Exam I - CIIC 5045

October 10, 2018

NAME:
Student ID:

1. (a) (*12 points*) Define formally the concepts of regular language, regular expression, and explain the relation between them.

(b) (*24 points*) Decide whether the language

$$L = \{w \in \{0,1\}^* : w = \epsilon \text{ or starts and ends with the same character}\}$$

is regular, and if it is, find a regular expression for L.

2. (a) (*12 points*) Define formally DFSA and NFSA, and computation of an input string under each model.

(b) (*24 points*) Find a regular expression for the language of the NFSA

$$N = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_0, \{q_5\})$$

where δ is defined by the table,

δ	0	1	ϵ
q_0	$\emptyset$	$\{q_1, q_2\}$	$\emptyset$
q_1	$\{q_1\}$	$\emptyset$	$\{q_3\}$
q_2	$\{q_1, q_4\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\emptyset$	$\{q_5\}$
q_4	$\{q_5\}$	$\{q_4\}$	$\emptyset$
q_5	$\emptyset$	$\emptyset$	$\emptyset$

3. (a) (*12 points*) State formally the Pumping Lemma for Regular Languages and comment on its use for deciding the regularity of a language.

(b) (*24 points*) Use the Pumping Lemma for Regular Languages to demonstrate that the language

$$L = \{w \in \{0,1\}^* : w = 0^n 1^{2n+1} : n \text{ natural}\},$$

is not a regular language.

4. (*16 points*) Bob says that the language $L = \{w \in \{0,1\}^* : w = 0^n 1^n, n \text{ natural}\}$ is regular and gives the next argument:

"For each natural n consider the next construction:

$$A_0 = \{\epsilon\} \tag{1}$$

$$A_i = \{0\}A_{i-1}\{1\}, \text{for } i = 1, ..., n \tag{2}$$

$$B_n = \bigcup_{i=0}^{n} A_i. \tag{3}$$

Thus, for each n natural, A_i is regular because it is either the language consisting of the null string or a concatenation of $\{0\}$, A_{i-1} and $\{1\}$; which are all regular languages. And thus, for each n natural, B_n is also regular, because it is the union of the regular languages $A_0, A_1, ..., A_n$. Since this argument holds for every n natural, L is regular. "

What is wrong in Bob's argument? Justify your answer.