Marginal Treatment Effects Part II

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Recap of Last Lecture

$$\begin{aligned} Y_0 &= \mu_0 + U_0 & Z \sim \mathsf{Bernoulli}(q) \underline{\!\!\perp\!\!\!\perp} (V, U_0, U_1) \\ Y_1 &= \mu_1 + U_1 & \begin{bmatrix} V \\ U_0 \\ Y &= (1-D)Y_0 + DY_1 \end{bmatrix} \sim \mathsf{Normal} \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \sigma_0 \rho_0 & \sigma_1 \rho_1 \\ \sigma_0^2 & \sigma_{01} \\ & & \sigma_1^2 \end{bmatrix} \end{pmatrix}$$

The Good:

- ▶ Simple model with instrument $Z \in \{0,1\}$ and selection into treatment $D \in \{0,1\}$.
- \triangleright Treatment effects are heterogeneous and vary with "resistance" to treatment V.
- \blacktriangleright μ_0 , μ_1 , $\sigma_0\rho_0$, $\sigma_0\rho_0$, q, γ_0 and γ_1 point identified; Heckman 2-step Estimator.
- Beyond LATE: ATE, TOT, and TUT depend only on point identified parameters. . .

Recap of Last Lecture

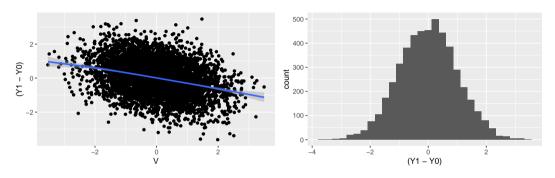
$$\mathsf{ATE} = \mu_1 - \mu_0$$

$$\mathsf{LATE} = \mathsf{ATE} - (\sigma_1 \rho_1 - \sigma_0 \rho_0) \left[\frac{\varphi(\gamma_0 + \gamma_1) - \varphi(\gamma_0)}{\Phi(\gamma_0 + \gamma_1) - \Phi(\gamma_0)} \right] = \frac{\mathbb{E}(Y|Z=1) - \mathbb{E}(Y|Z=0)}{\mathbb{E}(D|Z=1) - \mathbb{E}(D|Z=0)}$$

$$\mathsf{TOT} = \mathsf{ATE} - (\sigma_1 \rho_1 - \sigma_0 \rho_0) \left[\frac{(1-q)\varphi(\gamma_0) + q\varphi(\gamma_0 + \gamma_1)}{(1-q)\Phi(\gamma_0) + q\Phi(\gamma_0 + \gamma_1)} \right]$$

$$\mathsf{TUT} = \mathsf{ATE} + (\sigma_1 \rho_1 - \sigma_0 \rho_0) \left[\frac{(1-q)\varphi(\gamma_0) + q\varphi(\gamma_0 + \gamma_1)}{(1-q)\{1-\Phi(\gamma_0)\} + q\{1-\Phi(\gamma_0 + \gamma_1)\}} \right]$$

The Bad



Under Normality:

- 1. $E(Y_1 Y_0|V)$ is necessarily linear.
- 2. Unbounded ATEs for people with "extreme" values of V.

Relaxing Normality: the Latent Index Selection Model (LISM)

The Good:

- ▶ Simple model with instrument $Z \in \{0,1\}$ and selection into treatment $D \in \{0,1\}$.
- \triangleright Treatment effects are heterogeneous and vary with "resistance" to treatment V.
- ▶ No longer assume that (U_0, U_1, V) are jointly normal; mean zero WLOG.

Questions

- 1. How does this compare to the LATE model?
- 2. Is this model identified? If so can we estimate it?
- 3. If we can estimate it, does it allow us to go beyond late to ATE, TUT, TOT etc?

Assumptions of the Latent Index Selection Model

Treatment Take-up

$$D(Z) = 1\{\gamma_0 + \gamma_1 Z > V\}$$

Instrument Relevance

$$\mathbb{P}(\gamma_0 > V) \neq \mathbb{P}(\gamma_0 + \gamma_1 > V)$$

Instrument Exogeneity

$$Z \perp \!\!\! \perp (V, Y_0, Y_1)$$

- $ightharpoonup \gamma_0 + \gamma_1 Z$ is called the "latent index"
- ▶ We used relevance implicitly in our Heckman Two-step procedure.
- $ightharpoonup Z \perp \!\!\! \perp \!\!\! (V, U_0, U_1), \ Y_0 = \mu_0 + U_0, \ Y_1 = \mu_1 + U_1 \implies \text{exogeneity}$

Potential Treatments

- ▶ We described LATE model using "compliance type" variable $T \in \{n, a, c, d\}$
- ightharpoonup Equivalently, can describe using "potential treatments," a binary encoding: (D_0, D_1)

No Defiers aka Monotonicity

$$\mathbb{P}(T=d)=0 \iff \text{ either } D_0 \leq D_1 \text{ or } D_1 \leq D_0 \text{ with probability one.}$$

Unconfounded Type

$$Z \perp \!\!\! \perp T \iff Z \perp \!\!\! \perp (D_0, D_1)$$

(Slightly) Stronger Version of LATE Assumptions

Existence of Compliers in terms of Observables

$$\mathbb{P}(T=c) > 0 \iff \mathbb{E}[D|Z=1] \neq \mathbb{E}[D|Z=0]$$

No Defiers in terms of Potential Treatments Either $D_0 < D_1$ or $D_1 < D_0$ with probability one.

Replacement for Mean Exclusion

$$Z \perp \!\!\! \perp \!\!\! \perp (Y_0, Y_1, D_0, D_1)$$

- ▶ Equivalent to $Z \perp \!\!\! \perp (Y_0, Y_1, T)$
- ▶ Implies $Z \perp\!\!\!\perp (D_0, D_1)$, which is equivalent to unconfounded type.
- Implies but is slightly stronger than mean exclusion.

These two models are equivalent!

Latent Index Selection Model

- 1. $D = 1\{\gamma_0 + \gamma_1 Z > V\}$
- 2. $\mathbb{P}(\gamma_0 > V) \neq \mathbb{P}(\gamma_0 + \gamma_1 > V)$
- 3. $Z \perp \!\!\! \perp (Y_0, Y_1, V)$

Local Average Treatment Effects Model

- 1. Either $D_0 \leq D_1$ or $D_1 \leq D_0$ wp 1.
- 2. $\mathbb{E}[D|Z=1] \neq \mathbb{E}[D|Z=0]$
- 3. $Z \perp \!\!\! \perp (Y_0, Y_1, D_0, D_1)$

LISM Assumptions \Rightarrow LATE Assumptions

▶ Straightforward. Details follow on the next slide.

LATE Assumptions ⇒ LISM Assumptions

► A bit trickier. See: Glickman & Normand (2000) and Vytacil (2002)

$$Z \perp \!\!\! \perp (Y_0, Y_1, D_0, D_1)$$

- $ightharpoonup D = 1\{\gamma_0 + \gamma_1 > V\} \implies (D_0, D_1)$ are a function of V.
- ▶ In particular: $D_0 \equiv D(Z = 0) = 1\{\gamma_0 > V\}$, $D_1 \equiv D(Z = 1) = 1\{\gamma_0 + \gamma_1 > V\}$
- ▶ The LISM assumes $Z_{\perp \!\!\! \perp}(Y_0, Y_1, V)$, so by Decomposition: $Z_{\perp \!\!\! \perp}(Y_0, Y_1, D_0, D_1)$.

$$\mathbb{P}(D=1|Z=1)\neq \mathbb{P}(D=1|Z=0)$$

- lacktriangle The LISM assumes that $\mathbb{P}(\gamma_0 > V)
 eq \mathbb{P}(\gamma_0 + \gamma_1 > V)$
- $ightharpoonup \mathbb{P}(D=1|Z=0)=\mathbb{P}(\gamma_0>V),\ \mathbb{P}(D=1|Z=1)=\mathbb{P}(\gamma_0+\gamma_1>V)$

Either $D_0 \leq D_1$ or $D_1 \leq D_0$ with probability one.

- $ightharpoonup \mathbb{P}(\gamma_0 > V) \neq \mathbb{P}(\gamma_0 + \gamma_1 > V)$ rules out $\gamma_1 = 1$.

The Generalized Roy Model

Model

$$Y_0 = \mu_0(X) + U_0$$

 $Y_1 = \mu_1(X) + U_1$
 $Y = (1 - D)Y_0 + DY_1$

Assumptions

- 1. $D = 1\{\nu(X, Z) > V\}$
- 2. $Z \perp \!\!\! \perp (Y_0, Y_1, V) | X$
- 3. Distribution of V|X = x is continuous.
- Covariates X: observed heterogeneity; (U_0, U_1, V) : unobserved heterogeneity
- $lackbox{U}_0 \equiv Y_0 \mathbb{E}(Y_0|X); \ U_1 \equiv Y_1 \mathbb{E}(Y_1|X)$ so both are mean zero.
- ightharpoonup Z may not be be binary; unknown function $u(\cdot)$

Monotonicity

Model

$$Y_0 = \mu_0(X) + U_0$$

 $Y_1 = \mu_1(X) + U_1$
 $Y = (1 - D)Y_0 + DY_1$

Assumptions

- 1. $D = 1\{\nu(X, Z) > V\}$
- 2. $Z \perp \!\!\! \perp (Y_0, Y_1, V) | X$
- 3. Distribution of V|X = x is continuous.
- ▶ Holding X fixed, we can shift $\nu(X,Z)$ by changing Z without affecting V.
- ▶ Why? Conditional on X, Z and V are independent and V doesn't enter $\nu(\cdot)$.
- For a given shift in Z, two people with the same observed characteristics X experience the same shift in $\nu(\cdot)$ regardless of whether they have different resistance to treatment V

Normalization: Transform V to Uniform(0,1)

- ▶ For any continuous RV W with CDF H, $\widetilde{W} \equiv H^{-1}(W) \sim \text{Uniform}(0,1)$
- ▶ Condition on (X = x); let F_x be the conditional dist of V | X = x (continuous)
- \triangleright Remember: conditional on X, Z and V are independent!

$$D|(X = x) = 1 \left\{ \nu(x, Z) > V_x \right\} = 1 \left\{ F_x^{-1} \left(\nu(x, Z) \right) > F_x^{-1} (V_x) \right\}$$
$$= 1 \left\{ F_x^{-1} (\nu(x, Z)) > \widetilde{V}_x \right\} = 1 \left\{ g(x, Z) > \text{Uniform} \right\}$$

▶ If $W \sim \mathsf{Uniform}(0,1)$ then $\mathbb{P}(W < c) = c$.

$$\pi(x,z) \equiv \mathbb{P}(D=1|X=x,Z=z) = \mathbb{P}(g(x,z) > \mathsf{Uniform}) = g(x,z)$$

- ▶ WLOG normalize $V|X = x \sim \text{Uniform}(0,1) \implies V|(X = x, Z = z)$ also uniform
- ▶ The function $\nu(\cdot)$ becomes the **propensity score** $\pi(X, Z)$.

Generalized Roy Model

Model

$$Y_0 = \mu_0(X) + U_0$$

 $Y_1 = \mu_1(X) + U_1$
 $Y = (1 - D)Y_0 + DY_1$
 $\pi(X, Z) \equiv \mathbb{P}(D = 1|X, Z)$

Assumptions

- 1. $D = 1\{\pi(X, Z) > V\}$
- 2. $Z \perp \!\!\! \perp (Y_0, Y_1, V) | X$
- 3. $V|(X=x,Z=z) \sim \mathsf{Uniform}(0,1)$

ATE, TOT and TUT in the Generalized Roy Model

$$\begin{aligned} \mathsf{ATE}(x) &\equiv \mathbb{E}[Y_1 - Y_0 | X = x] = \mu_1(x) - \mu_0(x) \\ \mathsf{TOT}(x) &\equiv \mathbb{E}[Y_1 - Y_0 | X = x, D = 1] = \mu_1(x) - \mu_0(x) + \mathbb{E}[U_1 - U_0 | X = x, D = 1] \\ \mathsf{TUT}(x) &\equiv \mathbb{E}[Y_1 - Y_0 | X = x, D = 0] = \mu_1(x) - \mu_0(x) + \mathbb{E}[U_1 - U_0 | X = x, D = 0] \end{aligned}$$

- \triangleright Same definitions as before, but now we are conditioning on X.
- Average over the distribution of X to obtain unconditional versions.

Policy-Relevant Treatment Effects (PRTEs)

$$\mathsf{PRTE}(x) \equiv \frac{\mathbb{E}[Y_i | X_i = x, \mathsf{New Policy}] - \mathbb{E}[Y_i | X_i = x, \mathsf{Old Policy}]}{\mathbb{E}[D_i | X_i = x, \mathsf{New Policy}] - \mathbb{E}[D_i | X_i = x, \mathsf{Old Policy}]}$$

- ightharpoonup Compare a new policy to old one; average over X to obtain unconditional version.
- Policy \equiv change in the propensity score p(Z,X) that changes who is treated without affecting (Y_1, Y_0, V) .
- ▶ PRTE is the average change in *Y* per person shifted into treatment.
- At some values of x, people may be shifted out of treatment
- ► A LATE is a PRTE, but a given LATE may not answer *your* policy question!

Marginal Treatment Effects (MTEs)

Textbook Normal Model

- Any treatment effect of interest can be calculated from $(\gamma_0, \gamma_1, \mu_0, \mu_1, \delta)$.
- ▶ These parameters are identified: Heckman Two-step approach

Generalized Roy Model

 Any treatment effect can be calculated as from knowledge of the Marginal Treatment Effect (MTE) function

$$\mathsf{MTE}(v,x) \equiv \mathbb{E}(Y_1 - Y_0 | X = x, V = v)$$

- ightharpoonup How do treatment effects vary with observed (x) and unobserved (v) heterogeneity?
- ightharpoonup No unobserved heterogeneity \implies MTE is *constant* as a function of v.
- ▶ Like textbook model parameters, MTE does *not* depend on the instrument Z.

From MTE Function to Target Parameters

Target Parameters

► ATE, TOT, TUT, PRTEs, LATE, etc.

General Approach

Any of the above (and more!) can be computed as a weighted average of the MTE.

Example: ATE from MTE

$$\begin{aligned} \mathsf{ATE}(x) &\equiv \mathbb{E}[Y_1 - Y_0 | X = x] = \mathbb{E}_{V | X = x} [\mathbb{E}(Y_1 - Y_0 | X = x, V = v)] \\ &= \mathbb{E}_{V | X = x} [\mathsf{MTE}(X, V)] = \int \mathsf{MTE}(x, v) \, dF_{V | X = x}(v) \\ &= \int_0^1 \mathsf{MTE}(v, x) \times 1 \, dv \end{aligned}$$

- ▶ Follows because $V|X = x \sim \text{Uniform}(0,1)$.
- See Mogstad & Torgovitsky (2018) for other weighting functions.

How can we identify the MTE function? Notation

$$m(p,x) \equiv \mathbb{E}[Y|\pi(X,Z) = p, X = x]$$

$$m_0(p,x) \equiv \mathbb{E}[Y|\pi(X,Z) = p, X = x, D = 0]$$

$$m_1(p,x) \equiv \mathbb{E}[Y|\pi(X,Z) = p, X = x, D = 1]$$

Two Approaches

1. Local Instrumental Variables

$$\mathsf{MTE}(p,x) = \frac{\partial}{\partial p} m(p,x)$$

2. Separate Estimation

$$\mathsf{MTE}(p,x) = [m_0(p,x) - m_1(p,x)] + p \frac{\partial}{\partial p} m_1(p,x) + (1-p) \frac{\partial}{\partial p} m_0(p,x)$$

The Local Instrumental Variables Approach

Can Show that

$$m(p,x) \equiv E[Y|\pi(X,Z) = p, X = x] = \mu_0(x) + p[\mu_1(x) - \mu_0(x)] + K(p,x)$$

 $K(p,x) \equiv pE(U_1 - U_0|V \le p, X = x) = \int_0^p E(U_1 - U_0|X = x, V = v) dv.$

Differentiating with respect to p

$$\frac{\partial}{\partial p} E[Y|P(X,Z) = p, X = x] = \mu_1(x) - \mu_0(x) + \frac{\partial}{\partial p} K(p,x)$$

$$= \mu_1(x) - \mu_0(x) + E(U_1 - U_0|X = x, V = p)$$

$$\equiv \mathsf{MTE}(p,x)$$

 \triangleright 2nd-to-last equality: definition of K(p,x) and Fundamental Theorem of Calculus.

Theory Versus Practice

- ▶ Both local IV and separate estimation approaches involve non-parametric regression of *Y* on *X* and the propensity score.
- ▶ This is extremely challenging in practice even if X is low-dimensional!
- Need variation in propensity score for fixed X; this comes from Z.
- To non-parametrically identify the full MTE function, need an instrument that allows $\pi(X, Z)$ to vary over the **full range** [0, 1] for any value of X!
- ▶ In practice, researchers make simplifying assumptions and carry out semi-parametric or flexible parametric estimation.
- ► This invariably involves interpolation / extrapolation to some degree!
- ► See Mogstad & Torgovitsky (2018) for a partial identification approach.

Cornelissen et al (QJE; 2018) - Who Benefits from Universal Child Care?

Background

- ► Major policy question: causal effect of early childhood interventions, including state-provided day care.
- Some studies of highly-targeted programs (e.g. Head Start / Perry Preschool) find sizable positive effects.
- Evidence for universal provision is mixed: some find sizable negative effects (Quebec study).
- How to rationalize these conflicting findings?
- ▶ Maybe targeted programs enroll children *most likely to benefit*, i.e. those with an adverse home environment.

Cornelissen et al (QJE; 2018) - Who Benefits from Universal Child Care?

This Study

- ▶ Study provision of universal preschool/childcare in Germany using MTE approach.
- Treatment is early attendance, defined as attending for at least three years.
- ▶ Instrument is a staggered roll-out of 1990s policy reform that affected the number of slots for publicly-provided childcare in different places.
- ▶ Main outcome is a universal school readiness exam administered at age 6.

Cornelissen et al (QJE; 2018) - Who Benefits from Universal Child Care?

Main Findings

- Evidence of reverse selection on gains from observed characteristics.
- ▶ Minorities benefit most from childcare but are least likely to enroll.
- Similar selection on unobservables: "high resistance" children benefit most.
- Effect is so strong that TUT > ATE > 0 > TOT!
- \triangleright Evidence that treatment effect heterogeneity comes from Y_0 rather than Y_1 .

The Rest of the Lecture

- ▶ We'll focus on their **implementation** of MTE methods.
- Also talk a bit about policy counterfactuals.
- See the paper for more details.

A Simplified MTE Model

Additive Separability

- lacksquare $\mathbb{E}[U_0|V,X]=\mathbb{E}[U_0|V]$ and $\mathbb{E}[U_1|V,X]=\mathbb{E}[U_1|V]$
- Changing X only affects the intercept of the MTE, viewed as a function of v.
- Still allows V to vary with X.

Linearity

- $ightharpoonup \mathbb{E}[Y_0|X=x]=x'eta_0$ and $\mathbb{E}[Y_1|X=x]=x'eta_1$
- Restricts the way that covariates affect the intercept of the MTE function.

Implications of Separability and Linearity

MTE Function

$$\begin{split} \mathsf{MTE}(p,x) &= \mu_1(x) - \mu_0(x) + E(U_1 - U_0|X = x, V = p) \\ &= \mu_1(x) - \mu_0(x) + E(U_1 - U_0|V = p) \\ &= x'(\beta_1 - \beta_0) + E(U_1 - U_0|V = p) \\ &= x'(\beta_1 - \beta_0) + \frac{d}{dp}K(p) \end{split} \tag{Linearity}$$

Observed Conditional Mean Function

$$\mathbb{E}[Y|\pi(X,Z) = p, X = x] = \mu_0(x) + p[\mu_1(x) - \mu_0(x)] + K(p,x)$$
$$= x'\beta_0 + x'(\beta_1 - \beta_0)p + K(p)$$

▶ This is a **semi-parametric model**: linear regression plus unknown function K(p)

A Parametric Approximation

- ► Could choose to carry out semi-parametric estimation, but Cornelissen et al (2018) take a simpler approach.
- Model K(p) as a polynomial in p; don't include constant or first-order term since they're already in the regression:

$$\mathbb{E}[Y|\pi(X,Z) = p, X = x] = x'\beta_0 + x'(\beta_1 - \beta_0)p + \sum_{j=2}^{J} \alpha_j p^j$$

▶ If we knew p, we could run this regression; unfortunately we don't know it!

Implementation

- 1. Run probit/logit of D_i on (X_i, Z_i) to estimate the propensity scores \hat{p}_i .
- 2. Estimate β_0, β_1, α from the following regression:

$$Y_i = X_i \beta_0 + X_i' (\beta_1 - \beta_0) \widehat{\rho}_i + \sum_{j=2}^J \alpha_j \widehat{\rho}_i^j + \epsilon_i$$

3. Construct the estimated MTE function as follows:

$$\widehat{\mathsf{MTE}}(p,x) = x'\widehat{\beta}_0 + x'(\widehat{\beta}_1 - \widehat{\beta}_0)p + \sum_{i=2}^J \widehat{\alpha}_j p^j$$

4. Take weighted average of MTE(p,x) to construct desired target parameter.

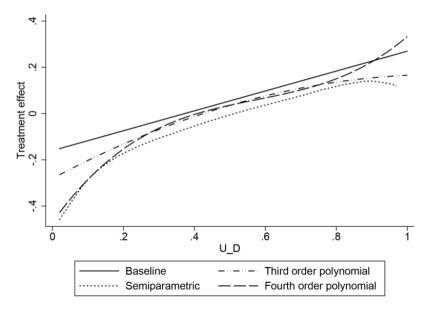
Some Specifics from Cornelissen et al (2018)

▶ Add municipality (*R*) and exam cohort (*T*) dummies:

$$Y = X\beta_0 + \alpha R + \tau T + X(\beta_1 - \beta_0)\hat{p} + \sum_{j=1}^{J} \alpha_j \hat{p}^j + \epsilon$$

- \blacktriangleright Experiment with J=2, J=3, J=4, and a semi-parametric specification.
- Remember: we differentiate to get the MTE, so J=2 is a linear specification for $\mathbb{E}(U_1-U_0|V)$. Sound familiar?
- ightharpoonup Similar results across the different specifications of K(p) in this case.

Treatment effects **increase** with resistance to treatment!



Policy Counterfactuals

TABLE 9
POLICY-RELEVANT TREATMENT EFFECTS

	PRTE (1)	Propensity Score	
		Baseline (2)	Policy (3)
1. Bring 2002 $P(Z)$ to .9 by adding .275	.160* (.085)	.67	.90
2. Bring 2002 $P(Z)$ to .9 by multiplying 1.5	.165* (.087)	.67	.90
3. Lift 2002 cohort's coverage rate (Z) to 1 if < 1	.123 (.077)	.67	.71
4. Add .4 to 2002 cohort's coverage rate (Z)	.141* (.086)	.67	.72