Lab: Drag – Part I

Drag Model

Although we often ignore it, air exerts a force on all objects as they move through it. Air resistance, *a.k.a. drag*, has two basic properties:

1. It is opposite in direction to the object's velocity
2. It increases in magnitude as the object's speed increases

For objects moving through air near Earth's surface, we can use a simple model to determine the magnitude of the drag:

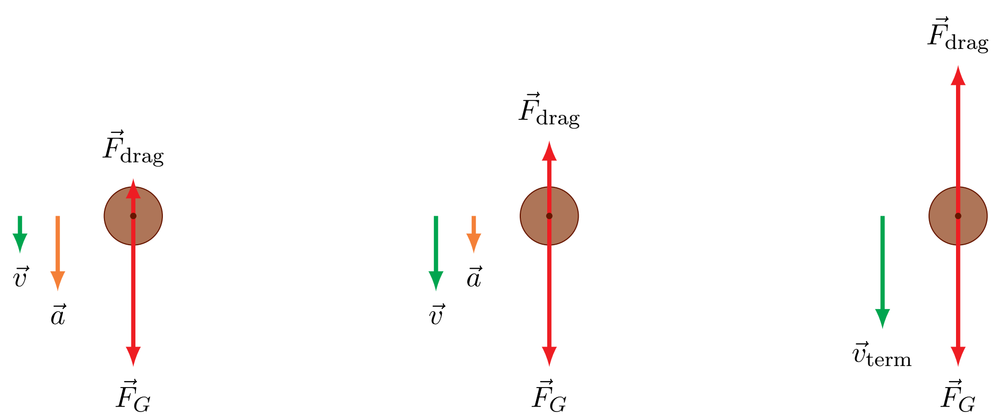
$$F_{\text{drag}} = \frac{1}{2} C \rho A v^2 \tag{1}$$

where v is the speed of the object, A is the cross-section area of the object, ρ is the density of air, and C is the drag coefficient. The drag coefficient is a dimensionless constant that represents the complex details of how the object interacts with the air as it moves. For most objects the drag coefficient is determined experimentally.

If we account for drag, an object does *not* experience a constant acceleration as it falls. As the speed of a falling object increases, the drag force acting on it increases as well. Eventually the drag force has a magnitude equal to the force of gravity on the object. At that point the object is in dynamic equilibrium and will continue to move with a constant speed. We call this constant speed the terminal speed:

$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}} \tag{2}$$

where m is the mass of the object and g is the acceleration due to gravity.



At low speeds drag is small, therefore $a \approx g$.

As speed increases so does drag, therefore $a < g$.

At the terminal speed drag is equal to the force of gravity, therefore $a = 0$.

Drag coefficient

We will use the trials with one coffee filter to determine the drag coefficient. You will need to:

- Determine the average v_{term} using all 10 trials.
- Use software (*ImageJ*) and your photograph to determine the cross-section area of one filter.
- Determine the mass of one filter using your measurement from the scale.
- Look up the density of air at room temperature (20°C).
- Rearrange Equation (2) and find the drag coefficient C.

Simulation using Euler's method

For an object falling downwards and experiencing drag the net force is

$$F_{\text{net,y}} = -mg + \frac{1}{2}C\rho A v^2$$

and therefore its acceleration is

$$a_y = \frac{F_{\text{net,y}}}{m} = -g + \frac{C\rho A v^2}{2m}$$

To determine the velocity as a function of time or the position as a function of time we must integrate this equation. Although it is possible to do the integral analytically, it is not an easy task! We would rather integrate the acceleration *numerically* using a process called *Euler's method*.

Euler's method tells us that if we take a very small interval of time ($\Delta t \ll 1$) we can simply calculate what the next values of acceleration, velocity, and position are using the equations of motion. We then iterate this process over the entire motion. The algorithm is something like this:

1. Set the initial values for y and v_y
2. Calculate the value of a_y using the current value of v_y
3. Calculate the next values of y and v_y using a small time interval Δt
4. Iterate steps 2 and 3 until the motion is finished

We will code Euler's method to simulate one of the trials where **more than one coffee filter** was dropped and compare it to the real data (you can choose which one). You will need to:

- Define all the constant values (m, g, ρ, A, C)
- Define the initial values for y and v_y and define the time step Δt
- Define an array to keep track of the values of y
- Write a loop that implements Euler's method and records the value of y for each step until the filter reaches the ground
- **Make a graph of position as a function of time from your simulation and overlay the real data for that trial**