline{y^3} = \beta^2 \mathcal{W}(p_k - p_\ell) + 3\beta \mathcal{W}\mu_{k\ell}^* + 3\mathcal{W}\lambda_{k\ell}^*$$

$$\mathbb{E}(y^2 T|z_k) - \mathbb{E}(y^2 T|z_\ell) \equiv$$

$$\Delta \overline{y^2 T} = \beta(1 - \alpha_1) \mathcal{W}(p_k - p_\ell) + 2(1 - \alpha_1) \mathcal{W}\mu_{k\ell}^* + \lambda_{k\ell}^*$$

## Theorem: $\beta$ , $\alpha_0$ and $\alpha_1$ identified

Adding  $E[\varepsilon^3|z] = E[\varepsilon^3]$ ,  $z$  need only be binary.

Solve for  $\lambda_{k\ell}^*$ , substitute and rearrange. After further substitutions:

$$2\mathcal{W}^2(1 - \alpha_1)^2 + 2\mathcal{R}\mathcal{W}(1 - \alpha_1) + (\mathcal{S} - \mathcal{R}^2) = 0$$

where

$$\mathcal{S} \equiv \frac{\Delta \overline{y^3} - 3\mathcal{W} [\Delta \overline{y^2 T} + \mathcal{R} \Delta \overline{y T}]}{\mathcal{W}(p_k - p_\ell)}$$

- ▶ Quadratic in  $(1 - \alpha_1)$  and observables only
- ▶ Always two real roots: one is  $(1 - \alpha_1)$  and the other is  $\alpha_0$ .
- ▶ To tell which is which, need  $\alpha_0 + \alpha_1 < 1$ .



## Recap of Results

1. Using first-moment information alone,  $\beta$  is unidentified regardless of how many values the instrument takes on.
2. Using second moment information  $\alpha_1 - \alpha_0$  is identified
  - ▶ Partial identification bound for  $\beta$
  - ▶ Identifies  $\beta$  if  $\alpha_0$  is known (e.g. smoking/birthweight example)
3. Using third moment information  $\beta$ ,  $\alpha_0$  and  $\alpha_1$  are identified so long as  $\alpha_0 + \alpha_1 < 1$ .

# Empirical Illustration: Schooling and Test Scores

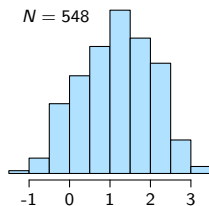
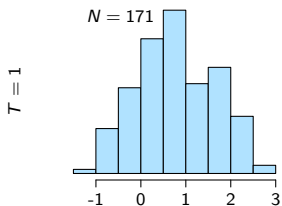
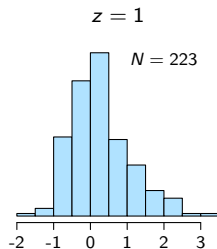
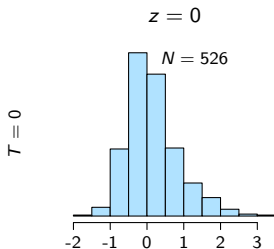
Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters.  
Randomly choose 6 and build a school in each village of these clusters ( $N = 1468$ ).

- ▶  $y$  – Child's score on math and language test
- ▶  $T^*$  – Child's true school attendance
- ▶  $T$  – Parent's report of child's school attendance
- ▶  $x$  – Child and household characteristics
- ▶  $z$  – School built in village

# Empirical Illustration: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)



# Empirical Illustration: Schooling and Test Scores

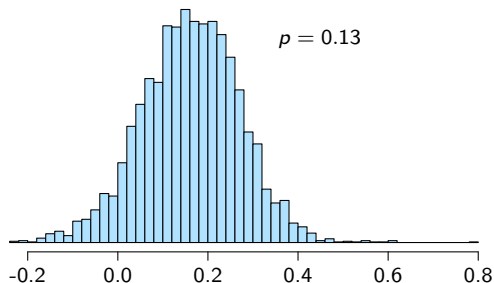
Burde & Linden (2013, AEJ Applied)

## Cluster Bootstrap Distribution of $\hat{\alpha}_1 - \hat{\alpha}_0$

$$\hat{\beta}_{OLS} = 0.88$$

$$\hat{\beta}_{IV} = 1.27$$

$$\hat{\alpha}_1 - \hat{\alpha}_0 = 0.18$$



But what if  $z$  is endogenous?

## Recall: Unrestricted System

$$\tilde{y}_{0k} = (\beta + m_{1k}^*)\alpha_1 p_k^* + (1 - \alpha_0)(1 - p_k^*)m_{0k}^*$$

$$\tilde{y}_{1k} = (\beta + m_{1k}^*)(1 - \alpha_1)p_k^* + \alpha_0(1 - p_k^*)m_{0k}^*$$

## Intelligible Quantities

$$\delta_{T^*} \equiv \mathbb{E}[u|T^* = 1] - \mathbb{E}[u|T^* = 0]$$

$$\delta_z \equiv \mathbb{E}[u|z = 1] - \mathbb{E}[u|z = 0]$$

... both are linear functions of  $m_{tk}^*$ .

## Identified Set for $(\alpha_0, \alpha_1, \delta_{T^*}, \delta_z)$

### First Moment Information

$$\delta_z = C(\alpha_0, \alpha_1 | \mathbf{p}, \mathbf{q}, \bar{\mathbf{y}}) - \left( \frac{p_1 - p_2}{1 - \alpha_0 - \alpha_1} \right) \delta_{T^*}$$

### Second Moment Information

$$\text{Var}(u | T = t, z = k) > 0$$

$$\implies [\text{Var}(y | T = t, z = k) - Q_{tk}(\delta_{T^*}, \delta_z, \alpha_0, \alpha_1 | \mathbf{p}, \mathbf{q}, \bar{\mathbf{y}})] > 0$$

# Approaches to (Partial) Identification

## Identification

- ▶  $\delta_z = 0, \alpha_0 = \alpha_1 = 0 \Rightarrow$  Wald Estimator
- ▶ Joint Exogeneity ( $\Rightarrow \delta_{T^*} = \delta_z = 0$ )  
Kane et al. (1999), Black et al. (2000), Mahajan (2006)...

## Partial Identification

- ▶ Frazis & Loewenstein (2003):  $\delta_z = 0, (\alpha_0 + \alpha_1) \in [\ell, u]$
- ▶ Conley et al. (2012):  $\delta_z \in [\underline{\delta}_z, \bar{\delta}_z], \alpha_0 = \alpha_1 = 0$
- ▶ Nevo & Rosen (2012):  $\delta_T^* > \delta_z, \delta_T^* \delta_z > 0, \alpha_0 = \alpha_1 = 0$



# Our Proposed Approach

## Elicit Beliefs

Ask researcher for bounds on  $\alpha_0, \alpha_1, \delta_{T^*}, \delta_z$

## Discipline Beliefs

Are these beliefs mutually consistent? Explore joint constraints implied by identified set.

## Incorporate Beliefs

Carry out (Bayesian) inference for  $\beta$  using beliefs, constraints, and accounting for sampling uncertainty.

# Example: Vouchers for Private Schooling (PACES)

Angrist et al. (2002, AER)

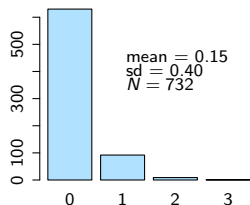
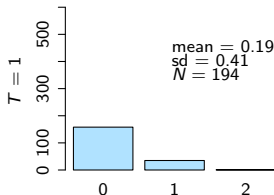
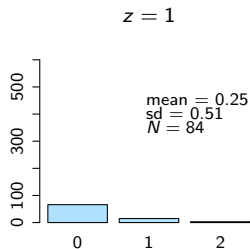
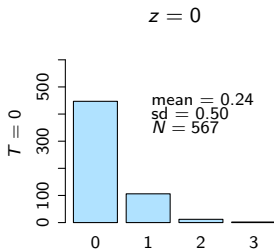
Data from Colombia: vouchers to attend private school awarded by lottery to poor, primary school-aged children ( $N = 1577$ ).

- ▶  $y$  – # of grades repeated after lottery
- ▶  $T^*$  – Scholarship use
- ▶  $T$  – Self-reported Scholarship use
- ▶  $\mathbf{x}$  – Demographic controls
- ▶  $z$  – Offered scholarship through lottery

Authors raise concerns about the lottery in one of the two cities. . .

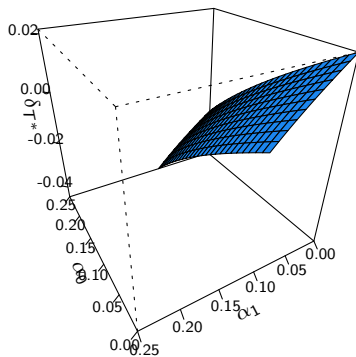
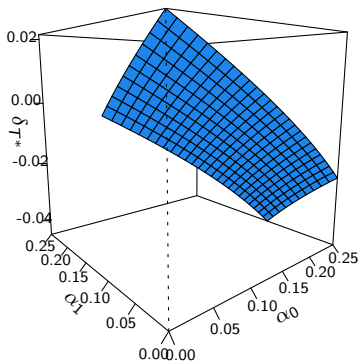
# Example: Vouchers for Private Schooling (PACES)

Overall: Mean = 0.19, SD = 0.45

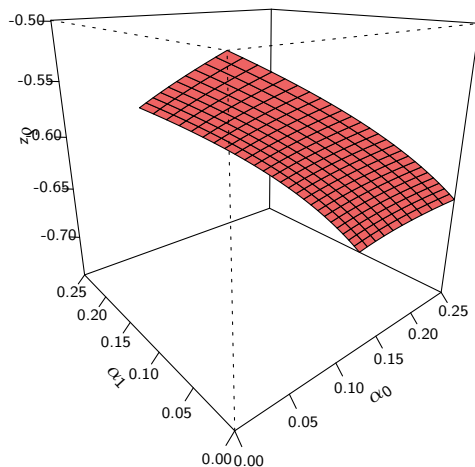


# Implications of Valid IV: $\delta_z = 0$

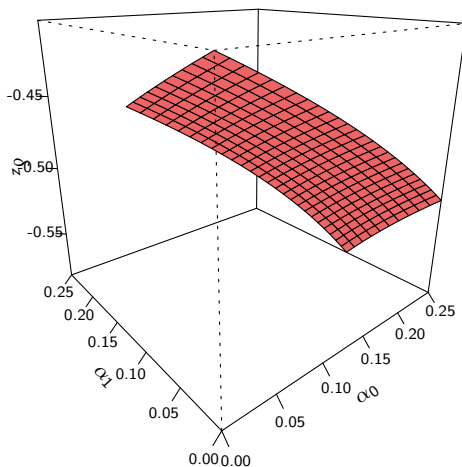
Angrist et al. (2002)



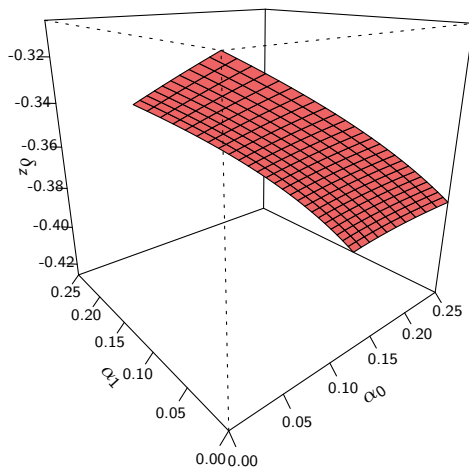
## Implications of Negative Selection: $\delta_{T^*} = -0.75$



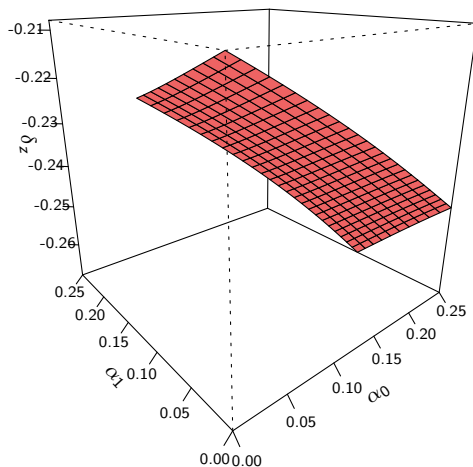
## Implications of Negative Selection: $\delta_{T^*} = -0.60$



## Implications of Negative Selection: $\delta_{T^*} = -0.45$

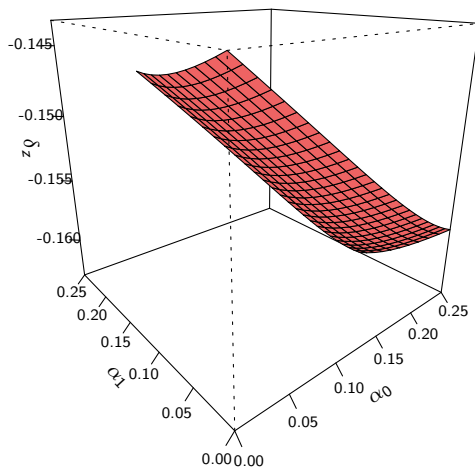


## Implications of Negative Selection: $\delta_{T^*} = -0.30$

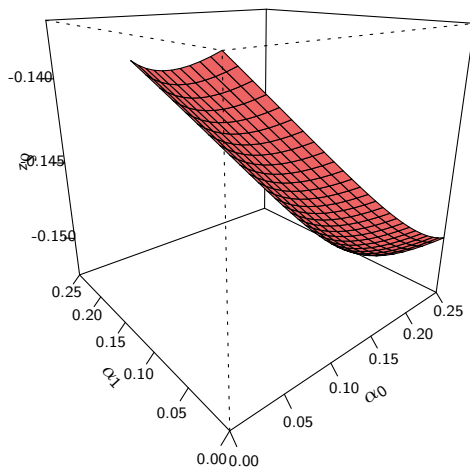




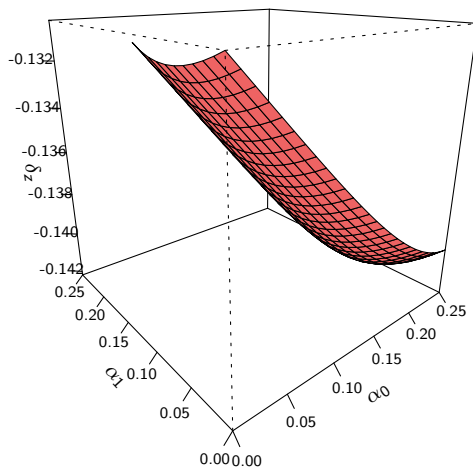
## Implications of Negative Selection: $\delta_{T^*} = -0.20$



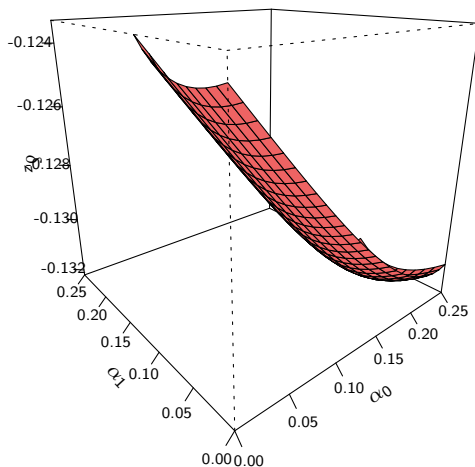
## Implications of Negative Selection: $\delta_{T^*} = -0.19$



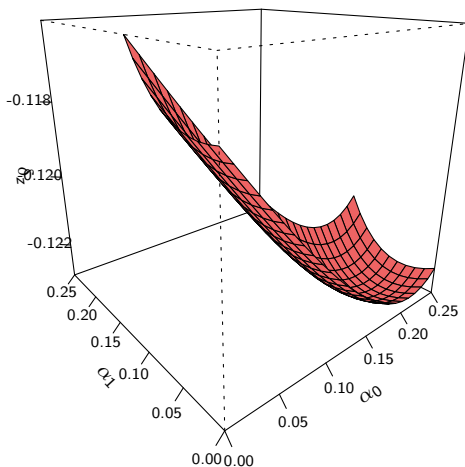
## Implications of Negative Selection: $\delta_{T^*} = -0.18$



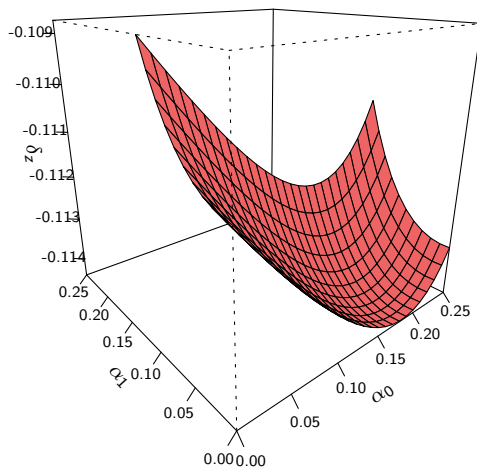
## Implications of Negative Selection: $\delta_{T^*} = -0.17$



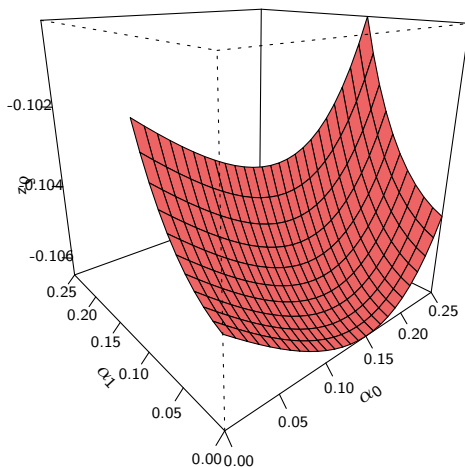
## Implications of Negative Selection: $\delta_{T^*} = -0.16$



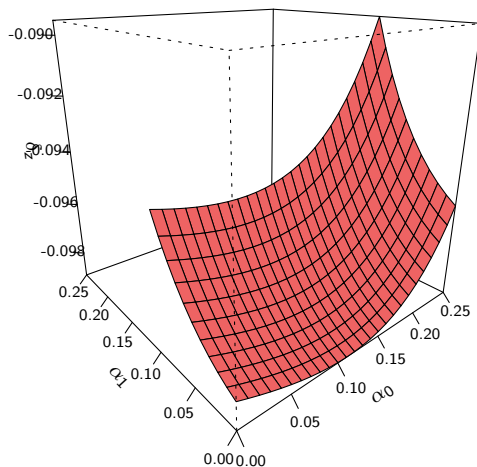
## Implications of Negative Selection: $\delta_{T^*} = -0.15$



## Implications of Negative Selection: $\delta_{T^*} = -0.14$

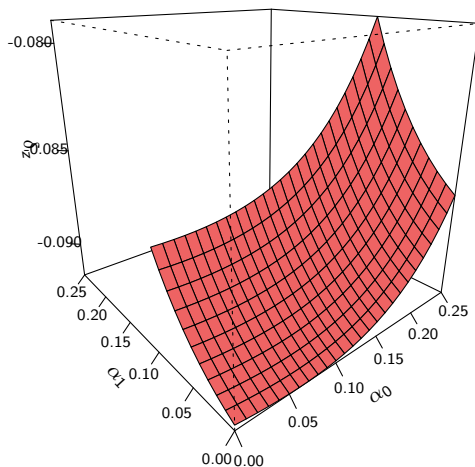


## Implications of Negative Selection: $\delta_{T^*} = -0.13$

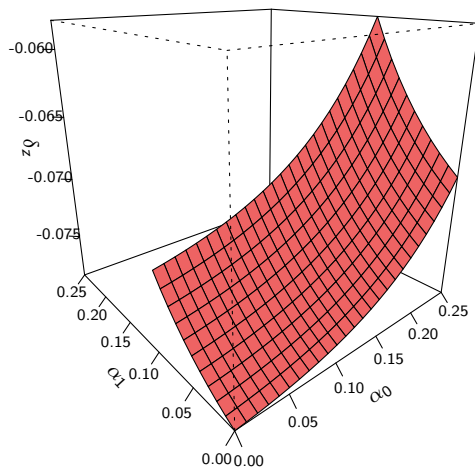




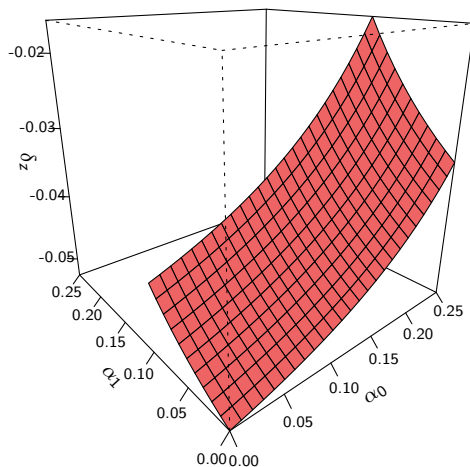
## Implications of Negative Selection: $\delta_{T^*} = -0.12$



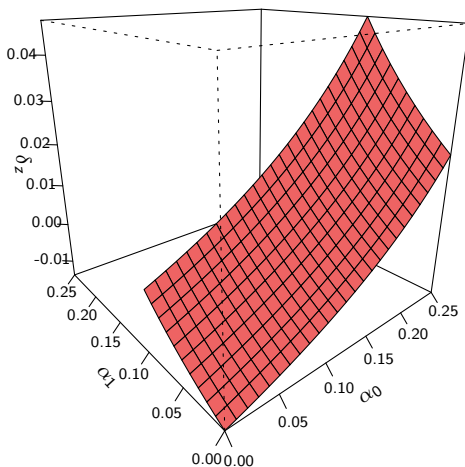
## Implications of Negative Selection: $\delta_{T^*} = -0.10$



## Implications of Negative Selection: $\delta_{T^*} = -0.06$



## Implications of Negative Selection: $\delta_{T^*} = 0$



# (Bayesian) Inference via Transparent Parameterization

## Reduced Form Parameters: $(\mathbf{p}, \mathbf{q}, \bar{\mathbf{y}}, \sigma^2)$

- ▶ Draw reduced form parameters from Bayesian posterior constructed to match usual large-sample frequentist inference.

## Structural Parameters $(\delta_{T^*}, \delta_z, \alpha_0, \alpha_1)$

- ▶ Updated by data only *through* reduced form parameters.
- ▶ Impose prior beliefs on structural parameters
- ▶ E.g.  $\alpha_0, \alpha_1 \sim \text{iid } U(0, 0.25)$ ,  $\delta_{T^*} | (\alpha_0, \alpha_1) \sim U(\ell, u)$

## “Blue Bar” Picture

## Implications for Beta

Bayes histogram next to Frequentist histogram (picture of lower bound)

Probability that frequentist identified sets include zero? Something like that

# Conclusion





# Simulation Study

## Simulation Study: $y = \beta T^* + \varepsilon$

- ▶  $(\varepsilon, \eta) \sim$  jointly normal, mean 0, variance 1, corr. 0.3.
- ▶ First stage:  $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$ 
  - ▶ Half of subjects have  $z = 1$ , the rest have  $z = 0$ .
  - ▶  $\gamma_0 = \Phi^{-1}(\delta)$
  - ▶  $\gamma_1 = \Phi^{-1}(1 - \delta) - \Phi(\delta)$
  - ▶  $\delta$  equals fraction of those offered treatment who fail to take it up *and* fraction of those not offered treatment who do.
- ▶ Generate  $T$  as follows:
  - ▶  $T^* = 0 \implies T = 0$ , i.e.  $\alpha_0 = 0$
  - ▶  $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$
  - ▶  $\alpha_0, \alpha_1$  unknown to econometrician.

Sampling Distribution of  $\hat{\alpha}_1 - \hat{\alpha}_0$