In this paper is studied Lagrangian relative equilibrium motions of the three bodies of masses m_1, m_2, m_3 in spaces of constant gaussian curvature κ. The spaces considered are the Eclidean plane $\mathbb{R}^2$ ($\kappa = 0$), the 2-dimensional sphere $\mathbb{S}^2_\kappa$ ($\kappa > 0$) and the hyperbolic space $\mathbb{H}^2_\kappa$ ($\kappa < 0$). The gravitational force attracting two bodies is assumed inversely proportional to the area of a ball of radius equal to the disntace between the bodies.

In first instance the authors derived in Section IV the equations of motion for each geometry.

The existence of relative equilibria in $\mathbb{S}^2_\kappa$ is studied in Section V. There is proved that for every acute scalene triangle configurations inscribed in the equator of $\mathbb{S}^2_\kappa$ there exists masses $m_1, m_2, m_3 > 0$, which if placed at the vertices of the triangle form a relative equilibrium that rotates around the equator with any chosen nonzero angular velocity. In another Theorem, is proved that if three bodies is in a relative equilibrium motion in some closed hemisphere of $\mathbb{S}^2_\kappa$ and one of the bodies is on the equator, then all three bodies must move on the equator. Finally, it is shown a relative equilibria that move on non-great circles parallel with the plane of the equator given by isosceles non-equilateral triangles. These relative equilibria occur for masses $m_2 = m_3 > m_1/2$ in two pairs of bands symmetric to the equator, as shown in Figure 2.

The authors studied the existence of relative equilibria in $\mathbb{H}^2_\kappa$ parallel with the x, y-plane. They prove that there are no isosceles relative equilibria parallel with the x, y-plane other than the Lagrangian solutions of equal masses.

Next an alternative form of the equations of motion that is more suitable for the study of Lagrangian (equilateral) relative equilibria is introduced. Then it is studied the planetary problem, i.e. in which two masses are negligible, and it is proved that there occur no bifurcations of the Lagrangian relative equilibria when passing from $\mathbb{S}^2_\kappa$ to $\mathbb{R}^2$ to $\mathbb{H}^2_\kappa$, as κ decrease on $(\infty, -\infty)$.

Finally the authors consider the case when $m_1, m_2 > 0$ and $m_3 = 0$. Ii is assumed that two bodies of equal mass move on a non-equatorial circle of the sphere $\mathbb{S}^2_\kappa$, being always diametrically opposed, and form a Lagrangian relative equilibrium with a third body, which has negligible mass. Then, it is proved that the circle on which the two bodies move must have its radius equal to $(2\kappa)^{-1/2}$ and the third body must move on the equator. In the last result is established that, if one of the three masses is negligible, then there are no Lagrangian relative equilibria in $\mathbb{H}^2_\kappa$ and there are no Lagrangian relative equilibria in $\mathbb{S}^2_\kappa$ either if the curvature κ is sufficiently small, unless the two non-negligible masses are equal.