Practice Problems: Treatment Effects

Advanced Econometrics 1
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This document contains some practice problems to help you prepare for the upcoming exam. I will discuss these in my revision lecture and post full solutions on the course website `https://treatment-effects.com/syllabus`.

1. Let Y be an outcome, D be a binary treatment, and (Y_0, Y_1) be the associated potential outcomes. For an observed vector of covariates $\boldsymbol{X}$, define the propensity score $p(\boldsymbol{X}) \equiv \mathbb{P}(D = 1|\boldsymbol{X})$ and regression functions $m_d(\boldsymbol{X}) \equiv \mathbb{E}(Y|D = d, \boldsymbol{X})$ for $d = 0, 1$. Suppose that $\boldsymbol{X}$ satisfies both the *selection on observables* and *overlap* assumptions.

   (a) Show that $\mathbb{E}(Y_0) = \mathbb{E}\left[\frac{(1-D)Y}{1-p(\boldsymbol{X})}\right]$ and $\mathbb{E}(Y_1) = \mathbb{E}\left[\frac{DY}{p(\boldsymbol{X})}\right]$. Use this result to propose a way of identifying the ATE of D on Y.

   (b) Show that $\mathbb{E}(Y_0) = \mathbb{E}[m_0(\boldsymbol{X})]$ and $\mathbb{E}(Y_1) = \mathbb{E}[m_1(\boldsymbol{X})]$. Use this result to propose a way of identifying the ATE of D on Y.

   (c) Show that $\mathbb{E}(Y_0) = \mathbb{E}\left[\frac{(1-D)m_0(\boldsymbol{X})}{1-p(\mathbf{X})}\right]$ and $\mathbb{E}(Y_1) = \mathbb{E}\left[\frac{Dm_1(\boldsymbol{X})}{p(\boldsymbol{X})}\right]$. While this result could be used to identify the ATE of D on Y, doing so is much less convenient that using one of the methods from the preceding two parts. Explain why.

   (d) Let $\pi(\cdot)$ be a function that satisfies $0 < \pi(\boldsymbol{X}) < 1$ but *may or may not* equal the propensity score function $p(\cdot)$ defined above. Show that

   $$\text{ATE} = \mathbb{E}\left[\frac{DY}{\pi(\boldsymbol{X})} + \left\{1 - \frac{D}{\pi(\boldsymbol{X})}\right\} m_1(\boldsymbol{X}) - \frac{(1-D)Y}{1-\pi(\boldsymbol{X})} - \left\{1 - \frac{1-D}{1-\pi(\boldsymbol{X})}\right\} m_0(\boldsymbol{X})\right].$$

   (e) Let $\mu_0(\cdot)$ and $\mu_1(\cdot)$ be two functions of $\boldsymbol{X}$ that *may or may not* equal the regression functions $m_0(\cdot)$ and $m_1(\cdot)$ defined above. Show that

   $$\text{ATE} = \mathbb{E}\left[\frac{DY}{p(\boldsymbol{X})} + \left\{1 - \frac{D}{p(\boldsymbol{X})}\right\} \mu_1(\boldsymbol{X}) - \frac{(1-D)Y}{1-p(\boldsymbol{X})} - \left\{1 - \frac{1-D}{1-p(\boldsymbol{X})}\right\} \mu_0(\boldsymbol{X})\right].$$

   (f) Using the expressions given in the preceding two parts, propose a method for identifying the ATE of D on Y that allows the propensity score to be misspecified as long as the regression functions are correctly specified, and vice-versa. (FYI: this property is called *double robustness*.)

2. Consider an experiment in which unemployed workers are randomly offered the chance to participate in a job training program. Let $Z = 1$ if a worker is offered training and $Z = 0$ otherwise Let $D = 1$ if a worker *actually attends* job training and $D = 0$ otherwise. Finally, let $Y = 1$ if a worker is employed 18 months after the experiment and $Y = 0$ otherwise. Suppose that only workers who are offered job training can attend the program, so that $Z = 0$ implies $D = 0$. Further suppose that the *unconfounded type* and *mean exclusion* restrictions hold.

   (a) Does $\mathbb{E}[Y|D = 1] - \mathbb{E}[Y|D = 0]$ identify the ATE of job training on later employment? If not, what does it identify? Explain briefly.

   (b) Does $\mathbb{E}[Y|Z = 1] - \mathbb{E}[Y|Z = 0]$ identify the ATE of job training on later employment? If not, what does it identify? Explain briefly.

   (c) The question statement failed to mention the *no defiers* assumption. As it turns out, this assumption holds automatically. Explain how we know that there are neither defiers nor always-takers in this example.

   For the remainder of this question, let $T \in \{n, c\}$ indicate a person's *compliance type*, where n denotes never-taker and c denotes complier. As explained in the preceding part, there are no always-takers in this example. Further let (Y_0, Y_1) denote the potential outcomes from *attending* job training.

   (d) Show that $\mathbb{E}[Y|Z = 1] = \mathbb{P}(T = n)\mathbb{E}(Y_0|T = n) + \mathbb{P}(T = c)\mathbb{E}(Y_1|T = c)$.

   (e) Show that $\mathbb{E}(Y|Z = 0) = \mathbb{P}(T = n)\mathbb{E}(Y_0|T = n) + \mathbb{P}(T = c)\mathbb{E}(Y_0|T = c)$

   (f) Suppose that $\mathbb{P}(T = c) > 0$. Based on the results of the preceding two parts, what causal effect does the Wald estimand identify in this example? Explain the economic meaning of this effect in the present context.