Periodic solutions for a Sitnikov restricted four-body problem with primaries in a colinear configuration

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Abstract

1 Introduction

In this paper we obtain existence of periodic solutions for the following restricted nonplanar Newtonian four-body problem (see figure 1):

- We have three primary bodies of masses m_1, m_2, m_3. The fourth body is masless.
- The primary bodies are in a central colinear rigid motion (see [Llibre et al., 2015, Section 2.9]). This motion is carried out in a plane Π.

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- The massless particle is moving on tha perpendicular line to Π passing through the center of masses.

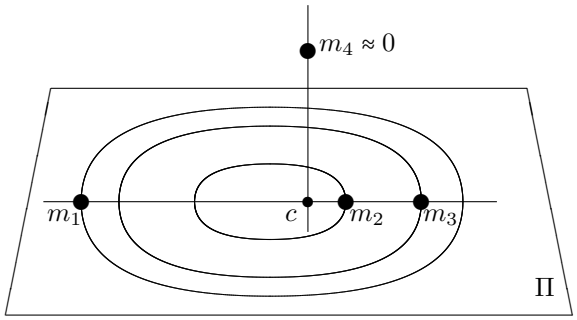

Figure 1: Four-body problem

Problems like the one presented have been extensively discussed in the literature. In [Sitnikov, 1960] K. Sitnikov considered the problem of two body in a Keplerian motion and a massless particle moving in the perpendicular line to the orbital plane passing through the center of masses. Sitnikov obtained deep results about existence of solutions, some of them periodic (see [Moser, 1973, III(5)]. Since then many other authors have studied Sitnikov problem, for instance Llibre and Ortega [Llibre and Ortega, 2008], Chesley [Chesley, 1999], Hagel and Trnkler [Hagel and Trenkler, 1993], Dvorak [Dvorak, 1993], Pérez, Jiménez and Lacomba [Pérez and Lacomba, 2009], Liu, Zhou, and Sun [Liu et al., 1991], Llibre, Meyer and Soler [Llibre et al., 1999], Jiménez-Lara a and Escalona-Buendía [Jiménez-Lara and Escalona-Buendía, 2001], Dankowicz and Holmes [Dankowicz and Holmes, 1995],

Acknowledgments

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