Mis-classified, Binary, Endogenous Regressors: Identification and Inference*

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This Version: August 23, 2017, First Version: October 31, 2015

Abstract

This paper studies identification and inference for the effect of a mis-classified, binary, endogenous regressor when a discrete-valued instrumental variable is available. We begin by showing that the only existing point identification result for this model is incorrect. We go on to derive the sharp identified set under mean independence assumptions for the instrument and measurement error, and find that these fail to point identify the effect of interest. This motivates us to consider alternative and slightly stronger assumptions: we show that adding second and third moment independence assumptions suffices to identify the model. We then turn our attention to inference. We show that both our model, and related models from the literature that assume regressor exogeneity, suffer from weak identification when the effect of interest is small. To address this difficulty, we exploit the inequality restrictions that emerge from our derivation of the sharp identified set under mean independence only. These restrictions remain informative irrespective of the strength of identification. Combining these with the moment equalities that emerge from our identification result, we propose a robust inference procedure using tools from the moment inequality literature. Our method performs well in simulations, both for the exogenous and endogenous regressor case.

Keywords: Instrumental variables, Measurement error, Endogeneity, Weak identification, Moment inequalities

JEL Codes: C10, C18, C25, C26

*We thank Daron Acemoglu, Manuel Arellano, Kristy Buzard, Xu Cheng, Bernardo da Silveira, Bo Honoré, Sophocles Mavroeidis, Yuya Takahashi, and seminar participants at Cambridge, CEMFI, Chicago Booth, Manchester, Northwestern, Oxford, Penn State, Princeton, UCL, and the 2017 North American Summer Meeting of the Econometric Society for valuable comments and suggestions. This document supersedes an earlier version entitled "On Mis-measured Binary Regressors: New Results and Some Comments on the Literature."

1 Introduction

Write an intro!

Old Introduction: Many treatments of interest in applied work are binary. To take a particularly prominent example, consider treatment status in a randomized controlled trial. Even if the randomization is pristine, which yields a valid binary instrument (the offer of treatment), subjects may select into treatment based on unobservables, and given the many real-world complications that arise in the field, measurement error may be an important concern. This paper studies the use of a discrete instrumental variable to identify the causal effect of an endogenous, mis-measured, binary treatment in a model with additively separable errors. Specifically, we consider the following model

$$y = h(T^*, \mathbf{x}) + \varepsilon \tag{1}$$

where $T^* \in \{0, 1\}$ is a mis-measured, endogenous treatment, $\mathbf{x}$ is a vector of exogenous controls, and ε is a mean-zero error. Since T^* is potentially endogenous, $\mathbb{E}[\varepsilon|T^*, \mathbf{x}]$ may not be zero. Our goal is to non-parametrically estimate the average treatment effect (ATE) function

$$\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x}). \tag{2}$$

using a single discrete instrumental variable $z \in \{z_k\}_{k=1}^K$. We assume throughout that z is a relevant instrument for T^*, in other words

$$\mathbb{P}(T^* = 1|z_j, \mathbf{x}) \neq \mathbb{P}(T^* = 1|z_k, \mathbf{x}), \quad \forall k \neq j. \tag{3}$$

While the structural relationship involves T^*, we observe only a noisy measure T, polluted by non-differential measurement error. In particular, we assume that

$$\mathbb{P}(T = 1|T^* = 0, z, \mathbf{x}) = \alpha_0(\mathbf{x}) \tag{4}$$

$$\mathbb{P}(T = 0|T^* = 1, z, \mathbf{x}) = \alpha_1(\mathbf{x}) \tag{5}$$

where the mis-classification error rates $\alpha_0(\mathbf{x})$ and $\alpha_1(\mathbf{x})$ can depend on $\mathbf{x}$ but not z, and additionally that, conditional on true treatment status, observed treatment status provides no additional information about the error term. In other words, we assume that

$$\mathbb{E}[\varepsilon|T^*, T, z, \mathbf{x}] = \mathbb{E}[\varepsilon|T^*, z, \mathbf{x}]. \tag{6}$$

Although a relevant case for applied work, the setting we consider here has received little attention in the literature. The only existing result for the case of an endogenous treatment appears in an important paper by Mahajan (2006), who is primarily concerned with the case of an exogenous treatment. As we show below, Mahajan's identification result for the endogenous treatment case is incorrect. As far as we are aware, this leaves the problem considered in this paper completely unsolved.

We begin by showing that the proof in Appendix A.2 of Mahajan (2006) leads to a contradiction. Throughout his paper, Mahajan (2006) maintains an assumption (Assumption 4) which he calls the "Dependency Condition." This assumption requires that the instrumental variable be relevant, namely that it generates variation in true treatment status. When extending his result for an exogenous treatment to the more general case of an endogenous one, however, he must impose an additional condition on the model (Equation 11), which turns out to violate the Dependency Condition. Since one cannot impose the condition in Equation 11 of Mahajan (2006), we go on to study the prospects for identification in this model more broadly. We consider two possibilities. First, since Mahajan's identification results require only a binary instrument, we borrow an idea from Lewbel (2007) and explore whether expanding the support of the instrument yields identification based on moment equations similar to those used by Mahajan (2006). While allowing the instrument to take on additional values does increase the number of available moment conditions, we show that these moments cannot point identify the treatment effect, regardless of how many (finite) values the instrument takes on.

We then consider a new source of identifying information that arises from imposing stronger assumptions on the instrumental variable. If the instrument is not merely mean independent but in fact *statistically independent* of the regression error term, as in a randomized controlled trial or a true natural experiment, additional moment conditions become available. We show that adding a conditional second moment independence assumption on the instrument identifies the *difference* of mis-classification rates $\alpha_1(\mathbf{x}) - \alpha_0(\mathbf{x})$. Because these rates must equal each other when there is no mis-classification error, our result can be used to test a necessary condition for the absence of measurement error. It can also be used to construct simple and informative partial identification bounds for the treatment effect. When one of the mis-classification rates is known, this identifies the treatment effect. More generally, however, this is not the case. We go on to show that a conditional third moment independence assumption on the instrument point identifies both $\alpha_0(\mathbf{x})$ and $\alpha_1(\mathbf{x})$ and hence the ATE function $\tau(\mathbf{x})$. Both our point identification and partial identification results require only a binary instrument, and lead to simple, closed-form method of moments estimators.

This project is still in progress. The present draft focuses on establishing identification in the simplest possible way: by directly solving a set of equations implied by conditional moment restrictions on ε. Additional results regarding efficient estimation and sharp bounds for α_0, α_1 under weaker conditions on the instrumental variable are currently in progress. For some additional discussion of these results, see our conclusion below in Section 4.

The remainder of this paper is organized as follows. In section 2 we discuss the literature in relation to the problem considered here. Section 3 introduces notation and assumptions, and presents our main results. Section 4 concludes. All proofs appear in the Appendix.

Old Literature Review: Measurement error is a pervasive feature of economic data, motivating a long tradition of measurement error modelling in econometrics. The textbook case considers a continuous regressor (treatment) subject to classical measurement error in a linear model. In this setting, the measurement error is assumed to be unrelated to the true, unobserved, value of the treatment of interest. Regardless of whether this unobserved treatment is exogenous or endogenous, a single valid instrument suffices to identify its effect. When an instrument is unavailable, Lewbel (1997) shows that higher moment assumptions can be used to construct one, provided that the mis-measured treatment is exogenous. When it is endogenous, Lewbel (2012) uses a heteroskedasticity assumption to obtain identification.

Departures from the linear, classical measurement error setting pose serious identification challenges. One strand of the literature considers relaxing the assumption of linearity while maintaining that of classical measurement error. Schennach (2004), for example, uses repeated measures of each mis-measured treatment to obtain identification, while Schennach (2007) uses an instrumental variable. Both papers consider the case of exogenous treatments.[1] More recently, Song et al. (2015) rely on a repeated measure of the mis-measured treatment and the existence of a set of additional regressors, conditional upon which the treatment of interest is unrelated to the unobservables, to obtain identification. Another strand of the literature considers relaxing the assumption of classical measurement error, by allowing the measurement error to be related to the true value of the unobserved treatment. Chen et al. (2005) obtain identification in a general class of moment condition models with mis-measured data by relying on the existence of an auxiliary dataset from which they can estimate the measurement error process. In contrast, Hu and Shennach (2008) and Song (2015) rely on an instrumental variable and an additional conditional location assumption on the measurement error distribution. More recently, Hu et al. (2015) use a continuous instrument to identify the ratio of partial effects of two continuous regressors, one measured

[1] For comprehensive reviews of the challenges of addressing measurement error in non-linear models, see Chen et al. (2011) and Schennach (2013).

with error, in a linear single index model.

Many treatments of interest in economics, however, are binary, and in this case classical measurement error is impossible. Because a true 1 can only be mis-measured as a 0 and a true 0 can only be mis-measured as a 1, the measurement error must be *negatively* correlated with the true treatment status (Aigner, 1973; Bollinger, 1996). For this reason, even in a textbook linear model, the instrumental variables estimator can only remove the effect of endogeneity, not that of measurement error (Frazis and Loewenstein, 2003). Measurement error in a discrete variable is usually called mis-classification.[2] The simplest form of mis-classification is so-called *non-differential* measurement error. In this case, conditional on true treatment status, and possibly a set of exogenous covariates, the measurement error is assumed to be unrelated to all other variables in the system.

A number of papers have studied this problem without the use of instrumental variables under the assumption that the mis-measured binary treatment is exogenous. The first to address this problem was Aigner (1973), who characterized the asymptotic bias of the OLS estimator in this setting, and proposed a technique for correcting it using outside information on the mis-classification process. Another early contribution by Bollinger (1996) provides partial identification bounds. More recently, Chen et al. (2008a) use higher moment assumptions to obtain identification in a linear regression model, and Chen et al. (2008b) extend these results to the non-parametric setting. van Hasselt and Bollinger (2012) and Bollinger and van Hasselt (2015) provide additional partial identification results.

Continuing under the assumption of an exogenous treatment, a number of other papers in the literature have considered the identifying power of an instrumental variable, or something like one. Black et al. (2000) and Kane et al. (1999) more-or-less simultaneously pointed out that when *two* alternative measures of treatment are available, both subject to non-differential measurement error, a non-linear GMM estimator can be used to recover the treatment effect. In essence, one measure serves as an instrument for the other although the estimator is quite different from IV.[3] Subsequently, Frazis and Loewenstein (2003) correctly note that an instrumental variable can take the place of one of the measures of treatment in a linear model with an exogenous treatment, allowing one to implement a variant of the GMM estimator proposed by Black et al. (2000) and Kane et al. (1999). However, as we will show below, the assumptions required to obtain this result are stronger than Frazis and

[2]For general results on the partial identification of discrete probability distributions using mis-classified observations, see Molinari (2008).

[3]Ignoring covariates, the observable moments in this case are the joint probability distribution of the two binary treatment measures and the conditional means of the outcome variable given the two measures. Although the system is highly non-linear, it can be manipulated to yield an explicit solution for the treatment effect provided that the true treatment is exogenous.

Loewenstein (2003) appear to realize: the usual IV assumption that the instrument is mean independent of the regression error is insufficient for identification.

Mahajan (2006) extends the results of Black et al. (2000) and Kane et al. (1999) to a more general nonparametric regression setting using a binary instrument in place of one of the treatment measures. Although unaware of Frazis and Loewenstein (2003), Mahajan (2006) makes the correct assumption over the instrument and treatment to guarantee identification of the conditional mean function. When the treatment is in fact exogenous, this coincides with the treatment effect. Hu (2008) derives related results when the mis-classified discrete regressor may take on more than two values. Lewbel (2007) provides an identification result for the same model as Mahajan (2006) under different assumptions. In particular, the variable that plays the role of the "instrument" need not satisfy the exclusion restriction provided that it does not interact with the treatment and takes on at least three distinct values.

Much less is known about the case in which a binary, or discrete, treatment is not only mis-measured but endogenous. Frazis and Loewenstein (2003) briefly discuss the prospects for identification in this setting. Although they do not provide a formal proof they argue, in the context of their parametric linear model, that the treatment effect is unlikely to be identified unless one is willing to impose strong and somewhat unnatural conditions.[4] The first paper to provide a formal result for this case is Mahajan (2006). He extends his main result to the case of an endogenous treatment, providing an explicit proof of identification under the usual IV assumption in a model with additively separable errors. As we show below, however, Mahajan's proof is incorrect.

The results we derive here most closely relate to the setting considered in Mahajan (2006) in that we study non-parametric identification of the effect of a binary, endogenous treatment, using a discrete instrument. Unlike Mahajan (2006) we consider and indeed show the necessity of using higher-moment information to identify the causal effect of interest. Unlike Kreider et al. (2012), who partially identify the effects of food stamps on health outcomes of children under weak measurement error assumptions, we do not rely on auxiliary data. Unlike Shiu (2015), who considers a sample selection model with a discrete, mis-measured, endogenous regressor, we do not rely on a parametric assumption about the form of the first-stage. Finally unlike Ura (2015), who studies local average treatment effects under very general forms of mis-classification but presents only partial identification results, we point identify an average treatment effect under non-differential measurement

[4]For example, one could consider using the results of Hausman et al. (1998), who study regressions with a mis-classified, discrete *outcome* variable, as a first-stage in an IV setting. In principle, this approach would fully identify the mis-classification error process. Using these results, however, requires either an explicit, nonlinear, parametric model for the first stage, or an identification at infinity argument.

error. Moreover, unlike the identification strategies from the existing literature described above, we do not rely upon continuity of the instrument, a large support condition, or restrictions on the relationship between the true, unmeasured treatment and its observed surrogate, subject to the condition that the measurement error process is non-differential.

2 Coverage and Width of Confidence Intervals

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	90	90	90	91	90	91	90	90
	0.1	91	93	94	94	94	94	90	89
	0.2	92	93	94	94	94	94	92	90
	0.3	93	93	94	94	94	93	92	91
0.1	0.0	92	93	93	94	94	93	90	87
	0.1	93	95	96	97	97	96	92	87
	0.2	95	96	97	98	97	96	92	87
	0.3	96	98	98	98	98	95	92	88
0.2	0.0	93	93	93	93	93	93	92	89
	0.1	95	96	98	98	97	95	93	89
	0.2	97	97	98	98	97	95	92	89
	0.3	98	98	98	98	97	95	93	91
0.3	0.0	93	94	94	94	94	93	92	91
	0.1	97	97	98	98	97	95	93	89
	0.2	98	98	98	98	97	94	93	91
	0.3	99	99	99	98	98	96	95	94

Table 1: Coverage (1 - size) of 90% GMS joint test for α_0 and α_1: $n = 1000$. Based on 10000 simulation replications.

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	90	90	90	91	90	91	90	90
	0.1	91	93	94	94	94	94	90	89
	0.2	92	93	94	94	94	94	92	90
	0.3	93	93	94	94	94	93	92	91
0.1	0.0	92	93	93	94	94	93	90	87
	0.1	93	95	96	97	97	96	92	87
	0.2	95	96	97	98	97	96	92	87
	0.3	96	98	98	98	98	95	92	88
0.2	0.0	93	93	93	93	93	93	92	89
	0.1	95	96	98	98	97	95	93	89
	0.2	97	97	98	98	97	95	92	89
	0.3	98	98	98	98	97	95	93	91
0.3	0.0	93	94	94	94	94	93	92	91
	0.1	97	97	98	98	97	95	93	89
	0.2	98	98	98	98	97	94	93	91
	0.3	99	99	99	98	98	96	95	94

Table 2: Coverage (1 - size) of 90% GMS joint test for α_0 and α_1: $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	95	95	95	96	96	96	95	95
	0.1	96	97	97	97	97	97	95	94
	0.2	96	97	98	98	97	97	96	95
	0.3	97	97	97	98	97	97	96	95
0.1	0.0	96	97	97	97	97	97	95	93
	0.1	97	98	99	99	99	98	96	92
	0.2	98	99	99	99	99	98	96	93
	0.3	99	99	99	99	99	98	96	94
0.2	0.0	97	97	97	97	97	96	96	94
	0.1	98	99	99	99	99	98	96	94
	0.2	99	99	99	99	99	98	96	94
	0.3	99	100	100	99	99	98	97	95
0.3	0.0	97	97	97	97	97	96	96	95
	0.1	99	99	99	99	99	98	97	94
	0.2	99	99	99	99	99	98	97	96
	0.3	100	100	100	99	99	98	98	97

Table 3: Coverage (1 - size) of 95% GMS joint test for α_0 and α_1: $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	95	95	95	96	96	96	95	95
	0.1	96	97	97	97	97	97	95	94
	0.2	96	97	98	98	97	97	96	95
	0.3	97	97	97	98	97	97	96	95
0.1	0.0	96	97	97	97	97	97	95	93
	0.1	97	98	99	99	99	98	96	92
	0.2	98	99	99	99	99	98	96	93
	0.3	99	99	99	99	99	98	96	94
0.2	0.0	97	97	97	97	97	96	96	94
	0.1	98	99	99	99	99	98	96	94
	0.2	99	99	99	99	99	98	96	94
	0.3	99	100	100	99	99	98	97	95
0.3	0.0	97	97	97	97	97	96	96	95
	0.1	99	99	99	99	99	98	97	94
	0.2	99	99	99	99	99	98	97	96
	0.3	100	100	100	99	99	98	98	97

Table 4: Coverage (1 - size) of 95% GMS joint test for α_0 and α_1: $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	97.7	97.7	97.6	97.7	98.0	98.0	97.4	97.9
	0.1	98.0	98.7	98.8	99.1	98.8	98.4	97.1	96.4
	0.2	98.4	98.5	98.9	98.9	98.8	98.6	98.0	97.0
	0.3	98.5	98.8	98.8	99.0	98.7	98.4	97.8	97.5
0.1	0.0	98.1	98.5	98.3	98.8	98.8	98.4	96.8	95.7
	0.1	98.6	99.1	99.5	99.6	99.6	98.8	97.7	95.2
	0.2	99.0	99.3	99.7	99.8	99.7	98.9	97.5	95.7
	0.3	99.4	99.7	99.8	99.8	99.6	99.0	98.2	96.7
0.2	0.0	98.6	98.5	98.6	98.9	98.7	98.2	97.7	97.0
	0.1	99.0	99.5	99.7	99.7	99.4	99.0	98.1	96.5
	0.2	99.5	99.7	99.8	99.7	99.4	99.0	97.8	96.8
	0.3	99.7	99.8	99.8	99.8	99.5	99.0	98.7	97.7
0.3	0.0	98.7	98.7	98.8	98.7	98.7	98.2	98.1	97.6
	0.1	99.4	99.6	99.6	99.7	99.4	98.9	98.3	96.8
	0.2	99.8	99.8	99.7	99.8	99.5	99.1	98.5	97.8
	0.3	100.0	99.9	99.9	99.8	99.6	99.5	99.1	98.8

Table 5: Coverage (1 - size) of 97.5% GMS joint test for α_0 and α_1: $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	97.7	97.7	97.6	97.7	98.0	98.0	97.4	97.9
	0.1	98.0	98.7	98.8	99.1	98.8	98.4	97.1	96.4
	0.2	98.4	98.5	98.9	98.9	98.8	98.6	98.0	97.0
	0.3	98.5	98.8	98.8	99.0	98.7	98.4	97.8	97.5
0.1	0.0	98.1	98.5	98.3	98.8	98.8	98.4	96.8	95.7
	0.1	98.6	99.1	99.5	99.6	99.6	98.8	97.7	95.2
	0.2	99.0	99.3	99.7	99.8	99.7	98.9	97.5	95.7
	0.3	99.4	99.7	99.8	99.8	99.6	99.0	98.2	96.7
0.2	0.0	98.6	98.5	98.6	98.9	98.7	98.2	97.7	97.0
	0.1	99.0	99.5	99.7	99.7	99.4	99.0	98.1	96.5
	0.2	99.5	99.7	99.8	99.7	99.4	99.0	97.8	96.8
	0.3	99.7	99.8	99.8	99.8	99.5	99.0	98.7	97.7
0.3	0.0	98.7	98.7	98.8	98.7	98.7	98.2	98.1	97.6
	0.1	99.4	99.6	99.6	99.7	99.4	98.9	98.3	96.8
	0.2	99.8	99.8	99.7	99.8	99.5	99.1	98.5	97.8
	0.3	100.0	99.9	99.9	99.8	99.6	99.5	99.1	98.8

Table 6: Coverage (1 - size) of 97.5% GMS joint test for α_0 and α_1: $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	27	33	30	14	1	0	0	0
	0.1	27	32	29	13	2	0	0	0
	0.2	26	33	32	15	4	0	0	0
	0.3	26	34	30	17	5	0	0	0
0.1	0.0	26	32	31	14	2	0	0	0
	0.1	26	36	32	16	4	0	0	0
	0.2	27	35	31	18	8	0	0	0
	0.3	25	35	32	21	11	1	0	0
0.2	0.0	26	33	30	15	3	0	0	0
	0.1	26	33	30	19	6	0	0	0
	0.2	26	35	33	22	12	1	0	0
	0.3	26	35	33	26	15	3	0	0
0.3	0.0	26	32	32	16	6	0	0	0
	0.1	24	35	33	21	11	1	0	0
	0.2	26	32	35	27	15	4	0	0
	0.3	26	35	35	28	21	7	2	0

Table 7: Percentage of simulation replications for which the standard GMM confidence interval fails to exist, either becuase the point estimate is NaN or the asymptotic covariance matrix is numerically singular ($n = 1000$). Based on 2000 simulation replications.

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	25	36	29	7	0	0	0	0
	0.1	28	36	29	7	0	0	0	0
	0.2	28	37	28	10	1	0	0	0
	0.3	27	36	28	12	2	0	0	0
0.1	0.0	27	36	27	10	0	0	0	0
	0.1	26	36	29	9	1	0	0	0
	0.2	28	38	29	13	2	0	0	0
	0.3	24	36	31	15	5	0	0	0
0.2	0.0	26	36	30	9	1	0	0	0
	0.1	25	37	29	12	2	0	0	0
	0.2	27	38	32	17	4	0	0	0
	0.3	25	39	34	20	9	1	0	0
0.3	0.0	26	37	30	10	2	0	0	0
	0.1	25	38	31	16	4	0	0	0
	0.2	27	38	34	19	9	0	0	0
	0.3	27	36	36	23	13	2	0	0

Table 8: Percentage of simulation replications for which the standard GMM confidence interval fails to exist, either becuase the point estimate is NaN or the asymptotic covariance matrix is numerically singular ($n = 2000$)

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	100	95	92	93	94	95	94	95
	0.1	100	94	91	93	94	95	96	95
	0.2	99	94	92	92	94	96	96	96
	0.3	99	94	92	92	94	96	96	95
0.1	0.0	99	95	92	92	94	95	96	96
	0.1	100	94	91	93	94	95	95	94
	0.2	100	93	91	93	94	95	95	94
	0.3	99	93	91	91	93	95	96	96
0.2	0.0	99	94	90	92	94	95	96	96
	0.1	100	93	91	92	93	95	96	94
	0.2	100	93	90	92	92	95	95	95
	0.3	100	93	90	91	93	95	95	96
0.3	0.0	100	94	91	92	95	95	96	96
	0.1	99	94	91	92	93	94	96	95
	0.2	99	93	91	92	93	94	96	96
	0.3	99	93	90	92	92	94	95	96

Table 9: Coverage of nominal 95% GMM Intervals with $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	100	90	91	94	95	94	96	95
	0.1	100	90	91	93	95	95	95	96
	0.2	100	90	92	94	95	95	95	94
	0.3	100	90	92	93	95	95	95	94
0.1	0.0	100	90	92	92	94	95	94	96
	0.1	100	91	92	93	94	95	95	95
	0.2	100	90	92	92	95	96	95	95
	0.3	99	89	90	92	95	95	95	95
0.2	0.0	100	90	91	93	94	96	95	94
	0.1	99	91	92	93	94	96	95	96
	0.2	100	90	92	92	95	96	96	95
	0.3	99	90	91	93	95	95	96	96
0.3	0.0	100	90	91	93	94	97	95	95
	0.1	100	90	91	94	94	95	96	96
	0.2	99	90	91	92	94	96	96	96
	0.3	100	89	90	92	94	95	96	96

Table 10: Coverage of nominal 95% GMM Intervals with $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	83.99	7.45	3.01	1.52	0.88	0.47	0.37	0.35
	0.1	85.51	7.2	3.01	1.62	1.01	0.61	0.51	0.46
	0.2	74.92	7.79	3.21	1.72	1.13	0.76	0.65	0.58
	0.3	76.66	8.02	3.19	1.78	1.29	0.91	0.79	0.7
0.1	0.0	76.59	7.46	3.2	1.59	0.99	0.61	0.51	0.46
	0.1	78.46	8.07	3.21	1.72	1.17	0.78	0.67	0.6
	0.2	77.79	7.9	3.26	1.95	1.33	0.97	0.85	0.75
	0.3	65.63	8.2	3.5	2.13	1.59	1.18	1.04	0.92
0.2	0.0	69.39	7.52	3.26	1.7	1.14	0.75	0.65	0.58
	0.1	81.48	7.79	3.27	1.95	1.34	0.97	0.84	0.75
	0.2	79.96	7.94	3.58	2.16	1.64	1.21	1.06	0.95
	0.3	85.95	8.14	3.7	2.54	1.96	1.53	1.33	1.19
0.3	0.0	87.95	7.44	3.17	1.84	1.31	0.9	0.79	0.7
	0.1	72.15	8.01	3.45	2.15	1.61	1.18	1.04	0.92
	0.2	67.84	7.6	3.75	2.63	2	1.55	1.35	1.19
	0.3	84.13	8.47	4.23	3.07	2.55	1.98	1.77	1.55

Table 11: Median Width of nominal 95% GMM Intervals with $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	74.47	5.17	2.16	1.06	0.62	0.33	0.27	0.24
	0.1	86.02	5.07	2.25	1.13	0.7	0.43	0.36	0.33
	0.2	87.06	5.28	2.3	1.24	0.81	0.53	0.46	0.41
	0.3	83.3	5.08	2.33	1.34	0.92	0.65	0.56	0.5
0.1	0.0	71.27	5.38	2.24	1.14	0.71	0.43	0.36	0.33
	0.1	64.58	4.95	2.41	1.25	0.83	0.56	0.48	0.43
	0.2	80.03	5.31	2.39	1.38	0.98	0.69	0.6	0.53
	0.3	70.37	4.78	2.54	1.55	1.14	0.84	0.73	0.65
0.2	0.0	65.65	4.99	2.44	1.23	0.81	0.54	0.46	0.41
	0.1	71.25	5.31	2.49	1.35	0.98	0.69	0.6	0.54
	0.2	83.96	5.57	2.63	1.61	1.17	0.86	0.76	0.67
	0.3	75.88	5.83	2.88	1.85	1.44	1.09	0.95	0.85
0.3	0.0	74.62	5.23	2.41	1.32	0.92	0.65	0.56	0.5
	0.1	76.36	5.69	2.54	1.57	1.15	0.84	0.74	0.65
	0.2	91.87	5.44	2.96	1.82	1.42	1.08	0.96	0.85
	0.3	73.3	5.17	3.16	2.24	1.85	1.43	1.24	1.1

Table 12: Median Width of nominal 95% GMM Intervals with $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	96	97	97	96	97	97	95	96
	0.1	97	99	99	99	99	100	100	99
	0.2	98	99	99	100	100	100	100	100
	0.3	97	100	100	100	100	100	100	100
0.1	0.0	97	99	99	99	100	100	100	98
	0.1	98	100	100	100	100	100	100	100
	0.2	98	100	100	100	100	100	100	100
	0.3	97	100	100	100	100	100	100	100
0.2	0.0	97	99	99	100	100	100	100	100
	0.1	98	100	100	100	100	100	100	100
	0.2	98	100	100	100	100	100	100	100
	0.3	98	100	100	100	100	100	100	100
0.3	0.0	97	99	100	100	100	100	100	100
	0.1	97	100	100	100	100	100	100	100
	0.2	98	100	100	100	100	100	100	100
	0.3	98	100	100	100	100	100	100	100

Table 13: Coverage of nominal $> 95\%$ Bonferroni Intervals with $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	96	97	96	97	96	96	95	95
	0.1	97	98	99	100	100	100	100	99
	0.2	97	99	99	100	100	100	100	100
	0.3	97	99	100	100	100	100	100	100
0.1	0.0	97	99	99	99	100	100	100	99
	0.1	98	100	100	100	100	100	100	100
	0.2	98	100	100	100	100	100	100	100
	0.3	98	100	100	100	100	100	100	100
0.2	0.0	97	99	99	100	100	100	100	99
	0.1	98	100	100	100	100	100	100	100
	0.2	98	100	100	100	100	100	100	100
	0.3	98	100	100	100	100	100	100	100
0.3	0.0	97	100	100	100	100	100	100	100
	0.1	97	100	100	100	100	100	100	100
	0.2	97	100	100	100	100	100	100	100
	0.3	97	100	100	100	100	100	100	100

Table 14: Coverage of nominal $> 95\%$ Bonferroni Intervals with $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	0.4	0.41	0.43	0.43	0.43	0.42	0.41	0.41
	0.1	0.45	0.47	0.54	0.59	0.63	0.7	0.75	0.86
	0.2	0.51	0.54	0.65	0.76	0.85	0.95	1.01	1.17
	0.3	0.58	0.62	0.79	0.95	1.07	1.17	1.24	1.48
0.1	0.0	0.45	0.47	0.54	0.59	0.63	0.7	0.76	0.88
	0.1	0.51	0.54	0.66	0.77	0.86	1.03	1.18	1.46
	0.2	0.58	0.63	0.8	0.98	1.12	1.38	1.55	1.88
	0.3	0.67	0.75	1	1.25	1.46	1.74	1.94	2.4
0.2	0.0	0.51	0.54	0.65	0.76	0.86	0.96	1.02	1.19
	0.1	0.58	0.63	0.81	0.99	1.14	1.42	1.64	2.08
	0.2	0.67	0.75	1.01	1.29	1.54	1.97	2.33	2.9
	0.3	0.81	0.91	1.3	1.7	2.09	2.73	3.13	3.9
0.3	0.0	0.58	0.62	0.8	0.95	1.09	1.18	1.25	1.5
	0.1	0.68	0.74	1.01	1.26	1.49	1.84	2.13	2.78
	0.2	0.81	0.91	1.3	1.7	2.11	2.8	3.4	4.48
	0.3	1.01	1.16	1.74	2.35	2.93	4.17	5.2	6.85

Table 15: Median Width of nominal $> 95\%$ Bonferroni Intervals with $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	0.29	0.3	0.31	0.31	0.31	0.3	0.29	0.29
	0.1	0.32	0.35	0.4	0.44	0.48	0.53	0.55	0.61
	0.2	0.36	0.41	0.51	0.59	0.65	0.67	0.69	0.81
	0.3	0.41	0.48	0.64	0.76	0.81	0.8	0.85	1.01
0.1	0.0	0.32	0.35	0.4	0.44	0.48	0.53	0.56	0.62
	0.1	0.36	0.41	0.51	0.6	0.69	0.82	0.88	1.02
	0.2	0.41	0.48	0.64	0.79	0.91	1.04	1.08	1.27
	0.3	0.48	0.59	0.82	1.02	1.16	1.25	1.33	1.61
0.2	0.0	0.36	0.41	0.51	0.59	0.65	0.67	0.7	0.82
	0.1	0.41	0.48	0.65	0.79	0.92	1.09	1.21	1.52
	0.2	0.48	0.59	0.83	1.05	1.24	1.49	1.61	1.96
	0.3	0.57	0.73	1.09	1.43	1.69	1.9	2.08	2.6
0.3	0.0	0.41	0.48	0.64	0.77	0.82	0.78	0.84	1.02
	0.1	0.48	0.59	0.83	1.03	1.18	1.36	1.57	2.06
	0.2	0.57	0.73	1.1	1.43	1.71	2.11	2.45	3.18
	0.3	0.72	0.95	1.5	2.03	2.53	3.15	3.56	4.56

Table 16: Median Width of nominal $> 95\%$ Bonferroni Intervals with $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	96	97	97	96	97	97	95	93
	0.1	97	99	99	99	99	98	96	95
	0.2	98	99	99	100	100	97	96	96
	0.3	97	100	100	100	99	96	96	96
0.1	0.0	97	99	99	99	100	98	97	95
	0.1	98	100	100	100	100	96	96	96
	0.2	98	100	100	100	99	96	96	95
	0.3	97	100	100	100	97	95	96	96
0.2	0.0	97	99	99	100	100	96	96	96
	0.1	98	100	100	100	99	96	96	96
	0.2	98	100	100	100	96	95	95	96
	0.3	98	100	100	98	95	95	95	96
0.3	0.0	97	99	100	100	100	95	96	97
	0.1	97	100	100	100	97	94	96	96
	0.2	98	100	100	98	94	94	96	96
	0.3	98	100	99	96	92	94	95	96

Table 17: Coverage of two-step CI constructed from nominal 95% GMM and nominal > 95% Bonferroni intervals: $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	96	97	96	97	96	96	95	93
	0.1	97	98	99	100	100	98	97	96
	0.2	97	99	99	100	100	97	96	95
	0.3	97	99	100	100	99	96	96	96
0.1	0.0	97	99	99	99	100	98	96	95
	0.1	98	100	100	100	100	96	96	97
	0.2	98	100	100	100	99	96	96	97
	0.3	98	100	100	99	97	95	96	96
0.2	0.0	97	99	99	100	100	97	96	95
	0.1	98	100	100	100	98	96	96	97
	0.2	98	100	100	100	96	96	96	96
	0.3	98	100	100	97	95	95	96	96
0.3	0.0	97	100	100	100	99	98	97	96
	0.1	97	100	100	100	96	95	96	97
	0.2	97	100	100	97	94	96	96	97
	0.3	97	100	100	94	94	95	96	96

Table 18: Coverage of two-step CI constructed from nominal 95% GMM and nominal > 95% Bonferroni intervals: $n = 2000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	0.4	0.41	0.43	0.43	0.43	0.42	0.4	0.35
	0.1	0.45	0.47	0.54	0.59	0.63	0.67	0.52	0.46
	0.2	0.51	0.54	0.65	0.76	0.84	0.82	0.65	0.58
	0.3	0.58	0.62	0.79	0.95	1.05	0.96	0.79	0.7
0.1	0.0	0.45	0.47	0.54	0.59	0.63	0.67	0.51	0.46
	0.1	0.51	0.54	0.66	0.77	0.86	0.92	0.69	0.61
	0.2	0.58	0.63	0.8	0.97	1.11	1.17	0.87	0.75
	0.3	0.67	0.75	1	1.25	1.4	1.4	1.06	0.92
0.2	0.0	0.51	0.54	0.65	0.76	0.85	0.83	0.65	0.58
	0.1	0.58	0.63	0.81	0.99	1.12	1.18	0.86	0.75
	0.2	0.67	0.75	1.01	1.29	1.48	1.56	1.08	0.95
	0.3	0.81	0.91	1.3	1.67	1.95	1.77	1.35	1.2
0.3	0.0	0.58	0.62	0.8	0.95	1.07	0.95	0.8	0.7
	0.1	0.68	0.74	1.01	1.26	1.43	1.48	1.06	0.93
	0.2	0.81	0.91	1.3	1.66	1.98	1.94	1.37	1.19
	0.3	1.01	1.16	1.73	2.24	2.71	2.33	1.78	1.55

Table 19: Median width of two-step CI constructed from nominal 95% GMM and nominal $> 95\%$ Bonferroni intervals: $n = 1000$

		β							
α_0	α_1	0	0.25	0.5	0.75	1	1.5	2	3
0.0	0.0	0.29	0.3	0.31	0.31	0.31	0.3	0.29	0.25
	0.1	0.32	0.35	0.4	0.44	0.48	0.48	0.36	0.33
	0.2	0.36	0.41	0.51	0.59	0.65	0.57	0.46	0.41
	0.3	0.41	0.48	0.64	0.76	0.79	0.68	0.56	0.5
0.1	0.0	0.32	0.35	0.4	0.44	0.48	0.48	0.37	0.33
	0.1	0.36	0.41	0.51	0.6	0.68	0.65	0.48	0.43
	0.2	0.41	0.48	0.64	0.78	0.89	0.83	0.61	0.54
	0.3	0.48	0.59	0.82	1.02	1.09	0.98	0.75	0.65
0.2	0.0	0.36	0.41	0.51	0.59	0.65	0.58	0.46	0.41
	0.1	0.41	0.48	0.65	0.79	0.9	0.89	0.61	0.54
	0.2	0.48	0.59	0.83	1.05	1.2	1.22	0.77	0.67
	0.3	0.57	0.73	1.09	1.4	1.58	1.53	0.97	0.85
0.3	0.0	0.41	0.48	0.64	0.77	0.8	0.69	0.56	0.5
	0.1	0.48	0.59	0.83	1.02	1.13	1.19	0.75	0.65
	0.2	0.57	0.73	1.1	1.4	1.62	1.79	0.97	0.85
	0.3	0.72	0.95	1.49	1.93	2.36	1.58	1.25	1.1

Table 20: Median width of two-step CI constructed from nominal 95% GMM and nominal $> 95\%$ Bonferroni intervals: $n = 2000$

3 Conclusion

A Mahajan's Approach

Here we show that Mahajan's proof of identification for an endogenous treatment is incorrect. The problem is subtle so we give his argument in full detail. We continue to supress dependence on the exogenous covariates $\mathbf{x}$.

The first step of Mahajan's argument is to show that if one could recover the conditional mean function of y given T^*, then a valid and relevant binary instrument would suffice to identify the treatment effect.

Assumption A.1 (Mahajan A2)**.** *Suppose that* $y = c + \beta T^* + \varepsilon$ *where*

(i) $\mathbb{E}[\varepsilon|z] = 0$

(ii) $\mathbb{P}(T^* = 1|z_k) \neq \mathbb{P}(T^* = 1|z_\ell)$ *for all* $k \neq \ell$

(iii) $\mathbb{P}(T = 1|T^* = 0, z) = \alpha_0$, $\mathbb{P}(T = 0|T^* = 1, z) = \alpha_1$

(iv) $\alpha_0 + \alpha_1 \neq 1$

(v) $\beta \neq 0$

Lemma A.1 (Mahajan A2)**.** *Under Assumption ???, knowledge of the mis-classification error rates* α_0, α_1 *suffices to identify* β*.*

In his Theorem 1, Mahajan (2006) proves that α_0, α_1 can in fact be identified under the following assumptions.[5]

Assumption A.2 (Mahajan A1)**.** *Define* $\nu = y - \mathbb{E}[y|T^*]$ *so that by construction we have* $\mathbb{E}[\nu|T^*] = 0$*. Assume that*

(i) $\mathbb{E}[\nu|T^*, T, z] = 0$.[6]

(ii) $\mathbb{P}(T^* = 1|z_k) \neq \mathbb{P}(T^* = 1|z_\ell)$ *for all* $k \neq \ell$

(iii) $\mathbb{P}(T = 1|T^* = 0, z) = \alpha_0$, $\mathbb{P}(T = 0|T^* = 1, z) = \alpha_1$

(iv) $\alpha_0 + \alpha_1 < 1$

(v) $\mathbb{E}[y|T^* = 0] \neq \mathbb{E}[y|T^* = 1]$

[5] Technically, one additional assumption is required, namely that the conditional mean of y given T^* and any covariates would be identified if T^* were observed.

[6] This is Mahajan's Equation (I).

Lemma A.2 (Mahajan Theorem 1)**.** *Under Assumptions ???, the error rates* α_0, α_1 *are identified as is the conditional mean function* $\mathbb{E}[y|T^*]$*.*

Notice that the identification of the error rates in Lemma ??? does not depend on the interpretation of the conditional mean function $\mathbb{E}[y|T^*]$. If T^* is an exogenous treatment, the conditional mean coincides with the treatment effect; if it is endogenous, this is not the case. Either way, the meaning of α_0, α_1 is unchanged: these parameters simply characterize the mis-classification process. Based on this observation, Mahajan (2006) claims that he can rely on Lemma ??? to identify α_0, α_1 and thus the causal effect β when the treatment is endogenous via Lemma ???. To do this, he must build a bridge between Assumption ??? and Assumption ??? that allows T^* to be endogenous. Mahajan (2006) does this by imposing one additional assumption: Equation 11 in his paper.

Assumption A.3 (Mahajan Equation 11)**.** *Let* $y = c + \beta T^* + \varepsilon$ *where* $\mathbb{E}[\varepsilon|T^*]$ *may not be zero and suppose that*

$$\mathbb{E}[\varepsilon|T^*, T, z] = \mathbb{E}[\varepsilon|T^*].$$

Lemma A.3. *Suppose that* $y = c + \beta T^* + \varepsilon$ *where* $E[\varepsilon|z] = 0$ *and define the unobserved projection error* $\nu = y - \mathbb{E}[y|T^*]$*. Then Assumption ??? implies that* $E[\nu|T^*, T, z] = 0$*, which is Assumption ???.*

To summarize, Mahajan's claim is equivalent to the proposition that under Assumptions ??? β is identified even if T^* is endogenous. although Lemmas ??? are all correct, Mahajan's claim is not.[7] While Assumption ??? does guarantee that Assumption ??? holds, when combined with Assumption ??? it also implies that ??? fails if T^* is endogenous. The failure of Assumption ??? in turn leads to a division by zero in the solution to the linear system following Mahajan's displayed Equation 26: the system no longer has a unique solution so identification fails.[8]

Proposition A.1 (Lack of a First Stage)**.** *Suppose that Assumptions ??? hold and* $\mathbb{E}[\varepsilon|T^*] \neq 0$*. Then* $\mathbb{P}(T^* = 1|z_1) = \mathbb{P}(T^* = 1|z_2)$*, violating Assumption ???.*

[7] Our Lemma ??? does not in fact appear in Mahajan (2006), but it is an implicit step in his proof in Appendix A2.

[8] Notice that the root of the problem is the attempt to use *one* instrument to solve both the measurement error and endogeneity problems. In a setting where one had a second mis-measured surrogate for T^* *in addition* to an instrument that is conditionally mean independent of ε one could use the second surrogate as an instrument for the first to estimate α_0 and α_1 via Lemma ??? and then use the additional instrumental variable to estimate $\beta/(1 - \alpha_0 - \alpha_1)$ via the familiar Wald IV estimator. This is effectively the approach used by Battistin et al. (2014) to evaluate the returns to schooling in a setting with multiple misreported measures of educational qualifications.

To understand the economic intuition behind Proposition ???, consider a simple example in which we randomize the offer of a job training program to a sample of workers to study the impact on future earnings. In this context z indicates whether a particular individual is *offered* job training by the experimenter while T^* indicates whether she actually *obtains* job training from any source, inside or outside of the experiment. We observe not T^* but a self-report T that is measured with error. In this example u contains all of the unobservable factors that determine an individual's wage.

Assumption ??? allows for endogenous treatment receipt: $\mathbb{E}[u|T^* = 1]$ may be different from $\mathbb{E}[u|T^* = 0]$. We might expect, for example, that individuals who obtain job training are more motivated than those who do not, and hence earn higher wages on average. However, Assumption ??? imposes that $\mathbb{E}[u|T^* = t, z_1] = \mathbb{E}[u|T^* = t, z_2]$ for $t = 0, 1$. This has two implications. First, it means that, among those who do not obtain job training, the average value of u is the same for those who were offered training and those who were not. Second, it means that, among those who *did* obtain job training, the average value of u is the same for those who were offered training and those who were not. In other words, Assumption ??? requires that there is *no selection on unobservables*. This is exactly the opposite of what we would expect in the job training setting. For example, individuals who are offered job training but refuse it, are likely to be very different from those who are not offered training and fail to obtain it from an outside source. And herein lies the problem: Assumption ??? simultaneously allows endogeneity and rules out selection. Given that the offer of job training is randomly assigned, and hence a valid instrument, the only way to avoid a contradiction is if there is no first stage: the fraction of individuals who take up job training cannot depend on the offer of training.

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