Home Work 2

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Question 1.4.3

Is it the case that for each $i \in \mathbb{N}, L^i \cap L^{i+1} = \emptyset$? If this indeed the case, prove it. Otherwise provide a counterexample.

If L is not a string alphabet then this statement is False. A counterexample would be $L = \{0, 00\}$, then $L^2 = \{00, 000, 000, 0000\}$. Thus we have arrived at a contradiction.
This would be true if Σ were a string alphabet. As such:
Let L be a formal language over an alphabet Σ .
Such that $L = \{l : \text{ l is a string over } \Sigma\}$

Proof: We first demonstrate by induction that $\forall a \in L^n, |a| = n$.

Then the proof will go as follows:

Base Case:

For $n = 1$. Since L is a language over an alphabet Σ each $a \in L$ is a symbol, and therefore $|a| = 1$.

Inductive Hypothesis: $\forall a \in L^n, |a| = n$. Then,
$L^{n+1} = L^n L = \{w : w = xy | x \in L^n \wedge y \in L\}$ by property of concatenation.
Since $|xy| = |x| + |y|$, using the inductive hypothesis we get $|xy| = n + 1$.

By proving that every element in L^n cannot possibly be L^{n+1} and viceversa, the elements of each of these formal languages have different string length, we can conclude:

$i \in \mathbb{N}, L^i \cap L^{i+1} = \emptyset$