

Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

Francis J. DiTraglia
Camilo Garcia-Jimeno

University of Pennsylvania

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What is the causal effect of T^* ?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- ▶ y – Outcome of interest
- ▶ h – Unknown function that *does not depend on i*
- ▶ T^* – Unobserved, endogenous binary treatment
- ▶ T – Observed, mis-measured binary surrogate for T^*
- ▶ \mathbf{x} – Exogenous covariates
- ▶ ε – Mean-zero error term
- ▶ z – Discrete instrumental variable

Target of Inference:

ATE function: $\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$

Example: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- ▶ y – Log wage
- ▶ T^* – True training attendance
- ▶ T – Self-reported training attendance
- ▶ x – Individual characteristics
- ▶ z – Offer of job training

Example: Returns to Schooling

Oreopoulos (2006, AER)

Fuzzy RD: minimum school-leaving age in UK increased from 14 to 15 in 01947 but some already stayed until 15 before the law and others failed to comply after it.

- ▶ y – Log wage
- ▶ T^* – School attendance at age 15
- ▶ T – Self-report of school attendance at age 15
- ▶ x – Individual characteristics
- ▶ z – Indicator: born in or after 1933

Related Literature

Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015)

Model: $y = c + \beta T^* + \varepsilon$

First-stage

$$p_k^* \equiv \mathbb{P}(T^* = 1 | z = z_k) \neq \mathbb{P}(T^* = 1 | z = z_\ell) \equiv p_\ell^*, k \neq \ell$$

Measurement Error

- ▶ Non-differential: $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$
- ▶ Does not depend on z :

$$\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$$

$$\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$$

Notation

Define error term that absorbs constant: $u = c + \varepsilon$

Observable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	\dots	$z = K$
$T = 0$	\bar{y}_{01} ρ_{01}	\bar{y}_{02} ρ_{02}	\dots	\bar{y}_{0K} ρ_{0K}
$T = 1$	\bar{y}_{11} ρ_{11}	\bar{y}_{12} ρ_{12}	\dots	\bar{y}_{1K} ρ_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y | T = t, z = z_k], \quad \rho_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1 | z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	\dots	$z = K$
$T^* = 0$	m_{01}^* p_{01}^*	m_{02}^* p_{02}^*	\dots	m_{0K}^* p_{0K}^*
$T^* = 1$	m_{11}^* p_{11}^*	m_{12}^* p_{12}^*	\dots	m_{1K}^* p_{1K}^*

$$m_{tk}^* = \mathbb{E}[u | T^* = t, z = z_k], \quad p_{tk}^* = q_k p_k^*$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$$

Unrestricted System of Equations

$$(1 - p_k)\bar{y}_{0k} \equiv \tilde{y}_{0k} = (\beta + m_{1k}^*)\alpha_1 p_k^* + (1 - \alpha_0)(1 - p_k^*)m_{0k}^*$$

$$p_k\bar{y}_{1k} \equiv \tilde{y}_{1k} = (\beta + m_{1k}^*)(1 - \alpha_1)p_k^* + \alpha_0(1 - p_k^*)m_{0k}^*$$

$$p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}$$

Possible Restrictions On m_{tk}^*

Joint Exogeneity: $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m_{tk}^* = c \quad \text{for all } t, k$$

Exogenous Treatment: $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p_{tk}^* m_{tk}^* = c \quad \text{for all } t$$

Exogenous Instrument: $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p_k^*) m_{0k}^* + p_k^* m_{1k}^* = c \quad \text{for all } k$$

Mahajan (2006, ECTA)

Regression Model

$$y = \mathbb{E}[y|T^*] + \nu$$

$$\mathbb{E}[\nu|T^*] = 0 \text{ by construction}$$

Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon|T^*] \neq 0$$

Main Result (Correct) – Exogenous Treatment

Relevant binary instrument z ($p_1^* \neq p_2^*$) identifies α_0, α_1 and $\mathbb{E}[y|T^*]$ provided that $\mathbb{E}[\nu|T^*, T, z] = 0$ and $\alpha_0 + \alpha_1 < 1$.

Extension (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$, $p_1^* \neq p_2^*$, $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$ identified.

► Details

What about increasing the support of z ?

$\mathbb{E}[\varepsilon|z] = 0 \implies$ *pair of equations for each $k = 1, \dots, K$*

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0) \left(\frac{\beta}{1 - \alpha_0 - \alpha_1} \right) + (1 - \alpha_0)c - (p_k - \alpha_0)m_{1k}^*$$

$$\hat{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0) \left(\frac{\beta}{1 - \alpha_0 - \alpha_1} \right) + \alpha_0 c + (p_k - \alpha_0)m_{1k}^*$$

where $\hat{y}_{0k} = (1 - p_k)\bar{y}_{0k}$ and $\hat{y}_{1k} = p_k\bar{y}_{1k}$

2K Equations in $K + 4$ Unknowns

Theorem: β is unidentified regardless of K .

Proof of special case: $\alpha_0 = 0$

1. System of equations:

$$\tilde{y}_{0k} = c + p_k \beta \left(\frac{\alpha_1}{1 - \alpha_1} \right) - p_k m_{1k}^*$$

$$\tilde{y}_{1k} = p_k \beta + p_k m_{1k}^*$$

2. $\beta/(1 - \alpha_1) \equiv \mathcal{W}$ identified, $\beta\alpha_1/(1 - \alpha_1) = \mathcal{W} - \beta \implies$

$$(c + p_k \mathcal{W} - \tilde{y}_{0k})/p_k = \beta + m_{1k}^*$$

$$\tilde{y}_{1k}/p_k = \beta + m_{1k}^*$$

3. Sum equations from 1 $\implies (c + p_k \mathcal{W} - \tilde{y}_{0k}) = \tilde{y}_{1k}$

Conditional *Second* Moment Independence.

New Assumption

Homoskedastic errors w.r.t. the *instrument*: $E[\varepsilon^2|z] = E[\varepsilon^2]$

Reasonable?

Makes sense in an RCT or a true natural experiment.

New Moment Conditions

Defining $\mu_{k\ell}^* = (p_k - \alpha_0)m_{1k}^* - (p_\ell - \alpha_0)m_{k\ell}^*$,

$$\mathbb{E}(y^2|z_k) - \mathbb{E}(y^2|z_\ell) \equiv \Delta \overline{y^2} = \beta \mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu_{k\ell}^*$$

$$\mathbb{E}(yT|z_k) - \mathbb{E}(yT|z_\ell) \equiv \Delta \overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu_{k\ell}^*$$

Theorem: $(\alpha_1 - \alpha_0)$ is Identified if $E[\varepsilon^2|z] = E[\varepsilon^2]$

Requires only binary z

Solve for $\mu_{k\ell}^*$, substitute $\beta = \mathcal{W}(1 - \alpha_0 - \alpha_1)$, rearrange to find

$$\alpha_1 - \alpha_0 = 1 + \mathcal{R}/\mathcal{W}, \quad \text{where} \quad \mathcal{R} \equiv \frac{\Delta \bar{y}^2 - 2\mathcal{W}\Delta \bar{y}\bar{T}}{\mathcal{W}(p_k - p_\ell)}.$$

What good is $(\alpha_1 - \alpha_0)$?

- ▶ Test necessary condition for *no mis-classification*: $\alpha_0 = \alpha_1$
- ▶ Simple, tighter partial identification bounds for β
- ▶ If α_0 known, e.g. zero $\implies \beta$ point identified

Conditional *Third* Moment Independence

New Assumption

Third moment independence w.r.t instrument: $E[\varepsilon^3|z] = E[\varepsilon^3]$

New Moment Conditions

Define $\lambda_{k\ell}^* = (p_k - \alpha_0)v_{1k}^* - (p_\ell - \alpha_0)v_{1\ell}^*$

where $v_{tk}^* = \mathbb{E}(u^2 | T^* = t, z_k)$. Then

$$\mathbb{E}(y^3|z_k) - \mathbb{E}(y^3|z_\ell) \equiv$$

$$\Delta \overline{y^3} = \beta^2 \mathcal{W}(p_k - p_\ell) + 3\beta \mathcal{W}\mu_{k\ell}^* + 3\mathcal{W}\lambda_{k\ell}^*$$

$$\mathbb{E}(y^2 T|z_k) - \mathbb{E}(y^2 T|z_\ell) \equiv$$

$$\Delta \overline{y^2 T} = \beta(1 - \alpha_1) \mathcal{W}(p_k - p_\ell) + 2(1 - \alpha_1) \mathcal{W}\mu_{k\ell}^* + \lambda_{k\ell}^*$$

Theorem: β , α_0 and α_1 identified

Adding $E[\varepsilon^3|z] = E[\varepsilon^3]$, z need only be binary.

Solve for $\lambda_{k\ell}^*$, substitute and rearrange. After further substitutions:

$$2\mathcal{W}^2(1 - \alpha_1)^2 + 2\mathcal{R}\mathcal{W}(1 - \alpha_1) + (\mathcal{S} - \mathcal{R}^2) = 0$$

where

$$\mathcal{S} \equiv \frac{\Delta \overline{y^3} - 3\mathcal{W} [\Delta \overline{y^2 T} + \mathcal{R} \Delta \overline{y T}]}{\mathcal{W}(p_k - p_\ell)}$$

- ▶ Quadratic in $(1 - \alpha_1)$ and observables only
- ▶ Always two real roots: one is $(1 - \alpha_1)$ and the other is α_0 .
- ▶ To tell which is which, need $\alpha_0 + \alpha_1 < 1$.

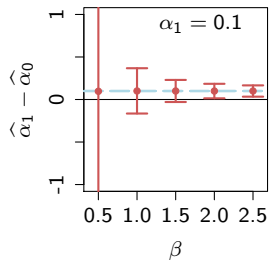
Simulation Study

Simulation Study: $y = \beta T^* + \varepsilon$

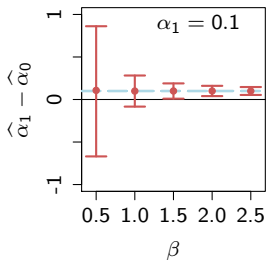
- ▶ $(\varepsilon, \eta) \sim$ jointly normal, mean 0, variance 1, corr. 0.3.
- ▶ First stage: $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
 - ▶ Half of subjects have $z = 1$, the rest have $z = 0$.
 - ▶ $\gamma_0 = \Phi^{-1}(\delta)$
 - ▶ $\gamma_1 = \Phi^{-1}(1 - \delta) - \Phi(\delta)$
 - ▶ δ equals fraction of those offered treatment who fail to take it up *and* fraction of those not offered treatment who do.
- ▶ Generate T as follows:
 - ▶ $T^* = 0 \implies T = 0$, i.e. $\alpha_0 = 0$
 - ▶ $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$
 - ▶ α_0, α_1 unknown to econometrician.

Sampling Distribution of $\hat{\alpha}_1 - \hat{\alpha}_0$

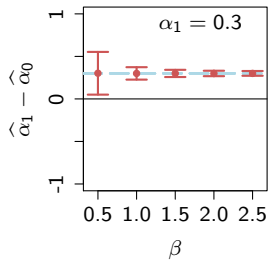
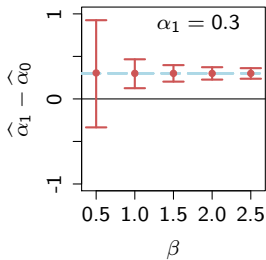
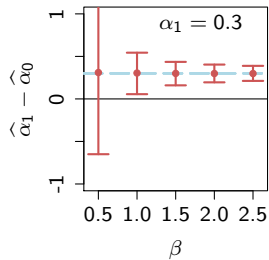
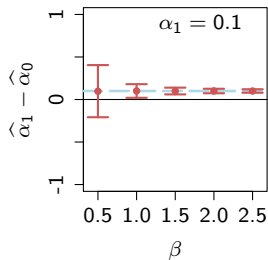
(a) $N = 500, \delta = 0.1$



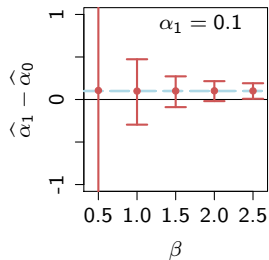
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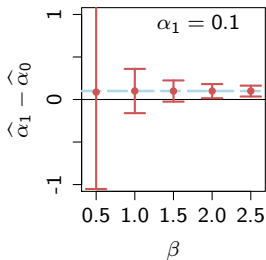
(c) $N = 5000, \delta = 0.1$



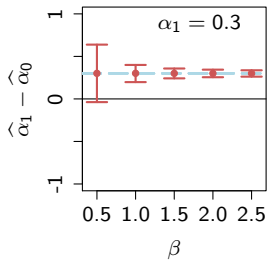
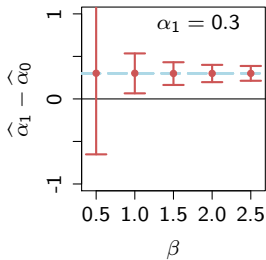
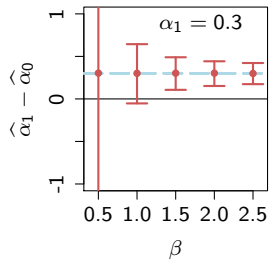
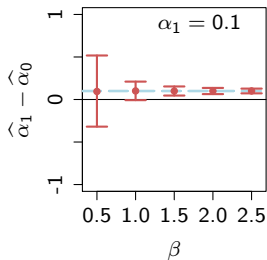
(a) $N = 500, \delta = 0.2$



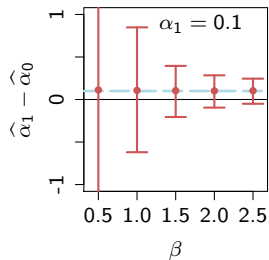
(b) $N = 1000, \delta = 0.2$



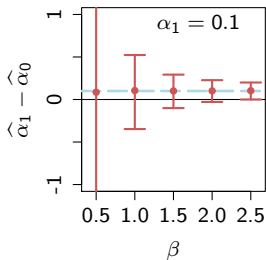
(c) $N = 5000, \delta = 0.2$



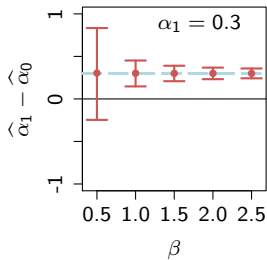
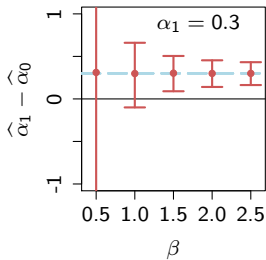
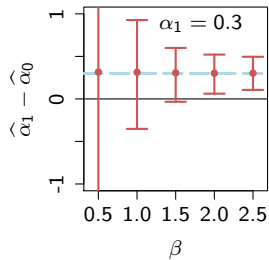
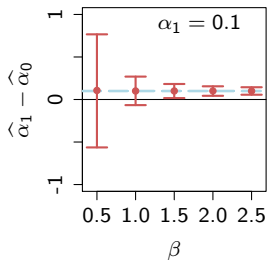
(a) $N = 500, \delta = 0.3$



(b) $N = 1000, \delta = 0.3$



(c) $N = 5000, \delta = 0.3$

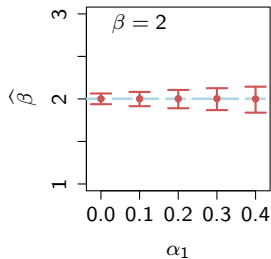
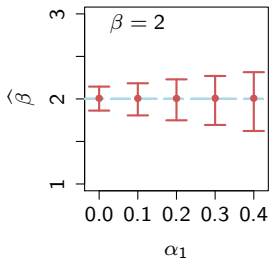
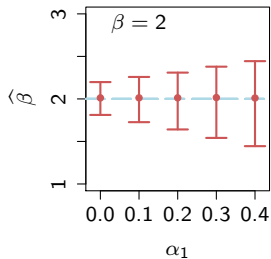
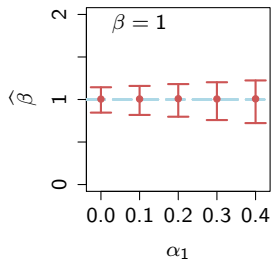
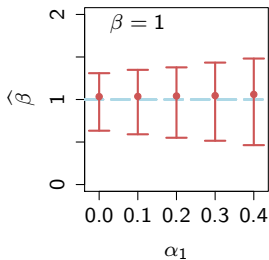
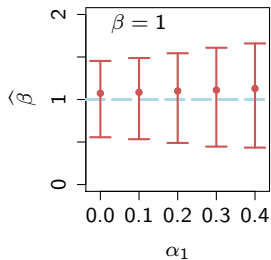


Sampling Distribution of $\hat{\beta} = (1 - \hat{\alpha}_0 - \hat{\alpha}_1)\hat{\beta}_{IV}$

(a) $N = 500, \delta = 0.1$

(b) $N = 1000, \delta = 0.1$

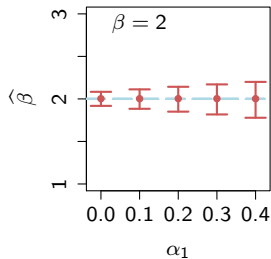
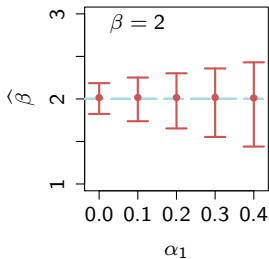
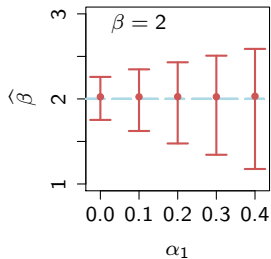
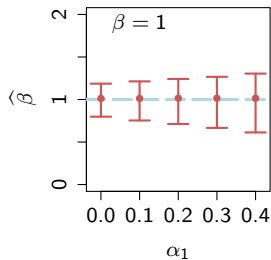
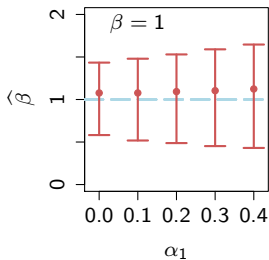
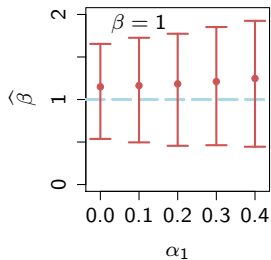
(c) $N = 5000, \delta = 0.1$



(a) $N = 500, \delta = 0.2$

(b) $N = 1000, \delta = 0.2$

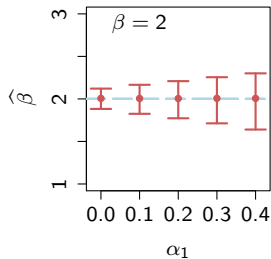
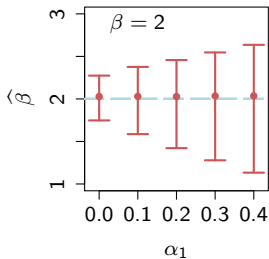
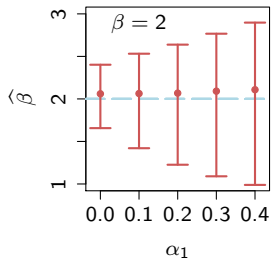
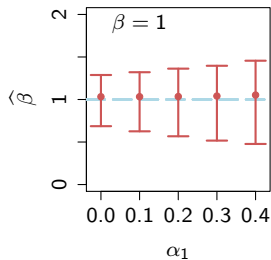
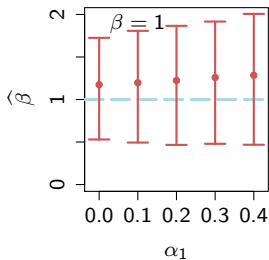
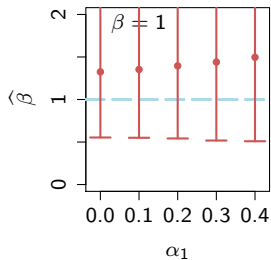
(c) $N = 5000, \delta = 0.2$



(a) $N = 500, \delta = 0.3$

(b) $N = 1000, \delta = 0.3$

(c) $N = 5000, \delta = 0.3$



Conclusion

- ▶ Endogenous, mis-measured binary treatment.
- ▶ Important in applied work but no solution in the literature.
- ▶ Usual (1st moment) IV assumption fails to identify β
- ▶ 2nd moment assumption identifies $\alpha_1 - \alpha_0$
- ▶ 3rd moment assumption identifies β

Mahajan's Argument

Regression Model

$$y = \mathbb{E}[y | T^*] + \nu$$

$$\mathbb{E}[\nu | T^*] = 0 \text{ by construction}$$

Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon | T^*] \neq 0$$

Ingredients

1. If $p_1^* \neq p_2^*$, $\mathbb{E}[\varepsilon | z] = 0$ then, since $\beta_{IV} = \beta / (1 - \alpha_0 - \alpha_1)$, knowledge of α_0, α_1 is sufficient to recover β . (Correct)
2. If $p_1^* \neq p_2^*$, $\mathbb{E}[\nu | T^*, T, z] = 0$, α_0, α_1 are identified. (Correct)

How to satisfy both 1 and 2 while allowing $\mathbb{E}[\varepsilon | T^*] \neq 0$?

3. Assume that $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$
(i.e. $m_{01}^* = m_{02}^*$ and $m_{11}^* = m_{12}^*$)

The Flaw in Mahajan's Argument

Proposition

If $\mathbb{E}[\varepsilon | T^*] \neq 0$ then $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$ combined with $\mathbb{E}[\varepsilon | z] = 0$ implies $p_1^* = p_2^*$, i.e. z is irrelevant for T^* .

Proof

Recall that $\mathbb{E}[\varepsilon | z] = 0$ implies

$$(1 - p_1^*)m_{01}^* + p_1^*m_{11}^* = c$$

$$(1 - p_2^*)m_{02}^* + p_2^*m_{12}^* = c$$

while Mahajan's assumption implies $m_{01}^* = m_{02}^*$ and $m_{11}^* = m_{12}^*$.

Therefore either $m_{01}^* = m_{02}^* = m_{11}^* = m_{12}^* = c$, which is ruled out by $E[\varepsilon | T^*] = 0$, or $p_1^* = p_2^*$.