

# Estimating the Effect of a Mis-measured, Endogenous, Binary Treatment

Francis J. DiTraglia  
Camilo Garcia-Jimeno

University of Pennsylvania

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## What is the causal effect of $T^*$ ?

$$y_i = h(T_i^*, \mathbf{x}_i) + \varepsilon_i$$

- ▶  $y$  – Outcome of interest
- ▶  $h$  – Unknown function that *does not depend on*  $i$
- ▶  $T^*$  – Unobserved, endogenous binary treatment
- ▶  $T$  – Observed, mis-measured binary surrogate for  $T^*$
- ▶  $\mathbf{x}$  – Exogenous covariates
- ▶  $\varepsilon$  – Mean-zero error term
- ▶  $z$  – Discrete instrumental variable

# Example 1: Smoking and Birthweight (SNAP Trial)

Coleman et al. (N Engl J Med, 2012)

RCT with 1050 pregnant smokers in England: 521 given nicotine patches, the rest given placebo patches.

- ▶  $y$  – Birthweight
- ▶  $T^*$  – True smoking behavior
- ▶  $T$  – Self-reported smoking behavior
- ▶  $\mathbf{x}$  – Mother characteristics
- ▶  $z$  – Indicator of nicotine patch

## Example 2: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters. Randomly choose 6 and build a school in each village of these clusters.

- ▶  $y$  – Child's score on math and language test
- ▶  $T^*$  – Child's true school attendance
- ▶  $T$  – Parent's report of child's school attendance
- ▶  $\mathbf{x}$  – Child and household characteristics
- ▶  $z$  – School built in village

## Example 3: Job Training Partnership Act (JPTA)

Heckman et al. (2000, QJE)

Randomized offer of job training, but about 30% of those *not* offered also obtain training and about 40% of those offered training don't attend. Estimate causal effect of *training* rather than *offer* of training.

- ▶  $y$  – Log wage
- ▶  $T^*$  – True training attendance
- ▶  $T$  – Self-reported training attendance
- ▶  $\mathbf{x}$  – Individual characteristics
- ▶  $z$  – Offer of job training

# Related Literature

## Continuous Treatment

Lewbel (1997, 2012), Schennach (2004, 2007), Chen et al. (2005), Hu & Schennach (2008)...

## Binary, Exogenous Treatment

Aigner (1973), Bollinger (1996), Kane et al. (1999), Black et al. (2000), Frazis & Loewenstein (2003), Mahajan (2006), Lewbel (2007)

## Binary, Endogenous Treatment

Only existing result in Mahajan (2006)

Model:  $y = h(T^*, \mathbf{x}) + \varepsilon$

## ATE Function

$$\tau(\mathbf{x}) = h(1, \mathbf{x}) - h(0, \mathbf{x})$$

## First-stage

$$p_k^*(\mathbf{x}) \equiv \mathbb{P}(T^* = 1 | z = z_k, \mathbf{x}) \neq \mathbb{P}(T^* = 1 | z = z_\ell) \equiv p_\ell^*(\mathbf{x}), \quad k \neq \ell$$

## Measurement Error

Non-differential,  $\mathbb{E}[\varepsilon | T^*, T, z, \mathbf{x}] = \mathbb{E}[\varepsilon | T^*, z, \mathbf{x}]$ , and does not depend on  $z$ :

$$\alpha_0(\mathbf{x}) = \mathbb{P}(T = 1 | T^* = 0, z, \mathbf{x})$$

$$\alpha_1(\mathbf{x}) = \mathbb{P}(T = 0 | T^* = 1, z, \mathbf{x})$$

## Notation

- Treat exog. covariates  $\mathbf{x}$  non-parametrically: hold fixed at  $\mathbf{x}_a$  throughout:

$$y = \beta T^* + u$$

$$u = \varepsilon + c$$

where  $\beta = \tau(\mathbf{x}_a)$  and  $c = h(0, \mathbf{x}_a)$ .

- Similarly:

$$\alpha_0 = \mathbb{P}(T = 1 | T^* = 0)$$

$$\alpha_1 = \mathbb{P}(T = 0 | T^* = 1)$$

$$p_k^* = \mathbb{P}(T^* = 1 | z = z_k)$$



Observable Moments:  $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T = 0$	$\bar{y}_{01}$ $p_{01}$	$\bar{y}_{02}$ $p_{02}$	$\dots$	$\bar{y}_{0K}$ $p_{0K}$
$T = 1$	$\bar{y}_{11}$ $p_{11}$	$\bar{y}_{12}$ $p_{12}$	$\dots$	$\bar{y}_{1K}$ $p_{1K}$

$$\bar{y}_{tk} = \mathbb{E}[y | T = t, z = z_k], \quad p_{tk} = q_k p_k$$

$$q_k = \mathbb{P}(z = z_k), \quad p_k = \mathbb{P}(T = 1 | z = z_k)$$

# Unobservable Moments: $y = \beta T^* + u$

	$z = 1$	$z = 2$	$\dots$	$z = K$
$T^* = 0$	$m_{01}^*$ $p_{01}^*$	$m_{02}^*$ $p_{02}^*$	$\dots$	$m_{0K}^*$ $p_{0K}^*$
$T^* = 1$	$m_{11}^*$ $p_{11}^*$	$m_{12}^*$ $p_{12}^*$	$\dots$	$m_{1K}^*$ $p_{1K}^*$

$$m_{tk}^* = \mathbb{E}[u | T^* = t, z = z_k], \quad p_{tk}^* = q_k p_k^*$$

$$p_k^* = \mathbb{P}(T^* = 1 | z = z_k) = \frac{p_k - \alpha_0}{1 - \alpha_0 - \alpha_1}$$

## Possible Assumptions On $m_{tk}^*$

Joint Exogeneity:  $\mathbb{E}[\varepsilon | T^*, z] = 0$

$$\implies m_{tk}^* = c \quad \text{for all } t, k$$

Exogenous Treatment:  $\mathbb{E}[\varepsilon | T^*] = 0$

$$\implies \frac{1}{\mathbb{P}(T^* = t)} \sum_k p_{tk}^* m_{tk}^* = c \quad \text{for all } t$$

Exogenous Instrument:  $\mathbb{E}[\varepsilon | z] = 0$

$$\implies (1 - p_k^*) m_{0k}^* + p_k^* m_{1k}^* = c \quad \text{for all } k$$

# Mahajan (2006, Econometrica)

## Regression Model

$$y = \mathbb{E}[y|T^*] + \nu$$

$$\mathbb{E}[\nu|T^*] = 0 \text{ by construction}$$

## Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon|T^*] \neq 0$$

## Main Result (Correct) – Exogenous Treatment

Relevant binary instrument  $z$  ( $p_1^* \neq p_2^*$ ) identifies  $\alpha_0, \alpha_1$  and  $\mathbb{E}[y|T^*]$  provided that  $\mathbb{E}[\nu|T^*, T, z] = 0$ .

## Mahajan (2006, Econometrica)

### Regression Model

$$y = \mathbb{E}[y|T^*] + \nu$$

$$\mathbb{E}[\nu|T^*] = 0 \text{ by construction}$$

### Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon|T^*] \neq 0$$

### Additional Result (Incorrect) – Endogenous Treatment

$$\mathbb{E}[\varepsilon|z] = 0, p_1^* \neq p_2^*, \mathbb{E}[\varepsilon|T, T^*, z] = \mathbb{E}[\varepsilon|T^*] \implies \beta \text{ identified.}$$

# Mahajan's Argument

## Regression Model

$$y = \mathbb{E}[y | T^*] + \nu$$

$$\mathbb{E}[\nu | T^*] = 0 \text{ by construction}$$

## Causal Model

$$y = c + \beta T^* + \varepsilon$$

$$\mathbb{E}[\varepsilon | T^*] \neq 0$$

## Ingredients

1. If  $p_1^* \neq p_2^*$ ,  $\mathbb{E}[\varepsilon | z] = 0$  then, since  $\beta_{IV} = \beta / (1 - \alpha_0 - \alpha_1)$ , knowledge of  $\alpha_0, \alpha_1$  is sufficient to recover  $\beta$ . (Correct)
2. If  $p_1^* \neq p_2^*$ ,  $\mathbb{E}[\nu | T^*, T, z] = 0$ ,  $\alpha_0, \alpha_1$  are identified. (Correct)

How to satisfy both 1 and 2 while allowing  $\mathbb{E}[\varepsilon | T^*] \neq 0$ ?

3. Assume that  $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$   
(i.e.  $m_{01}^* = m_{02}^*$  and  $m_{11}^* = m_{12}^*$ )

# Mahajan's Argument

## Proposition

If  $\mathbb{E}[\varepsilon | T^*] \neq 0$  then  $\mathbb{E}[\varepsilon | T^*, T, z] = \mathbb{E}[\varepsilon | T^*]$  combined with  $\mathbb{E}[\varepsilon | z] = 0$  implies  $p_1^* = p_2^*$ , i.e.  $z$  is irrelevant for  $T^*$ .

## Proof

Recall that  $\mathbb{E}[\varepsilon | z] = 0$  implies

$$(1 - p_1^*)m_{01}^* + p_1^*m_{11}^* = c$$

$$(1 - p_2^*)m_{02}^* + p_2^*m_{12}^* = c$$

while Mahajan's assumption implies  $m_{01}^* = m_{02}^*$  and  $m_{11}^* = m_{12}^*$ .

Therefore either  $m_{01}^* = m_{02}^* = m_{11}^* = m_{12}^* = c$ , which is ruled out by  $E[\varepsilon | T^*] = 0$ , or  $p_1^* = p_2^*$ .

What about increasing the support of  $z$ ?

$\mathbb{E}[\varepsilon|z] = 0 \implies$  *pair of equations for each  $k = 1, \dots, K$*

$$\hat{y}_{0k} = \alpha_1(p_k - \alpha_0) \left( \frac{\beta}{1 - \alpha_0 - \alpha_1} \right) + (1 - \alpha_0)c - (p_k - \alpha_0)m_{1k}^*$$

$$\hat{y}_{1k} = (1 - \alpha_1)(p_k - \alpha_0) \left( \frac{\beta}{1 - \alpha_0 - \alpha_1} \right) + \alpha_0 c + (p_k - \alpha_0)m_{1k}^*$$

where  $\hat{y}_{0k} = (1 - p_k)\bar{y}_{0k}$  and  $\hat{y}_{1k} = p_k\bar{y}_{1k}$

**2K Equations in  $K + 4$  Unknowns**



*Theorem:*  $\beta$  is unidentified regardless of  $K$ .

(For general case, see paper.)

Proof of special case:  $\alpha_0 = 0$

1. System of equations simplifies to

$$\hat{y}_{0k} = c + p_k \beta \left( \frac{\alpha_1}{1 - \alpha_1} \right) - p_k m_{1k}^*$$

$$\hat{y}_{1k} = p_k \beta + p_k m_{1k}^*$$

2.  $\beta/(1 - \alpha_1) \equiv \mathcal{W}$  is identified and imposing this, algebra gives  $\beta\alpha_1/(1 - \alpha_1) = \mathcal{W} - \beta$ .

*Theorem:*  $\beta$  is unidentified regardless of  $K$ .

(For general case, see paper.)

Proof of special case:  $\alpha_0 = 0$  continued...

3. Substituting:

$$(c + p_k \mathcal{W} - \hat{y}_{0k})/p_k = \beta + m_{1k}^*$$

$$\hat{y}_{1k}/p_k = \beta + m_{1k}^*$$

4. Linear system in  $(\beta, m_{1k}^*)$  – no solution or  $\infty$  of solutions.

5. Sum original pair of equations  $\implies c + p_k \mathcal{W} - \hat{y}_{0k} = \hat{y}_{1k}$   
thus  $\infty$  of solutions. The model is unidentified.

# Conditional *Second* Moment Independence.

## New Assumption

Homoskedastic errors w.r.t. the *instrument*:  $E[\varepsilon^2|z] = E[\varepsilon^2]$

## Not Crazy!

Holds in an RCT or a *true* natural experiment.

## New Moment Conditions

Defining  $\mu_{k\ell}^* = (p_k - \alpha_0)m_{1k}^* - (p_\ell - \alpha_0)m_{k\ell}^*$ ,

$$\mathbb{E}(y^2|z_k) - \mathbb{E}(y^2|z_\ell) \equiv \Delta \overline{y^2} = \beta \mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu_{k\ell}^*$$

$$\mathbb{E}(yT|z_k) - \mathbb{E}(yT|z_\ell) \equiv \Delta \overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu_{k\ell}^*$$

*Theorem:*  $(\alpha_1 - \alpha_0)$  is Identified.

(Requires only binary  $z$ )

Proof

$$\Delta \overline{y^2} = \beta \mathcal{W}(p_k - p_\ell) + 2\mathcal{W}\mu_{k\ell}^*$$

$$\Delta \overline{yT} = (1 - \alpha_1)\mathcal{W}(p_k - p_\ell) + \mu_{k\ell}^*$$

Solve for  $\mu_{k\ell}^*$ , substitute and rearrange:

$$\mathcal{R} \equiv \beta - 2(1 - \alpha_1)\mathcal{W} = \frac{\Delta \overline{y^2} - 2\mathcal{W}\Delta \overline{yT}}{\mathcal{W}(p_k - p_\ell)}.$$

Rearrange and substitute  $\beta = \mathcal{W}(1 - \alpha_0 - \alpha_1)$  to find

$$\alpha_1 - \alpha_0 = 1 + \mathcal{R}/\mathcal{W}.$$

## What Good is $(\alpha_1 - \alpha_0)$ ?

- ▶ Test a necessary condition for *no mis-classification*:  $\alpha_0 = \alpha_1$
- ▶ Simple, tighter partial identification bounds for  $\beta$
- ▶ In some settings, one of the mis-classification probabilities is known to be zero  $\implies \beta$  point identified

## Conditional *Third* Moment Independence

### New Assumption

Third moment independence w.r.t instrument:  $E[\varepsilon^3|z] = E[\varepsilon^3]$

### New Moment Conditions

Define  $\lambda_{k\ell}^* = (p_k - \alpha_0)v_{1k}^* - (p_\ell - \alpha_0)v_{1\ell}^*$

where  $v_{tk}^* = \mathbb{E}(u^2 | T^* = t, z_k)$ . Then

$$\mathbb{E}(y^3|z_k) - \mathbb{E}(y^3|z_\ell) \equiv$$

$$\Delta \overline{y^3} = \beta^2 \mathcal{W}(p_k - p_\ell) + 3\beta \mathcal{W}\mu_{k\ell}^* + 3\mathcal{W}\lambda_{k\ell}^*$$

$$\mathbb{E}(y^2 T|z_k) - \mathbb{E}(y^2 T|z_\ell) \equiv$$

$$\Delta \overline{y^2 T} = \beta(1 - \alpha_1) \mathcal{W}(p_k - p_\ell) + 2(1 - \alpha_1) \mathcal{W}\mu_{k\ell}^* + \lambda_{k\ell}^*$$

*Theorem:*  $\beta$ ,  $\alpha_0$  and  $\alpha_1$  are identified.

Requires  $\alpha_0 + \alpha_1 < 1$ , but  $z$  need only be binary.

Proof

$$\Delta \overline{y^3} = \beta^2 \mathcal{W}(p_k - p_\ell) + 3\beta \mathcal{W} \mu_{k\ell}^* + 3\mathcal{W} \lambda_{k\ell}^*$$

$$\Delta \overline{y^2 T} = \beta(1 - \alpha_1) \mathcal{W}(p_k - p_\ell) + 2(1 - \alpha_1) \mathcal{W} \mu_{k\ell}^* + \lambda_{k\ell}^*$$

Solve for  $\lambda_{k\ell}^*$ , substitute and rearrange:

$$\mathcal{S} \equiv \beta^2 - 3\mathcal{W}(1 - \alpha_1)(\beta + \mathcal{R}) = \frac{\Delta \overline{y^3} - 3\mathcal{W} [\Delta \overline{y^2 T} + \mathcal{R} \Delta \overline{y T}]}{\mathcal{W}(p_k - p_\ell)}.$$

*Theorem:*  $\beta$ ,  $\alpha_0$  and  $\alpha_1$  are identified.

Requires  $\alpha_0 + \alpha_1 < 1$ , but  $z$  need only be binary.

Proof continued...

$$\mathcal{S} \equiv \beta^2 - 3\mathcal{W}(1 - \alpha_1)(\beta + \mathcal{R}) = \frac{\Delta \bar{y}^3 - 3\mathcal{W} [\Delta \bar{y}^2 \bar{T} + \mathcal{R} \Delta \bar{y} \bar{T}]}{\mathcal{W}(p_k - p_\ell)}$$

Use the fact that  $\mathcal{R} = \beta - 2(1 - \alpha_1)\mathcal{W}$  to eliminate  $\beta$  from  $\mathcal{S}$ :

$$2\mathcal{W}^2(1 - \alpha_1)^2 + 2\mathcal{R}\mathcal{W}(1 - \alpha_1) + (\mathcal{S} - \mathcal{R}^2) = 0$$

which is a quadratic in  $(1 - \alpha_1)$  and observables only! Can show that there are always two real roots: one is  $(1 - \alpha_1)$  and the other is  $\alpha_0$ . To tell which is which, need  $\alpha_0 + \alpha_1 < 1$ .



## Recap of Results

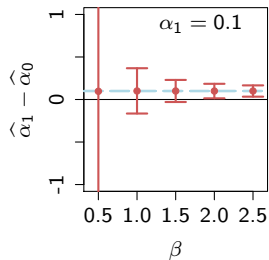
1. Using first-moment information alone,  $\beta$  is unidentified regardless of how many values the instrument takes on.
2. Using second moment information  $\alpha_1 - \alpha_0$  is identified
  - ▶ Partial identification bound for  $\beta$
  - ▶ Identifies  $\beta$  if  $\alpha_0$  is known (e.g. smoking/birthweight example)
3. Using third moment information  $\beta$ ,  $\alpha_0$  and  $\alpha_1$  are identified so long as  $\alpha_0 + \alpha_1 < 1$ .

## Simulation Study: $y = \beta T^* + \varepsilon$

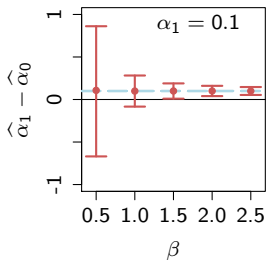
- ▶  $(\varepsilon, \eta) \sim$  jointly normal, mean 0, variance 1, corr. 0.3.
- ▶ First stage:  $T^* = \mathbf{1}\{\gamma_0 + \gamma_1 z + \eta > 0\}$ 
  - ▶ Half of subjects have  $z = 1$ , the rest have  $z = 0$ .
  - ▶  $\gamma_0 = \Phi^{-1}(\delta)$
  - ▶  $\gamma_1 = \Phi^{-1}(1 - \delta) - \Phi(\delta)$
  - ▶  $\delta$  equals fraction of those offered treatment who fail to take it up *and* fraction of those not offered treatment who do.
- ▶ Generate  $T$  as follows:
  - ▶  $T^* = 0 \implies T = 0$ , i.e.  $\alpha_0 = 0$
  - ▶  $T|T^* = 1 \sim \text{Bernoulli}(1 - \alpha_1)$
  - ▶  $\alpha_0, \alpha_1$  unknown to econometrician.

Sampling Distribution of  $\hat{\alpha}_1 - \hat{\alpha}_0$

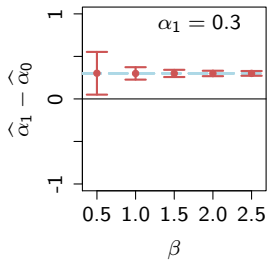
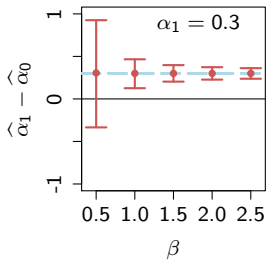
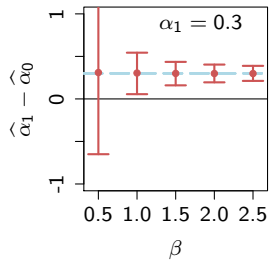
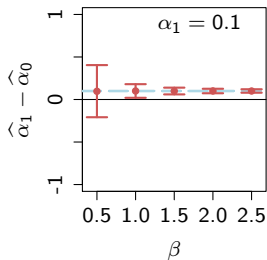
(a)  $N = 500, \delta = 0.1$



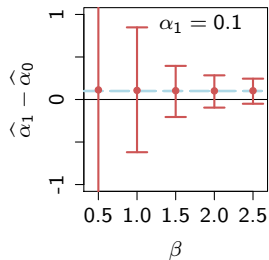
(b)  $N = 1000, \delta = 0.1$



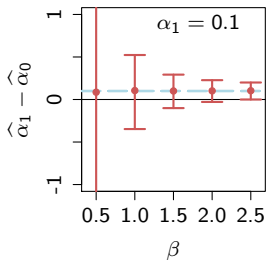
(c)  $N = 5000, \delta = 0.1$



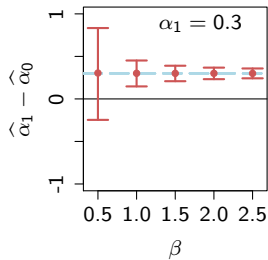
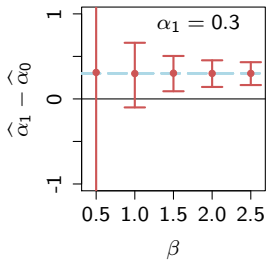
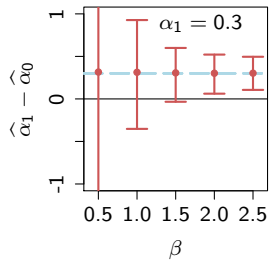
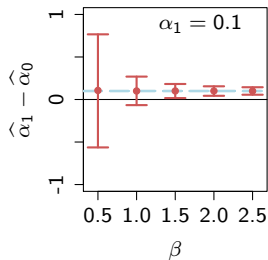
(a)  $N = 500, \delta = 0.3$



(b)  $N = 1000, \delta = 0.3$



(c)  $N = 5000, \delta = 0.3$

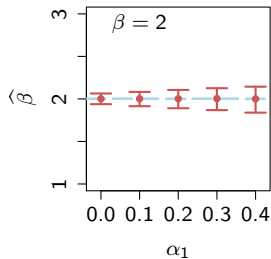
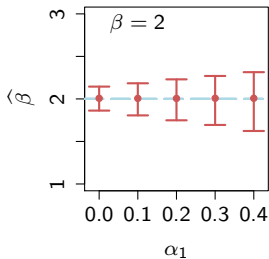
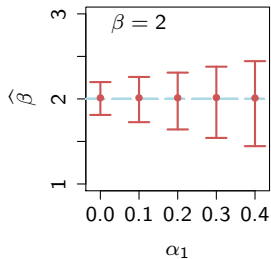
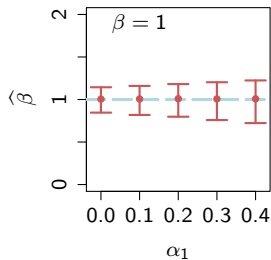
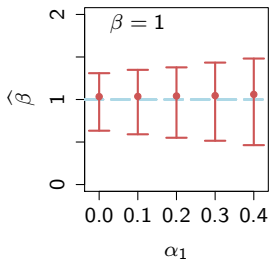
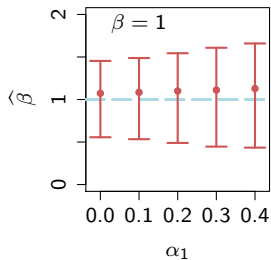


Sampling Distribution of  $\hat{\beta} = (1 - \hat{\alpha}_0 - \hat{\alpha}_1)\hat{\beta}_{IV}$

(a)  $N = 500, \delta = 0.1$

(b)  $N = 1000, \delta = 0.1$

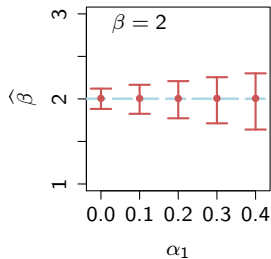
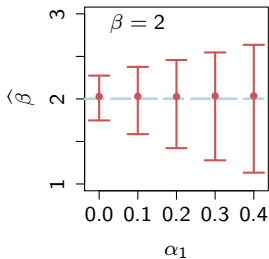
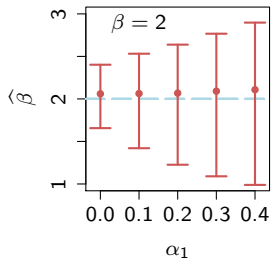
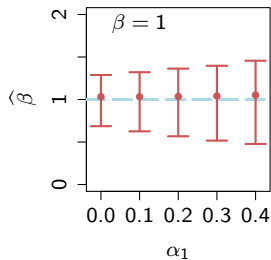
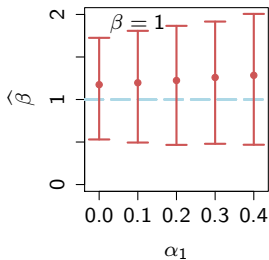
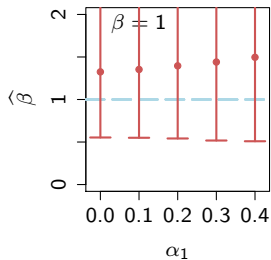
(c)  $N = 5000, \delta = 0.1$



(a)  $N = 500, \delta = 0.3$

(b)  $N = 1000, \delta = 0.3$

(c)  $N = 5000, \delta = 0.3$





# Empirical Illustration: Schooling and Test Scores

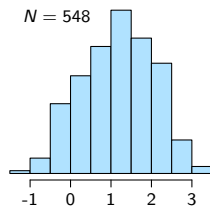
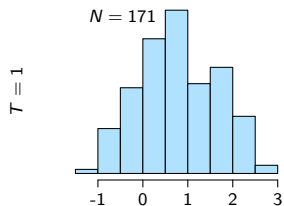
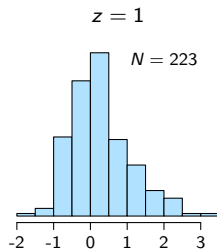
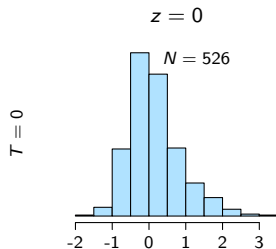
Burde & Linden (2013, AEJ Applied)

RCT in Afghanistan: 32 villages divided into 11 clusters.  
Randomly choose 6 and build a school in each village of these clusters ( $N = 1468$ ).

- ▶  $y$  – Child's score on math and language test
- ▶  $T^*$  – Child's true school attendance
- ▶  $T$  – Parent's report of child's school attendance
- ▶  $x$  – Child and household characteristics
- ▶  $z$  – School built in village

# Empirical Illustration: Schooling and Test Scores

Burde & Linden (2013, AEJ Applied)



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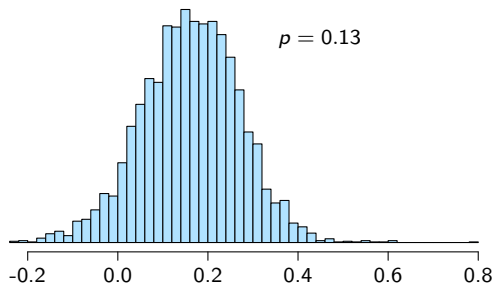
Burde & Linden (2013, AEJ Applied)

## Cluster Bootstrap Distribution of $\hat{\alpha}_1 - \hat{\alpha}_0$

$$\hat{\beta}_{OLS} = 0.88$$

$$\hat{\beta}_{IV} = 1.27$$

$$\hat{\alpha}_1 - \hat{\alpha}_0 = 0.18$$



# Conclusion

- ▶ Study causal effect of an endogenous, mis-measured, binary treatment.
- ▶ Important case in applied work, only existing result is incorrect. Identification requires going beyond first moments.
- ▶ New partial and point identification results by exploiting higher moments of outcome variable.
- ▶ Test necessary condition for absence of measurement error.
- ▶ Explore sampling distribution of our simple closed-form method of moments estimator in a simulation experiment.
- ▶ Detect evidence of measurement error in real-world example.