Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

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February 27th, 2017

What is the causal effect of T^*?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Unknown function that *does not depend on i*
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete (typically binary) instrumental variable

Target of Inference:

ATE function: $\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$

Example: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with 1050 pregnant smokers in England: 521 given nicotine patches, the rest given placebo patches.

- y – Birthweight
- T^* – True smoking behavior
- T – Self-reported smoking behavior
- $\mathbf{x}$ – Mother characteristics
- z – Indicator of nicotine patch

Example: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters. Randomly choose 6 and set up school in each village of these clusters.

- y – Girl's score on math and language test
- T^* – Girl's true school attendance
- T – Parent's report of child's school attendance
- $\mathbf{x}$ – Child and household characteristics
- z – School built in village

Related Literature

Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015), Denteh et al. (2016)

Mahajan Details

Model: $y = c + \beta T^* + \varepsilon$

Valid Instrument

$\mathbb{E}[\varepsilon|z] = 0.$

First-stage

$p_k^* \equiv \mathbb{P}(T^* = 1|z = z_k) \neq \mathbb{P}(T^* = 1|z = z_\ell) \equiv p_\ell^*$, $k \neq \ell$

Non-differential Measurement Error

- $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$
- $\alpha_0 + \alpha_1 < 1$

Observable Moments: $y = c + \beta T^* + \varepsilon$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ / p_{01}	$\bar{y}_{02}$ / p_{02}	$\dots$	$\bar{y}_{0K}$ / p_{0K}
$T = 1$	$\bar{y}_{11}$ / p_{11}	$\bar{y}_{12}$ / p_{12}	$\dots$	$\bar{y}_{1K}$ / p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|\,T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

Define error term that absorbs constant: $u = c + \varepsilon$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T^* = 0$	m^*_{01} / p^*_{01}	m^*_{02} / p^*_{02}	$\dots$	m^*_{0K} / p^*_{0K}
$T^* = 1$	m^*_{11} / p^*_{11}	m^*_{12} / p^*_{12}	$\dots$	m^*_{1K} / p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u|T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p^*_k = \mathbb{P}(T^* = 1|z = z_k)$$

System of Equations given $E[\varepsilon|z] = 0$

$\boxed{\mathbb{E}[\varepsilon|z] = 0 \implies \textit{pair} \text{ of equations for each } k = 1, \ldots, K}$

$$(1 - p_k)\bar{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + (1 - \alpha_0)c - (p_k - \alpha_0)m^*_{1k}$$

$$p_k\bar{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m^*_{1k}$$

Theorem

$2K$ equations in $K + 4$ unknowns, but β is unidentified *regardless* of K.

Intuition

Using $E[\varepsilon|z] = 0$ to eliminate m^*_{0k} from the system "entangles" the equations such that each pair only provides one restriction.

Bounds for Mis-classification Probabilities

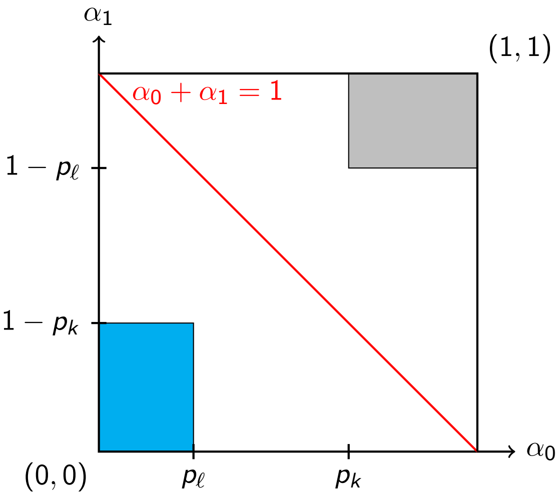
$\alpha_0(z) = \alpha_0, \ \alpha_1(z) = \alpha_1$

$$\Longrightarrow \quad p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}, \quad 1 - p_k^* = \frac{1 - p_k - \alpha_1}{1 - \alpha_0 - \alpha_1}$$

$\alpha_0 + \alpha_1 < 1 \iff \mathrm{Cor}(T, T^*) > 0 \iff (1 - \alpha_0 - \alpha_1) > 0$

$\alpha_0 < \min_k\{p_k\}, \ \alpha_1 < \min_k\{1 - p_k\}$

$\alpha_0 \leq \min_k \{p_k\}, \quad \alpha_1 \leq \min_k \{1 - p_k\}$



Bounds for β

$\mathbb{E}[\varepsilon|z] = 0$

$\implies \beta_{RF} = \mathbb{E}[y|z_k] - \mathbb{E}[y|z_\ell] = \beta(p_k^* - p_\ell^*)$

Mis-classification

$\implies p_k^* - p_\ell^* = (p_k - p_\ell)/(1 - \alpha_0 - \alpha_1)$

Combining: $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$

$\alpha_0 + \alpha_1 < 1 \implies$

- β is between β_{RF} and β_{IV}
- β_{IV} *inflated* but has correct sign
- β_{RF} bound equivalent to substituting α_0, α_1 bounds

Strengthening the Measurement Error Assumptions

- $\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$
- $\alpha_0 + \alpha_1 < 1$
- $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$

Additional Assumption

$\mathbb{E}[\varepsilon^2 | T^*, T, z] = \mathbb{E}[\varepsilon^2 | T^*, z]$

Improve bounds for α_0, α_1 to tighten lower bound for β...

Tighter Bounds for α_0, α_1 from Conditional Variances

Assume

$\mathbb{E}[\varepsilon^2 | T^*, T, z] = \mathbb{E}[\varepsilon^2 | T^*, z]$

Observables

$\sigma_{tk}^2 = \text{Var}(y | T = t, z = k)$

Constrain Unobservables

$s_{tk}^{*2} = Var(u | T^* = t, z_k) > 0$

$$(p_k - \alpha_0)\left[(1-\alpha_0)p_k\sigma_{1k}^2 - \alpha_0(1-p_k)\sigma_{0k}^2\right] > \alpha_0(1-\alpha_0)p_k(1-p_k)(\bar{y}_{1k} - \bar{y}_{0k})^2$$
$$(1 - p_k - \alpha_1)\left[(1-\alpha_1)(1-p_k)\sigma_{0k}^2 - \alpha_1 p_k\sigma_{1k}^2\right] > \alpha_1(1-\alpha_1)p_k(1-p_k)(\bar{y}_{1k} - \bar{y}_{0k})^2$$

Schooling and Test Scores – Afghan RCT

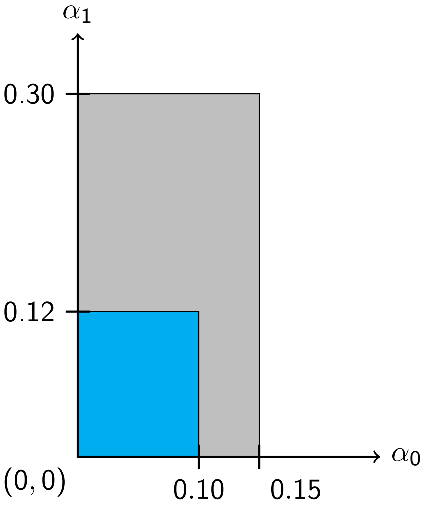
Burde & Linden (2013, AEJ Applied)

“Weak” Bounds

$\beta \in [0.65 \times \beta_{IV},\ \beta_{IV}]$

Add 2nd Moments

$\beta \in [0.78 \times \beta_{IV},\ \beta_{IV}]$



Independence Assumption: $\varepsilon \perp T | (T^*, z)$

Define $F_{tk}(\tau) = \mathbb{P}(Y \leq \tau | T = t, z_k)$ and $F_k(\tau) = \mathbb{P}(Y \leq \tau | z_k)$

$$\alpha_0 \leq p_k \inf_{\tau} \left\{ \left[\frac{F_{1k}(\tau)}{F_k(\tau)} \right] \wedge \left[\frac{1 - F_{1k}(\tau)}{1 - F_k(\tau)} \right] \right\} \leq p_k$$

$$\alpha_1 \leq (1 - p_k) \inf_{\tau} \left\{ \left[\frac{F_{0k}(\tau)}{F_k(\tau)} \right] \wedge \left[\frac{1 - F_{0k}(\tau)}{1 - F_k(\tau)} \right] \right\} \leq (1 - p_k)$$

Bounds for (α_0, α_1) do *not* require z to be a valid instrument!

Sufficient Conditions To Identify α_0, α_1, and β

Baseline Assumptions

- $\mathbb{E}[\varepsilon|z] = 0$
- $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z),\ \alpha_1 = \mathbb{P}(T = 0|T^* = 1, z),\ \alpha_0 + \alpha_1 < 1$

Strengthen IV Assumption

- $\mathbb{E}[\varepsilon^2|z] = \mathbb{E}[\varepsilon^2]$
- $\mathbb{E}[\varepsilon^3|z] = \mathbb{E}[\varepsilon^3]$

Strengthen Measurement Error Assumption

- $\mathbb{E}[\varepsilon^2|T^*, T, z] = \mathbb{E}[\varepsilon^2|T^*, z]$
- $\mathbb{E}[\varepsilon^3|T^*, T, z] = \mathbb{E}[\varepsilon^3|T^*, z]$

First Moment Condition

Assumptions

- $\mathbb{E}[\varepsilon|z] = 0$
- $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$

Moment Condition

$$\mathrm{Cov}(y, z) - \left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right)\mathrm{Cov}(T, z) = 0$$

MC # 1 identifies $\beta/(1 - \alpha_0 - \alpha_1)$

Second Moment Condition

Additional Assumptions

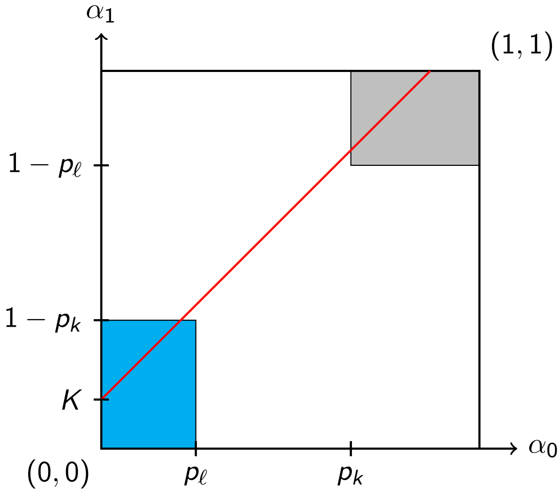
- $\mathbb{E}[\varepsilon^2|z] = \mathbb{E}[\varepsilon^2]$
- $\mathbb{E}[\varepsilon^2|T^*, T, z] = \mathbb{E}[\varepsilon^2|T^*, z]$

Moment Condition

$$\mathrm{Cov}(y^2, z) - \frac{\beta}{1-\alpha_0-\alpha_1}\left\{2\mathrm{Cov}(yT, z) - \beta\mathrm{Cov}(T, z)\left(\frac{1+\alpha_0-\alpha_1}{1-\alpha_0-\alpha_1}\right)\right\} = 0$$

Given MC #1, MC #2 identifies $(\alpha_1 - \alpha_0)$

$\alpha_1 - \alpha_0 = K$



Third Moment Condition

Additional Assumptions

- $\mathbb{E}[\varepsilon^3 | z] = \mathbb{E}[\varepsilon^3]$
- $\mathbb{E}[\varepsilon^3 | T^*, T, z] = \mathbb{E}[\varepsilon^3 | T^*, z]$

Moment Condition

$$\begin{aligned}\mathrm{Cov}(y^3, z) - \left(\frac{\beta}{1-\alpha_0-\alpha_1}\right) \Bigg\{ & \beta^2 \left[1 + \frac{6\alpha_0(1-\alpha_1)}{(1-\alpha_0-\alpha_1)^2}\right] \mathrm{Cov}(T, z) \\ & -3\beta \left[\frac{1-(\alpha_1-\alpha_0)}{1-\alpha_0-\alpha_1}\right] \mathrm{Cov}(yT, z) + 3\mathrm{Cov}(y^2 T, z) \Bigg\} = 0\end{aligned}$$

Theorem

Third moment suffice to identify the model provided that $\beta \neq 0$. If $\beta = 0$, the reduced form identifies β.

Weak Identification in Simple Special Case: $\alpha_0 = 0$

$$\text{Cov}(y,z) - \left(\frac{\beta}{1-\alpha_1}\right)\text{Cov}(T,z) = 0$$

$$\text{Cov}(y^2,z) - \frac{\beta}{1-\alpha_1}\left\{2\text{Cov}(yT,z) - \beta\text{Cov}(T,z)\right\} = 0$$

$$\beta = \frac{2\text{Cov}(yT,z)}{\text{Cov}(T,z)} - \frac{\text{Cov}(y^2,z)}{\text{Cov}(y,z)}$$

Simulation DGP: $y = \beta T^* + \varepsilon$

Errors

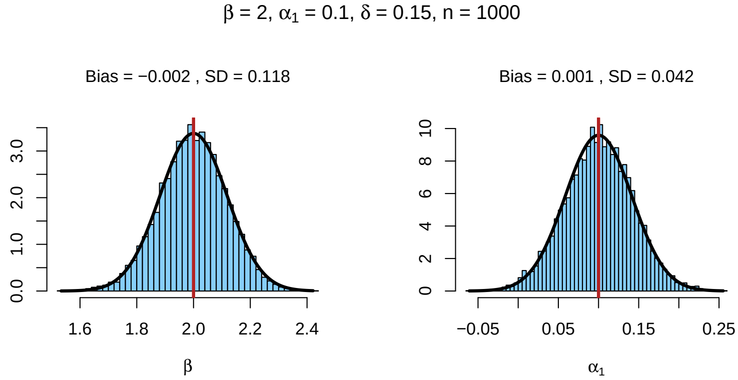
$(\varepsilon, \eta) \sim$ jointly normal, mean 0, variance 1, correlation 0.5.

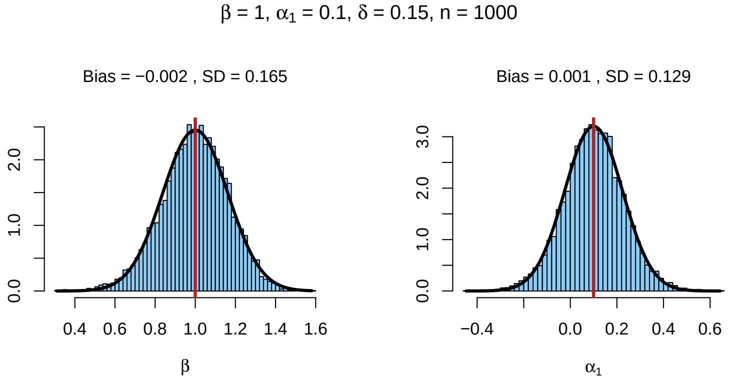
First-Stage

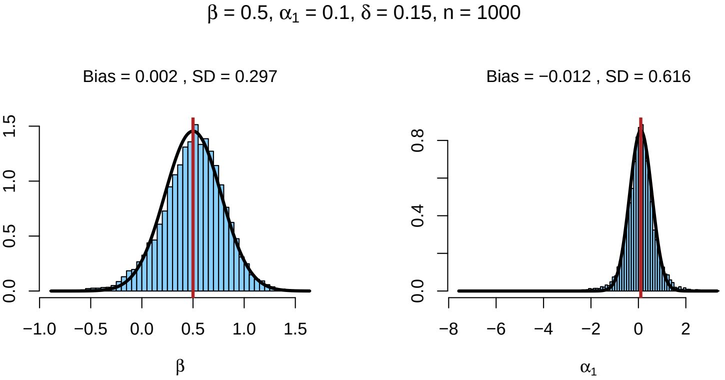
- Half of subjects have $z = 1$, the rest have $z = 0$.
- $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
- $\delta = \mathbb{P}(T^* = 0|z = 1) = \mathbb{P}(T^* = 1|z = 0)$

Mis-classification

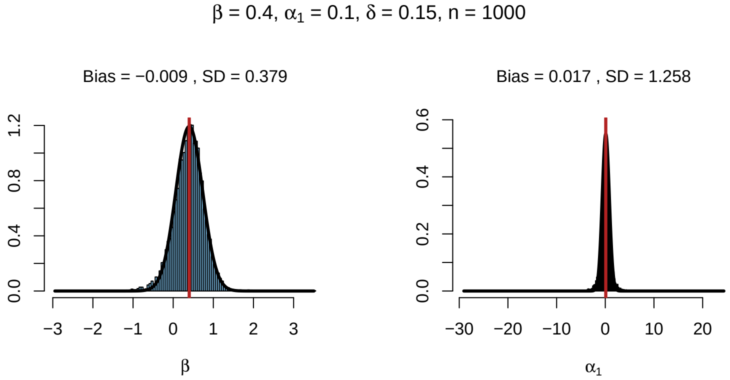
- Set $\alpha_0 = 0$
- $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$

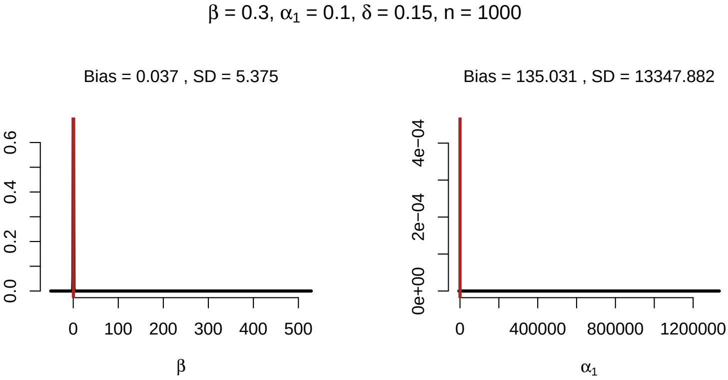

β = 2, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.118
0.0
1.0
2.0
3.0
1.6
1.8
2.0
2.2
2.4
β
Bias = 0.001 , SD = 0.042
0
2
4
6
8
10
−0.05
0.05
0.15
0.25
α1


β = 1, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.165
Bias = 0.001 , SD = 0.129
0.0
1.0
2.0
3.0
0.4
0.6
0.8
1.0
1.2
1.4
1.6
−0.4
0.0
0.2
0.4
0.6
β
α1



β = 0.5, α1 = 0.1, δ = 0.15, n = 1000
Bias = 0.002 , SD = 0.297
Bias = −0.012 , SD = 0.616
0.0
0.5
1.0
1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
β
0.0
0.4
0.8
−8
−6
−4
−2
0
2
α1


β = 0.4, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.009 , SD = 0.379
Bias = 0.017 , SD = 1.258
0.0
0.4
0.8
1.2
−3
−2
−1
0
1
2
3
β
0.0
0.2
0.4
0.6
−30
−20
−10
0
10
20
α1


β = 0.3, α1 = 0.1, δ = 0.15, n = 1000
Bias = 0.037 , SD = 5.375
Bias = 135.031 , SD = 13347.882
0.0
0.2
0.4
0.6
0
100
200
300
400
500
β
0e+00
2e−04
4e−04
0
400000
800000
1200000
α1

Coverage and Width of Nominal 95% CIs

$\alpha_1 = 0.1, \delta = 0.15, n = 1000, \rho = 0.5$

		Coverage		Width	
β	RF	RF	GMM	RF	GMM
2.00	1.400	0.95	0.95	0.35	0.23
1.50	1.050	0.95	0.95	0.32	0.26
1.00	0.700	0.95	0.95	0.29	0.32
0.50	0.350	0.95	0.96	0.27	0.55
0.25	0.175	0.95	0.98	0.26	1.08
0.20	0.140	0.95	0.99	0.25	1.40
0.15	0.105	0.95	0.99	0.25	1.86
0.10	0.070	0.95	1.00	0.25	3.04
0.05	0.035	0.95	1.00	0.25	4.76
0.01	0.007	0.95	1.00	0.25	5.92

Conclusion

Summary

- Endogenous, mis-measured binary treatment.
- Important in applied work but no solution in the literature.
- Usual (1st moment) IV assumption fails to identify β
- Bounds for mis-classification probabilities and β.
- Higher moment / independence restrictions identify β

Extensions / Work in Progress

- Weak Identification: Two-step Inference?
- Heterogeneous Treatment Effects

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Main Result (Correct) – Exogenous Treatment

Relevant binary instrument z ($p_k^* \neq p_\ell^*$) identifies α_0, α_1 and $\mathbb{E}[y|T^*]$ provided that $\mathbb{E}[\nu|T^*, T, z] = 0$ and $\alpha_0 + \alpha_1 < 1$.

Extension (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$, $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$ identified.

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Ingredients

1. If $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|z] = 0$ then, since $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$, knowledge of α_0, α_1 is sufficient to recover β. (Correct)
2. If $p_k^* \neq p_\ell^*$, $\mathbb{E}[\nu|T^*, T, z] = 0$, α_0, α_1 are identified. (Correct)

   How to satisfy both 1 and 2 while allowing $\mathbb{E}[\varepsilon|T^*] \neq 0$?

3. Assume that $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$

   (i.e. $m_{0k}^* = m_{0\ell}^*$ and $m_{1k}^* = m_{1\ell}^*$)

Flaw in the Argument

Proposition

If $\mathbb{E}[\varepsilon|T^*] \neq 0$ then $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$ combined with $\mathbb{E}[\varepsilon|z] = 0$ implies $p_k^* = p_\ell^*$, i.e. z is irrelevant for T^*.

Proof

$\mathbb{E}[\varepsilon|z] = 0$ implies

$$(1 - p_1^*)m_{0k}^* + p_1^* m_{1k}^* = c$$

$$(1 - p_2^*)m_{0\ell}^* + p_2^* m_{1\ell}^* = c$$

while Mahajan's assumption implies $m_{0k}^* = m_{0\ell}^*$ and $m_{1k}^* = m_{1\ell}^*$. Therefore either $m_{0k}^* = m_{0\ell}^* = m_{1k}^* = m_{1\ell}^* = c$, which is ruled out by $E[\varepsilon|T^*] = 0$, or $p_k^* = p_\ell^*$.

Back