Home Work 1

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Question 1.4.1

Can you find examples of unions of strings and alphabets that are not string alphabets?

Let $\Sigma_0 = \{0, 1\}$ and let $\Sigma_1 = \{00, 11\}$. Then $\Sigma = \Sigma_0 \cup \Sigma_1 \ = \ \{0, 1, 00, 11\}$. Then $\rho(\omega) = (0, 0, 1, 1)$ or $\rho(\omega) = (0, 0, 11)$ or $\rho(\omega) = (00, 11)$ or $\rho(\omega) = (00, 1, 1)$.
In this example the string mapping is not well defined, therefore $\Sigma_0 \cup \Sigma_1$ is not a *string alphabet*

Another example would be:
Let $\Sigma_2 = \{a, b\}$ and let $\Sigma_3 = \{baa\}$
Let $\Sigma = \Sigma_2 \cup \Sigma_3 \ = \ \{a, b, baa\}$
Let $\omega = baa$ which is a string of the alleged alphabet. Then $\rho(\omega) = (b, a, a)$ or $\rho(\omega) = (baa)$. Thus, this other example is not a *string alphabet.*

Question 1.4.2

Alice said that any finite union of string of the same length is a string alphabet. Is Alice right?

Proof:

Let P(x) =: "x is a string alphabet"
Let $\Sigma_0 = \{a1, a2, \ldots, ak\}$ and let $\Sigma_1 = \{b1, b2, \ldots, bn\}$

Let $\Sigma = \{\omega \mid (\omega = \Sigma_0 \cup \Sigma_1) \wedge ((\forall a \in \Sigma_0)(\forall b \in \Sigma_1)|a| = |b|)\}$

$(\forall \omega \in \Sigma)P(\omega)$

$(\forall \omega_0 \in \Sigma_0)(\forall \omega_1 \in \Sigma_1) \quad \omega_0 \notin \Sigma \iff (\omega_0 = \omega 1) \wedge (\omega_1 \in \Sigma)$, by property of sets. Thus $\omega \in \Sigma$ is unique and of the same length. Since every element in Σ ,when given a tuple $\rho(\omega^*) \ = \mathrm{T}$, then $(\forall t \in T)$ will have a single mapping to the *string alphabet* Σ .

Q.E.D