Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

Francis J. DiTraglia

Camilo Garcia-Jimeno

University of Pennsylvania

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Estimating the Effect of T^*

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- y – Outcome of interest
- h – Known or Unknown function
- T^* – Unobserved, endogenous binary treatment
- T – Observed, mis-measured binary surrogate for T^*
- $\mathbf{x}$ – Exogenous covariates
- ε – Mean-zero error term
- z – Discrete (typically binary) instrumental variable

Target of Inference: $\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$

Example: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- y – Log wage
- T^* – True training attendence
- T – Self-reported training attendance
- $\mathbf{x}$ – Individual characteristics
- z – Offer of job training

Related Literature

Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008), Song (2015), Hu et al. (2015)...

Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

Binary, Endogenous Treatment

Mahajan (2006), Shiu (2015), Ura (2015), Denteh et al. (2016)

Mahajan Details

Model: $y = c + \beta T^* + \varepsilon$

Valid Instrument

$\mathbb{E}[\varepsilon | z] = 0.$

First-stage

$p_k^* \equiv \mathbb{P}(T^* = 1 | z = z_k) \neq \mathbb{P}(T^* = 1 | z = z_\ell) \equiv p_\ell^*$, $k \neq \ell$

Non-differential Measurement Error

- $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$
- $\alpha_0 + \alpha_1 < 1$

Observable Moments: $y = c + \beta T^* + \varepsilon$

	$z = 1$	$z = 2$	$\ldots$	$z = K$
$T = 0$	$\bar{y}_{01}$ p_{01}	$\bar{y}_{02}$ p_{02}	$\ldots$	$\bar{y}_{0K}$ p_{0K}
$T = 1$	$\bar{y}_{11}$ p_{11}	$\bar{y}_{12}$ p_{12}	$\ldots$	$\bar{y}_{1K}$ p_{1K}

$$\bar{y}_{tk} = \mathbb{E}[y|T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1|z = z_k)$$

Unobservable Moments: $y = \beta T^* + u$

Define error term that absorbs constant: $u = c + \varepsilon$

	$z=1$	$z=2$	$\dots$	$z=K$
$T^*=0$	m^*_{01} p^*_{01}	m^*_{02} p^*_{02}	$\dots$	m^*_{0K} p^*_{0K}
$T^*=1$	m^*_{11} p^*_{11}	m^*_{12} p^*_{12}	$\dots$	m^*_{1K} p^*_{1K}

$$m^*_{tk} = \mathbb{E}[u|T^* = t, z = z_k], \quad p^*_{tk} = q_k p^*_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p^*_k = \mathbb{P}(T^* = 1|z = z_k)$$

System of Equations given $E[\varepsilon|z] = 0$

$\boxed{\mathbb{E}[\varepsilon|z] = 0 \implies \textit{pair} \text{ of equations for each } k = 1, \ldots, K}$

$$(1 - p_k)\bar{y}_{0k} = \alpha_1(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + (1 - \alpha_0)c - (p_k - \alpha_0)m^*_{1k}$$

$$p_k\bar{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0)\left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) + \alpha_0 c + (p_k - \alpha_0)m^*_{1k}$$

Theorem

$2K$ equations in $K + 4$ unknowns, but β is unidentified *regardless* of K.

Intuition

Using $E[\varepsilon|z] = 0$ to eliminate m^*_{0k} from the system "entangles" the equations such that each pair only provides one restriction.

First Moment Condition

Assumptions

- $\mathbb{E}[\varepsilon|z] = 0$
- $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*, z]$
- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0|T^* = 1, z)$

Moment Condition

$$\text{Cov}(y, z) - \left(\frac{\beta}{1 - \alpha_0 - \alpha_1}\right) \text{Cov}(T, z) = 0$$

MC # 1 identifies $\beta/(1 - \alpha_0 - \alpha_1)$

Second Moment Condition

Additional Assumptions

- $\mathbb{E}[\varepsilon^2|z] = \mathbb{E}[\varepsilon^2]$
- $\mathbb{E}[\varepsilon^2|T^*, T, z] = \mathbb{E}[\varepsilon^2|T^*, z]$

Moment Condition

$$\mathrm{Cov}(y^2, z) - \frac{\beta}{1-\alpha_0-\alpha_1}\left\{2\mathrm{Cov}(yT, z) - \beta\mathrm{Cov}(T, z)\left(\frac{1+\alpha_0-\alpha_1}{1-\alpha_0-\alpha_1}\right)\right\} = 0$$

Given MC #1, MC #2 identifies $(\alpha_1 - \alpha_0)$

Third Moment Condition

Additional Assumptions

- $\mathbb{E}[\varepsilon^3|z] = \mathbb{E}[\varepsilon^3]$
- $\mathbb{E}[\varepsilon^3|T^*, T, z] = \mathbb{E}[\varepsilon^3|T^*, z]$

Moment Condition

$$\begin{aligned}\mathrm{Cov}(y^3, z) - \left(\frac{\beta}{1-\alpha_0-\alpha_1}\right)\Bigg\{ & \beta^2\left[1+\frac{6\alpha_0(1-\alpha_1)}{(1-\alpha_0-\alpha_1)^2}\right]\mathrm{Cov}(T, z) \\ & -3\beta\left[\frac{1-(\alpha_1-\alpha_0)}{1-\alpha_0-\alpha_1}\right]\mathrm{Cov}(yT, z) + 3\mathrm{Cov}(y^2T, z)\Bigg\} = 0\end{aligned}$$

Theorem

Third moment suffice to identify the model provided that $\beta \neq 0$. If $\beta = 0$, the reduced form identifies β.

GMM Estimator in Simple Special Case: $\alpha_0 = 0$

$$\text{Cov}(y,z) - \left(\frac{\beta}{1-\alpha_1}\right)\text{Cov}(T,z) = 0$$

$$\text{Cov}(y^2,z) - \frac{\beta}{1-\alpha_1}\left\{2\text{Cov}(yT,z) - \beta\text{Cov}(T,z)\right\} = 0$$

$$\beta = \frac{2\text{Cov}(yT,z)}{\text{Cov}(T,z)} - \frac{\text{Cov}(y^2,z)}{\text{Cov}(y,z)}$$

Bounds for Mis-classification Probabilities

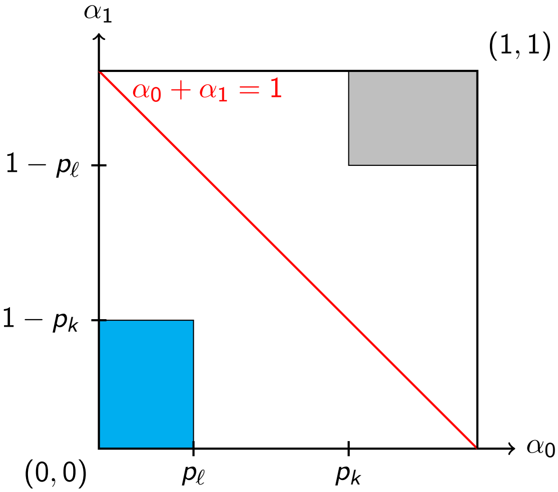
$\alpha_0(z) = \alpha_0,\ \alpha_1(z) = \alpha_1$

$$\Longrightarrow \quad p_k^* = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}, \quad 1 - p_k^* = \frac{1 - p_k - \alpha_1}{1 - \alpha_0 - \alpha_1}$$

$\alpha_0 + \alpha_1 < 1 \iff \mathrm{Cor}(T, T^*) > 0 \iff (1 - \alpha_0 - \alpha_1) > 0$

$\alpha_0 < \min_k\{p_k\},\ \alpha_1 < \min_k\{1 - p_k\}$

$\alpha_0 \leq \min_k \{p_k\}, \quad \alpha_1 \leq \min_k \{1 - p_k\}$



Bounds for β

$\mathbb{E}[\varepsilon|z] = 0$

$\implies \beta_{RF} = \mathbb{E}[y|z_k] - \mathbb{E}[y|z_\ell] = \beta(p_k^* - p_\ell^*)$

Mis-classification

$\implies p_k^* - p_\ell^* = (p_k - p_\ell)/(1 - \alpha_0 - \alpha_1)$

Combining: $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$

$\alpha_0 + \alpha_1 < 1 \implies$

- β is between β_{RF} and β_{IV}
- β_{IV} *inflated* but has correct sign
- β_{RF} bound equivalent to substituting α_0, α_1 bounds

Strengthening the Measurement Error Assumptions

- $\alpha_0 = \mathbb{P}(T = 1 | T^* = 0, z)$
- $\alpha_1 = \mathbb{P}(T = 0 | T^* = 1, z)$
- $\alpha_0 + \alpha_1 < 1$
- $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*, z]$

Additional Assumption

$\mathbb{E}[\varepsilon^2 | T^*, T, z] = \mathbb{E}[\varepsilon^2 | T^*, z]$

Improve bounds for α_0, α_1 to tighten lower bound for β...

Tighter Bounds for α_0, α_1 from Conditional Variances

Assume

$\mathbb{E}[\varepsilon^2 | T^*, T, z] = \mathbb{E}[\varepsilon^2 | T^*, z]$

Observables

$\sigma^2_{tk} = \text{Var}(y | T = t, z = k)$

Constrain Unobservables

$s^{*2}_{tk} = Var(u | T^* = t, z_k) > 0$

$$(p_k - \alpha_0)\left[(1-\alpha_0)p_k\sigma^2_{1k} - \alpha_0(1-p_k)\sigma^2_{0k}\right] > \alpha_0(1-\alpha_0)p_k(1-p_k)(\bar{y}_{1k} - \bar{y}_{0k})^2$$
$$(1 - p_k - \alpha_1)\left[(1-\alpha_1)(1-p_k)\sigma^2_{0k} - \alpha_1 p_k\sigma^2_{1k}\right] > \alpha_1(1-\alpha_1)p_k(1-p_k)(\bar{y}_{1k} - \bar{y}_{0k})^2$$

Simulation DGP: $y = \beta T^* + \varepsilon$

Errors

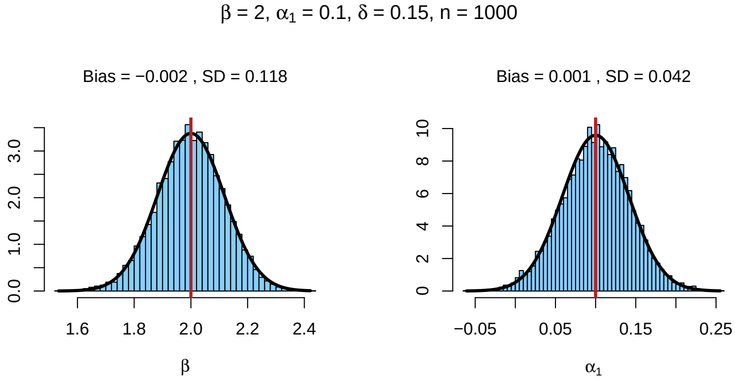
$(\varepsilon, \eta) \sim$ jointly normal, mean 0, variance 1, correlation 0.5.

First-Stage

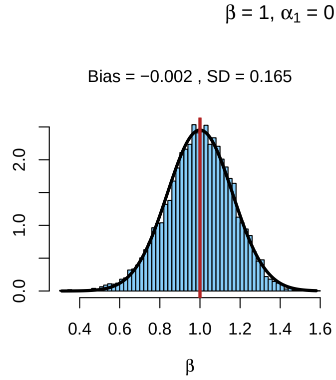
- Half of subjects have $z = 1$, the rest have $z = 0$.
- $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$
- $\delta = \mathbb{P}(T^* = 0 | z = 1) = \mathbb{P}(T^* = 1 | z = 0)$

Mis-classification

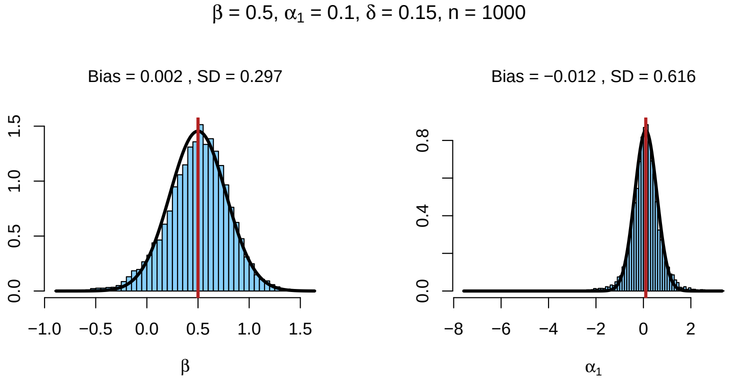
- Set $\alpha_0 = 0$
- $T | T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$

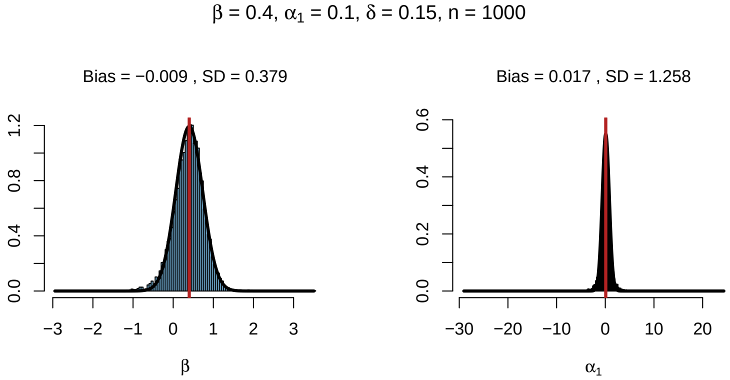


β = 2, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.118
0.0
1.0
2.0
3.0
1.6
1.8
2.0
2.2
2.4
β
Bias = 0.001 , SD = 0.042
0
2
4
6
8
10
−0.05
0.05
0.15
0.25
α1

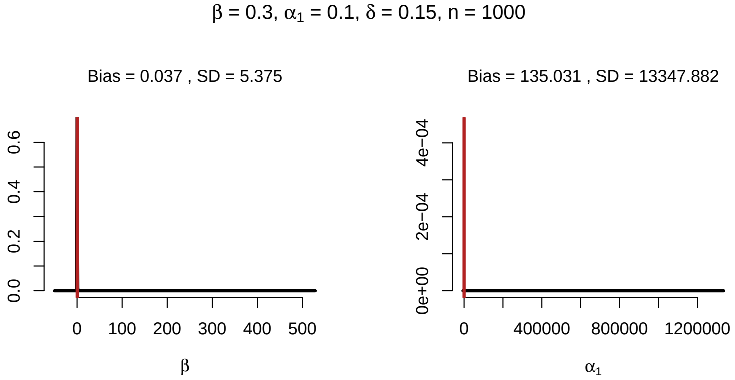


β = 1, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.002 , SD = 0.165
Bias = 0.001 , SD = 0.129
0.0
1.0
2.0
0.4
0.6
0.8
1.0
1.2
1.4
1.6
β
0.0
1.0
2.0
3.0
−0.4
0.0
0.2
0.4
0.6
α1


β = 0.5, α1 = 0.1, δ = 0.15, n = 1000
Bias = 0.002 , SD = 0.297
Bias = −0.012 , SD = 0.616
1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
1.5
β
0.8
0.4
0.0
−8
−6
−4
−2
0
2
α1



β = 0.4, α1 = 0.1, δ = 0.15, n = 1000
Bias = −0.009 , SD = 0.379
1.2
0.8
0.4
0.0
−3
−2
−1
0
1
2
3
β
Bias = 0.017 , SD = 1.258
0.6
0.4
0.2
0.0
−30
−20
−10
0
10
20
α1


β = 0.3, α1 = 0.1, δ = 0.15, n = 1000
Bias = 0.037 , SD = 5.375
0.0
0.2
0.4
0.6
0
100
200
300
400
500
β
Bias = 135.031 , SD = 13347.882
0e+00
2e−04
4e−04
0
400000
800000
1200000
α1

Coverage and Width of Nominal 95% CIs

$\alpha_1 = 0.1, \delta = 0.15, n = 1000, \rho = 0.5$

		Coverage		Width	
β	RF	RF	GMM	RF	GMM
2.00	1.400	0.95	0.95	0.35	0.23
1.50	1.050	0.95	0.95	0.32	0.26
1.00	0.700	0.95	0.95	0.29	0.32
0.50	0.350	0.95	0.96	0.27	0.55
0.25	0.175	0.95	0.98	0.26	1.08
0.20	0.140	0.95	0.99	0.25	1.40
0.15	0.105	0.95	0.99	0.25	1.86
0.10	0.070	0.95	1.00	0.25	3.04
0.05	0.035	0.95	1.00	0.25	4.76
0.01	0.007	0.95	1.00	0.25	5.92

Conclusion

- Endogenous, mis-measured binary treatment.
- Important in applied work but no solution in the literature.
- Usual (1st moment) IV assumption fails to identify β
- Higher moment / independence restrictions identify β
- Identification-Robust Inference incorportating additional inequality moment conditions.

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Main Result (Correct) – Exogenous Treatment

Relevant binary instrument z ($p_k^* \neq p_\ell^*$) identifies α_0, α_1 and $\mathbb{E}[y|T^*]$ provided that $\mathbb{E}[\nu|T^*, T, z] = 0$ and $\alpha_0 + \alpha_1 < 1$.

Extension (Incorrect) – Endogenous Treatment

$\mathbb{E}[\varepsilon|z] = 0$, $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta$ identified.

Mahajan (2006, ECTA)

Regression Model

$y = \mathbb{E}[y|T^*] + \nu$

$\mathbb{E}[\nu|T^*] = 0$ by construction

Causal Model

$y = c + \beta T^* + \varepsilon$

$\mathbb{E}[\varepsilon|T^*] \neq 0$

Ingredients

1. If $p_k^* \neq p_\ell^*$, $\mathbb{E}[\varepsilon|z] = 0$ then, since $\beta_{IV} = \beta/(1 - \alpha_0 - \alpha_1)$, knowledge of α_0, α_1 is sufficient to recover β. (Correct)
2. If $p_k^* \neq p_\ell^*$, $\mathbb{E}[\nu|T^*, T, z] = 0$, α_0, α_1 are identified. (Correct)

   How to satisfy both 1 and 2 while allowing $\mathbb{E}[\varepsilon|T^*] \neq 0$?
3. Assume that $\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*]$

   (i.e. $m_{0k}^* = m_{0\ell}^*$ and $m_{1k}^* = m_{1\ell}^*$)

Flaw in the Argument

Proposition

If $\mathbb{E}[\varepsilon | T^*] \neq 0$ then $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$ combined with $\mathbb{E}[\varepsilon | z] = 0$ implies $p_k^* = p_\ell^*$, i.e. z is irrelevant for T^*.

Proof

$\mathbb{E}[\varepsilon | z] = 0$ implies

$$(1 - p_1^*) m_{0k}^* + p_1^* m_{1k}^* = c$$

$$(1 - p_2^*) m_{0\ell}^* + p_2^* m_{1\ell}^* = c$$

while Mahajan's assumption implies $m_{0k}^* = m_{0\ell}^*$ and $m_{1k}^* = m_{1\ell}^*$. Therefore either $m_{0k}^* = m_{0\ell}^* = m_{1k}^* = m_{1\ell}^* = c$, which is ruled out by $E[\varepsilon | T^*] = 0$, or $p_k^* = p_\ell^*$.

Back