OSCILLATORS

An oscillator is a circuit, which produces a periodic signal without any input signal. It converts DC power (from the supply) to a periodic signal.

Oscillators are extensively used in both receive and transmit paths. They are used to provide the local oscillation for the mixers for up and down conversion.

On this topic, we cover:

1. Oscillator basics
2. Oscillator topologies
3. Phase noise issues
4. Oscillator implementations and design

Oscillator types:

1. Crystal oscillators
2. Active-RC and Gm-C oscillators
3. Ring oscillators
4. LC timed oscillators
5. Relaxation oscillators.
   … etc.

Voltage Controlled Oscillators:

VCO's are used in RF applications, usually, *in a Phase locked loop* to *provide Different LO frequencies* needed for channel selection. Besides frequency synthesis it is used for other applications.

OSCILLATOR BASICS

Most RF oscillators can be viewed as + ve feedback system.
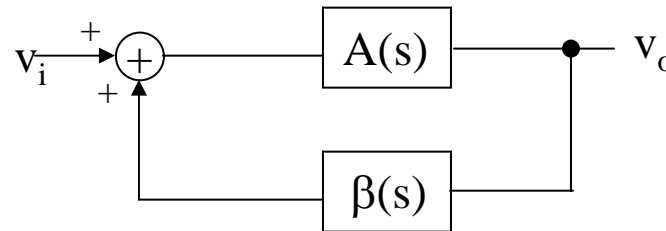
$$v_o(s) = \frac{A(s)}{1 - A(s)\beta(s)} v_i(s)$$

The Barkhausen Criterion states that sustained oscillation can be achieved.

if $\qquad |A(s_o)\beta(s_o)| = 1$ $\qquad$ Loop gain equal to unity.

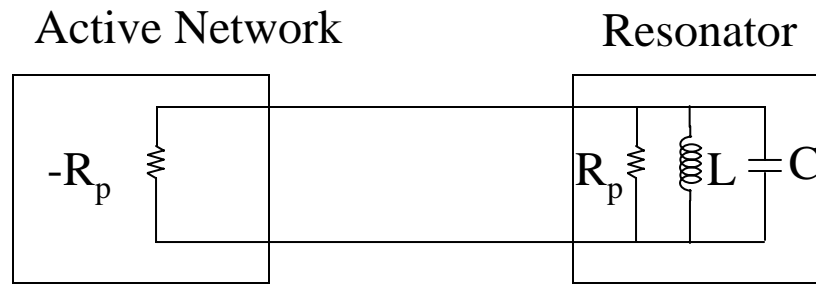and $\qquad |A(s_o)\beta(s_o)| = 0^o$

For zero v_i the output will be finite at a given frequency (ω_o).
For loop gain >1 oscillation will grow.

Two-Port vs One-Port Models

The previously shown model is known as the two-port model, because the A(s) and β(s) Networks are both two-port nets in a closed loop.

The one-port model of oscillators is shown below:

Active Network   Resonator

$-R_p$ ⧢ R_p ⧢ L C

The resonator (LC tank for example) has parasitic resistances (R_P) which prevents the resonator from oscillating because the stored energy will leak through the resistance. To compensate for this loss a positive feedback negative parallel resistances ($-R_p$) will be added to the resonator so that the energy loss in R_p is replenished by the negative resistance. The negative resistance's implementation is typically an active network.

What sets the Frequency and Amplitude of an Oscillator?

The frequency is usually set by Barkhausen Criterion. It is the frequency at which the loop gain is greater or equal to 1 and the phase is zero.

In many cases, as in LC tuned oscillators, the ideal oscillation frequency is determined by the LC tank.
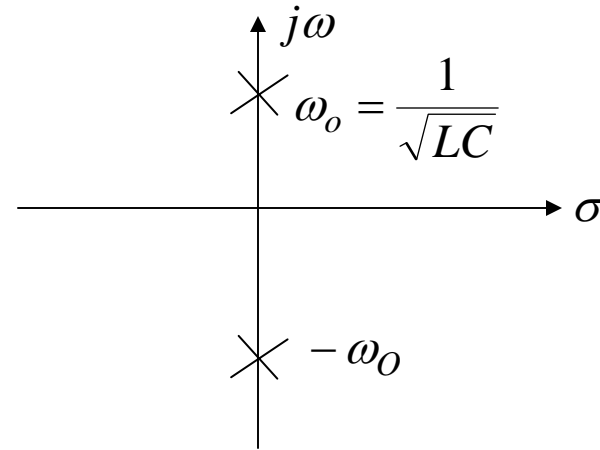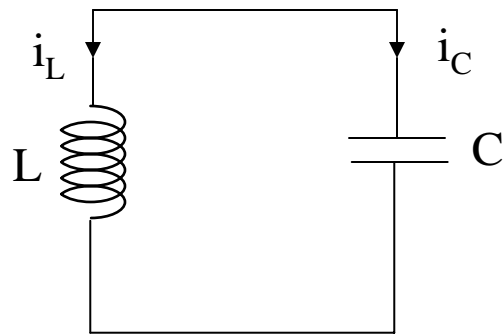
$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

The amplitude is, however, more a complicated parameter to set.

More on Oscillator Theory:

Ideal Oscillator:

Once the tank is excited (by a current pulse for example) a certain amount of energy will be conserved. This energy will alternate between magnetic and electric forms.

In the time domain:

$$v = L\frac{di_L}{dt}$$

but $i_L = -i_c$

and $i_c = C\frac{dv}{dt}$

$$\Rightarrow v = -LC\frac{d_v^2}{dt^2}$$

or $\dfrac{d_v^2}{dt^2} + \omega_o^2 v = 0$

where $\omega_o = \dfrac{1}{\sqrt{LC}}$

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The Differential Equation of the Ideal Oscillator

$$\frac{d_v^2}{dt^2} + \omega_o^2 v = 0$$

When solved yields:

$$v(t) = V_1 e^{j\omega_o t} + V_2 e^{-j\omega_o t}$$

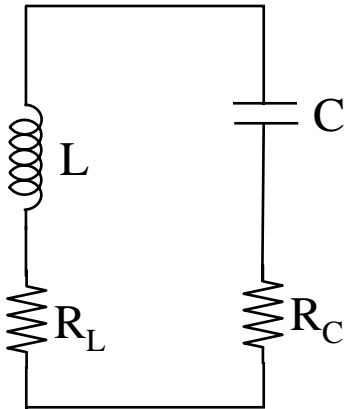Choosing the case where the phase angle is zero at t=0

$$v(t) = V \sin \omega_o t$$

In the frequency domain:

$$V(s) = sL\big(i_L(s) + i_L(o)\big) \quad \text{but} \qquad I_c(s) = -i_L(s)$$

$$I_c = sC \cdot v(s)$$

$$\Rightarrow V(s) = \frac{sL\omega_o^2 i_L(o)}{s^2 + \omega_o^2}$$

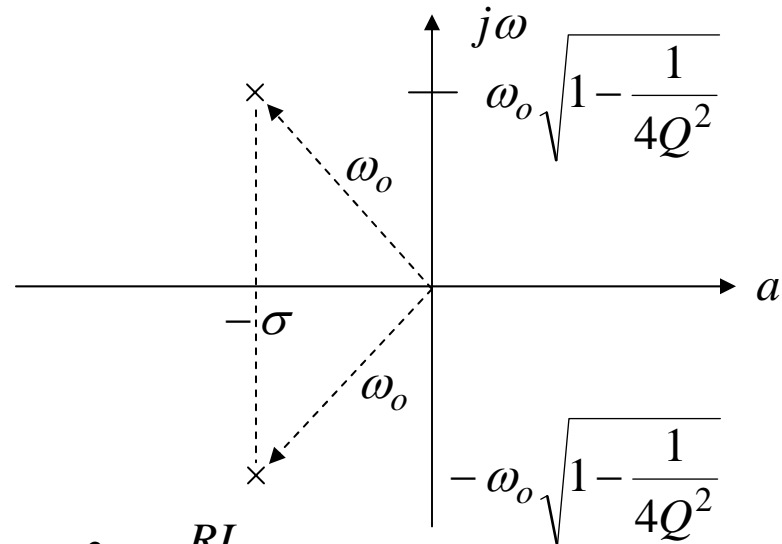Both poles are at $\pm j\omega_o$ as expected !!

Real Oscillator:

Characteristic equation

$$s^2 LC R_C + s(L + R_L R_C C) + R_L$$

or

$$s^2 + s\frac{L + R_L R_C C}{LCR_C} + \frac{R_L}{LCR_C}$$

Initial excitation well start some oscillation. This will die out after a while because the Energy leaks through the parasitic resistances.

$$\omega_o^2 = \frac{RL}{L_C R_C}$$

$$\tau = \frac{L + R_L R_C C}{2LCR_C}$$

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If oscillation is to be maintained. The energy loss (due to parasitic resistance) is to be compensated for. This energy is usually provided from the power supply via a negative resistance, a nonlinear conductance or regenerative feedback, which convert dc power to signal power.
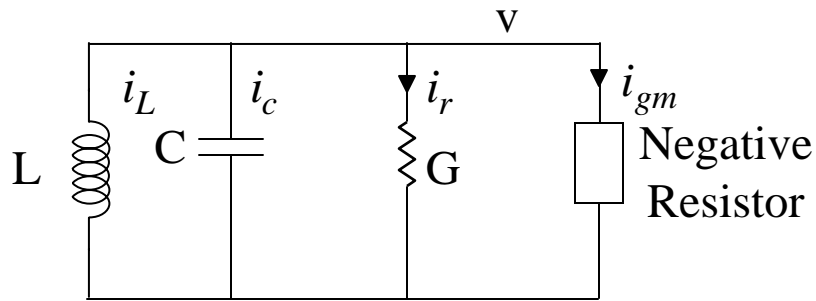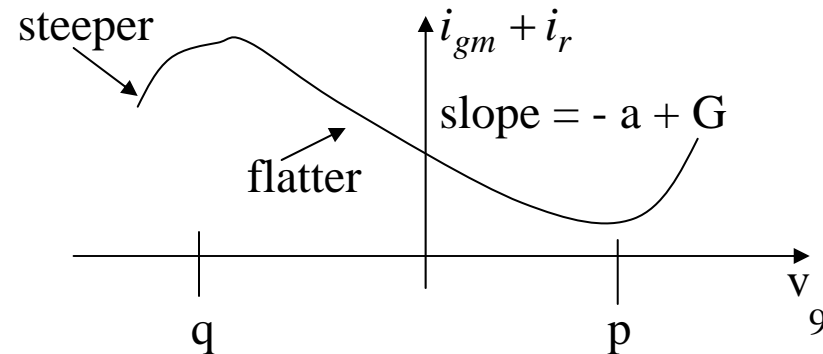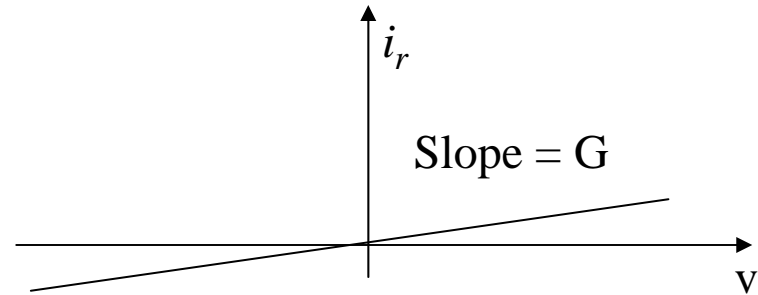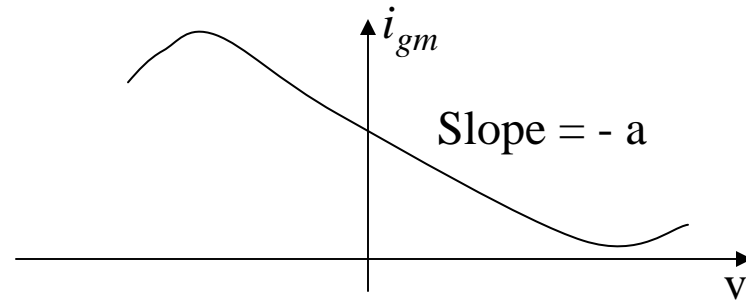

$$G = \frac{1}{Q_L^2 R_L \,//\, Q_c^2 R_c}$$

R_L and R_c are the series parasitic R_s of the L and C, respectively.

where $Q_L = \dfrac{\omega_o L}{R_L}$

$$Q_C = \frac{1}{\omega_o C R_C}$$

Slope = - a

Slope = G

steeper

flatter

$i_{gm} + i_r$

slope = - a + G

q   p

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The following model equation can be written

$$i_L + i_c + i_r + i_{gm} = 0$$

$$i_L = \frac{1}{L} \int v\, dT$$

$$i_c = C\frac{dv}{dt}$$

$$i_r = G_v$$

Differentiating both sides of the above equation:

$$\frac{1}{L}v + C\frac{d^2v}{dt^2} + G\frac{dv}{dt} + \frac{di_{gm}}{dt} = 0$$

$$\boxed{LC\frac{d^2v}{dt^2} + L\frac{d}{dt}\left[G_v + i_{gm}\right] + v = 0}$$

The above DE describes the behaviour of many oscillator implementations.

At the quiescent bias point (which is the equilibrium point)

$$\frac{d}{dt}\left(i_{gm}\right)\Bigg|_{v=0} = -a$$

$$\implies \quad LC\frac{d^2v}{dt} + L(G-a) + v = 0$$

In the frequency domain, the characteristic equation at the equilibrium point is given by:

$$LCs^2 + L(G-a)s + 1 = 0$$

Solving for the poles of the system:

$$s_{1,2} = -\left(\frac{G-a}{2C}\right) \pm j\sqrt{\frac{1}{LC} - \left(\frac{G-a}{2C}\right)^2}$$

$$= \quad \alpha \quad \pm \quad j\beta$$

In the time domain:

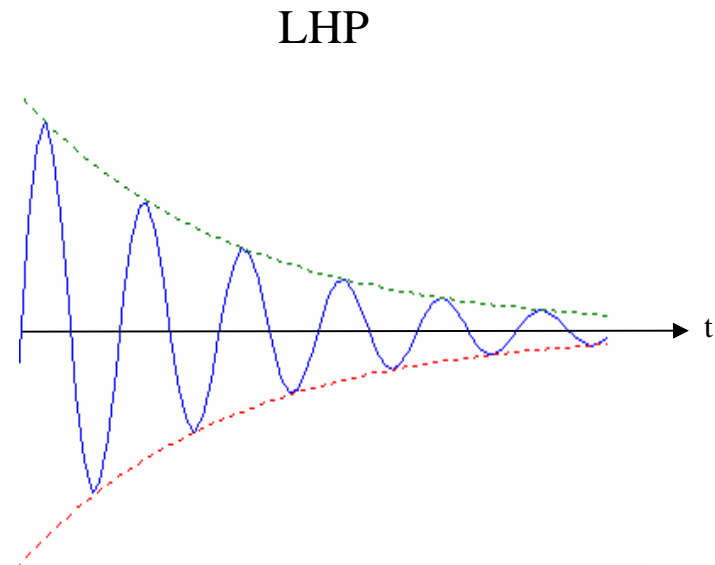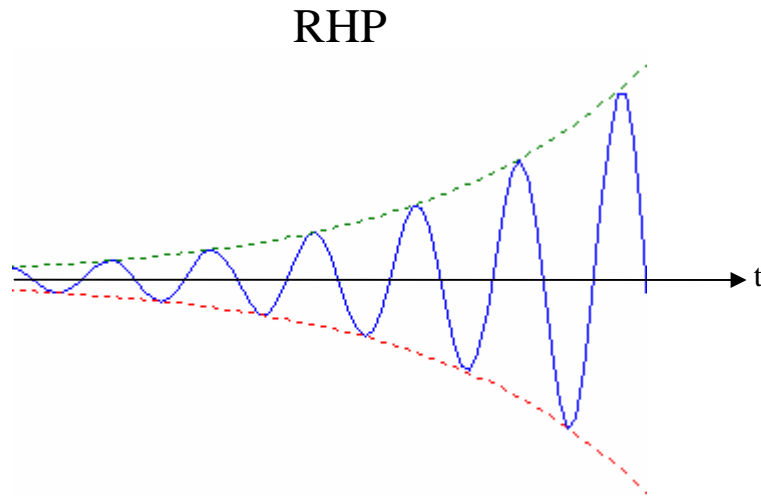$$\boxed{v(t) = A\,e^{\alpha t}\cos\beta t}$$

Oscillation envelope   Oscillation

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$$v(t) = A\, e^{\alpha t} \cos \beta t$$

$$\text{where } \alpha = \frac{a - G}{2C}$$

From the above equ., the following observations can be made:

1. The oscillator frequency is determined by $\beta \cong \dfrac{1}{\sqrt{LC}}$

2. if $\alpha > o$ $\quad (G < a) \longrightarrow$ growing oscillation.

3. if $\alpha < o$ $\quad (G > a)$ Decaying oscillation

RHP

LHP

$G < a$

$G > a$

In practical oscillators, the slope of the negative resistance or nonlinear element (a) is made greater than G $(a \cong 3G)$. This results in growing oscillations as long as the swing is limited between points p and q.

If the oscillation swing grows beyond p or q, the slope (a-G) or α become negative and the poles move the LHP causing the oscillation to decay tempe. Until the swing drops within below p and/or q. This is how a sustained oscillation is produced and a steady sate can be reached.

$-(a-G)$

p   q
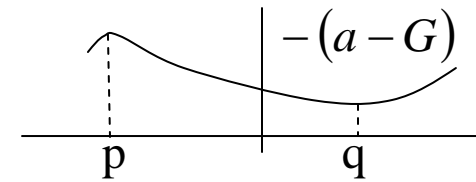
→ Note that the oscillations will not be pure sinusoid. The harmonics are Usually far out and can be easily filtered.

→ Also note that it is hard to determine the oscillation swing exactly!

The van der Pol Approximation

In the 1920's , Van der Pol proposed to model the total I-V Ch<u>c</u> $\left[\left(i_{gm}+i_r\right)vs\ v\right]$
is the simplest possible manner. He assumed a cubic approximation.

$$i(v)=-a_1v+b_1v^3$$
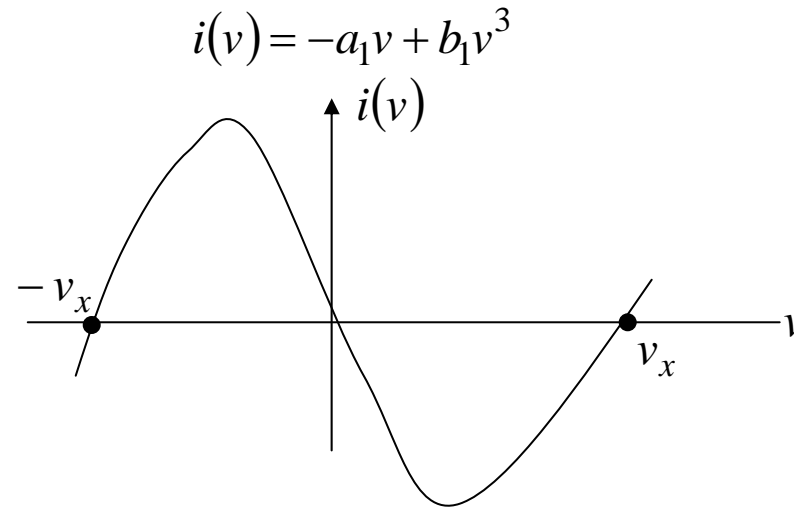
where

$$a_1 = a - G$$

$$b_1 = \frac{a-G}{v_x^2}$$

Van der Pol's analysis leads to the following time domain expression for the
Oscillation voltage.

$$v(t)=\sqrt{\frac{4\,a_1}{3\,b_1}}\ \left(1+e^{\frac{-(t-t_o)\varepsilon}{\sqrt{LC}}}\right)^{-\frac{1}{2}}\cos\left(\frac{t}{\sqrt{LC}}+\phi_o\right)$$

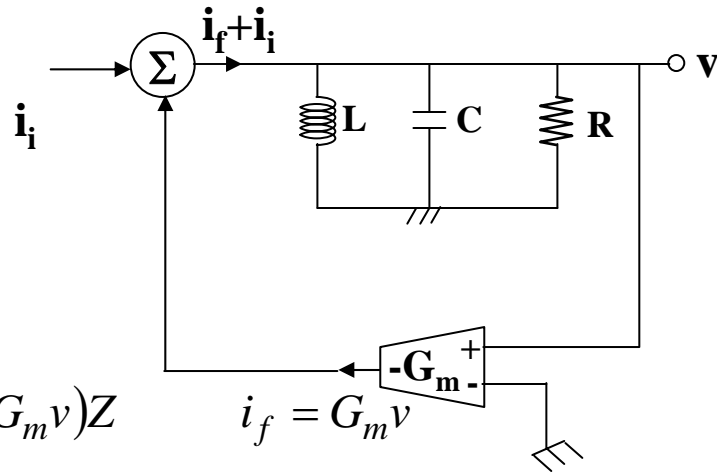$$\text{where}\ \ \varepsilon = a_1\sqrt{\frac{L}{C}}$$

Φ_o is introduced to provide the proper phase in relation to the constant to

At steady state, the zero-to-peak voltage amplitude reaches a maximum value of

$$v_{max} = \sqrt{\frac{4}{3}\frac{a_1}{b_1}} = 1.15\, v_x$$

Basic LC Oscillator Topologies

(a) Feedback model:

$$v = (i_f + i_i)Z = (i_i + G_m v)Z \qquad i_f = G_m v$$

$$v(1 - G_m Z) = i_i Z$$

$$v = i_i \frac{Z}{1 - G_m Z} \qquad\qquad \text{when } Z = \frac{1}{G + sC + \dfrac{1}{sL}}$$
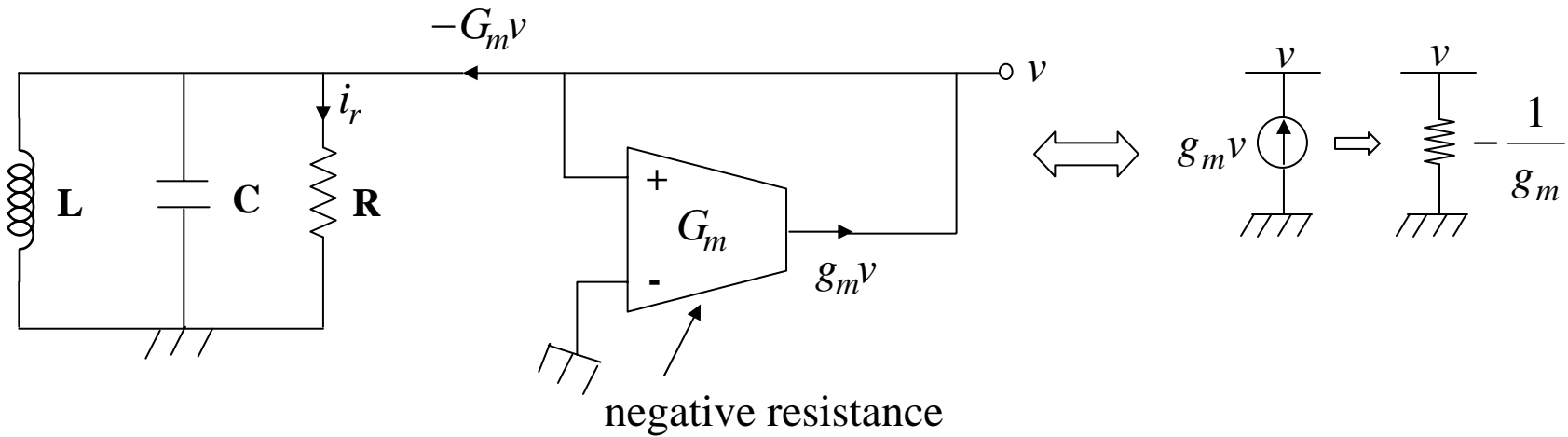
$$= \frac{s/c}{s^2 + s\dfrac{G}{C} + \omega_o^2}$$

$$v = i_i \frac{s/c}{s^2 + s(G - G_m)/c + \omega_o^2} = i_i \frac{sL}{s^2 LC + sL(G - G_m) + 1}$$
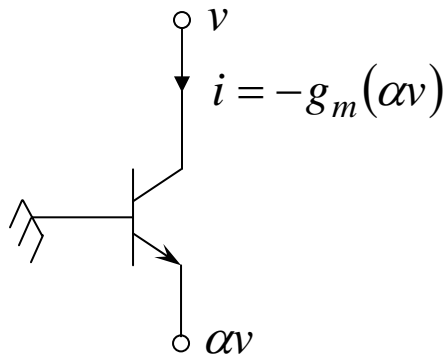
The characteristic :

$$s^2 LC + L(G - G_m)s + 1 = 0 \qquad \text{NOTE: Similarly to that on pp 11}$$
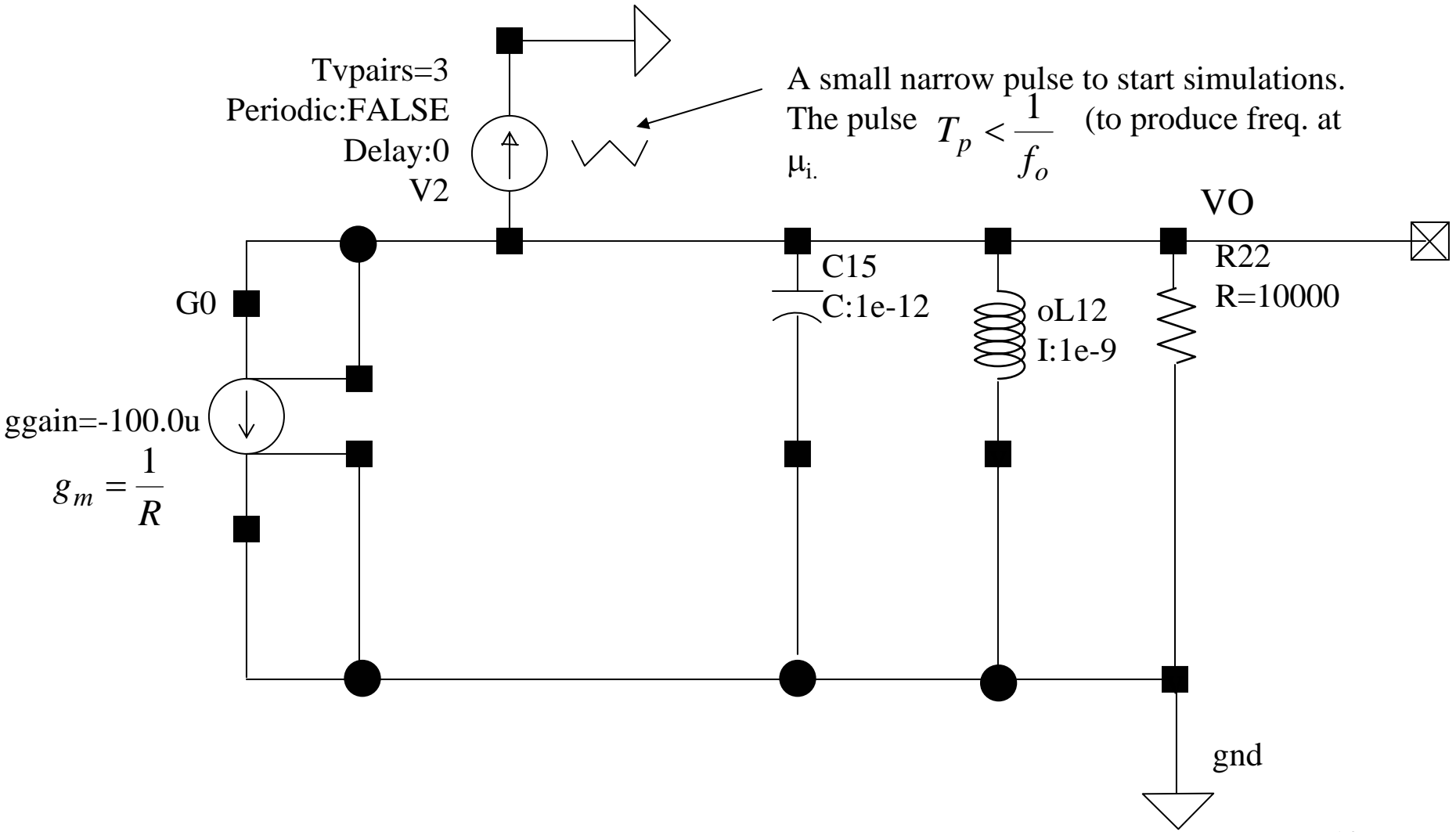
(b)   Negative Resistance Model:

$-G_m v$

i_r

L   C   R

+

G_m

-

$g_m v$

negative resistance

$\Longleftrightarrow$ $g_m v$ $\Rightarrow$ $-\dfrac{1}{g_m}$

v v

Negative Resistance Realization:

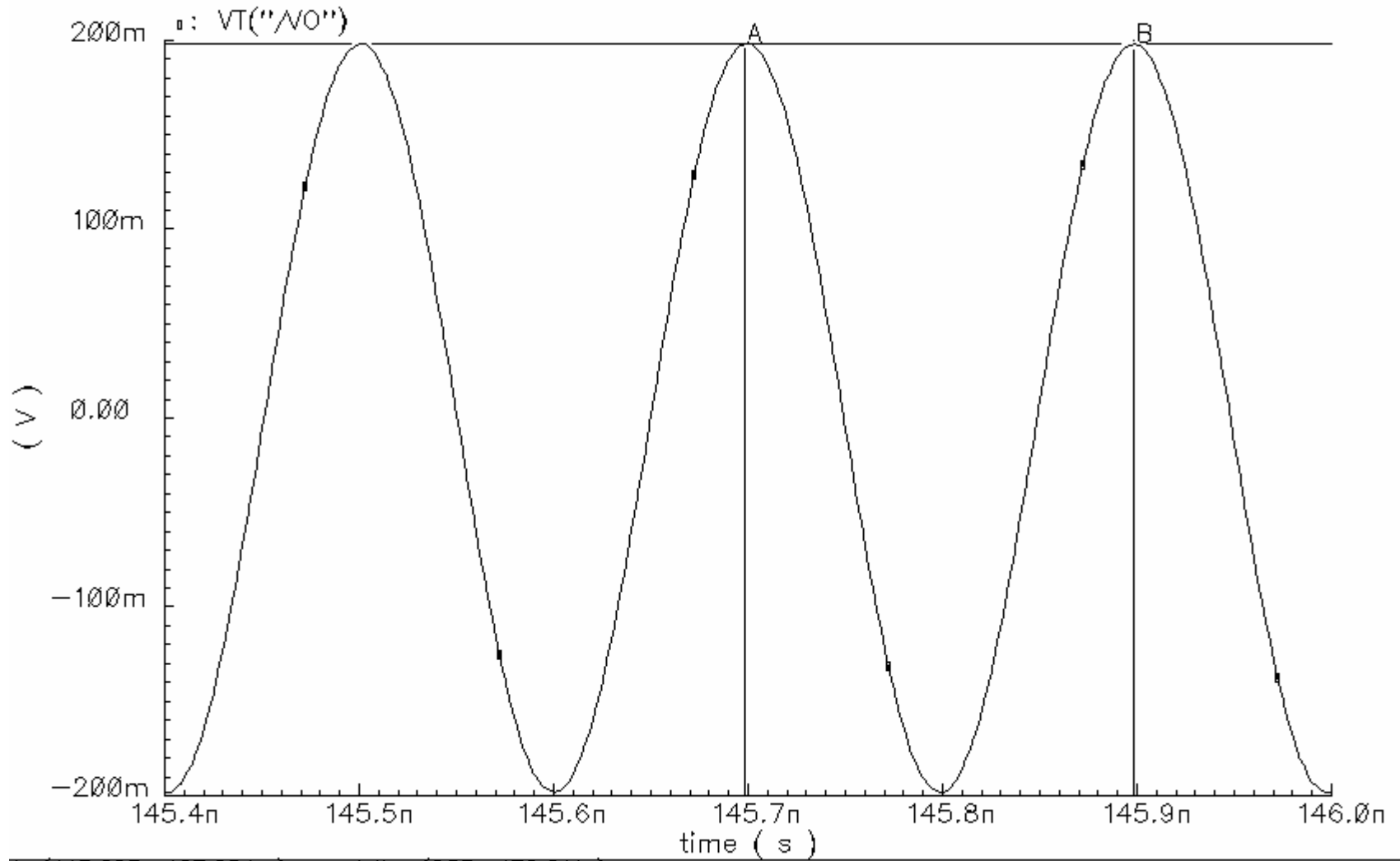v

$i = -g_m(\alpha v)$

αv

$$G_m = \frac{i}{v(1-\alpha)} = -g_m \frac{\alpha}{1-\alpha} \ , \quad \alpha < 1$$

A MOS device can also be used.

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See the sustained on the next page. The frequency $\dfrac{1}{\sqrt{LC}}$

Tvpairs=3
Periodic:FALSE
Delay:0
V2

A small narrow pulse to start simulations. The pulse $T_p < \dfrac{1}{f_o}$ (to produce freq. at μ_i.

VO

G0

C15
C:1e-12

oL12
I:1e-9

R22
R=10000

ggain=-100.0u

$g_m = \dfrac{1}{R}$

gnd

□ : VT("/VO")

A: (145.698n 197.954m)   delta: (200p 179.611u)
B: (145.898n 198.134m)   slope: 898.056K

□ : VT("/VO")

(V)

200m

100m

0.00

−100m

−200m

0.0   20n   40n   60n   80n

time (s)
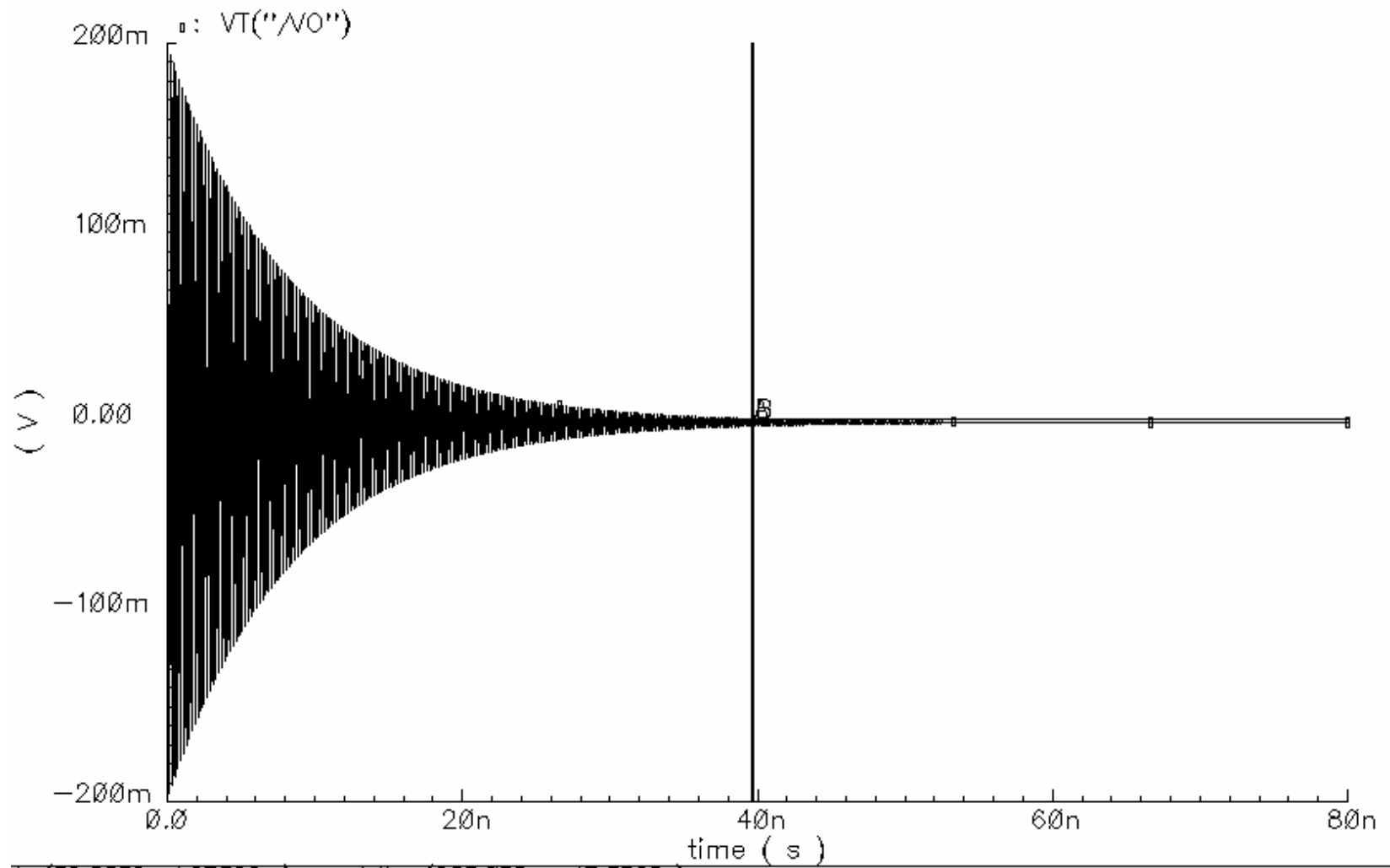
A: (39.5632n 1.97096m)   delta: (200.035p −47.3866u)
B: (39.7632n 1.92358m)   slope: −236.892K
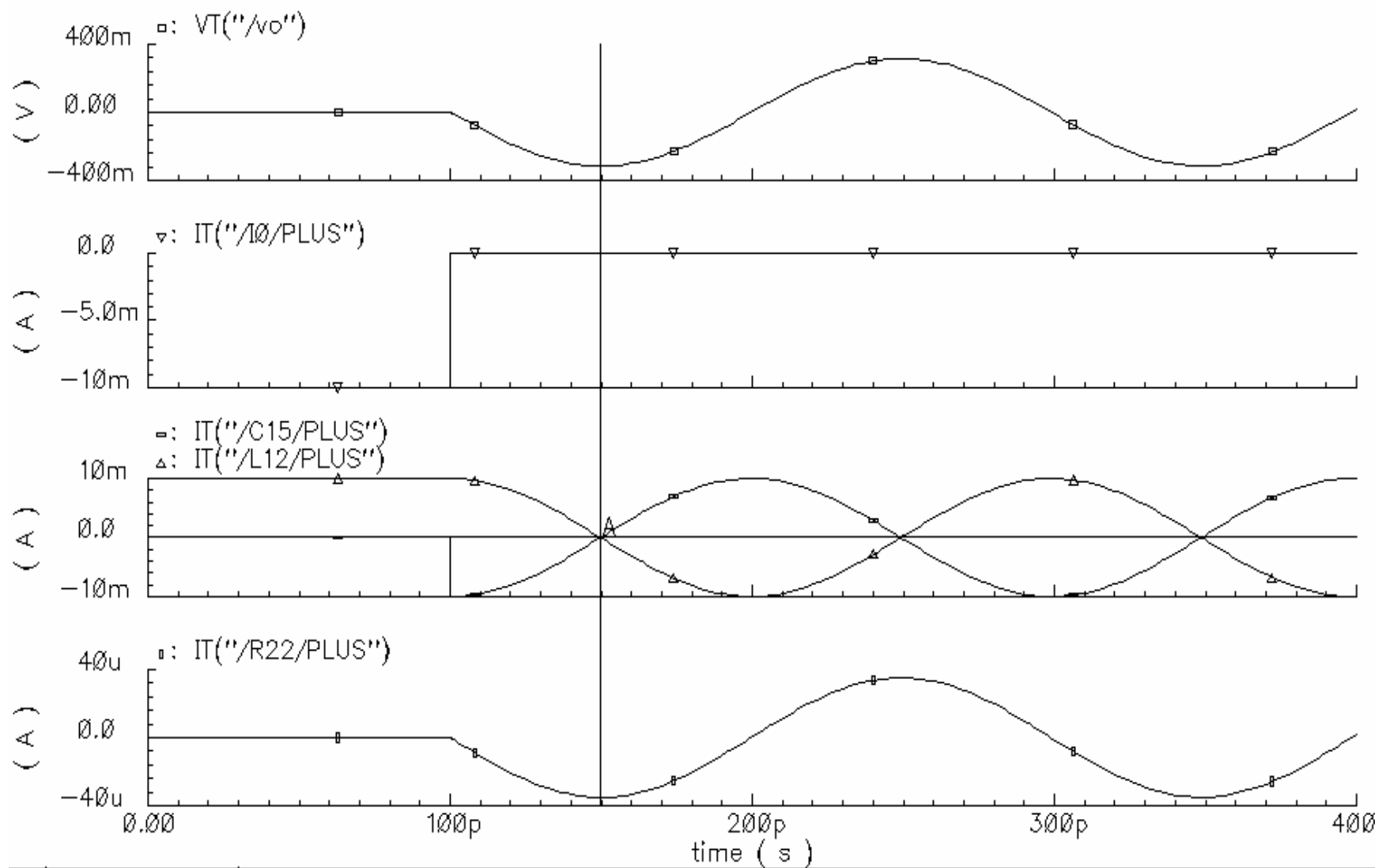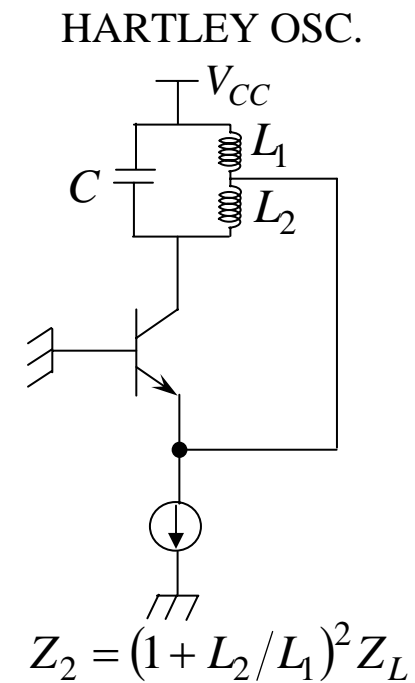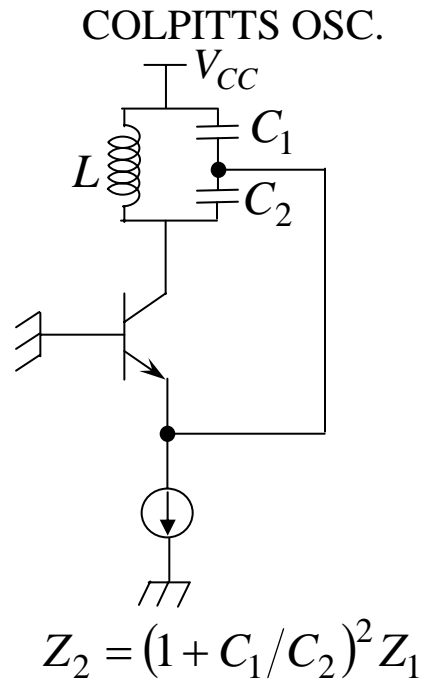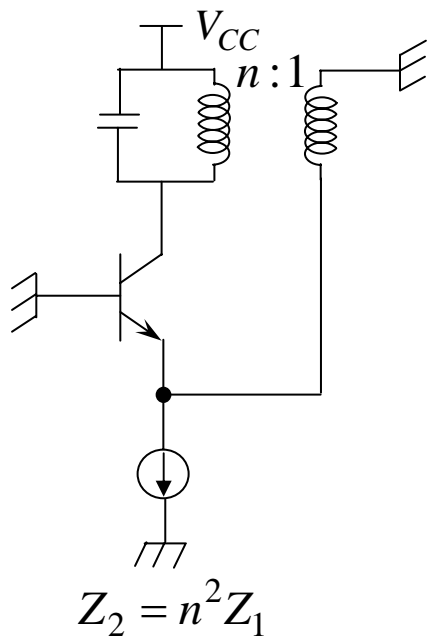
A: (149.647p 15.1135u)

21
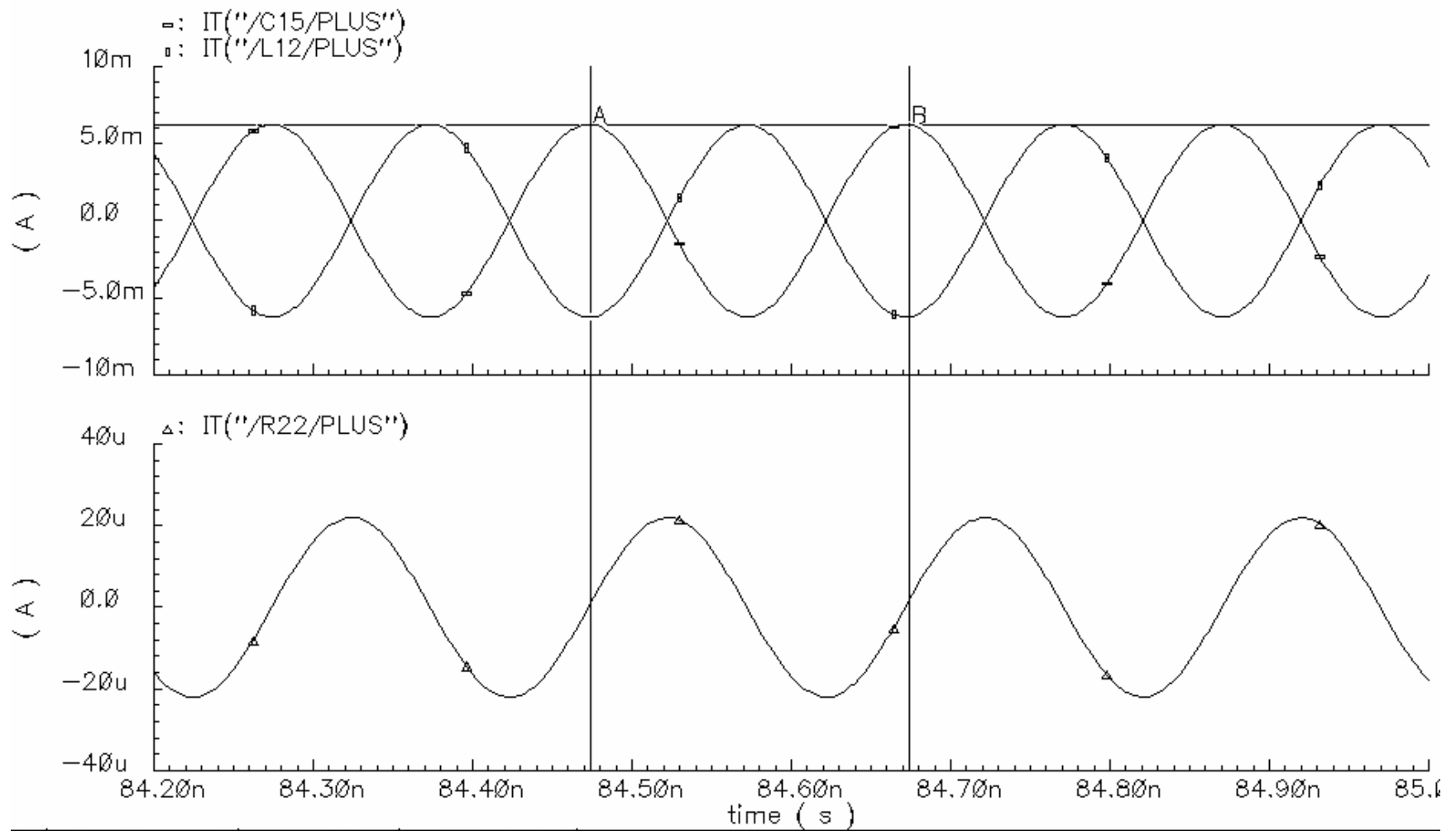
Full Oscillator Realization:
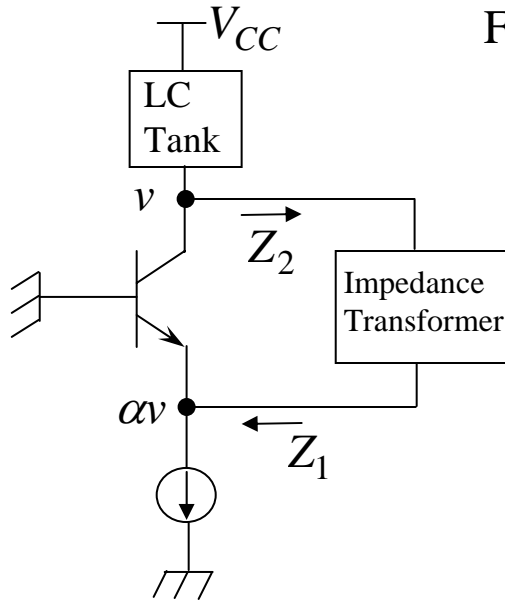
To avoid loading the LC tank (to prevent reducing its Q), an impedance transformer to up-convert the impedance $(Z_2 > Z_1)$

Different realization for the impedance transformation are shown below:

COLPITTS OSC.   HARTLEY OSC.

$Z_2 = n^2 Z_1$ $Z_2 = (1 + C_1/C_2)^2 Z_1$ $Z_2 = (1 + L_2/L_1)^2 Z_L$

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Note that a resonance, the phase shift of the loop is supposed to be zero (according to The Barkhausan Criterion). That is why the output signal is fed back to the emitter. The Zero-phase condition may be satisfied if the output signal is inverted and then fed back to the base as shown below:
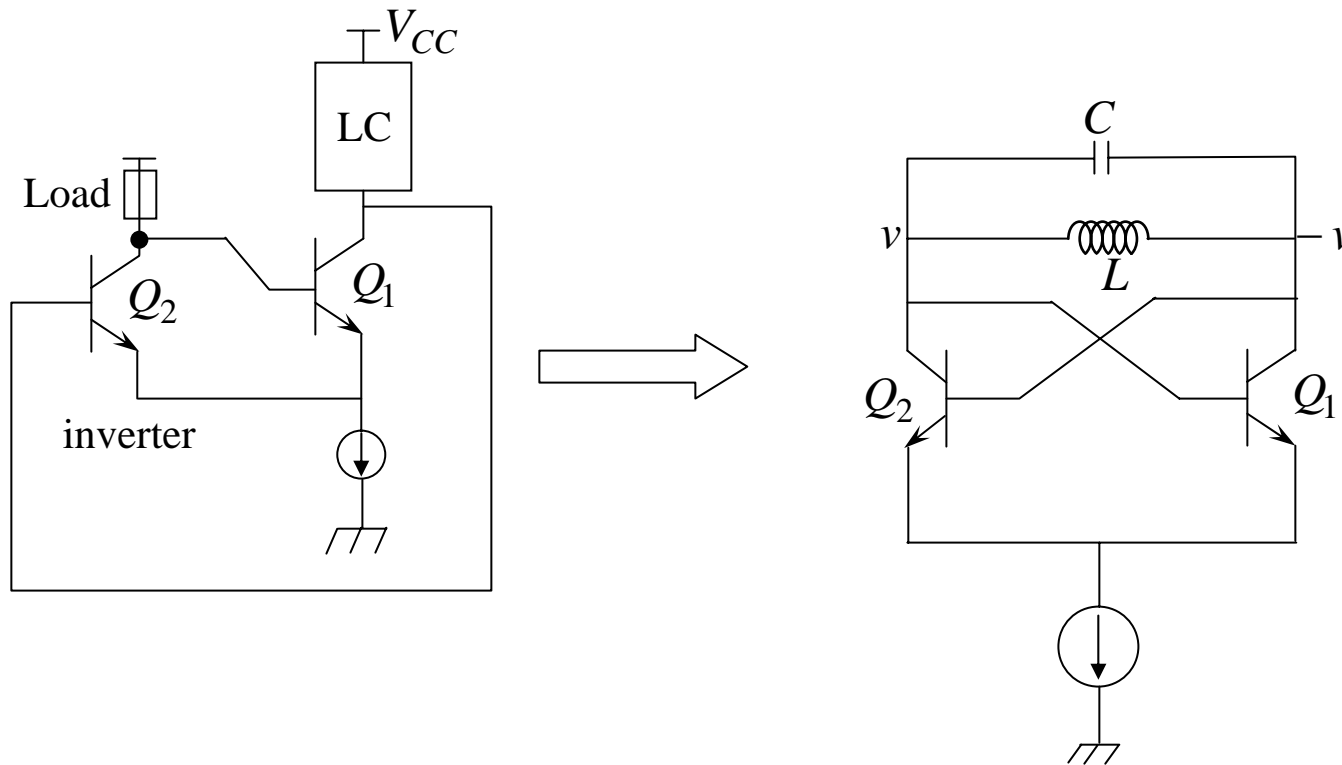
This topology is widely used and is known as the $-v$ Gm oscillator

It is interesting to note that the cross-coupled pair (BJT an MOS) presents a negative resistance to the tank. That is why it can be classified as a negative resistance oscillator.
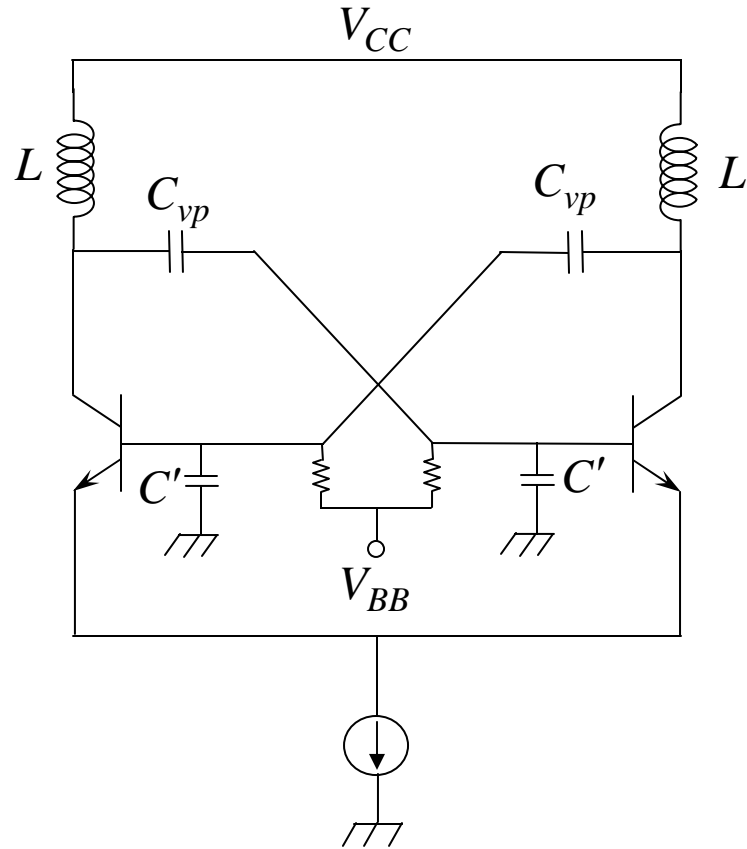
Since it is a fully differential circuit we may model the cross-coupled pair as follows:
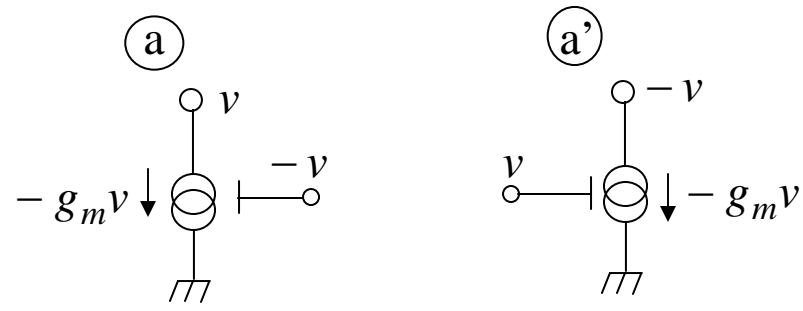
The resistance between a and a' can be expressed as:

$$R = \frac{v - (-v)}{-g_m v} = -\frac{2}{g_m}$$

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1 – BJT Realization:

The capacitor divider is used to reduce the voltage swing at the base compared to that at the collector to avoid saturating the BJTs.

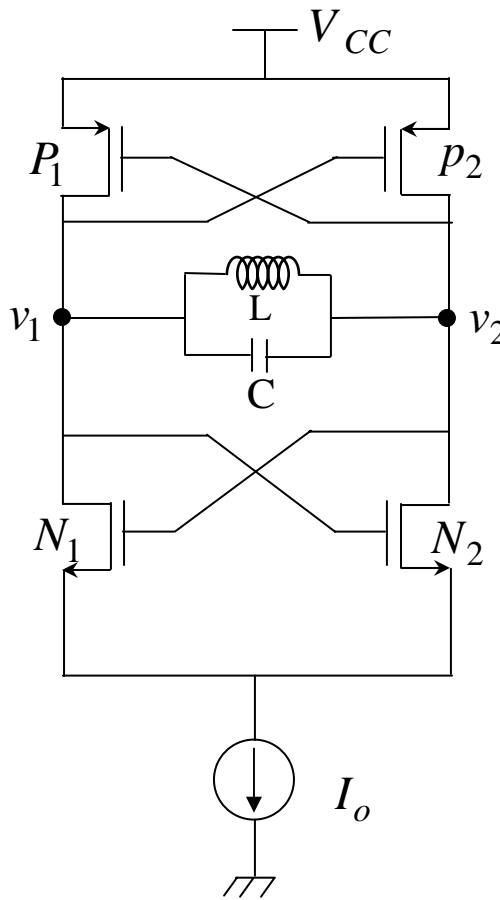$$v_{base} = v_{collector} \times \frac{C_{vp}}{C + C'} \qquad C' > C_{vp}$$

The AC coupling resistors add noise.

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2 – NMOS (or PMOS) Realization

It is similar to the BJT except that it does not need capacitive division.

3 – CMOS Realization:

Current Limited Oscillation

In this case the current of the tail current sources is fully switched from one side of the pair to the other at a frequency $\omega_o = \dfrac{1}{\sqrt{LC}}$ Assuming that the BJT or MOS switches are fast enough, the current waveform in each branch is a square wave with 50% duty cycle. Assuming that the tail current is I_o, the current wave form in one side of the pair has the following shape.
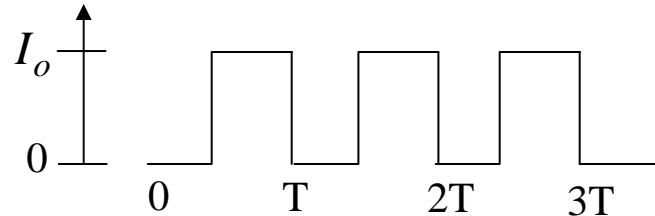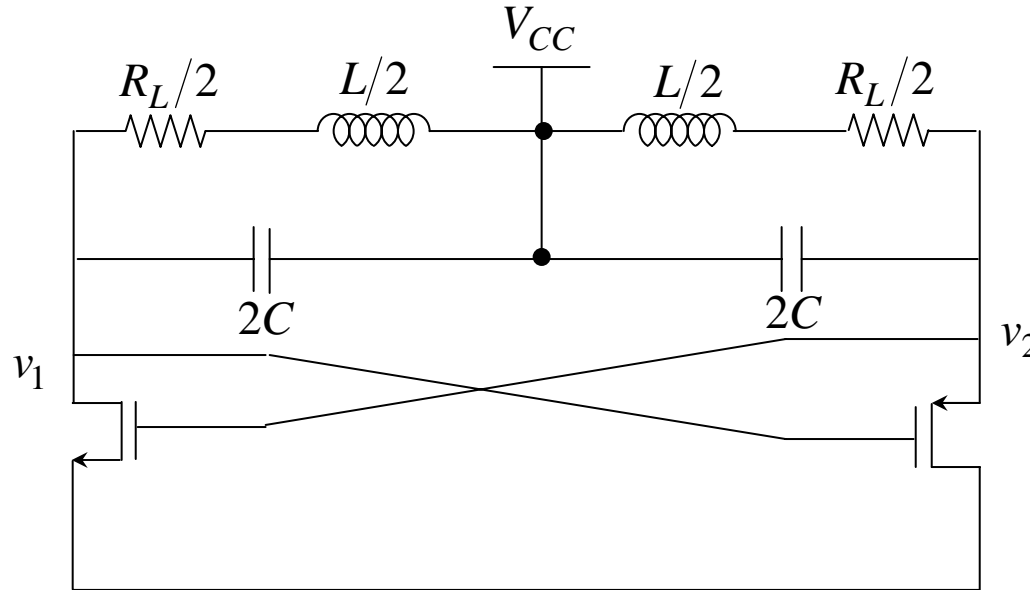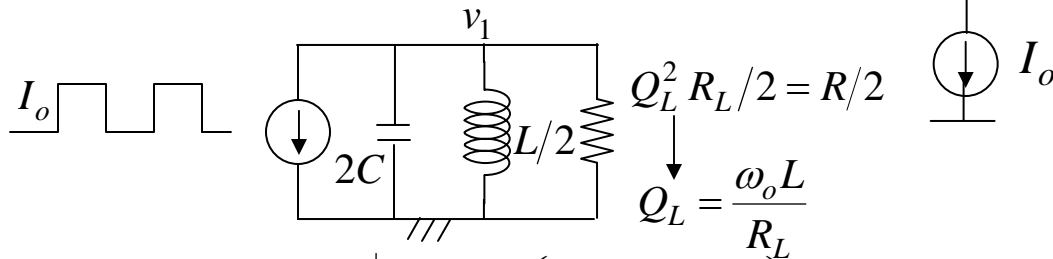

This can be described by its power series as

$$i_{(f)} = \frac{1}{2} I_o \left(1 + \frac{4}{\pi} \sin \omega_o t + \frac{4}{3\pi} \sin 3\omega_o t + \cdots \right)$$

NOTE: If the current wave form deviate from a square wave which is typically the case the $\dfrac{4}{\pi}$ factor will change to some other number depending on the on the current waveform shape!!

Consider the NMOS oscillator:

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$Q_L^2 R_L/2 = R/2$$

$$Q_L = \frac{\omega_o L}{R_L}$$

$$v_1(t)\Big|_{\omega=\omega_o} = \left(\frac{2}{\pi} I_o \sin \omega_o t\right) \times Z\Big|_{\omega=\omega_o} = \frac{2}{\pi} I_o \frac{R}{2} \sin \omega_o t = \frac{I_o R}{\pi} \sin \omega_o t$$

$$v_{12p} = v_{1p} - v_{2p} = \frac{2}{\pi} I_o \cdot R \qquad\qquad R = Q_L^2 \frac{R_L}{2} // Q_C^2 \frac{R_C}{2}$$

NOTE: The current components at $3\omega_o$, $5\omega_o$, $7\omega_o$ … will produce voltage components with small power because the impedance at these frequencies is small.
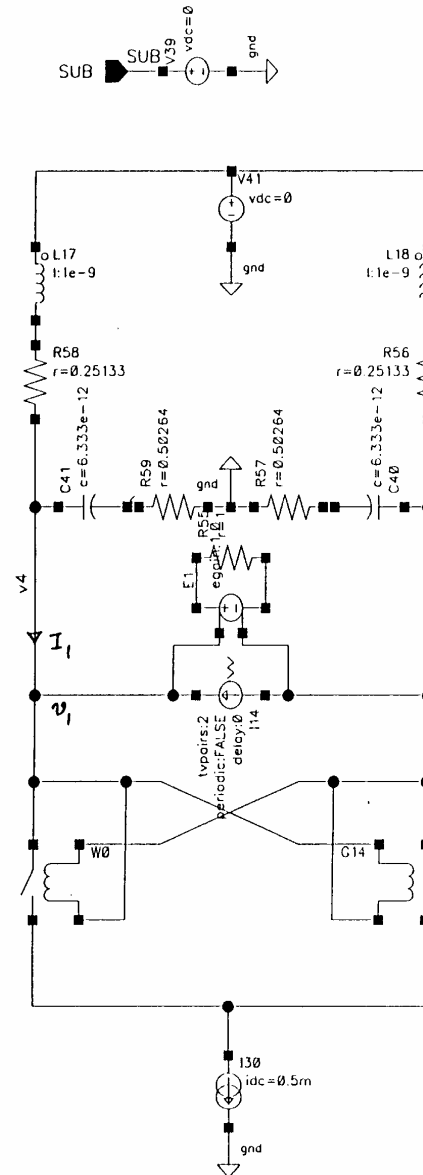
$$\omega_o = \frac{1}{\sqrt{LC}} = 12.56 \, GHz$$

$$Q_L = 50 \qquad\qquad Q_L^2 = 628.3$$

$$Q_C = 25 \qquad\qquad Q_C^L = 314.2$$

$$\frac{R}{2} = 209C$$

Switching of square wave current (pulses)

△: IT("/WØ/N+")

600u

400u

200u

0.00

(A)

I_1

■: VT("/v4")

80m

30m

−20m

−70m

(V)

ν_1

$-66.45mV = \frac{2}{\pi} I_1 \times R = \frac{2}{\pi} \times 0.51\overset{-3}{10} \times 209 = 66.5mV$

2.40n   2.50n   2.60n   2.70n   2.80n   2.90n   3.00n   3.10n   3.20n

time (s)
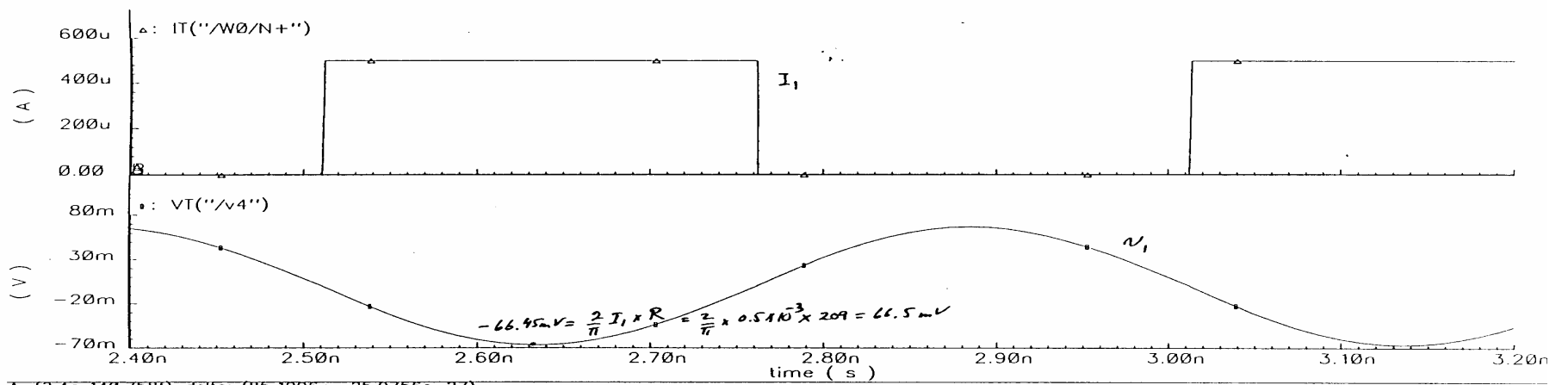
A: (2.4n 140.758f) delta: (85.1996y −25.9756e−27)
B: (2.4n 140.758f) slope: −304.879u

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Now consider the CMOS oscillator with a similar tank as that used for the NMOS Oscillator (pp 24).

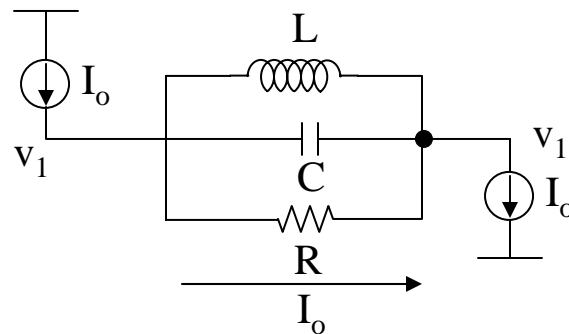In one half cycle, the devices P1 and N2 are on while N1 and P2 are off, so the oscillator can be modeled as follows:

On the second half cycle, the current through R is still I_o but flows in the opposite direction. So the current waveform (assuming that the PMOS and NMOS devices switch enough) looks as follows:
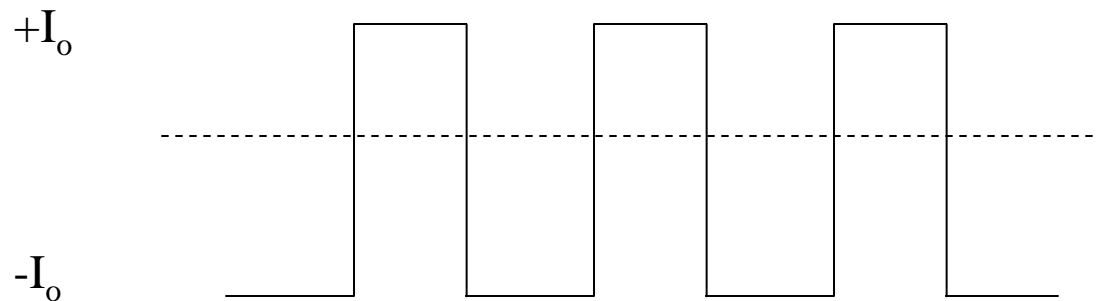
This is different from the wave form previously shown, because it is bipolar. The Fourier expansion for the CMOS oscillator is:

$$i(t) = I_o\left(-\frac{4}{\pi}\sin\omega_o t + \frac{4}{3\pi}\sin 3\omega_o t + \frac{4}{5\pi}\sin 5\omega_o t\cdots\right)$$

The differential voltage of the CMOS oscillator is hence, given by:

$$v_{12}(t)\Big|_{\omega=\omega_O} = \frac{4I_o}{\pi}\cdot R\sin\omega_o t$$

$$v_{12p} = \frac{4}{\pi}I_o R$$

- Note that swing of the CMOS oscillator is double that of the NMOS (or PMOS) oscillator as seen before.
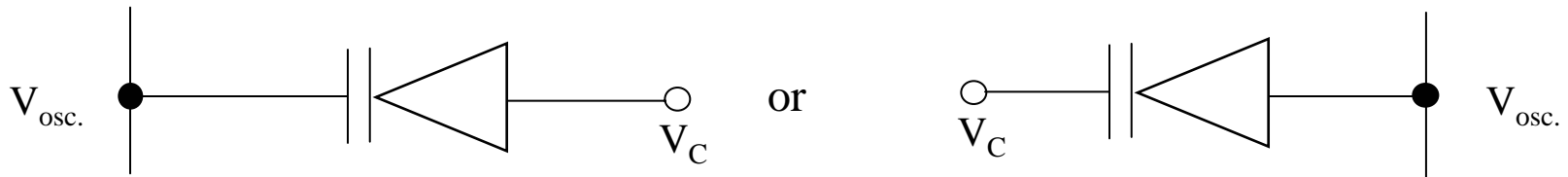
Some Important Points:

1. In the oscillators cases we considered so far we have assumed that the swing is small enough so that the swing is determined by the current and the tank parallel resistance. If this is the case, we consider the oscillator to be current limited. If The swing grows too much, the swing will be limited by the supply voltage and in the oscillator is known to be voltage limited.

2. In current limited cases, the swing increases by increasing the current (tail current) or by using high Q tank (which increases the resistance.).

3. The switching speed of the core devices will determine the current waveform shape which in turn determines magnitude of I at ω_o (fundamental component).
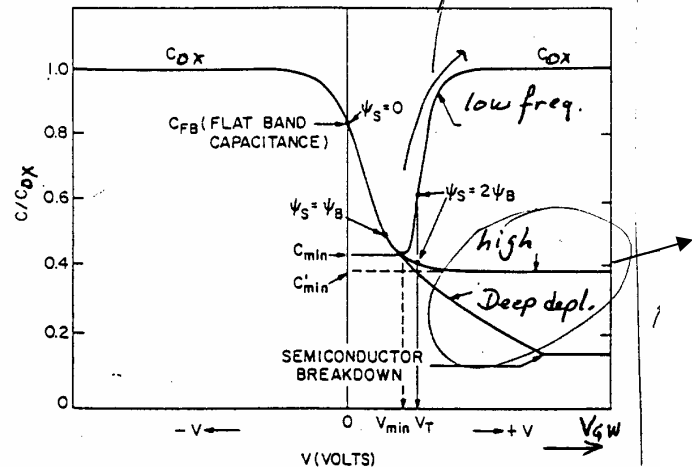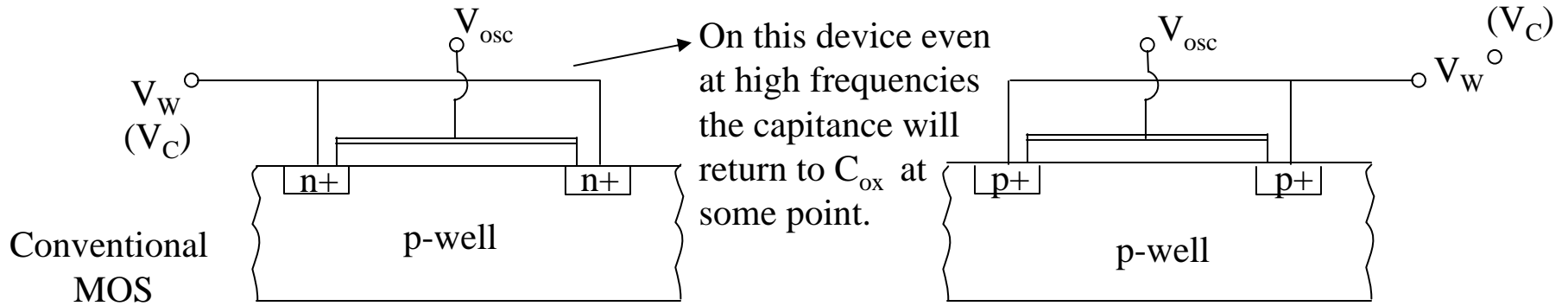
Voltage Controlled Oscillators

This is one of the most critical blocks in any RF frequency synthesizers. Adjustable oscillators are used for tuning to desired frequencies. A VCO is used as part of a PLL to obtain precise frequencies.

The frequency tuning is done by changing the LC tank capacitance value. This is more practical than varying the inductance. Different types of variations have been used to obtain a voltage controlled capacitance. Among them are pm junction varactors and MOS varactors and MOS varactors.
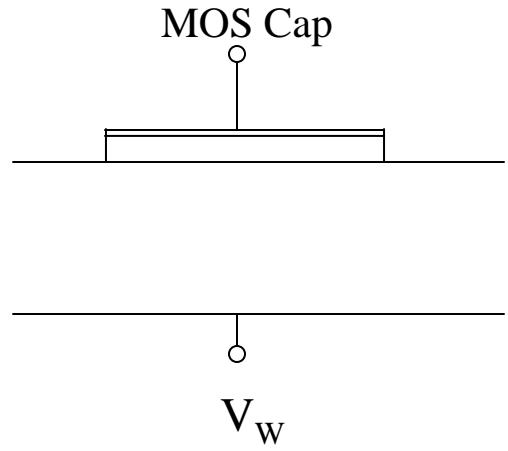
In a pn junction varactor the oscillator output node is connected to either the p or n side and the controlling voltage is applied to the opposite terminal

MOS varactor come in two types. Let us consider NMOS varactors.

V_{osc}

V_W
(V_C)

n+   n+

p-well

Conventional
MOS

On this device even
at high frequencies
the capitance will
return to C_{ox} at
some point.

V_{osc}

V_W

(V_C)

p+   p+

p-well

MOS Cap

V_W

For high
frequency

C_{ox}

1.0

0.8

C_{FB} (FLAT BAND
CAPACITANCE)

$\psi_s = 0$

low freq.

C_{OX}

0.6

$\psi_s = \psi_B$

$\psi_s = 2\psi_B$

C/C_{OX}

0.4

C_{min}

high

C'_{min}

Deep depl.

0.2

SEMICONDUCTOR
BREAKDOWN

0

$-V$

0   V_{min} V_T

$+V$

$V_G W$

V (VOLTS)

For MOS Cap:

$V_{GW} < 0 \implies$ Accumulation mode

$C = C_{OX}$

M   O   S

C_{ox}

Q_s

$\Delta V \to \Delta Q_m$ & ΔQ_s
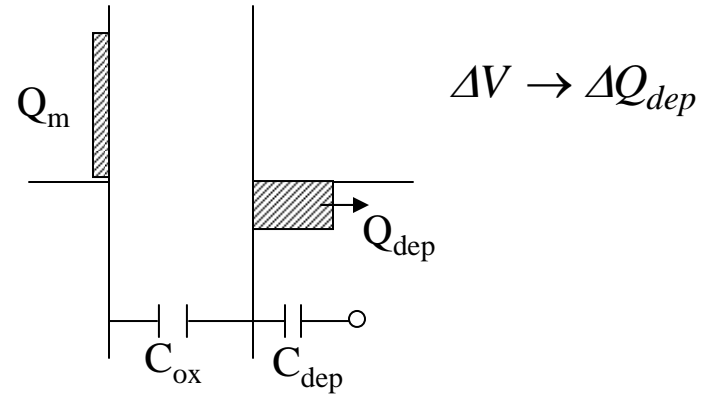
Q_m
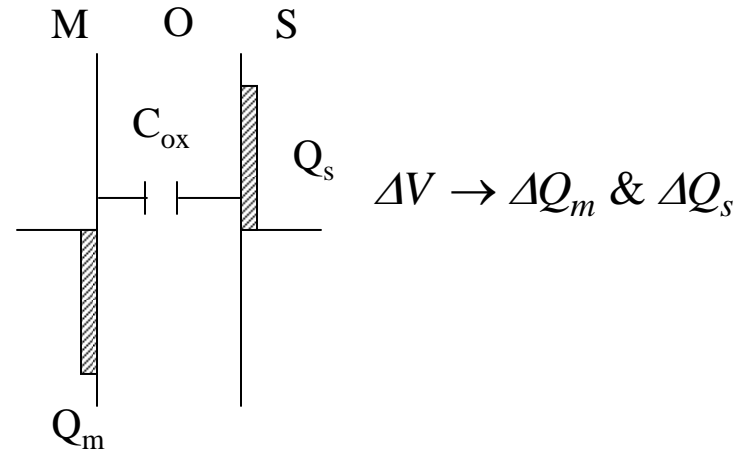
$V_T > V_{GW} > 0 \implies$ Depletion mode

$C = \dfrac{C_{ox}C_{dep}}{C_{ox} + C_{dep}}$

$C_{dep} = \dfrac{dQ_{dep}}{dV}$

Q_m

$\Delta V \to \Delta Q_{dep}$

Q_{dep}

C_{ox} C_{dep}

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$$V_{GW} > V_T$$

At low frequency:

Since the frequency is low, the minority charges have time to be pulled from the bulk to the surface to make up the inv. layer and the depletion layer does not change any more so

$$C = \frac{dQ_{inv}}{dV} = C_{ox}$$

At high frequency:

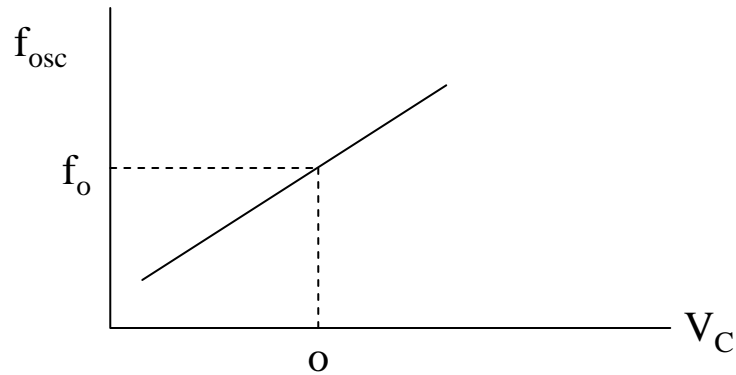There is no time to attract minority changes to the surface. So the change comes form the dep layer

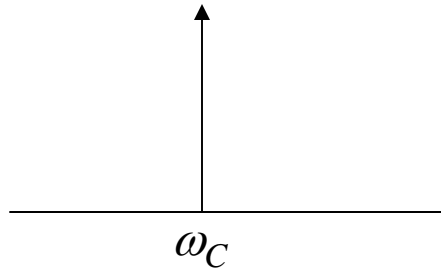$$C = \frac{C_{ox}C_{dep}}{C_{ox} + C_{dep}}$$

Q_m

$\Delta V \rightarrow \Delta Q_{inv} \ \Delta Q_{dep} = 0$

Q_{dep}

Q_{inv}
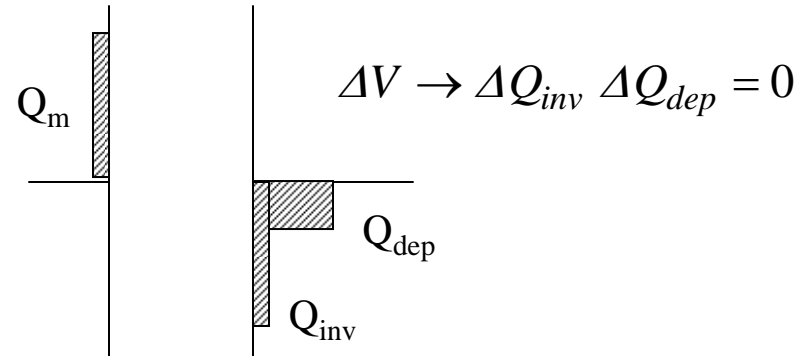
An ideal VCO has the following law

$$f_{osc} = f_o + K_{VCO}V_C$$

Control
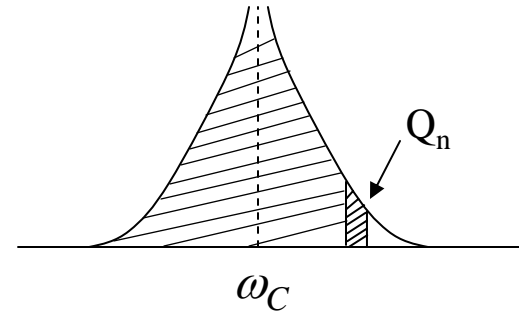
Free running
frequency

VCO gain
(Hz/V)

The spectrum of $\phi_n(t)$ is up-converted to $\pm\,\omega_C$

ω_C

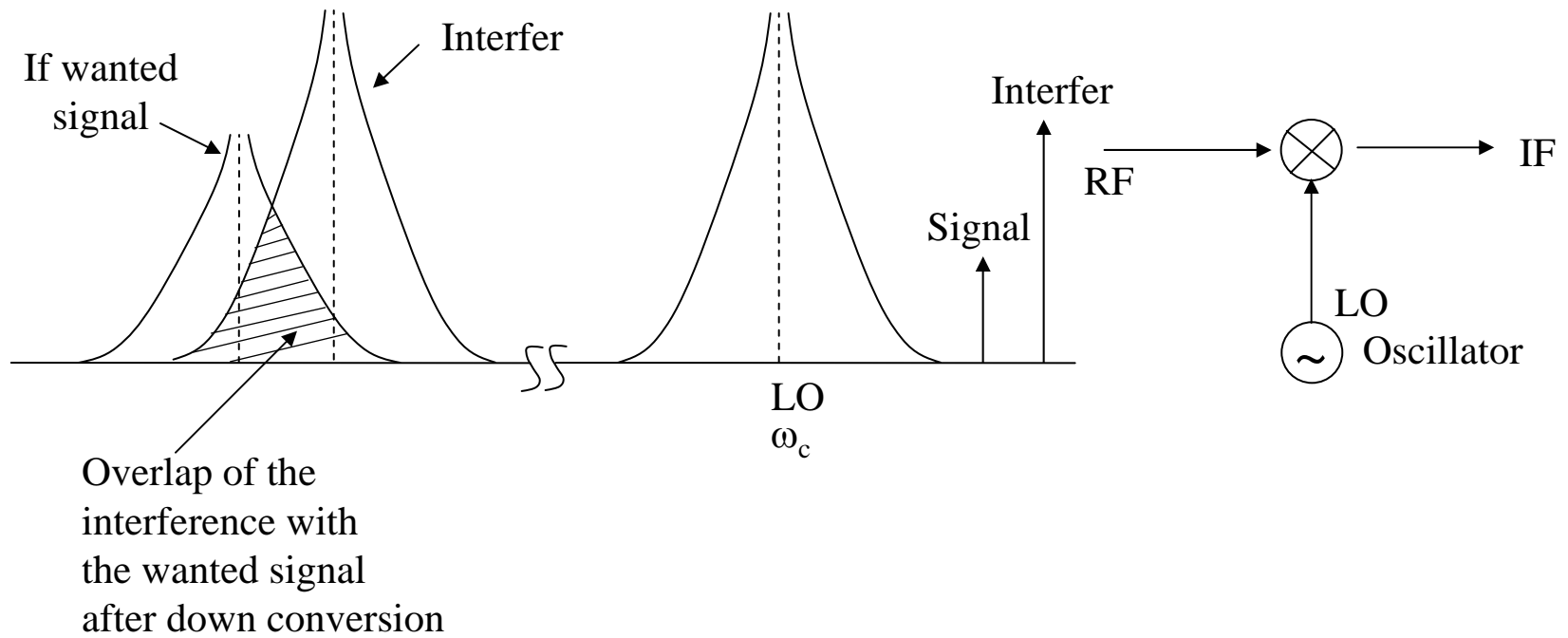Ideal Oscillator

Q_n

ω_C

Real Oscillator

The phase noise is measured by dividing the power in 1Hz bandwidth at an offset Δf by the power of the carrier (f_c).

$$L(\Delta f) = \text{Phase noise} = 40\log\left(\frac{\text{power in 1Hz at } \Delta f \text{ offset}}{\text{power of the carrier}}\right) \quad dB_c/Hz$$

Implications of Phase Noise

In the case of receiver:

If wanted signal

Interfer

Interfer

Signal

RF

IF

LO
ω_c

LO
Oscillator

Overlap of the interference with the wanted signal after down conversion
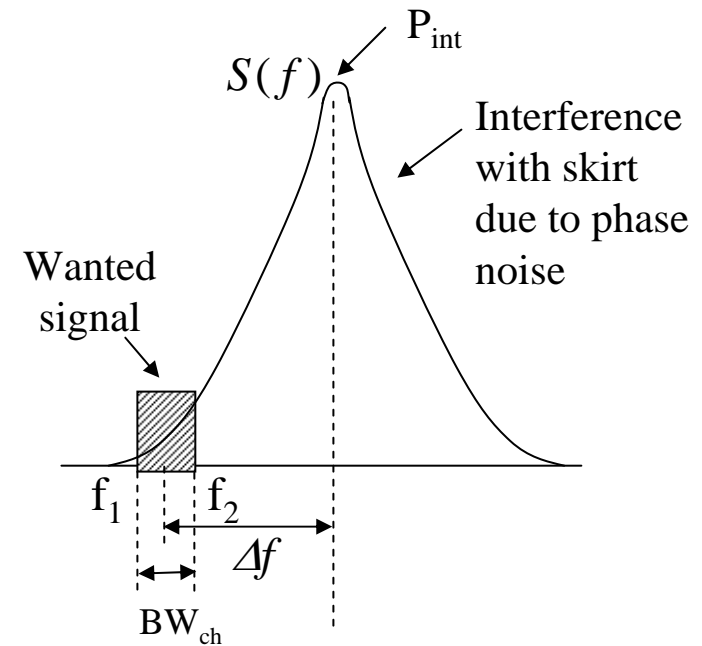
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In the case of transmitter:

If the phase noise of the LO at the transmitters side is large, the skirt of the RF transmitted signal overlaps nearby wanted signals and hence reduces the SNR.
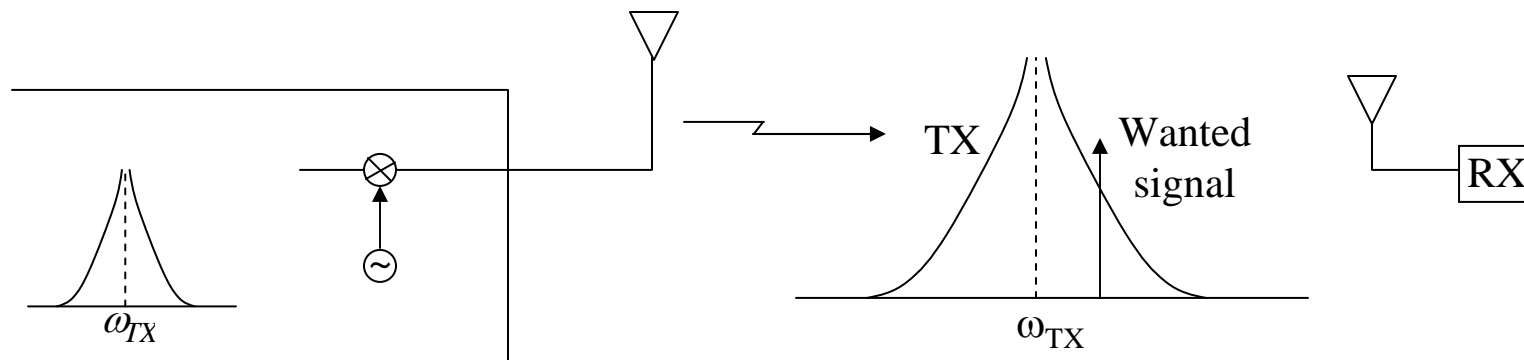
Assume that, from the blocking specs, that

$P_{int} - P_{sig} = YdB$

P_{sig} = the signal power integrated over

     channel BW.

Assume also that a given signal to noise ratio (SNR) is to be met.

    The noise power $= P_n = \int_{f_1}^{f_2} S_n(f)df$

    for simplicity assume the $S_n(f)$ is constant (P_{no})

    between f_1 & f_2.

    $\therefore P_n = P_{no}(f_2 - f_1) = P_{no} \times BW_{ch}$
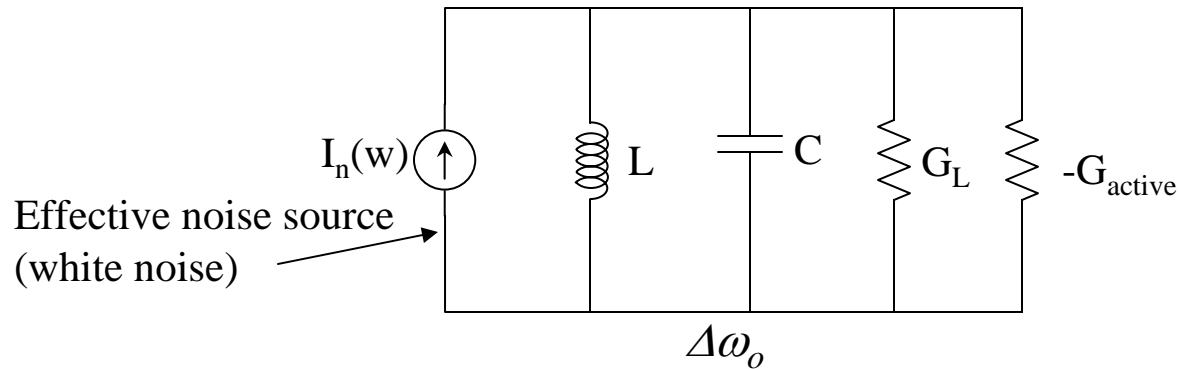
    $\phi_n = \dfrac{P_{no}}{P_{int}} = \dfrac{P_n/BW_{ch}}{P_{int}} = \dfrac{P_n}{BW_{ch}} \times \dfrac{1}{P_{int}} = \dfrac{1}{BW_{ch}} \dfrac{P_n}{P_{sig}} \times \dfrac{P_{sig}}{P_{int}}$

    $\phi_n = (-10 \log BW_{ch} - SNR - Y)dB_c/Hz$

    $\phi_n = -(10 \log BW_{ch} + SNR + Y)dB_c/Hz$

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LEESON'S MODEL

Based on a linear time in variant approach for timed LC tank oscillators, one can derive an expression for the phase noise.

It can be shown that impedance seen by the effective noise source at $\omega_o + \Delta\omega_o$
(where $\Delta\omega \ll \omega_o$ is given by:

$$Z(\omega_o + \Delta\omega_o) = -j\frac{1}{G_L}\frac{\omega_o}{2Q_L\Delta\omega}$$

G_L is the parallel parasitic conductance of the tank

Q_L is the loaded Q of the tank

The phase noise (for the case of white noise) can, hence be given by:

$$L(BW) = 10 \log \frac{\overline{v_{noise}^2}}{v_{sig}^2}$$

$$= 10 \log \frac{\frac{1}{2}\left| Z(\omega_o + BW)^2 \cdot \overline{i_n^2} \right|}{\frac{1}{2}\hat{V}_{sig}^2}$$

$$= 10 \log \frac{\frac{1}{2}\overline{i_n^2}}{\frac{1}{2}\hat{V}_{sig}} \frac{1}{G_L^2}\left(\frac{\omega_o}{2Q_L \Delta\omega} \right)$$

$\overline{i_n^2}$ can be written as $4\,F\,KT\,G_L$ this expression for the effective noise curent density is not physical. (Just a model!) F is difficult to derive and is just a fitting parameter!

$$L(\Delta\omega) = 10 \log\left(\frac{2FKT}{P_{sig}}\left(\frac{\omega_o}{2Q_L \Delta\omega} \right)^2 \right)$$

$$P_s = \frac{1}{2} = i_o \hat{V}_{sig} G_L$$

The rms power dissipated in G_L.

Even though the LTI model is not accurate, it yields important facts for oscillator design:

1. $L(\Delta\omega) \quad \alpha \quad \dfrac{1}{Q_L^2} \quad \Rightarrow \quad$ increase Q_L

2. $L(\Delta\omega) \quad \alpha \quad \dfrac{1}{P_s} \quad \Rightarrow \quad$ increase P_s by increasing Q_L and I

3. $L(\Delta\omega) \quad \alpha \quad F \quad \Rightarrow \quad$ reduce noise inside the oscillator.

Leeson's model is not accurate and fails top account for the large signal Behavior of the oscillator. Yet it gives insight for oscillator design.