Periodic solutions for a Sitnikov restricted four-body problem with primaries in a colinear configuration

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Abstract

1 Introduction

In this paper we obtain existence of periodic solutions for the following restricted nonplanar Newtonian four-body problem P (see figure 1):

P_1 We have three primary bodies of masses m_1, m_2, m_3. The fourth body is masless.

P_2 The primary bodies are in a central colinear configuration rigid motion (see [11, Section 2.9]). This motion is carried out in a plane Π.

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P_3 The massless particle is moving on tha perpendicular line to Π passing through the center of masses.

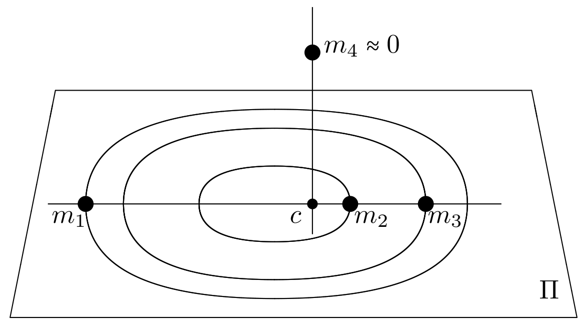

Figure 1: Four-body problem

Problems like the one presented have been extensively discussed in the literature. In [16] K. Sitnikov considered the problem of two body in a Keplerian motion and a massless particle moving in the perpendicular line to the orbital plane passing through the center of masses. Sitnikov obtained deep results about existence of solutions, some of them periodic (see [13, III(5)]. Since then many other authors have studied Sitnikov problem, for instance Liu, Zhou, and Sun [9], Hagel and Trenkler [6], Dvorak [5], Dankowicz and Holmes [4], Llibre, Meyer and Soler [10], Chesley [3], Jiménez-Lara a and Escalona-Buendía [7], Llibre and Ortega [12], Pérez, Jiménez and Lacomba [15].

Problems like the Sitnikov problem for four bodies were addressed more recently. In [17] Soulis, Papadakis and Bountis studied existence, linear stability and bifurcations for a problem similar to P, where in place to have a Eulerian colinear configuration they had a Lagrangian equilateral triangle configuration for the primaries bodies, which are supposed to have the same mass $m_1 = m_2 = m_3$. Later, In [1] Baltagiannis nad Papadakis considered more general masses and in [14] Pandey and Ahmad extend the analysis started in [17] to the case when the primaries are oblate (not mass points). In [19], Zhao and Zhang proved existence of periodic solutions for a problem similar to the one dealt with in [17]. They used a variational approach. In the present paper we extend the analysis in [19] to the case of a colinear central configuration for the primaries. In [8] Li, Zhang and Zhao studied a special type of restricted circular $N+1$-body problem with equal masses for the primaries.

Given that in our problem the primaries are no longer equidistant and their relative position is determined by a polynomial equation of fifth degree, the calculations involved here are tedious to reproduce completely and difficult for that the reader to check them by hand. For this reason we have prepared a jupyter-notebook (see [2]) with some of these calculations. With a little knowledge of Python-Sympy (see [18]) the reader can check and reproduce them easily.

2 Preliminaries and Main Results

We start considering n bodies, $n > 2$, of masses $m_1, \ldots, m_n$ moving in a Euclidean 3-dimensional space according to Newton's laws of motion. We assume that $x_1(t), \ldots, x_n(t)$ are the coordinates of the bodies in some inertial Cartesian coordinate system. Then the system satisfies the Newtonian equations of motion

$$m_j \ddot{x}_j = \sum_{i \neq j} \frac{m_i m_j (x_i - x_j)}{r_{ij}^3}, \quad j = 1, \ldots, n, \tag{1}$$

where $r_{ij} = |x_i - x_j|$ is the Euclidean distance between x_i and x_j. As is customary we introduce the potential function

$$U(x) = \sum_{i<j} \frac{m_i m_j}{r_{ij}} \tag{2}$$

Acknowledgments

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