

Periodic solutions for a Sitnikov restricted four-body problem with primaries in a colinear configuration

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Abstract

1 Introduction

In this paper we obtain existence of periodic solutions for the following restricted nonplanar Newtonian four-body problem P (see figure 1):

P_1 We have three primary bodies of masses m_1, m_2, m_3 . The fourth body is masless.

P_2 The primary bodies are in a central colinear configuration rigid motion (see [10, Section 2.9]). This motion is carried out in a plane Π .

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P_3 The massless particle is moving on the perpendicular line to Π passing through the center of masses.

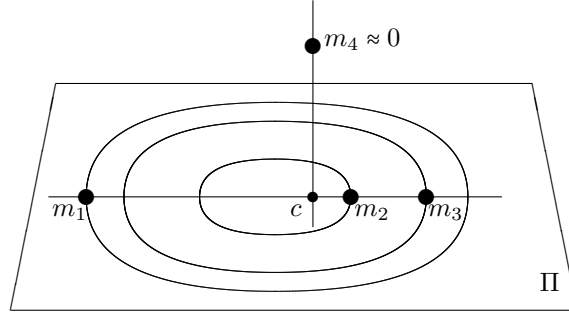


Figure 1: Four-body problem

Problems like the one presented have been extensively discussed in the literature. In [15] K. Sitnikov considered the problem of two body in a Keplerian motion and a massless particle moving in the perpendicular line to the orbital plane passing through the center of masses. Sitnikov obtained deep results about existence of solutions, some of them periodic (see [12, III(5)]). Since then many other authors have studied Sitnikov problem, for instance Liu, Zhou, and Sun [8], Hagel and Trenkler [6], Dvorak [5], Dankowicz and Holmes [4], Llibre, Meyer and Soler [9], Chesley [3], Jiménez-Lara a and Escalona-Buendía [7], Llibre and Ortega [11], Pérez, Jiménez and Lacomba [14].

Problems like the Sitnikov problem for four bodies were addressed more recently. In [16] Soulis, Papadakis and Bountis studied existence, linear stability and bifurcations for a problem similar to P , where in place to have a Eulerian colinear configuration they had a Lagrangian equilateral triangle configuration for the primaries bodies, which are supposed to have the same mass $m_1 = m_2 = m_3$. Later, In [1] Baltagiannis nad Papadakis considered more general masses and in [13] Pandey and Ahmad extend the analysis started in [16] to the case when the primaries are oblate (not mass points). In [18], Zhao and Zhang proved existence of periodic solutions for a problem similar to the one dealt with in [16]. They used a variational approach. In the present paper we extend the analysis in [18] to the case of a colinear central configuration for the primaries. Given that in our problem the primaries are no longer equidistant and their relative position is determined by a polynomial equation of the fifth degree, the calculations involved here are tedious to reproduce completely and difficult for the reader to check them by hand. For this reason we have prepared a jupyter-notebook (see [2]) with some of these calculations. With a little knowledge of Python-Sympy (see [17]) the reader can check and reproduce them easily.

2 Preliminaries and Main Results

Acknowledgments

References

- [1] AN Baltagiannis and KE Papadakis. Families of periodic orbits in the restricted four-body problem. *Astrophysics and Space Science*, 336(2):357–367, 2011.
- [2] Gastón Beltritti, Fernando Mazzone, and Martina Oviedo. Auxiliary calculations. URL: <https://github.com/fdmazzone/ArchivosProyecto/blob/master/Mecanica%20Celeste/CalculosPaper.ipynb> [cited 29.12.2016].
- [3] Steven R Chesley. A global analysis of the generalized sitnikov problem. *Celestial Mechanics and Dynamical Astronomy*, 73(1-4):291–302, 1999.
- [4] Harry Dankowicz and Philip Holmes. The existence of transverse homoclinic points in the sitnikov problem. *Journal of differential equations*, 116(2):468–483, 1995.
- [5] R Dvorak. Numerical results to the sitnikov-problem. In *Qualitative and Quantitative Behaviour of Planetary Systems*, pages 71–80. Springer, 1993.
- [6] Johannes Hagel and Thomas Trenkler. A computer aided analysis of the sitnikov problem. In *Qualitative and Quantitative Behaviour of Planetary Systems*, pages 81–98. Springer, 1993.
- [7] Lidia Jiménez-Lara and Adolfo Escalona-Buendía. Symmetries and bifurcations in the sitnikov problem. *Celestial Mechanics and Dynamical Astronomy*, 79(2):97–117, 2001.
- [8] Jie Liu, Ji-lin Zhou, and Yi-sui Sun. Numerical research on the sitnikov problem. *Chinese astronomy and astrophysics*, 15(3):339–344, 1991.
- [9] Jaume Llibre, Kenneth R Meyer, and Jaume Soler. Bridges between the generalized sitnikov family and the lyapunov family of periodic orbits. *journal of differential equations*, 154(1):140–156, 1999.
- [10] Jaume Llibre, Richard Moeckel, and Carles Simó. *Central Configurations, Periodic Orbits, and Hamiltonian Systems*. Advanced Courses in Mathematics - CRM Barcelona. Birkhäuser, 2015, nov 2015.
- [11] Jaume Llibre and Rafael Ortega. On the families of periodic orbits of the sitnikov problem. *SIAM Journal on Applied Dynamical Systems*, 7(2):561–576, 2008.

- [12] J. Moser. *Stable and Random Motions in Dynamical Systems: With Special Emphasis on Celestial Mechanics*. Annals Mathematics Studies. Princeton University Press, 1973.
- [13] LP Pandey and I Ahmad. Periodic orbits and bifurcations in the sitnikov four-body problem when all primaries are oblate. *Astrophysics and Space Science*, 345(1):73–83, 2013.
- [14] Hugo Jiménez Pérez and Ernesto A Lacomba. On the periodic orbits of the circular double sitnikov problem. *Comptes Rendus Mathématique*, 347(5):333–336, 2009.
- [15] K Sitnikov. The existence of oscillatory motions in the three-body problem. In *Dokl. Akad. Nauk SSSR*, volume 133, pages 303–306, 1960.
- [16] PS Soulis, KE Papadakis, and T Bountis. Periodic orbits and bifurcations in the sitnikov four-body problem. *Celestial Mechanics and Dynamical Astronomy*, 100(4):251–266, 2008.
- [17] SymPy Development Team. *SymPy: Python library for symbolic mathematics*, 2016. URL: <http://www.sympy.org>.
- [18] Xiaoxiao Zhao and Shiqing Zhang. Nonplanar periodic solutions for spatial restricted 3-body and 4-body problems. *Boundary Value Problems*, 2015(1):1, 2015.