FIN 372 / STA 372

Optimization Methods in Finance: Homework 1

Non-graded problems

This assignment is graded on Credit/No-Credit.

If you complete the homework, and it is acceptable, you will get credit. If you do not submit or if the submitted work is not acceptable, you will not get credit. Getting a credit is required to obtain a grade for the group project that follows.

Problem 1:

Use "for loop(s)" and "if statement" within the loop to generate a 20 by 20 Lehmer matrix 'A' where

$$A_{ij} = \begin{cases} \frac{i}{j}, & j \geq i \\ \frac{j}{i}, & j < i. \end{cases}$$

(Hint: First generate a 4 by 4 matrix with all the elements being NA. Then use for loop(s) and if statement to definite the Lehmer matrix. Find the 4 by 4 Lehmer matrix and use the Wikipedia to check. Then you can change the code to generate a 20 by 20 matrix)

You don't need to print the matrix out.

Problem 2:

Test whether A is symmetric or not. (Symmetric means A's transpose is equal to A)

Hint: You may use function all.equal.

Problem 3:

Calculate the inverse of A and assign it to 'C'. Test whether the inverse is correct. That is, calculate (C *A) to see whether the product is identity matrix or not.

Hint: Identity matrix can be generated using function diag. You may use function all.equal to compare C*A and identity matrix.

Problem 4:

Given the square matrices

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 3 \\ 3 & -2 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -6 & -3 \\ 7 & -14 & -7 \\ -1 & 2 & 1 \end{bmatrix}$$

Verify that AB = 0 although neither A nor B is. Also calculate BA. Is AB = BA?

Please do the multiplication by hand first, and then use R to verify your answer.

Problem 5:

A bank makes four kinds of loans to its personal customers, and these loans yield the following annual interest rates to the Bank:

- First mortgage 14%
- Second mortgage 20%
- Home improvement 20%
- Personal overdraft 10%

We are interested in the bank's lending strategy. The information we know is as following:

1. In total $250 million is lent out.
2. First mortgages are 55% of all mortgages (i.e. the first and the second mortgage) issued.
3. Second mortgages are 25% of all loans issued.
4. The average interest rate on all loans is 15%.

Calculate the lending strategy using matrix inversion.

Deliverables

Name your homework as hw1.pdf and your code as hw1.R. Please submit it on Canvas before the deadline.