d_1, d_2, d_3 positive integer. $\Omega \subset \mathbb{R}^{d_1}$, $m(\Omega) < \infty$ y $f : \Omega \times \mathbb{R}^{d_2} \to \mathbb{R}^{d_3}$.

For Orlicz Spaces we will use the Orlicz norm.

Nemitski operator

$$\boldsymbol{f}u(x) = f(x, u(x))$$

maps $\mathbb{R}^{d_2}$-valued functions defined on $\Omega \subset \mathbb{R}^{d_1}$ into $\mathbb{R}^{d_3}$-valued funcitions defined on $\Omega \subset \mathbb{R}^{d_1}$. Measurability seems follows the same lines that [2, p. 349].???

Let $\Phi_1 : \mathbb{R}^{d_2} \to \mathbb{R}$ and $\Phi_2 : \mathbb{R}^{d_3} \to \mathbb{R}$ be anisotropic N-functions.

Theorem 0.1. *Deber #1 Theorem similar to [1, Lemma 17.2]*

Proof. Let $u \in \Pi(E^{\Phi}(_{,}\mathbb{R}^{d_2}), r)$ be. Adapting the [3, Prop. 3, p. 92-93] to $\mathbb{R}^d$-valued functions (see also [] By [4, Th. 5.5] for Luxemburg norm and N-functions defined on infinite dimensional Banach space) (???) we obtain $\|u_0\|_{L^{\Phi}(\Omega, \mathbb{R}^{d_2})} < r$ where $u_0 = u - \chi_A u$ for certain set $A \subset \Omega$ where $\|u\|_{L^{\infty}(A, \mathbb{R}^{d_2})} < \infty$, in particular $\chi_A u \in E^{\Phi}(\Omega, \mathbb{R}^{d_2})$. From [4, Th. 5.4] we obtain that $\chi_A u$ has an absolutely continuous norm. Therefore we can finf $u_1, \ldots, u_n$ with $u_i u_j \equiv 0$, for $i \neq j$, $\|u_i\|_{L^{\Phi}(\Omega, \mathbb{R}^{d_2})} < r$, $i = 1, \ldots, n$ and $\chi_A u = u_1 + \cdots + u_n$. The result follows from the identity (see [2, p. 353])

$$\boldsymbol{f}(u_0 + \cdots + u_n) = \boldsymbol{f}u_0 + \cdots \boldsymbol{f}u_n - (n-1)\boldsymbol{f}0.$$

Deber #2 completar otros enunciados □

Deber #3 Acotacion y continuidad

Deber #4 Nemitsky asociado a $f = \Phi'$.

Lemma 0.2 (Lemma 9.1 KR). *Suppose that* $d \geq 1$ *and* $\|u\|_{L^{\Phi}(\Omega, \mathbb{R}^d)} \leq 1$. *Then* $v(x) = \nabla\Phi(u(x))$ *belong to* $\tilde{L}^{\Psi}(\Omega, \mathbb{R}^d)$ *and* $\rho_{\Psi}(v) \leq 1$.

References

[1] M. A. Krasnosel′skiĭ and Ja. B. Rutickiĭ. *Convex functions and Orlicz spaces*. P. Noordhoff Ltd., Groningen, 1961.

[2] M.A. Krasnosel'skii, P.P. Zabreyko, E.I. Pustylnik, and P.E. Sobolevski. *Integral operators in spaces of summable functions*. Mechanics: Analysis. Springer Netherlands, 2011.

[3] M. M. Rao and Z. D. Ren. *Theory of Orlicz spaces*, volume 146. Marcel Dekker, Inc., New York, 1991.

[4] Gudrun Schappacher. A notion of orlicz spaces for vector valued functions. *Applications of Mathematics*, 50(4):355–386, 2005.